NPTEL

NPTEL ONLINE COURSE

# Transform Techniques for Engineers

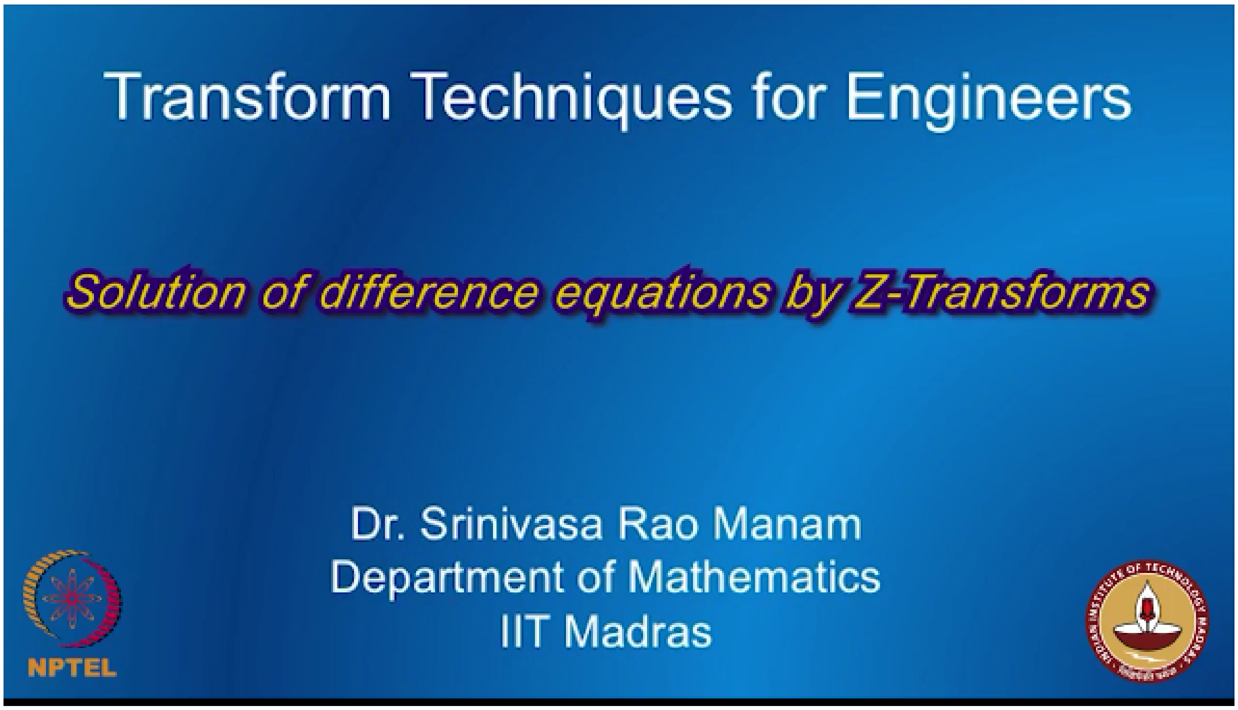
## Solution of difference equations by Z-Transforms

With

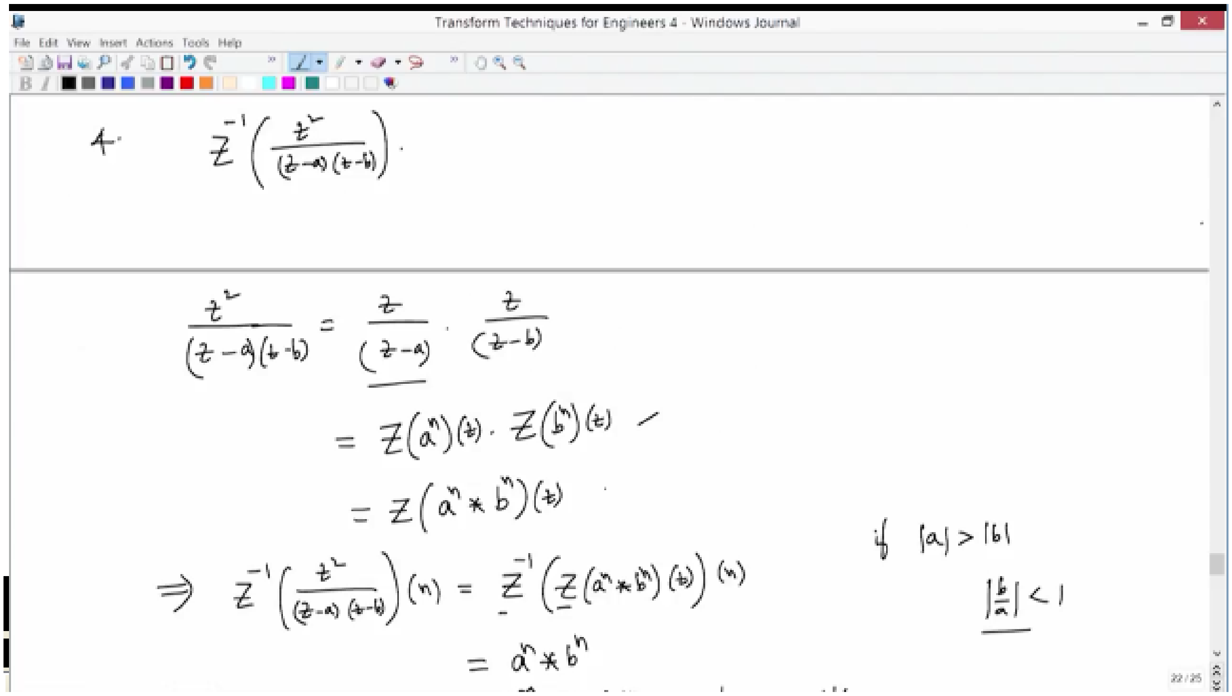
Dr. Srinivasa Rao Manam

Department of Mathematics

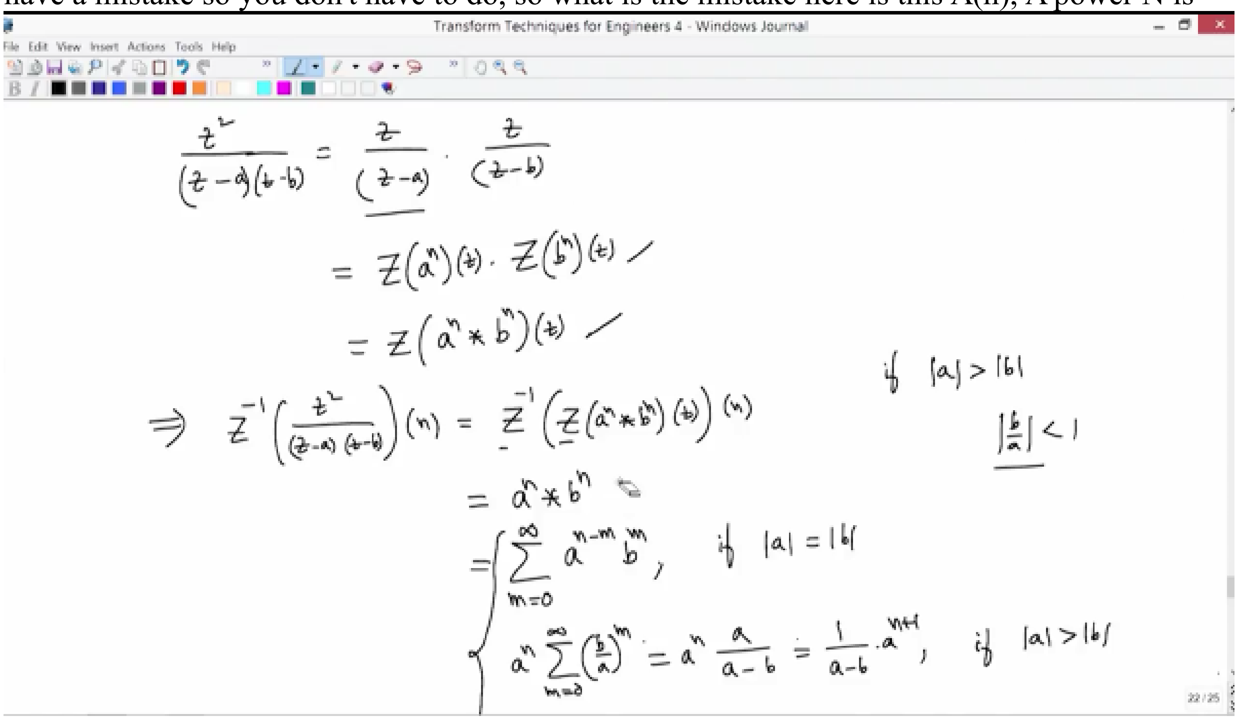
IIT Madras



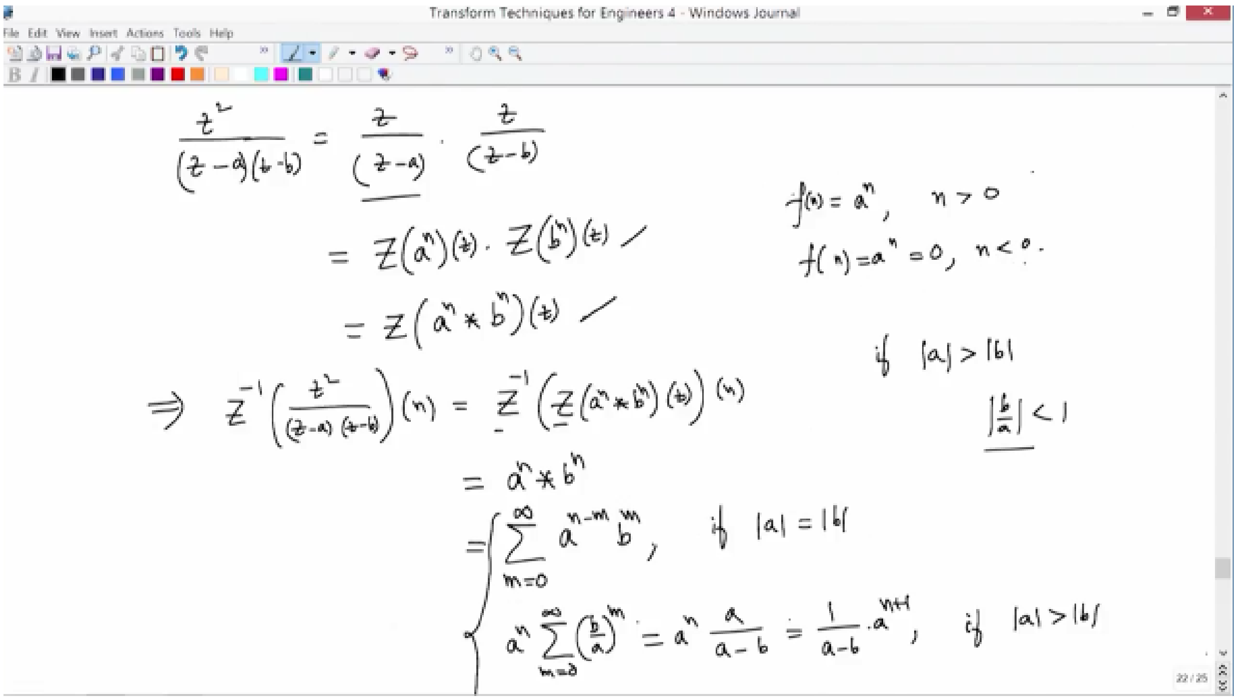

Welcome back, in the last video we were discussing about in finding inverse transforms of certain complex valued functions, so let's continue to do that when this, in some of the examples we made use of this convolution property, so that's where we made a slight error. Let's look into the problem where we have done these inverse transforms, so to start with Z inverse of Z square/Z-A into Z-B, so that’s a fourth example that we have done, if you look at this, so finding this we are trying to find this Z inverse Z transform and you write like this and then you use the convolution property, and finally up to here is fine, okay, so this is fine, and this is fine.




