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Transform Techniques for Engineers
Inverse Z-Transforms
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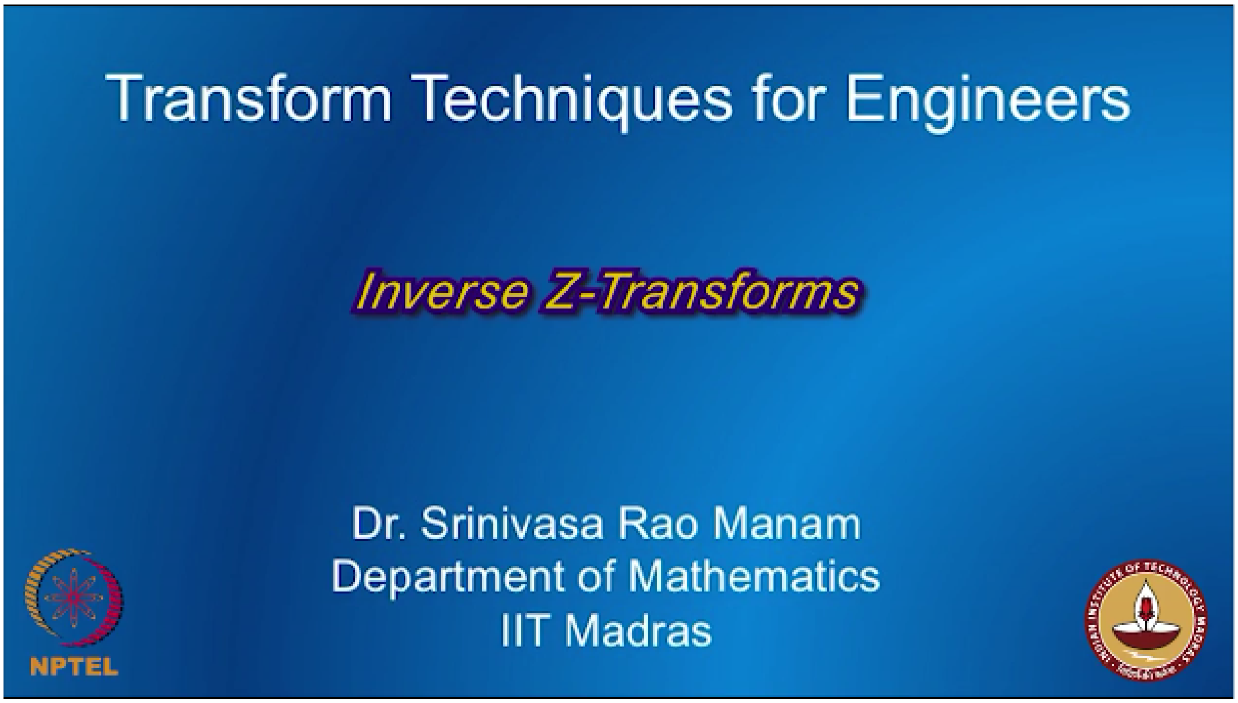


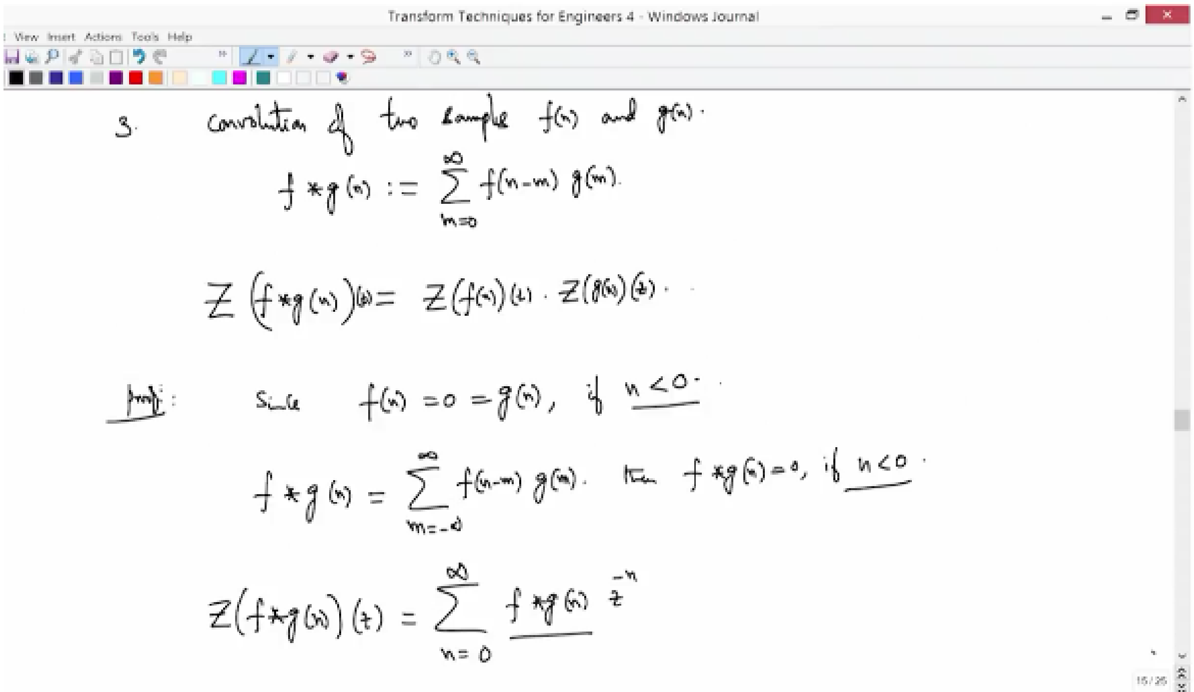
Welcome back, in the last video we were discussing about properties of Z transforms, we have seen two such properties, let’s look into some other properties, let’s look into some other properties in this video, and along with inverse transforms, before we apply this Z transform technique to solve difference equations.

So let's look into third property that we want to do, this is the analogous to this convolution, so convolution of two samples, first I define what are those two samples F(n) and G(n), so F star G(n) this is by definition so the same way we do this convolution I define as sigma F(n-m) and G(m) M is running from 0 to infinity, so this is how I do.

So I take one function N-M other function M so you sum it up with M, M is running from 0 to infinity, so this if you have this form, so if this is your convolution Laplace transform of, sorry Z transform of F star G(n) this convolution is actually is equal to Z transform of F(n) as a function of Z this is also function of Z times Z transform of G(n) as a function of Z, okay.

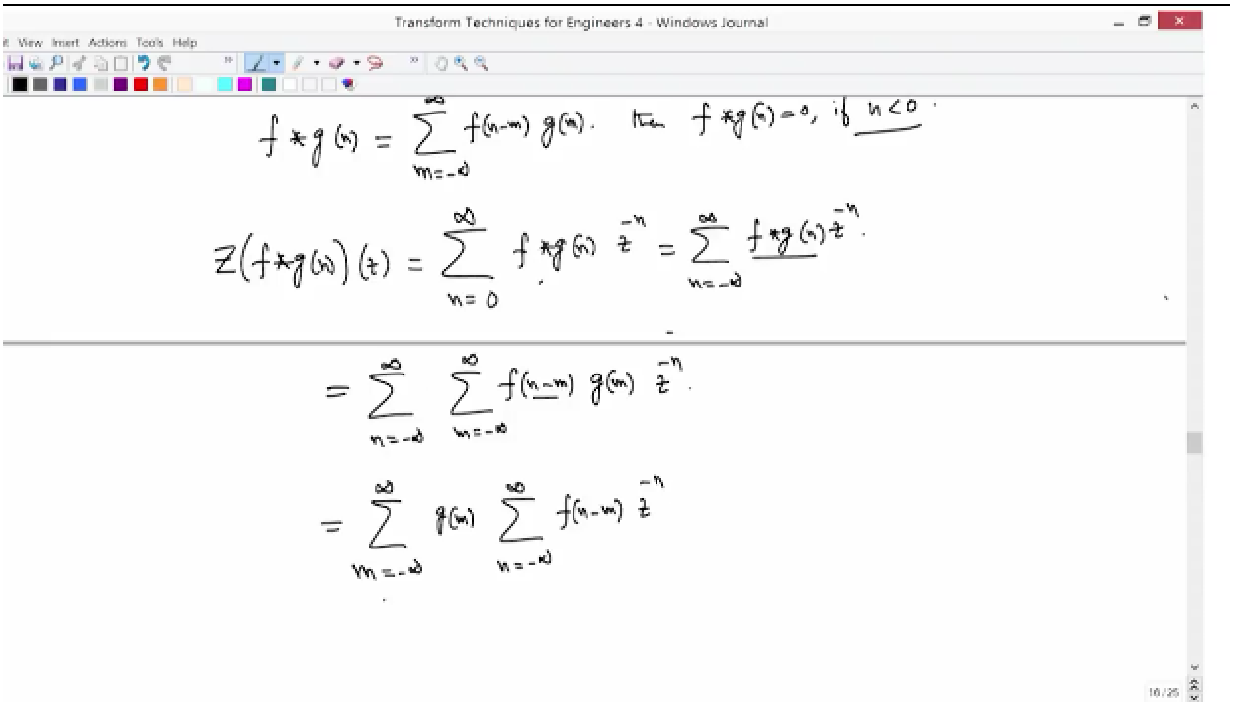
So this we can easily see a proof, so what is the common? What is that if you actually see the domain of this convolution, Z transform as a function of Z where is this validity of Z, this is the common domain between the validity of a domain of this Z transform of F, and Z transform of G, so the common, so intersection area, intersection domain of these, both the domains is actually a domain of definition for this Z transform of this convolution of these two samples, okay.

So let's do in, it's easy way so since F(n) is 0, and G(n) is also 0 if N is negative, negative integer so we can think of this Z transform we can think of F convolution G(n) is actually same as running from M is from -infinity to infinity, F(n-m) G(m) okay, so N is running from, so this is same as that, okay, because if you consider any negative powers that will become 0, if you choose so I can work with -infinity to infinity.



Then you apply Z transform of F convolving with G(n) as a function of Z is simply, apply by definition N is from 0 to infinity, this F convolving with G(n) times Z power –N, so again this if I have, N negative this is 2 or 0, N negative this is also 0, okay, if this convolution is this then F convolving with G(n) is 0, if N is negative integer because of this, okay, so if N is negative integer, M is running from -infinity to infinity, and G is, if this is positive and this will be negative in that case, if N is negative M is positive still you can get only positive side, so if N is negative for example -1, and if N is negative integer so F(n-1) n-1 M is always negative so that is because of this is 0, so any negative N this is 0, okay, this you can easily see, so because of this now this definition I can extend it as N is from -infinity to infinity, F convolving with G(n) Z power –N.

