NPTEL

NPTEL ONLINE COURSE

Transform Techniques for Engineers

Properties of Z-Transforms

With

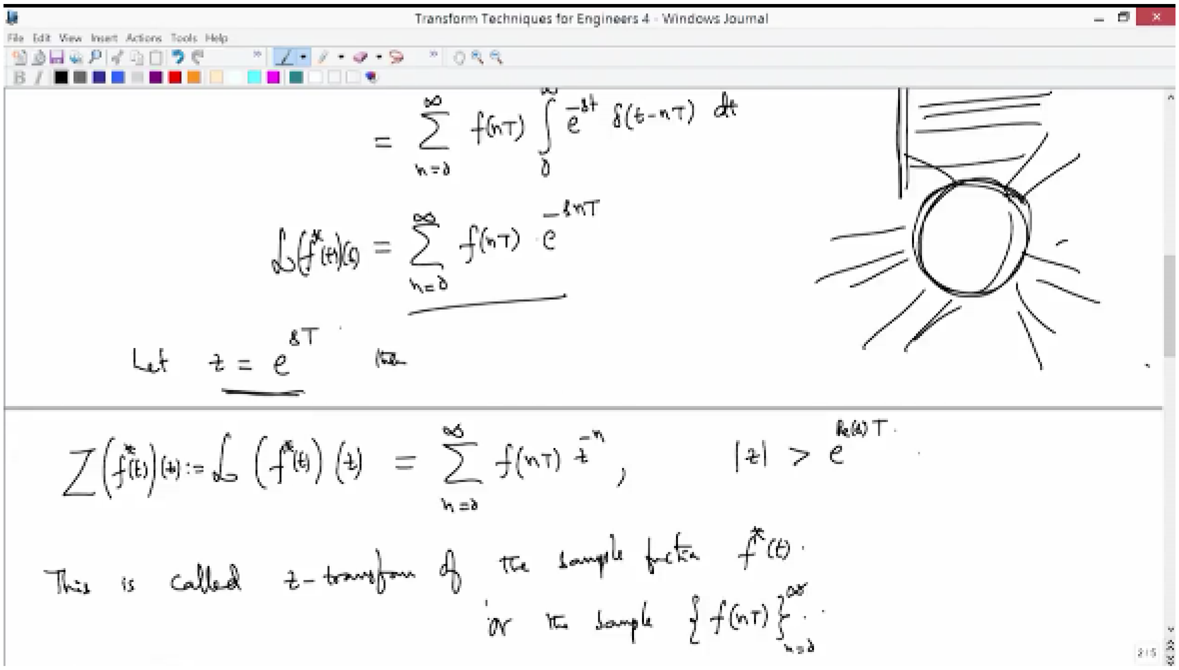
Dr. Srinivasa Rao Manam

Department of Mathematics

IIT Madras

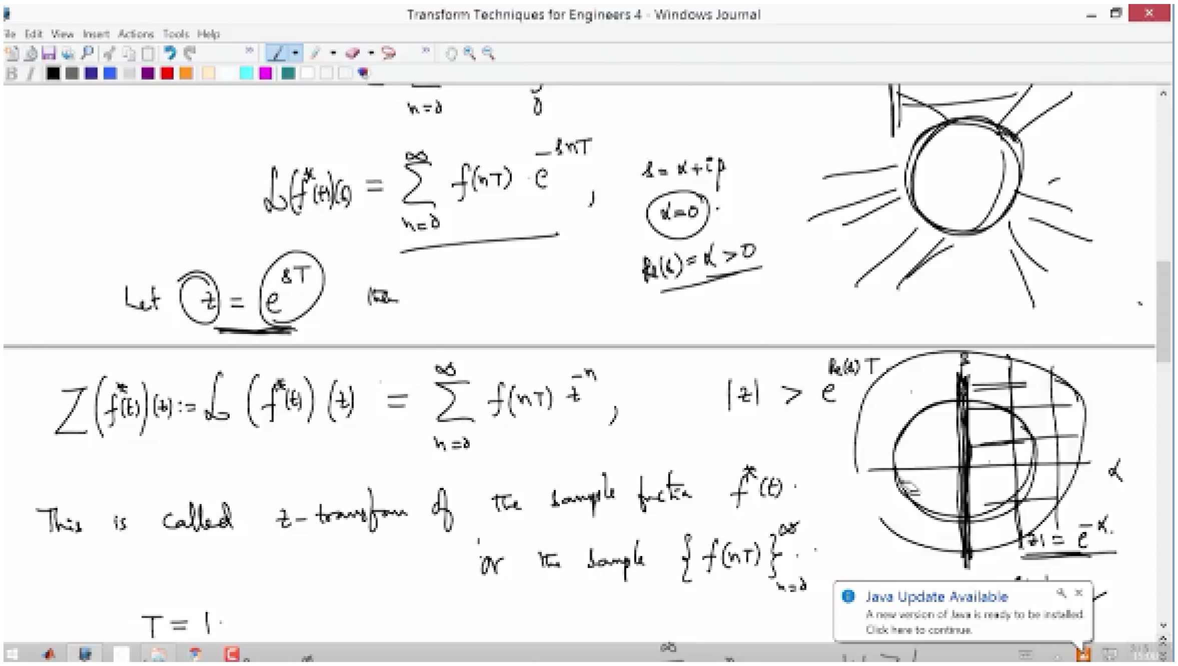
Welcome back, in the last video we have seen how to define a Z transform for a discrete signal from continuous signal you extract a sample, sample of discrete values and then you put it as a sample function so that you can apply, as a sample function is a continuous time variable T so that you can apply as the formally if you apply the Laplace transform, so from the Laplace transform formal definition of Laplace transform if we applied to this sample function you end up getting a series of signal values and exponential function.

So if you look at this, this is your Z transform that's how you define your Laplace transform of that function, so Laplace transform of the signal function F star(t) as a function of S is this, this

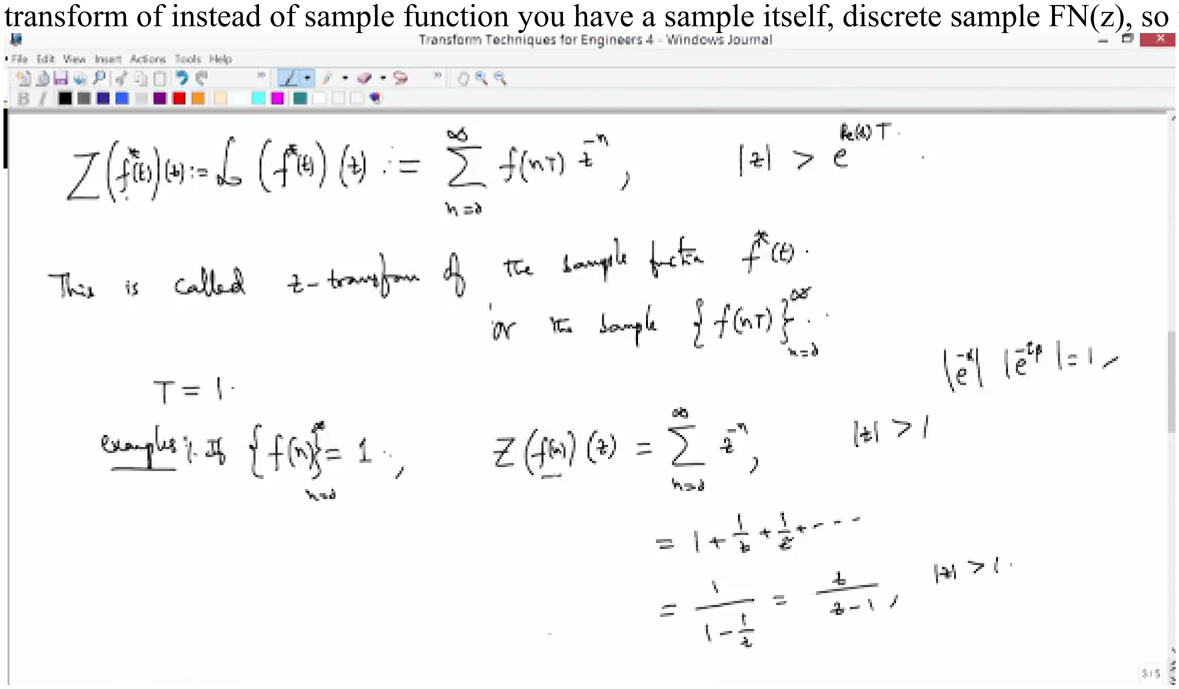


is what you have and instead of this form you substitute Z = E power ST, so once you have this you have applied the Laplace transform, so you have real part of S, so S you have, S is a complex variable, S = alpha + I beta for example, it's a complex plane in the complex plane this is what it is, so you have alpha is either positive or negative or 0, so if alpha = 0 for example if alpha = 0 and what you have is this is the line, this is the alpha, this is the beta line, and this is alpha + I beta is your S complex plane, if alpha = 0 that is this imaginary axis, if this imaginary axis if you see after transformation what you get is this summation, this summation E power, if I define Z as E power ST what you end up is F(nt) times series from 0 to infinity, Z is E power ST, E power ST is alpha is 0, E power I beta, so E power ST if you take T = 1, so if you take T = 1 what actually happens is E power –SN, so E power S is your Z so you have a Z power, E power S, E power S is your Z so you have Z power N, so if you substitute, if you substitute what happens, modulus of E power -S and T, T = 1, S = simply I beta and then you have N, so this is actually 1, so that means basically this is transformed in the Z variable plane, so this S plane, S complex plane to Z complex plane what you have is simply mod Z = 1, of course you have, this multiplied with whatever comes out, okay.