So what is this one? When you write this the convolution of this is this, and this is where you have a mistake so you don't have to do, so what is the mistake here is this A(n), A power N is

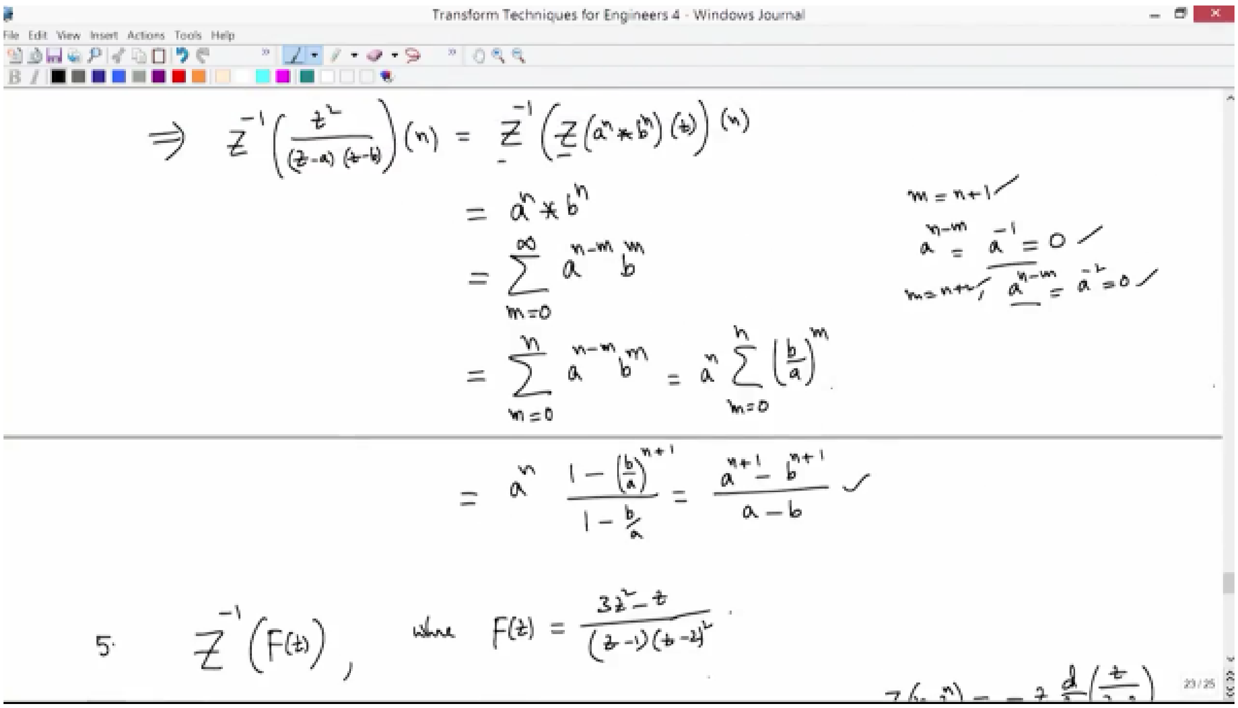



only defined for N positive, okay, so A power N is actually 0, so if you call this F(n) which is equal to A power N, if N is positive, N is positive this is the case, F(-n) that is A power -N is actually 0 you have to consider this is 0 for N positive so that means to say A power N = 0 for

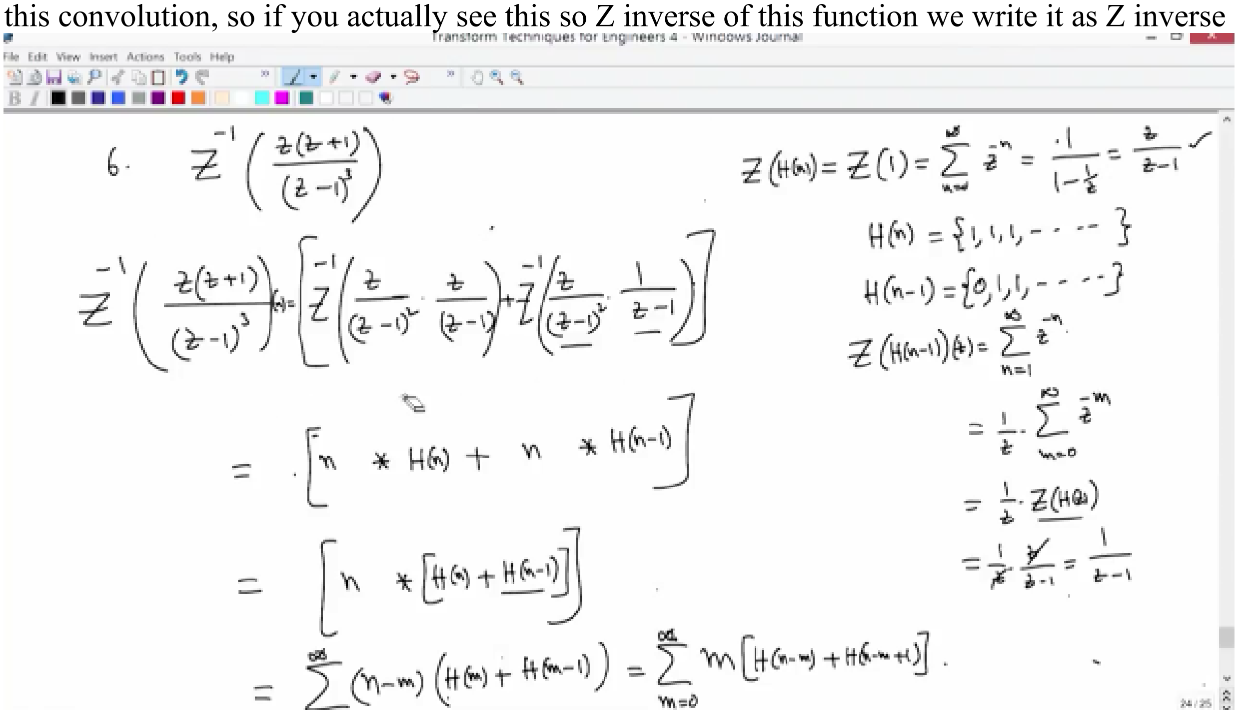



N negative, so this we have not made use of this here, okay, if you use, so let's say you need not worry about what is bigger of, A is bigger or small, modulus of A is bigger or modulus A is smaller, so let’s look at this convolution, so convolution as such you have this, let's not look at this which one is bigger, which one is smaller, so let's just look at directly what is the convolution, so this is your convolution and if you see M is running from 0 to infinity A power N-M and B(m), so what is, when M = N+1 what happens, if M is N+1 A power N-M is actually A power -1 which is actually 0 because A negative powers are 0, so M = N+2 also you see that A power N-M is actually A power -2 and so on, so which is actually 0, so this is same as because of that you can write this as M is from 0 to N, up to N there is no issue, so only from N+1 and N+2 and these all becoming 0 because A power N-M is 0 so you have 0 times B power M, so these are all 0, so only N sums, only n terms you have M is running from 0 to N, N+1 terms basically, okay, so this sum this finite sum is the result of this Z inverse, okay.

So this if you calculate you can write this as A power N is common, and B power N is also common this is M, okay, so you have a sum which is running from B/A this is A power N times M is running from 0 to N, B/A over M, so this is what it is, so this sum if you write it M = 0, this is 1, so 1-A times the first term times 1- this is your finite sum, geometric sum, so what you have is B/A power you have 0 to M, how many you have? How many terms? So you have N, so you have N+1 divided by 1-B/A so this is the result, so what you have here is A power N, A power N goes, and A A cancels so you get A power N finally the result is A power N+1/A-B okay, -B power N+1, because this A power N+1 so you have A power N+1, A power N cancel and we have only A, A you can cancel here, so you get this is your results, so this is the result of this Z transforms, Z inverse of this, okay, Z inverse if you want you get this is what is your result.

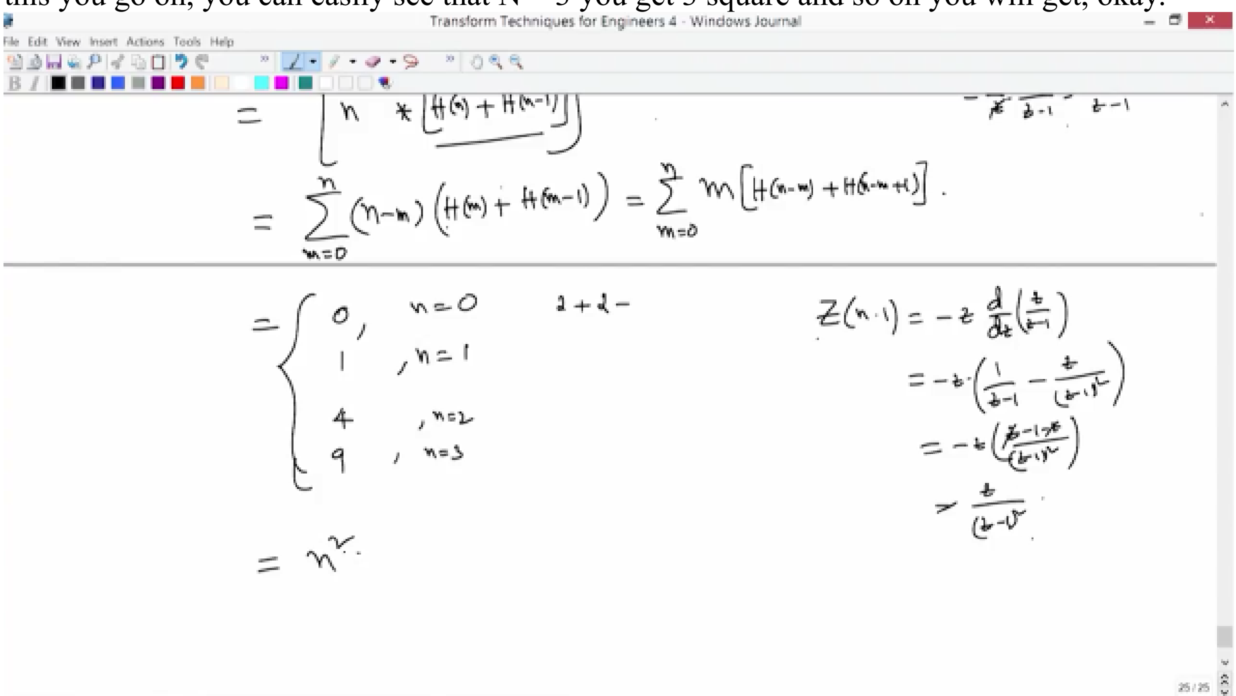



So the same thing we can do here, similar thing and we have done, we have evaluated this inverse Z-transform of this function in the last video and there we have not used the convolution so this problem of 5 there is no issue, and problem number 6 that is where we use this convolution, so if you actually see this so Z inverse of this function we write it as Z inverse




of this + this and you see that each of these is a convolution of this, and you can write it this as a convolution of this together N convolving with this if you write, convolution of these two functions, so you have seen that this convolution if again, so here A, this is you're a, this is your F(n) F(n-m) and this is your G(n), so you have a G(m), you have to write G(m) okay, and you have to take this summation M is from 0 to infinity, so if M equal to up to N, and F is having positive arguments, if M = M+1 onwards, F(n-m) is actually F of negative, negative arguments because of that these are all 0, so you have only negative side is 0, okay, inverse any signal will have 0, you'll only have, you're only looking for positive side so if you assume that these are all 0, and what you have is this sum will be again M is running from this, so this convolution sum will be M is running from M to N, so M to N you have this so you write M to N, sigma M to N this is what you have, either of this is true, now you can easily see that this is equal to N square, so actually look at this, how do I do this? So you pick up, you fix any N you pick up, so let's say some N, so you have if you pick up N, M = N this is 0, so this M = N, so M = N-1 this is 1, and M = N-1 and you have this value is 2, right.

