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NPTEL ONLINE COURSE

Transform Techniques for Engineers

Introduction to Z-Transforms

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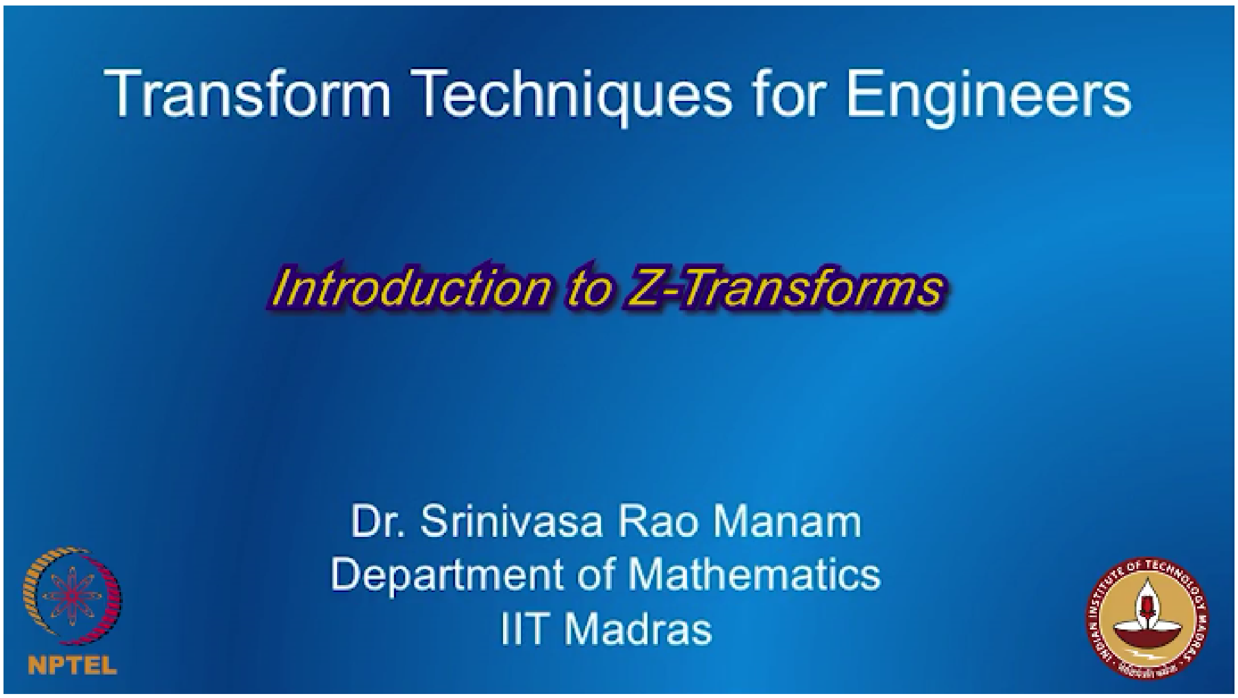
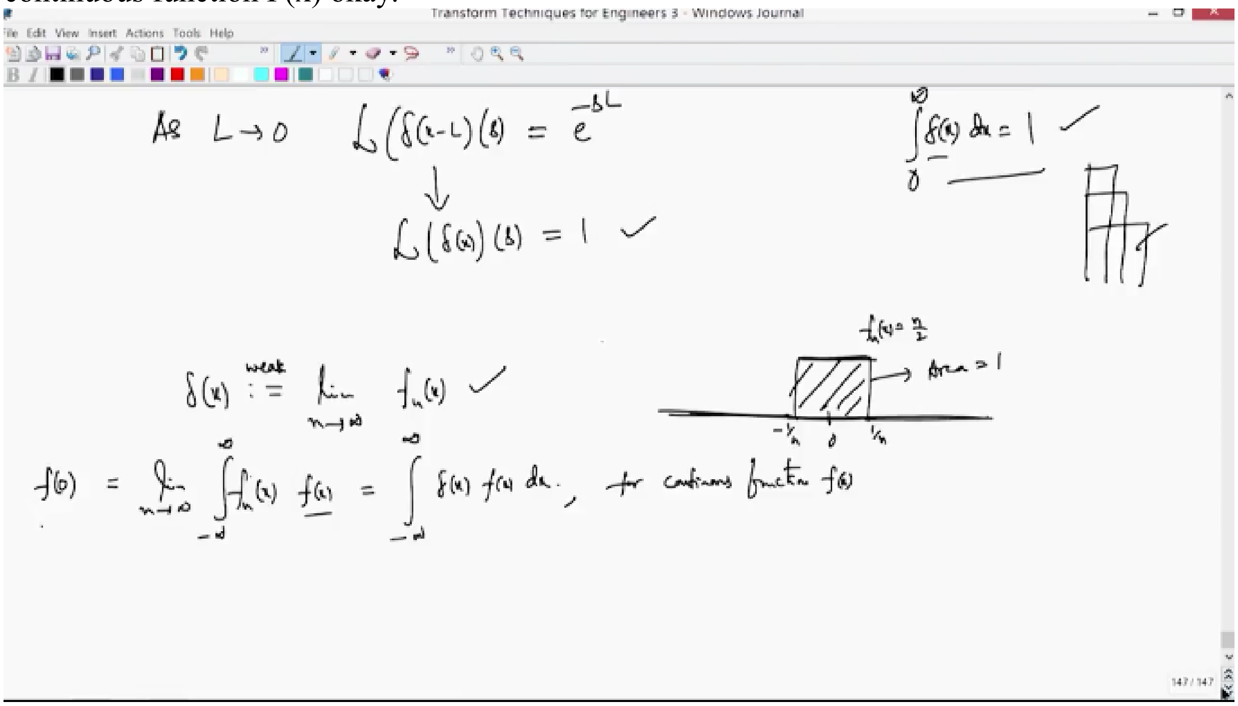


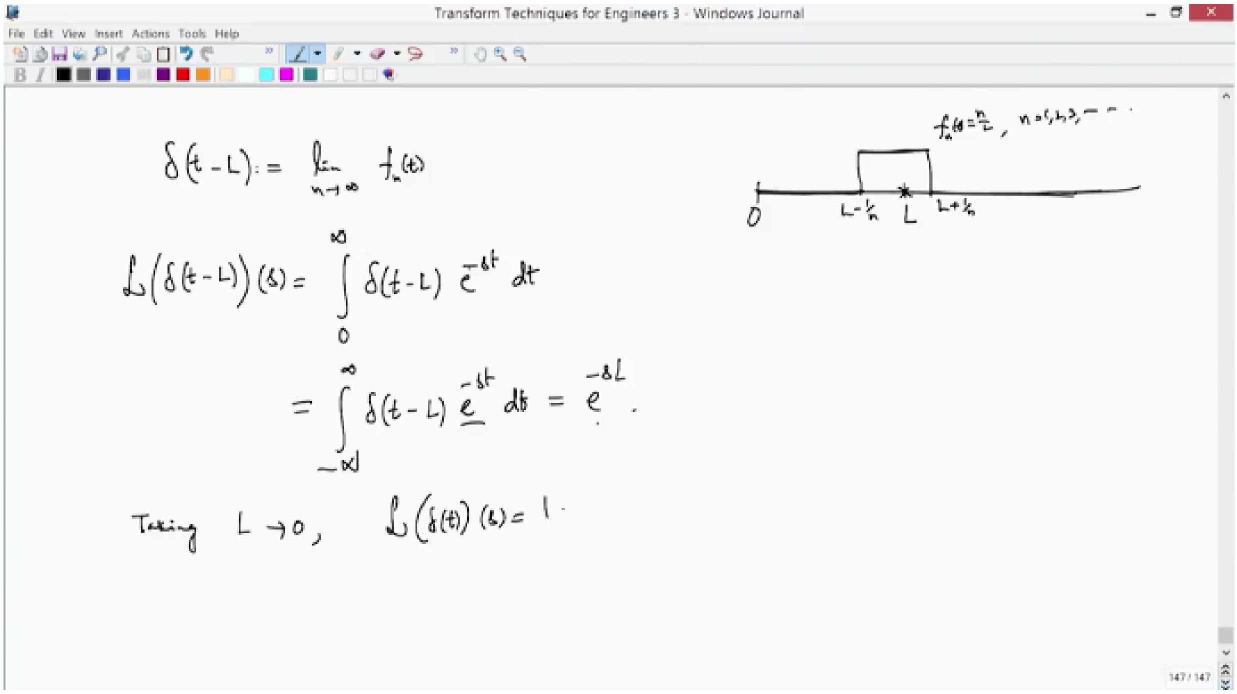
Welcome back, in the last video we were discussing about the applications of Laplace transform to ordinary differential equations and right hand side are the forcing function, is not a continuous function.

We have seen examples of piecewise continuous functions, so when you have ordinary differential equations with forcing function that is the right hand side is impulse function, when you can also solve by Laplace transform technique, so when you apply the Laplace transform you need to know what is the Laplace transform of this delta function.