So basically this line is becoming mod Z = 1, if you have a line here this becomes E power T = 1, so you have a -alpha N, so -alpha power N, so this is modulus power N = 1, so this is a mod Z, Z is only without any N okay, so that is your Z so you have this becomes E power –alpha, so mod Z = -Z alpha E power -alpha is actually a circle again, so if you have, if your transform Laplace transform is valid from positive to infinity I mean alpha from positive, real part of S is positive if it's valid real part of S is alpha that is positive, if it is valid here that means it is analytic in this in this part of the plane, this imaginary axis is transformed to a circle because mod Z = 1, anything outside that means these lines is transformed to outside the circle, that is mod Z = E power -alpha okay, so this is how you define.



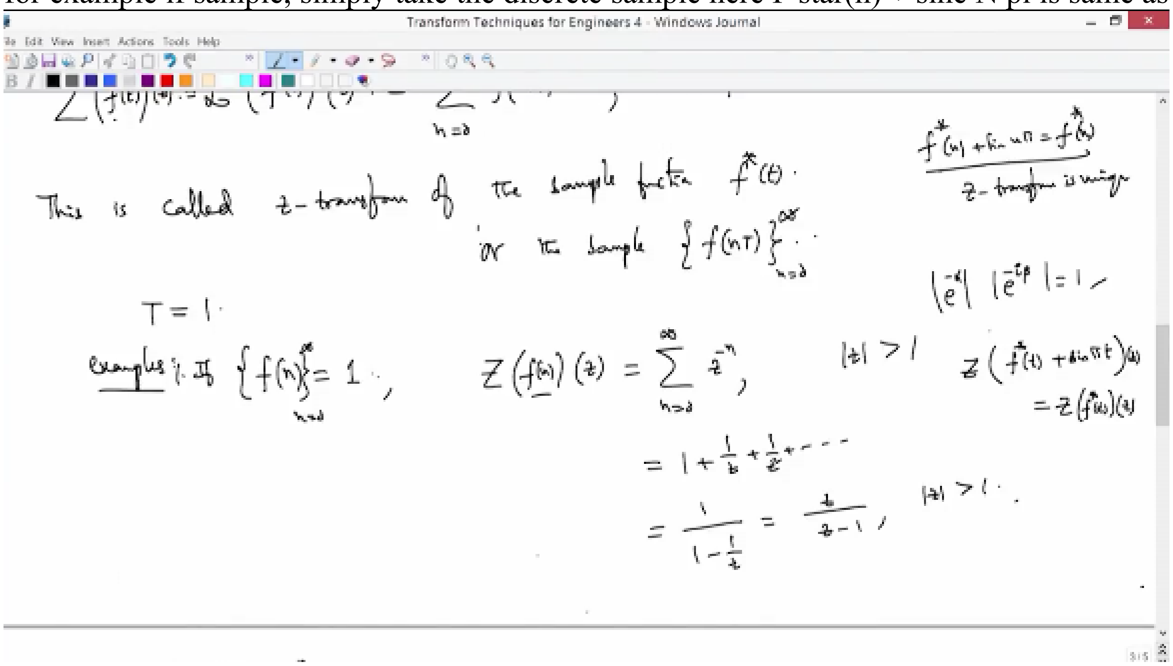
I'll just remove this, this is how you define your Laplace transform to the sample function so that you get your Z transform, so whatever Laplace transform you have I denote in this way Z transform of this sample function which is a function of Z, as a function of Z you have this as your definition, so this is your definition of your Z transform and also you can also write Z transform of instead of sample function you have a sample itself, discrete sample FN(z), so for



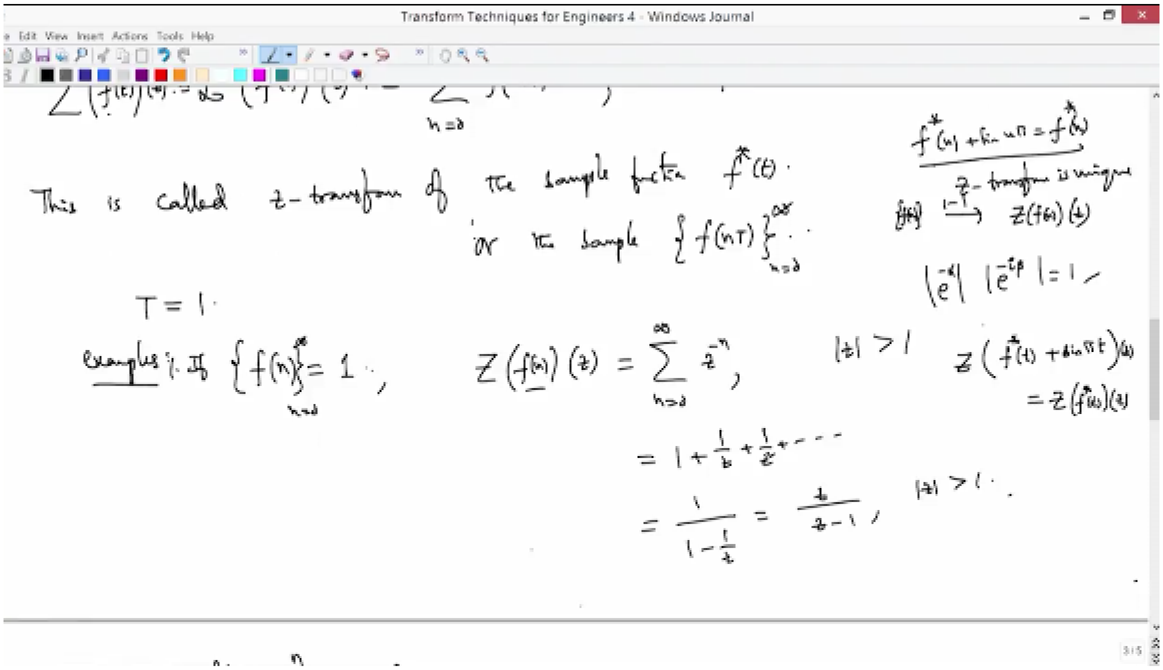
this if you take the sample clearly if you take, if you have a sample you have Z transform as a series, you have a complex valued function that is valid. For example here if I take as 1, Z transform is actually valid from mod Z is greater than 1, so specifically you have this domain, in this domain you have this Z transform as a complex valued function is that plane.

And then if I consider, if I instead of taking this discrete sample if I consider as a sample function for example, F star(t) and if I add sine pi T, sine pi T to it if I add to that, what is this,

okay, if I add to this F, instead of this Z transform of this, this is actually if you see this is actually same as Z transform of F star(t) as a function of Z, because of sine pi T you are just replacing instead of sine pi, if F(n) instead of T you are putting N’s, right, F(t) instead of T you're putting N so you have a sine pi N that is equal to 0, so instead of working with the sample functions you have, you work with only a sample data, so discrete data and then it is unique, okay, so you have 2 from these functions F star(t) + sine pi T, and F star(t) these are two different functions, sample functions that go to same Z transform, same Z transform, okay, so that means it's not one-to-one, okay, it's not one-to-one function, it's not one-to-one from the space of functions to Z transform in the Z variable.
So to, but if you consider only samples, discrete samples and you will get, okay, and sampling theorem will enables you to give you, to make this mapping as one-to-one, okay, that sampling theorem I will not state it but just don't worry about this you have sample function which, different sample function will take you to the same sample data, but here if you take the sample, for example if sample, simply take the discrete sample here F star(n) + sine N pi is same as F



star(n), so both are same, so if you work with the sample, discrete sample then your Z transform is unique, okay, Z transform is unique, so that means it's not one-to-one from this samples, discrete samples that means sequences of these FN’s to Z transforms, so these have a complex function Z of F(n) as a function of Z this is one-to-one mapping, okay.

