NPTEL
NPTEL ONLINE COURSE
Transform Techniques for Engineers
Evaluation of Integrals by Laplace Transform
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Welcome back, this could perhaps be the last video on applications of Laplace transform. So far we have seen applications of solving, applications of Laplace transform solving the ordinary differential equations with initial conditions.

So far we have seen only right-hand side, when you consider the ordinary differential equations with a non-homogeneous term being a continuous function or elementary function that is expressed, that is as a function that is defined as a single function for all T positive and you have solved them with initial and boundary, initial conditions or boundary conditions you know how to solve this simple ordinary differential equation by Laplace transform technique.

And then we move on to have applications of this Laplace transform for partial differential equations, and could solve typical, one of the typical equations which you have seen which is a hyperbolic type, which is a wave equation and heat equation which is of parabolic type, and these are basically it involves, both the equations involve time, that means initial value you have to provide, initial value or values you have to provide for these boundary value problems, initial boundary value.

So when you provide these initial conditions you say that this is initial boundary value problem for the partial differential equation which is either hyperbolic type or parabolic type, so mostly we have seen wave equation or general hyperbolic equation we have seen with initial conditions when you provide how to solve in certain domains and also a heat equation mainly, we have considered heat equation only for, as a typical equation for the parabolic type for which you have given initial condition and boundary conditions and you could see how you get the solutions by the Laplace transform.

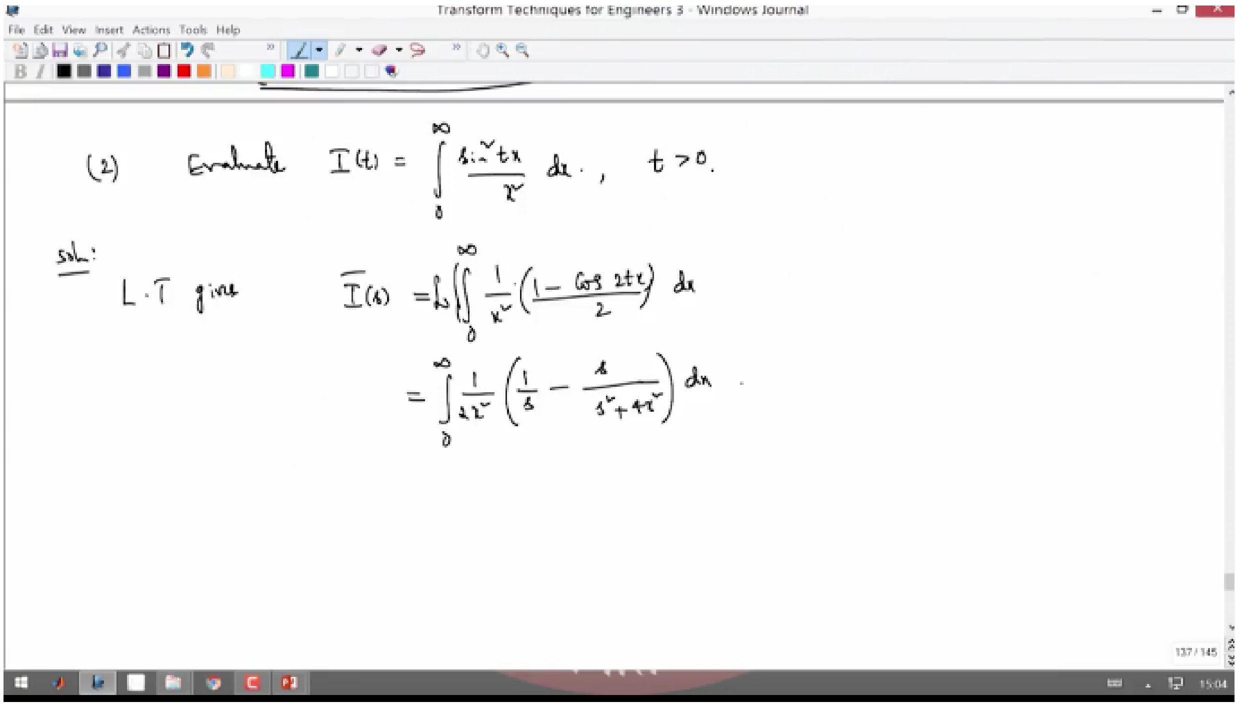
And it's tempting to apply the same technique to other type which is elliptic type of equations, for example Laplace equation if you consider, for elliptic type of equation if you provide so when you apply the Laplace transform for example Laplace equation you have UXX and UYY = 0, when you apply with respect to X variable Laplace equation is UXX + UYY = 0, so 2 derivatives you have and if you apply Laplace transform to X variable you need U(0,y) and UX(0,y) these are the two initial values of U, and U and U derivative with respect to X at X = 0 you need, but when you provide these initial conditions, if you provide for the elliptic type of equation which is a Laplace equation and you may see that equations need not be a well posed problems you may have, you may not have a solutions and may not be unique, so what do you mean by a well posedness is basically when you have a solution which is there as a solution, and which is a unique solution, so if at all there is a solution that should be unique, it should be there as a solution and if you perturb your initial data your solution also will not, if you perturb little in your initial data your solution also should not deviate much, so corresponding to those initial values, so that is called stability, so that is also called continuous dependence on the initial values, okay.

So if this conditions, these three things having for your equation, for example solution, uniqueness, and continuous dependence on the initial data, you perturb little in your initial data your solution also will not deviate much, is also little, so that can be controlled with disturbance in your initial data, okay, so that's a continuous dependence on the initial data, so this continuous dependence one of these conditions will fail if you provide initial data to the elliptic equations, okay, so for Laplace equation if you provide initial conditions the problem is not well posed problem, so it's a ill posed problem, so you see, you have an examples we can give but it's not required so if you are studying, if you have studied already differential equations you know, you might have seen for a Laplace equation if you provide some initial conditions and you perturb very little in that initial data you see that solution deviates away from, so one solution is 0, other solution is you give some nonzero perturb very closed to 0 you give initial values, and you see that solution of this perturbed initial data you will see that the solution of the Laplace equation is away from 0, so much there's a lot of deviation from 0, so that means it's not stable, it's not ill-posed problem, so we don't solve, we don't apply the Laplace transform technique to solve elliptic type of equations with the initial data.

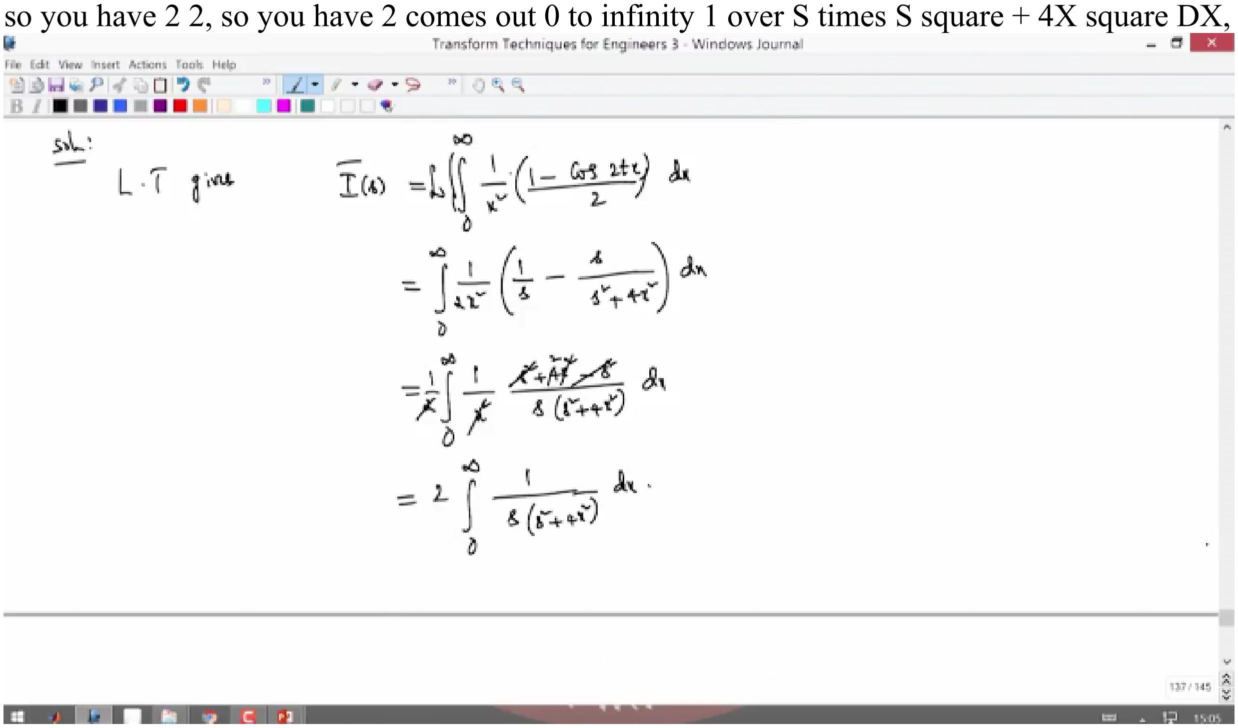
Even if you consider U(x,0) U(0,y) you take and other initial data you don't use it, you keep them as unknowns and at the end you try to invert it and get the solution and then you apply these conditions, and then so basically for the Laplace equation what is required is a boundary data, only boundary data is required if you use the boundary data to, in order to use the boundary data you can only use, so for example if you use the boundary data the solution you get, for the Laplace equation, the boundary value problem for the Laplace equation is well posed, if you provide only, if you provide only boundary data for the domain, for the Laplace, for the unknown function U(x,y), so you have to provide that data only on the boundary, so if you apply the Laplace transform to the X variable in the Laplace equation you may need U(0,y) and UX(0,y), so these are the two quantities you may require.

For example U (0,y) you use it it's given as a boundary data, okay, but UX(0,y) is not known you assume that it is an arbitrary function of Y and you as though it is given function, so it's an unknown function you keep it as it is and apply the Laplace transform technique and get the invert it and get the solution in terms of this unknown function.

And finally you use other boundary data whatever is given other boundary data instead of UX(0,y) you can use it to get this unknown function but still you may have, may get into trouble if you apply if you consider such problems, such boundary value problems only and then use this Laplace transform technique to get the solution, so when you apply this Laplace transform to the boundary value problem for the elliptic equation which is a Laplace equation you may end up difficulties because it's only boundary value problem, so Laplace transform you need to know when you apply the derivative, Laplace transform to the derivative, you may require these initial data, so it is advisable to use the Laplace transform technique to initial boundary value problems for the partial differential equations which you see only for hyperbolic and parabolic equations. So we'll not do any problems for the Laplace equation which is elliptic type because a problem you usually not well posed problem, okay.
So and then we have seen other applications, for example how to solve certain type of Volterra type of integral equations, you can solve by Laplace transform technique in the last video, also you have seen how to evaluate certain integrals involving a parameter, okay, so we will continue to do that and also we'll get back to the ordinary differential equations when the right hand side being some kind of piecewise continuous functions, for example heavyside function or in fact its generalized function for example delta function when you have, how do you get, we have not seen what is a Laplace transform of delta function, what is the Laplace transform of heavyside function, we have seen only Laplace transform of heavyside function which is anyway 1/S and you have not seen Laplace transforms of the delta function which is a Laplace transform of and of generalized function, okay, so we'll do that and then solve some ordinary differential equations when the right hand side, non-homogeneous term being non continuous or for example piecewise continuous function or a generalized function, for example delta function, okay.
So when you have this kind of problems when it comes in the applications, Laplace transform is a very handy, comes very handy to solve this kind of problems, otherwise these problems cannot be solved in the usual techniques of differential equations, okay.
So we'll see, we'll continue to do this evolution of these integrals with the second example, we'll do this here, evaluate I(t) as integral 0 to infinity, sine square TX/X square DX, so how do I evaluate this IT, and T being positive, okay, so T is positive, so that you can apply Laplace transform to this, okay, Laplace transform gives a solution is, Laplace transform gives I bar(s) when you apply the Laplace transform which is equal to, when you apply the Laplace transform you could take this inside, assume that you can take and you have 1/X square DX times, let's write, times so when you apply this sine square TX, sine square TX you don't know really so we will try to write this as Laplace transform for this 1/X square, 1 - cos 2TX/2 right, so you have by 2 is your sine square DX, so this if you apply Laplace transform for this whole thing we could take it inside Laplace transform of this that is 1, 1/S X square -, 1/X square is common or 1/X square is common you apply the Laplace transform to 1/2, 1/2X square, 1 is 1/S-cos thing is, cos 2TX is S square divided by S square, S divided by S square + 4T square, so that is the Laplace transform of cos 2X, cos 2TX, okay, so this should be X square, so this is DX.

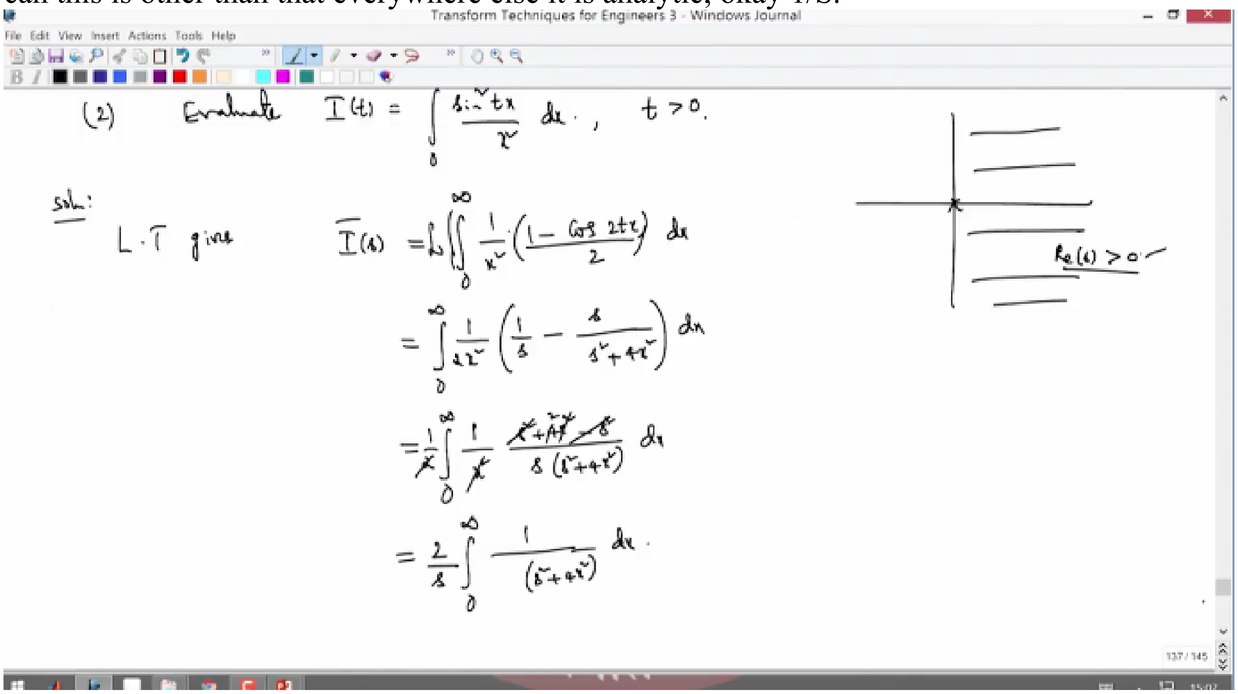


So this one you try to evaluate now that T is removed, now you can evaluate this as 1/S, S is nothing to do with this integral so you have 1/S, you simplify further so what you see is 1/2 comes down, 1/X square by S(S square + 4X square) so you have S square + 4X square - here S square, so you have S square S square goes and you have DX, so this X square X square goes so you have 2 2, so you have 2 comes out 0 to infinity 1 over S times S square + 4X square DX,