So now I write the definition of this convolution inside that, so -infinity infinity, F convolution this I write it from – infinity, M is from -infinity infinity F(n-m) G(m) that is for F convolution G and with Z power –N, so what is the next step, next step is just to change N-M as R, so in this index if you change N-M is M = N-R, so if I put M = -infinity, R is, N is also –infinity, M is also –infinity, N and M both takes -infinity to +infinity so R also we'll take -infinity to +infinity, so let me write this, so Z power –N inside this if I take R, if I replace M/M+R so R is the index now, so M is N-R, N is fixed for each fixed N here you have M is varying, M is varying means R is varying, M is varying from -infinity to infinity, so is or -infinity infinity, so you can write F(n-m) as F(r) times G(n-r), no, so again you're getting back the same, so let me not use this technique, so what I do is I simply write this, I interchange these sums, because the sum is, this sum is once it is finite this double sum is convergent sum, so you write this interchange these limits so you have this one, and you have G(m) and inside G(m) is nothing to do with N, N is from –infinity infinity, I interchange these two sums, so you have F(n-m) Z power –N, so this is what into, this is what you have if I interchange.



Now this is equal to M is from -infinity infinity, G(m) times Z power -M will give me I know that this is Z transform of G(m), so this times if I multiply this I have to multiply with Z power M, so that I will add it here F(n-m) Z power –N+M, so that is -N –M okay, so this N-M if we put as R, so R is also running from -infinity to infinity, so this you can replace with R, so this is nothing but so you can see that M is -infinity infinity, G(n) so you can write N, N is anyway dummy, so G(n) Z power –N times this is, now if I can replace R = -infinity infinity F(r) times Z power –R, because N-M = R and then N is running, so you have N is, for each fixed M you have N, N is running from -infinity to infinity, so R is running from between -infinity infinity, so you have to write N = R + N, anyway it's not required so we have N-M, N-M, so there's nothing, this is not required, so if I change this R as dummy into N this is with N, so this becomes simply Z transform of F(n) as a function of Z times, Z transform of G(n) as a function of Z, so this is exactly what you want to prove.

$$= \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} f(n-m)\, g(m)\, z^{-n}.$$

$$= \sum_{m=-\infty}^{\infty} g(m) \sum_{n=-\infty}^{\infty} f(n-m)\, z^{-n}.$$

$$= \sum_{m=-\infty}^{\infty} g(m)\, z^{-m} \sum_{\underset{\kappa}{n}=-\infty}^{\infty} f(\underset{\kappa}{\underline{n-m}})\, z^{-\underset{\kappa}{(n-m)}} \qquad n-m=\kappa.$$

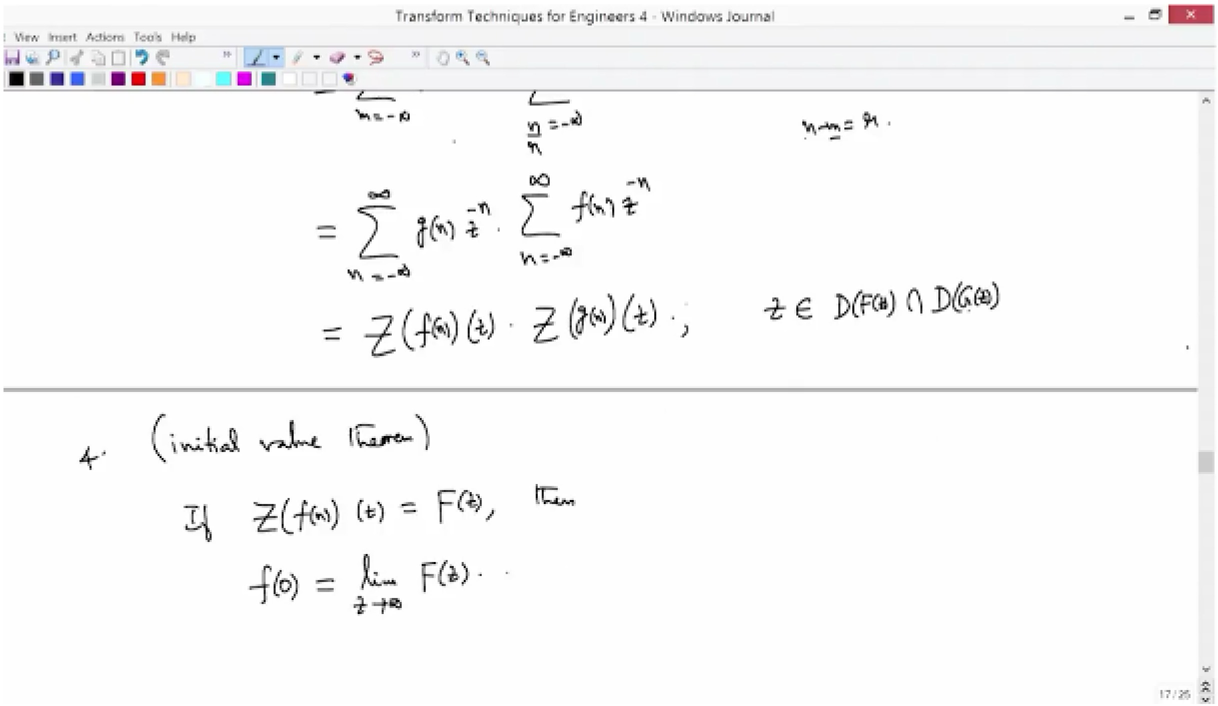
$$= \sum_{n=-\infty}^{\infty} g(n)\, z^{-n} \cdot \sum_{n=-\infty}^{\infty} f(n)\, z^{-n}$$

$$= Z(f(n))(z) \cdot Z(g(n))(z).$$

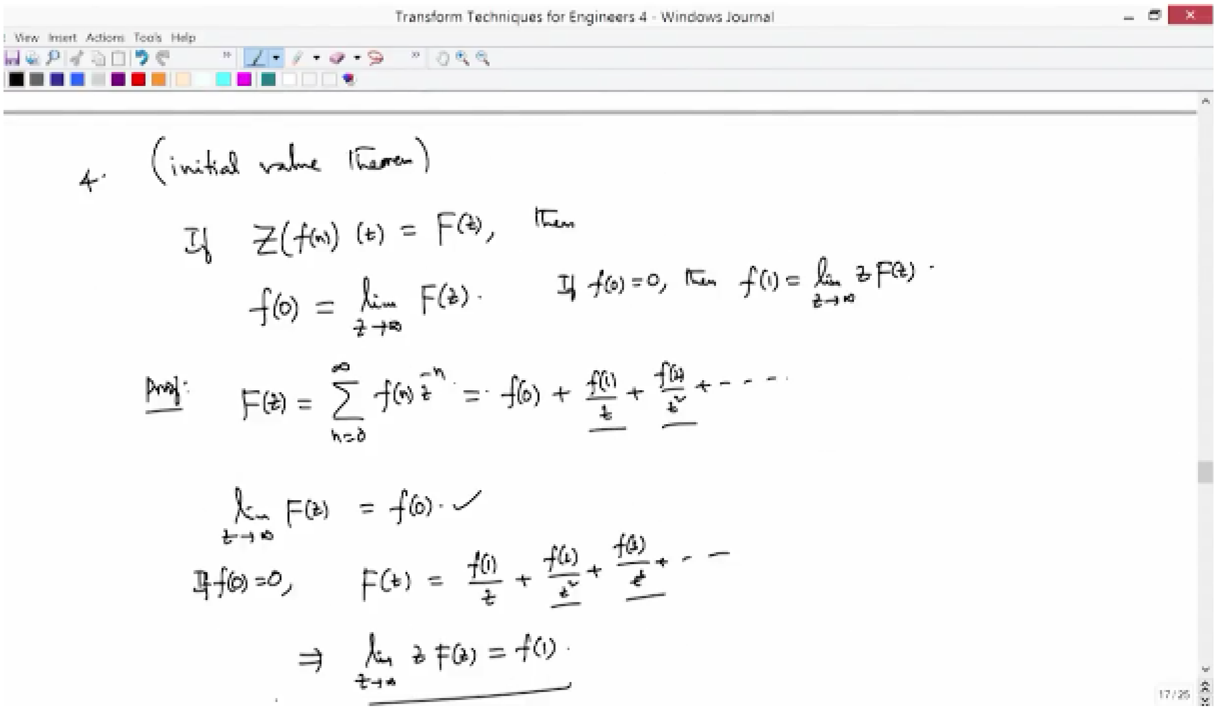
So this is simple to prove this property, this we can use, make use to find the Z transform, Z inverse, inverse Z-transform of certain complex valued functions which you can find using this convolution theorem just like what, the same thing we have done for a Laplace transform and Fourier transform if you recall, then also we can use this convolution theorem to find the inverse Laplace transform, so the same technique we use here.

So where is the domain of validity? Domain of this, so Z belongs to domain of capital F(z) intersection domain of capital G(z) where capital F and G are Z transforms of small samples F(n) and G(n) okay, F(z) G(z) are Z transforms of samples F(n) and G(n), so this is the domain of this intersection, so that is where this is valid, okay.

So we'll see other properties, this is one we use, other one is just like in a just similar to what we have done for the Laplace transforms, we can also have some initial value theorem and final value theorem, so let's look into that, so what is these versions here? Initial value theorem if you see is actually simple, so if Z-transform of the sample F(n) is a function of Z is capital F(z) then F(0) initial value, that is similar to what we have done there, so this is actually limit of their S tends to infinity F bar(s), right, that is what is the initial condition there, initial value theorem, so here we get Z goes to infinity, here F(z), so this is very simple to prove, proof is straightforward, so what we have is, and also if F(0) is 0 then you can also have F1 easily as a limit Z goes to F(z) so this is also you can get it from this, this is a deduction from out of this initial value theorem, so we will see these two in this proof.