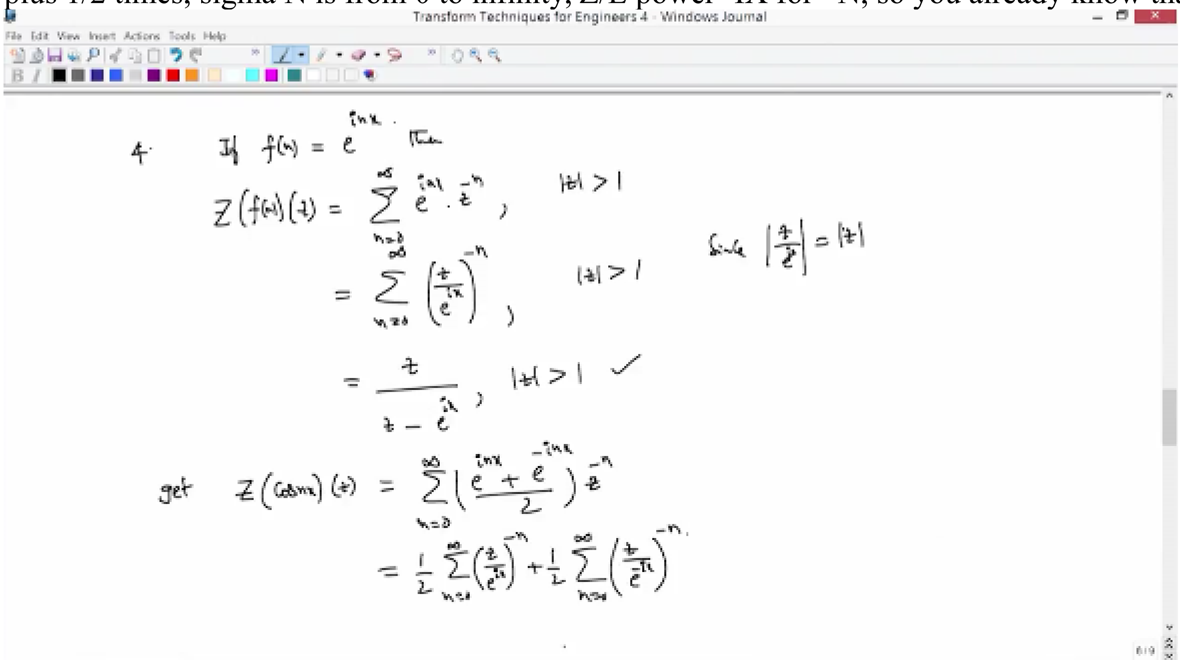


So this, how do we ensure this? This is actually sample function, sample thing, there is a sampling theorem that tells you that if your transform variable if you, for example for Fourier transform there is a sampling theorem that tells you that if your Fourier transform is, takes only finitely many values, finite range in the transform variable you can get your, you can recover your signal by only discrete values of the signal, you can recover from this sample, provided under certain conditions on the sample, for example the frequency of the sample so you have to choose some, that's a theorem it's called sampling theorem that tells you that, Fourier transform of a sample some function, for example F(t) if it’s, if once you take the Fourier transform or transform variable is xi, zeta, let's say xi, xi is your Fourier transform variable, and it’s range is only finite, then F(t) I can recover from, if I work with only a sample from this F(t) that means only discrete values I can choose so that I can just recover F(t) out of that, so that is the basic idea of signal processing and that's basic idea of these discrete transforms.

For example discrete Fourier transform, fast Fourier transform, all these are, first Fourier transform is simply algorithm to compute the discrete Fourier transform, the same way you have this any discrete transforms and you have this, you have to make use of this sampling theorem to get back the original signal which is a continuous variable, so you need not worry about that, so what do we have is if we work with only this a sample, discrete sample your transform is unique which is a function of Z which is unique, so that's what you need to understand, so we have calculated a few samples or what is it's Z transform, let's do some more examples and then its properties.

So let me do one more, so forth one, so if I use, if F(n) = E power INX, then what is your Z transform? Z of F(n)(z) which is equal to sigma N is from 0 to infinity E power INX times Z power –N, so this you can rewrite so you can think of writing this as sigma N is from 0 to infinity you have E power Z/E power IX power –N, okay, so this is equal to, so clearly mod Z is greater than 1 this is valid, mod Z is greater than 1, so because of this geometric series this is valid, this converges in the mod Z greater than 1, so you can easily see that because mod Z/E power IX = mod Z, so okay, so you have this, this you can sum it up this becomes 1/1 over -E power IX/Z, so that is you have Z/Z- this, Z/Z-E power IX, this is valid from mod Z is greater than 1, so this is your Z transform.

So from this you can easily see that Z transform of cos NX and sine NX, okay, so get Z transform of cos NX is a function of Z is simply real part of this, a real part of this you can get it as Z, if you rewrite, so if you write this as, this becomes simply real part, right, so this is a real part of Z/Z-, not real part, so how do I get this? So once you have this so you can get this one as you can apply this definition, instead of cos NX you write E power INX + E power –INX divided by 2 times Z power –N, so you have 2 sums, so sum of 1, sum is 1/2 times sigma N is from 0 to infinity Z/E power –IX so you have E power IX times power N –N, and then plus 1/2 times, sigma N is from 0 to infinity, Z/E power -IX for –N, so you already know that



this is, first this sum is that so that will give you 1/2 times Z/Z-E power IX this will be plus into Z/Z-E power –IX, again this is valid from mod Z is greater than 1.

So if you actually calculate together so 1/2 is common so you have Z square, so Z square -Z times E power IX + E power -X so we have 2 cos X, and then if you multiply together so you get 1, in the numerator you have Z times Z into, Z is common anyway, so you have Z comes out and you have Z –E power -IX + Z - E power IX, so you have 2Z this is Z goes and you have, this becomes again cos X, this is -2 cos X, so 2 2 goes and you end up getting Z square by, sorry Z into Z - cos X, Z into Z - cos X/Z square – 2Z cos X + 1, so this is again, this is your Z transform of cosine X.

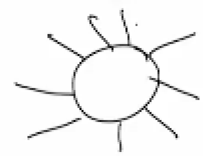
$$= \frac{\quad}{z - e^{ix}}, \quad |z| > 1$$

$$\text{get} \quad \underline{Z(\cos nx)}(z) = \sum_{n=0}^{\infty} \left(\frac{e^{inx} + e^{-inx}}{2} \right) z^{-n}$$

$$= \frac{1}{2} \sum_{n=0}^{\infty} \left(\frac{z}{e^{ix}} \right)^{-n} + \frac{1}{2} \sum_{n=0}^{\infty} \left(\frac{z}{e^{-ix}} \right)^{-n}$$

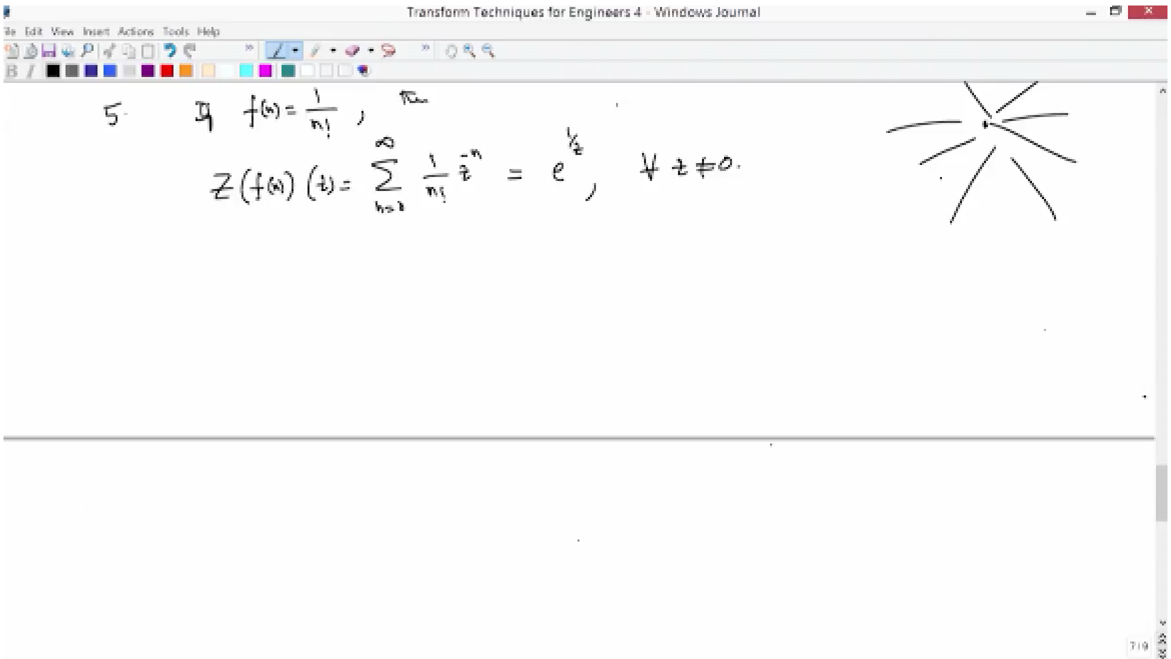
$$= \frac{1}{2} \cdot \frac{z}{z - e^{ix}} + \frac{1}{2} \cdot \frac{z}{z - e^{-ix}}; \quad |z| > 1$$

$$= \frac{z}{\cancel{2}} \frac{2z - \cancel{2} \cos x}{z^2 - 2z \cos x + 1} = \frac{z(z - \cos x)}{z^2 - 2z \cos x + 1}, \quad |z| > 1$$



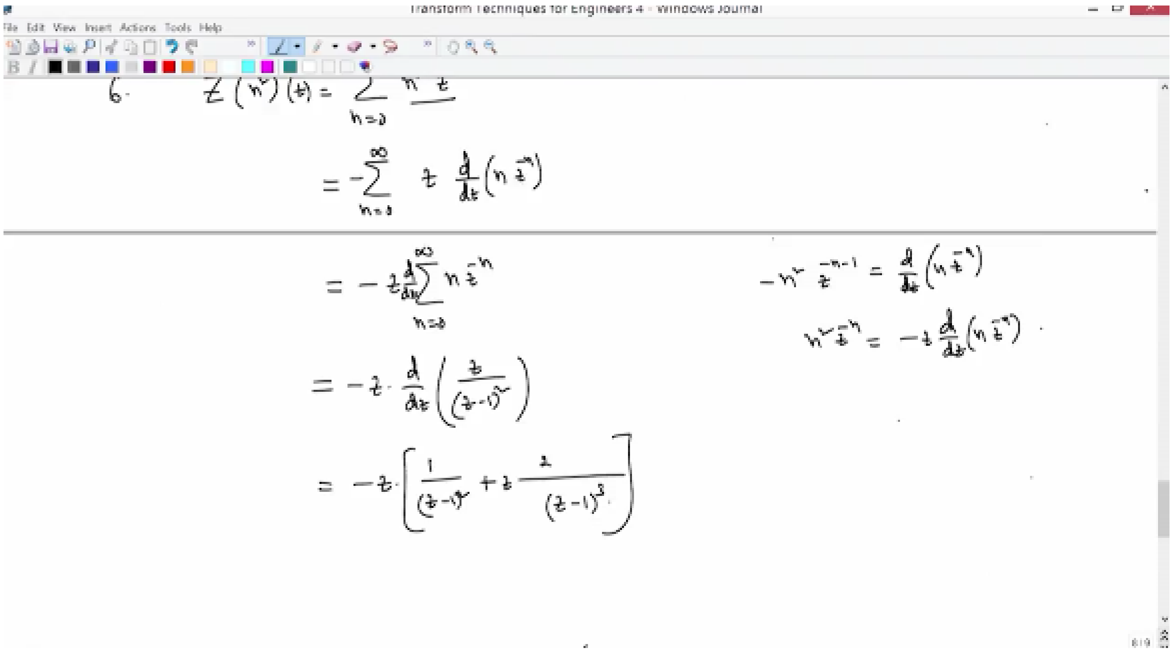
$$Z(\sin nx)(z) = \frac{z \sin x}{z^2 - 2z \cos x + 1}, \quad |z| > 1$$