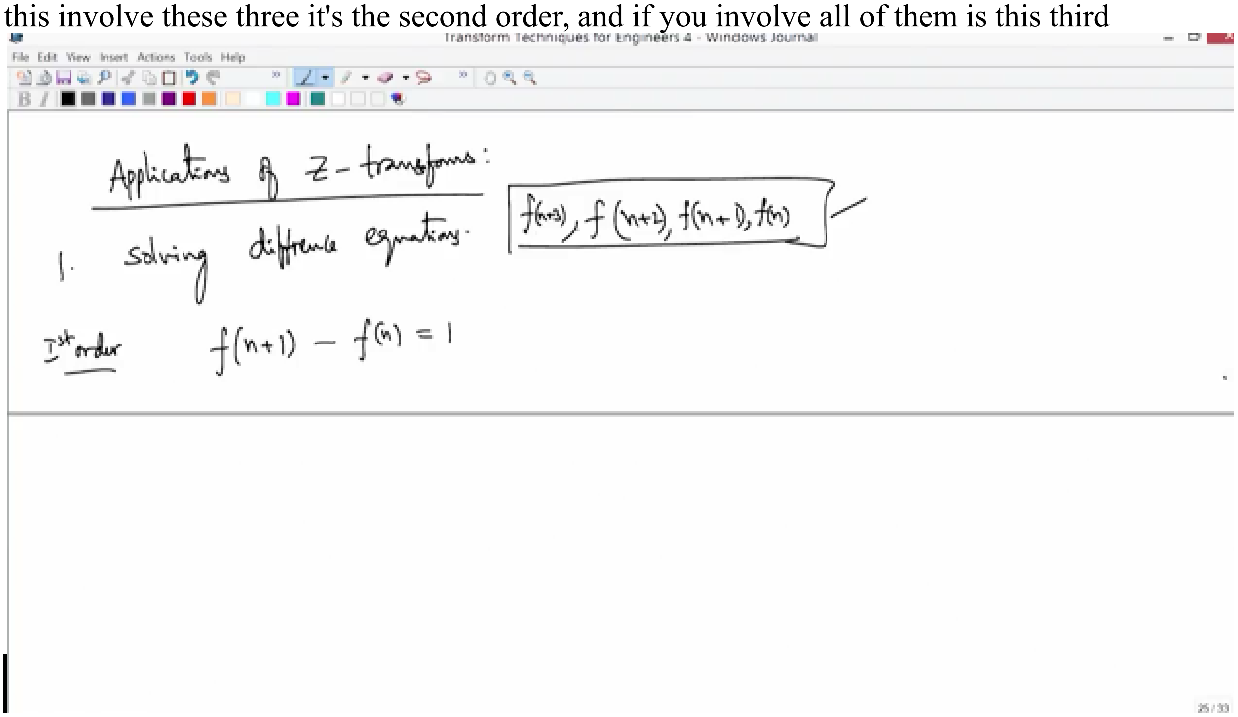
So how do I see this? This is equal to N square, let's look at term by term, so let's fix N = 1, okay, first of all 0, if you put 0, then M is 0, 0 into 0 so finally you get 0, so answer is 0, N = 0 this value is 0, okay, let me write like this, so this is like, this is 0 if N = 0, N = 1 if you look at it, N = 1 if you put M is running from 0 to 1, M = 0, 1-0 that is 1, and H(0) is 1 okay + H(negative) that is 0, so you have simply 1, 1 + M = 1, 1 - 1 that is 0, so anyway 0, so it's together its 1, N = 1 it is, N = 2 if you calculate you can easily see that 2 – M = 0, M = 0, 2-0 so 2 times M = 0 H(0) + negative so that is 1, + M = 1 or 2-1 that is 1 times and this will be your H(1) and H(0) that is 2, so you have 2+ M = 2 finally, so 2-2 that is 0 so you have 4 now, so like this you go on, you can easily see that N = 3 you get 3 square and so on you will get, okay.




So by induction one can show that or some other means you can easily see that is actually N square, okay, so the minor mistake that we had is we have not considered, when you write the convolution you have to consider only a negative, you have to consider only positive, only positive, N positive integers only this is true, negative wherever you look at the quantity that is negative it has to be taken as 0, okay, so you view this as F(n) and G(n), F(n) G(n) has to be 0 in the negative side, okay, so this is how you can see this N square.

Now let's look at the applications of these Z transform, so we can solve this, or you can use these Z transforms to solve some difference equations, difference equations are basically discretized version of differential equations so they are like a recurrence relation kind of things so you can just look at some difference equations, so applications of Z transform, so I've basically give two applications, one is solving this Z-transforms, I mean one is solving difference equations other one is using these Z transforms you can evaluate certain sums, there we used Laplace transform to evaluate certain integrals involving a parameter, here we use the Z transforms to evaluate certain sums involving a parameter, so we will see that application before after finishing these difference equations.

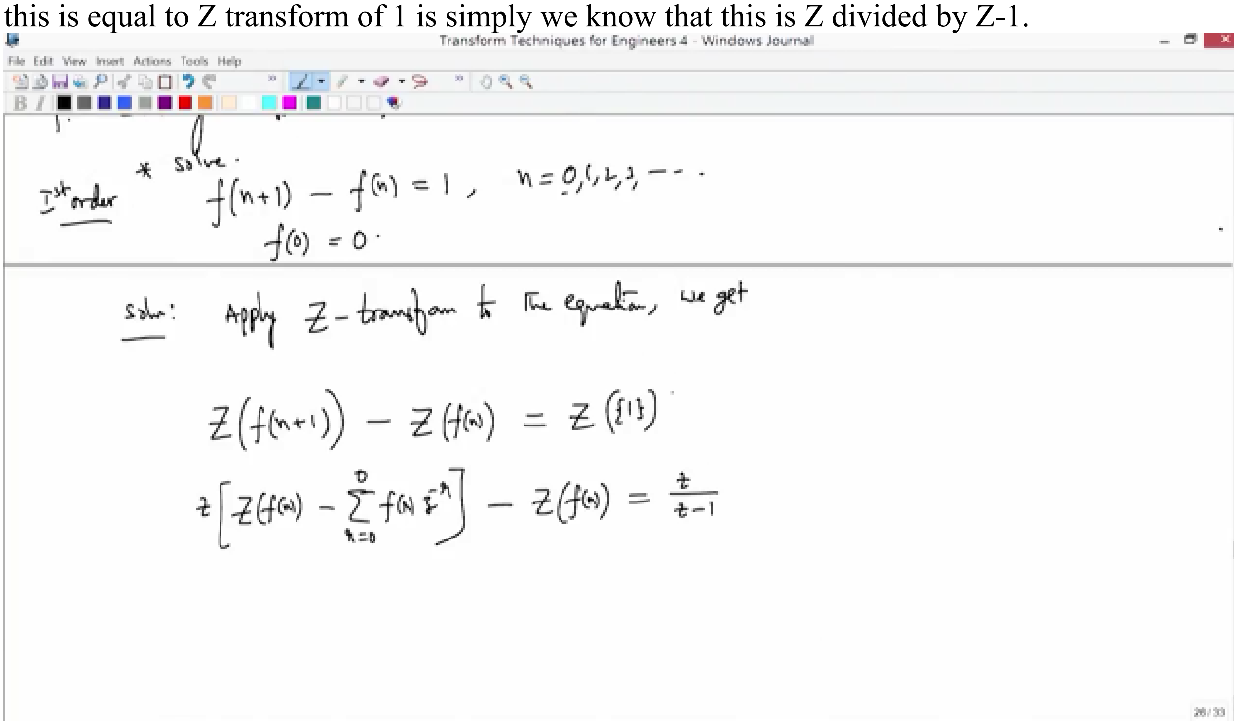
So let's see solving, so first application is that we see we will do only two applications one such is solving difference equations, so let's look at difference equation looks like, so let's look at the first order if you have any function so unknown function is F(n+1) – F(n)= 1, so if you look at F(n+1) N so this is N+1 if it is this this is a first order, okay, first order, if it is N+2 this is second order, okay, so this is actually F(n+1), N+1, N involving so functions involving N+1 and N arguments that's first order, if you have N+2, N+1 and N involving, so any combination of this F(n+2) F(n+1) and F(n), so function of this 3 is a second order difference equation, if you involve, if you involve N+3 that is third order difference equation, so any combination of these things is called third order difference equations, if you involve with these two is the first order, this involve these three it's the second order, and if you involve all of them is this third




order and so on, so we can see that this is the first order difference equation, so once you have this differential equation this is running from N is from 0, 1, 2, 3 and so on, okay, so that is what is given, you need to find what is (n)? Since I told this is a discretized version of difference equation, differential equation and when you solve this differential equation, corresponding differential equation in the differential equations when you solve the first order differential equation you get one arbitrary constant, to account for that arbitrary constant you give the initial condition, here also you provide here similar way initial condition, what is the initial condition here? Instead of N = 0 that is the initial condition F(0) you can provide, so you'll be given that, so let's take it as 0.

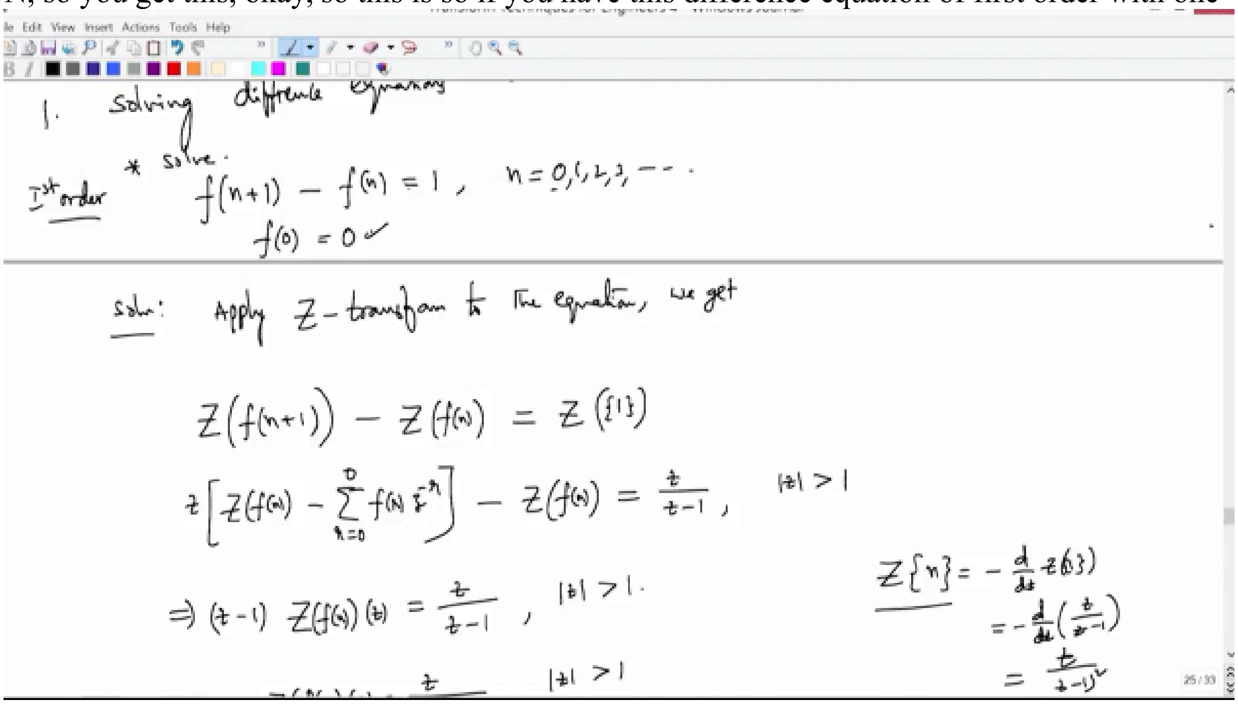
So how do I solve this? So let's solve this one, okay, first order this, this is to solve, this difference equation with this initial condition, solution, so how do I apply? Here I apply Z transform, apply Z transform to the equation, we get, what we get is F(n+1), what is this Z transform? You have seen already on the properties of Z transforms when you apply F(n+1) that is F(n+m) when you apply, Z transform of F of, because of linear property first of all F(n+1) this is Z transform of this -Z transform of F(n) and is equal to Z transform of this function, this is you consider this as a sequence for each N you have this is one constant, so it's a kind of constant sequence, okay, so what you have is this is a sequence, when you see this constant the basically sequence, so that is Z transform of the sequence 1, okay, so constant sequence so this is what you have.

Now you can apply the property of the Z transform, Z transform of F(n+m) that is Z power M times, Z transform of F(n) - sigma R is running from 0 to M-1 F(r) into Z power –R, so you can use that property and write here because this is the 1, N+M = 1 you can write this as Z power on, Z power 1 times Z transform of F(n) as a function of Z, obviously and minus you go, you remove this R is from 0 to M, M is 1 here, and you have F(r) times Z power –R, so this is actually F(0) – F(1) times, F(1) divided by Z, sorry here M-1 go, you go up to M-1, M = 1 so this is R is from 0 to 0, okay, so what you have only first term so only one term, okay, so this one this is M = 1 so R is running from 0 to M-1, so M-1 is 1-1 that is 0, -Z transform of F(n) this is equal to Z transform of 1 is simply we know that this is Z divided by Z-1.

