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NPTEL ONLINE COURSE
Transform Techniques for Engineers
Solution of Integral Equations by Laplace Transform
With
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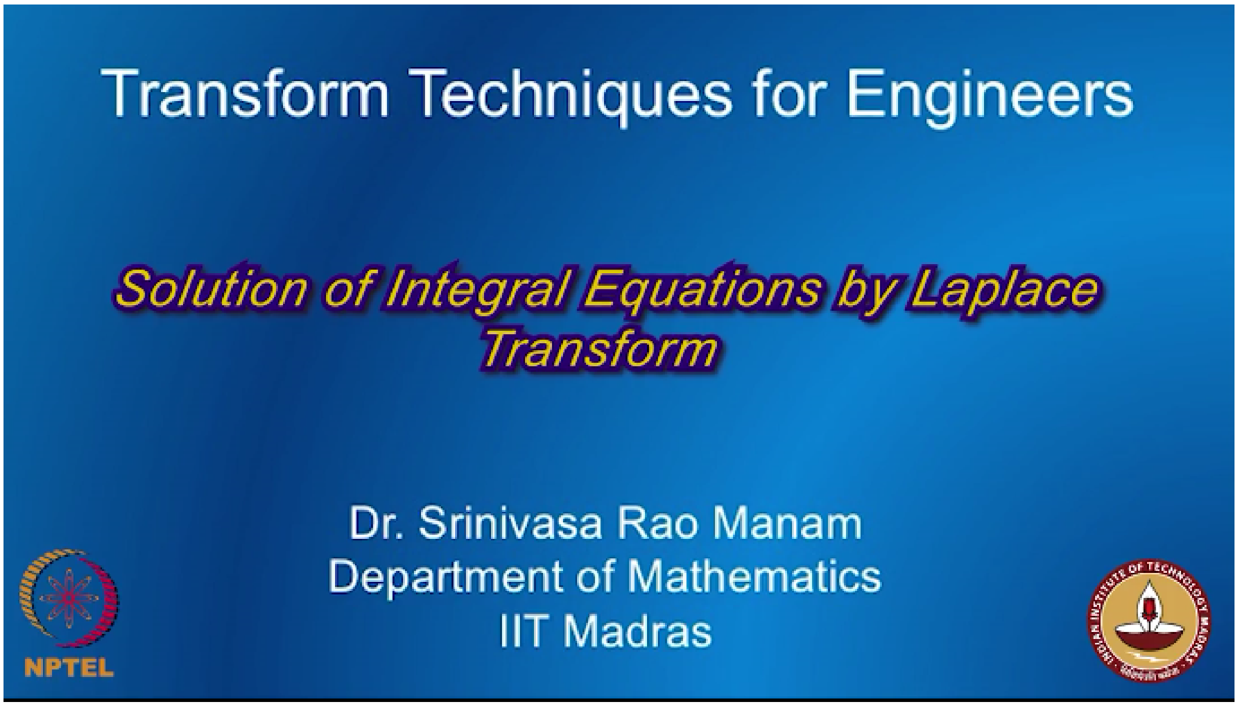
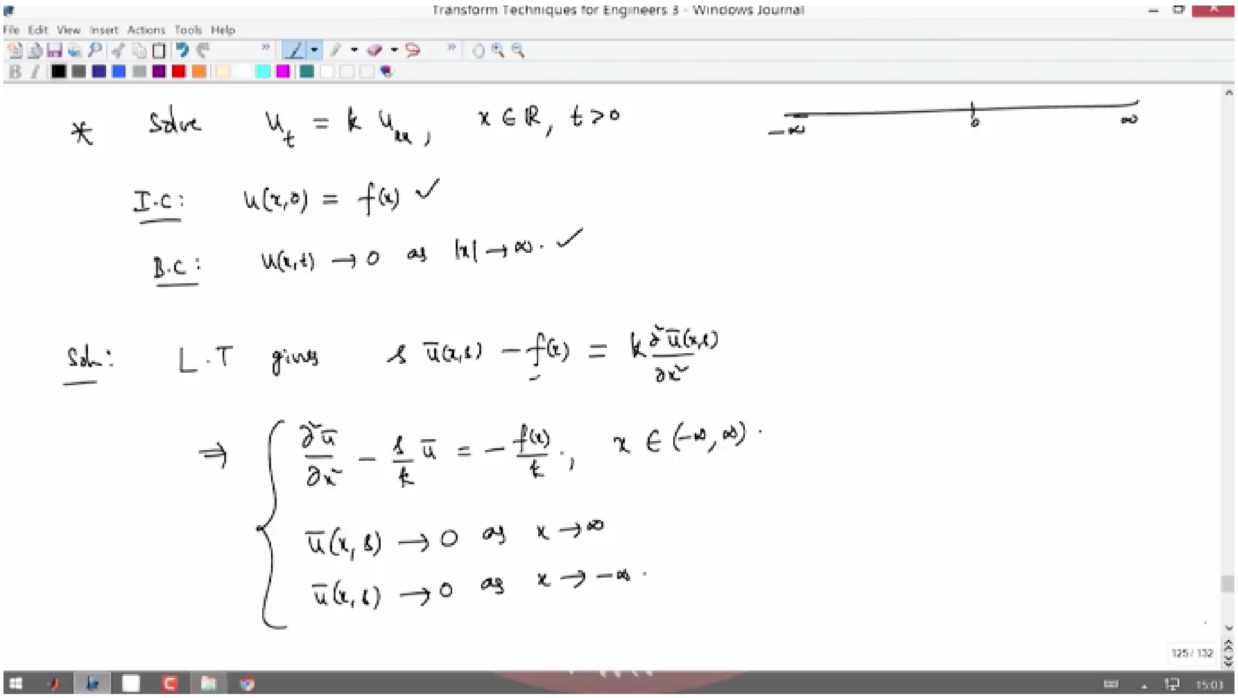


Welcome back, in the last few videos we were discussing about the applications of Laplace transform to heat equation in the process you may require Laplace inversion for some special functions, exponential of root S type of functions, we have given three such functions how to find a Laplace inversion, you don't have to just try to understand the technique how we derive these inverse transforms, otherwise you can make a table and use it wherever you come across these functions, its inverse transform.

So in the last video we started a problem for infinite rod for which you have initial condition and some boundary conditions, if you give initial condition what happens in the, initially you have a rod at some temperature what happens to it for all times when you see that at infinity, both the infinities you have temperature will be 0 as a boundary condition if you have so what happens is, this is a basic problem, basic partial differential equations, solution of the partial differential equation of heat equation, one-dimensional heat equation.

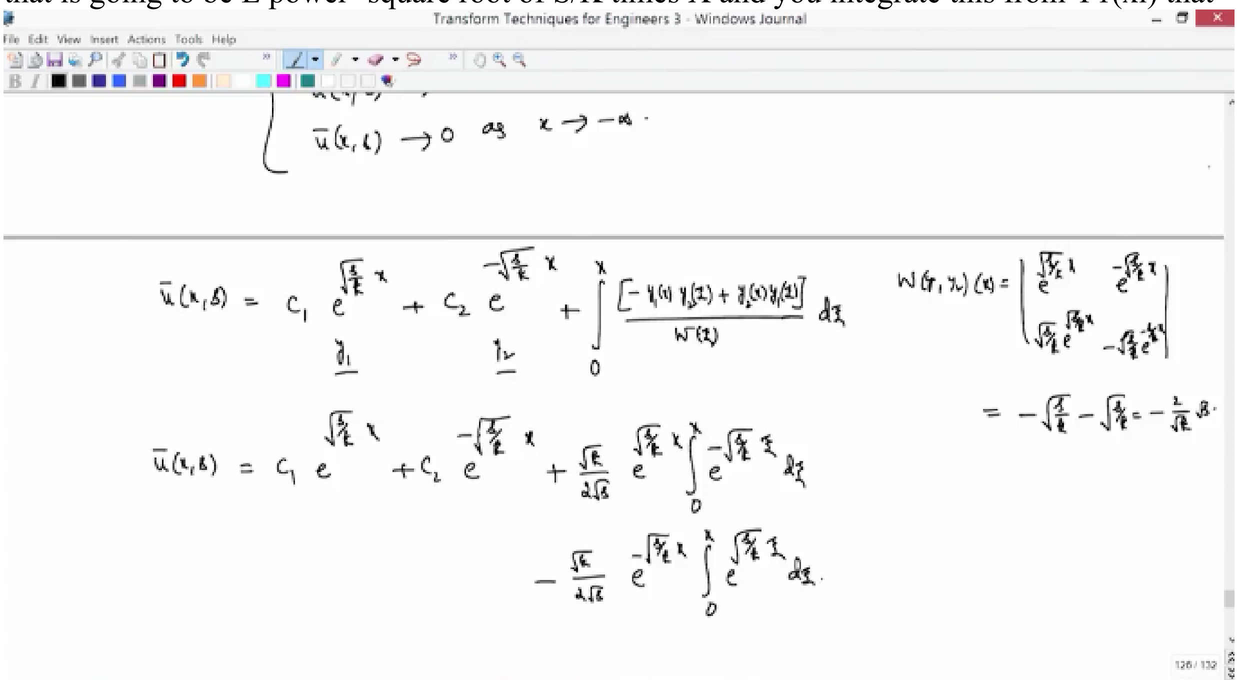
So let's look at this problem we will get similar, this is similar to what we have done there, we get in which, this is similar to what we have done in the last video where we got exponential of -root S A times root S as for which you need Laplace inversion, so that's what we will see, this is another example where you can find such a function for which you need inversion, so let’s look into this, we apply the Laplace transform for the heat equation this is what we get, and you