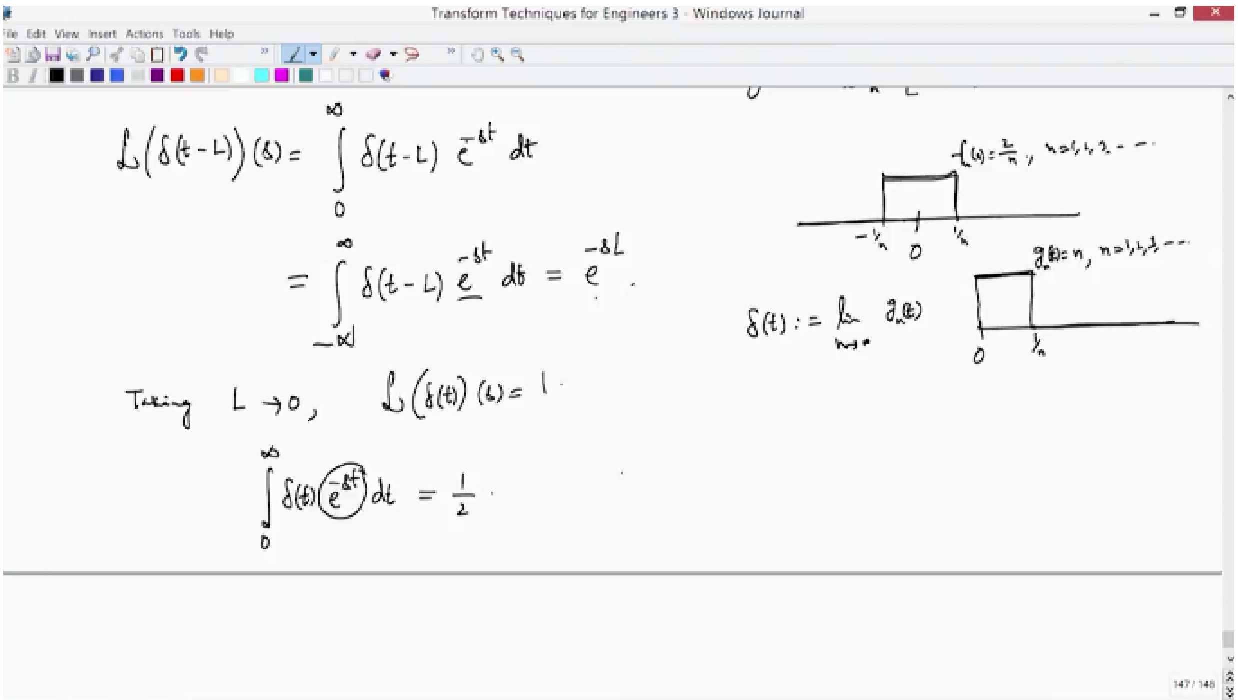
So before I do this what is the delta function? Delta function some sense you can see that it's a limit of these FN(x) those are step, kind of step functions as you see that it's 1 everywhere, if you consider delta function as usual, so let me write separately so if you think of delta function as I have defined earlier as a limit of FN(x) in a weak sense, as N goes to infinity these FN's are at 0 you make a step like this and 0 here, so this is your function 0 here and from between -1/N to 1/N and you have this is FN(x) as N or rather 2/N because this length is 2/N so here N/2, okay, so this length is, this length of this interval is 2/N so this you multiply with N/2, so I give this one so that the area is, that this area is equal to 1 okay, area is equal to 1, so in that sense so now only weak sense, a weak convergence, weak sense you have to take this that means, that means on an average this is true, so that means FN(x) and F(x) you integrate from –infinity infinity you take this limit N goes to infinity, this is actually -infinity infinity delta(x) F(x) DX, so if you actually calculate this one with any F(x) if you know F(x) so this is going to be, you can see that this is going to, this you can calculate because this is unknown, this is usual function, this is a given function, any function for every continuous function, so let's take for continuous function F(x) okay.



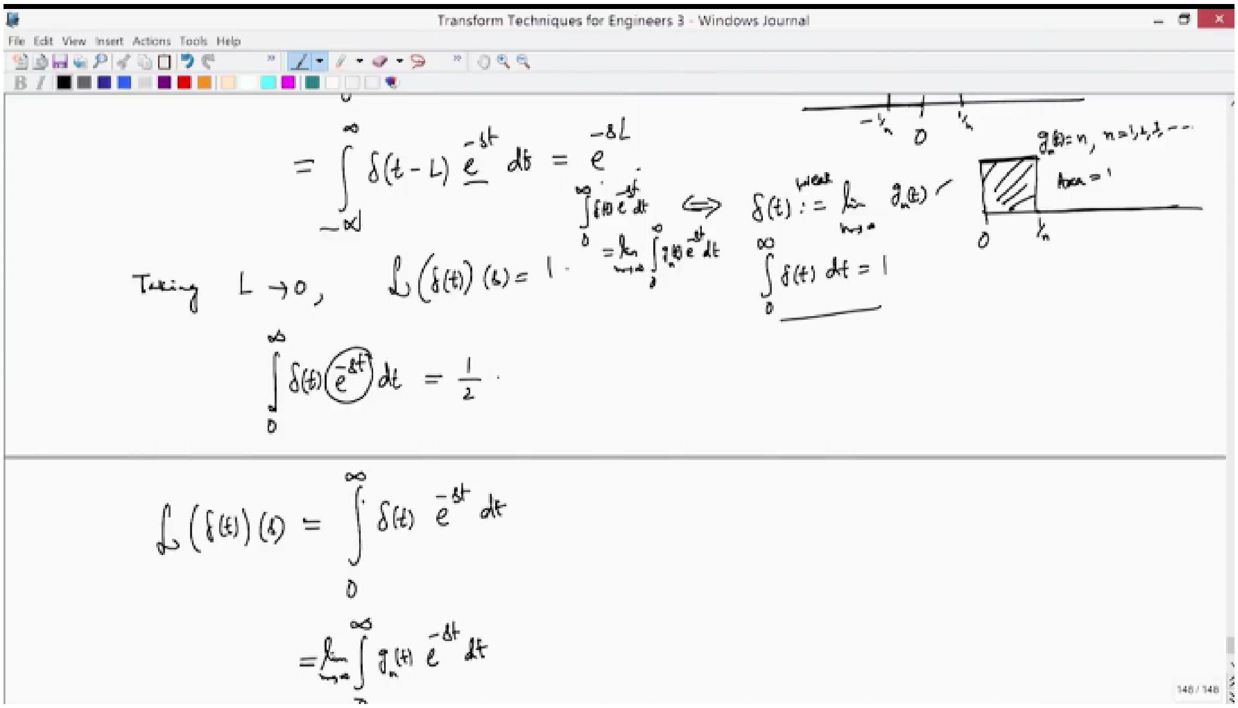
So if you take this one you can see that this value if you calculate it will be, it's going to be F(0), so in that sense that's what we have defined as a delta function in the earlier videos when we do the Fourier transforms. So in this case when you want to calculate Laplace transform of a delta function so Laplace transform is defined from 0 to infinity, so because it's 0 to infinity over your domain let's consider your impulse at let us say L, so you have a delta(x-l) this as a limit of this FN(x), now FN(x) as here this is between L-1/N to L+1/N, so here 0 to L-1/N is 0, function value is 0, and between this to this L-1/N to L+1/N function value is N/2, okay, let me write FN(x) = N/2 otherwise outside this, this is 0, 0 to infinity, so if you consider like this all your FN's you start with N = 1 onwards, N = 1, 2, 3 onwards, so this limit as N goes to infinity this is your definition of delta function, so when you apply Laplace transform of this, if you apply the Laplace transform, Laplace transform of this delta function of, let's I use T if you want, you can also use T as a variable this is your T, if I use T here so delta function T-L as a function of S you need so this is by definition 0 to infinity, delta(t-l) E power -ST DT, so as you see this is same as because this delta function it's valid, it's only valid as a limit of N goes to infinity it's defined positive side, so when you consider this negative side -infinity infinity you can think of same because outside this is 0, 0 to –infinity or -infinity to 0 this function is 0, so I can include here E power -ST DT, so this you already know as value of this T = L you have to put it here so E power –SL, so that is your Laplace transform of this delta function.



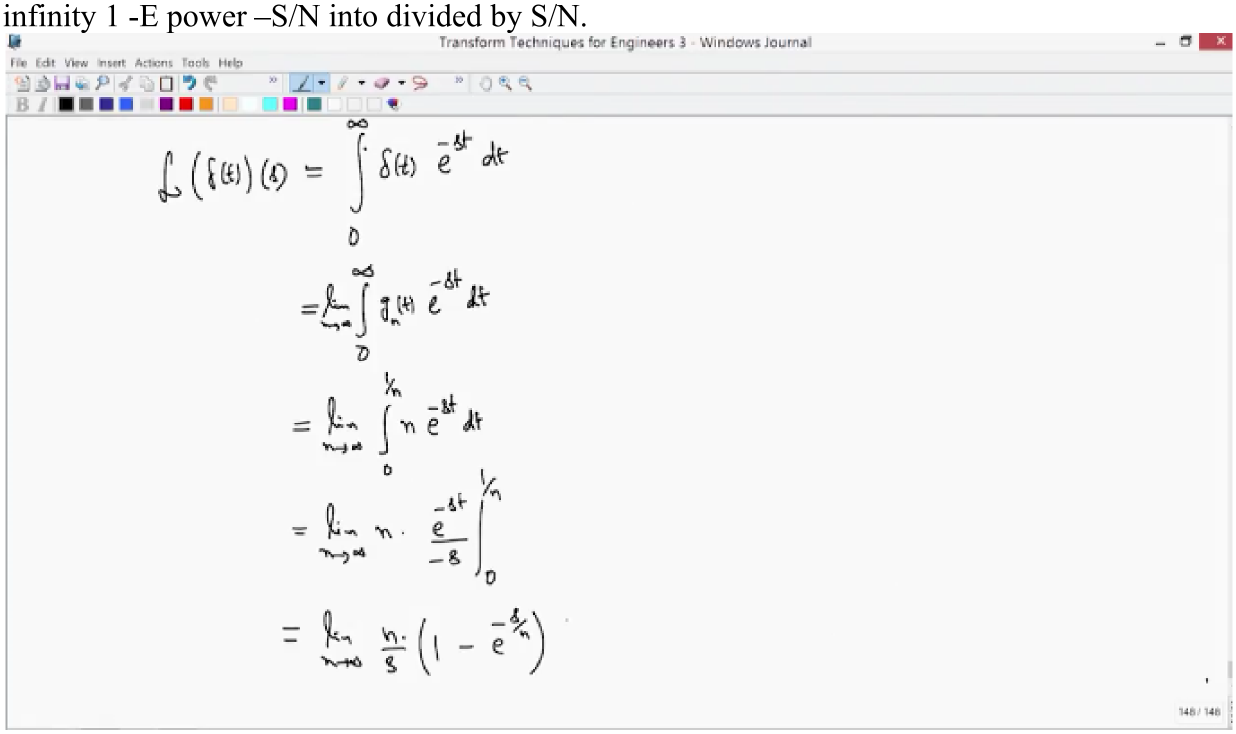
When this impulse is at point L inside, so you can think of you take the limit, taking limit, taking L goes to 0 what we see is the Laplace transform of delta function(t) = E power this is limit L goes to 0 so if you do this, this is going to be 1, so this you can easily see straightforward way, but if you actually see directly, why we get a confusion here is if you directly look at this one, this is actually 0 to infinity delta(t) E power -ST DT, earlier we have seen that when you have a delta, impulse function at 0, delta function from 0 to infinity we have seen that is value is actually equal to 1/2 of the value at this point so that is 1, so this is 1/2, okay, when you consider this as 1, we have seen that this is equal to 1/2, because of this contribution at 1/2 okay, so delta function this is only you have taken 1/2, so what my delta function is? If you have 0 here, if you take your function like this as your -1/N to 1/N FN(x) = 2/N, if I consider this as running from, N is running from 1, 2, 3 onwards then this is true, then this is actually true 1/2 okay, so when you take the Laplace transform you see that when you put T = 0 so it's going to be 1/2 this is actually true, okay, if your delta function is this, this is a limit of all this sequence which are spilling over to negative side, but if you don't have a domain here, so you don't have domain, so what my domain for the Laplace transform here your delta function S actually, delta function is limit of these FN(x) some GN(x) let me call these are your reference, GN(x) are like this is your 0, so I consider this is 0 to 1/N, other outside is 0, so it is GN(x) = N, N is from 1, 2, 3 onwards.