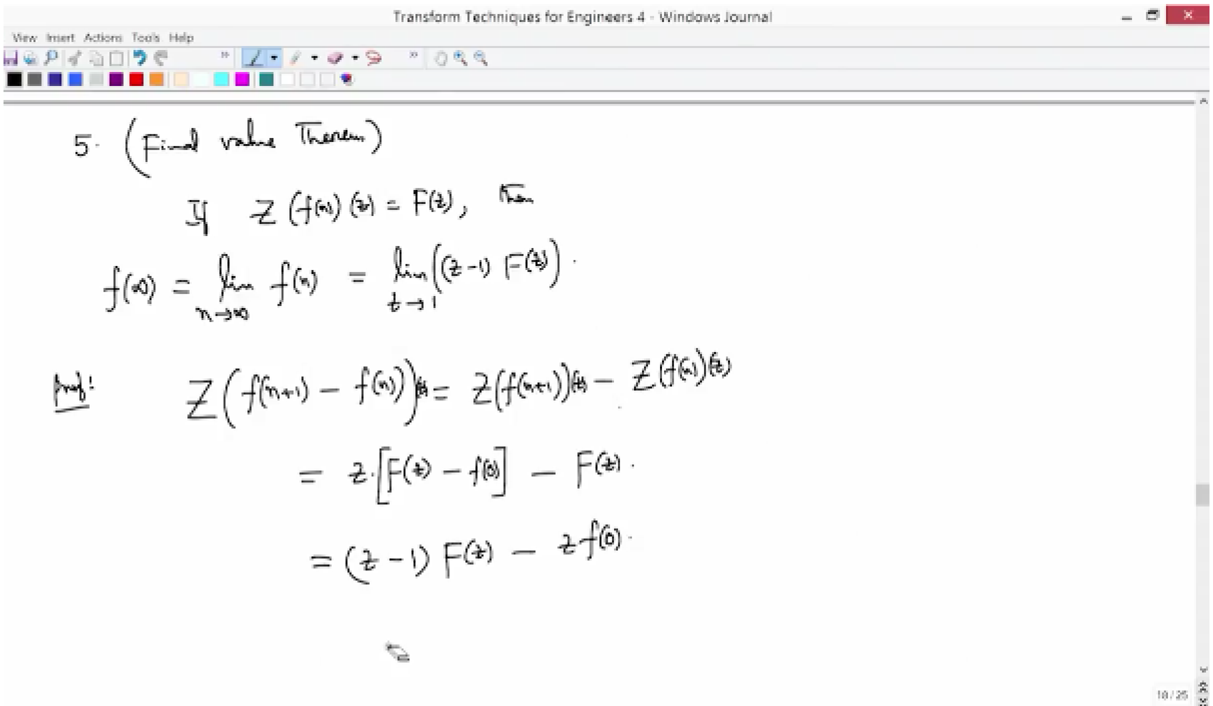


So what we do is what we know is this Z transform of the sample as F(z), capital F(z) is N is from 0 to infinity, F(n) Z power -N and then this you write it separately this if you write, this is equal to F(0) +F1/Z + F2/Z square and so on, so what happens to this, if you take this Z limit, Z goes to infinity you end up each of these will become 0 because Z goes to infinity you end F(0), so if F(0) is 0 so this is clear, this is very simple straightforward okay, so if F(0) is 0 then capital F(z) = F1/Z + F2/Z square + F3/Z cube and so on from this, okay, so this means Z into F(z) now if you take the limit Z goes to infinity this is actually equal to F(1) because all other terms Z into this is going to 0 as Z goes to infinity, so this is immediate from the definition of the Z transform that's called the initial value theorem version, the discrete case, in the discrete case, okay, this is the Z transform initial value theorem.

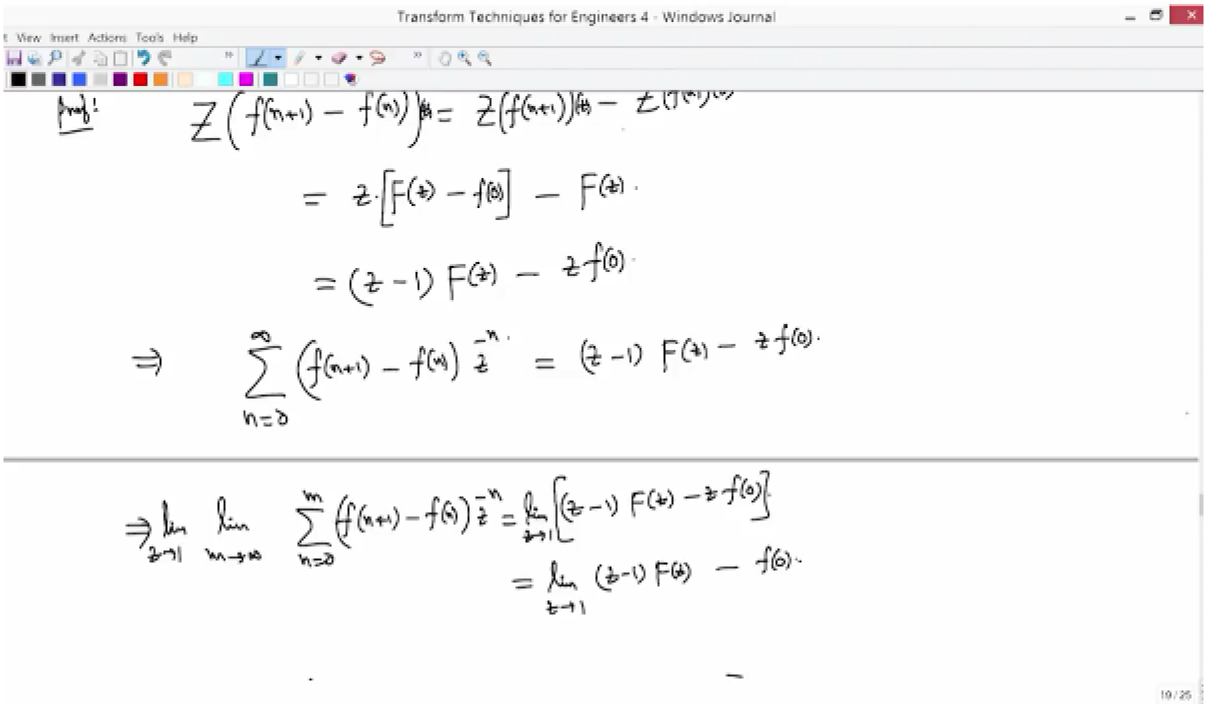


And the same way you can get the final value theorem that's what we have seen for Laplace transforms, final value theorem but this is not the same, we cannot, it may not be the same that we have done in the Laplace case. Final value theorem this tells you that if F of again, so if Z transform of F(n) the sample is a function of Z is F(z) capital F(z) then a limit F(n) as N goes to infinity this is same as a limit, Z goes to 1, Z-1 times F(z), capital F(z), so this limit will give you this F infinity which is the final value, okay.

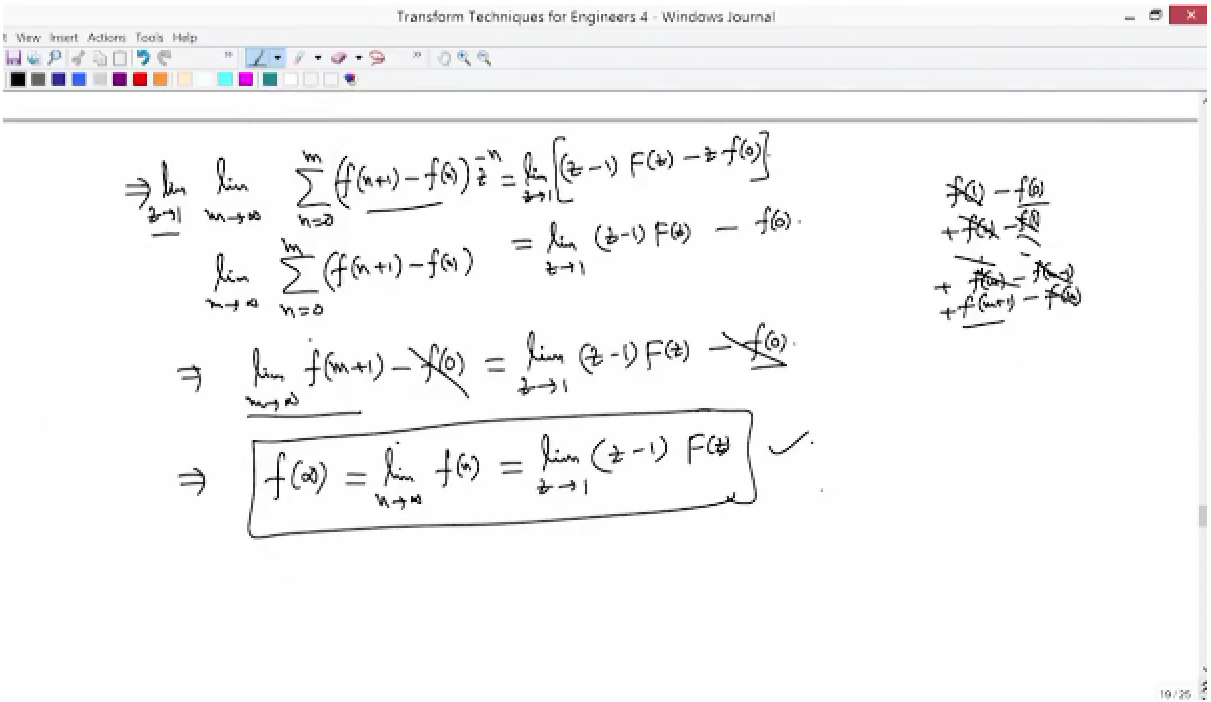
So how do I do this? To do this let me consider proof is a little tricky so this is, if I consider this Z transform of the difference, F(n+1) –F (n) this sample, 2 sample differences if you take because of linear property which is a trivial as a Z transform of F(n+1) and –Z transform of F(n) as a function of Z of course, everything is function of Z at the end, so this is equal to F(n+1) you can use, make use of first property that is Z times capital F(z) okay, and then minus, what is N+1? N+1 is, okay, so this is not N-M so if you use this property, property 1 if you use, what is the property one? We use this one here, F(n+m) is simply Z power M times for Z transform - the remaining items up to M-1 items in the Z transform you negate it, so that is what you have, so if you do it so Z1, so Z times F(z) - it is 1 so up to 0 terms so that is F(0) okay.