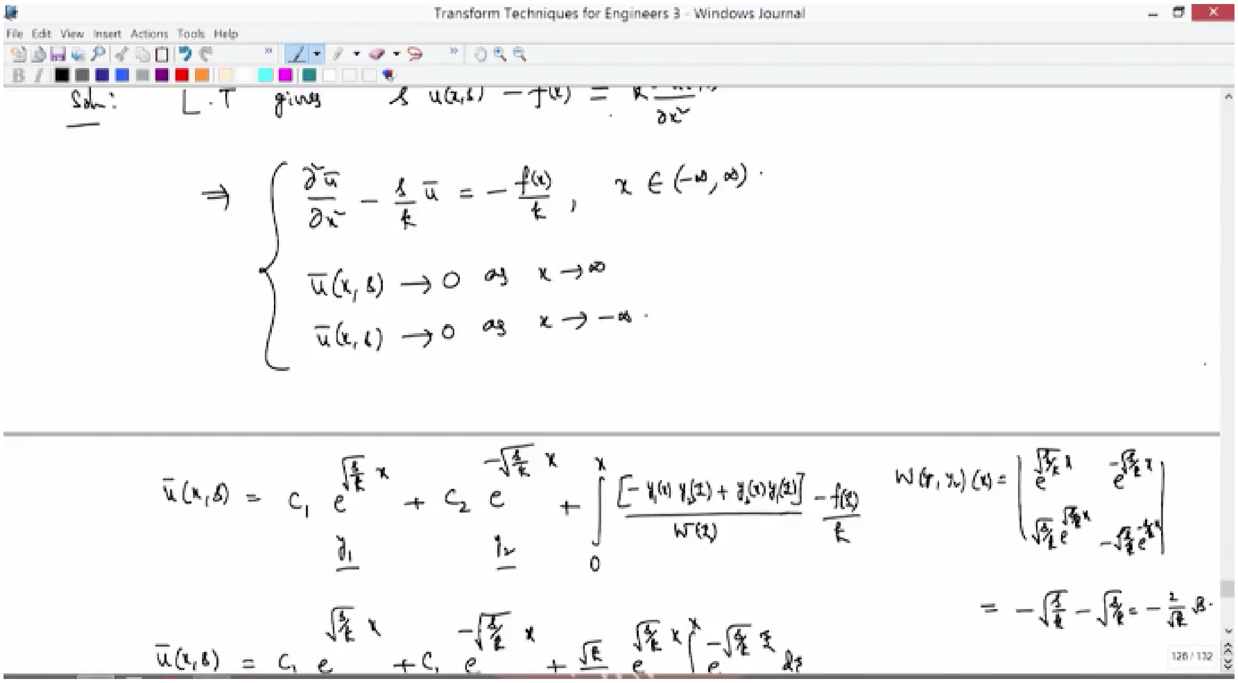
apply the boundary conditions where we use the initial condition, boundary conditions if you apply the Fourier transform a Laplace transform you get this boundary conditions at infinity, so you have this, you have these boundary conditions you need to solve this non-homogeneous ordinary differential equation, so if you solve this a solution will be U bar(x,s) this is going to be C1 times E power root S/K, for example full root S/K times X + C2 times E power -square root of S/K times X and then this is your, this is your complementary function that is, it will give you solution of the homogenous equation and to get a particular integral for this general function F we use a variation of parameters, so in the variation of parameters how do you get the variation of parameters? I use direct formula you can just look into any ordinary differential equation book to find what is the technique for variation of parameters when you have, these are the two linearly independent solutions exponential this function, other exponential of minus of this, so this if you take what you get is you need a Wronskian of these two functions it's called this Y1 and Y2 or two solutions of homogeneous equation, and Wronskian of Y1, Y2 if you calculate as a function of X that is a determinant of E power S/K under root X times and this one and -root S/K times X and if you differentiate this you get root S/K times E power root S/K times X, and this is minus root S/K and E power –S/K times X, so this if you calculate, if we just get this, this is -root S/K and minus, minus again so this side is root S/K, so you get is, what you get is -2/root K times root S, so that is your Wronskian, so if you calculate your particular solution by variation of parameters what you get is this is like a, you can write, I am writing a formula for this particular solution from the variation of parameters method using this Wronskian.

So in this you have this, this is how it goes, it looks Wronskian of X for example you have Wronskian of X here and numerator you have Y1, let's write Y1, so Y1 is this so you have let me write this as Y1(x) and Y2(xi) +, this is plus Y2(x) times Y1(xi) and you have here, instead you take this as xi, instead of W(x) you take W(xi) this is W(x) which you have here is anyway constant as a function of S, W(xi) which is basically a constant, so you don't have to worry but in the formula it's a W(xi) and this you do integrate with respect to xi from 0 to X, then it will be a function of X, okay, so this one is your solution of your, particular solution of this nonhomogeneous equation so that when you sum with homogeneous solution which is this plus this particular solution will give the general solution of this nonhomogeneous linear ordinary differential equation.

So let me use, let me give what it is, so you get U bar(x,s) as C1 times E power square root of S/K times X + C2 times E power -square root of S/K times X + if you actually calculate so you get you have a -root K/2 root S that is the denominator and you have this integration from 0 to X, so let's split this integral this part Y1(x) it's nothing to do with the xi, so I write that as so -Y1 that makes it + Y1(x) is E power square root of S/K times X, and this you integrate with respect to X times Y2(xi) that is E power - square root of S/K times xi D xi, and then other one and again I have a -sign root K/2 root S that is because of W xi, and you have Y2(x) comes out that is going to be E power -square root of S/K times X and you integrate this from Y1(xi) that

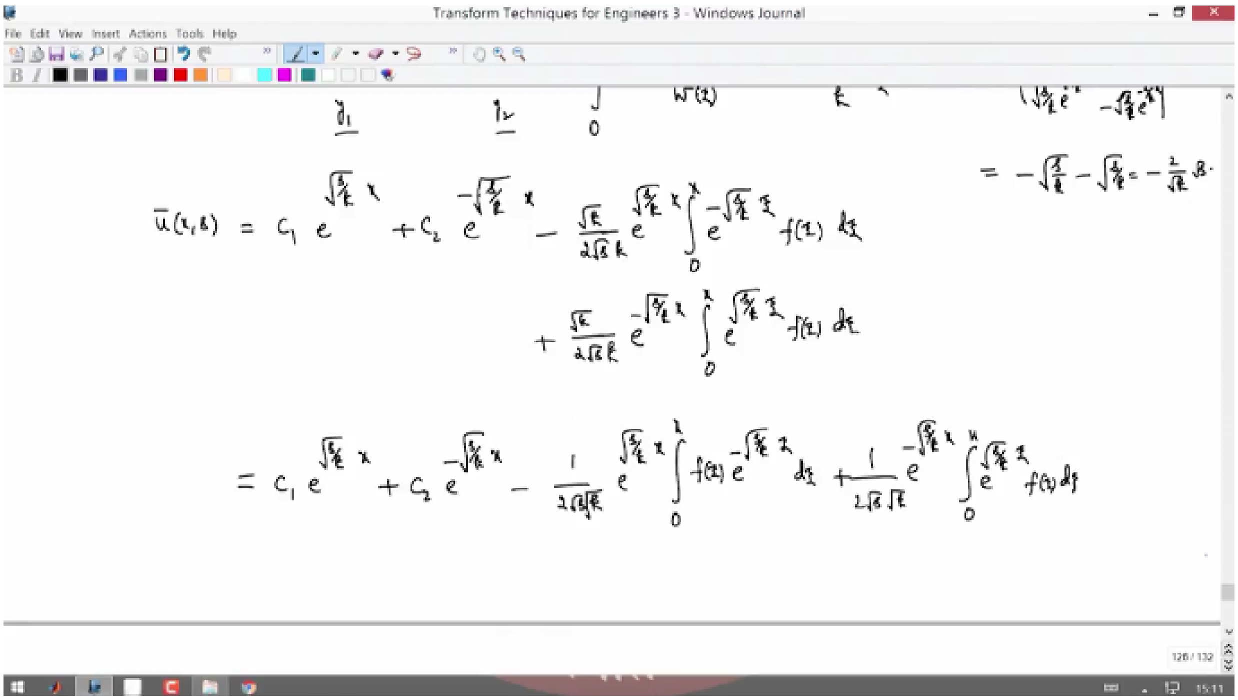


is E power a square root of S/K times xi D xi, so if you simplify this you get C1 times E power square root of S/K times X + 0 times E power -square root of S/K times X, and this one will be root K/2 root S this if you integrate this part you get root K -, - sign will have root K/root S, sorry I think I missed one more thing so this is, this you have to multiply with your right hand side, so this is your this thing, this you have to multiply whole thing with –F(xi)/K that is your