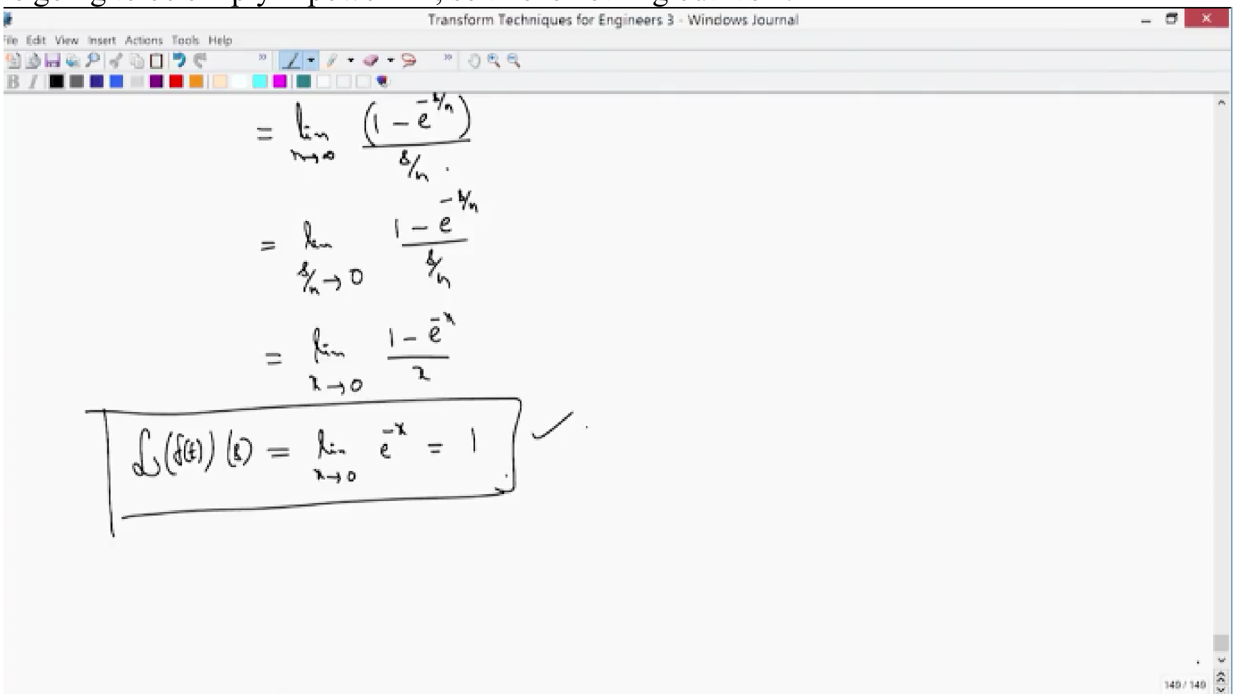
So if I choose like this GN(t) okay, GN(t) is N here, value is N between 0 to 1/N, outside 1/N to infinity this is 0, so if you take like this then what happens, and I have to choose is if we choose in such a way that, why I choose this is here N so that integral, this area is 1, area = 1, so you have an integral delta T between 0 to T DT is always maintained as 1, so because of this you see that if I calculate Laplace transform of delta(t)(s) which is equal to Laplace, which is 0 to infinity delta(t) E power –ST DT, okay, so this I can write it as 0 to infinity a limit of this GN(t) okay, this means what, this means if I multiply E power ST which is a continuous function so this means this is actually same as integral 0 to infinity, delta(t) E power -ST DT is equal to limit as N goes to infinity 0 to infinity, GN(t) E power – ST DT okay, this is a weak sense, on an average this is true, that is the meaning of this convergence, because this is not usual function, okay, so because of that so this I write it as limit of N goes to infinity GN(t) E power



–ST DT, now I know what is my GN(t), GN(t) is N between 0 to 1/N, limit N goes to infinity between 0 to 1/N this is N E power -ST DT, outside is 0, so this is equal to limit N goes to infinity and if N comes out of the integral because nothing to do with this, so you have E power -ST/-S you apply into the limits, and you have this limit N goes to infinity times N times, this is when you put 0, T = 0 that is 1-1/S -, so rather N/S comes out, so you have 1, 0 thing, because of minus is 1, and when I substitute 1/N that is E power –S/N, so this is what, this limit if you look at this, how do you get this limit? This limit I rewrite as, I rewrite as a limit of N goes to infinity 1 -E power –S/N into divided by S/N.

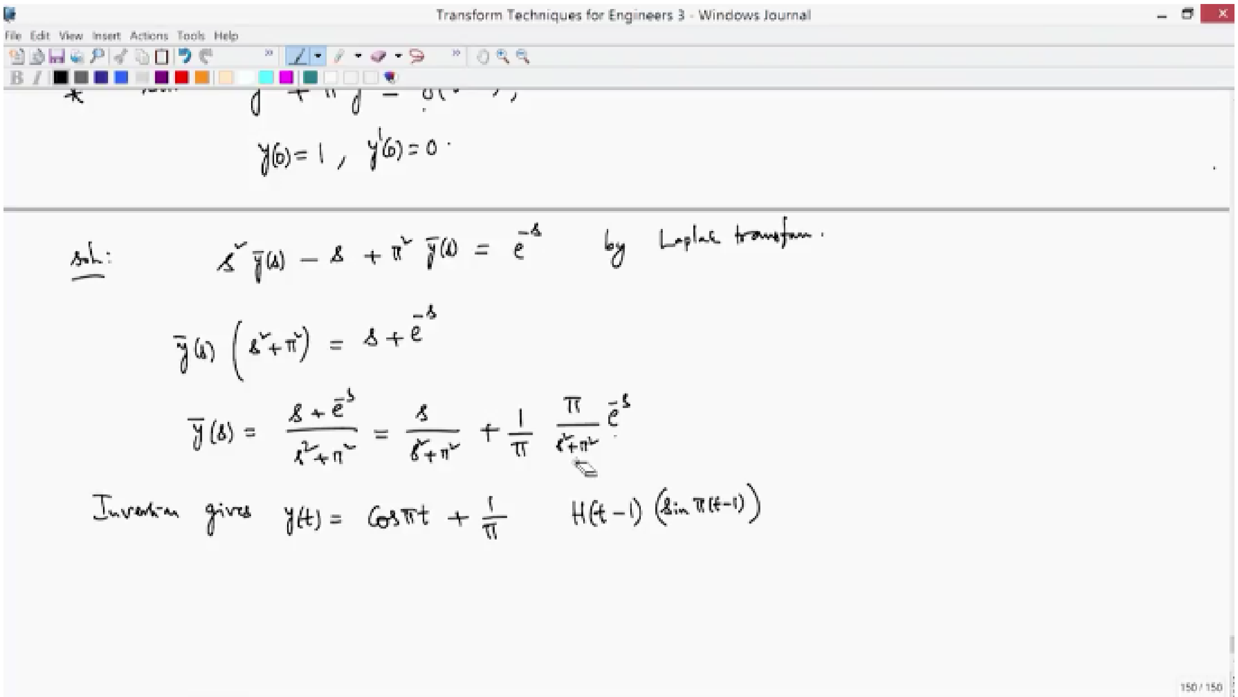


If N goes to infinity S/N whatever maybe, S is fixed constant, so S is S/N goes to 0, so this is same as limit S/N goes to 0 as N goes to infinity S/N goes to 0 you have 1 -E power –S/N by S/N, so if I replace this as some X, S/N as some variable S, so let's write this as X, X goes to 0 1 -E power –X/X, so this is as X goes to 0 this is like a Laplace rule you can 0/0 form, so if you apply this is same as limit X goes to 0, if you differentiate this it's going to be, this is 0 and this is going to be simply E power –X, so this is nothing but it's 1.