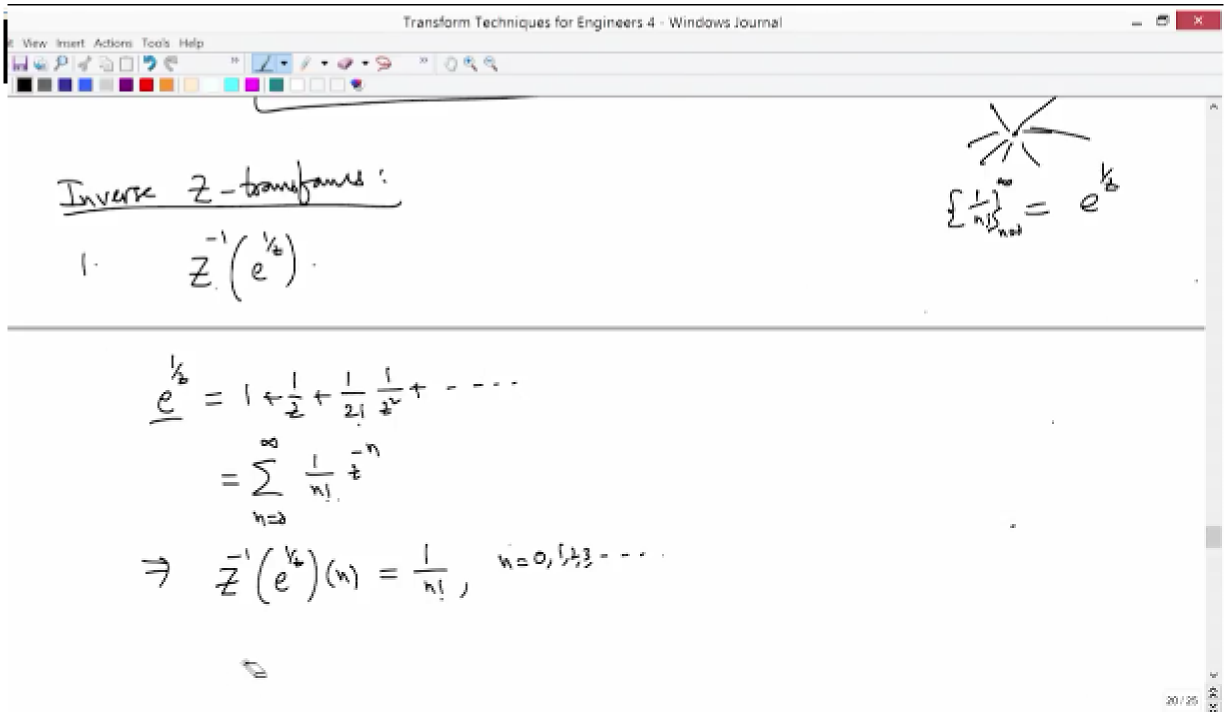
And then this is for this and you have -capital F(z) so what you have is here is Z-1 times F(z) - Z times F(0), small f(0) sample value, so what happens? Now if I multiply so you write what is the Z transform of left hand side, so this implies the left hand side is by definition N is from 0 to infinity F(n+1) – F(n) times Z power -N that is the definition which is equal to Z-1 F(z) -Z times F(0), so this you can think of writing as a partial sums, M goes to infinity, sigma N is from 0 to M, F(n+1) – F(n) times Z power –N and this is equal to Z-1 times F(z) - Z times, small f(0), so this gives me what this you can easily calculate, if I put Z =1, okay, so if I take now both sides limit Z goes to 1, so what happens here? This if I take Z goes to 1, so what you get is the whole thing if you consider Z goes to 1 this is nothing but limit Z goes to 1, Z-1 times capital F(z) and here you get F(0) because Z goes to 1 that is the value.



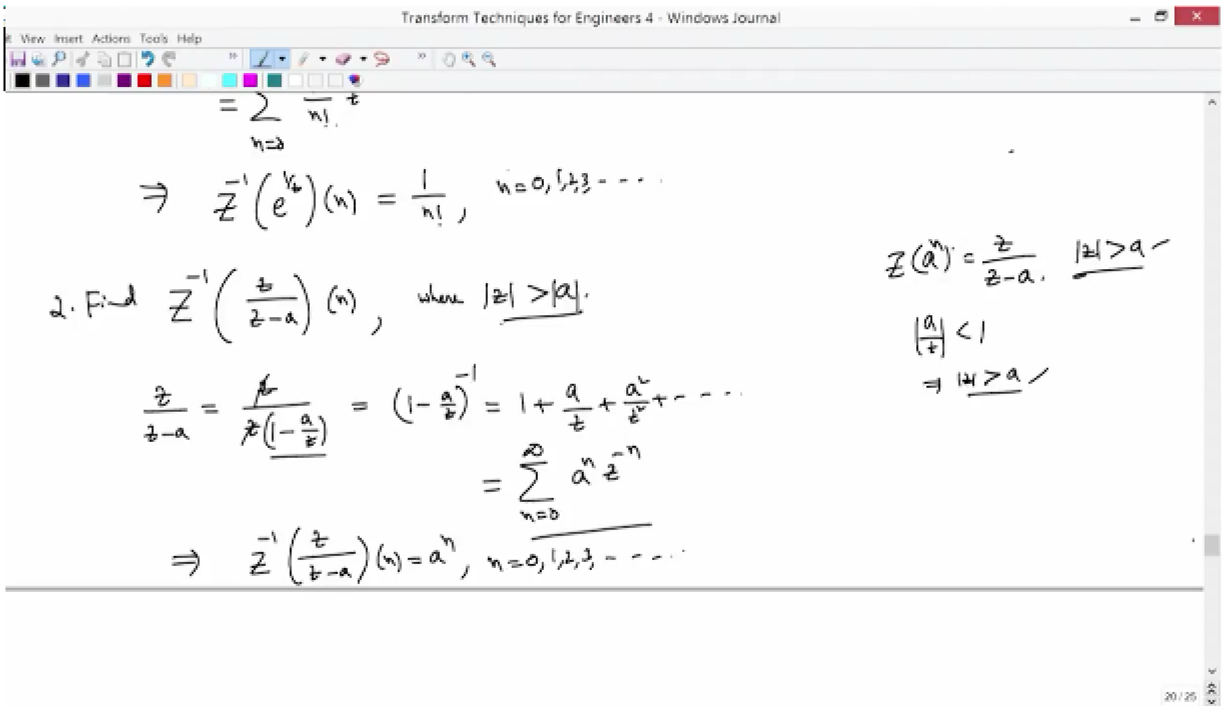
Now what happens to the left hand side this part you can write as F(m+1) everything gets cancelled this is when once I allow this Z goes to 1 this is simply 1 and you end up getting as a limit M goes to infinity, sigma N goes to M, F(m+1) rather(n+1) – F(n) first of all this and this is, this implies this limit M goes to infinity this sum if you actually do this summation addition you start with N = 0 is F(1)-F(0) + F(2)- F(1) and so on up to F(m+1) – F(m), and the earlier term is F(m) – F(m-1) so it goes like this end up, and you see that this goes here, so you end up getting this term and this term will remain, so you have F(m+1) – F(0) equal to this limit Z goes to 1, Z-1 times capital F(z) – F(0), small f(0), so this is a finite quantity, this is just from a sample, so you can cancel both sides so you end up getting, limit of M goes to 0 these quantities itself okay, so limit only goes here, up to here, so this means F(infinity) this is same as limit F(n) as N goes to infinity, the same as limit Z goes to 1, Z-1 times capital F(z), so this is your final value theorem.



So this may be useful when you apply a difference equations, so as and when it comes we can use, make use of this to the applications, just like what we have done in the Laplace transform case, okay, let's find these are the typical properties that we have for Z transforms, now let's see what is inverse Z transform given complex valued function F(z) with domain of definition, so let's give inverse transform, so let’s look into inverse transforms, inverse Z transforms, so what we have is, I’ll start with one, so let's find Z inverse of E power 1/Z, this is valid everywhere we have seen, okay, so for what function you have? If you remember if I take 1/N factorial as your sequence, N is from 0 to infinity then you have seen that E power 1/Z is your, E power 1/Z is a Z transform, so Z transform of this is E power 1/Z so Z inverse of this is actually it goes, this is what it is, you can directly if you remember you can do it, so otherwise what you do is to get this E power 1/Z you try to expand this 1+1/Z + 1/Z square, 1/2 factorial into 1/Z square and so on, you write like this, this gives what is your F(n)? F(n) is, so this is sigma N is from 0 to infinity, some constant that is 1/N factorial times Z power –N, this is your Z transform of this quantity is actually this, so this is one way, so this implies Z inverse of E power 1/Z, this is a trivial thing, this is as a function of N is simply 1/N factorial, so N is running from 0, 1, 2, 3 onwards, 0 factorial is 1.



So second example, let's look at the inverse transform of, Z transform of, inverse Z transform of Z into Z –or rather Z/Z-A as a function of N, so this is again you don't have to remember Z/Z-A you have seen already A power N if you consider Z transform of this you have found that this Z/Z-A, okay, where domain of validity is this, mod Z is greater than A, so this one to get this you start with the functions Z/Z-A, so Z and divide by Z you take it out and you have 1-A/Z, so if you do like this A/Z has to be less than 1, so that means mod Z is greater than A, okay, so mod Z is greater than A, so clearly so if it is given, once it is given like this validity of this, they should be given, so where mod Z is this, find just like finding this where this is this, then only I can write A/Z otherwise I may have to use Z/A, if mod Z is than A I have to choose Z/A, Z/A then only Z/A is less than 1, then only I can expand this one, 1 over 1-Z/A because this is Z/A is less than 1, so here A/Z is less than 1 that is clearly because of this mod A/Z okay, mod A/Z is actually mod you have to choose, mod A/Z is less than 1, so because of that Z Z goes you have 1/1-, so this is 1 by A/Z inverse, mod A/Z is less than 1, so I have I can write A/Z + A square/Z square and so on, so what you have is simply A power N, Z power –N, and N goes to 0 to infinity, so this is Z transform of A power N, so this implies, so this is Z inverse of Z/Z-A is equal, as a function of N is A power N, N is running from 0, 1, 2, 3 onwards, so there's one simple things on the third one.



You can also we use partial fractions to find the Z transforms, okay, Z inverse of F(z) where F(z) is as a function of N, where F(z) let me write as Z/Z square-6Z+8, so how do I get this? So if I write this F(z) as Z over it has 2 roots the denominator that you can write Z-3 and or rather Z-2 and Z-4 okay, so that you have a -6Z so that is what it is and this you make a partial fractions to see that this is going to be 1/Z, Z/Z-4 –Z/Z-2, so if you actually see that Z square -2Z -Z square +4Z, so that 2Z comes up so you have a Z here, so you have 2 2 cancel, so this is the form for this, so if you try to get this Z inverse of F(z) there's a function of N this is again it's a Z transform is linear, Z inverse is also linear, so you can because of the linear property Z inverse of this 1/2 times Z/Z-4 - Z inverse of 1/2 times Z/Z-2 so this is 1/2 comes out, because of linearity again Z/Z- 4 is A power N so you have 4 power N and you have here for this you have 1/2 times 2 power N, so this is what is your inverse Z-transform okay for every N.