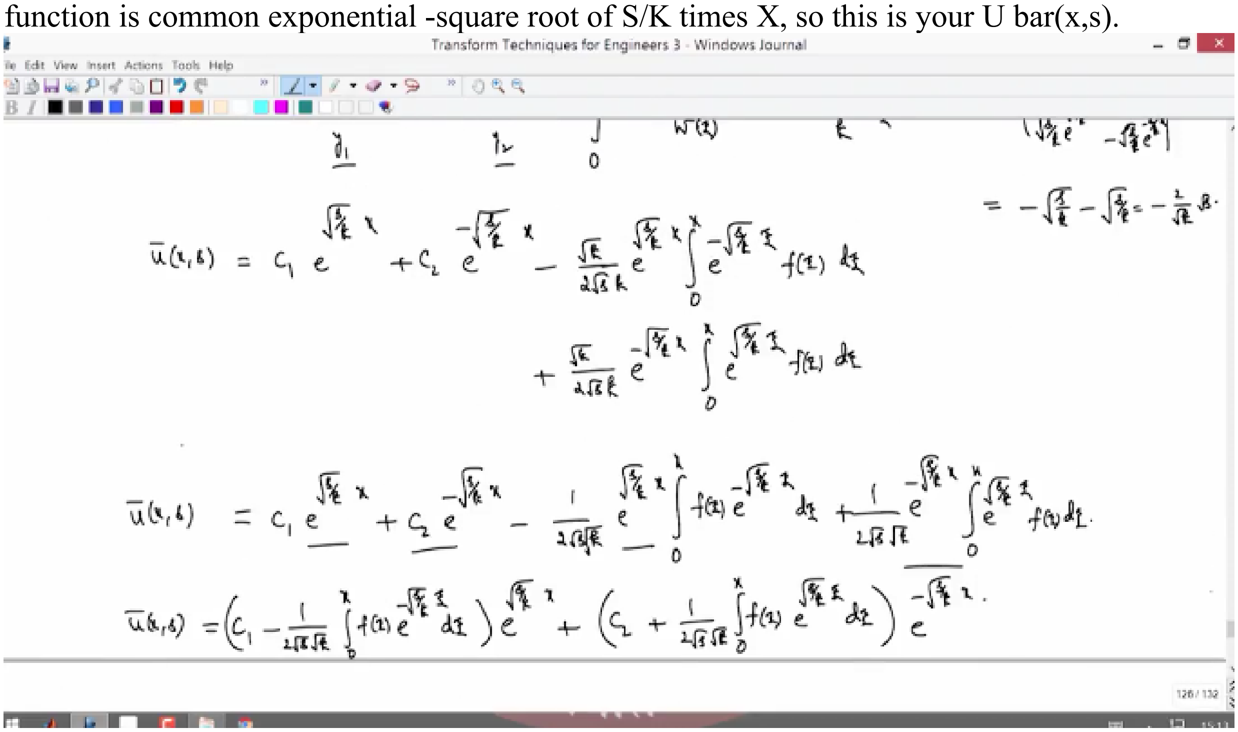


right hand side, so this is your right hand side so that you have to integrate, so I missed that so if you do that you have F(xi) here, F(xi) minus sign that makes it minus here and you have a K comes out so that makes it K here.

And similarly here also you have to multiply F(xi) D xi and with of course minus sign that makes it plus here and you have a K here, so if you simplify this you have minus SK times let me write E power square root of S/K times X and this will be square root of S/K, sorry you cannot integrate it, right, so this times this is simply integral as such I leave it here, so this you cannot integrate because F is unknown, so you have F(xi) times E power -square root of S/K times xi D xi minus so you can simplify this so that you have denominator you have this one, and here also you have 1/2 root S times root K times E power –square root of S/K times X and this you integrate from 0 to X E power square root of S/K times xi F(xi) D xi, so this is your

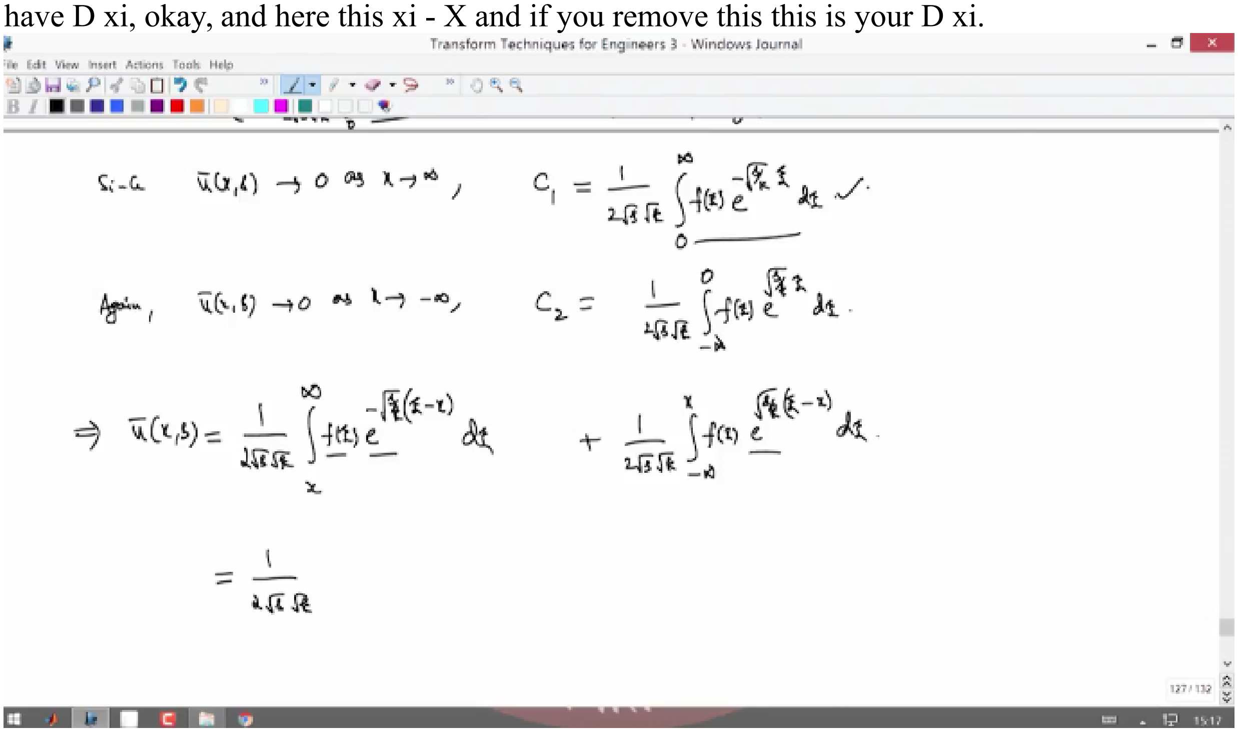


function as U bar(x,s), so you see that this is common here and here, okay, so you can combine them to see that you have C1 E power, C1 minus this part, so whatever you have 1/2 root S root K and you have this integral 0 to X times F(xi) E power -square root of S/xi X S/K times xi D xi, this product with E power square root of S/K times X, and you have the other part this you combine with this so you have a + C2- C2 here plus you get 1/2 root S root K you have integral 0 to X, F(xi) E power square root of S/K xi D xi, this you combine with, this is common, this function is common exponential -square root of S/K times X, so this is your U bar(x,s).

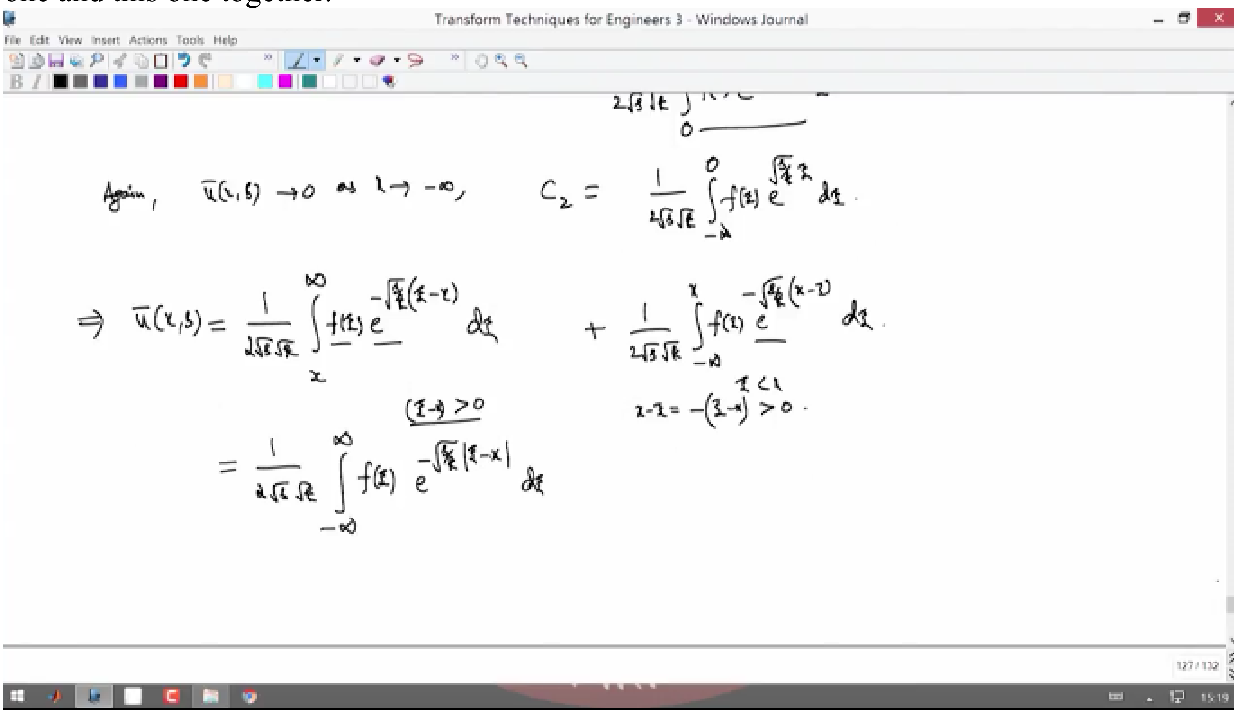


Now you apply for this, this is your general solution of this non-homogeneous ODE and you apply this boundary condition now, if you apply this as X goes to infinity, if X goes to infinity