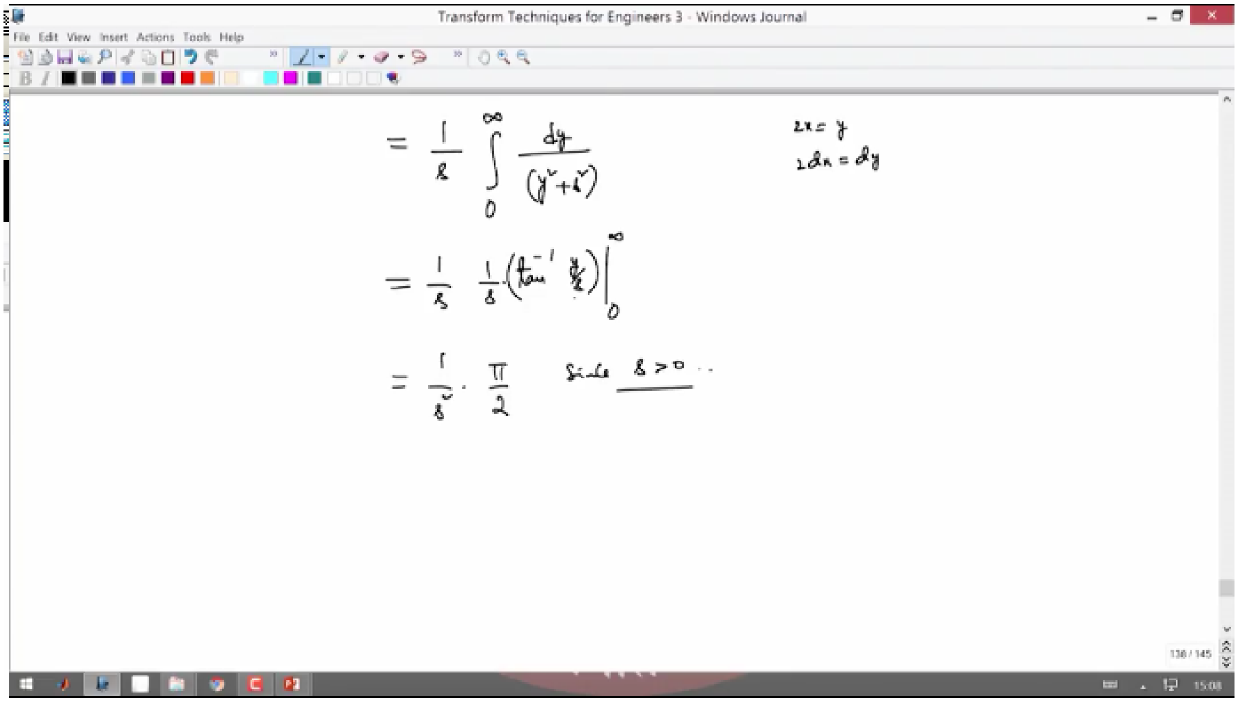


so this is the integral you need to evaluate, so S is nothing to do with this so you can put it outside, and you see that this is going to be, how do I evaluate this? This you can write it as 0 to infinity, 1/X square so that means, so you take 2X as Y, for example if you take 2 DX as your DY, so I write DY, so 2DX as your DY, and you have this will be Y square + S square, and

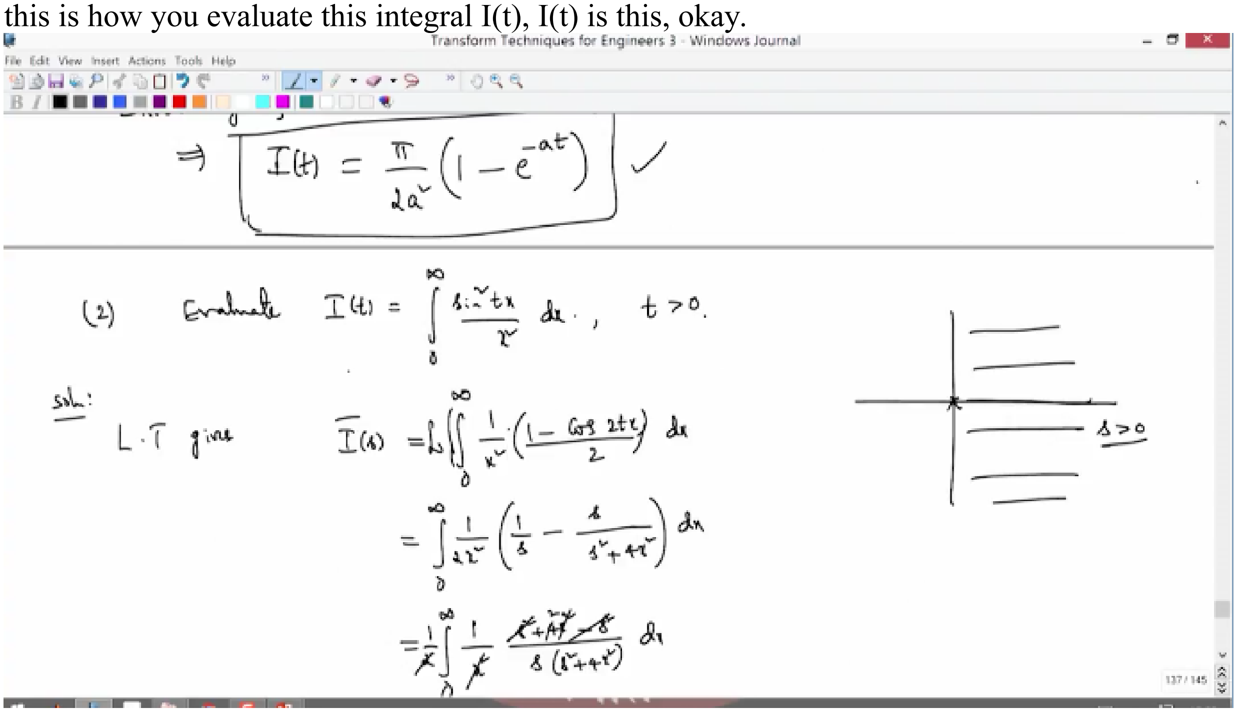
when you substitute X = 0, Y is 0, X = infinity, Y is infinity, so limits won't be changed so you have this, so this is equal to 1/S this if you do this you have tan inverse, this is going to be 1/S times tan inverse Y/S and for which you apply the limits you get 1/S square, tan inverse Y/S is simply you get pi/2 + or – pi/2 depending on the value of, depending on S, okay, depending on this value of S, okay, so S is positive you have pi/2 otherwise –pi/2, if S is negative.
So we will see what is your S? S is, when you apply this for example you applied for this one so S is when you apply 1/S in the complex plane, you have 1/S is we applied the transform for 1 which is 1/S, so 0 is the point so you have so S is always real part of S is positive, okay, so you can this is other than that everywhere else it is analytic, okay 1/S.



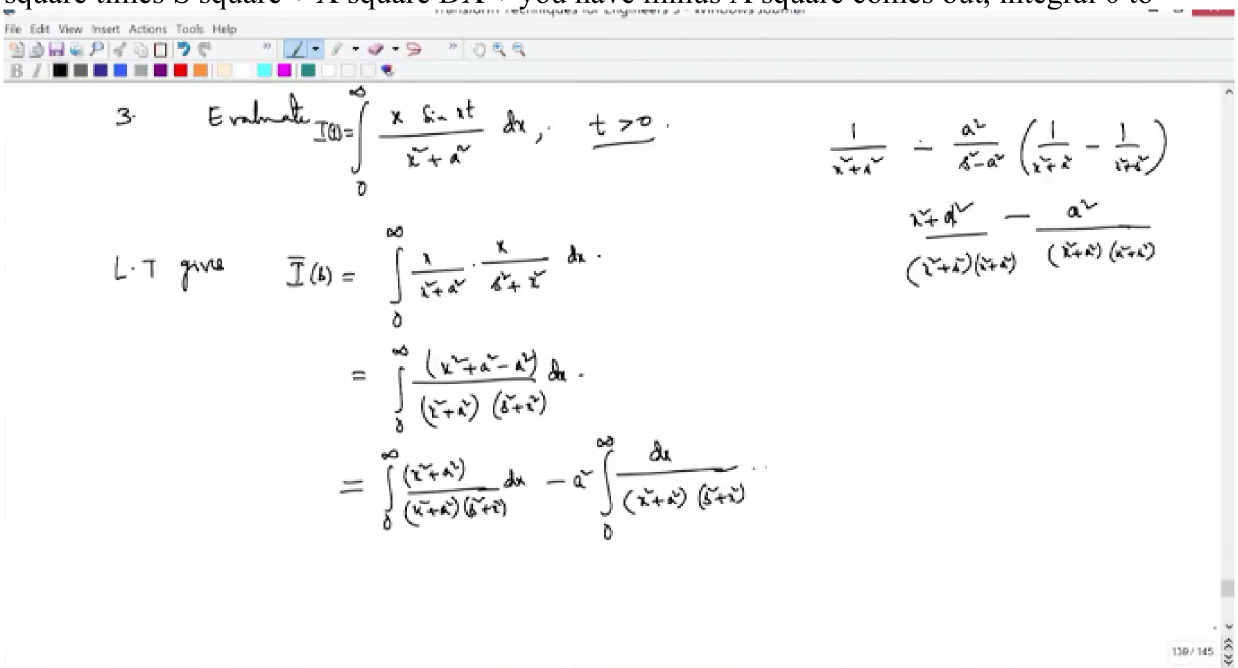
Similarly S/S square + 4S square that is also in this 1/2 plane real part of S is positive in this, it’s analytic, so we can safely see that S is positive, so S is only positive, so real part of S is positive means if you are on the real line S is positive, so you can think of this as S is positive, so let me not use this so we have this, since S is positive you have this.



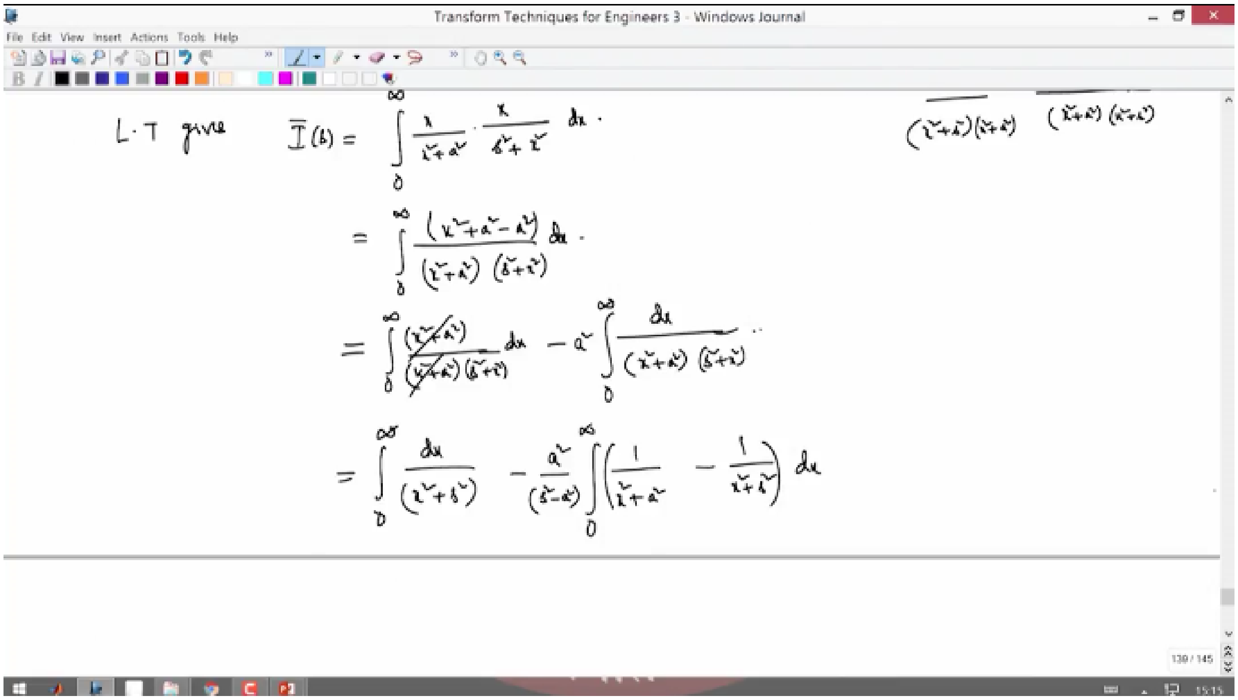
So now we take the inverse transform this is your I bar(s), inverse transform now gives I(t) that is the required integral value is pi/2 times T into just T, right, this is your T, so T is 1/S square, Laplace transform T is 1/S square so inversion is T, so this is what you have for T positive, so this is how you evaluate this integral I(t), I(t) is this, okay.