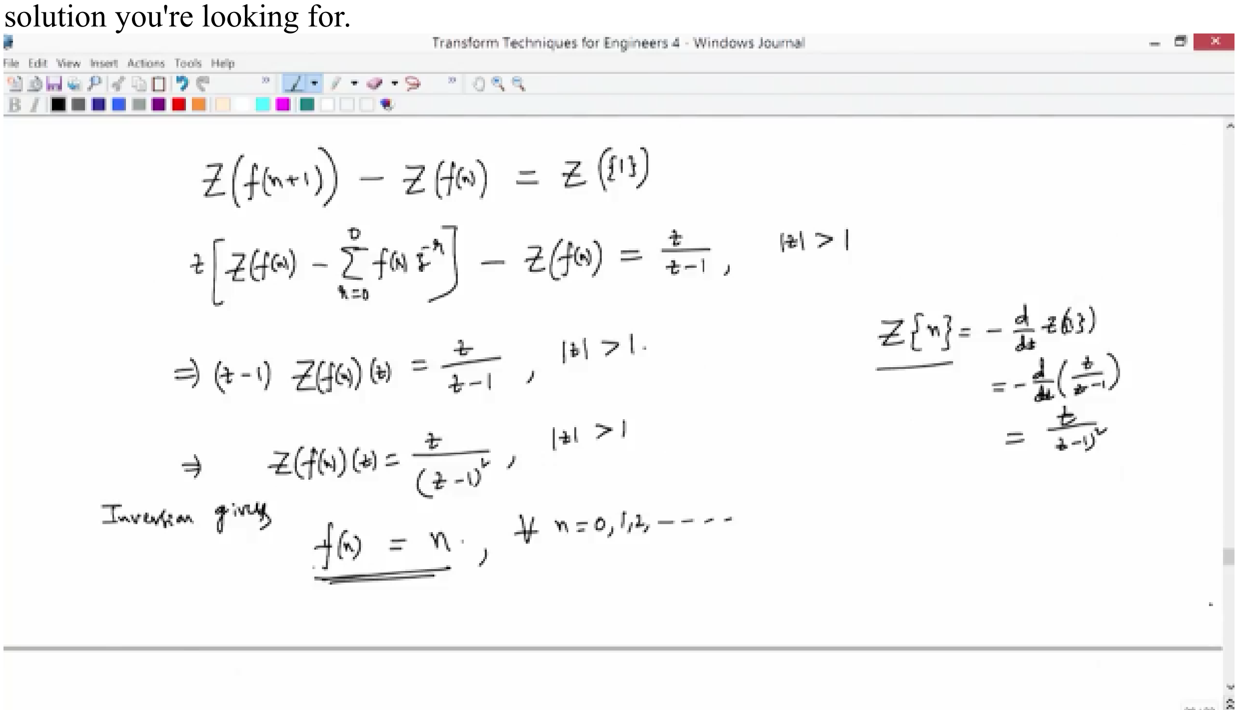



So what is this one? This is actually so because of this quantity this is mod Z is greater than 1 this is valid, we know for this one we know because of this unknown, because this is unknown so we can take the same domain over which this is true, so you have a Z times, Z transform of F(n) this is a function of Z –F(0) this is F(0) that is 0 is given, so you have this is, this 1-Z transform, so Z transform of the same thing so you have, we can combine this with this and Z-1 times this equal to Z divided by Z-1 this is valid here.

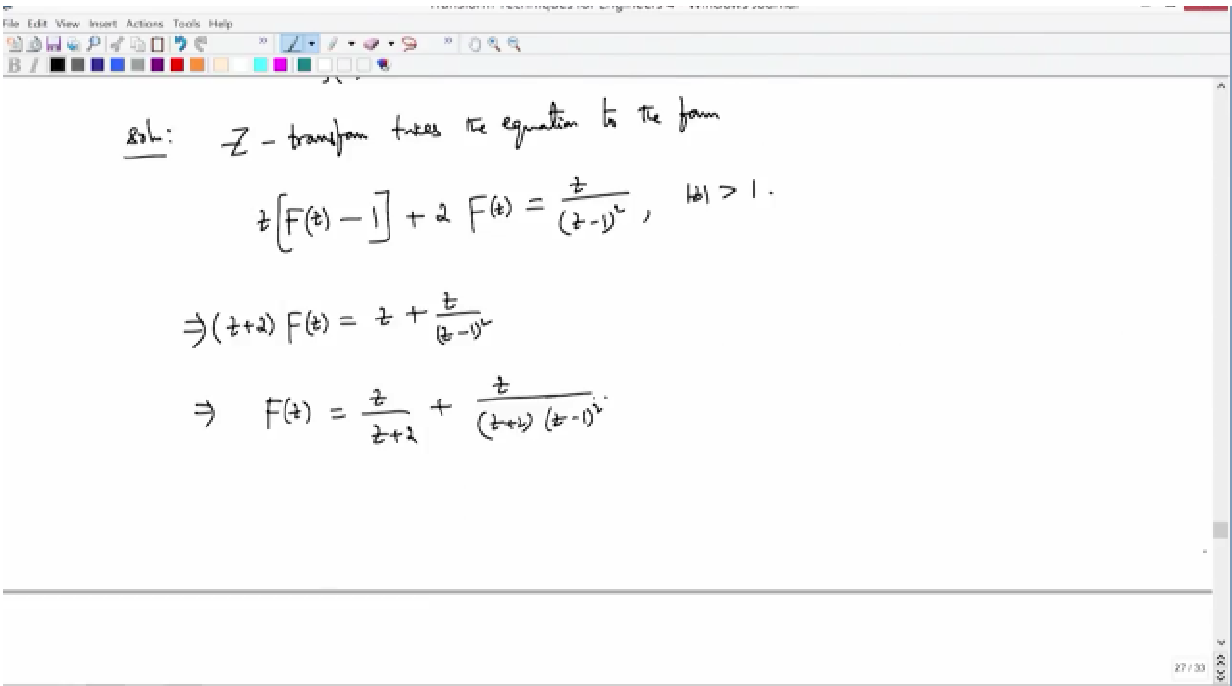
So if we actually see what is Z transform of F(n), as a function of Z is Z/Z-1 whole square in this domain, so this you have already seen that this is, if you actually see earlier Z transform of F(n) which is N, if you actually see that is value is, Z times Z-1 whole square, so this is N into 1 this is –D/DZ of Z transform of 1, the sequence 1 that is –D/DZ of Z/Z-1 this is actually same as Z divide by Z-1 whole square, so you can easily see that this is true for every N, 0, 1, 2, 3, onwards okay, so Z inverse, so what is that? F(n), so inverse transform gives, inversion gives left hand side this is F(n) because this is inverse transform of Z transform, and right hand side inverse transform is, because of this so inverse transform of this quantity is same as N, simply N, so you get this, okay, so this is so if you have this difference equation of first order with one



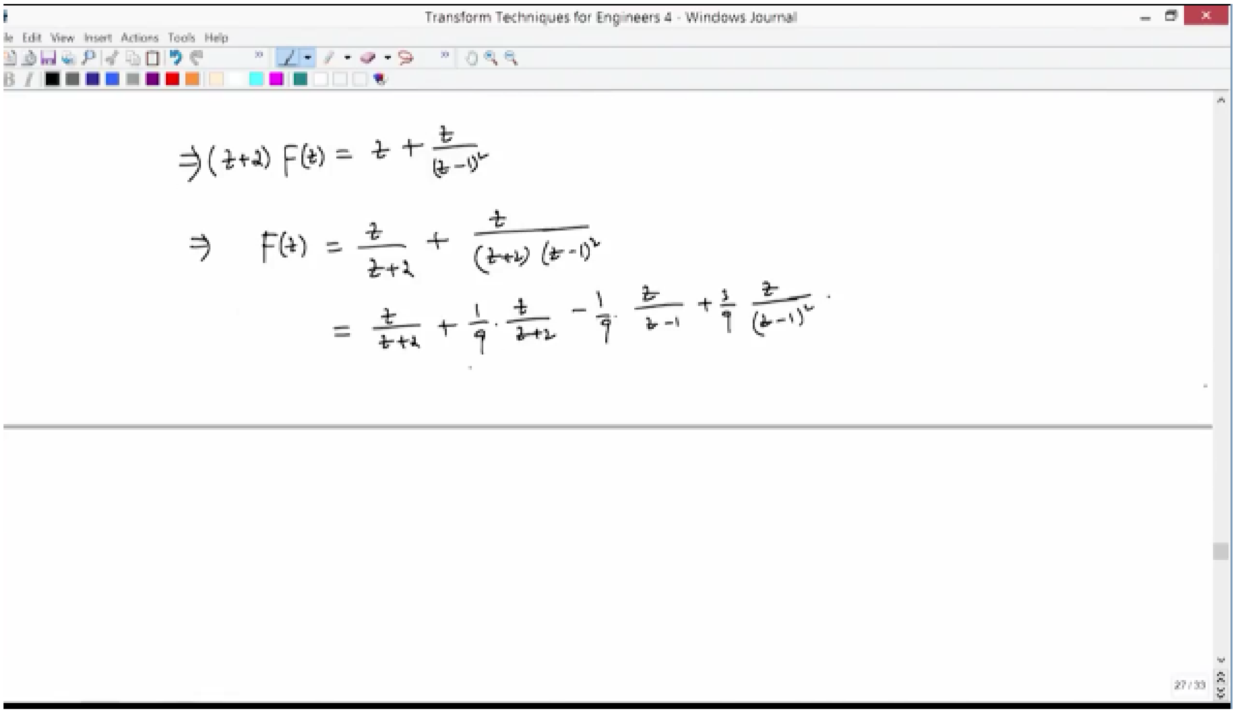

initial condition that is F(0)is given here and you can calculate what is the Z transform of F(n) and this domain, and you can once you invert it you can get your F(n) that is what is the solution you're looking for.



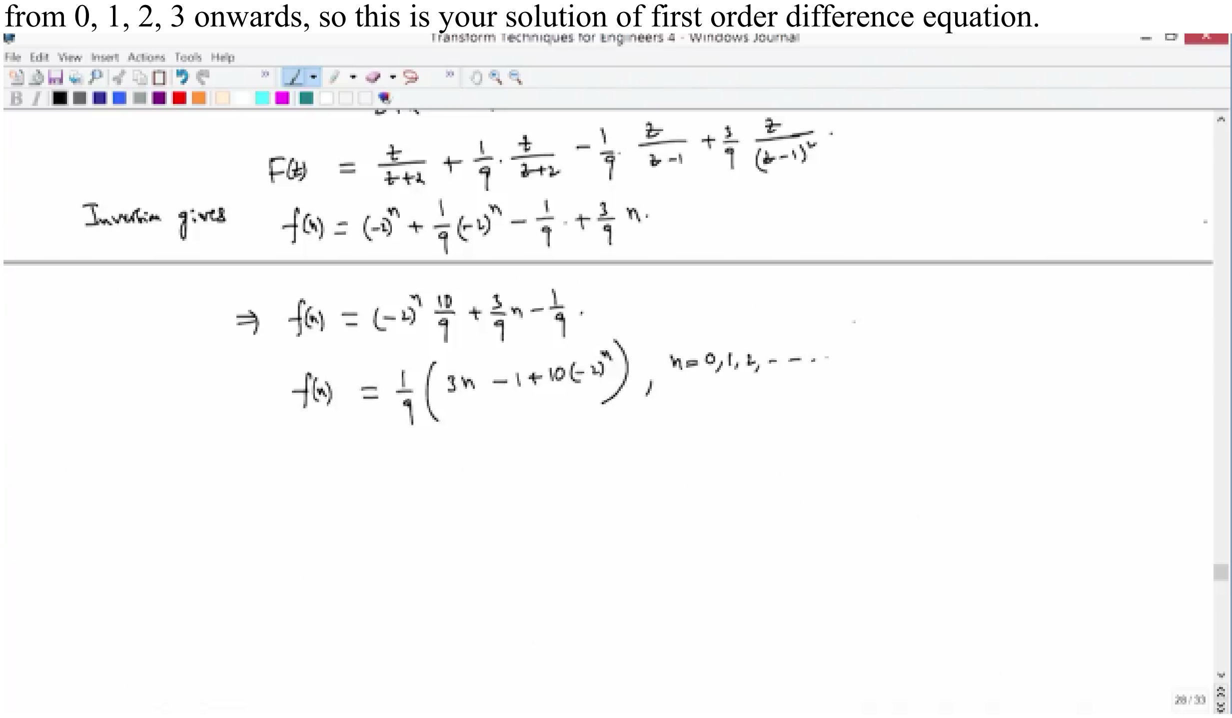

So let's look at some other examples, let's solve another first order difference equations, let's solve F(n+1) + 2 times F(n) = N, given that so N is running from 0, 1, 2, 3 onwards given that initial condition because the first order F(0) is 1, let's say this is 1 given, okay, the solution if you look for Z transform application, Z transform takes the equation to this form, to the Z transform of F(n+1) is Z times Z transform of F(n) that let me call this F capital F(z) okay, if I call this Z transform of F(n) is Z, instead of denoting Z transform of F(n) as a function of Z, I am writing capital F(z) so this one minus here F(0) because the first order F(0) that is 1, and then +2 times Z transform of F(n) is if I denoted with this, this is equal to Z transform of N that just now we have seen that it's value Z divided by Z-1 whole square, because right hand side valid in this domain, so we can consider this equation in this domain, so this implies because F(z) we don't know where is actually valid, okay, so because it involves a left hand side it is involving this F(z), right hand side is this in this domain, so it has to be F(z) also should be in the same domain.