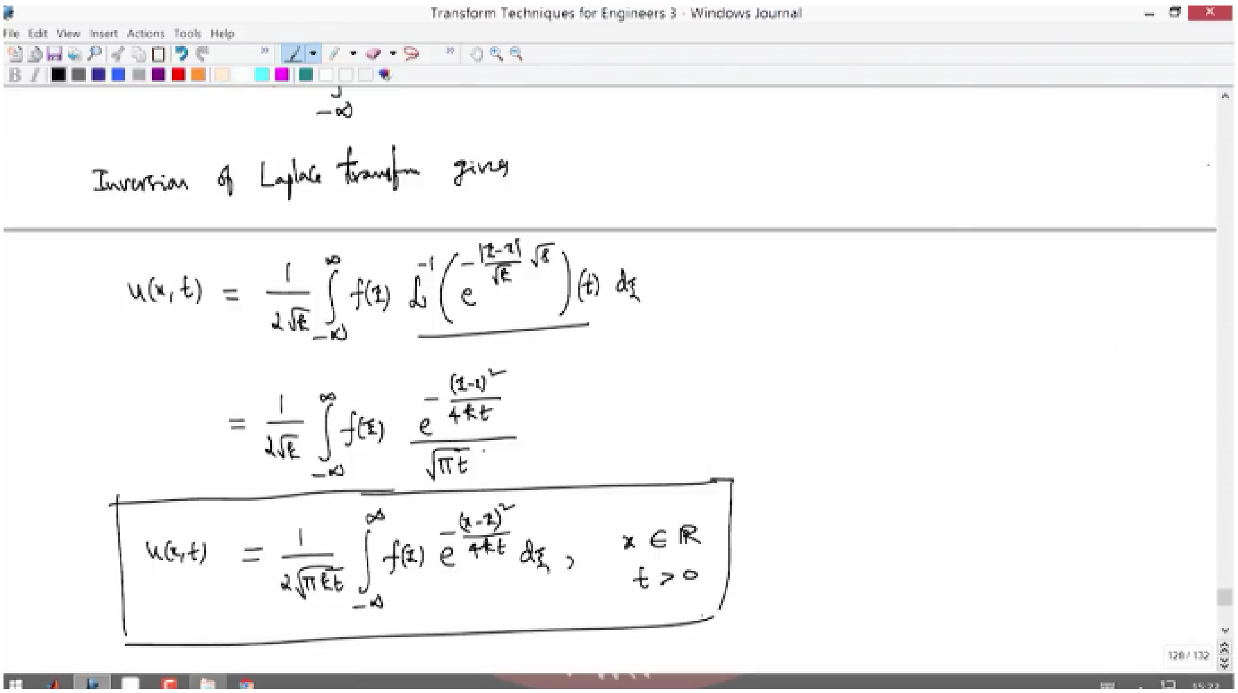
that has to be finite, so that has to go to 0, so that means this has to go to 0, okay, otherwise as X goes to infinity this part is going to 0 anyway, so for this to go to 0 since U bar(x,s) goes to 0 as X goes to infinity gives, this is because of this you have C1 has to be equal to 1/2 root S times root K this is as X goes to infinity this has to go to 0, so as X put E, wherever X is there this one, infinity F xi E power -square root of S/K times xi D xi, so you get what is your C1, and now again, you apply other boundary condition U bar(x,s) that goes to 0 as X goes to -infinity that makes it so you know what it is C1 here or if I go, X goes to –infinity this is anyway going to 0, so and this part has to be go to 0 that makes it C2 as -1/2 square root of S root K times 0 to –infinity, so you can write with plus -infinity to 0, F(xi) E power square root of S/K times xi D xi, this is how we get, now you put it into the equation, I put it into the general solution, so that is U(x,s) that is C1 is this, so if you combine it, C1 if you combine here what you get is, if you combine this, this is same integral as this so if you do it from 0 to infinity -0 to X, so it is going to be C1- of this whole thing is actually equal to 1/2 root S root K, now the integral is from X to infinity F(xi) E power -square root of S/K xi D xi.
And similarly C2+ this one -infinity to 0, -infinity to –X, -infinity to 0, so when you combine this part and this part together addition, will give you –infinity to X F(xi), of course this whole thing with, first of all this quantity into exponential of square root of S/K times X+C this if you combine it and you get 1/2 root S times root K times -infinity to X, F(xi) E power square root of S/K xi D xi this times E power -square root of S/K times X, so this is what you have, so if you combine this what you get is you can combine both of this together as 1/2 root S root K, how do I combine this? F(xi) is common and you have -infinity to X, and X to infinity F(xi) and this is, this if you combine this one and this one together, so we put them together you have D xi here, you can take them inside D xi, so if you combine this what you have is xi-X here so that you have D xi, okay, and here this xi - X and if you remove this this is your D xi.



So this if you combine what you get is xi is between here in this case xi is between, xi is bigger than X here, xi is less than X, so xi - X is negative here, and here xi -X is positive, so –xi –X is negative, so you can write, this you can combine as or rather this is, you can leave it as such, this is xi-X is positive here, because xi – X is here is negative, you write minus of this will be positive, okay, so you can combine this, this you can write it as a minus of xi-X, this is X- xi, this is X- xi is positive, okay, so if you combine this what you have is -infinity to infinity F(xi) E power -square root of S/K and this is because this is positive you can put it as a modulus of this with D xi, if this is negative xi - X for some X, xi is always bigger than, so okay this is always positive, this is always positive, so you have a minus and X - xi is positive, so in any case if you write modulus of xi X and is anyway because this is negative it will become minus of this, and that is the modulus, okay, so this is D xi, so this is how I combine this together this one and this one together.

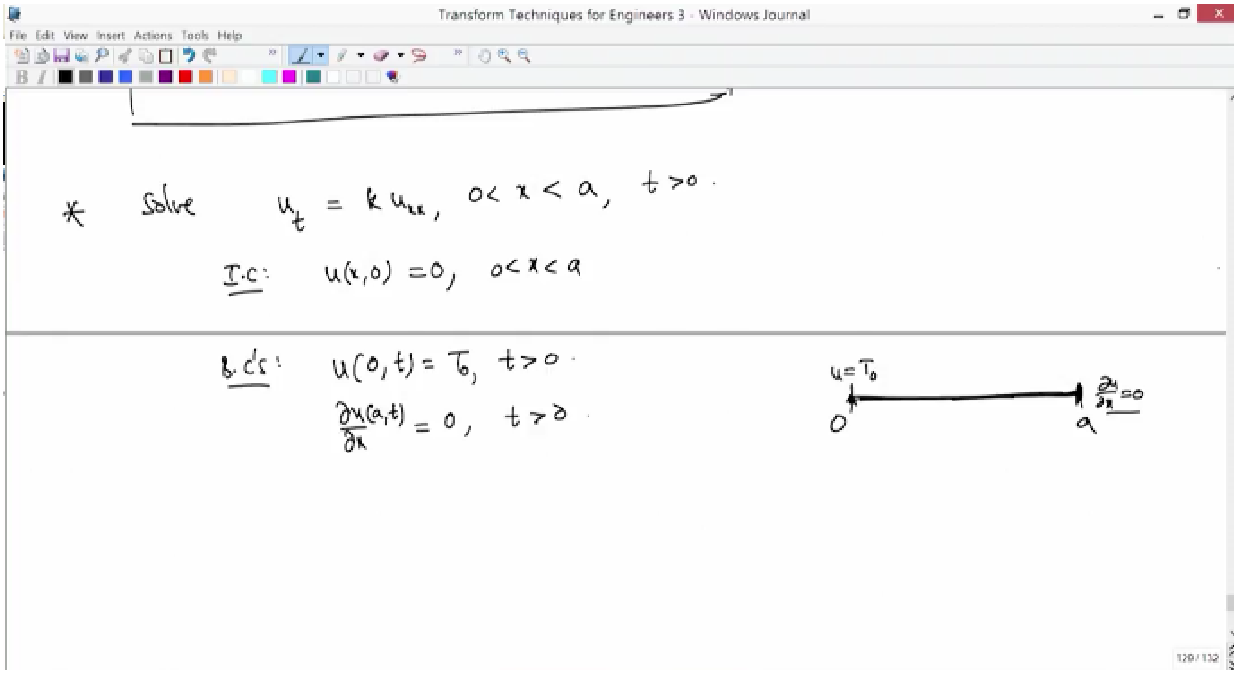


This is my U bar(x,s) now this is the part you have to take inversion, inversion will give, inversion of Laplace transform gives U(x,t) the solution as 1/2 root K this infinity -infinity here F(xi) I can take this Laplace transform inside, Laplace inversion inside okay, so if you do that for what you have is Laplace inversion of E power -square root of S, of course A is modulus of xi-X/root K, this is your A times root S, this is your exponential function, this is the function of what you get is function of T, okay, and but eventually it's a function of xi that you are integrating inside, okay, so this is how you get your Laplace inversion and this you already know what is your Laplace inversion of this you already know that we can use and put it here, Laplace inversion of this I directly write it as 2 root K this inversion –infinity infinity F(xi) and the Laplace inversion of this is simply E power-, A is xi-X modulus square, so that is nothing but this you can put it like this, X - xi anything you like, this square divided by K, divided by K is your square, A square of 4T, 4T together will give 4KT, and this whole thing divided by a square root of pi T, so what you get is finally as U(x,t) is your solution is 1/2 root pi KT times this –infinity infinity F(xi) E power- X–xi or xi-X, let me use X-xi whole square both are same by 4KT D xi, so this is your solution for X belongs to full R, and T positive, so this is the required solution, this is the standard solution that we know for an infinite rod having initially temperature for F(x) and eventually as T goes to infinity this is what you have for all times this is how the solution looks, okay.