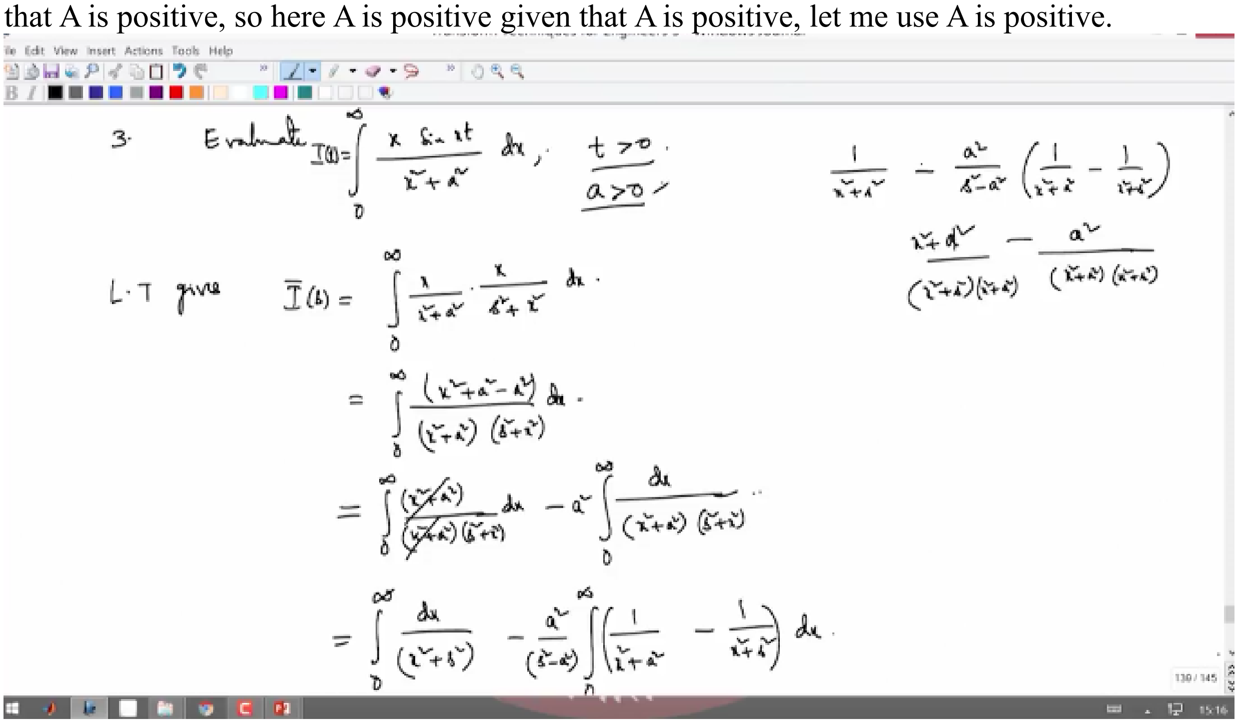
So we'll do one more example, for example let me, so let's evaluate integral 0 to infinity X sine XT/X square + A square DX, what is this value? So T is positive, of course T should be positive, this parameter T which is positive I(t) for example if I call this, so Laplace transform gives I bar(s) if you apply and you see this is going to be X divided by X square +A square and this is sine XT is X/S square + X square DX, this is actually X square/X square + A square times S square + X square DX, how do I do this? 1/X square + A square - A square/S square – A square times 1/X square + A square – 1/X square + S square, so this is X square + A square, X square + S square, so you have - X square goes, so you have X square - A square goes, so you have – A square 1/X square + A square X square + A square, so what I do is I just add A square and -A square and you get this if you split it 0 to infinity X square + A square/X square + A square times S square + X square DX + you have minus A square comes out, integral 0 to



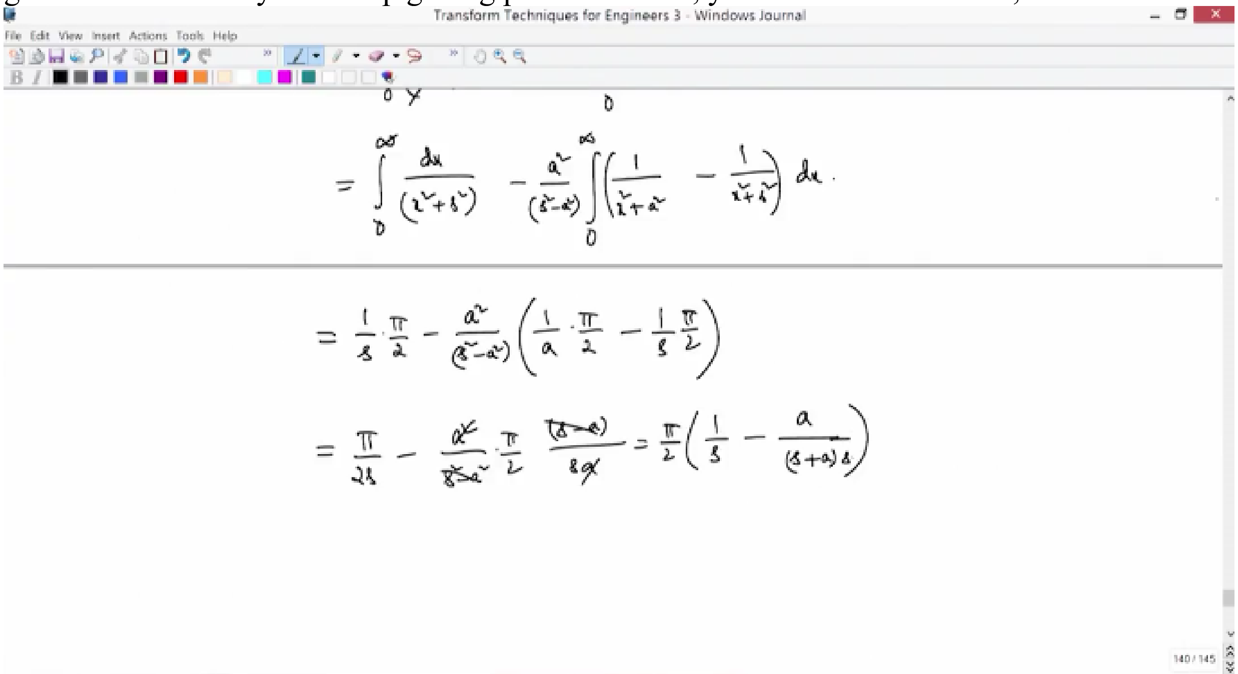
infinity DX/X square + A square times S square + X square, so this you use partial fractions here, so this gets cancelled so you get 0 to infinity DX/X square + S square and then this one will be A square, if you do the partial fractions here this will be X square + A square -1/X square + S square, then you have A square A square goes, so you have S square - A square, comes on the top, so you have this, this is your DX, and this is your let's put it outside, so this is



together DX, so this is equal to, you can now integrate this as 1/S times pi/2 because S is positive again, because you applied for sine XT, so when you do that S is always positive, so you have this -A square/S square -A square times, here 1/A times pi/2, so because A is positive again, right, so depending on A is positive or negative, so minus here again 1/S pi/2, so assume that A is positive, so here A is positive given that A is positive, let me use A is positive.



Then I have this otherwise I may have to replace -1/A times pi/2, if A is negative, okay, so if I use A is positive this is what is the case, because I use this tan inverse, tan inverse X/A, so if I use that I have this, this for each of these integrals, okay, so this is equal to pi/2S - A square/S square – A square pi/2 comes out as a thing and you have SA times S-A, so A A goes here, S-A goes here once and you end up getting pi/2 times 1/S-, you have A here A/S+A, S into S+A

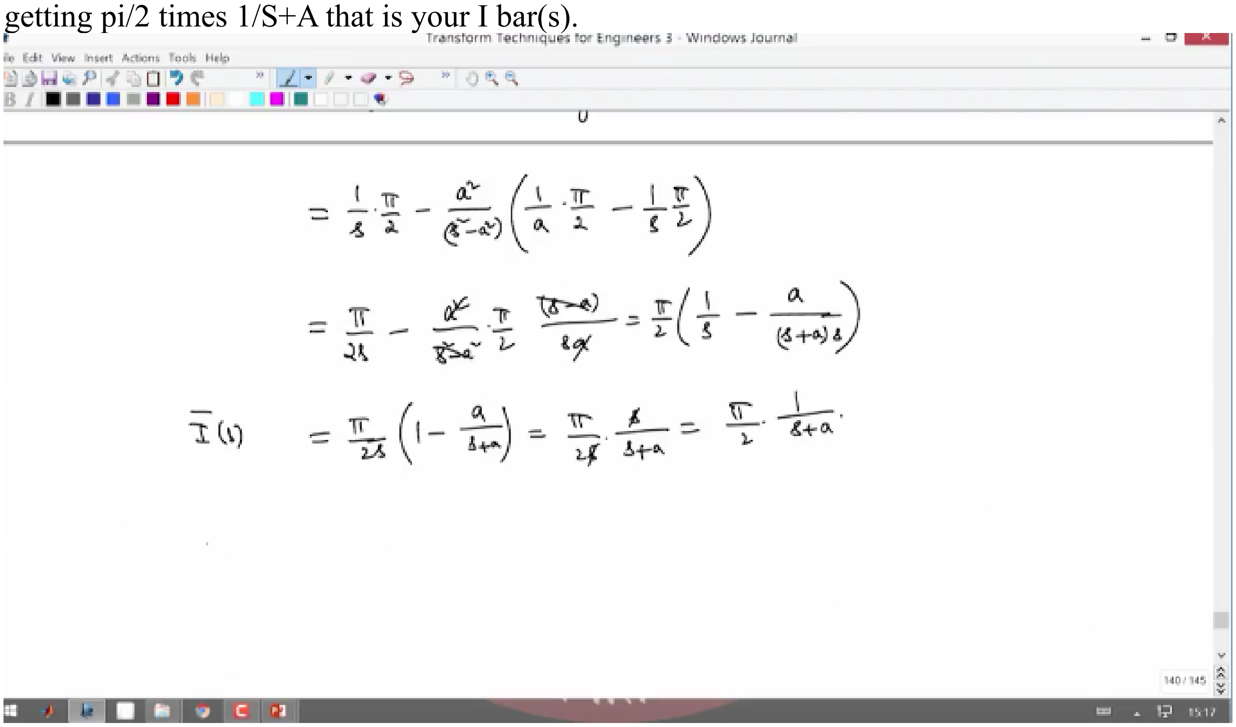


$$= \int_0^\infty \frac{du}{(u^2+s^2)} - \frac{a^2}{(s^2-a^2)} \int_0^\infty \left(\frac{1}{u^2+a^2} - \frac{1}{u^2+s^2} \right) du .$$

$$= \frac{1}{s} \cdot \frac{\pi}{2} - \frac{a^2}{(s^2-a^2)} \left(\frac{1}{a} \cdot \frac{\pi}{2} - \frac{1}{s} \frac{\pi}{2} \right)$$

$$= \frac{\pi}{2s} - \frac{a^{\cancel{2}}}{\cancel{s^2-a^2}} \cdot \frac{\pi}{2} \frac{\cancel{(s-a)}}{s\cancel{a}} = \frac{\pi}{2} \left(\frac{1}{s} - \frac{a}{(s+a)s} \right)$$

rather, okay, so this is what you have, so this you can write it like pi/2 times so 2S times, take S out and you have 1 – A/S+A so this is pi/2S times S+A-A that is S/S+A, S S goes so you end up getting pi/2 times 1/S+A that is your I bar(s).

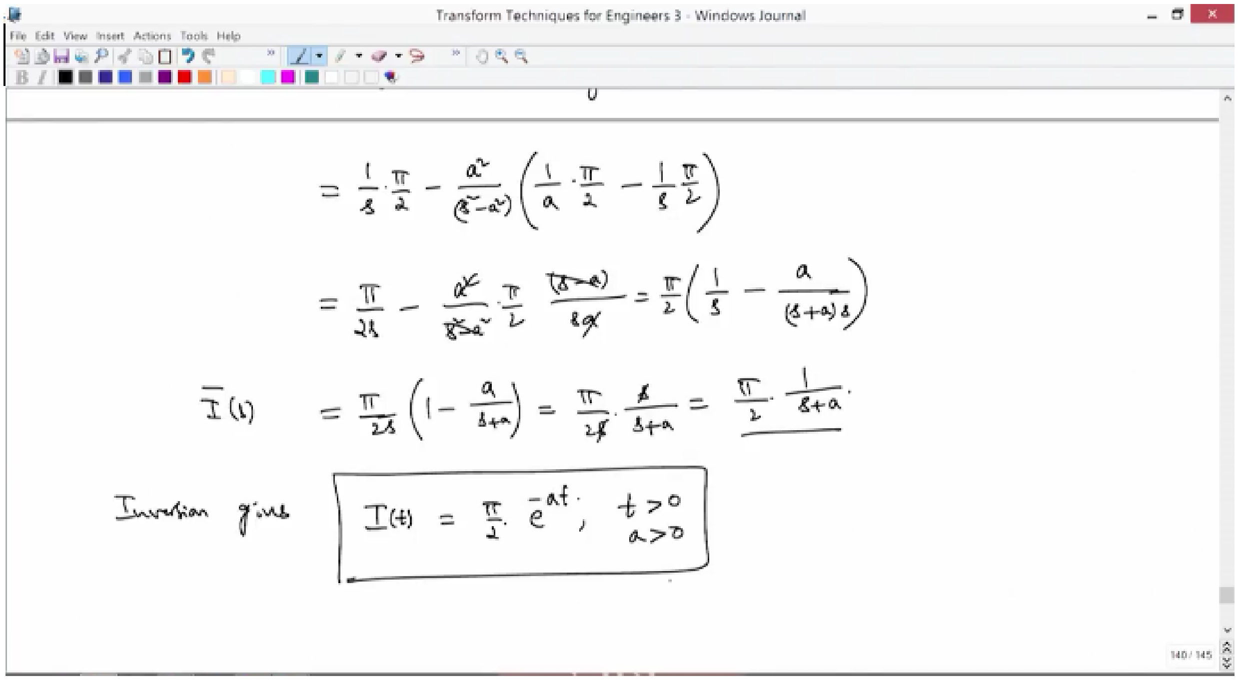


$$= \frac{1}{s} \cdot \frac{\pi}{2} - \frac{a^2}{(s^2-a^2)} \left(\frac{1}{a} \cdot \frac{\pi}{2} - \frac{1}{s} \frac{\pi}{2} \right)$$

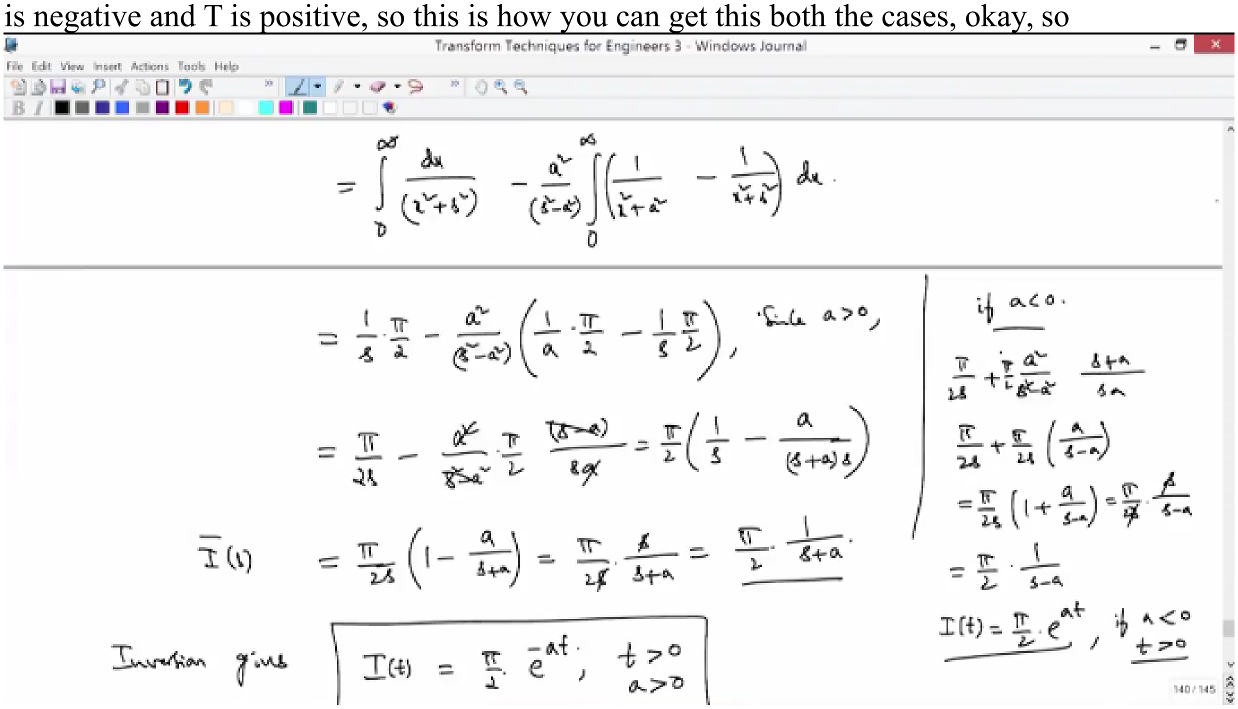
$$= \frac{\pi}{2s} - \frac{a^{\cancel{2}}}{\cancel{s^2-a^2}} \cdot \frac{\pi}{2} \frac{\cancel{(s-a)}}{s\cancel{a}} = \frac{\pi}{2} \left(\frac{1}{s} - \frac{a}{(s+a)s} \right)$$

$$\bar{I}(s) = \frac{\pi}{2s} \left(1 - \frac{a}{s+a} \right) = \frac{\pi}{2\cancel{s}} \cdot \frac{\cancel{s}}{s+a} = \frac{\pi}{2} \cdot \frac{1}{s+a} .$$