So what is F(z)? F(z) is Z+2 and then -equal to Z+, you take this –Z to the other side Z+Z/Z-1 whole square so this is okay, so let's look at this, this implies F(z) is Z divided by Z+2, so this plus Z divided by Z+2 times Z-1 whole square, why I'm not combining is I know this Z divided by Z+2 I can invert it easily, so inversion will give before I invert this let's write this as, this as it is because I know it's inversion and this you put it in the partial fractions method, you can use a partial fraction method here because this degree is smaller than denominator so you have, if you write this as a partial fractions 1 is over Z+2, so it's going to be 1/9 times Z divided by Z+2, I'm writing directly 3/9 times Z divided by Z-1 so actually -1/9 times Z/Z-1 + 3/9 times Z divided by Z-1 whole square, so this is how you get the partial fractions as these 3 for this second term.



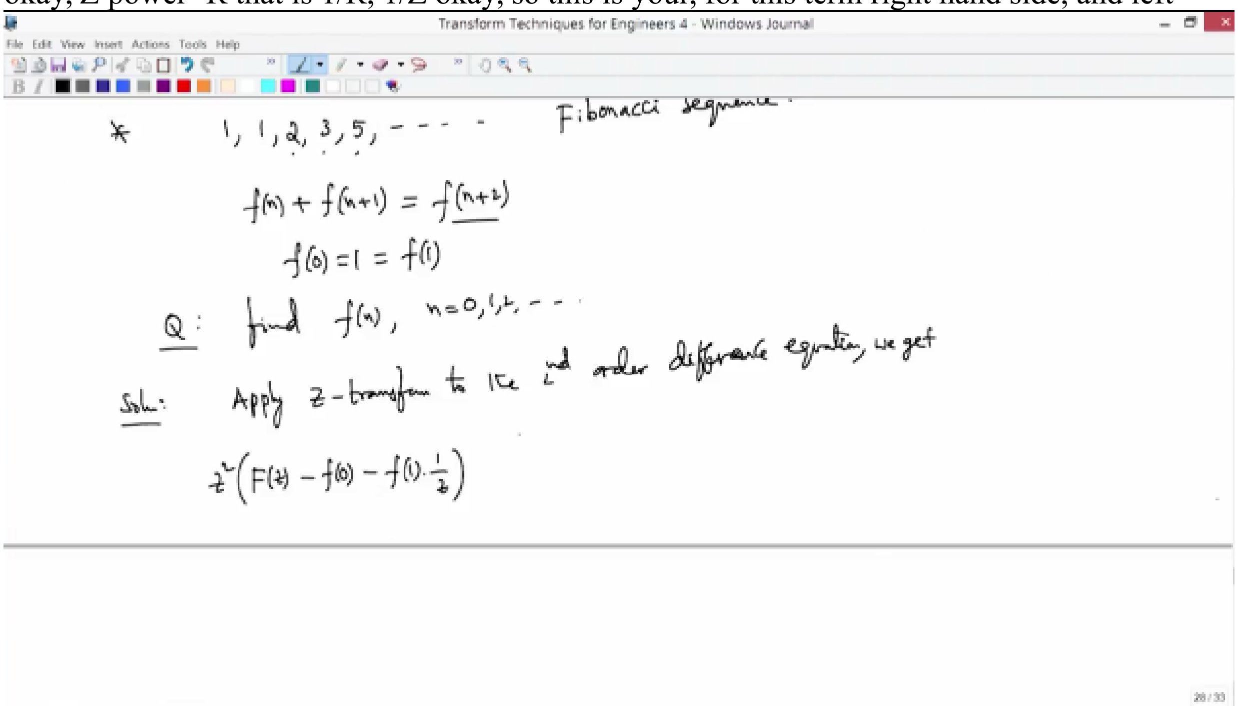

Now we can invert this is your F(z), so inversion gives F(n) running from 0, 1, 2 onwards, this is easily you can see that this is actually, what is Z/Z+2 this is A power N, so you have 2 power N, right, so Z- -2 that is -2 power N + 1/9 -2 power N, here 1/9 times Z/Z-1 is simply 1, okay, 1 power N so that is 1 + 3/9 times Z/Z-1 square is simply N, so this is equal to F(n) = -2 power N is common so you have 1 + 1/9 is 10/9 and you have + 3/9 N -1/9, so you can write it nicely -2 power N times 10 or you have 3N - 1 + 10 times - 2 power N, so this is your F(n), N is running from 0, 1, 2, 3 onwards, so this is your solution of first order difference equation.



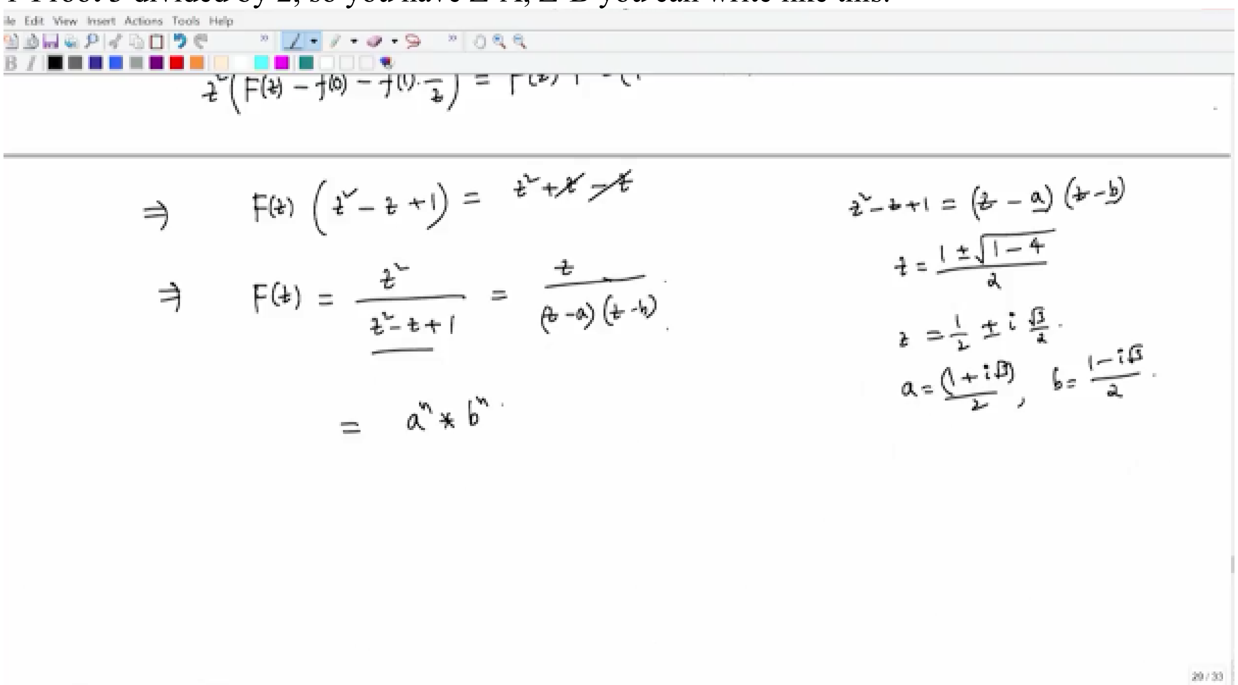

So let's look at some more examples, we'll do only these examples this is how you apply your properties of Z transforms, apply the Z transform and use the properties of these Z transforms to get the inversion and that your solution of, and make use of this initial data whatever is given, so we have seen some Fibonacci series that is starting with 1 1 and you just add this, so you need to Fibonacci sequence starts with 1 1 and you add this you get 2, and then again you add these two you will get next term, last two terms you add you get a next term and so on, so this is your Fibonacci sequence, so if I put it this so the basic idea is last two terms that if I call this $F(n) + F(n+1)$ so let's say N equal to, N you start somewhere N+1 and this is N+ second term so that is your $F(n+2)$ second term.

And what is known is initial values that is $F(0) = 1$, and $F(1) = 1$, so this is $F(n+2)$ because of $F(n+2)$ this is second order difference equation, so you can solve this with this initial data, so now question is finding how to find $F(n)$, N is from as a formula, okay, so this you already know this is your sequence you can instead of giving like this as the numbers you can give nth term of this Fibonacci series, that is your solution of this difference equation.

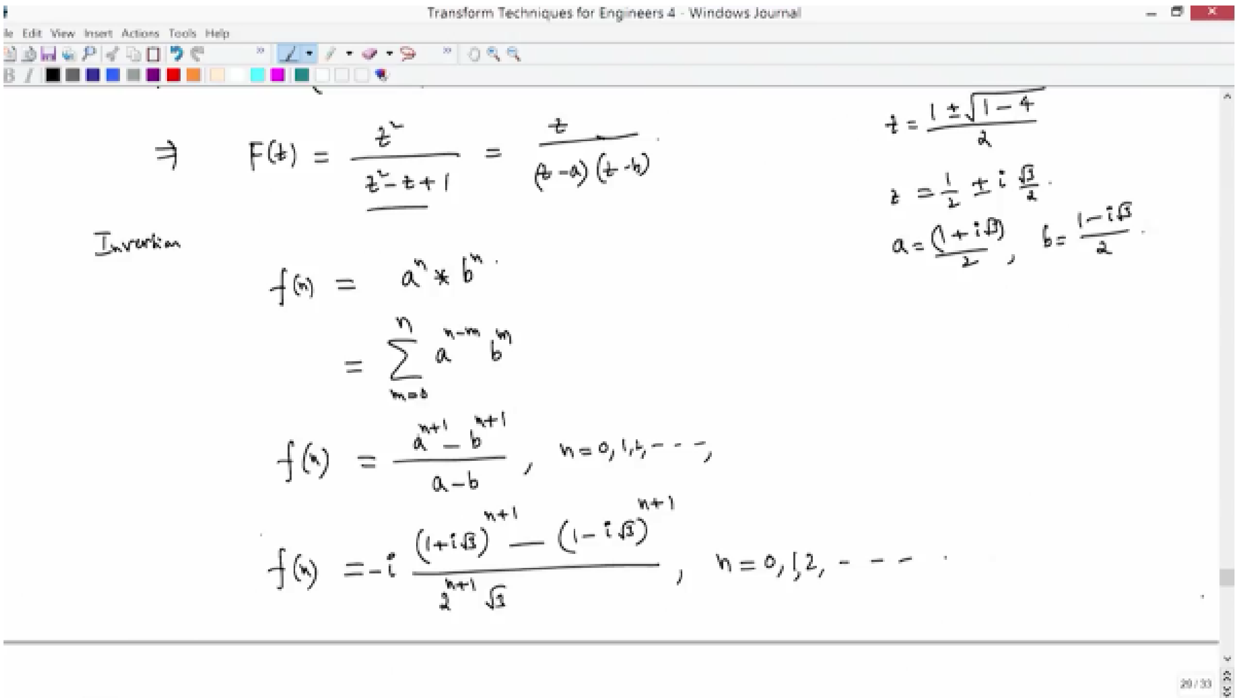
So let's find out, how do we do this? Apply Z transform to the, apply Z transform to the equation, to the second order equation, difference equation, we get, what we get? If you apply this for example $F(n+2)$ if you apply $F(n+2)$ you get Z square times capital $F(z)$ that is Z transform of $F(n)$ - M = 0, $F(0) – F(1)$ times that is M equal to, because this is the second order M is, summation is R is running from 0 to M-1, M is 2, 2-1 is 1, so we have $F(1)$ times 1/Z okay, Z power -R that is 1/R, 1/Z okay, so this is your, for this term right hand side, and left




hand side capital $F(z)$ + Z times $F(z)$ that is a Z transform of F – $F(0)$ that is 1 anyway, so if you actually see both sides and combine, what is the $F(z)$? $F(z)$ if you take the term Z square here you bring it this side that is going to be –Z +1, you bring these two terms to the other side and keep the other side, so if you keep here that is going to be Z square -F naught, so you have Z square comes this side and you have here Z square into 1/Z that is Z, so you have +Z here, so these two terms will become Z square +Z if I use this and here – Z into $F(0)$ so that is simply Z, so you have Z Z goes, so that is what is this, so this implies $F(z)$ is simply Z square divided by Z square - Z + 1, you can use your convolution theorem okay, so if I write this as Z/Z-, so you have 2 roots here Z square – Z + 1 has 0, has two roots, you can write this as Z-, so let's call this some A and into Z-B so if you do that these roots you can find A and B, and A or B is actually, roots are what is your Z? Z is 1 + or – square root of B square that is 1 -4AC, 4AC is 4 divided by 2, so you have actually 1/2 + or - I root 3/2, so these are your roots, so A is that, B is with minus sign, so you consider this as Z-, so we call this A = 1 + root 3, I root 3 divided by 2, B is 1-I root 3 divided by 2, so you have Z-A, Z-B you can write like this.