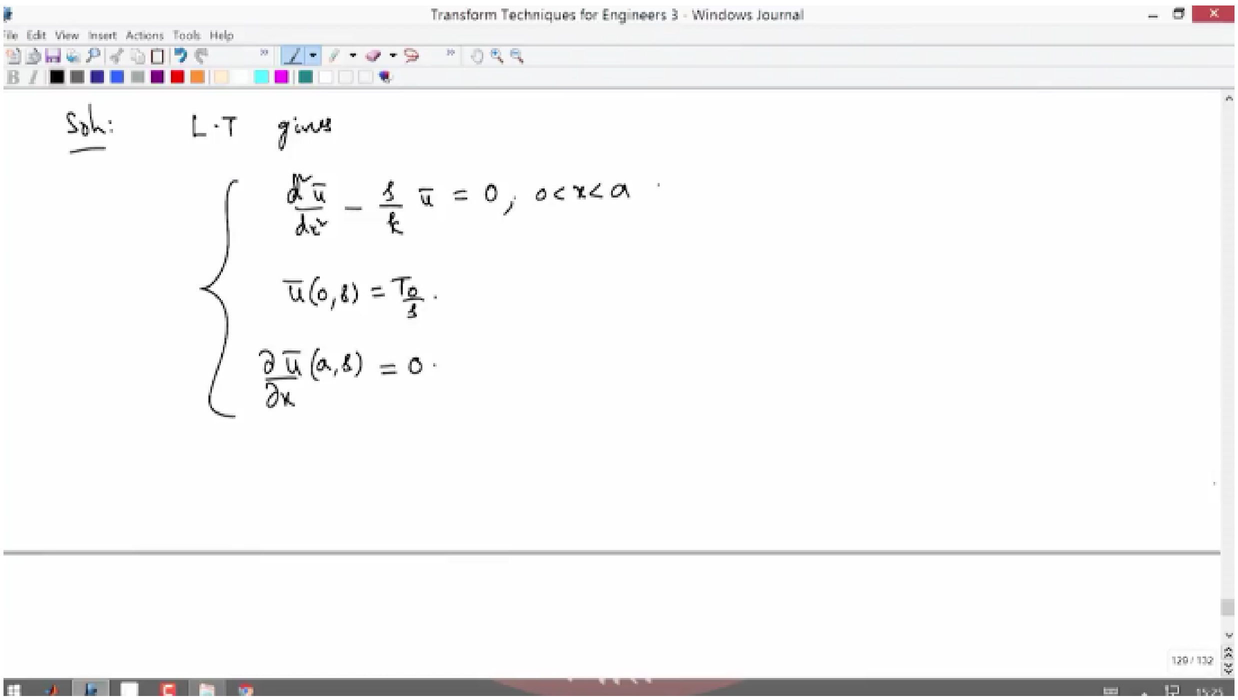
This is a well-defined, this is a nice integral it's convergent integral so you don't have worry, you don't have problem to evaluate this for all X and T, so this is how we solve this, but the process of applying the Laplace transform we have used again once again we use the inverse Laplace transform of E power –A root S/root S okay.

So let's move on to other applications, we’ll look at some more problems on heat equation, so let's look at finite medium, let's get back to the finite medium, so let's solve the heat equation UT = K times UXX, now I go only finite rod let us say between 0 to A and T is positive so you have a rod of finite length that is between 0 to A, okay, so this is your rod of length A, and initial condition is so initially a rod is at 0 temperature, so this is initially at 0 temperature that is 0 for X between 0 to A, boundary conditions you have to provide so that those are U(0,t) I give some let us say some T naught and UX, other end so here I maintain the temperature 0, T naught, okay, so other end I insulate it, I insulate it so that no heat flux goes out that means dou U/dou X = 0 at this point, no, I insulated so that I closed, insulated so that there's no heat passing out, okay, so if you write this as the boundary condition dou U/dou X(a,t) = 0 for all times, so this is also for all times I maintain it both the end points.

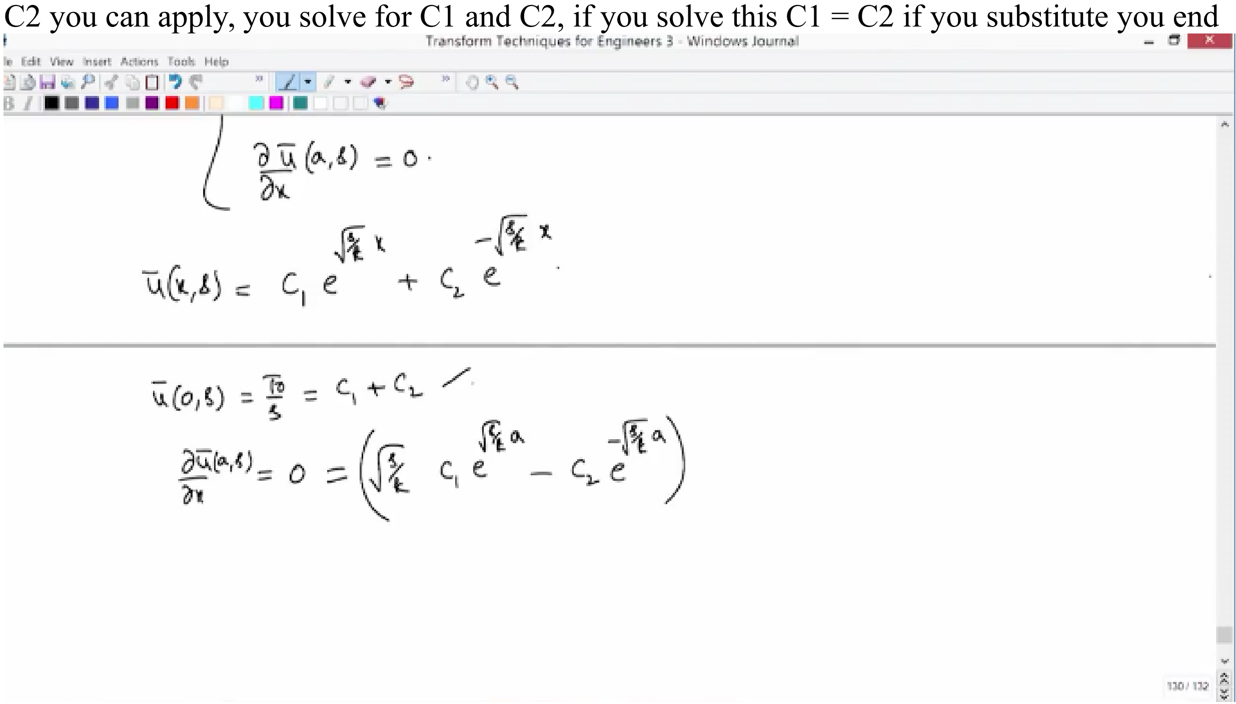


Then what is the solution that you look for by, we apply again Laplace transform to get this solution, so if you apply Laplace transform gives as usual the heat equation I write directly D square U bar/DX square –S/K U bar = 0, because of the initial condition 0 so I used here and so that I get I don't, I don't get this in the right hand side, if you have initial condition, initially it's rod is at some temperature so that will reflect in your homogeneous term, okay, so this is X positive, X is between 0 to A this is your domain of the differential equation, ordinary differential equation.

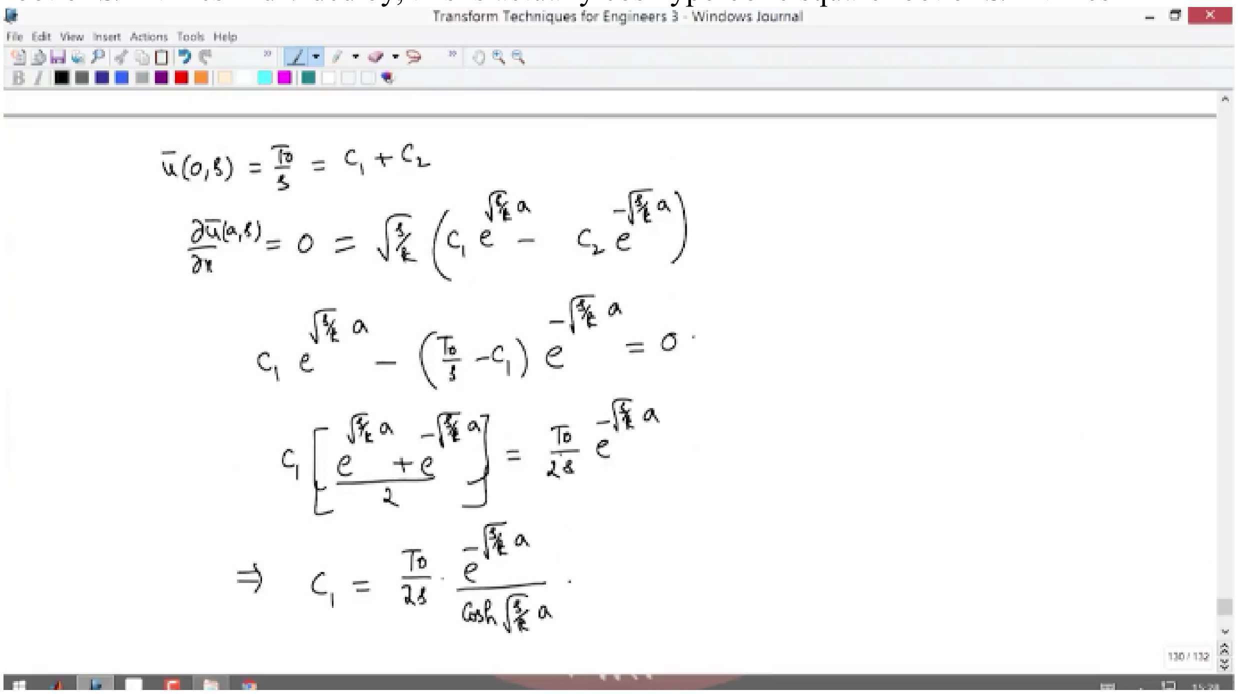
Now boundary conditions will give U bar(0,s) is T naught/S because of Laplace transform of the constant is 1/S, and Laplace transform of A,S when you differentiate this is, you’re differentiating with respect to X, nothing to do with that, so this is equal to 0 at A, so these are the boundary conditions you have, so this is your boundary value problem for the ordinary differential equation.