So now inversion will give, inversion gives I(t) that is integral is equal to the inversion of this is pi/2 times E power –AT, so this is what it is, this is what is your integral for T positive and A positive, if A is negative you may have to take a minus sign here okay, so this is for A positive,

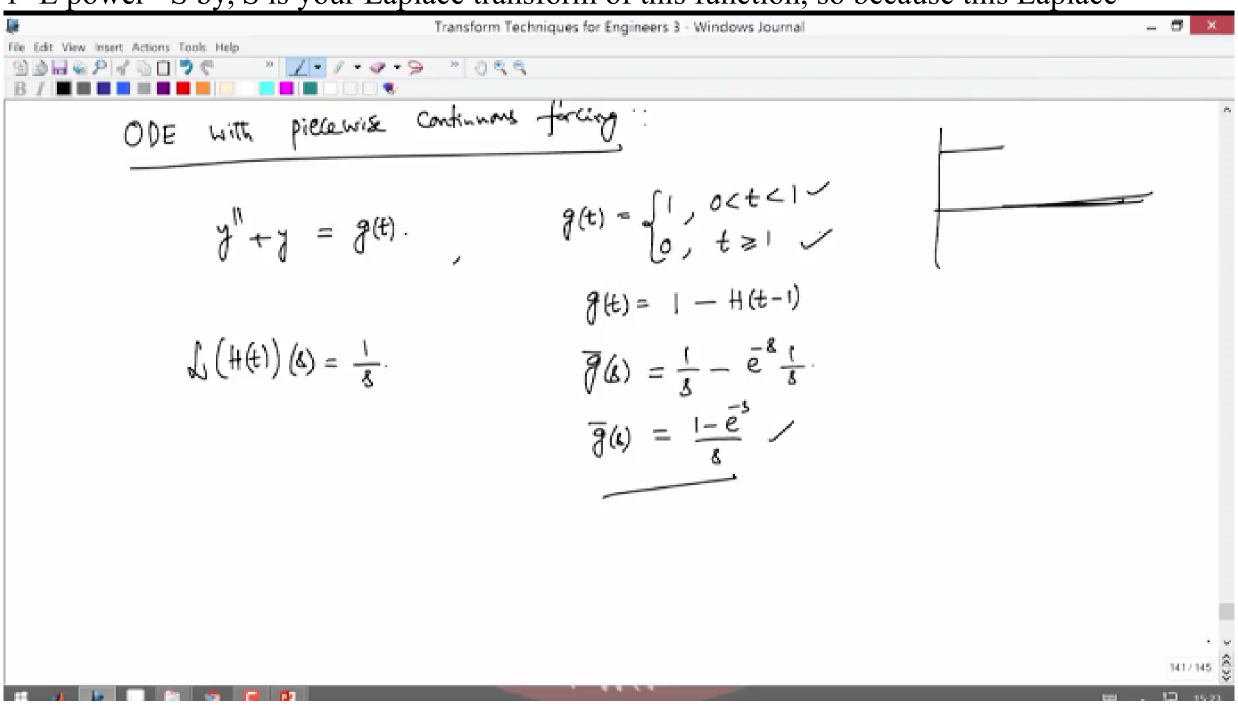


if side-by-side you can do, if this is, A is since, rather since A is positive here, if A is negative so what is the corresponding thing we have is pi/2S -A square/S square -A square times pi/2 comes out again here and you have a minus minus plus, so you have S into A and you have S+A, so pi/2S + pi/2S times and you have, what you have is A goes, A divided by S-A. So you have pi/2S times 1+A divided by S-A that is pi/2S times S –A, S-A + A that is S, so S S goes pi/2 times 1/S-A, so inversion will give in this case pi(t) is pi/2 times E power AT, if A is positive, A is negative and T is positive, so this is how you can get this both the cases, okay, so



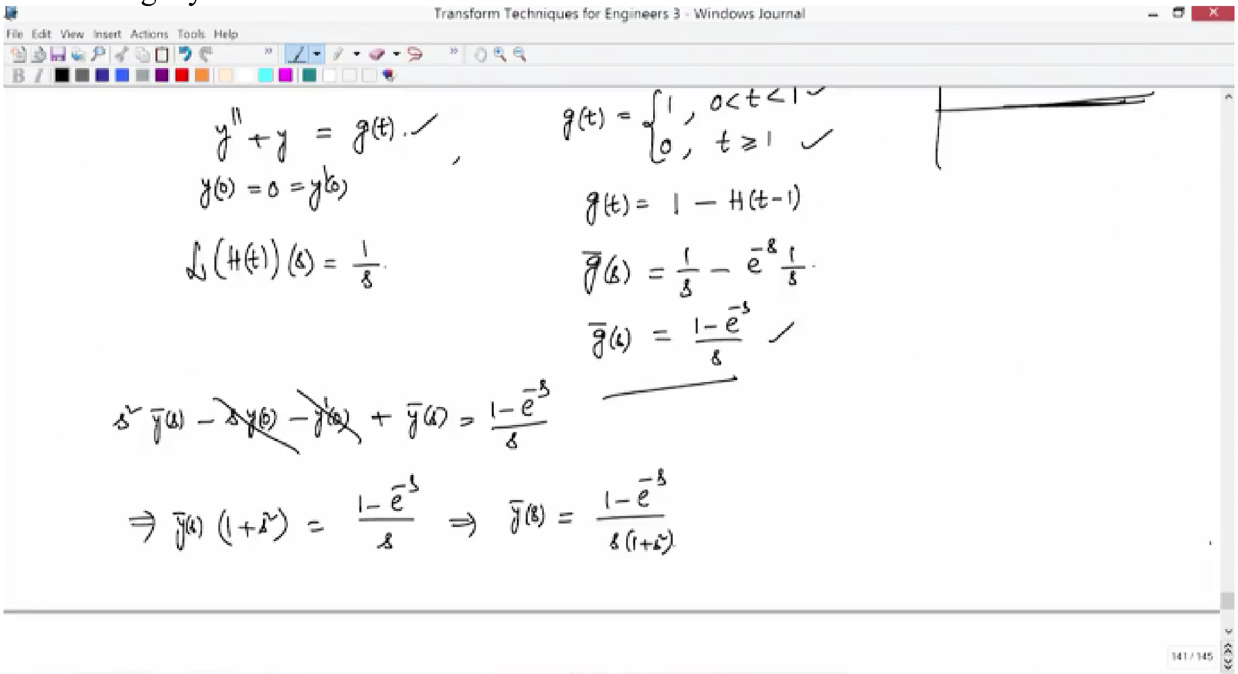
I can remove this restriction A positive in the problem, if A is positive only this one, if A is anything depending on this if A is positive you get this, if A is negative you have this result, so this is how we can evaluate certain some integrals involving a parameter, that parameter should be a range of that parameter should be 0 to infinity so that you can apply Laplace transform, and the idea is you can bring this Laplace transform integral, that integral you can put it inside, change the order of integration or rather the Laplace transform you can put, take it inside the integral if that is possible you can do this technique to evaluate certain integrals like this.

So let's get back to this ordinary differential equations with nonhomogeneous term being not continuous or piecewise continuous, or not usual functions, okay, so let's ODE with non-homogenous with piecewise continuous forcing, so what does it mean? So you have, you consider, for example so if you simply consider Y double dash + Y = some G(t), so G(t) is a continuous function you can apply directly your Laplace transform and get your G bar(s), okay, on the right hand side, if your right hand side G(t) for example if it is like this 1, if it's T is less than 1 and then otherwise 0, T is greater than 1, so what you do? So this is, if you plot this actually what you have is this is 1 here and then 0 here, so it's a discontinuous function, okay, it's a piecewise continuous forcing, still you can apply the Laplace transform here if you apply the Laplace transform here so idea is you know what is the Laplace transform of H(t), right, H(t) as a function of S is simply 1/S you have seen, okay, so we make use of this as here in this case, because you can write G(t) as 1- H(t-1), if I use this if T is between 0 to 1 and this is going to be 0 so you have 1 that is verified, and if it is T is greater than 1 this is 1, so 1-1 0, is always 0, okay, so this is you can always write like G(t) in this fashion for which you can apply the Laplace transform G bar(s) is 1/S- and this we have seen, this when you see this property of the Laplace transform and you see that is going to be –E power - this constant 1 times S, it's the Laplace transform of H(t) that is 1/S, so this is exactly what your E power, 1-E power –S, sorry, 1 -E power –S by, S is your Laplace transform of this function, so because this Laplace

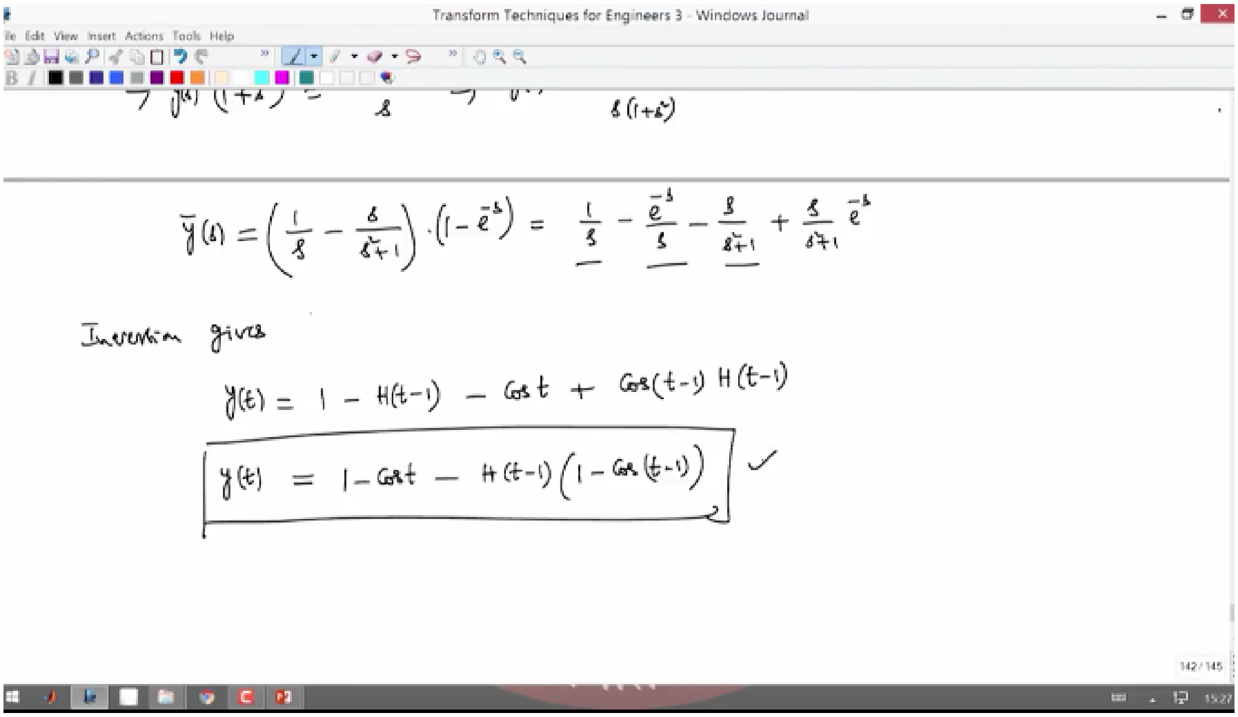


transform exists you can apply here and get your solution, so S times S square Y bar(s) – S times Y(0) - Y dash(0) so this is what is the Laplace transform when you apply for the second derivative.

So of course if you provide the initial conditions as 0, and Y dash(0) and these things can be 0 and which is equal, +Y bar(s) for this one, this term, you have G bar(s) which is for this if your G is here you can write E power –S/S, so this implies 1 + S square times Y bar(s) = 1- E powere power –S/S so this implies Y bar(s) = 1 – E power –S/S times 1 + S square, so this you try to invert and get your solution.