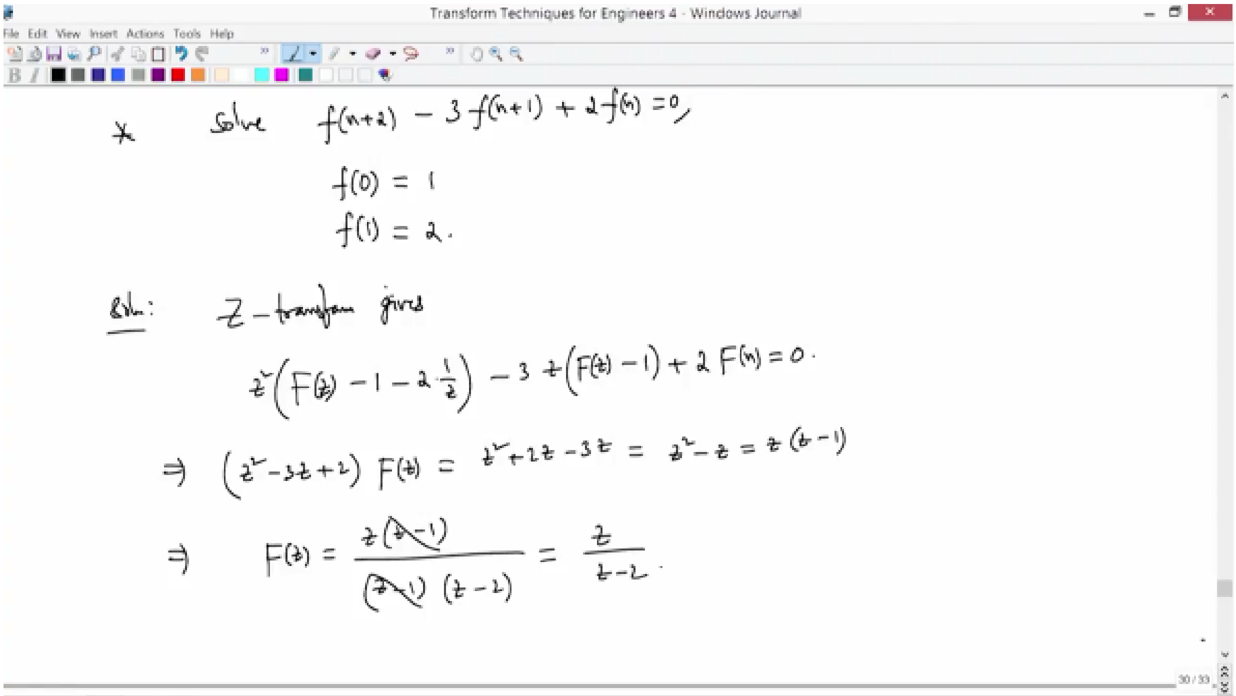

So we have seen that this is actually equal to A power N convolving with B power N earlier you have seen, okay, so that is the convolution, few inversion, inversion directly gives Z inversion of, Z inverse of this function you have earlier seen that is actually, this is convolution, this is equal to convolution as you have seen M is running from 0 to N, A power N-M, B power M, okay, so this we have already seen that it is actually A-B, A power N+1 – B power N+1, so N is running from 0, 1, 2, 3 onwards, so this is your, where B is, where A and B is, A and B are this, so let me simplify if you do that what happens is that 1 + I root 3 power N+1 -1 -I root 3 power N + 1 divided by so 2 power N+1, okay, so because of this and here the denominator what you have is this A – B that is A-B if you do, 1 1 goes, 2I root 3 divided by 2, so I root 3, so this is exactly, so you have if you write this root 3 here and you have –I, bring that I up so this is your Fibonacci sequence formula, nth term of the Fibonacci sequence is this, okay, so



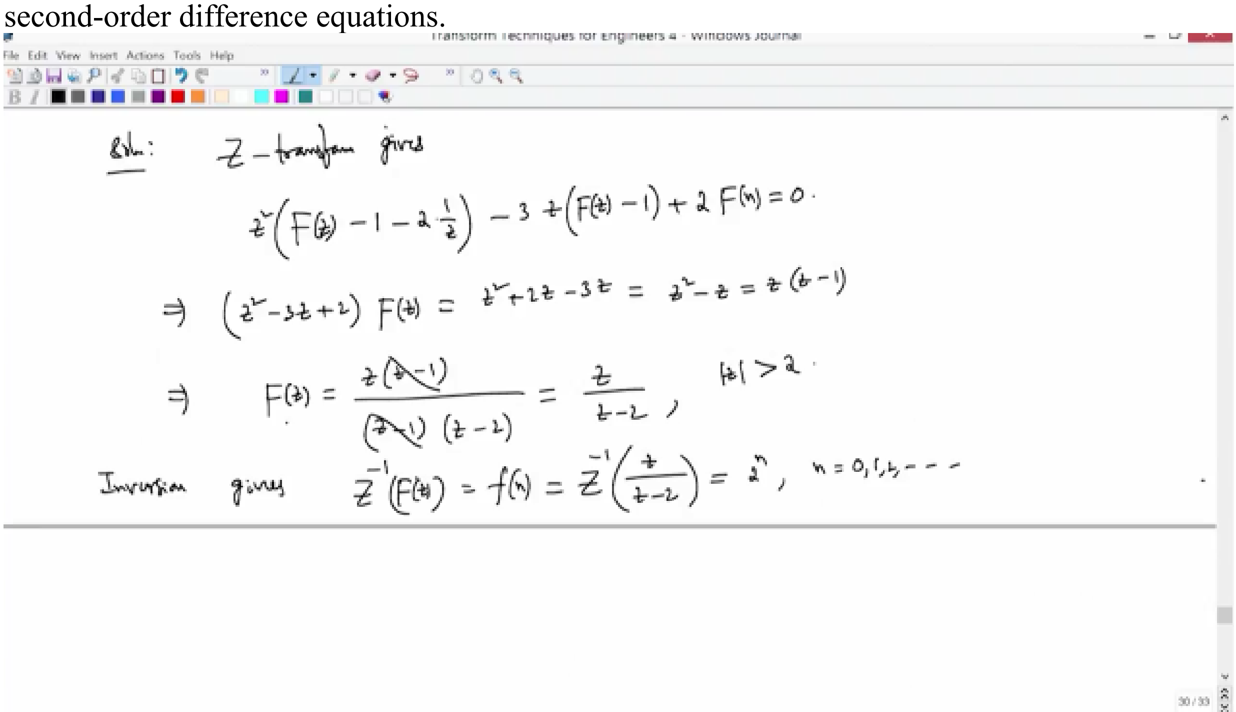

this term will give a, if you keep writing N = 0, N = 1 and so on you will get this sequence, okay, so this is how we can solve this second order difference equation which happen to be a, which happened to model of Fibonacci sequence, a Fibonacci sequence is a solution of this second order difference equation.

So let's look at some more second order equations, one more let's look it, so solve the initial-value problem that is solved F(n+2) -3 times F(n+1) and then you have +2 times F(n) = 0, if I want to solve this is the second order I have to give you initial conditions, two initial conditions F(0) and F(1) should be prescribed, let me take this as 1 and 2 okay, so this solution the same technique we apply, Z transform gives F(n+1) that is Z square times capital F(z), Z transform of F(n) – F(0) that is 1, and then -2 times 1/Z, F(1) into Z power -1, so you have this and this 1 -3 times F(n+1) is Z times Z transform of F(n), F(z) – F(1) is 1, so this is your Z transform for this second term.

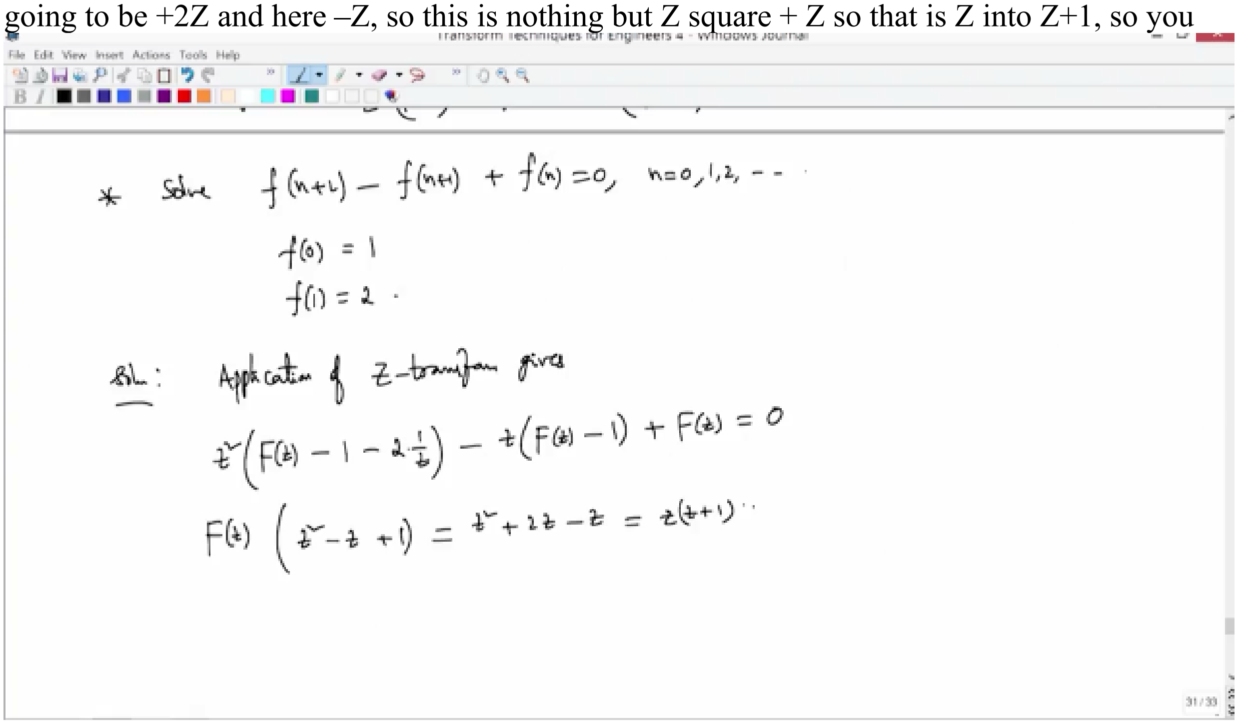
Third term is simply Z transform, 2 times Z transform of F(n) = 0, okay, so this implies Z square you combine F(z) so you have -3Z here + 2, so that is the coefficient of capital F(z) and other terms you bring it to the other side, so you have Z square comes out and that is your this term and here -2Z so it's going to be +2Z and this if you bring the other side this is going to be +3Z so that is going to be -3Z, okay, and then that's what it is, so you have this is equal to Z square –Z, so Z into Z-1, so this implies capital F(z) = Z into Z-1/Z square -3Z + 1, so that is Z-1 times Z-2, so if you actually write like that so you can easily see this one, so this is, this gets




canceled you have Z/Z-2 so inversion gives simply inversion gives a Z inversion of F(z) which is F(n) which I call as this is Z inverse of right hand side Z divided by Z-2, so this clearly I see that this is valid mod Z greater than 2, okay, that's why unknown also should be in that origin, so you have Z inversion of this is actually 2 times N, so N is running from 0, 1, 2, 3 onwards, so this is your solution, 2 power N is a solution of the second order difference equation with this initial data, okay, we change this initial data you may end up F(z) something else for which you have to find the inverse transform of the, inverse Z transform okay, so this is how we solve this second-order difference equations.