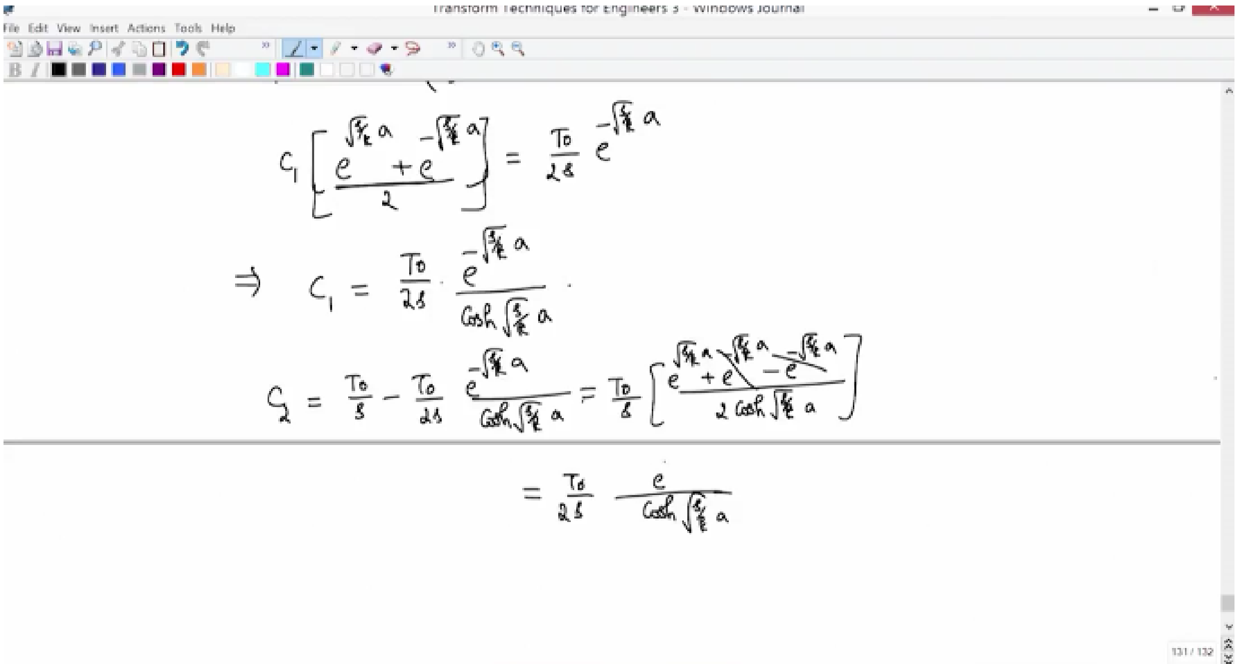
So how do we solve this? So we write this U bar(x,s), you can get the solutions as C1 times E power square root of S/K times X + C2 times E power -square root of S/K times X, if you apply the boundary conditions U bar(0,s) = T0/S, so which is equal to here if you substitute X = 0, you get C1 + C2 is that. And if you differentiate dou U bar/dou X(a,s) = 0, which is equal to, if you differentiate this with respect to, and put X = A, you have root S/K times C1 E power square root of S/K times A –C2 E power -square root of S/K times A, so if you use this C1 and C2 you can apply, you solve for C1 and C2, if you solve this C1 = C2 if you substitute you end



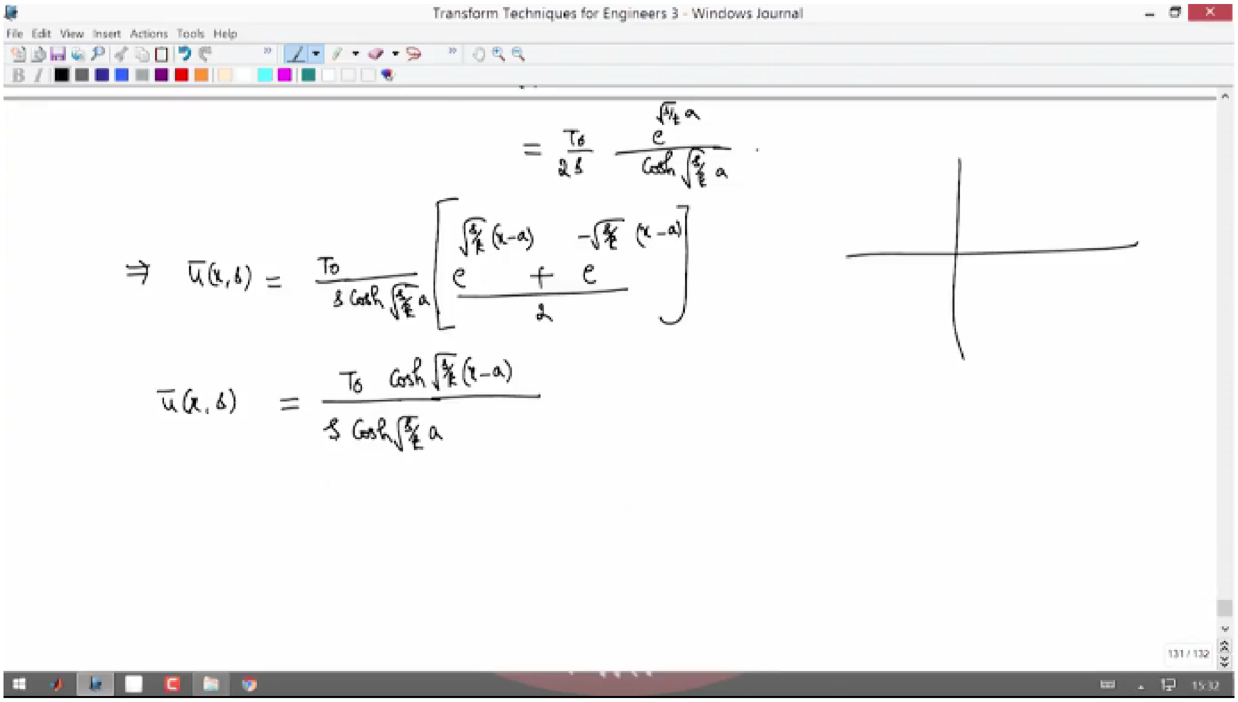
up getting, see if you do this way you will certainly ending up getting some nonzero constant, C1 and C2 both are nonzero constant, okay, if you actually see that you end up root S/K, here also root S/K comes out, so this is, you have two brackets here, okay, so this is what you get, and this is anyway constant, this cannot be 0 so this has to be 0 so if you put C1, C1 equal to, C1 E power square root of S/K times A - C2 is T naught/S - C1 times E power -square root of S/K times A = 0, okay, so what you get is C1 times E power square root of S/K times A, and this is +E power –square root of S/K times A = T0/S times E power -square root of S/K times A, so if you divide with 2 both sides you have 2S, so you get C1 as T0/2S times E power -square root of S/K times A divided by, this is actually cos hyperbolic square root of S/K times



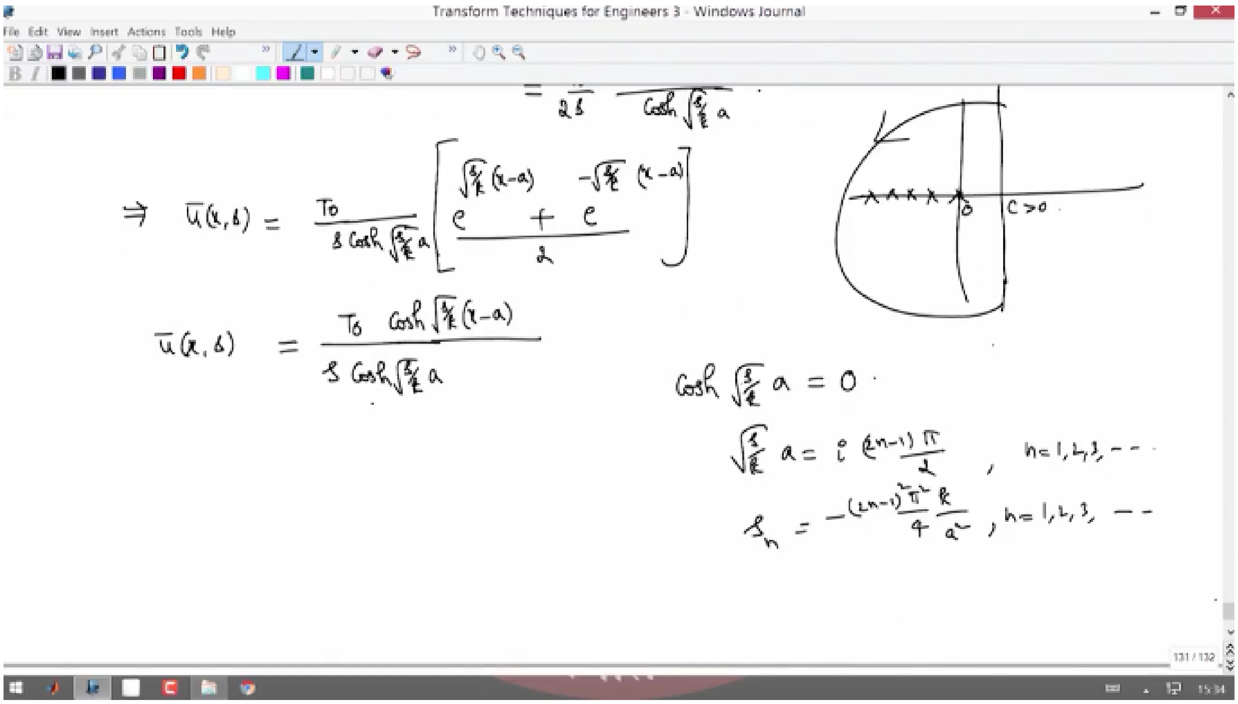
A, so once you know this you know what exactly is your C2, C2 is T0/S-C1 that is T0/2S times same one, so square root of S/K divided by A cos hyperbolic square root of S/K times A, so this if you simplify you get T0/S comes out 1-1/2 so you have 2 times cos hyperbolic square root of S/K times A comes as a denominator and you have here cos hyperbolic or rather 2 cos hyperbolic, 2 cos hyperbolic is numerator of this part, okay, numerator of this so that if I write it as E power square root of S/K A + E power -square root of S/K A and you have here minus simply E power -square root of S/K A, so this gets cancelled, so what you end up is simply C2 as T0/S or 2S, denominator as a cos hyperbolic square root of S/K times A, numerator is simply E power square root of S/K times A.