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3. $Z^{-1}\left(F(z)\right)(n)$, when $F(z) = \frac{z}{z^2-6z+8}$

$$F(z) = \frac{z}{(z-2)(z-4)} = \frac{1}{2}\left(\frac{z}{z-4} - \frac{z}{z-2}\right)$$

$$Z^{-1}\left(F(z)\right)(n) = Z^{-1}\left(\frac{1}{2}\,\frac{z}{z-4}\right) - Z^{-1}\left(\frac{1}{2}\,\frac{z}{z-2}\right)$$

$$= \frac{1}{2}\left(4^n - 2^n\right), \quad n = 0, 1, 2, 3, \ldots$$

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So next one is you can also use convolution theorem and you have these rational functions like this, what is the Z inverse of, if you choose Z square/Z-A Z-B, partial fractions sometimes will give you complicated, it's difficult to find maybe little difficult to find the partial fractions that case you can use the convolution theorem, so how do I find the convolution? Convolution theorem is, if I choose if I write Z square/Z-A into Z-B as Z/Z-A times Z/Z-B, so in this case this is my one and this is another function, so this is, how do I write this? This is, this I can write as a Z transform of A power N as a function of Z times, Z transform of B power N as a

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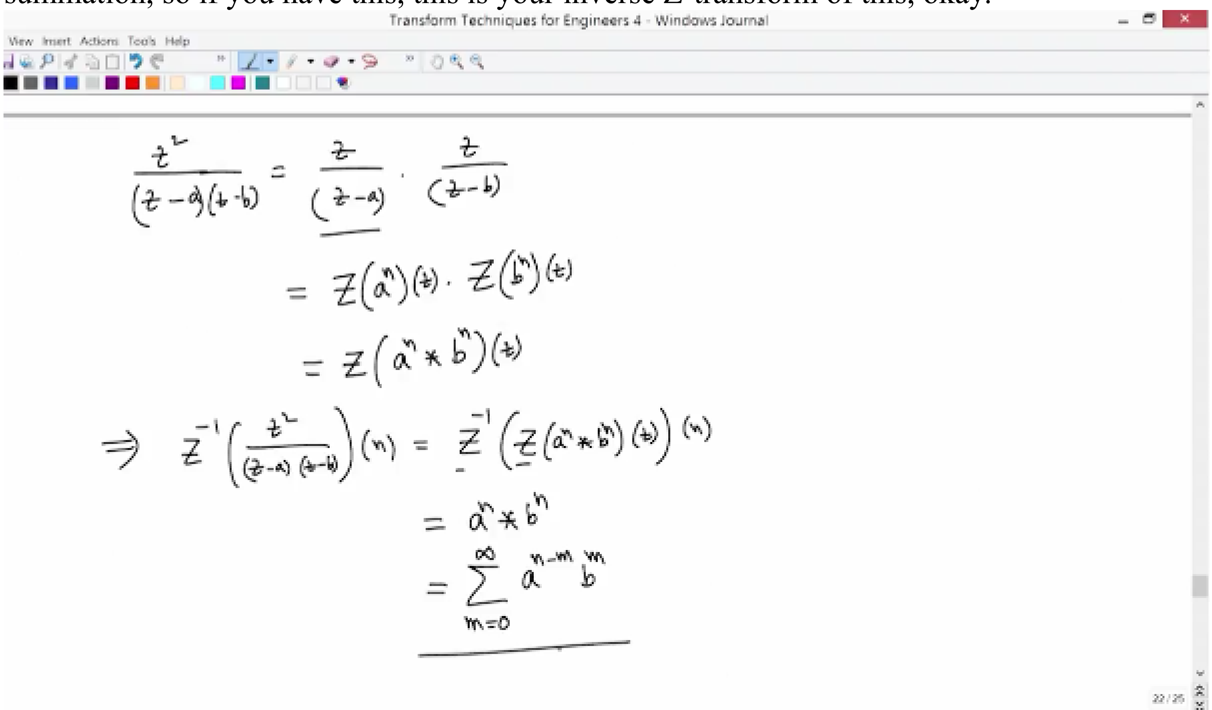
4. $Z^{-1}\left(\frac{z^2}{(z-a)(z-b)}\right)$.

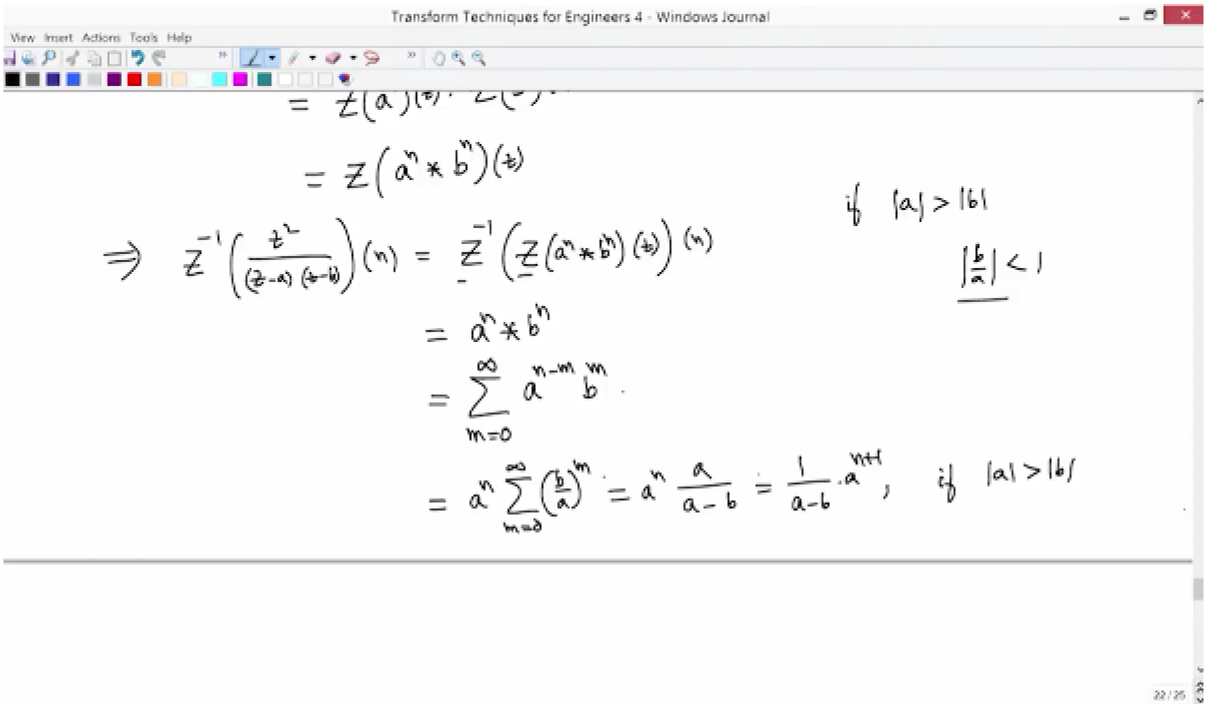
$$\frac{z^2}{(z-a)(z-b)} = \frac{z}{(z-a)} \cdot \frac{z}{(z-b)}$$

$$= Z\left(a^n\right)(z) \cdot Z\left(b^n\right)(z)$$

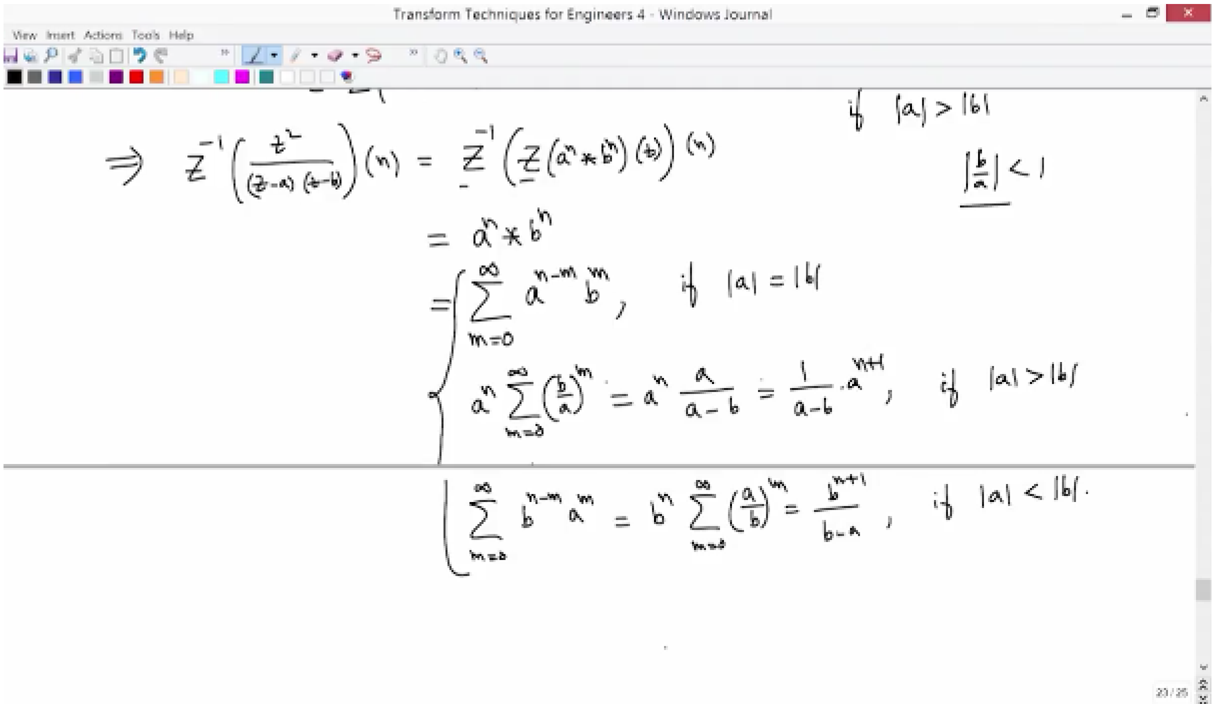
function of Z, and this is my given F(z) okay, this is F, this is my given F(z), so this is actually we know that this is actually same as Z transform of convolution between A convolving with B power N as a function of N which is a function of Z finally okay, so this we know by convolution theorem, so this implies so you take the both sides Z inverse, if you take both sides Z inverse, inverse both sides, so Z inverse of Z square/Z-A times Z-B, as a function of n is equal to Z inverse of Z(a power n x b n) as a function of Z and once you take this inverse this is becoming a function of N, so what is this one? So this Z Z goes you end up getting AN convolving with BN, there's a function of N, so this is summation, this is for should be function of N so you have running from 0 to infinity A power N-M, B power M that's it, that is your summation, so if you have this, this is your inverse Z-transform of this, okay.