So if you partial fractions if you use Y bar(s) I can write like 1/S – S/S square + 1 as 1 over S(1+S square) I can write like this into 1 -E power –S, so this is if you invert this so if you invert this one you get, so inversion gives here now you can apply the inversion, inversion gives Y(t) which is a solution of this differential equation with G being this, with this initial data so this is your initial value problem for this solution is Y(t) is 1 - E power –S/S you have already seen what is its Laplace transform of G that itself is your G, okay, so when you invert that is G(t) okay, minus, or you can split it so you can separate them and write 1/S -E power – S/S –S/S square + 1 and then +S/S square + 1 times E power –S, so you can apply now each of them separately, Laplace inversion you get 1/S is 1 minus this will be 1/S is 1 E power -S is H(t-1) right, if you actually see that is H(t-1) is E power –S/S, so you have this and here S/S square + 1 is a cos T, and the other one is here this becomes cos T-1 times H(t-1), because here this is the property this is a Laplace transform of this is E power –S times Laplace transform of cos T, okay, Laplace transform of this is same as this one, this is the property we have used this alone, okay, so if I use this Laplace transform of S is E power -S + S divided by S square + 1, so the inversion will be simply this one, so I use that property so you get this, so this is equal to 1 - cos T you can have H(t-1) common, and you can write like 1 and here -cos T-1, so this is

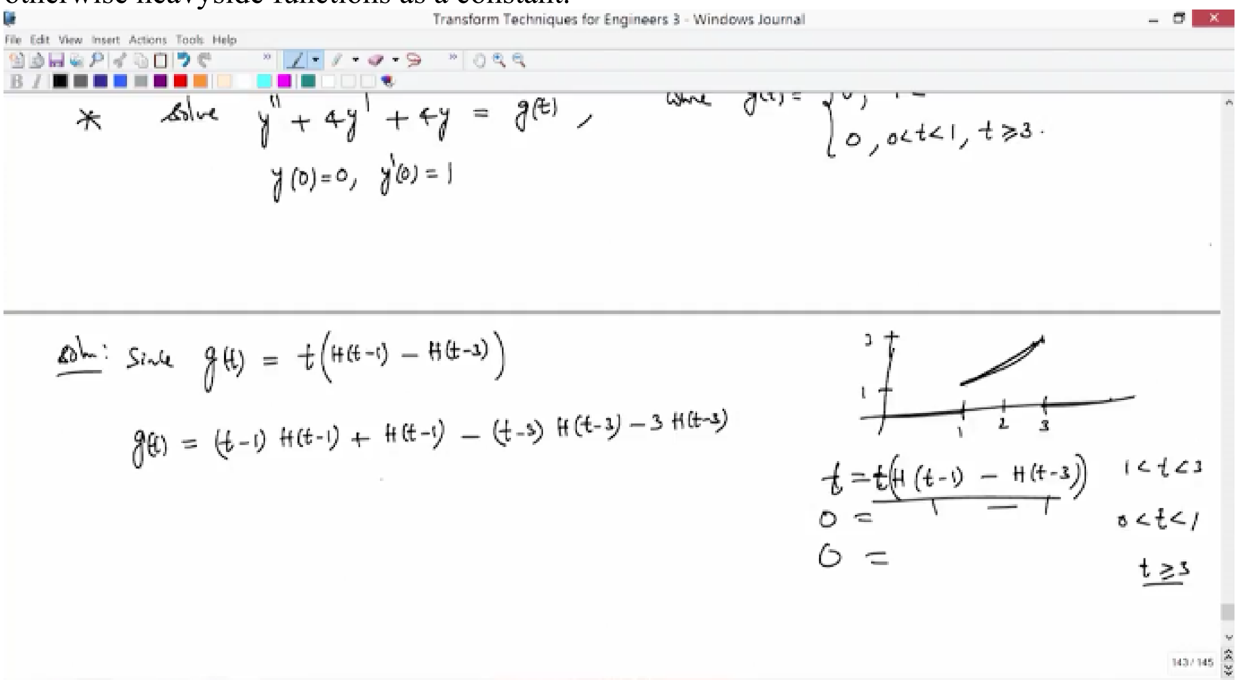


your solution of course you have a solution that is piecewise continuous function, you don't get it as a continuous function because Y double dash when you substitute finally because this is a piecewise function and you expect Y is also here you have a Y term you expect Y to be piecewise continuous function, okay, so this is all for T positive.

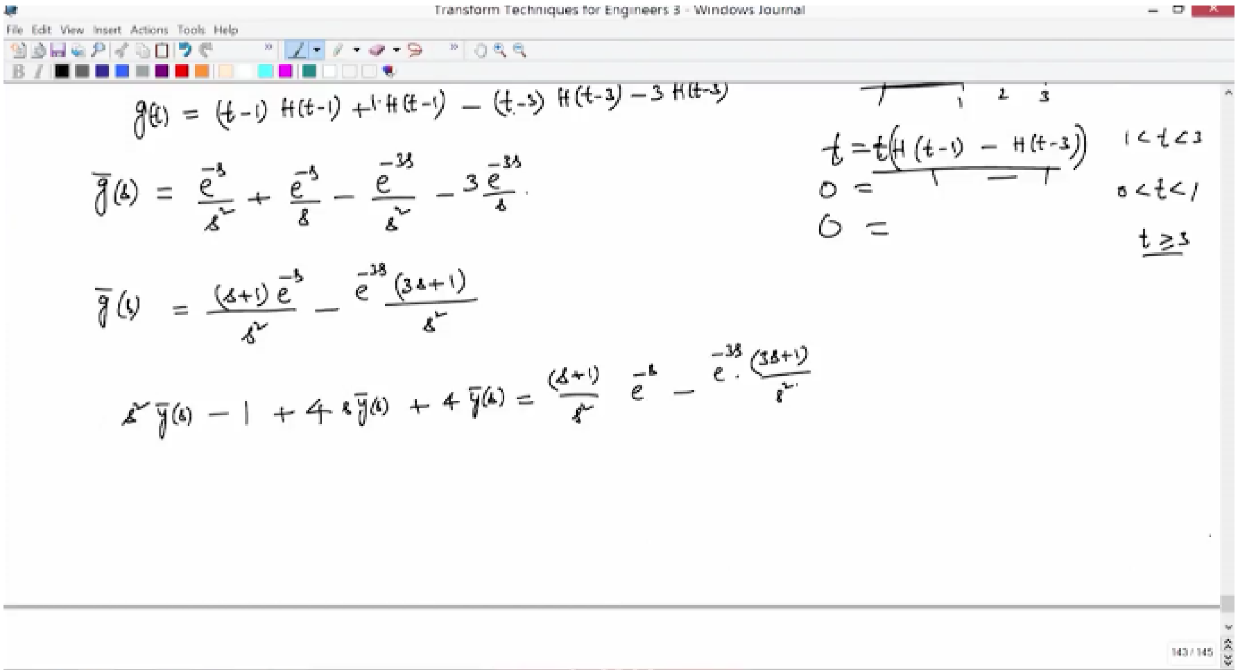
So this is how we can solve this ODE with right hand side being a piecewise continuous function, and not only this when you have a piecewise continuous functions like this if you are given you can always use, you can combine them with these heavyside function.

Let me give one more example, one more example let me do, let's solve Y double dash + 4Y dash + 4Y = G(t), where G(t) is if I use T between 1 T less than 3, 0, if T is between 0 to 1 and T is between, T greater than or equal to 3, so you have, this curve is it's between 1 and 3 it's 1, it's T, okay, so 1 and T is this is 1, so 1 to T this is the 3, up to 3, okay, so the linear function otherwise simply 0, so this is the curve you have, this is the curve, as the forcing curve you have and initial conditions you can give Y0 = 0, Y dash(0) = 1 let us say, so how do we solve? A solution you can apply this, so how do we write this G(t)? G(t) we can represent as, this you can easily see this is T times, if it is 1 here between 1 to 3, this is 1, and 2, 3, you can write this as H(t-1) – H(t-3), so what does this mean? This is a, if T is between 1 and 3 and you see that 3-1 so this is always positive, so this is 1- this is always at 0 so this is always 1, so for example this is always 1, this value is between T1 and 3 is always value is 1, okay, so if you multiply T both sides so you have T value here, okay, what happens when T is less than 1, T is less than 1 this is 0, and this is also 0, so multiplied with T is also 0 so you have 0, 0 if this is actually 0, if T is between 0 to 1 and T is greater than 1, again if T is greater than 3, or equal to 3 this is 1 - 1 that is also 0, okay, so all the cases you have so this is how you can represent.

So G(t) now I can represent as T times H(t-1) – H(t-3) with these heavyside functions you can represent since this is the case G bar(s) is T times H(t-1) you can apply Laplace transform that is E power S, A is 1 so S E power, my A is -1 so you have -S E power - S times Laplace transform T that is 1/S square, okay, and this one is again E power -3S Laplace transform by S square, no, you cannot apply, right T into H(t-1) you have to write it as T-1 times H(t-1) this is what we know, okay, so we should not write like this, so this before I do apply this Laplace transform I rewrite this function as in terms of T-1 times H(t-1) in the place of this, what is missing is that is extra is –H(t-1) so I make it H(t-1) as a plus term, I added it here I again T-3 times H(t-3) what I have is -T times H(t-3) I have a additionally + 3 times that I have to negate 3 times H(t-3), so I have written as T-1, argument of times, argument of this heavyside function, otherwise heavyside functions as a constant.

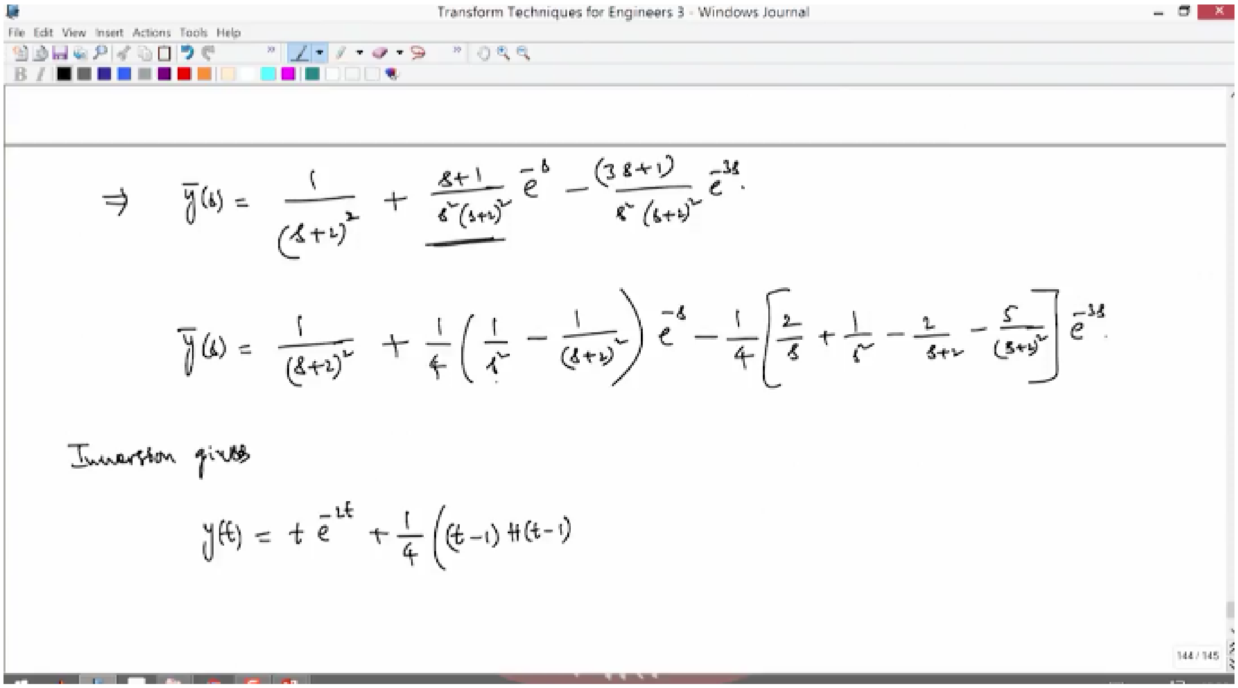


So here I can apply Laplace transform for G, so G(s) will be here this is E power –S/Laplace transform of T so that is S square + Laplace transform of this H(t-1), Laplace transform is E power –S/S because this is again you can think of 1 into Laplace transform of T-1, E power -S Laplace transform 1 is 1, 1/S okay, so here E power -3S/S square and here 3 times E power -3S/S, so if we actually combine them you get E power -S terms together you have S square S+ 1 times E power –S, and you have here E power -3S and you have again S square 3S+1 that is what you have, okay, that is your G bar(s) since this is the case you apply Laplace transform for the given equation now to see that S square Y bar(s) -S times Y(0), Y(0) is 0 so this term is 0 and -Y dash(0) so that is 1, okay, so 1 + 4 times this is 4 into Y dash, for this term if you apply the Laplace transform now, 4 times S into Y bar(s) –Y(0), Y(0) is 0 so I don't use this, this is this.



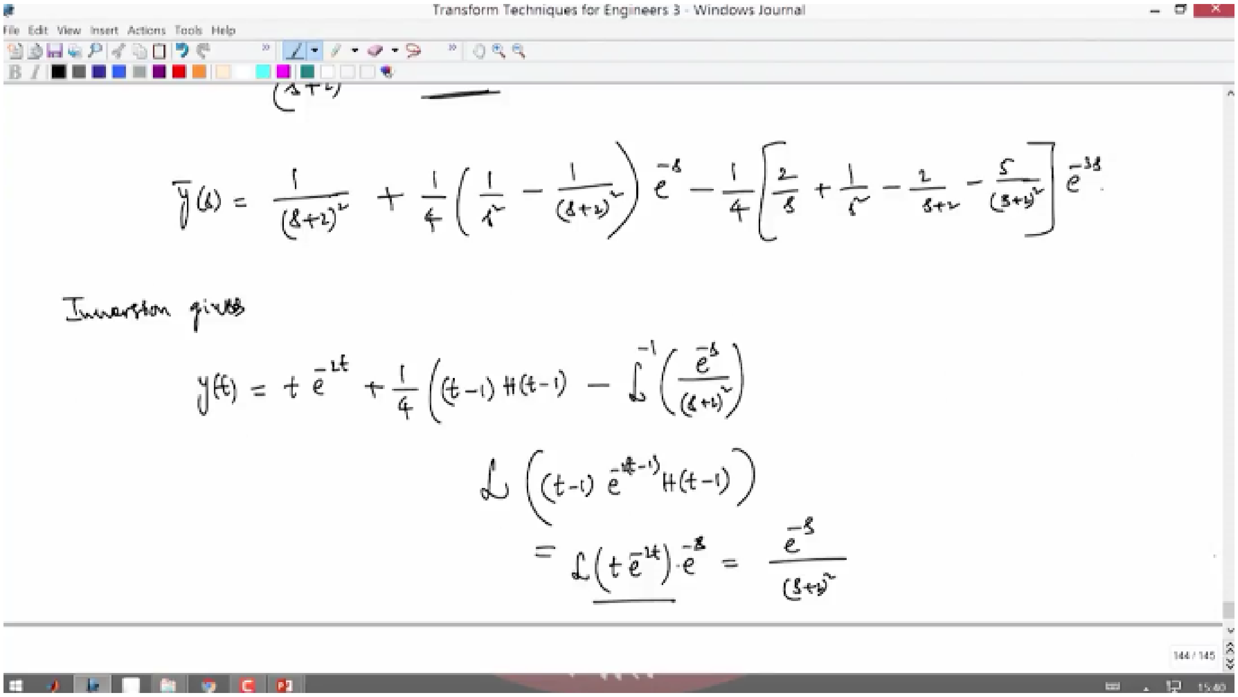
And then plus again 4Y the third time 4Y that is 4 times Y bar(s) which is equal to G bar(s) which is given here, so that is S+ 1/S square E power –S -E power -3S times 3S+1/S square, so you get this coefficient of Y bar(s) as S square + 4S + 4 = 1+ if you bring this -1 to the other side you have 1+ S+1/S square times E power -S -3S+1/S square times E power -3S, so you have this implies Y bar(s) is 1 over each term you divide with this, this is actually S +2 whole square, so if I divide this S+2 whole square + S+1/S square S +2 whole square times E power -S again here 3S +1/S square times S + 2 whole square E power -3S, so this is your Y bar(s), so inversion now gives you, inversion gives Y(t) as, so this you have to use your partial fractions before you do the inversion, so let me give you, let me take the partial fractions Y bar(s) as 1/S+2 whole square this one + this I can invert it so I keep as it is, this one I write this as 1/4 times 1/S square -1/S+2 whole square, so this will give me my S+1, so this quantity I can write like this by partial fractions in E power -S and here you can write again like 1/4 2/S + 1/S square -2/S +2 – 5/S+2 whole square, so you have to do get the partial fractions properly and if you do that you get this E power -3S.