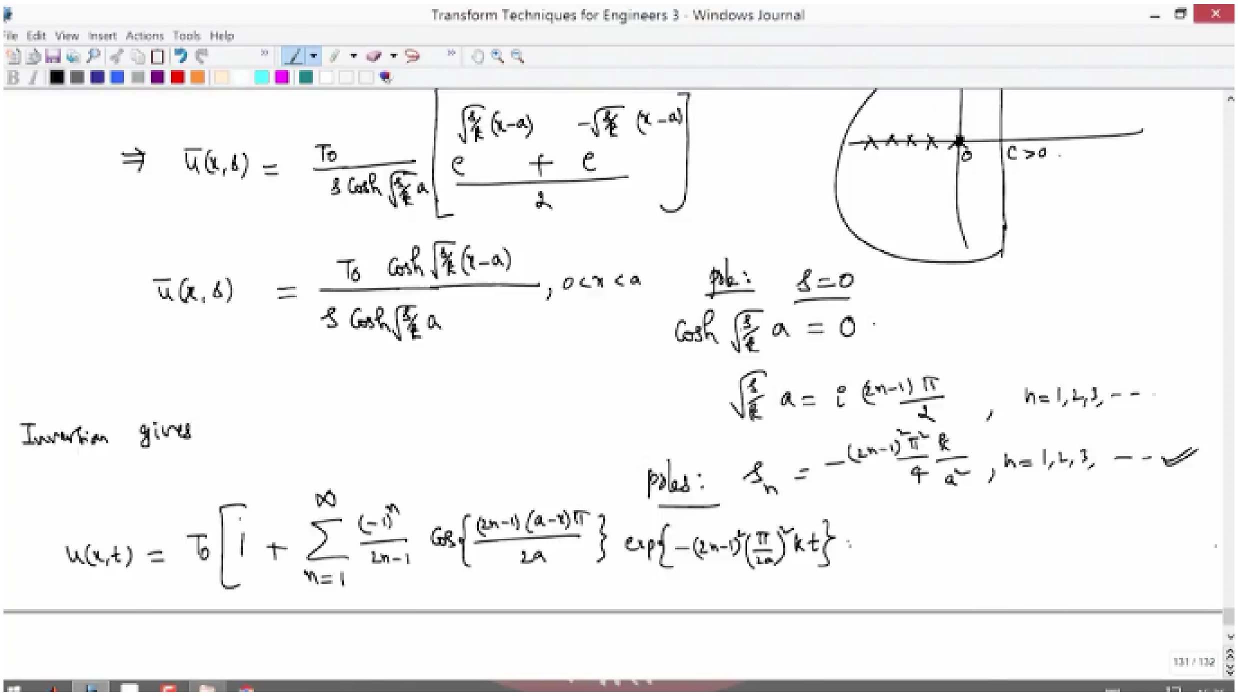
So use the C1 and C2 and substitute into here, so if you substitute C1, in the place of C1 so you get U bar(x,s), so C1 is a T0/2S that is common, cos hyperbolic square root of S/K times A that is also common, and you have E power -square root of S/K A times here C1 coefficient is E power square root of S/K into X, so you have X-A as, okay, so if we combine this that is what you have, so E power square root of S/K times X-A, and the other one is C2, C2 is again here so this is common, so T0/2S this denominator if you combine it here you get square root of S/K A here –X, so minus of X-A you can write, so this is nothing but if you combine with, divided by 2, you bring this 2 inside so you end up getting T0/S times cos hyperbolic square root of S/K times A here, cos hyperbolic square root of S/K times X-A, this is what is your U bar(x,s), so for which we need inversion, and you clearly see that if you see this function U bar(x,s) 0 is the, if you see 0 if you calculate, if you find, if you actually have expansion for, if you have a expansion for cos hyperbolic both sides, both numerator and denominator as exponential you



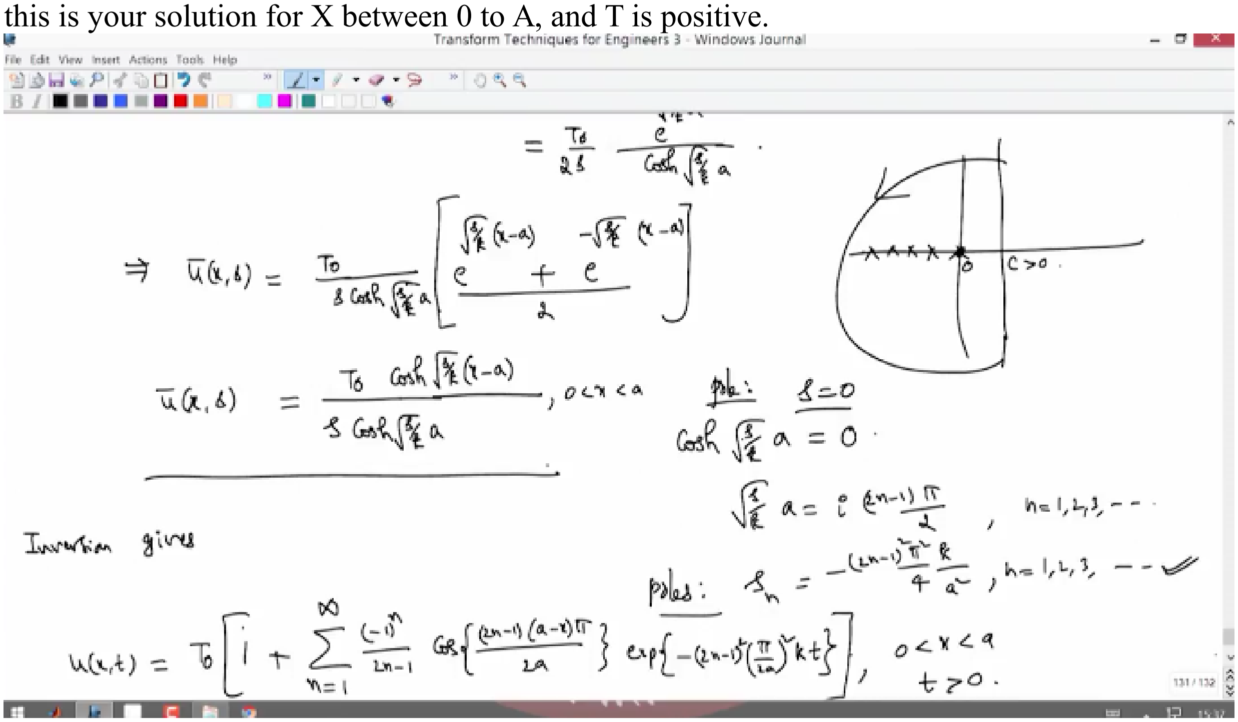
write, like this form if you write and try to expand and you see allow, allow S goes to 0, you end up seeing that only you multiply S times, S into this you see that is a constant, so if you look at only this part, this part alone if you take limit S goes to 0 it will be a constant by looking at a expansions and you allow S goes to 0 you see that is a constant, so that 0 is not a branch point, though you have a root S involved in the cos hyperbolic, it is not a branch point, but 1/S here makes 0 as a pole, so 0 is a pole, so outside this everywhere it's analytic function that U bar(x,s) so you can choose your C here, okay, C anything positive, and so that you can get this, using this Bromwich contour you have only one pole, as 0 is a pole and because of this cos hyperbolic, cos hyperbolic root S/K A = 0 that gives you what are the roots, okay, that you get S/K times A as I times N-1, what are the roots you have here, I times cos 2N-1, 2N-1 pi/2 that is N is from 1, 2, 3 onwards, N = 1 you have pi/2, cos pi/2 is 0, so you have Ith, because of I cos hyperbolic becomes cos, cos 2N-1 pi/2 that is 0, so because of this, if this is equal to this you have the square, so this will give me what are the, you square both sides you have S/K times A square = -2N-1 square pi square/4, and you bring this side K/A square, so these are your poles for N = 1, 2, 3, onwards, these are all negative side, on the negative real numbers you have poles, infinitely many poles you have using this Bromwich contour if you calculate its Laplace



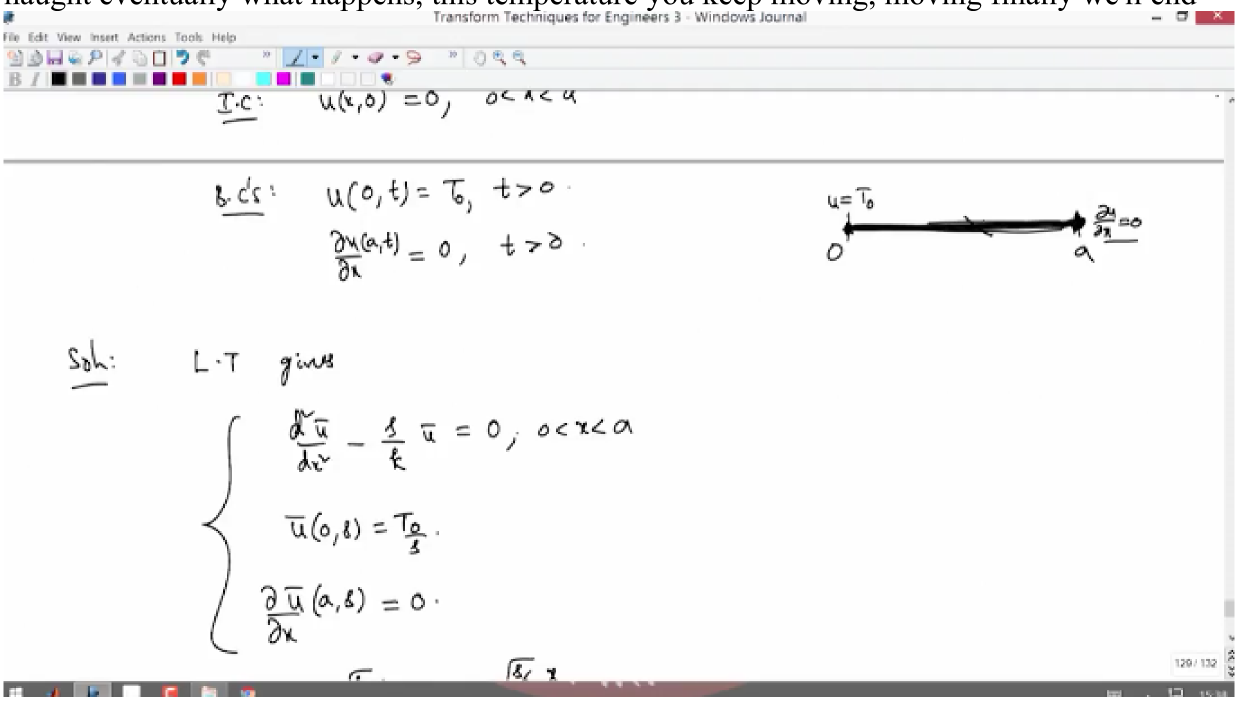
inversion you end up getting, you need to calculate only residues at these points, all the poles here and also at S = 0, S = 0 is a pole, and other poles are here, these are also poles.
At these points if you calculate the residues that is you multiply S and take the limit S goes to 0 this gets cancelled and what is this, limit of this as S goes to 0 will give me my first pole, so inversion will give me or by heavyside expansion theorem, if you apply as such as though you have infinitely many poles you get inversion gives you can see earlier method how to get this this thing, so X is anyway positive side, X is between here between 0 to A, so inversion gives U(x,t) as I write directly so this is simply T naught is a constant, first one you will get it as 1, okay, and the second one you have running all, for each N you have contribution N is 1 to infinity, and you have so you have a poles, right, so the contribution of the pole I write directly I'm writing, so directly as -1 power N divided by 2N-1 cos 2N-1 A-X pi/2A this is 1, and into exponential thing that is E power minus, let me write as exponential function of -2N-1 whole square pi/2A whole square times KT, so this is what exactly you get as your inversion, okay.