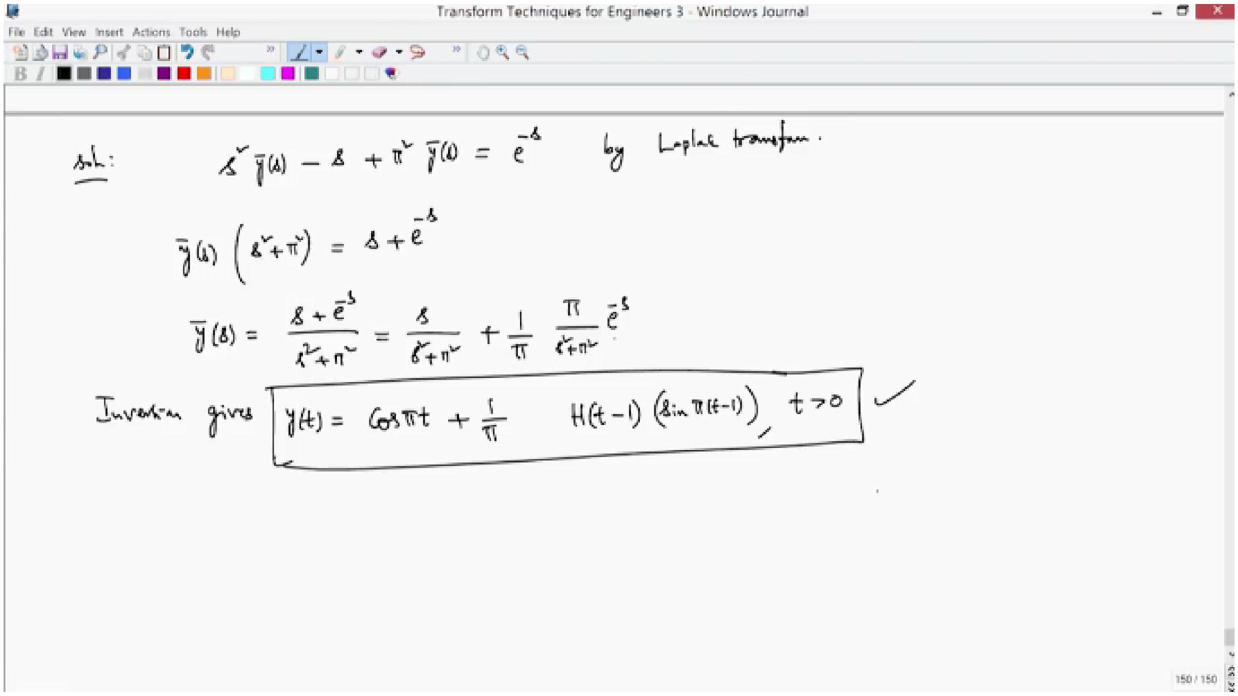
So that's how you get your Laplace transform of this, but you have to use your delta function as a limit of, sequence of usual functions, there are step functions like this so that, so that it's not spilling over to a negative side, negative side there is no spilling, okay, of your sequence of functions, so this is how you show this delta function Laplace transform of, delta function as a function of S is simply 1, okay, so this is how we show this one, so once you have this now we can apply this technique Laplace transform technique to solve some ordinary differential equations when the right hand side is having impulse function.

So let's solve this some examples solve Y square, solve Y double dash + pi square Y = delta function(t-1) so with this initial conditions, of course T is positive, D square Y/DT square, okay so and Y(0) given as 1, and Y dash(0) = 0, so when you have something like this it's may come in the applications of, when you see these kind of problems in the application you cannot usually solve by differential equation methods, so you can apply, that case you can apply this Laplace transform technique, so if you apply this S square Y bar(s) - S times Y(0) that is 1, and then you have a -Y dash(0) that is 0 so this is for Y double dash + pi square Y bar(s), and delta function of this, so if you apply this delta function E power - simply S, okay, if you apply the



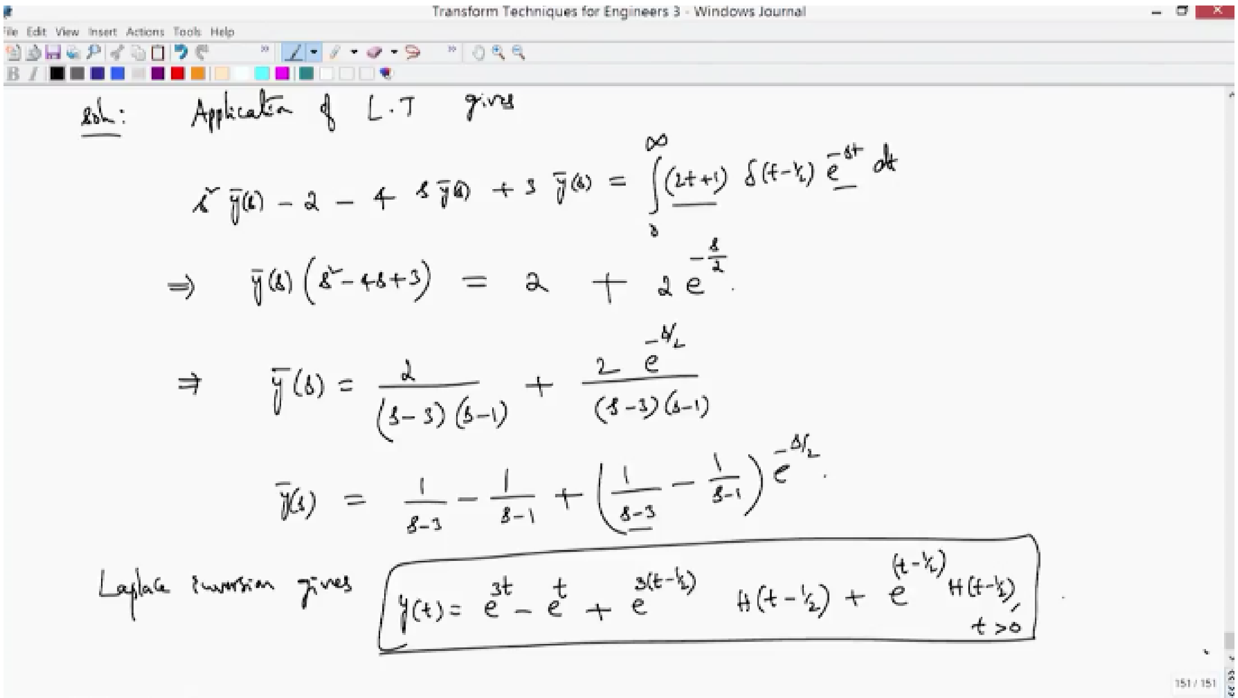
Laplace transform of delta function (t- 1) simply E power -S T = 1 so you have to replace, so this is what is from by Laplace transform, so you can see that Y bar(s) you can write as a product of S square + pi square = S + E power –S, so Y bar(s) = S + E power –S/S square + pi square, so this gives me, inversion gives now inversion, inversion will give Y(t) as S over S by, so this you can write it as S/S square + pi square + E power so you can write this as E power -S 1 over S square + pi square, so I use this as pi here, so that you have 1/pi, okay, so just constant.

So how do I get this inversion for this? This is simply cos pi T this is 1/pi times this is, this will give H(t-1) so that E power -S is accounted and this is Laplace transform of, this is Laplace transform of sine pi T, sine pi T so you have to write sine this into sine pi(t-1), so Laplace transform of this is E power -S times, Laplace transform of sine pi T which is this, okay, so that's what exactly your solution for T positive. So this is a piecewise continuous function which is the solution for this problem ordinary differential equation.



Let's solve one more problem, if I use Y double dash -4Y dash + 3Y = 2T+1 times delta function(t-1/2) T is positive, and then what you have is Y(0) = 0, Y dash(0) = 2, so what is the solution here? You apply the same technique, so application of Laplace transform gives S square Y bar(s) - S times, Y(0) that is 0 goes and you have –Y dash(2) that is 2 – 4 times for this term -4 times SY bar(s) – Y(0) that is 0, okay, and then +3 Y bar(s) equal to, now if you apply this right hand side 0 to infinity 2T + 1 times delta function(t- 1/2) E power -ST DT, so delta function for this whole function, so wherever T is there you put T = 1/2, so when you put T = 1/2, this is 2 and this is going to be E power –S/2, and this left hand side you have Y bar(s) as S square -4S+ and this is equal to 2 + this one, 2 you bring it to the other side this part so you have this, so this gives me Y bar(s) as 2 over this you can write it as S-3 times S-1 if we do like this you'll get this, this product is this and +2 over S-3 times S-1 times E power –S/2 so this you

have to use the partial fractions and get your solution, so how do I use this partial fractions 2 over, so let me write 1/S-3 – 1/S-1 then it is 2 right, that is actually this, this is a partial fraction so you have what 2, see this is also you can write the same way S-3 – 1/S-1 times E power –S/2, so this is your Y bar(s) so Laplace inversion gives solution Y(t) as 1/S-3 is E power 3T -E power T and then here this is because of this you have H(t- 1/2) and then this part, this part is actually E power 3 T-1/2, okay, this into this plus again, again for this you do the same thing, this is again going to be H(t- 1/2) for this part and 1/S-1 you have E power T, in the place of T you have to write T-1/2, so this is exactly what you have as a solution, okay, so this is your solution for T positive.



So this is how you get your solution of that ordinary differential equation with right hand side being some complicated function involving that delta function, so polynomial times delta function you can just or any continuous function times delta function you can apply this Laplace transform and get your solution.

So as a final example let me do one more example before wind up these applications of Laplace transform let me give you one more so, Y double dash + Y dash which is equal to T-1 – delta(t-2) Y(0) is 0, Y dash(0) is 0, so let me quickly solve this as a same technique S square Y bar(s) – S times Y(0) that is 0 - Y dash(0) both are 0 so this will contribute only to this, other one is Y dash is S times Y bar(s) – Y(0) that is 0, this is equal to this 1 will be E power -S and this is going to be E power -2S, so you have Y bar(s) as 1/S times S+1 E power -S –E power -2S, so this you apply this you can write it as 1/S - 1/S+1 times E power -S –E power -2S so you have a Y bar(s) this, so inversion gives, inversion gives a solution Y(t) as 1/S times E power -S is H(t) here, as a T-1 because if E power -S and 1/S is a Laplace transform of 1, right, so you have 1 here, so 1 as a function of T-1 itself is 1, okay, that is this and you have minus and E power this is E power -2S you can write it as H(t-2) and again one is that so you have H(t-1) and (t-2) for the first term into the whole thing, okay.