So now you can apply your inversion, inversion gives Y(t) the solution as T times E power -2T which is the Laplace inversion of 1/S+2 whole square + 1/4 times, 1/S is T, simply T – 1/S+, sorry, 1/S square into E power -S is T times H(t-1), right, rather you should have T-1 times H(t-1) Laplace transform is 1/S square times E power -S okay, that's what we have seen just now

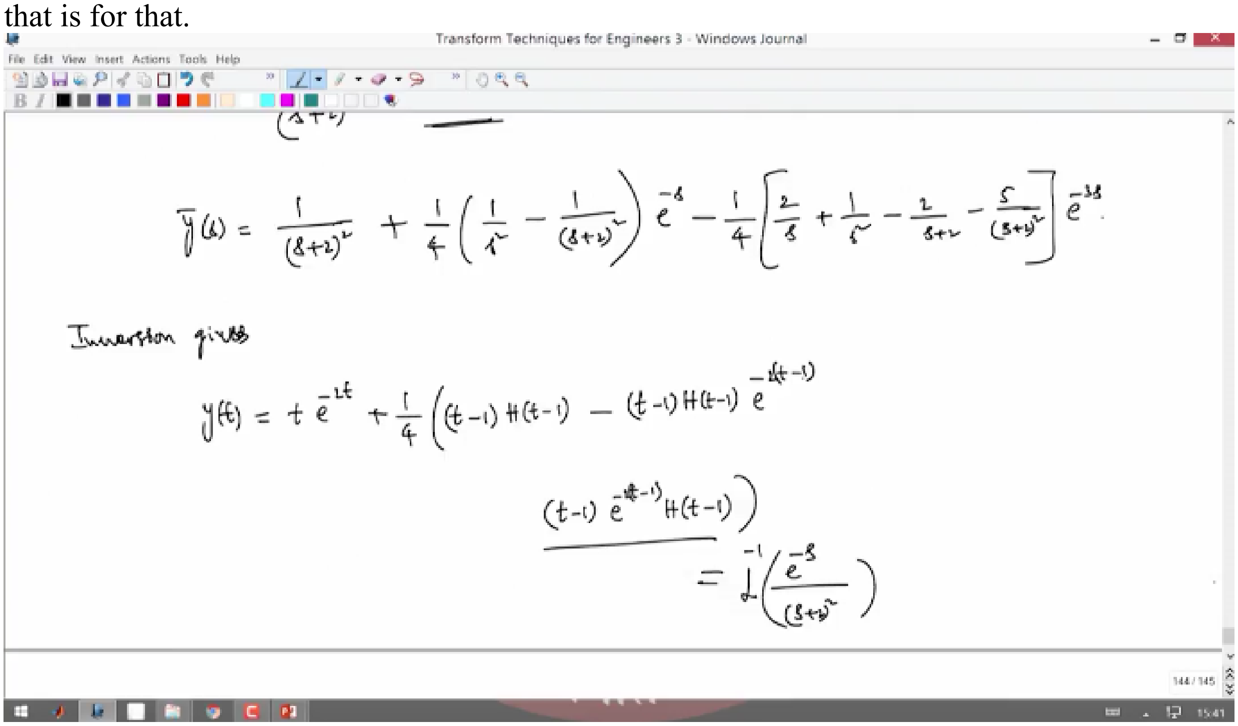


some time back, Laplace transform of this is E power –S/S square, so E power –S/S square as this one.

And similarly here E power S/S+2 whole square this you can write like, this you can write the Laplace inversion of 1/S square S+2 whole square times E power –S, okay, so let's see what it is, this is actually equal to H(t-1) times 1/S square if you get, if you get this as E power 1/S is simply 1/S square is simply T, right, so 1/S+2 means E power -2T, right, -2T and you have it in the place of T you should have T-1, Laplace transform of this is equal to Laplace transform of this E power -2T that is 1/S+2 and if you have to multiply with T of course T-1, so if you have T-1 times for this whole thing you have T times of this, Laplace transform of this into E power -S is equal to this Laplace transform of this is equal to Laplace transform of this which is E power – S/S+2 whole square, okay, so Laplace inversion of, this implies Laplace inversion of

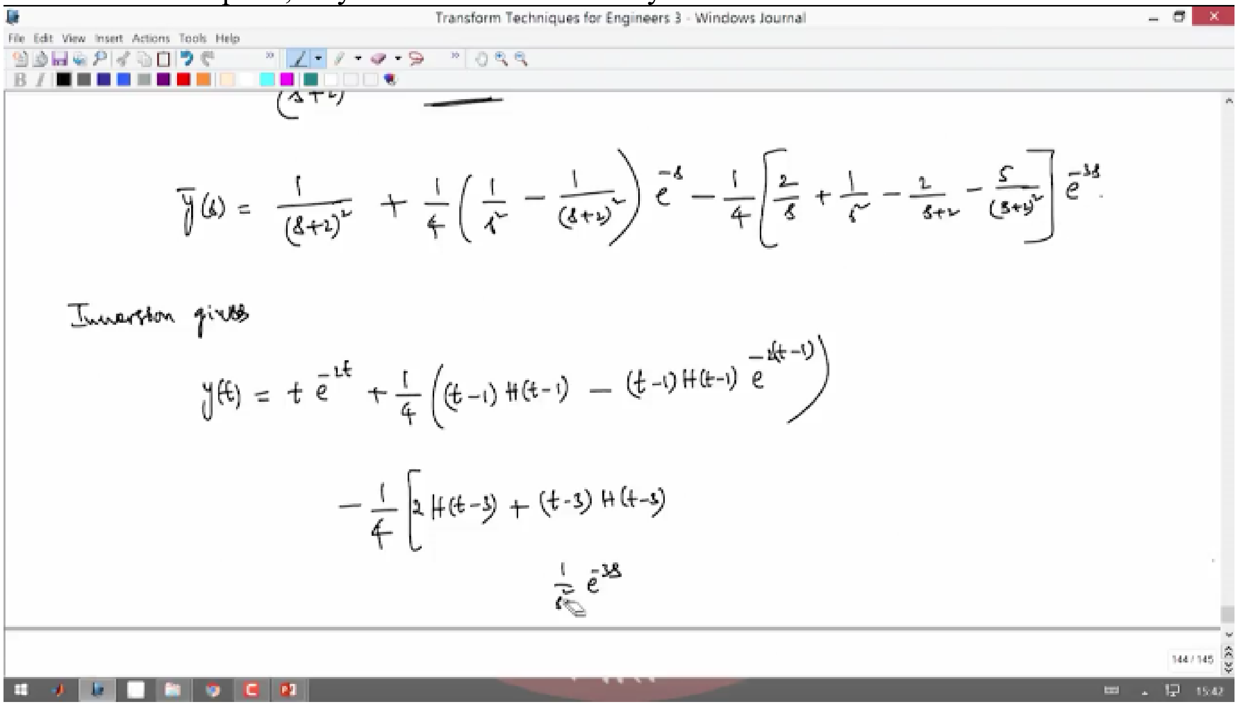


this is nothing but you simply remove this, okay, so this is this, so I'll write directly so use the same property, that property if you use you can have a T-1 times H(t-1) times E power -2T-1, that is for that.

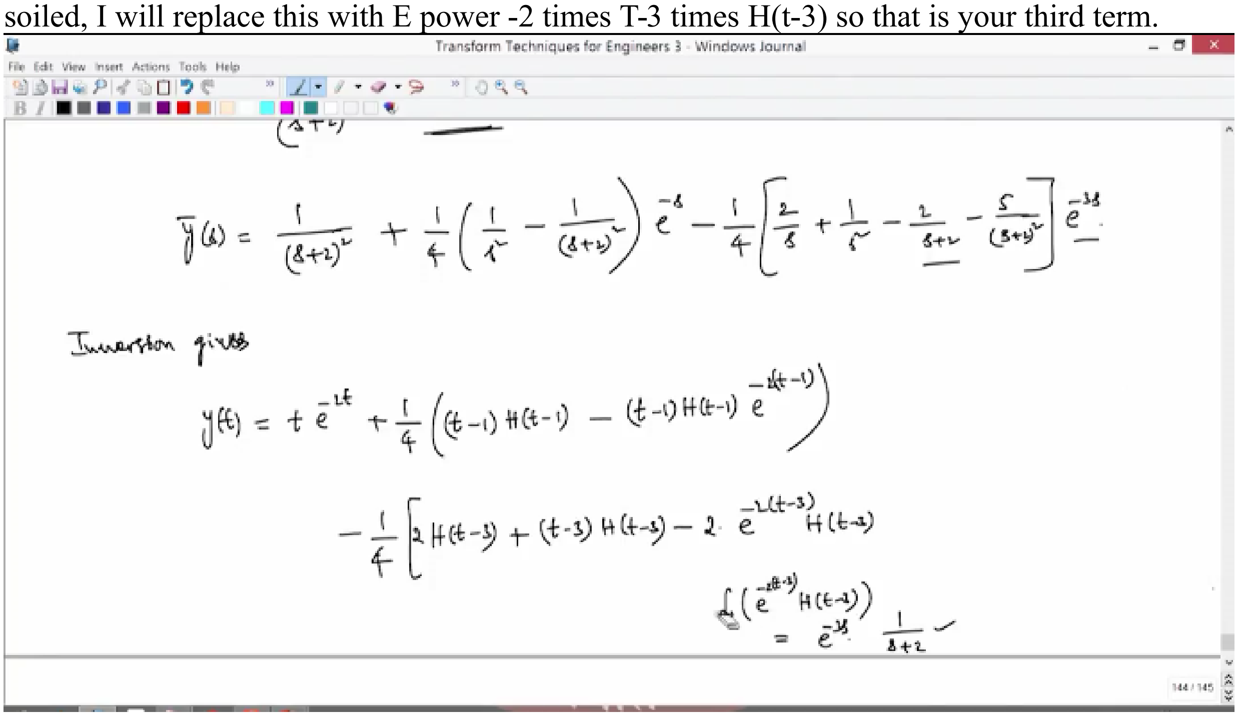


Same technique if you apply here, here also you can get that 1/4, so that is 1/2 times E power, so again so I write directly here 1/4 times you get so this is 1/S Laplace inversion of 1/S times E power -3S, this is H(t-3) times here you have simply 1 right so this is E power – 3S times Laplace transform of 1 that is 1/S, so you get, in the place of this you have H(t-3) into 2 is that, 2 is a constant so that is this, and then minus the next one is +1/S square that is going to be, so

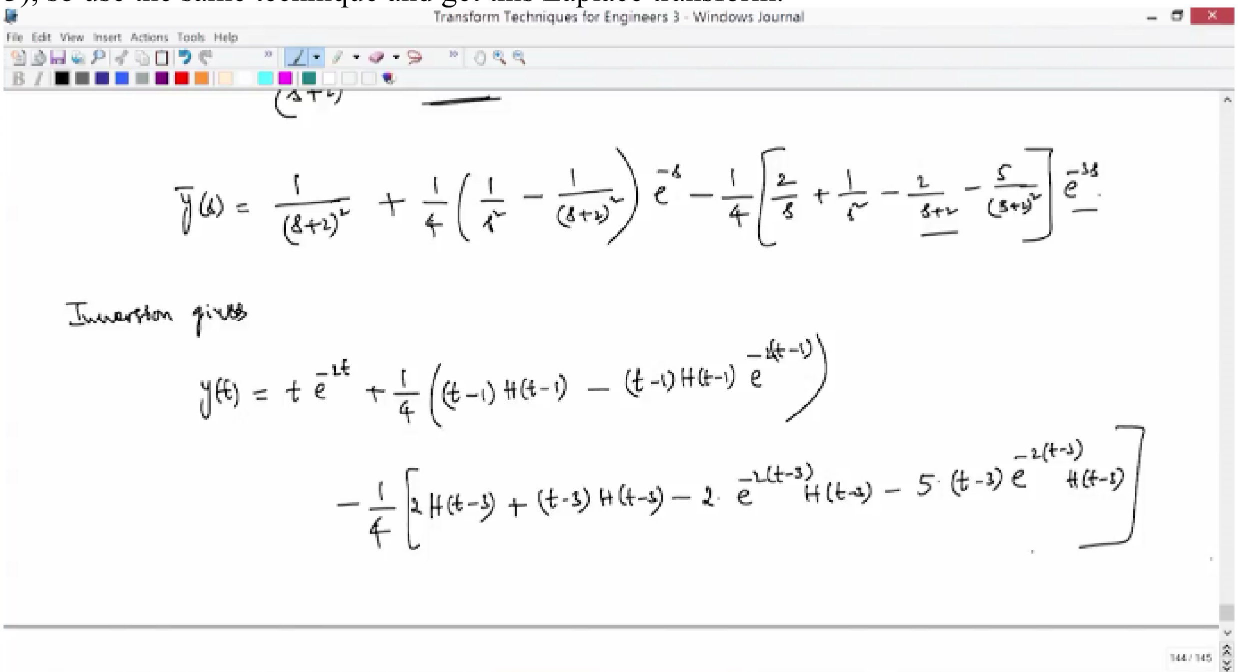
T-3 times H(t-3) will give me Laplace transform of this is E power -3S times Laplace transform of T that is 1/S square, so you have a second term you have that.



And third term you have -2 times 1/S+2, so it's going to be here Laplace inversion of 1/S+2 times E power -3S, this is going to be Laplace transform of H(t-3) times, so here you have T- 3 times rather E power -2 times T-3 if you put so this is actually Laplace transform of H(t-3) times this is actually E power -3S times Laplace transform of E power -2T, so E power Laplace transform of this is 1/S+2, so this is exactly my second term, so you have to replace this here as soiled, I will replace this with E power -2 times T-3 times H(t-3) so that is your third term.