In the same way you can get your, by just by writing -E power –INX divided by 2I you can get Z transform of sine NX as a function of Z you can just take it as an exercise you can get like this, by Z square -2Z cos X + 1, this is also valid from and outside this unit circle, okay, so this is one examples and we'll have one more, a simplest one we can get Z-transform is, if F(n) is having 1/N factorial, then Z transform of F(n) as a function of Z is simply N is from 0 to infinity, Z definition of Z transform F(n) is 1/N factorial times Z power –N, so this is simply E power 1/Z, so this is actually valid for every Z, for every Z right, for every Z this is valid, this is true, this is finite, okay, so you have convergence of this is true for all, so that means you have even at 0 at, except at 0, okay except at 0 you have this is valid, Z naught = 0 everywhere except that, Z is not equal to 0, that means you have this circular disk except that point 0 everywhere else you have this analyticity of this function, this is Z function, this is valid, the function is valid Z transform is valid everywhere except at that 0.



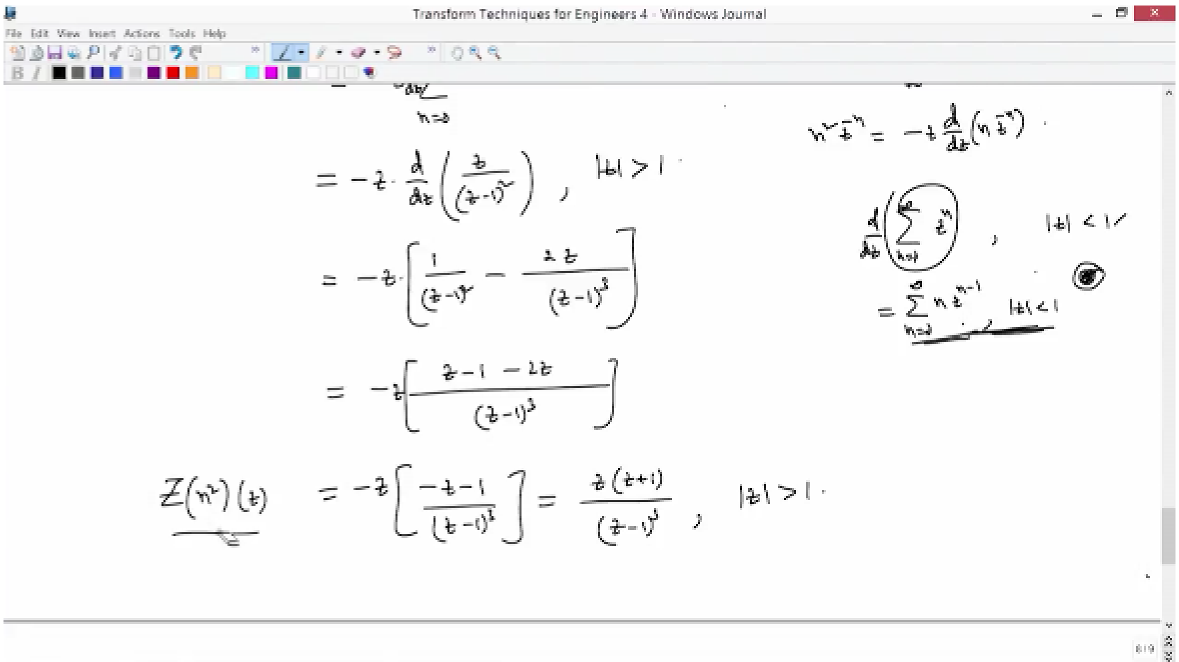
The same way you can get other examples, let me give some more examples that we may use later, Z transform of N square if you actually calculate, so this is as a function of Z this is sigma, N is from 0 to infinity, N square Z power -N and this one I can write, rewrite like N I can take it out, no, don't take N out, so what you have is N square, so how do I get this? So N square is this, so we can write sigma N is from 0 to infinity N times, N into Z power –N I can rewrite as N into, I rewrite as D/DZ (N into Z power –N), so what is this? This will give you minus of that, minus of this is actually will give you -N square right, what is this value? This value if you calculate this one this is actually equal to N into -N square Z power –N -1, right, so but you have only N square Z-N, so that is actually equal to so this means n square Z power -N this is actually equal to D/DZ (N into Z power –N) so you have N square Z power -N you can write it as –D/DZ(n into Z power-N) times and you can take this 1Z to the other side so you have Z here, so this you can replace with this -sign with Z, so this is –Z is anyway nothing to do with this sum, so you have this summation N is from 0 to infinity, so you can take this also outside D/DZ and this N into Z power –N, so you have already seen that this N into Z power -N is derivative D/DZ of this is Z transform of N.

Have we done this for N, Z transform of N we have seen that this is Z over Z/Z-1 whole square, so this is what we have done earlier, Z/Z-1 whole square if you do Z/Z-1 whole square, so for this if you differentiate with respect to Z you get 1/Z-1 whole square, and then you have, what you have is Z, if you differentiate this as Z here and you have a cube 2Z, 2Z/Z-1, Z you are differentiating cube and you have this actually 4 and this is a 0, minus minus plus and we have a 2Z-1 and so you have, this is what it is, so you have this gets cancelled with this so we have 3,



so this is what you have, so 2Z/Z-1 cube is the, this is the derivative so you have minus, so if you just calculate this Z-1 cube and you have Z-Z comes out in this, you have Z-1 + 2Z, so we have 3Z-1, sorry –Z comes when you differentiate you will get here – sign, so here you'll get Z-1 -2Z, so you have a minus that is going to be –Z, so Z-Z that is inside you have –Z -1/Z-1 cube, so minus minus cancel so what you get is Z into Z +1/Z -1 cube, so this is actually your Z transform of N square is a function of Z.

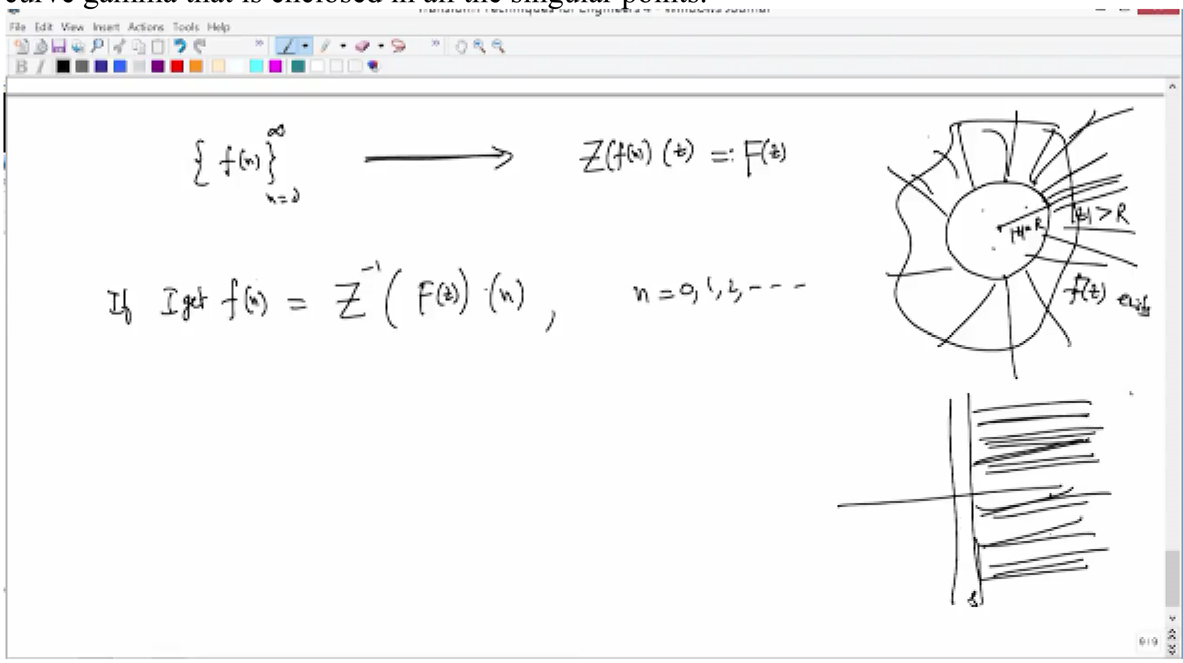
So this is again, where is this valid? A validity is again for, what we use is the, this is valid for mod Z is greater than 1, okay, have you seen Z transform(n) this is also valid here, okay, the same thing you are using as a derivative because uniformly convergent series so you have here this is also valid for mod Z is greater than 1 that means what I'm saying is if you have this series for example Z power N this is valid, let us say mod Z is less than 1, so there's derivative of this, if this is uniformly convergent which is so within this any closed interval, within this any closed disk inside, inside this unit disk if we take any closed disk or which you have this is uniformly, so I can do this term by term differentiation that comes as N into Z power N-1 so this is also valid from mod Z is less than 1, so that's what exactly I'm using, if it is greater than 1 it's valid this Z transform(n) then the same domain this derivative of that which is for Z transform(n square).