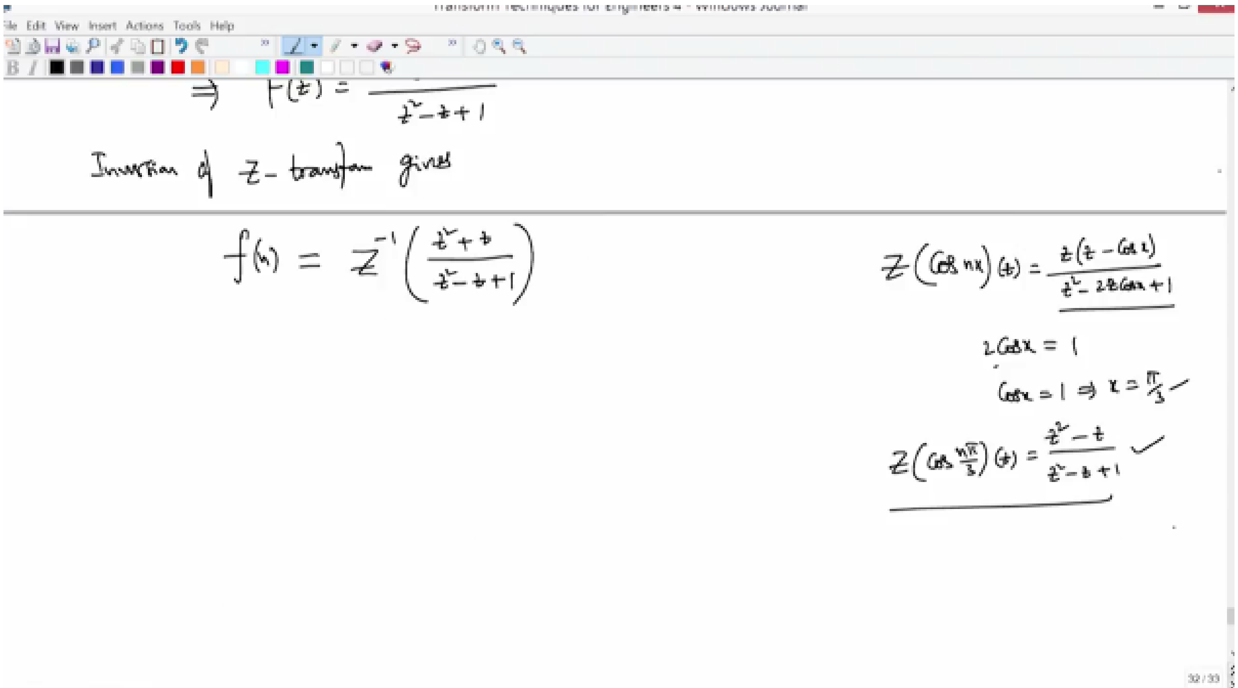


Let's do some more examples in second-order, solve F(n) + 2 – F(n+1) + F(n) = 0, N is running from 0, 1, 2, 3 onwards, and you have a second order equation so you should provide the initial conditions to F(0), F(1) let me take it as 2, okay, so what is the solution here? So if you apply the same technique, apply application of Z transform gives, what is the equation? Equation becomes Z square times capital F(z) - small f(0) that is 1 - F(1) that is 2 times 1/Z that is for this term, and you have second term -Z times F(z) – F(0) that is 1 + F(z) this becomes simply F(z) that is 0, so you get the coefficient of F(z) that has Z square here –Z here +1 equal to, and here if you bring Z square the other side this term so you have Z square, and here if you bring that is going to be +2Z and here –Z, so this is nothing but Z square + Z so that is Z into Z+1, so you

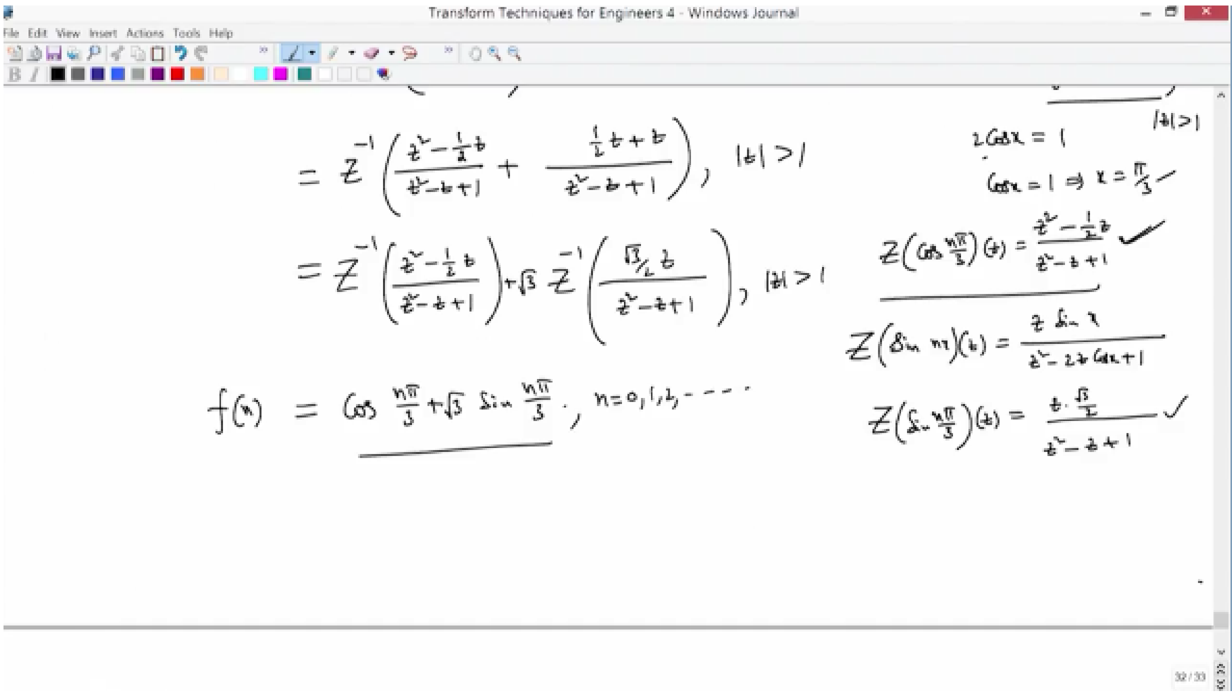



get F(z) as Z into Z+1/Z square - Z+1, so how do I do this? What is the Z transform? So inversion will give, inversion of Z transform gives left hand side small f(n) that is what is the solution that you are looking for, and this is Z inversion of Z square + Z/Z square – Z+1, so you might have seen some in the denominator when you have Z square - cos X or + cos XZ + 1, and the numerator Z square- or + cos X Z, so what is this transform, if you can remember, simply cos and sine, cos X Z transform if you use, Z transform of cos X is, let's recall those Z transform of cos X and sine X, Z transform of cos NX, okay, you need a sequence here, so cos NX as a function of Z this is equal to Z into Z-cos X/Z square -2Z cos X+1, so we want to see whether this is in this form, okay.

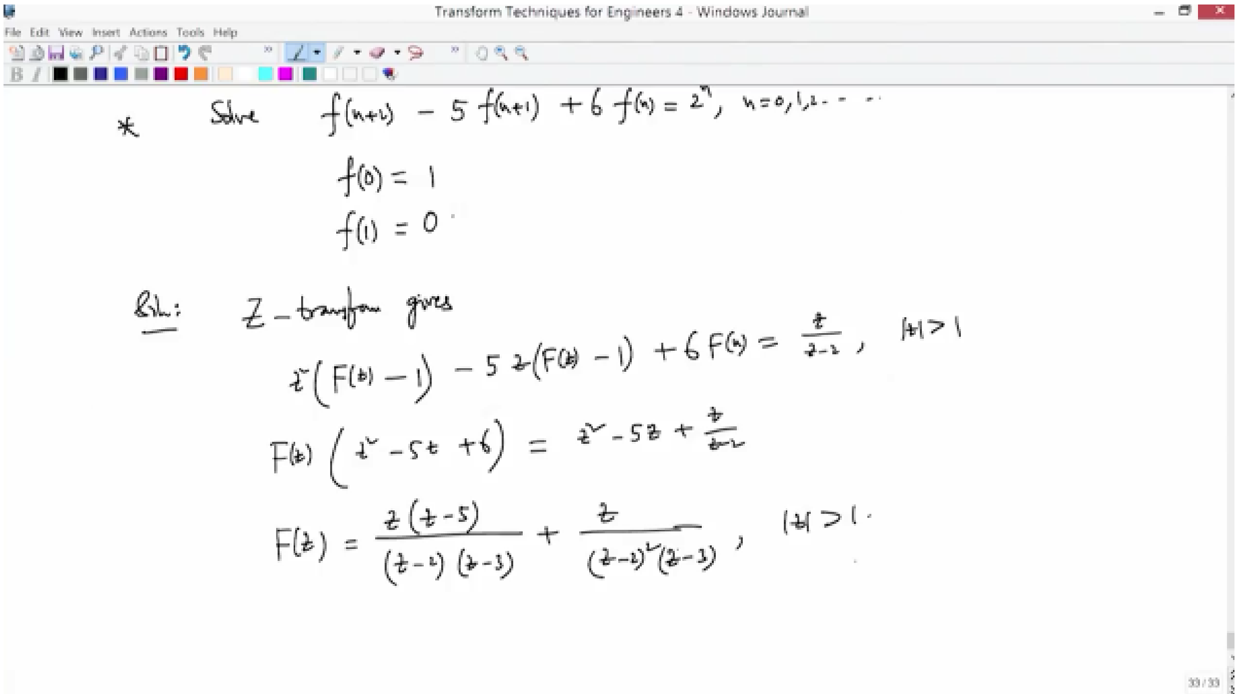
So Z square so how we can put this in the denominator so you have Z square - so you need -2 cos Z = -1, so 2 cos X should be 1, so cos X is 1 so this implies X equal to, so what is X? Cos equal to, so if X equal to 60 degrees that is pi/3, if it is pi/3 then we know that this is true, so X values is this so denominator will be same if X = pi/3, so Z transform of cos N pi/3 is a function of Z is Z square –Z times cos X is 1 so divided by Z square - 2 cos X is 1 so you have –Z +1, okay, so this is what we know from this Z transform of cos NX, so this you have to put it in that




form, so Z inversion of Z square - Z divided by Z square – Z+1 because I added I have to subtract it, so I subtracted so I have to add, okay, so cos pi/3 is basically cos pi/3 is what, cos pi/3 is basically 1/2, 1/2 Z so you need 1/2 Z so we have to put in this form, so to do that I'll write -1/2 Z okay, so that I have to add +1/2Z, so Z square + 1/2 Z and then you already have +Z divided by Z square –Z+1 okay, Z inversion is linear so you have a Z inversion of linear if you do write it you have Z square -1/2 Z divided by Z square –Z+1, so this we know this is a Z inversion of this that is cos N pi/3 + Z inversion of sine also if you look at it you can put that in this form, Z transform of sine NX(z) as a function of Z this is Z sine X divided by Z square -2 Z cos X+1 same denominator, so X = pi/3, sine pi/3 is root 3/2, root 3/2 divided by Z square -Z + 1 is Z transform of sine N pi/3 as a function of Z, okay, so this if we use here so you already have 3/2 Z square so you don't have any Z square here, so you have only this one, so I added this +Z so you have 3/2 Z you have 3/2 Z but you need is root 3/2 so that is only constant if you can take it out, so you can write root 3 here so that you have root 3/2 Z okay, so that one, so you can apply now this is for sine, and this is for cos, so you have Z inversion of this is actually cos N pi/3 + this is sine N pi/3, so where is this valid? Is actually valid for all Z again greater than 1, because of this, because this is mod Z greater than 1 this is valid and here also you can, these are all valid here, okay. So this is a complex domain we have this is your solution F(n), N is running from 0, 1, 2, 3, onwards, so this is your, okay I've got to missed this root 3, so you have root 3, root 3 times sine N pi/3 so this is your solution.