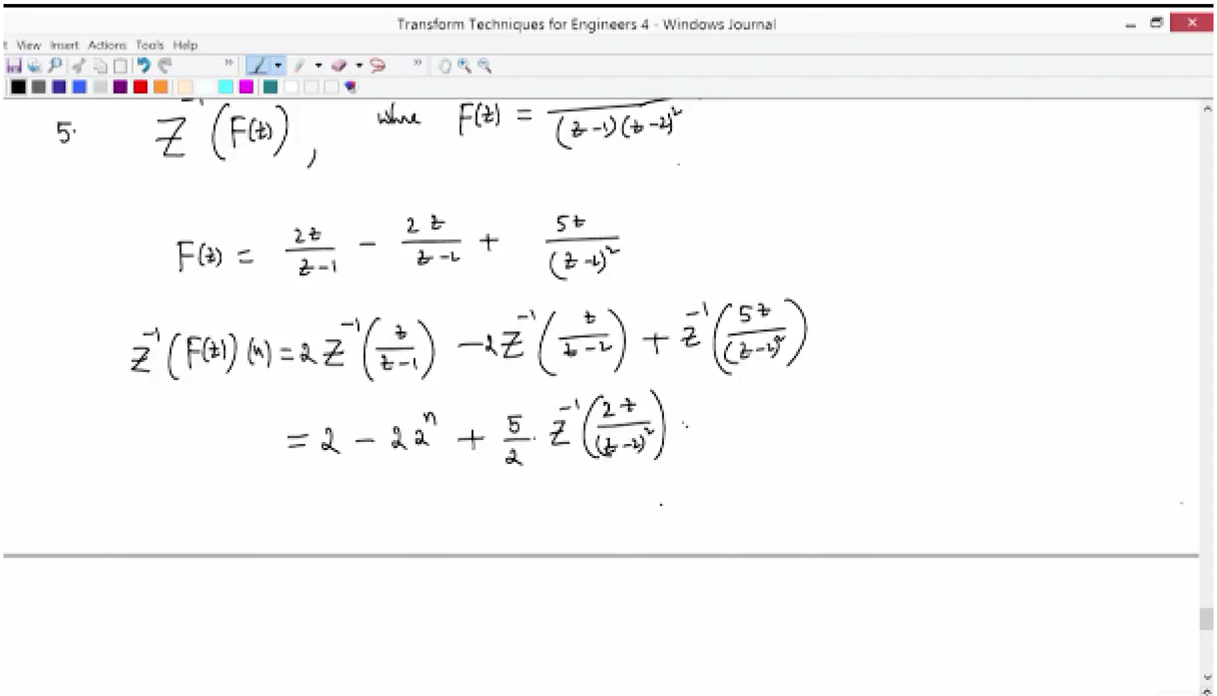
So you can simplify further by A power N bring it out this is from M is from 0 to infinity B/A power M provided so once you know A and B you know that mod A is same as mod B, if they are different, okay, so if they are same you cannot write like this, this is going to be infinity, so you cannot write A by, mod A is same as mod by B, mod A, mod B, you have to leave it like here, so if mod A is bigger than mod B then this will be mod B/mod A is less than 1, okay, so you choose, if you choose like this so that I can write this as A power N 1/1-B/A so you have A/A-B, so this is equal to 1/A-B times A power N+1, so this is true if mod A is bigger than mod B.



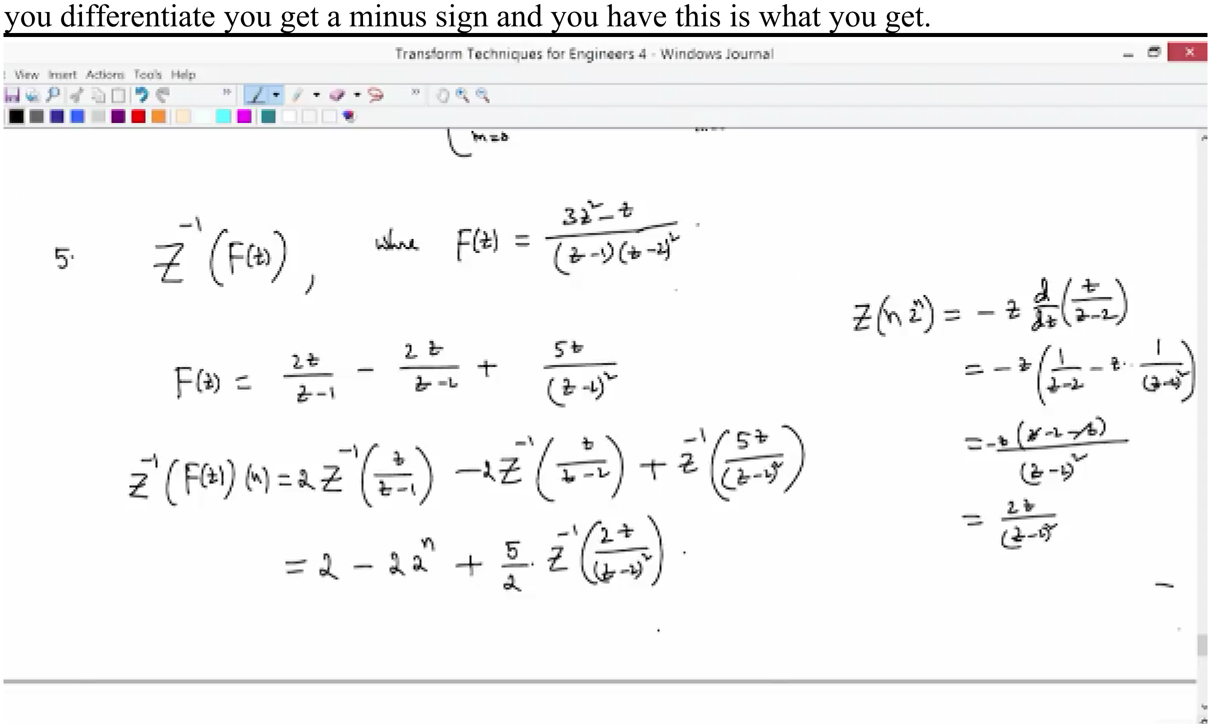
So what if you have mod A is, so this same thing we can write this is also equal to, we can also rewrite like instead of writing this way we can write, we can also write M is from 0 to infinity the convolution both sides is, both are same so you have B power N-M A power M, so this is also true and this will give me B power N sigma M is from 0 to infinity, A/B power M, so this is 1/1-A/B that is B-A/B into B power N so you have together B power N+1, so this is true if mod A is less than mod B, both the cases you can get this, okay, if mod A equal to this is, this is equal to sigma M is from 0 to infinity A power, so this is usually you can keep it if mod A is same as mod B, if mod A, so A, B can be complex numbers because these are a complex function so if A, B are this you leave it here as it is and if you have convolution you can choose if mod A is bigger than B you can modify this way, if you mod A is less than B you can modify this way, so this way you can get this Z inverse in all the cases, all the three cases, okay.



And then we look into some example, so what is the Laplace Z inversion of F(z), where F(z) is 3Z square – Z/Z-1 times Z-2 whole square, so use the partial fractions because the numerator is order is smaller, so if you write this F(z) as partial fractions I get 2Z/Z-1 - 2Z/Z-2 + 5/2 times 2Z/Z-2 whole square so you get something like this first initially, so because of this square Z-2 whole square, so this is your partial fractions, okay, 5Z this is how you can make a partial fractions and you write this so Z inverse of F(z) because of linear property you get as N so you have a Z inverse of 2Z/Z-1 – Z inverse of 2Z/Z-2 + Z inverse of 5Z/Z-2 whole square, so I have seen earlier that this is 2 comes out, you write 2 outside because we know Z/Z-1, what is that 1, so here also you can take 2 outside because of linearity so you see that Z inverse of this we know already that this is 2 into 1 power N so that is 1, -2 into here this is 2 power N, okay, so this second thing is this part is 2 power N, and what about this one? This is also again 5/2 times you can write Z inverse of 2Z/Z-2 whole square, so this one you can, you have seen already that

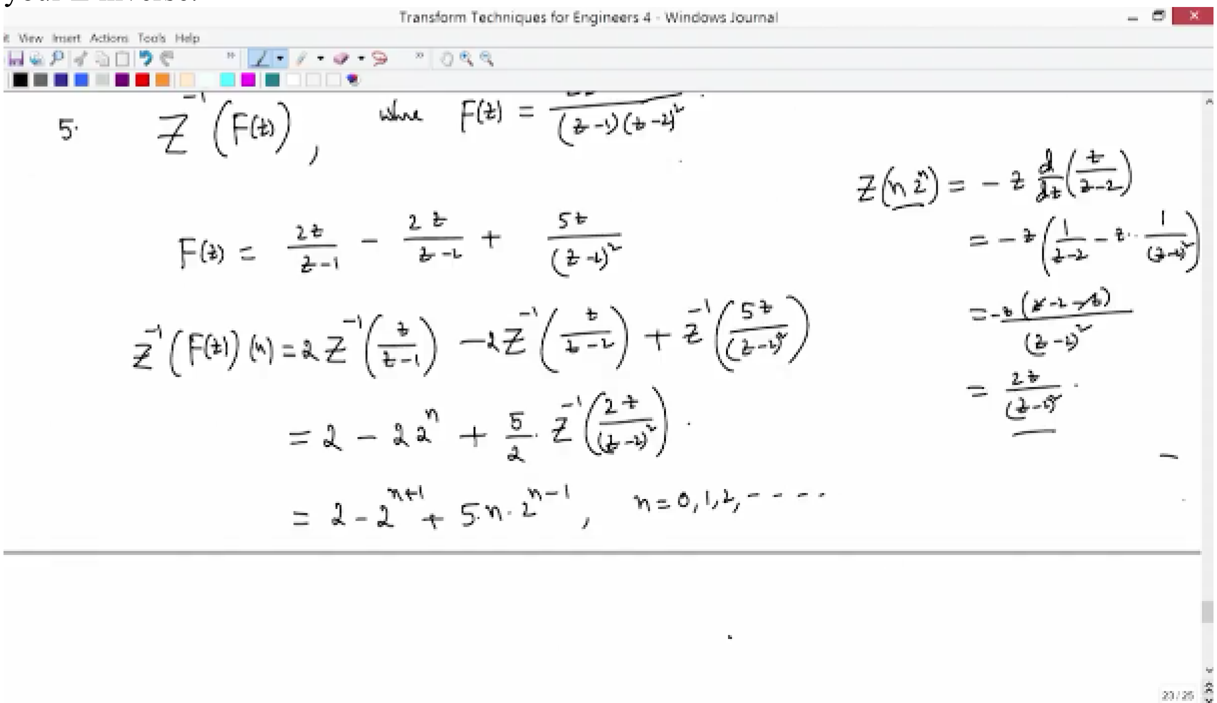


Z transform N into 2 power N is actually equal to, you have seen that this is this, if I use this one N into 2 power N as Z transform of this is, this is equal to -Z times D/DZ (Z transform of 2 power N) that is Z/Z-2, so if I do this you have –Z times 1/Z-2 + so, you have a minus here Z times, you have a, Z + Z the this will be –Z square and this I have 1/Z square, right, 1/Z-2 if you differentiate you get a minus sign and you have this is what you get.