So that is one, and let's use some more examples which are useful for you to do applications, ready to solve difference equations later on. So let's not do some more, these are your some of the very elementary Z transforms for these, there are some of the simple elementary sample functions as a function of N, some sequences for which you have these Z transforms, let's do some properties of these Z transforms and then move on to find the inverse Z transforms, and then we'll look into the applications of this Z transforms, that is to solve the difference equations.

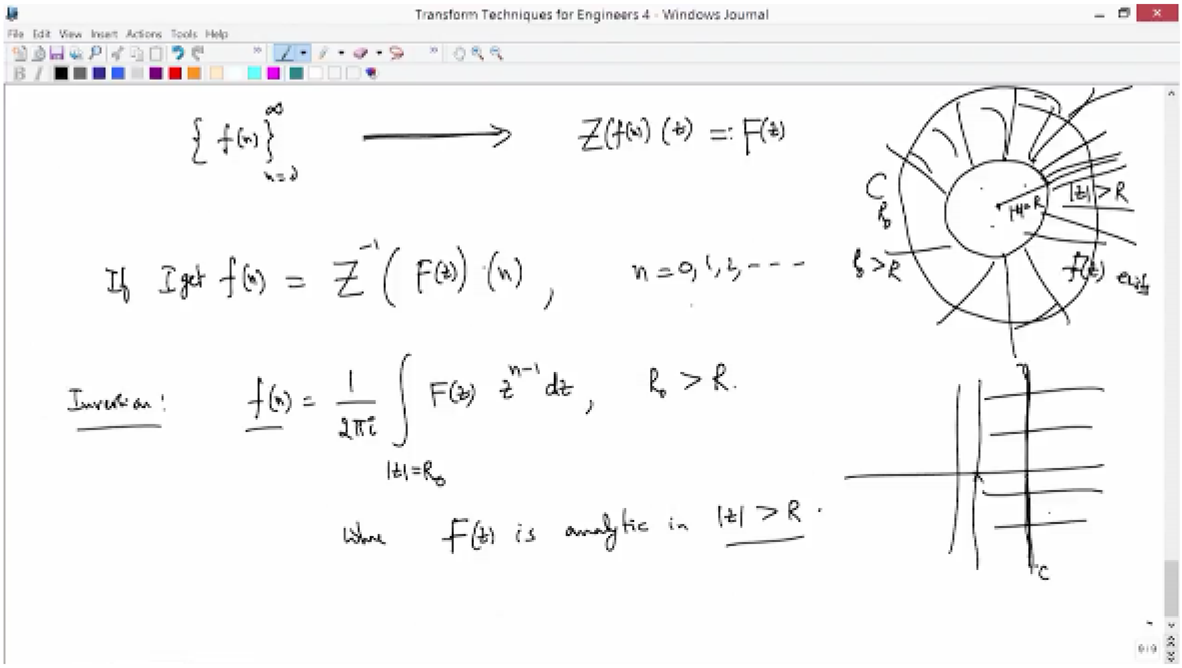
So let's look into the properties, properties of Z transform so as usual before I do this what is its, so far we calculated, given F(n) as a sequence from N is from 0 to, this is a sample I have Z transforms, Z transform is as a function of Z of F(n) as a function of Z, as a function of Z I have, so if I can call this as capital F because this is F, I write capital F(z) okay, so if I define this as a capital F(z) as the Z transform of F(n) the sequence, what do you have is this you give this one, I have this Z transform, okay.

So what is its inverse? Inversion is actually you have a 1 1, because of the 1 1 mapping with this thing you have this, because it's a Laplace transform it's a 1 1 mapping, and you have a inversion is F(n), so if you give this F(z), so the inversion is F(n), so you look for F(n) if you want, you want to, if you get this if I get F(n) that is Z inverse of any this F(z), Z transform of which is Z transform of F(n) okay, so how do I get this? This is as a function of N a sequence, okay, N is from, N belongs to, N is from 0, 1, 2, 3 onwards, so how do I get this F(n)? So if I write this as, so once this what I do is you know that you have a Z transform F(z) so that is validity is some let us say some Z, Z = 1, Z = R for example, you have within this, outside this you have let us say this is valid F(z) is valid, Z transform exists in this domain, okay, in this domain outside mod Z, mod Z is greater than R in this you have F(z) exists, if you look for that inversion I simply consider any closed curve its inside, which is inside, that encloses all your singular points of this F(z) clearly wherever this F(z) exist this is analytic, so that means this is analytic here, because this domain is nothing but your S variable on the right hand side, okay, wherever S exists from here that line beyond this right hand, right half plane is exactly that is where F is analytic, so that is mapped to this outside of this circular, outside this disk mod Z = R, okay, so mod Z are equal to outside this disk, that is mod Z is greater than R you have this function F(z) is analytic so all your singularities are inside only here, so if you consider any curve gamma that is enclosed in all the singular points.



So I will define F(n) as 1 by, so how did I choose this? This like in the complex plane a Laplace inversion is you choose any C that is where you have F(z) this Laplace transform is analytic, Laplace transform is analytic let us say from here onwards, then you have to consider your C outside this, C is bigger than this point so somewhere here, any C it will work, the same way I can choose this if you really this one, you consider this as big circle that encloses, let us say this is a circle of radius, let us say big R naught, R naught is where R naught is bigger than R, okay, so that is actually this line mapped to that circle, so I will just write it as directly from the Laplace inversion you see that C-I infinity to C+I infinity that is actually a circle here, so I write this as a circle mod Z = R naught which is any circle R naught, okay, which encloses all your singular points.

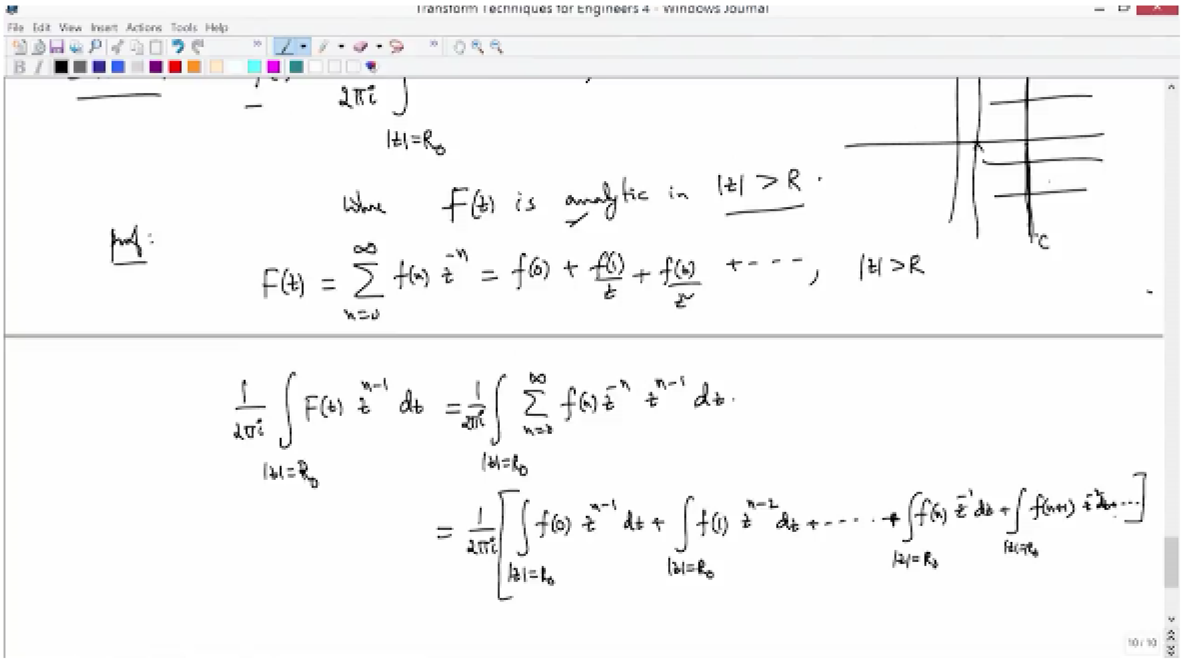
And then you have F(z), okay, and E power ST so that becomes, so this let me, I will not write exactly it's not exactly coming from Laplace transform, Laplace inversion so this will become that part E power ST becomes E power ST, E power -ST becomes in your definition of Z transform, if you actually see the definition of your Z transform, E power -ST becomes Z power –N, okay, and here E power SNT you will get a sentence you have Z power N, you have Z power N - 1 you will get, so the discrete version the Z transform is so inversion is, inversion looks like this FN is actually is this one where R naught is bigger than R, R is mod Z is that is where F(z) is analytic in the mod Z where F(z) is analytic the Z transform is analytic in mod Z is greater than R, okay, so say any closed curve, any circle if you consider that is what you will



get as your inversion, this is you can easily see this by complex function theory just by writing, how do I do this? You can write this as F(z), if you write F(z) which you know that this is a transform of F(n) so that is you have a F(n) Z transform of F(n), N is from 0 to infinity, so this is actually F(0) + F1(z), F1/Z and F2(Z square) Z-2 and so on, okay, by Z square you can put it if you don't like this, okay, so this is what is valid from mod Z is greater than R, that is what is the assumption, okay, once you have this Z transform it's inversion why is this F(n) we'll just prove it, okay, so I'm just trying to give you a kind of proof why this inversion is this one, so inverse function is basically F(n), right, so why is this inverse? So as you see F(0) is a complex valued function its inverse is F(n), so I need to find what is this F(n), F(n) I am claiming this to be this integral, this integral much little similar to Laplace inversion integral over that contour the contour here becomes the circle and F(z) is okay, and this part E power ST and DS, E power ST DS becomes this one.