And then other one is minus H(t-1) for this and 1/S + 1 is E power –T, so you have E power minus, in the place of T, T-1 so Laplace transform of this is E power –S for this and Laplace transform of E power -3 that is 1/S+1. And the same way this minus minus plus, now here H(t-2) and here again 1/S+1 you have to write E power –T, in the place of T you have to put T-2, so this is your solution of this problem.

$*\quad y'' + y' = \delta(t-1) - \delta(t-2),$

$y(0)=0,\quad y'(0)=0.$

Sol:

$$s^2\,\bar{y}(s) + s\,\bar{y}(s) = e^{-s} - e^{-2s}$$

$$\Rightarrow \bar{y}(s) = \frac{1}{s(s+1)}\left(e^{-s} - e^{-2s}\right)$$

$$\bar{y}(s) = \left(\frac{1}{s} - \frac{1}{s+1}\right)\left(e^{-s} - e^{-2s}\right)$$

Inversion gives $\boxed{y(t) = H(t-1) - H(t-2) - e^{-(t-1)}H(t-1) + e^{-(t-2)}H(t-2),\quad t>0}$

So you can directly apply your Laplace transform technique and get your solutions, so these are the applications I would like to present to you for ordinary differential equations PDE's and integral equations and you can evaluate some materials, okay.

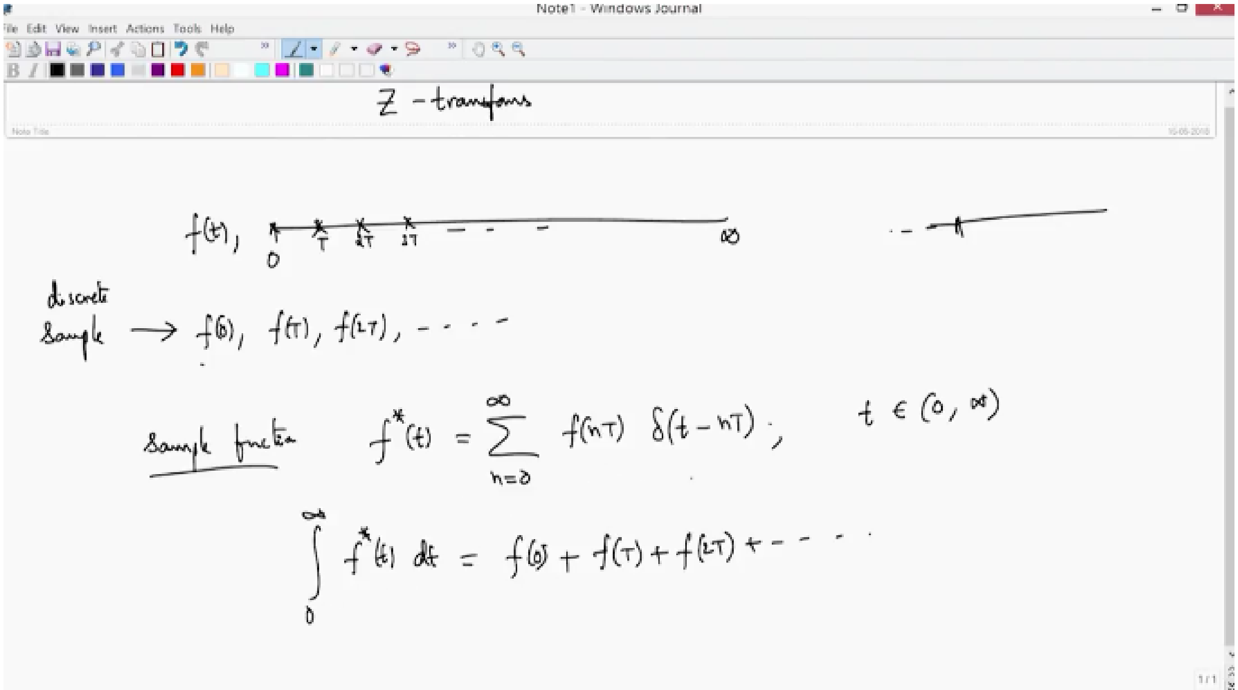
So we'll move on, we'll close this Laplace transform you can now your, you can use Laplace transform to solve, you can apply your Laplace transform and make use in your applications to solve some differential equations or integral equations in physics and engineering problems okay.

So let's move on to, so let's do Z transforms, okay Z transforms, so what is the Z transform? So we'll discuss as a final component we will discuss Z transforms in 1 or 2 or maximum 3, 3 to 4 videos, we will just give you a brief introduction and it's applications of Z transforms, Z transform is basically a discrete version of Laplace transform, so what we have seen so far is you started with Fourier transform of a periodic signal and then you went to, you defined what is the Fourier transform of non-periodic signal which you actually derived it's a transform and it's inversion you derived based on the periodic signal, so that's how you derived that thing, okay, you derived the Fourier transform. And then Laplace transform you derived based on the Fourier transform, so you use the Fourier integral theorem to derive the Laplace transform.

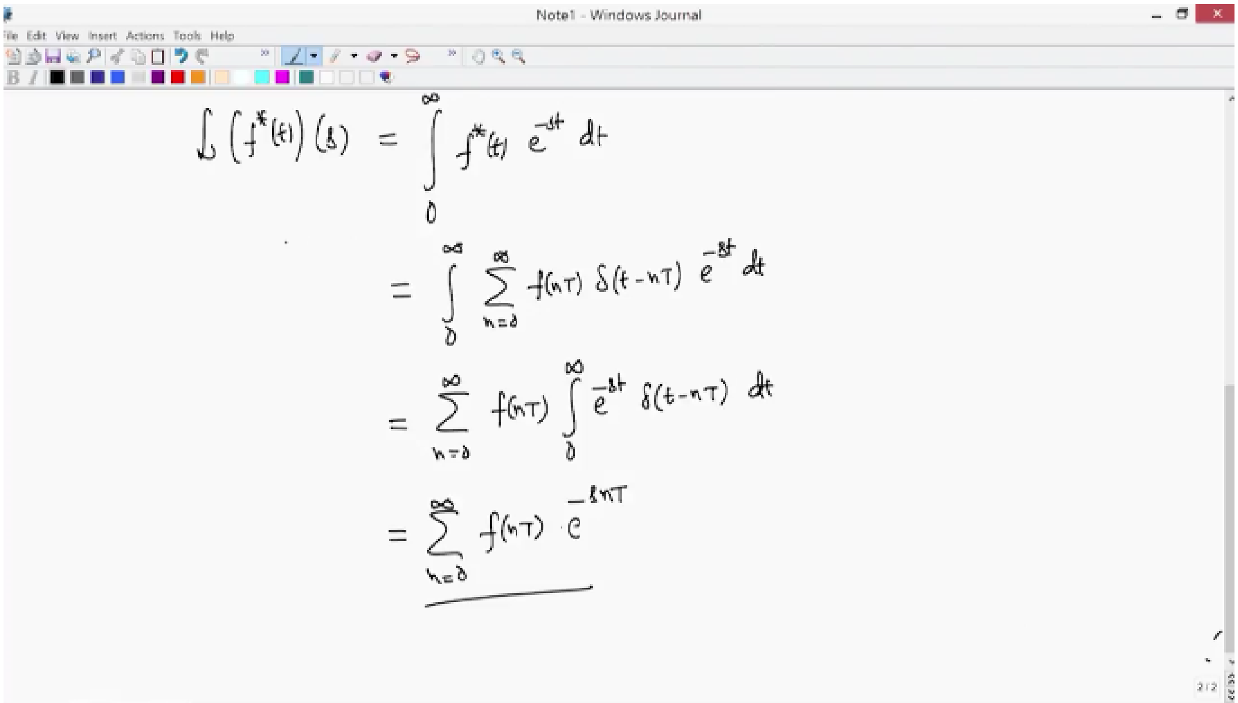
Now this Z transform is a simple discrete version of Laplace transform, so we use a Laplace transform and it's inverse transform to get, not inverse transform just use the Laplace transform which is defined as a continuous time which is between 0 to infinity, so in this between 0 to infinity you take only discrete points, discrete data for example if your function F(t) is given from 0 to infinity, so as a continuous variable, a continuous function, continuous variable T you have, for which you have a Laplace transform.

And now instead of F(t) you consider a values of F(t) at some discrete values, for example will say 0 at T, 2T, 3T, and so on, so T is a fixed constant, okay so if you do like that you have a discrete data so when you want to see this this is called a sample, so you have a sample, discrete sample so you can represent this sample as a continuous function in terms of impulses that means in terms of delta functions, okay, so we'll see how we get this if your function F(x) is given as F(t) let's say F(t), T is from 0 to infinity, so T is from 0 to infinity if it is given, if I want to choose only from 0, and as a discrete sample, let me take the sample, a discrete values if you take as a constant so let's take T, so this is your T and so that this is the same interval you can take, consider this 2T, 3T and so on, and so on you can consider as a sample, so you have F(0), F(t), F(2T) and so on, so this is your sample, a discrete sample okay, a discrete sample and this is your discrete sample, how do I represent, if I want to use for this you can represent this as, so sample function so let me write as a continuous function if you want to represent as a continuous variable, I will represent this data as a sample function, okay, so let me write it as F star(t) as a function of T now, this as sigma, N is from 0 to infinity because you have infinite data, N is from 0 to infinity, N = 0 you have F(nt) and N = 0, F(0) and you have a delta function that is T-NT, you can have like this.