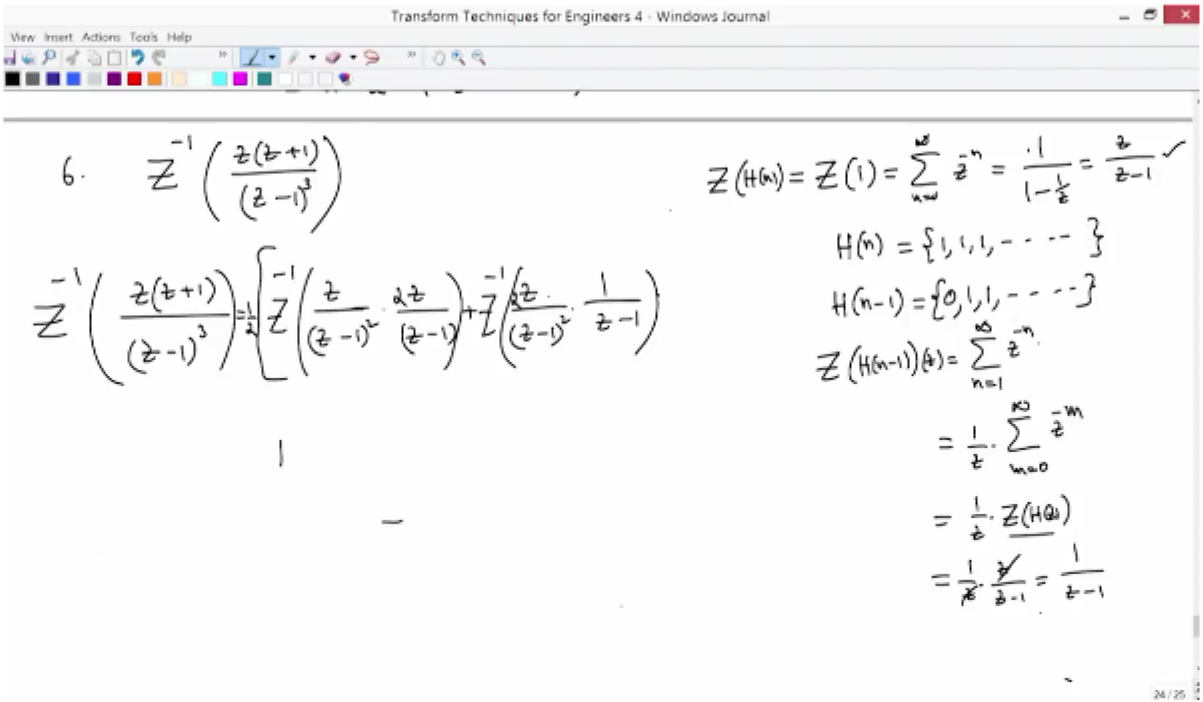


So if I use this you have Z-Z times, numerator is Z-2 and here –Z simple, -Z + Z goes here denominator is Z-2 whole square so you have 2Z/Z-2 whole square, so that's what we can use here, so we can see that 2-2 power N+1 + 5/2 times so you can write Z inverse of this is N into

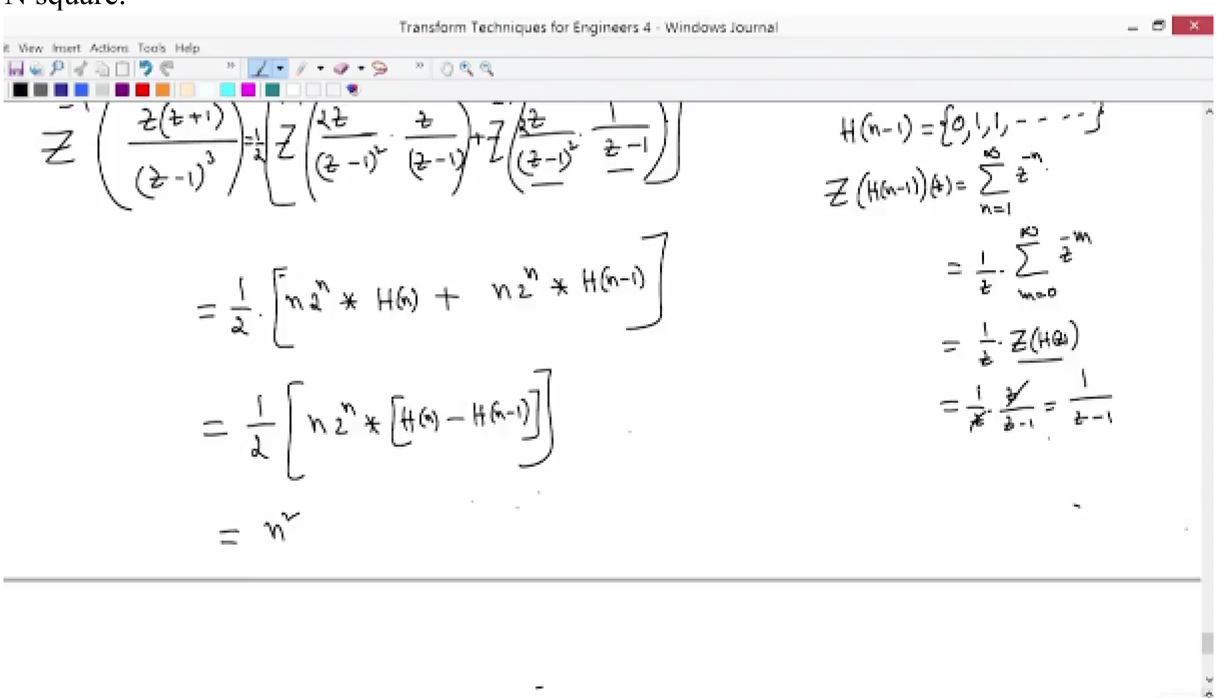
2 power N, so if you remove this you have N-1 for 5 times, so this is your validity, so this is your Z inverse.



And then let's look into some other examples like this you can go on doing Z inverse of, for example Z into Z+1/Z-1 power cube this also you can write, you can find this, so how do I write this? Z into Z+1/Z-1 cube I make a partial fractions here, first I write Z/Z-1 whole square into Z/Z-1 so because I know this one, okay, and also I know this part, okay, so these two I know this one and this one so I write it as first term, the second term I write it as, I write it as Z/Z-1 whole cube, so I write first as Z-2 and the one is Z-1, so 1/Z-1 if I, we already know that Z(1) if we actually calculate so you get 1/1-Z so you have that is constant right so you have Z power –N, N is from 0 to infinity this is Z/Z-1 so you have 1 over 1-1/Z so this is Z/Z-1, so if I choose this is the sequence of like H(n) okay, so this is H(n) is a heavyside function, your sequence is 1,1,1 so right from the beginning it's 1 1 1 constant.
So if I choose H(n-1) 1 onwards it is going to be 1 otherwise 0, so N = 0 its 0, 1, 1 and so on, so what is this one? Z of, so this implies Z of H(n) is actually equal to this H(n-1) if you calculate what happens, there's a function of Z this is N is from 0, it's from 1 to infinity Z power –N, so this if you calculate and you simply have 1/Z comes out and you end up getting N is from 1 to infinity, so if I calculate this is same as N-1, so N-1 if I take it as a new variable so as M so this is going to be as running from M is from 0 to infinity, so this is exactly 1/Z times Z transform of H(z), so this is 1/Z times Z transform H(z) is this, so I have Z/Z-1, so this gets cancelled, so 1/Z-1 is actually your Z transform of this heavyside function, but it's starting from 1 onwards okay, so if I know this, this one, this one I know, this one I know, so you can directly write your Z inverse, Z inverse of this is actually, Z inverse of this, okay, so Z inverse of both okay this is a product, okay, Z inverse of this + Z inverse of this function.
So you can make use of your, you can make use of convolution theorem here, so Z/Z-1 is 1 power N, so that is 1, and then Z/Z-1 whole square is 2Z/Z-1 whole square is N into 2 power, 2 that is N into 2 power N, okay, so you can write this as 1/2 here and you can put 2 here, you can