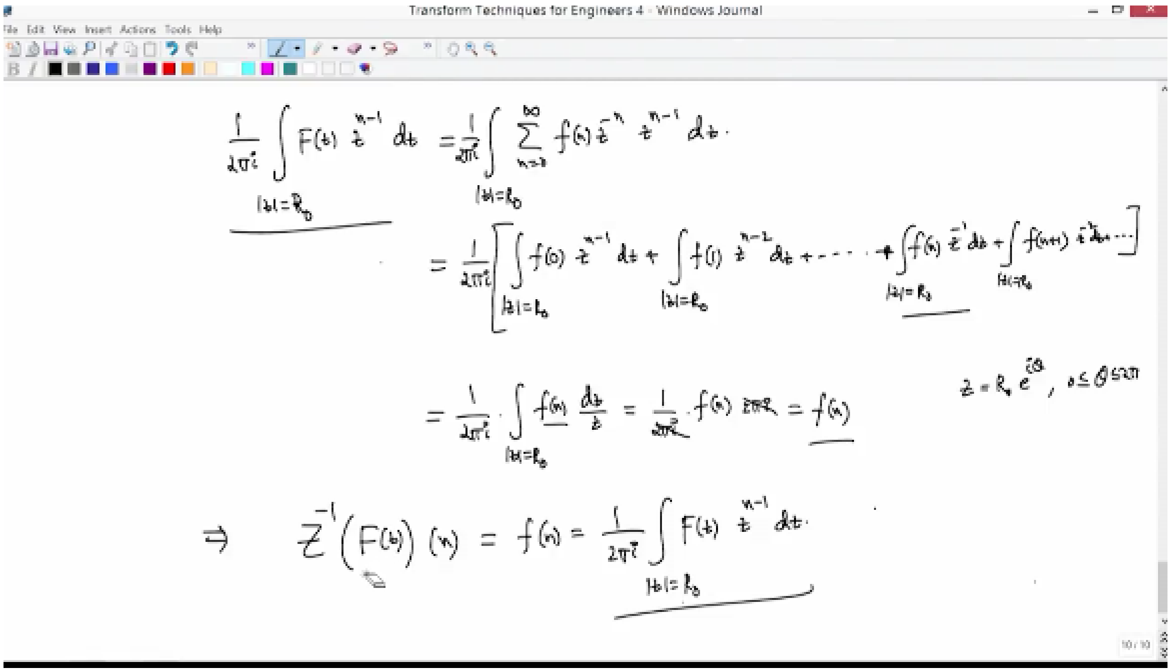
So that is E power S is, E power ST becomes Z power N and DS becomes so here DZ/Z so that is what is the you see, so how we prove this? Just by writing this one, so because F(z) is Z transform which is valid here and you try to multiply both sides 2 pi I times F(z) you integrate over mod Z = Z naught R naught and Z power N-1 DZ you multiply both sides whatever is required, so this is your F of, let us see what this is? This is equal to and the right hand side also you can do term by term 1/2 pi I times mod Z = R naught and you have F(n), this is sigma, N is from 0 to infinity, F(n) Z power -N into Z power into this is your F(z) into Z power N-1 DZ.

So this will give you actually what you have is 2 pi I times mod Z = R naught and you sum it up, so initially you have let me write this as sum of the integrals you have start with F(n) = F(0), Z power N-1 DZ + this is the closed, okay, so anyway mod Z = R naught that is the circle, circle of, this is F(1) times Z power N-2 DZ and so on, so you end up getting somewhere mod Z = R naught F(n) times DZ/Z, Z power -1DZ and so on, you'll go on getting mod Z = R naught



F(n+1) and you have Z power -2 DZ and so on, so this is what you have inside, okay, so assume that because this is a finite thing so this you know that F(z) is convergent, uniformly convergent so you can take this integration as well.

So the same thing same uniform convergence of this series, because this is validity mod Z is greater than 1, this is open set, any closed set inside this mod Z is greater than R is uniformly convergent, so you can do these term by term integration so you can take this inside, so you get this sum, and you see that if you calculate and this mod Z = R, so Z = R naught times E power I theta, theta is between 0 to 2 pi so this is your parameterization for this mod Z = R naught, so if you substitute this into each of these sum, these integrals only this will be nonzero, all other things are 0, so all these things are 0 except this one, so you have 1/2 pi I times integral, this is over C, this is only you have F(n) DZ/Z mod Z = R naught so this is equal to, so other things are 0, so you end up getting 2 pi I times so this is going to be F(n) is common this is nothing to do with Z, so F(n) is constant, DZ/Z value is simply 2 pi I, so that gets cancelled so you get F(n), so you see that this integral is nothing but F(n), so this implies Z inverse of F(n) capital F(n), sorry capital F(z) as a function of N is F(n) which we know that this is F(n) that is nothing but now I can get it from this result as 1/2 pi I times mod Z = R naught, capital F(z), Z power N-1 DZ.



$$\frac{1}{2\pi i}\oint_{|z|=R_0} F(z)\, z^{n-1}\, dz = \frac{1}{2\pi i}\oint_{|z|=R_0} \sum_{n=0}^{\infty} f(n)\, z^{-n}\, z^{n-1}\, dz$$

$$= \frac{1}{2\pi i}\left[\oint_{|z|=R_0} f(0)\, z^{n-1}\, dz + \oint_{|z|=R_0} f(1)\, z^{n-2}\, dz + \cdots + \underline{\oint_{|z|=R_0} f(n)\, z^{-1}\, dz} + \oint_{|z|=R_0} f(n+1)\, z^{-2}\, dz \cdots\right]$$

$$= \frac{1}{2\pi i}\oint_{|z|=R_0} f(n)\,\frac{dz}{z} = \frac{1}{2\pi i}\, f(n)\, 2\pi i = f(n) \qquad z = R_0 e^{i\theta},\ 0 \le \theta \le 2\pi$$

$$\Rightarrow \quad Z^{-1}\big(F(z)\big)(n) = f(n) = \frac{1}{2\pi i}\oint_{|z|=R_0} F(z)\, z^{n-1}\, dz.$$

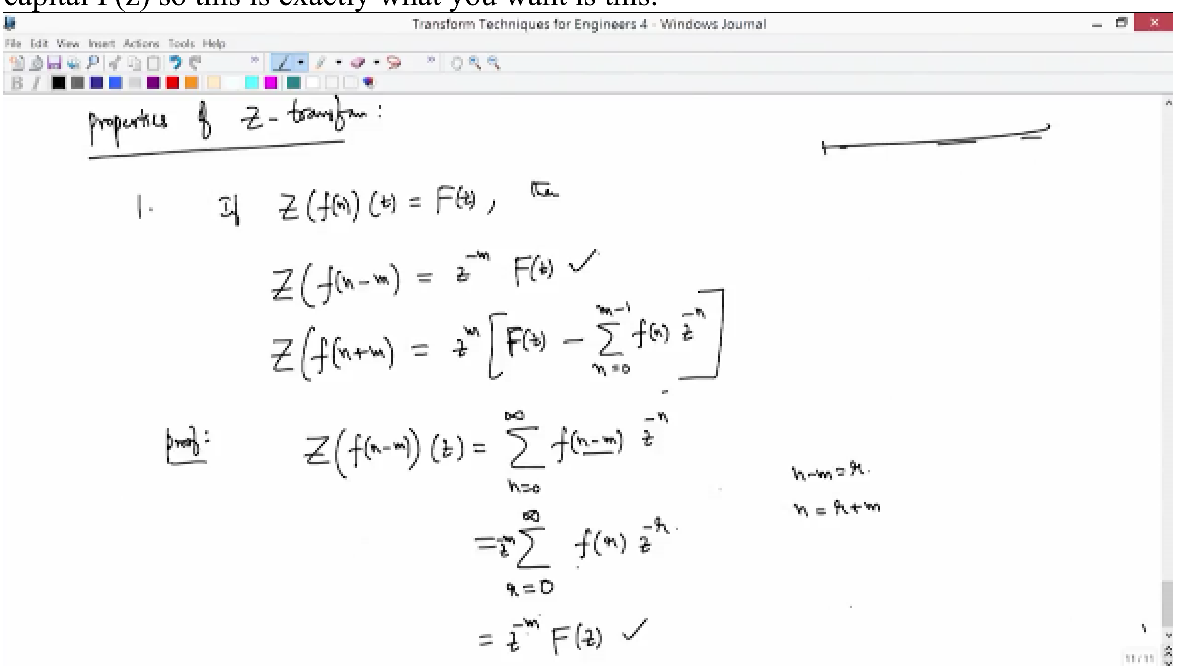
So for elementary functions you can recognize what is the inverse, and if your function F of, capital F(z) if it is a complicated function but it is you know that is, the domain of analyticity you can use this, you can consider, you can consider all its singular points and out, so that you choose some circle that encloses all these singular points of F(z) and you make this integral and if we evaluate this that is what is your F(n), so that is your inverse, okay, so this is how you can get the inverse, so let us make some, let's do some properties of this transforms, Z transforms, so you have this inversion you have, now we have defined Z transform and it's inverse transform, okay, so Z transform of a sample which is F(n), frequency F(n) which you get capital F(z) as your Z transform, and it's inverse transform is, if you start with F(n), if you already know F(n) then you know what is a inverse transform of capital F(z) that is F(n) itself okay, that's trivial, but if you don't know exactly what is your Z transform, as a capital F(z) then this formula will help you to evaluate that F(n), so that is your inverse, okay.