So this represents our delta, this represents a continuous variable for every T belongs to 0 to infinity, but what it does is when you consider this –infinity, let us say so if you choose this one, let's integrate from 0 to infinity to F star(t) DT if I use this, because of this delta function and what you get is simply F(0) + F of so, so delta function at 0, N = 0 again the way I have chosen earlier these are the limit of the only one side, okay, so this is not spilling out to the negative side, so that when you have this, when you apply this integration for N = 0, so what you have is only F(0), F(0) delta(t) DT, so you have F(0) F(t), sum of all these samples you will get, as a sum you will get, okay, so when you actually take the data that I instantaneously you take so basically you have, you will have a sometime to get this, so this is not exactly the point means it's not exactly the point it is some neighborhood of this point, exact point, so in that sense you represent this as delta functions and when you consider so you represent this as a sample function, as a continuous variable you can represent this data as a sample function which are in terms of this delta functions, so if you want to have a data you can get back your data by integrating over this T variable, so that's what you see by representing this as a delta function, okay, sum of all these delta functions.



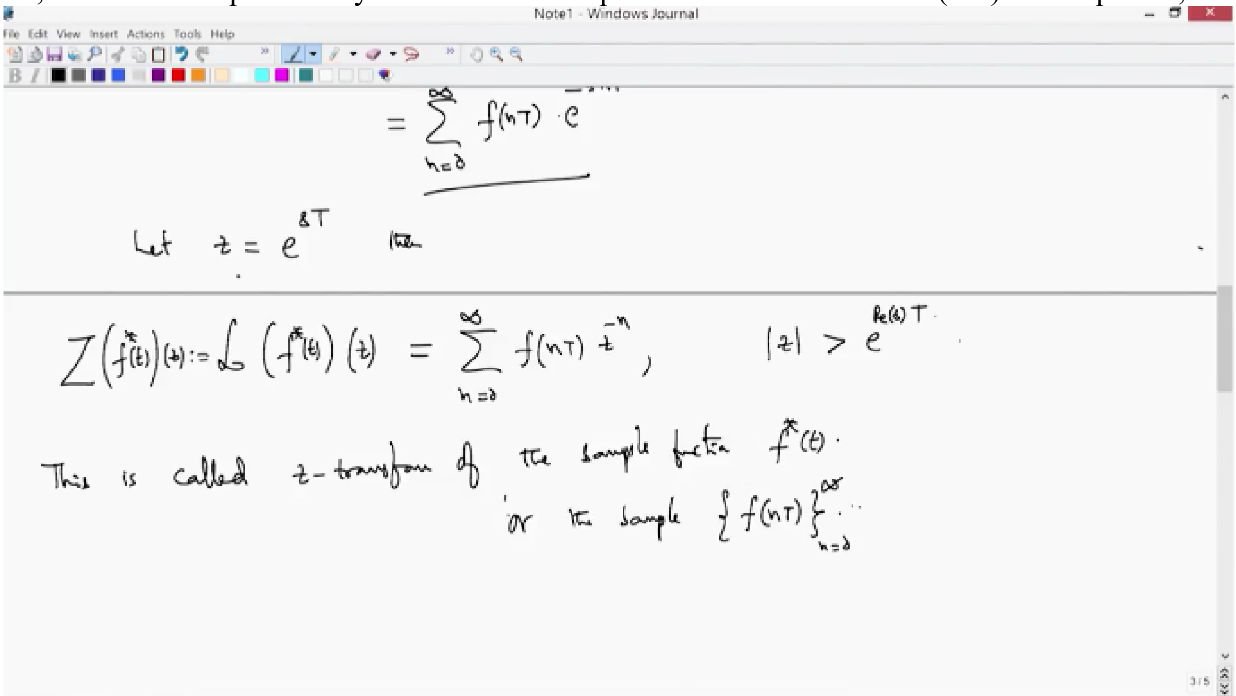
So for this function now this is a T variable which is a continuous variable, I can apply this delta function, so I can apply Laplace transform for this function, if I apply formally what is a Laplace transform of this F star(t) as a function of S let us, let's do this, if I apply this so which is I have to consider basically only a part of the data of F(t) okay, so originally for which you have a Laplace transform, so if I do this you have 0 to infinity, F star(t) E power -ST DT so this is nothing but 0 to infinity, the summation from N is from 0 to infinity, F(NT), delta(T- NT) times E power minus ST DT, assume that you can do this summation outside, you have F(NT) comes out as a constant, and this integration is from delta function E power –ST, delta function of T-NT DT so this is nothing but N is from 0 to infinity F(NT) times, now if you substitute T = NT E power –SNT, so this is your sum you get when you apply this Laplace transform, okay, formally, so formally if you apply Laplace transform this is what you get, it's just a motivation to get your Z transform.



So what I do is and now I consider this E power -ST as a sum Z if I choose, because S is a complex variable E power –ST is a complex number, so let Z be that then Z be minus, so E power ST, then what happens to your Laplace transform of F star(t), so Laplace transform of F star(t) it's a function of S, so this is, if it is a function of S if I substitute here this will be function of Z, okay, let me write this as a Z transform of, let me write as a Z transform of F star(t) which is function of Z, if I replace with Z by this E power ST by Z then this will be Z transform, Laplace becomes, I'm defining what is called Z transform, okay, whatever you get is this one, this is a function of Z, this is equal to or like initially it's actually Laplace transform okay and which is a function of Z so I'm calling this as your Z transform, okay, this is the definition of Z transform, Z transform is, so let me write this as Z, capital Z of F star(t) as a function of Z, this is the definition as this we, this is well-defined as if we do this, this is actually N is from 0 to infinity F(NT) times Z power –N, so where is this valid? So this is valid from, Z is a complex number so this is, mod Z is what is the minimum thing is mod Z so this should be mod Z is greater than E power ST, right, E power, real part of S rather let me write real part of S times T, okay, so this is well-defined for real part of S wherever real part of S for which you have a Laplace transform exist so that real part of S when you consider the mod Z what is left is E power real part of S times T, so this is exactly you have, so beyond which if this is finite at E power ST it is finite, beyond also this is true, okay, because it's 1/Z type, 1/Z power N okay.

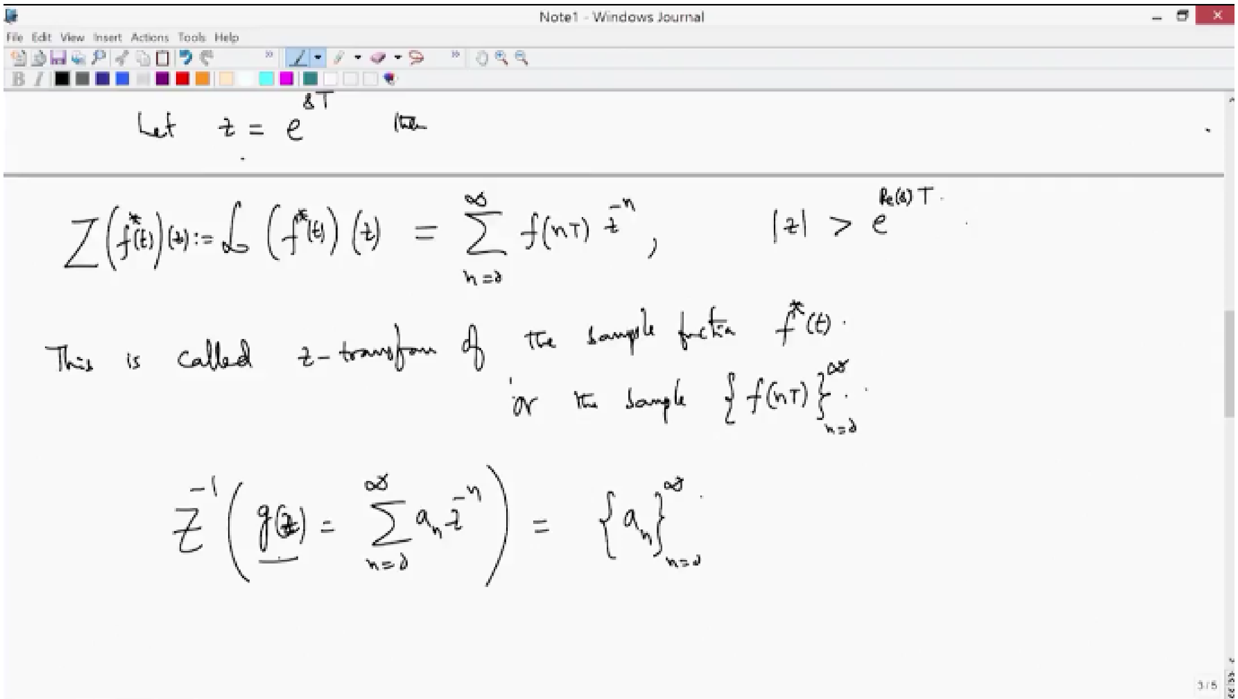
If it is valid at Z = Z naught, this series is finite at Z = Z naught then mod Z, Z is bigger than Z naught, this is always true because this is some negative powers, okay, so in that sense you have this, this is valid, for mod Z is bigger than this one.