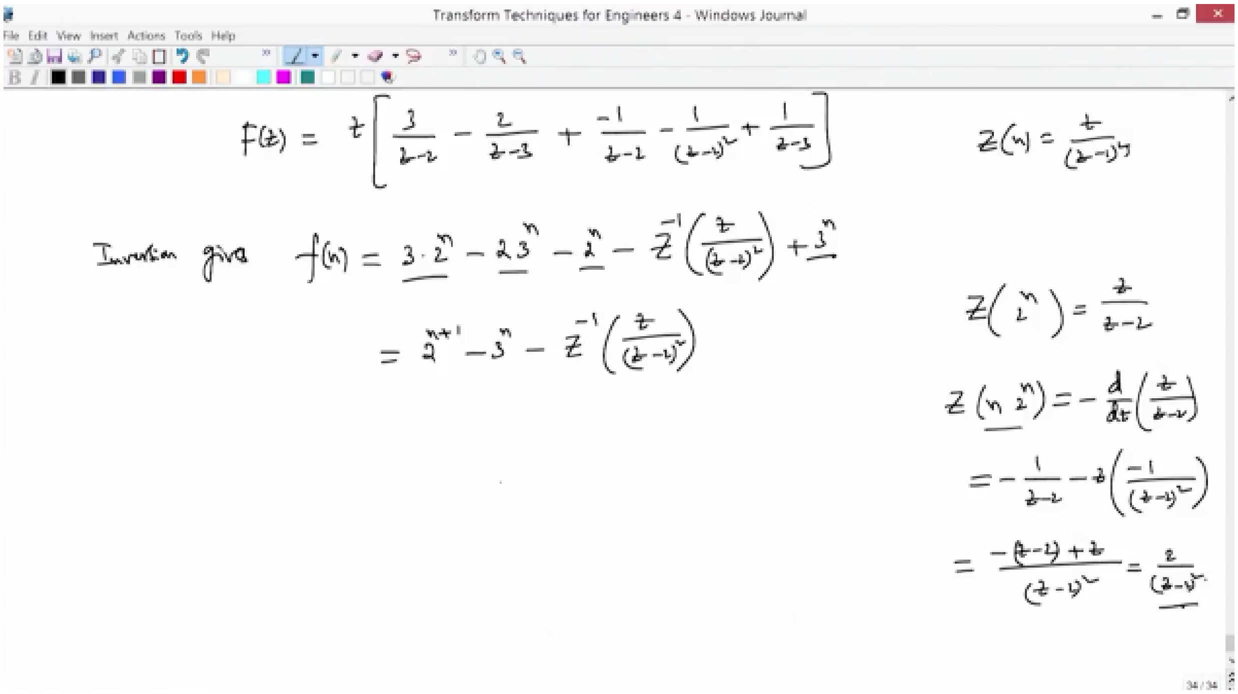



Let’s solve one more, then we'll look at the other application in the next video, so let's look at solving, solve F(n+2) - 5 times F(n+1) + 6 times F(n) = 2 power N, so N is running from 0, 1, 2, 3 onwards, and this is second order so you have to provide the initial conditions that as let's say 1, 0, so the solution so as usual Z transform gives, makes the equation as Z square times capital F(z) – F(0) is 1, other part is 0 so you don't have to worry, and you have 5 times Z times F(z) – F(0) that is 1 and + 6 F(n) = 2 power N is Z/Z-2, so this is valid mod Z greater than 1 right hand side so it should be left hand side also, so F(z) coefficient you bring it out so you have Z square -5Z + 6 = Z square and + 5Z that is going to be -5Z and +Z/Z-2, so what is your capital F(z) is? So you can write this Z square -5Z as division Z into Z-5 divided by Z-2 into Z-3, this is the product of Z-2, Z-3 + Z divided by Z-2 into Z-3 and you have one more time so square into Z-3, so this is what is your F(z) that is valid in this domain, so you can invert this to get your solution.

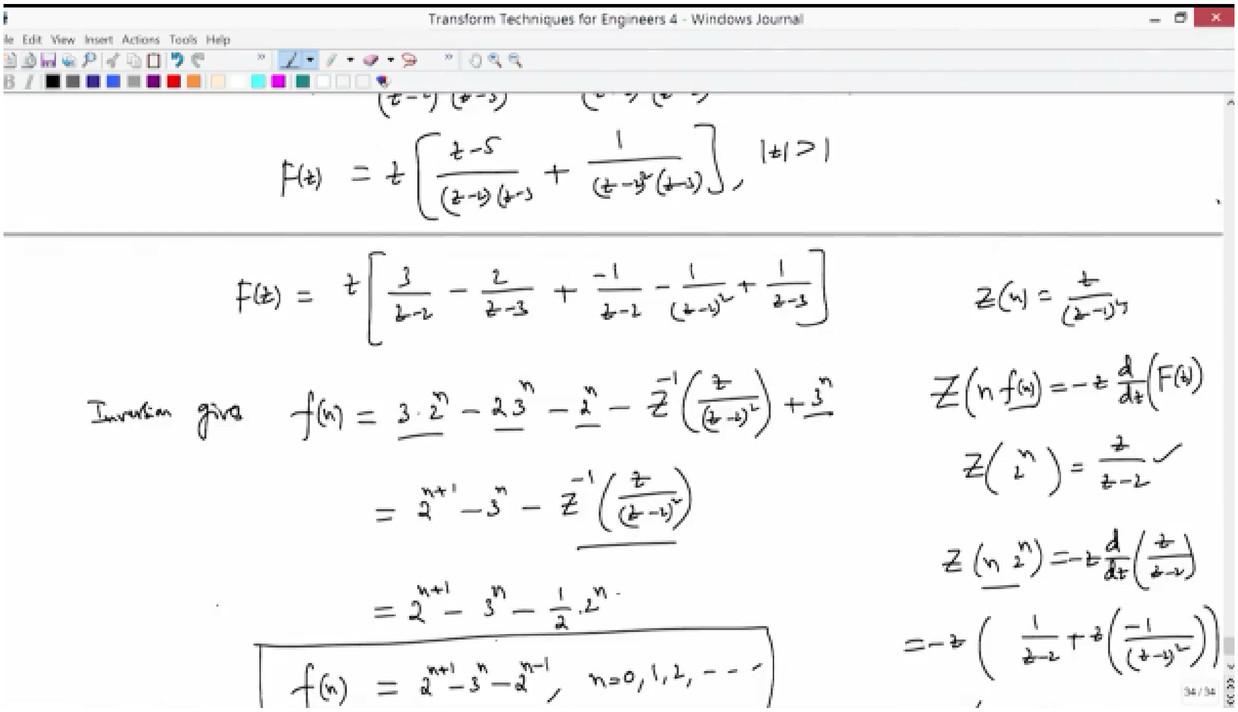



So inversion gives, how do I get the inversion? So before I do this before you get the inversion you simply 5Z and you have Z square, so Z is common so Z you take it out and you have Z-5/Z-2 times Z-3 + 1/Z-2 whole square Z-3, so let me write it like this, this has your F(z), then you write F(z), and you write it as a partial fractions you see that Z times this partial fractions if you write 3/Z-2 – 2/Z-3 as first term and here you get 1/Z – 1/Z-2 – 1/Z-2 whole square and then + 1/Z-3 so this is what you will get, if you sum add this this is your second term, so this whole thing is with Z so you can now invert it, inversion gives small f(n) that is a solution 3 times Z/Z-2 you have 2 power N and here 2 is constant Z/Z-3 is 3 power N and you have -1 constants Z/Z-2 inversion is 2 power N and here –Z/Z-2 power whole square is simply N, right, Z/Z-2 whole square, do we know that that is 1, that is actually Z/Z-1 whole square, okay, so we don't know this at, you have to be very careful this is not N, so Z/Z-1, Z transform of N is Z/Z-1 whole square, or Z-2 whole square but you have Z-2 whole square so you have a Z inversion of Z/Z-2 whole square whichever you don't know just write like this, and we can see later.

So this one will be Z/Z-3 you can easily see that is 3 power N, so if you combine this 3 power N will be, 3 power N - 2 power N, so you have -3 power N these two together and here this one together you have 2 power N+1, so 2 into 2 power N that is this if you combine you get 3-1 times 2 power N that is 2 into 2 power N, that is 2 power N+1 – Z inversion of Z/Z-2 whole square okay, so this one we can make use of, so how do you get this? We already know that Z transform of Z/Z-2 is equal to Z inversion, okay, or rather Z transform of 2 power N is Z/Z-2 okay, you know that Z transform of N times 2 power N = - D/DZ (Z/Z-2), so this one if you actually do you see that this is going to be -1/Z-2 -Z times minus minus plus 1/Z-2 whole square, okay, so if you actually combine at Z-2 whole square –Z-2 that is going to be +2 +Z, so it gets cancelled so you have, how do you get this? If you want to do carefully so let me do this, this as -1/Z-2 –Z times, now if you do this derivative 1/Z-2 whole square minus, you have -1, so you have, it's going to be combined it you see that –Z-2 and this is going to be +Z, so this is equal to, you are getting only 2 divided by Z-2 whole square, so N into 2 power N is this, but this is not what you have.




So how do I find this? So let me use properly some properties you have to be carefully you have to use this property to get this inversion, let me use how you have Z right, so I made a small mistake so you have a -Z times D/DZ (Z-2) okay, so that's fine, so you have –Z is there everywhere, okay so you have -Z everywhere so you have –Z, so -2Z/Z square, so let me do this carefully, so –Z times derivative of this is 1/Z-2 and you have +Z times derivative of 1/Z-2 is this, so if I actually do it it's going to be –Z times, Z-2 –Z/Z-2 whole square so Z Z goes you have -2, so it's going to be +2Z/Z-2 whole square, so inverse transform of Z/Z-2 whole square is 1/2 times 2 power N, so you have 2 power N+1 -3 power N -1/2 times 2 power N, so this is nothing but 2 power N+1, -3 power N - 2 power N-1, so this is your solution, N is from 0 to 1, 2, so here you made use of this property N into F(n) = -Z times D/DZ of capital F(z) which is the Z transform of this sequence, small f(n), so if I use f(n) as 2 power N since I know this so you have this, so you can see this is what for this, so this is your solution that you can get it as for this second order difference equation.




So this is how you solve any first order, second order difference equations with the initial data when it is provided, that is provided you can get the solution that is nth term of the sequence F(n) okay, nth term of the sequence that is F(n) as a solution of this difference equation you can get it just by using Z transform and the inverse Z transforms.

So in the next video we will see the applications of these Z transforms to find certain sums, when you have, when you can evaluate certain sums some infinite sums you can evaluate using these Z transform just like what we have done, just like you have evaluated integrals some integrals using Laplace transforms, so integral becomes series sum infinite series sum and instead of Laplace transform, so there it's expected because discretized version of Laplace transform is Z transform, so we just give that application as analogous application of evaluating infinite sums using Z transforms in the next video. Thank you for watching.


**Online Editing and Post Production**

Karthik

Ravichandran

Mohanarangan

Sribalaji

Komathi

Vignesh

Mahesh Kumar

**Web-Studio Team**

Anitha

Bharathi

Catherine

Clifford

Deepthi

Dhivya

Divya

Gayathri
Gokulsekar
Halid
Hemavathy
Jagadeeshwaran
Jayanthi
Kamala
Lakshmipriya
Libin
Madhu
Maria Neeta
Mohana
Mohana Sundari
Muralikrishnan
Nivetha
Parkavi
Poornika
Premkumar
Ragavi
Renuka
Saravanan
Sathya
Shirley
Sorna
Subhash
Suriyaprakash
Vinothini

**Executive Producer**

Kannan Krishnamurty

**NPTEL Coordinator**

Prof. Andrew Thangaraj
Prof. Prathap Haridoss

**IIT Madras Production**

Funded by
Department of Higher Education
Ministry of Human Resource Development
Government of India
www.nptel.ac.in