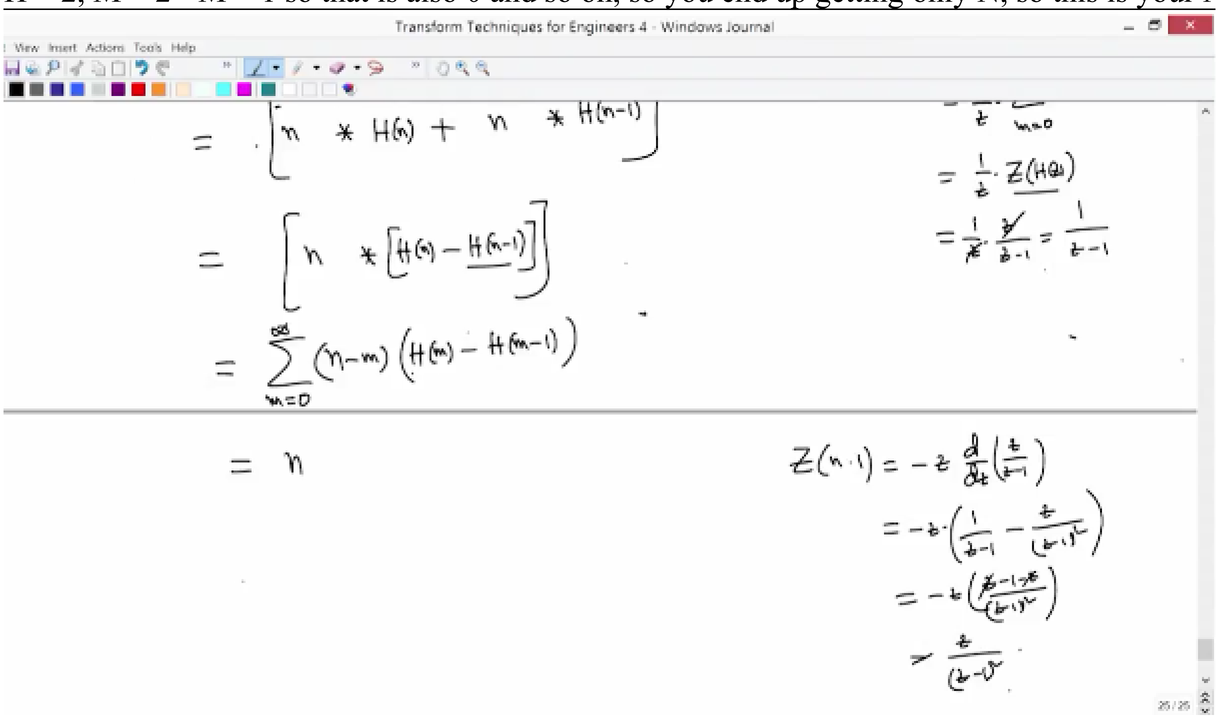


also make so that you can make 2 here, put a full whole bracket together, so if I use this so this is going to be 1/2 times, what is this one? This is convolution of this one so this is 2Z, sorry you have to put it here so this 2, so 2Z/Z-1 square is N into 2 power N convolving with its 1, this is H(n) rather, okay, this is H(n) because H(n) is Z/Z -1 + and here this is Z inverse of this product is this is also N into 2 power N convolving with, this is H(n-1) okay, so this is actually 1/N 1/2 times N into 2 power N convolving with H(n) – H(n-1), so this is what you have, so this you actually is to simplify and see this is actually you end up getting N square, so for each N you will get that only N square, so you sum it up and see that it's you'll see that it is end up getting N square.

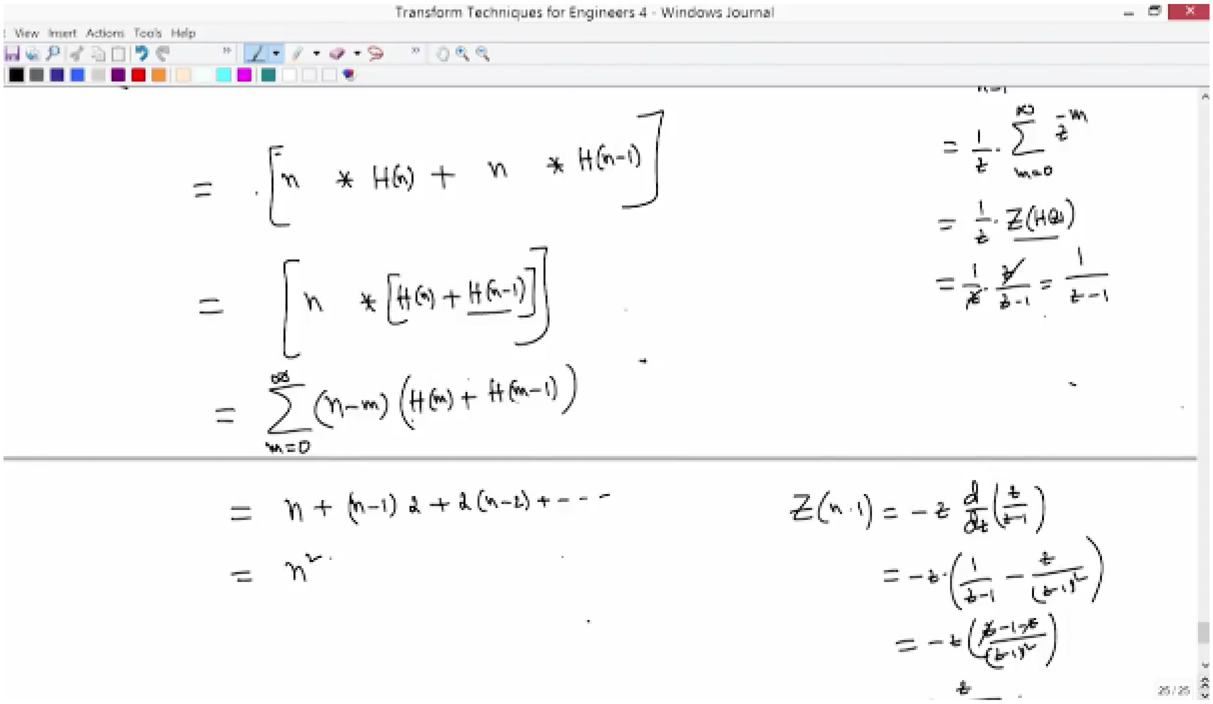


So how do I see this? So let me do this is 1/N into 2 power N, right, this is so, okay, no, no, this is N into 2 power Z is Z/Z-2 but I have only Z-1, so it's actually N into 1 power N, so this is 1 power N okay, so Z N into 1 power N that is 1, so this is simply N, N convolving with H(n) – H(n-1) so you have 1/2 times, so 2Z/Z-2 so you don't really require this 1/2 okay, so this 2 you need not right because Z/Z-1 square Z transform of N is actually equal to Z/Z-1 whole square, so how do I get this? This is equal to you can think of as 1 and this is same as –Z D/DZ of Z transform of 1 that is Z/Z-1, so this if you do this 1/Z-1 – Z/Z-1 whole square, so if you see this 1 -Z times Z-1 – Z/Z-1 whole square so this is minus this Z Z goes so you have finally Z/Z-1 whole square so that is simply N, so if I use that Z/Z-1 square is simply N, so you don't have 1/2 so you have this is actually N, if N = 0 this is negative and this is 1 and this is 1, so it's 1 okay, and if N = 2, so 2 multiplied with, 2 convolving with so H(2) – H(1), so if you actually do this convolution, if you do this convolution, so what is the convolution? N is from 0 to infinity N-M times H(m) – H(m-1).

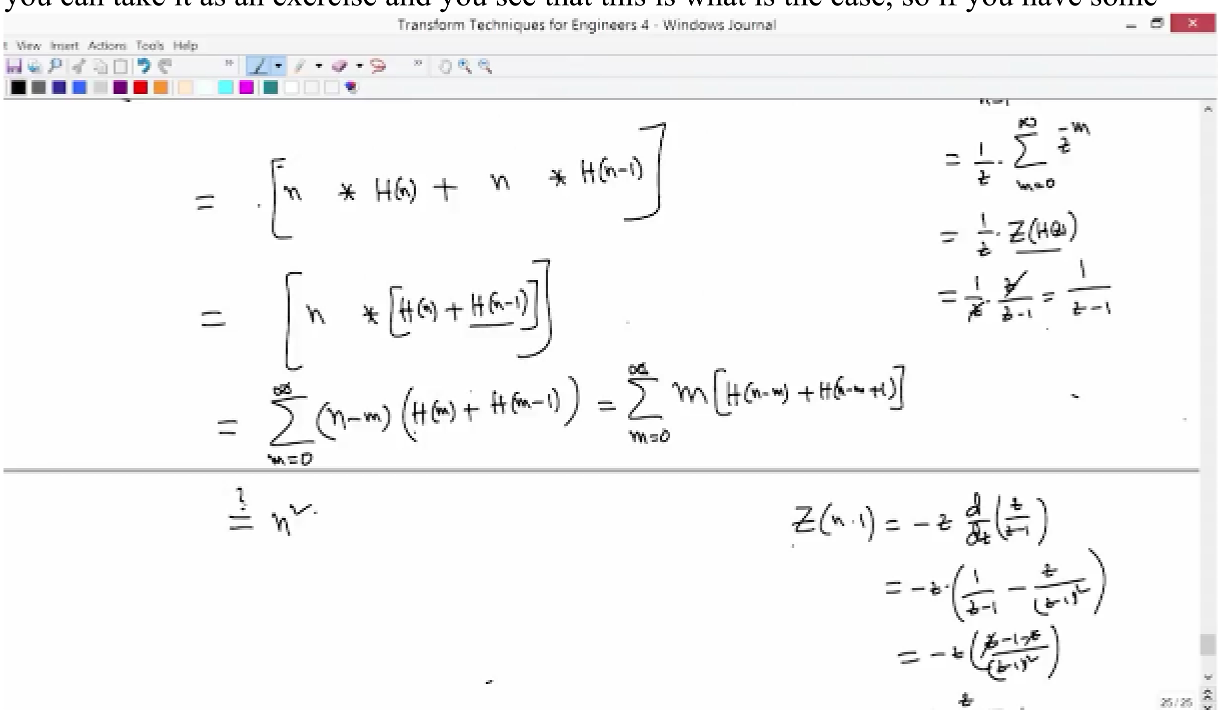
So M = 0, if you put M = 0, H(0) is 1 and this is only M, N right, so you have N, if you actually sum it up this is going to be, this is a convolution and if we convolve this is what it is the convolution, you have N+ M = 1, N-1 times and you have H(1) – H(0), H(1) – H(0) that is what is the value of H(0)? Is 1, H(1) is 1 so that is 0, this is simply 0, M = 1, H(1) – H(0) and then N-1, M = 1 is this, H(1) – H(0), so H(1) H(0) both the results are 1 1 that's 0, so it goes and then + H = 2, M = 2 - M = 1 so that is also 0 and so on, so you end up getting only N, so this is your N



so you should get only N, no, this is something is wrong, oh this is addition, right, so we have a addition here, so you have addition here, not subtraction, so this H(n) + H(n-1), so if you do this addition so you get N first H(m) so other one is N-1 times M = 1 this is 2 and again so 2 into N-2 and so on, so you end up getting N+, if you sum this up you end up getting, so it's you can see that this is going to be N square.



Let me convolve this is same as if, let's see if I can easily see that M is from 0 to infinity this is N and this is H(n-m) + H(n-m+1) so this is also true as a convolution, so if I do this way, so if I do this whether I see I get n square, so M = 0 that is 0, M = 1, H(n-1) + H(n), we'll just leave it here so this value you can verify that it is going to be N square this is the question we can see, you can take it as an exercise and you see that this is what is the case, so if you have some



rational functions like this you can make use of known Z transforms, Z inverse transforms and then convolution theorem you can apply and get this Laplace inversion, okay.

We will see some more examples and finally application of Z transform to the difference equations in the next video, so we'll see. Thank you very much.

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