So this is how you define your Z transform as a definition, this is your Z transform, this is called Z transform of the sample, sample function or sample F star(t) okay, so what is my sample? Our sample function of or sample, what is the sample I have? F(0), F(t) F(2T) and so on, so this is a sequence as you see this is a sequence if I write like this F(NT) is a sequence, N



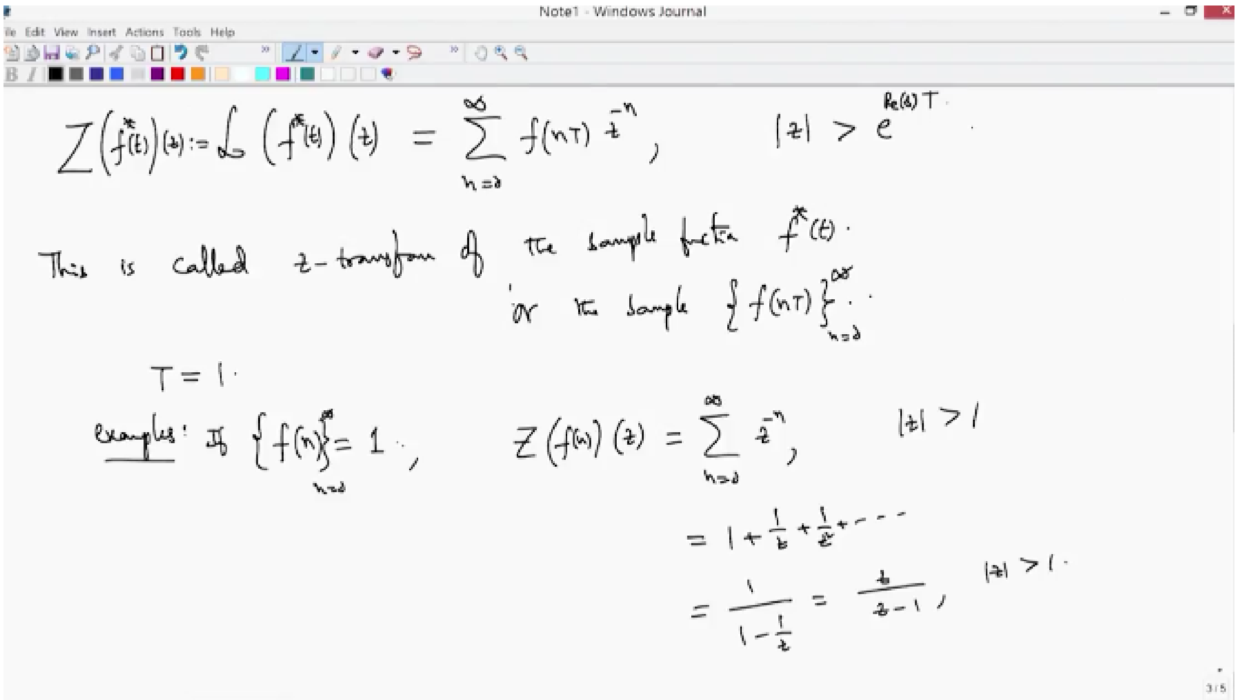
is from 0 to infinity okay, so this in the communication engineering you can see that you have a data that is recorded at specific intervals, equal intervals you can continually you record, you get as a discrete data, so this discrete data you can process it and then that's where when you want to process the transforms on the discrete data you get these finite discrete transforms, one such is this Z transform, okay, so we use this Z transform as a definition and what is it initially, what kind of functions, what kind of data you have this Z transforms exist, and then finally so you have this is your transform function you have this transform is this is a complex valued function you will get it as your as Z transform.

So inverse transform is if I know this, if your function of Z this complex valued function is this series, then if your complex valued function let us say some G(z) if I can write like as a sum, sum of Z power –N, N is from 0 to infinity then this Z inverse of this is actually equal to simply the constant sequence, N is from 0 to infinity because these are the coefficients, if it’s AN so I have AN’s here, okay, so because Z(an) is actually this that is given as G(z) that if you sum it



up that is, that as some G(z) okay, so to see this inverse and, to see before I see this inverse you can see, you can first find the Z transform of some samples, so some tribal samples, let's see some examples of Z transform, so one is a constants, okay.

So let's always take T = 1 first, before I do this let’s fixed as without loss of generality you can take it as T = 1 so that your sequence, so I’ll consider this sequence as F(n) as 1 for example, your sequence F(1) is constant, N is from 0 to infinity as a constant, okay, if this is the case what is Z transform of F(n) as a function of Z which is equal to this series by the definition, the series F(n) T = 1, N is 1 so you have Z power –N, so where is this valid? This is valid from mod Z, 1/mod Z is less than 1 so that means mod Z is greater than 1 this is, this we know that this is finite, this is by geometric series, okay, so Z transform of this is simply by, this is actually 1 + 1/Z + 1/Z square and so on, this is geometric series so you have 1 over 1 – 1/Z so this is simply Z over Z-1 so which is valid from mod Z greater than 1, so that is your Z transform for this samples.



So this is first example, so let's do some more, if I choose your F(n) as let's say A power N, and A is not equal to 0 if F(n) is this, what is the Z transform of F(n)? As a function of Z which is sigma, N is from 0 to infinity by definition you have, this is A power N times Z power –N, and this is again, this is like this is a series from 0 to infinity Z/A power -N so this is valid, Z/A modulus is greater than 1, so mod Z is greater than A, mod Z is greater than mod A, A even can be complex number, so once you have this and again we have 1 - 1 this is again 1 over 1-Z/A, okay, so you have Z/A or 1/Z so you have A/Z so you have a Z over Z-A and this mod Z is greater than mod A, so this is your Z transform, so like this you can go on get this.

Now let me do one more example before I close this, if F(n) is simply N, what is the Z transform? So then Z transform is Z transform of F(n) of Z which is equal to sigma, N is from 0 to infinity, this is N times Z power –N, so N equal to, see this is actually equal to 1/Z + 2/Z

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2. If $f(n) = a^n$, $a \neq 0$,

$$Z(f(n))(z) = \sum_{n=0}^{\infty} a^n z^{-n} = \sum_{n=0}^{\infty} \left(\frac{z}{a}\right)^{-n}, \quad |z| > |a|.$$

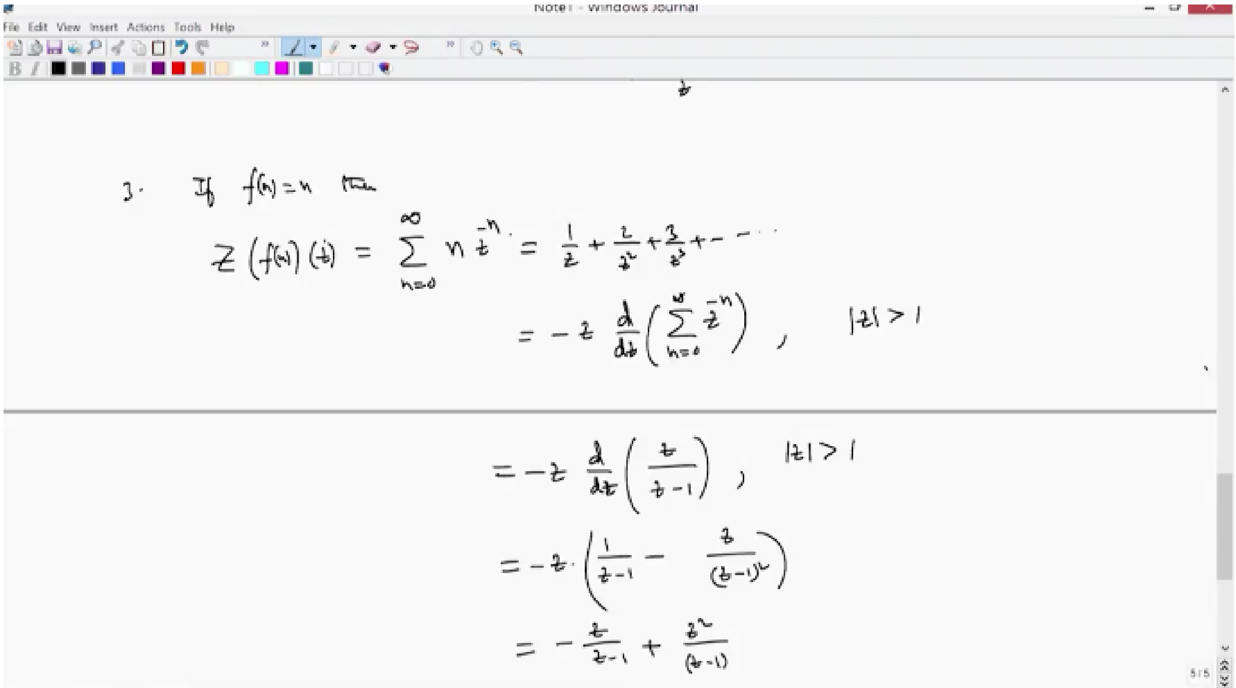
$$= \frac{1}{1 - \frac{a}{z}} = \frac{z}{z-a}, \quad |z| > |a|.$$

3. If $f(n) = n$ then

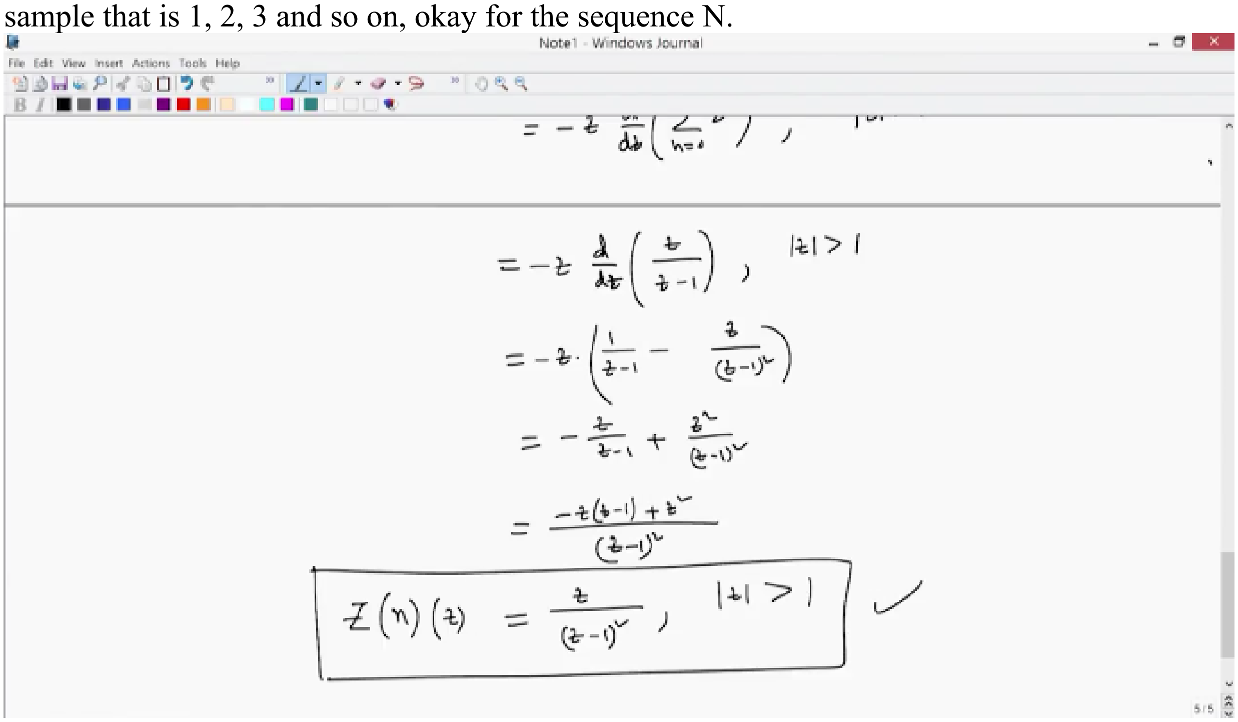
$$Z(f(n))(z) = \sum_{n=0}^{\infty} n z^{-n} = \frac{1}{z} + \frac{2}{z^2} + \frac{3}{z^3} + \cdots$$