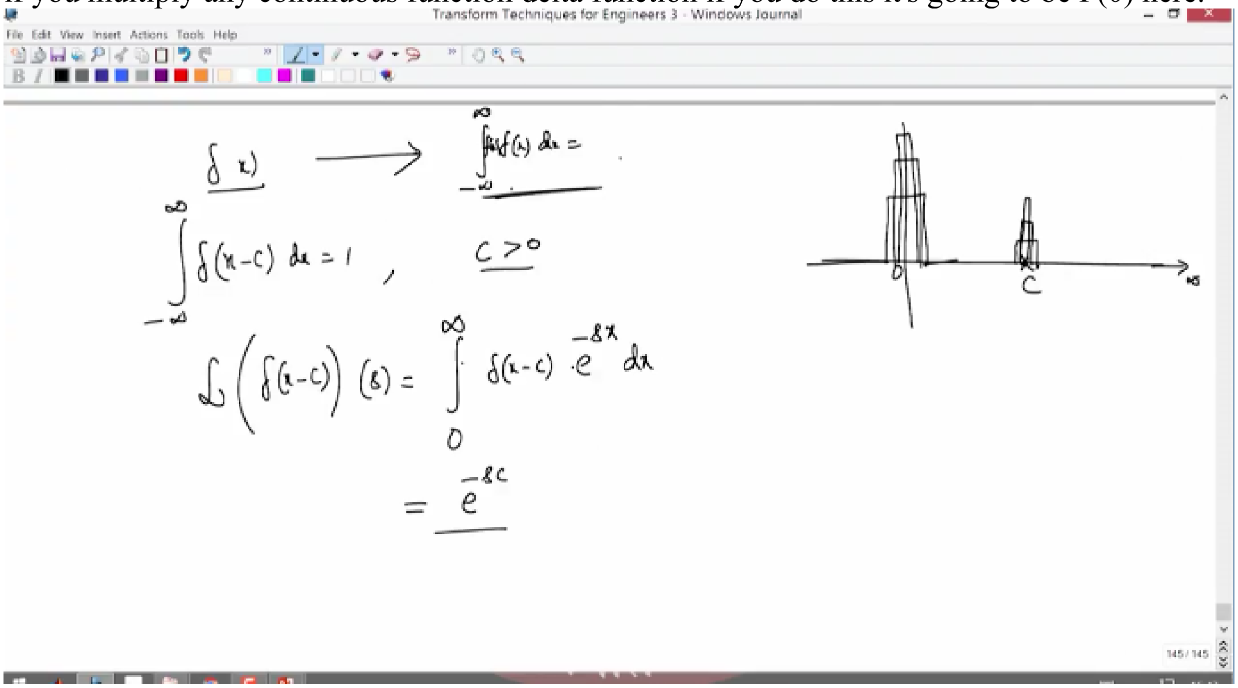


The fourth term is 5 times Laplace inversion of E power -3S times divided by S+2 whole square again here, so this is if you apply the same technique I'll write directly as T times T-3 times E power -2 T-3 that is for 1/S+2 whole square Laplace transform of T into E power -2T times H(t-3), so use the same technique and get this Laplace transform.



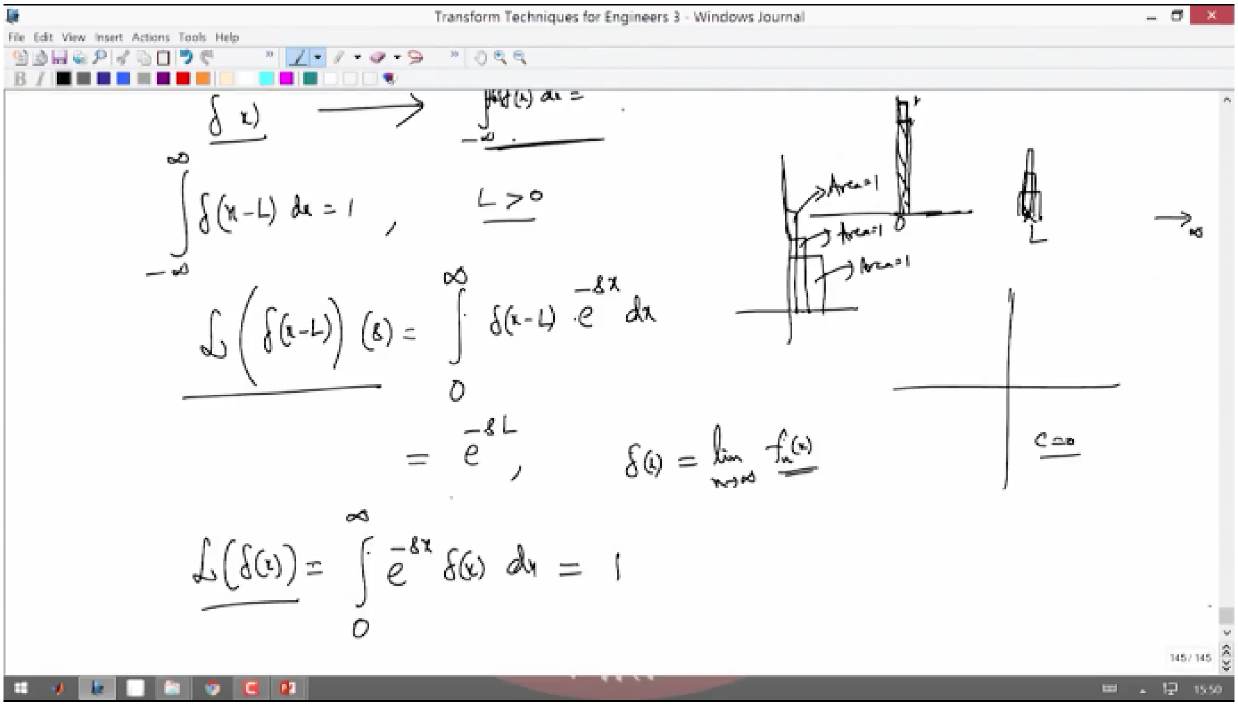
So if you combine this, if you can combine yourself and see that you combine this whole, all these things you can combine and see that you can see that for T positive, between 1 to 3 and T is less than 1, and T is greater than or equal to 3 you can get your discontinuous solution, piecewise continuous solution for the differential equation, so this is how you can solve this ordinary differential equations when the right hand side being piecewise continuous functions and the forcing is piecewise continuous function you cannot apply usual methods, you can apply that is where this Laplace transform comes handy you apply this Laplace transform to get your solution, as a piecewise continuous solution you can get for these problems, and not only this is piecewise continuous you can also get if your right hand side is having a delta function so you have seen the delta function, what is the delta function as we have seen earlier, in the earlier videos when we do the Fourier transform, Laplace transform when you apply, Laplace transform so the only thing is the Laplace transform of a delta function is 1, okay, so how we do this let's see, because our domain is between 0 to infinity.

If you take at C the Laplace transform for example lamp + C, X-C okay, if you consider X-C so you have a delta function X-C is, this is actually you see the limit of these functions, these functions at C, okay, so that this whole area for each of these functions area is 1, eventually the area of this function is -infinity to infinity DX = 1, similarly here -infinity infinity delta X equal to, DX = 1, so that is where, that is how we define my delta function, okay, so far right let me not use this, for this delta function you have this, this is where we have considered like this, the sequence of these functions otherwise 0, okay these sequence of these functions, these steps, step functions those are going to delta function in that sense such that each of this area is one so that eventually the limit is also, the condition is the integral of this value is equal to 1, so if I have such a limit of sequence of usual functions as a step functions like this at C, and so that -infinity to +infinity the whole integral is 1, so you see that Laplace transform of this, Laplace transform of this delta function as X-C, when C is positive as a delta function, so if you write the definition 0 to infinity, delta(x-c) times E power -SX DX, so you can see we have seen that because of this function this value is E power minus, we have seen that this is, if this is the case if you multiply any continuous function delta function if you do this it's going to be F(0) here.



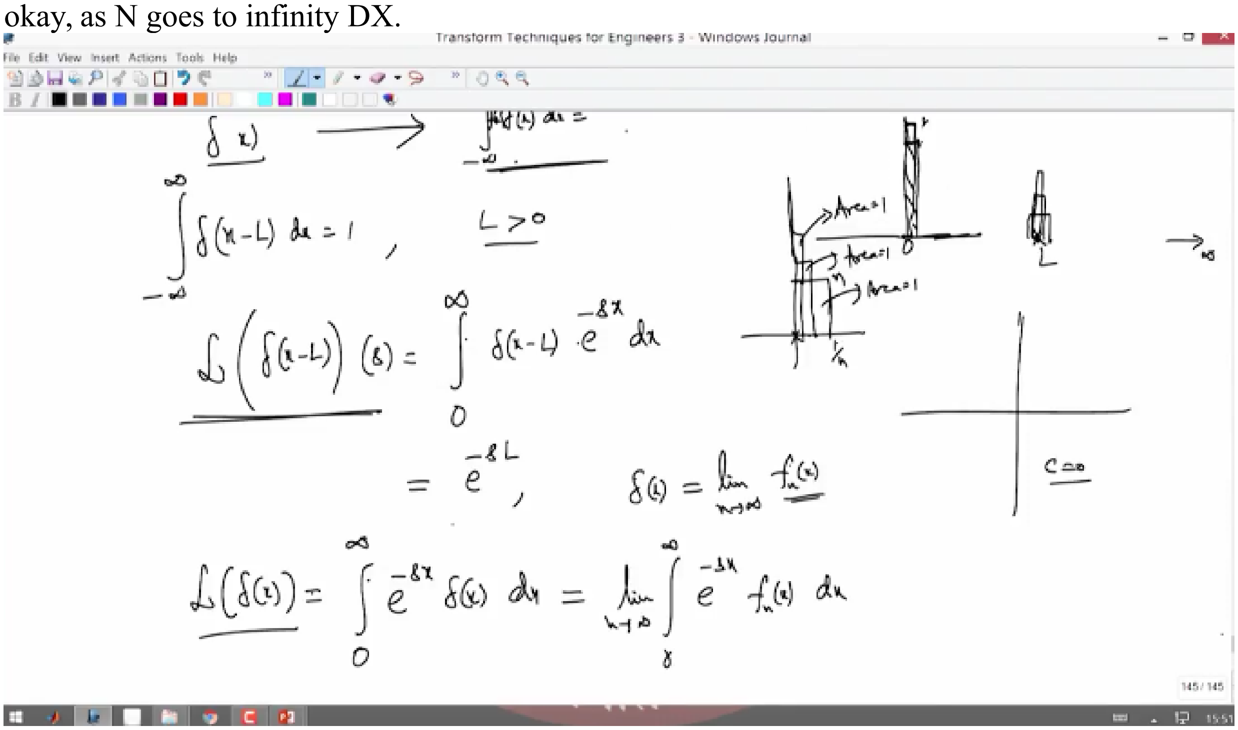
So similarly here F(c) you will get, so if I replace X/C you have SC that is what is your value of this, because the forcing, the instantaneous forcing the impulse is in your domain, positive side, that is Laplace transform of delta(x-c) is this one, okay, so since this is a continuous function this is analytic function E power –SC, so S you can always take it as 0 that line for inversion you can take it as S itself is 0, okay, this is C-I infinity the C itself is 0, C means that continuous curve, so the little me as this as another L, for example if I use this as L, okay, so you have SL so that in the inversion the C you can always take it as 0, so this let me use that L, okay, impulsive values that, impulse at L which is entirely in your domain.

So what happens to the, now the question is what is the Laplace transform of delta function, this delta function, to get this the delta function what I define as the impulsive which is minus, which is spilling over to negative side, now here I don't choose like that, so here what I do is at 0 my impulsive function is like this, so I choose values like this, so I have to choose like this, I have to choose my impulsive function like this so that this area is 1, okay, as a first one, second function this area is equal to 1 so as a third function okay, now the fourth function again this area is equal to 1, so like that this sequence of file is only from positive to this number up to this, okay, again I break it to half I go up to here so that otherwise 0, it's only from 0 to this value is, whatever be 1/N kind of thing, so if you do that this kind of impulse, the delta function if you define that is going to be, for a such a thing the Laplace transform is simply 0 to infinity E power –SX delta(x) DX, for such a sequence of functions delta X as a limit of sequence of these FN(x) as N goes to infinity so this FN(x) X's are only impulses here, one is this, other one is this, other one is another half and go up and so on, so these functions so that the area is always one, maintain as one, area is one, okay, so for all these areas equal to 1, so that is not spilling to a negative side so that if you do like this value is equal to simply the value at, if you do like this, this area if you do this simply 1, your value is simply 1, because impulsive

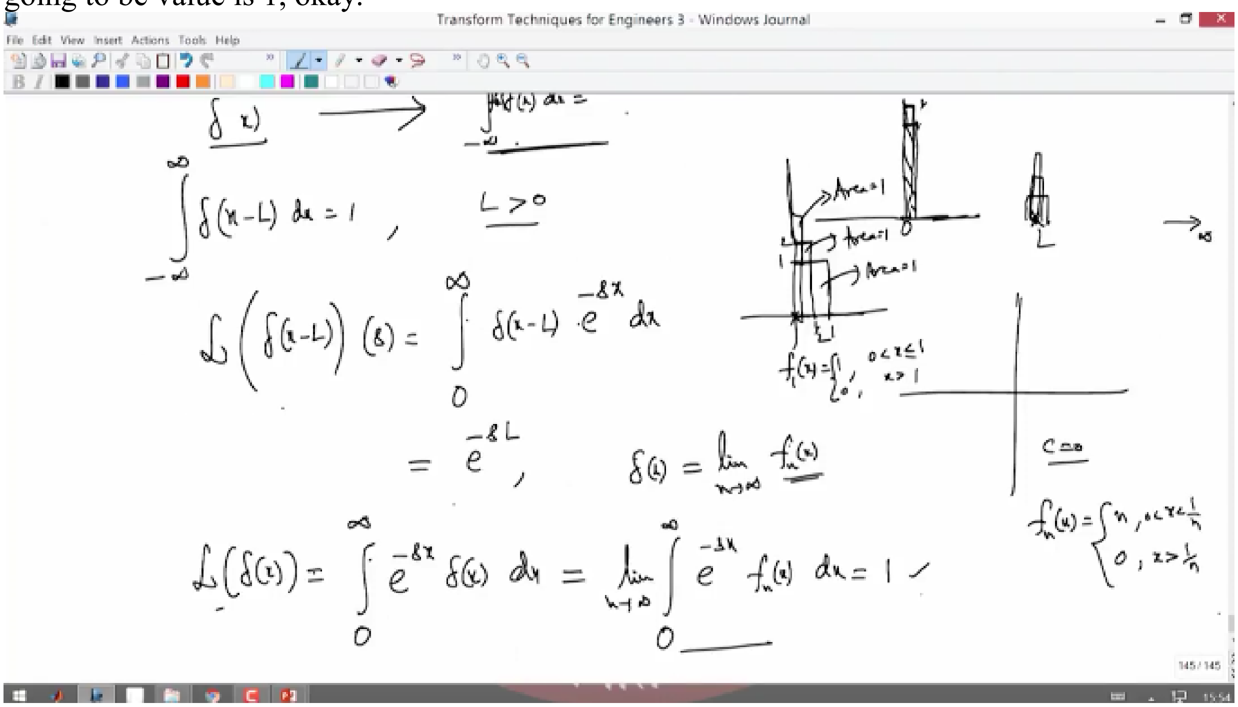


eventually it is going to be at 0, so you can show that is a value equal to 1, this you can see you can also see that as L goes to infinity this is very clear, so as L goes to infinity you can see that this delta function, this impulse is coming to, this impulse set L is coming to 0 and finally as L goes to 0 that is going to be 1, so this is what you can see.