So let's see the properties, properties of Z transform so we're just going analogous to the Laplace transform, that's where we have defined Laplace transform and it's inverse transform that's what I have done for the Z transform, so there once we define the Laplace transform and it's inverse transform we have seen the properties of Z transform, we have derived some Laplace transform for elementary functions, and then you see it's inverse function, inversion is clear for those functions, those Z transforms or those Laplace transform inversion is trivial, okay, if you know already what is the, you start with the function for which if we calculate the Laplace transform and whatever you calculated is your function of, for which if you want inversion that is simply whatever you started with is your Laplace transform.

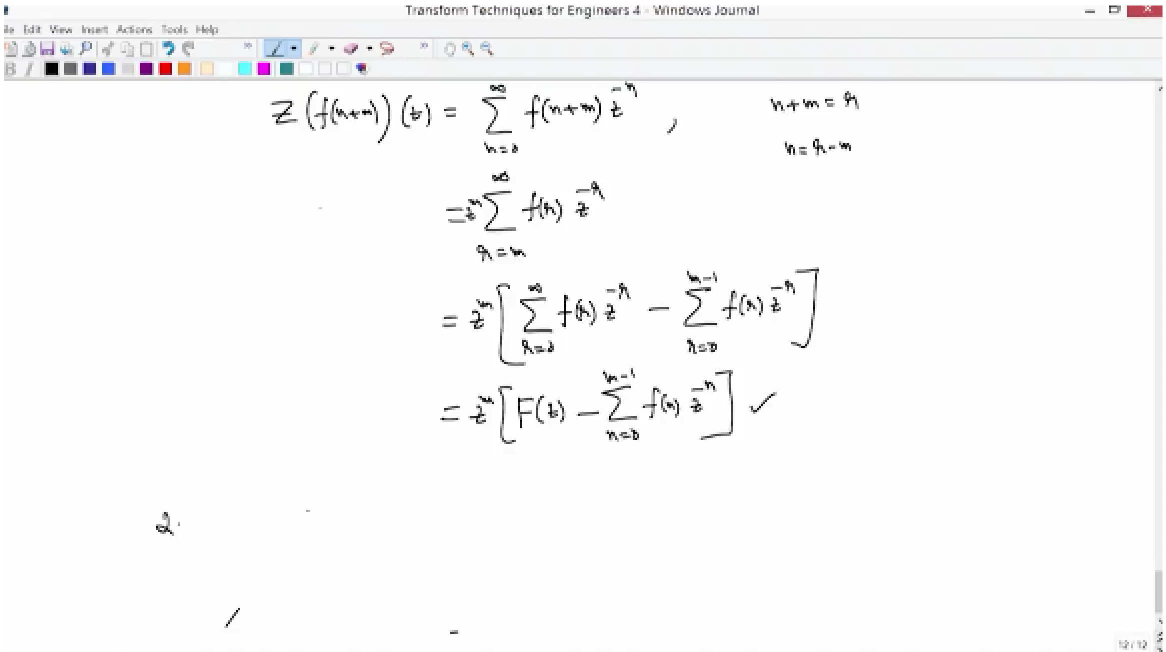
The same way here if you start with the sequence you end up getting some complex valued function, analytic function that function if it is given you can immediately say that it's a Laplace inversion is simply the sequence function F(n) of the sample function, okay.

So properties of Z transform is all, as usual like in the Laplace transform case, the linearity is valid so that will not do the linear property, so let's do some non-trivial properties that are not a simple property that not straightforward, okay, and that are useful later on, so let me do as one, first properties is if Z transform of F(n) as a function of Z is capital F(z) let us say, okay, then Z

transform of this translation F(n-m) = Z power -M times capital F(z) + this sigma, R is from –m to -1, F(r) times Z power –R, so this is how you get but we know that your signal, you don't have the negative values, it's valid only from 0, 1, 2, 3, infinity, so this will become 0 because F of all negative quantities are, small f of negative integers are 0, so you have this is your formula for F(n-m) and Z transform of F(n+m) if you calculate this one you get Z power M times, this is you have F(z), Z transform of F(n) and then minus you have to, this is required so N is from 0 to, or rather R is from 0 to M-1, now you can use N, N is from 0 to M-1 F(n) times Z power –N, so this part of Z transform you have to negate it out of this, and with Z power N, wherever this N+M, okay.
So validity of this is same as wherever this is valid, okay, the domain of this is same as the domain of this, so the proof we can see easily, proof is Z transform of F(n-m) as a function of Z this is by definition N is from 0 to infinity, F(n-m) times Z power –N, so this is your, so this will give me what I do is I replace N-M by this sigma F of, let us say R, N-M = R, okay, so you have N = R+M, so what is R? If I put N = 0, R is from -M to infinity, so negative values of function is 0 so it's running from, R is from 0 to infinity, negative values of F(r), F of negative values, negative integers are 0 anyway, so that's why I need not write, so R is from 0 to infinity and you have Z power - N is R+M so you have R and +M, R+M together so you write, you can write outside Z power –N outside, so this is nothing but simply you get Z power -M times capital F(z) so this is exactly what you want is this.

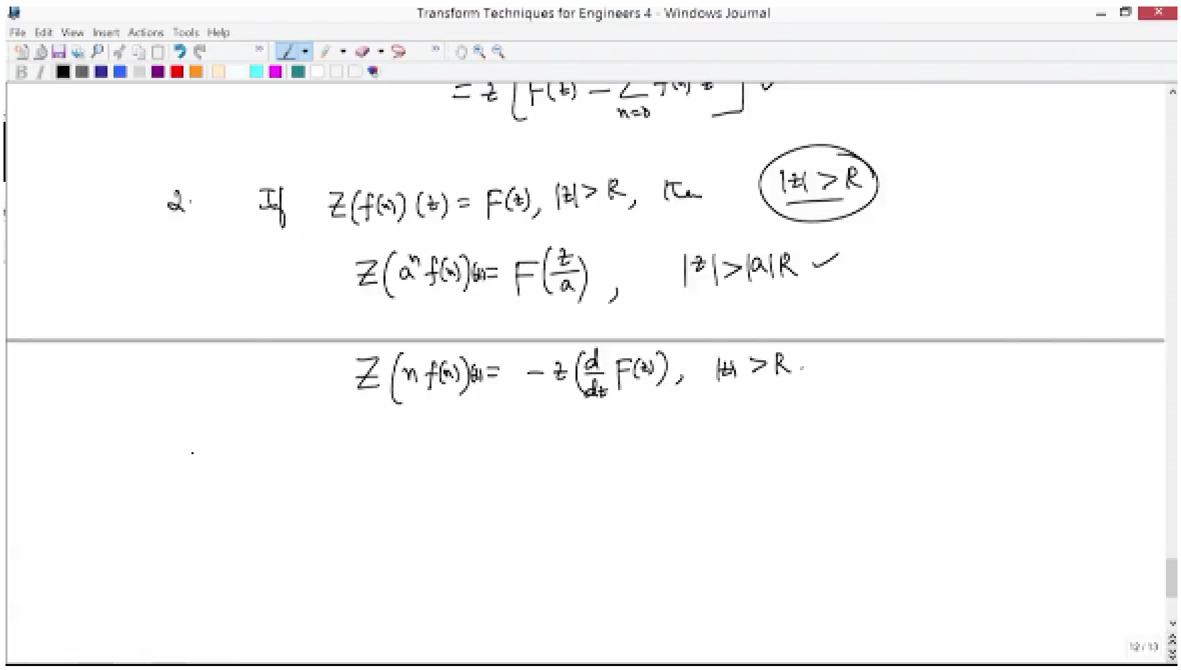


And then to look at the other result Z of F(n+m) as a function of Z is sigma N is from 0 to infinity, the same technique instead of N-M we write N+M times Z power -N and here you write N+M = R so that N = R-M, so if you write this N is from, N if you put 0, R = M, so R is from M to infinity and then you have F(r) times and Z is Z power –N –R and this is become minus minus plus R so you Z power M, positive sign you get, so this is nothing but Z power M times this sum I can write it like R is from 0 to infinity, F(r) small r, F(r) times Z power -R - up to M-1 term, so R is from 0 to M-1 F(r) into Z power –R, okay, so this is nothing but Z power M times, this is nothing but Z transform of F(n), so you have Z transform capital F(z) –this sum if I use N is from 0 to M-1, F(n) R power –N, sorry Z power –N, so this is exactly your second result what you want here, okay.



So this is the one property which we use when we solve the difference equations where when you apply this, you may need to apply Z transform to these translations N-M, N+M, okay, so the second property that useful for you is the same, similar way you can write if Z transform of F(n) of Z = F(z) then, so Z transform of A power N F(n) is if you have this, this is actually F of, capital F(z/a), if mod Z if this is valid mod Z is greater than A if this is the validity of this, let us say if this is the validity of this, if this is the validity of the domain for Z transform of this sample then this is valid from mod Z –A, Z/A is also greater than R, okay, mod Z is because, because F(z), capital F(z) mod Z is valid from, mod Z is greater than R, so you have this is valid, so you can write mod Z is A times mod A times R, that is where you have this validity here, okay, so this is one and also you can get some simple results.