4 / 5

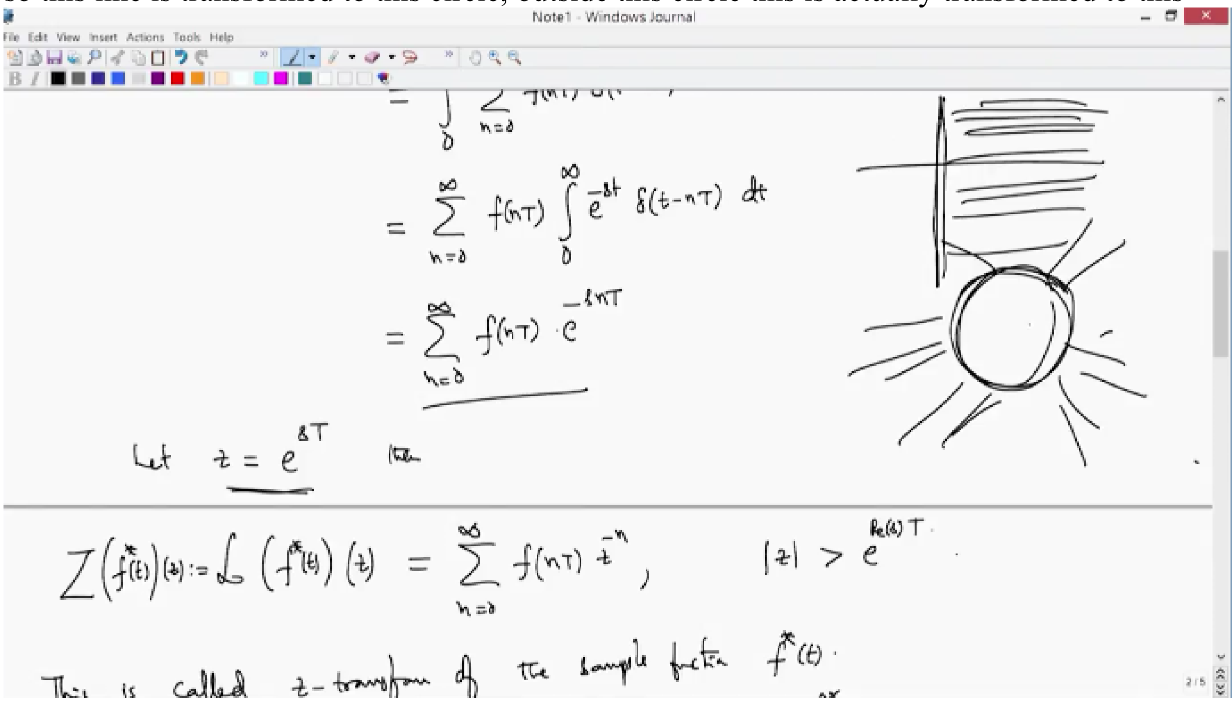
square + 3/Z cube and so on, so this is a series, this you can think of as you can take Z out, so what is this one? This is actually equal to, so I write directly if I write –Z times D/DZ of this series, N is from 0 to infinity Z power –N, so this is geometric series which is valid for mod Z is greater than 1, okay, for greater than 1 this series is finite we know that, okay, this series is valid when this is the case, as uniformly convergence you can do the term by term derivatives, so if you do the term by term derivative what you get is –N, -N comes out Z, Z power -N -1, so that Z Z goes you left with only this one, okay, so this is exactly, so you can differentiate this term by term, and what you get? So this is -Z times D/DZ of this we know already that this is Z power -N is Z divided by Z-1, so this is valid, mod Z greater than 1, so this is -Z times, so differentiate with respect to Z you have 1/Z-1 and you have + Z times 1/Z-1 square, Z-1 there's -1, so you have a –Z here, -Z/Z-1 square is its derivative, so you have –Z/Z-1 and you have + times Z square/Z-1 whole square.



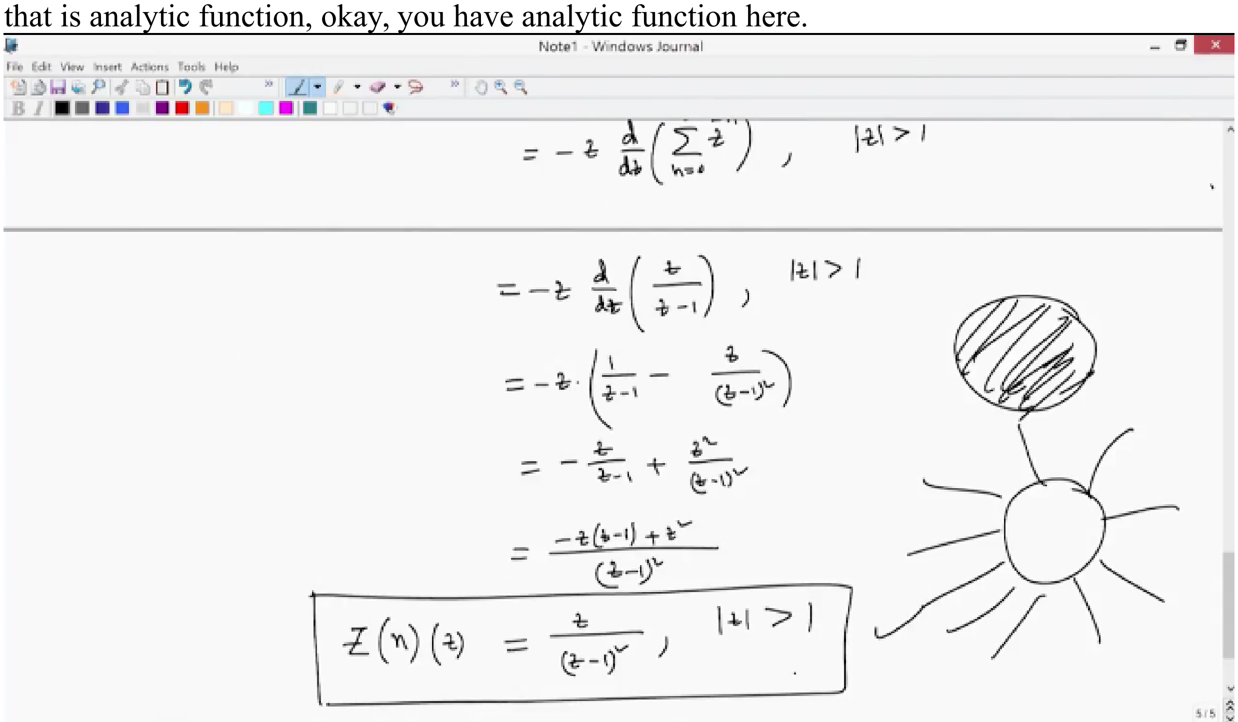
So this you can write it as Z-1 whole square and you have here Z into Z-1 + Z square, so Z square Z square goes so you have Z over Z-1 whole square, as mod Z is greater than 1 this is your Z transform (n) as a function of Z, okay, so this is how you can get your Z transform of sample that is 1, 2, 3 and so on, okay for the sequence N.



So you can go on doing given sequence, so we can find the Z transform like this as a function of Z, it's a complex valued function which is usually valid outside some interval, so outside some circuit, okay, so this is because as you see we have derived this from the Laplace transform and the convergence, when you have Laplace transform which is valid, this is over which basically this is a Laplace transform is convergent integral is analytic function in this interval, and if you consider this line, this line is transformed into a circle here, okay, a circle here because of this is Z = E power ST, E power ST, E power ST real part of S, E power real part of S is, this is transformed to a circle here because of mod Z, okay, mod Z is greater than 1, so this line is transformed to this circle, outside this circle this is actually transformed to this



one, this half plane that is where is analytic is actually transformed to outside this circle, so in that sense that's, for that reason you are always getting mod Z is greater than some 1 or some constant for these simple functions, so outside this circle you have the complex valued function that is analytic function, okay, you have analytic function here.



So this is how you can get your Z transform, we will see in the next video with many more examples and how to find its inverse also, and then its properties of this Z transform, so these properties are exactly analogous to your Laplace transform, exactly the same properties of Laplace transform you will see its versions in the discrete versions in the Z transform, and then we make use of them in the applications of solving applications where you solve our difference equations, instead of solving differential equations you discretize the differential equation you get a difference equation or recurrence relation type of equations that's where you can use the Z transforms and get your solutions, okay as a sequence. Thank you very much.

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