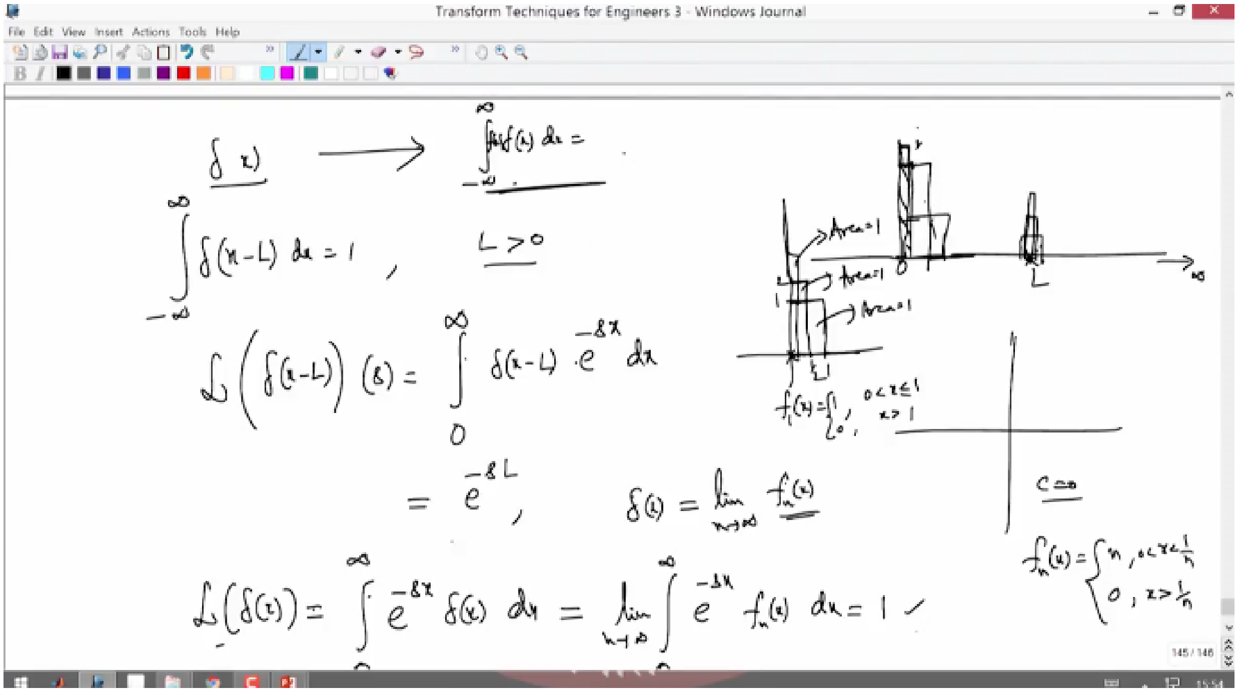
You can think of impulse at this point but only right hand side you have a sequence of functions or only right hand side as stuff functions whose area equal to 1, and finally you see that this value and you can think of this as a limit of integral 0 to infinity, your E power - SX FN(x) okay, as N goes to infinity DX.



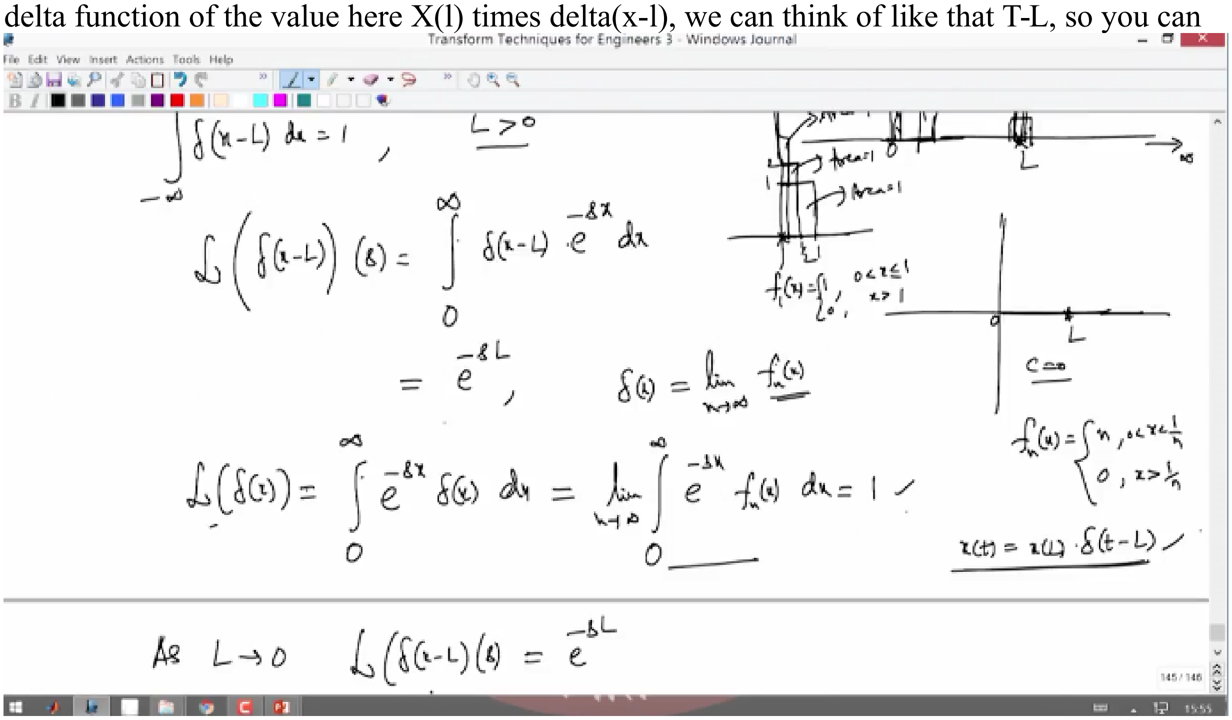
Now you have to make use of these step functions, you have to choose for example 1/N, if 1/N then what is the value of here, this is going to be N so that you have, so next one is 2/N, next one is 3/N, sorry, wait, this is 1 for example 1 the first one is 1, F1(x) is between 0 to 1 this value is 1, so that the area is together is 1, okay, and X greater than 1 is 0, okay, 0 otherwise is 1, so this is one like this, 1 between 0 to 1, 0 else, X positive, X greater than or equal to 1, okay, X put it X greater than 1 this is 0, F2 between you go 1/2, 0 to 1/2 it is 1, it's not 1, you have to take 2, this is 2 and otherwise 0, so that its area is maintained as 1, so if you do like this, this if you calculate this integral as a limit like earlier you have seen in the case of delta function, what we have done you will see that this value is actually equal to 1, you will see that is actually equal to 1, okay, because the impulse is at 0, so you will see that this integral you can calculate finally you take this general FN(x) is, general F1(x) is, this value is the N if I choose X is between 0 to 1/N otherwise 0 if X is greater than 1/N, so if I choose like this for X positive function, okay, you will see that this here N, 1/N, 0 to 1/N and this is going to be E power –X times 1/N, so E power, so that's you can easily see that so you will get, you will see that it is going to be value is 1, okay.



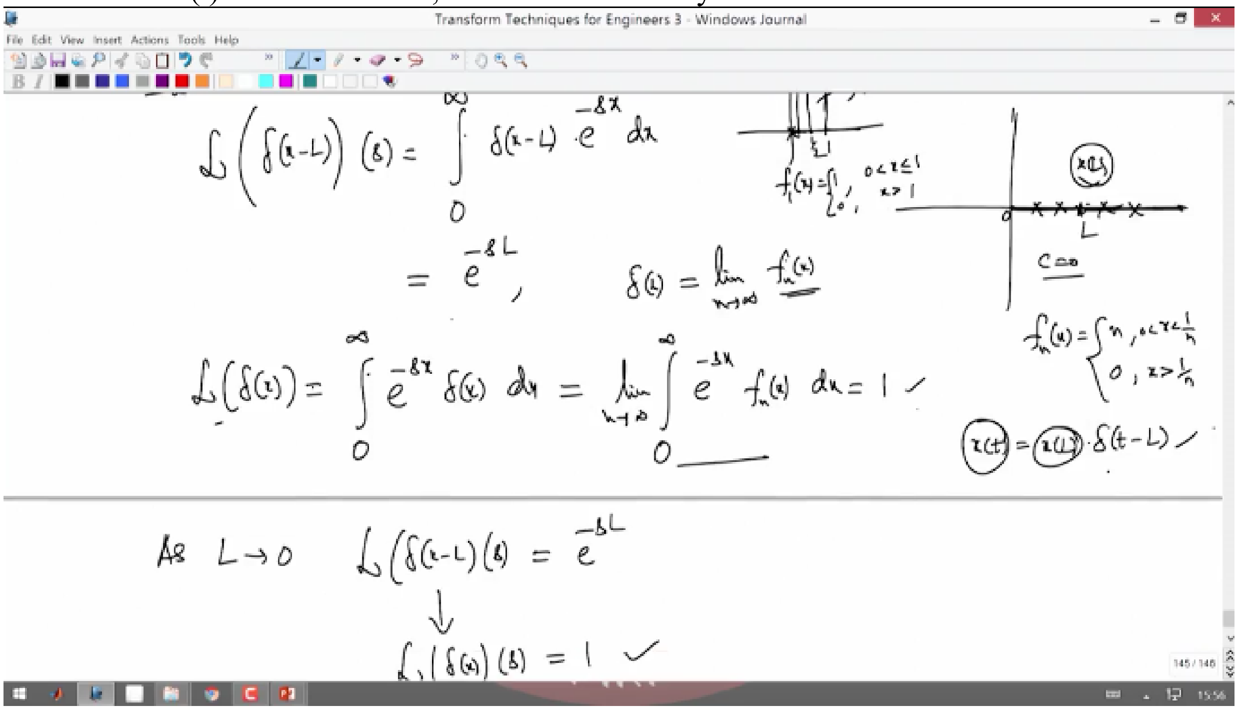
So that's how you can calculate, you can easily see that the Laplace transform of delta function, I am not calculating here you can do the same calculations and see that value is 1, Laplace transforms of this eventually you will see that it's 1, this can also be justified by just by taking L goes to 0, so as L goes to 0, Laplace transform of delta (X-L)(s) = E power – L, this goes to Laplace transform of the delta function(x) as a function of S this is going to be 1, so this impulse, this actually delta X you should not think of as a limit of sequence which are spilling over the negative side, okay, instead only positive side, so if you take here like this impulses at L as L goes to infinity these impulses you have to bring it only the positive side okay like this,



these are the sequence for which you can see that is value is 1, so if once you use this you may not come across this because you cannot have, you may not see in the applications and applications mostly in your domain 0 to infinity you have impulse, you can have a load at a particular value, at time you have this impulsive values at this you can put it as a lap, the delta function, so delta X-L okay, if this is your L you can have an impulse point load or you have a value at this point if you want to have you can represent that as a continuous function X(t) as delta function of the value here X(l) times delta(x-l), we can think of like that T-L, so you can

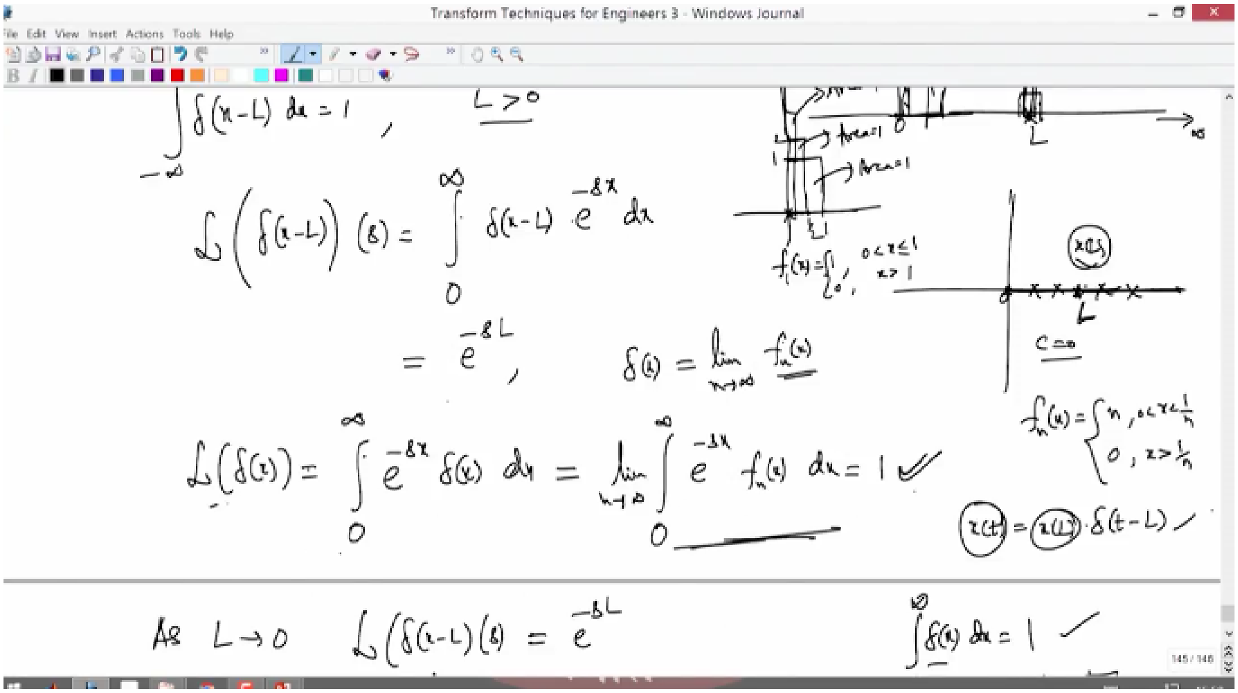


think of this as a point value, value at L is X(l) the sample value for this continuous function X(t) X(l) and you multiply with this impulsive function that will give as a continuous function you can represent as though it's not a continuous function, it's continuous variable T you can represent as a function of continuous variable you can represent this single value at L you have this value at X(l) otherwise it is 0, between 0 to infinity.



So like that you can have, if you have a sample of continuous values you take a sample, certain discrete values if you could consider you compose all these discrete values, you compose with these functions you can get your sample functions, if you take sample function like that so if you have L, if you can have usually your delta function will be, your impulsive function if you consider at T positive you can get this function as like this, so your differential equation you may come across applications, you have a differential equation right hand side being impulsive at some point in the T positive side, even if you have a, at 0 if you have impulsive functions so you can think of, you can take it as a delta function Laplace transform when you apply you should consider as 1, okay, if you, why I am saying that these impulsive functions are not spilling over to the left hand side, if they spill over to the left hand side what happens is that you know that delta function, if you consider as a limit of those functions FN which are spilling in the negative side this we have seen that earlier that is actually half, you get half, okay, because half if they are spilling to the negative side that this side area is only half that is the reason you're getting half here, but if you consider this only the negatives, only positive side you have these functions, not spilling over to the negative side but this value of the area is 1, so you have to have here 1, so such a limit of sequence of functions FN for which you have this delta function.

So in the Laplace transform the delta function means not the sequence of functions FN which are defined in the negative side, only positive side they are defined, and you have this sequence of functions for which the integral value is always 1 so limit of FN = delta X, only X positive side is defined, and so that FN of each of these functions integral values 0 to infinity is 1 so that eventually 0 to infinity the limit delta X of DX is also 1, so the area is maintained as 1 and so that you see that when you multiply some continuous function the value is also equal to 1, so this is what is a delta function of Laplace transform of delta function as 1.



So we will see this application and this application of this delta function in the ordinary differential equation in the next video, and then we will start Z transform, okay. So we will see in the next video. Thank you very much.

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