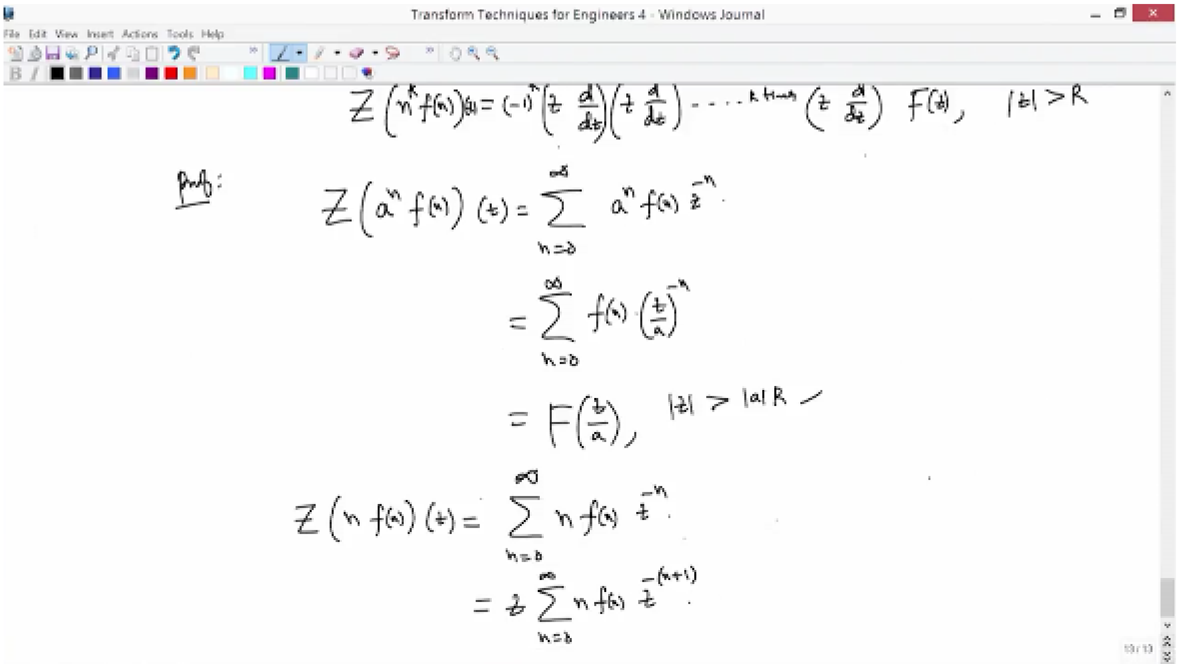
Let me use Z transform of N into F(n) which is equal to -Z times D/DZ of F(z), capital F(z), so let this is important, again mod Z is greater than R this is valid, okay, but this is valid Z transform as a function of Z is valid here, and this Z transform of N into F(n) is valid here, okay, so if Z is mod Z is greater than R is the validity of this, then this is what is true, okay, so



this proof we will see, so how do I prove this? So more generally I'll put let me write this N power K you will become -1 power K times this Z into DZ power K, so this you have to, you have to operate this differential operator on to this F(z) okay, this is valid for K is from 0, 1, 2, 3, onwards, so this means Z into D/DZ times, Z into D/DZ these are operators, okay, that many number of times, K times you operate, this you first operate on capital F(z) whatever you get it's a G(z) let us say, and then again you apply this like that you go on write iteratively till you apply this last operator, so that is the meaning of this one, it's not the power, okay.

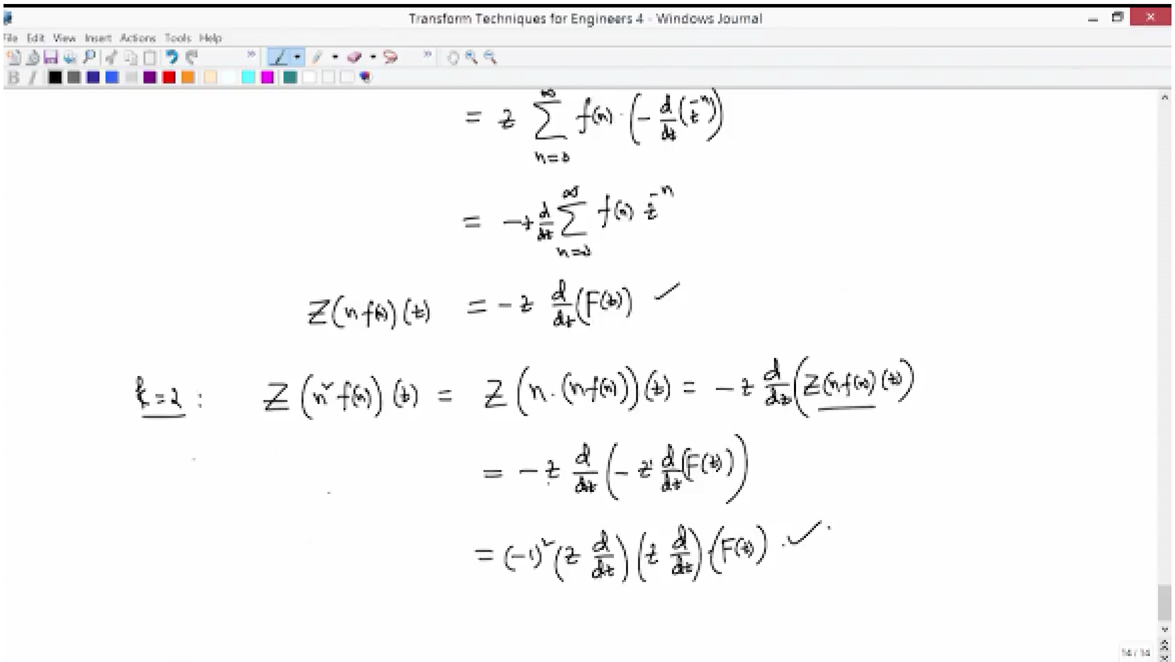
So if I write, so let me not write like this way it confuses you, so this times Z D/DZ up to K times okay, K times this operator D/DZ acting on F(z) this mod Z is greater than R is the same domain is valid for this, so proof is simple, so first you can apply this one, so first one is simple so you here write, Z of A power n F(n) this is as a function of Z this is sigma, N is from 0 to infinity, A power N times F(n) Z power –N, so this is simply N is from 0 to infinity, F(n) times Z/A power –N, so that's all, so you have this is nothing but capital F(z/a), so this is very simple so you have validity is mod A times R, so for every Z whose modulus is bigger than mod A times R is that is where its validity.

And now to look at the other one, so let me do for K = 1, so you have a Z times, Z transform of N into F(n), there's a function of Z is simply by definition, N is from 0 to infinity, N times F(n) times E power minus, sorry Z power –N, and this you write as N sigma, so Z you take it out, Z you take it out, N is from 0 to 1, Z you write it so if you write it I have to write N+1 here, F(n)



times N, okay, so I just rewrote whatever that sum, that sum is becomes this which is equal to Z times, inside N is from 0 to infinity this I can rewrite this as F(n), this is N into Z power N+1, I can rewrite this as –D/DZ of Z power –N, okay, what is this one? You have minus minus cancel, N times Z power –N -1, so N Z power -N –N -1, okay, so this is what I have written, so this I rewrite, these two terms N into this is nothing but this, so this is becomes -Z times sigma N is from 0 to infinity F(n) and you have Z power –N, D/DZ you can bring it out, so you have D/DZ, so this means so you have –Z D/DZ of capital F(z), capital F(z) is the Z transform of F(n) this sample.

So if you have this ZN F(n) of Z is this you keep on applying recursively, see if you want Z and of N square F(n) then what happens, this is again you can rewrite the same way, same technique you apply and get this, so you have this is Z times N into NF(n), NF(n) is a new F(n) okay, for this sample, what is your Z transform? So if you apply this mod Z D/DZ of Fourier, this Z transform of NF(n) of Z, so this is what you have to apply, so this is equal to -Z D/DZ of this we already know from this, so you have a -Z D/DZ of on capital F(z) which is Z transform of F(n) so you have -1 square Z D/DZ times Z D/DZ and you act these two operators on F(z), so for K = 2, so this is true, you can recursively prove that is true for any N = K, so you can show that this is trivial by induction argument, you can show that this is true for every K, okay.



So K is running from 0, 1, 2, 3 onwards, so this is the one property that, we have some more properties for this Z transform just like what we have seen in the Laplace transform to have a convolution of 2 samples we considered here, there we considered convolution of 2 functions and the Laplace transform of this convolution is product of Laplace transform of these two functions, so in the same way you have here Z transform of 2 sample, 2 sampled convolution, convolution of 2 samples Z transform of convolution of 2 samples is a product of Z transform of each of these samples, so that's what we will see in the next video, and we will see some inverse Z transforms, and then we will apply, we apply this technique whatever is, whatever we have learned how to find the Z transform and inversions, we apply to solve difference equations which are discretized version of differential equations, okay, that's what we will see in the next video. Thank you for watching.

Online Editing and Post Production

Karthik
Ravichandran
Mohanarangan
Sribalaji
Komathi
Vignesh
Mahesh Kumar

Web-Studio Team

Anitha
Bharathi
Catherine
Clifford
Deepthi
Dhivya
Divya
Gayathri
Gokulsekar

Halid
Hemavathy
Jagadeeshwaran
Jayanthi
Kamala
Lakshmipriya
Libin
Madhu
Maria Neeta
Mohana
Mohana Sundari
Muralikrishnan
Nivetha
Parkavi
Poornika
Premkumar
Ragavi
Renuka
Saravanan
Sathya
Shirley
Sorna
Subhash
Suriyaprakash
Vinothini

Executive Producer

Kannan Krishnamurty

NPTEL Coordinator

Prof. Andrew Thangaraj
Prof. Prathap Haridoss

IIT Madras Production

Funded by
Department of Higher Education
Ministry of Human Resource Development
Government of India
www.nptel.ac.in