So at this point you have this inversion, because of this, this is coming because of E power ST, S is this, SN(t), SN is this one pi square K/4, that is 4A square, so this is minus so this is coming from, this quantity is coming from E power ESNT times and this one, so you need to find the residue of this times E power ST, ST of this, okay, so if you do that you see ending, first one is this, this is your residue for the function this times E power ST, so you have this, so this is your solution for X between 0 to A, and T is positive.



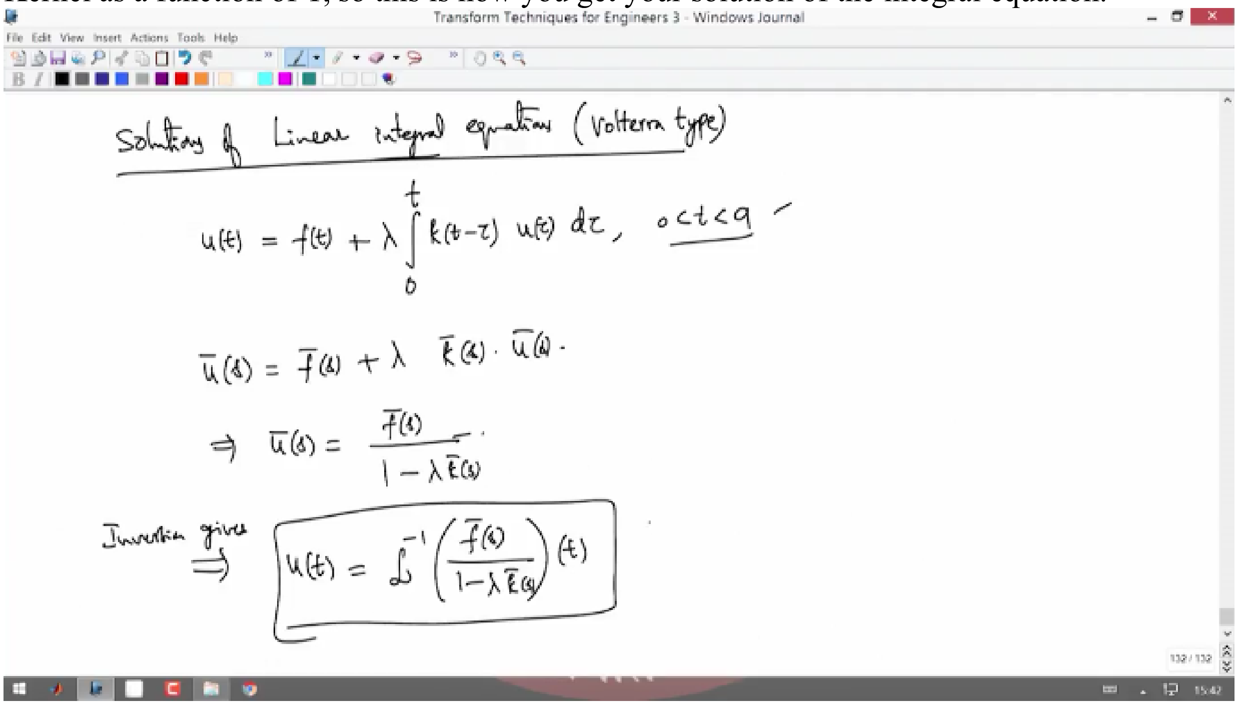
So this is how you can get the solution using a heavyside expansion theorem or directly using the Bromwich contour for which if you have, if you calculate your U bar(x,s) on a finite, infinite rod if you consider and initially at temperature, one side you keep at some temperature, other side you insulate it, so if initially is at rest, and temperature is 0, what do you expect is eventually as T goes to infinity, if you actually see if as T goes to infinity you see that this whole sum is going to become a 0 because of this exponential, so what remains is only T naught, so that is clearly expected from this physical thing of this infinite rod, one side every, so literally its insulated, it's all insulated only heat is going only one direction, and this side also insulated, so you keep pumping at this point you keep on adding, you keep the temperature at T naught eventually what happens, this temperature you keep moving, moving finally we'll end



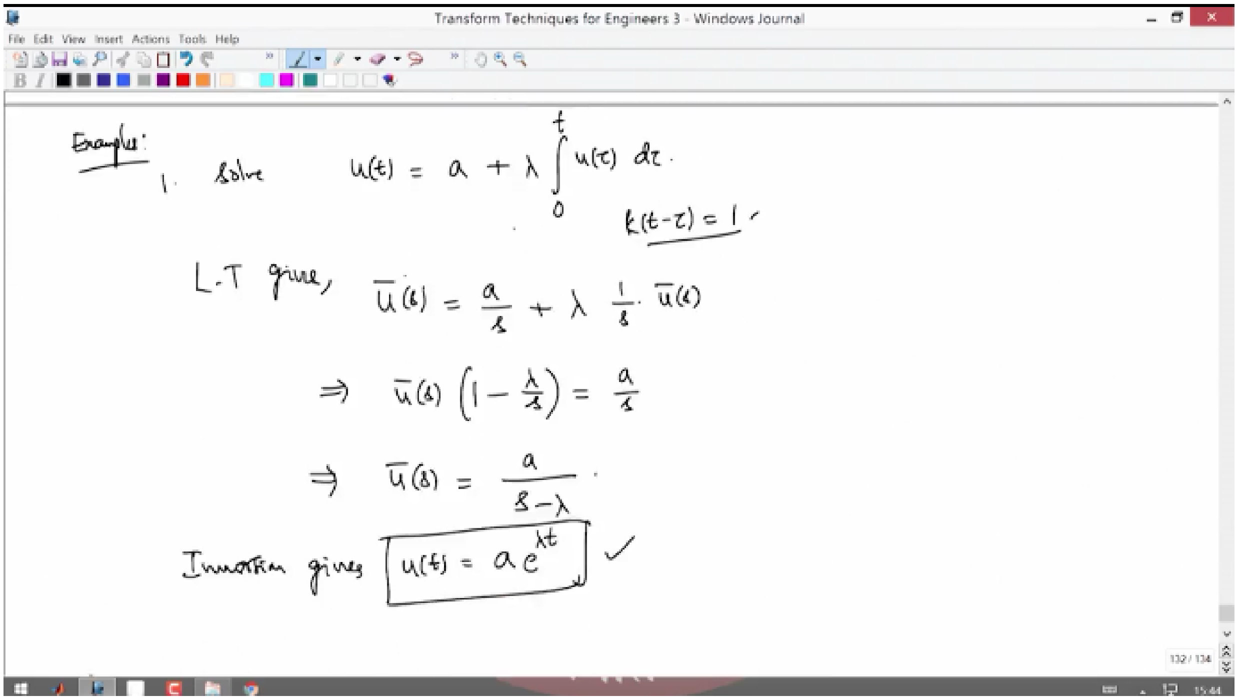
up getting as T goes to infinity at every point it'll become a temperature T naught, so constant temperature, okay, so that's how you get this solution that's clearly see as T goes to infinity, so this is how you solve this finite medium problem. So you can also solve similar such problems, any such problem on heat equation you can solve with initial and boundary conditions using this Laplace transform, okay.

Let's look at the other applications of this Laplace transform, other application says one can solve integral equations, we can solve solutions of linear integral equations Volterra type actually, you can only solve Volterra type, this is a type of integral equation, let me write what it is if you have unknown function as U(t) and if you have a function here, given function nonhomogeneous term + some lambda times 0 to T, this is where we use the convolution, convolution you know how to take its Laplace transform that is why this if you have from 0 to T that is called Volterra type if you fix from 0 to some fixed number that is different type of integral equation, so this is a Volterra type of integral equation so you have some Kernel, K of T,tau times, this also you need because of this convolution this K(t,tau) this is your Kernel times unknown function U tau D tau, so this function this Kernel this is called Kernel of the integral equation this also if you want to apply Laplace transform this has to be convolution of this K and K function and the unknown function, so because to make it a convolution you want to have your Kernel to be of this form, okay, so where T is between, T is positive, positive to any let us say some fixed A, okay, so T is between 0 to A.

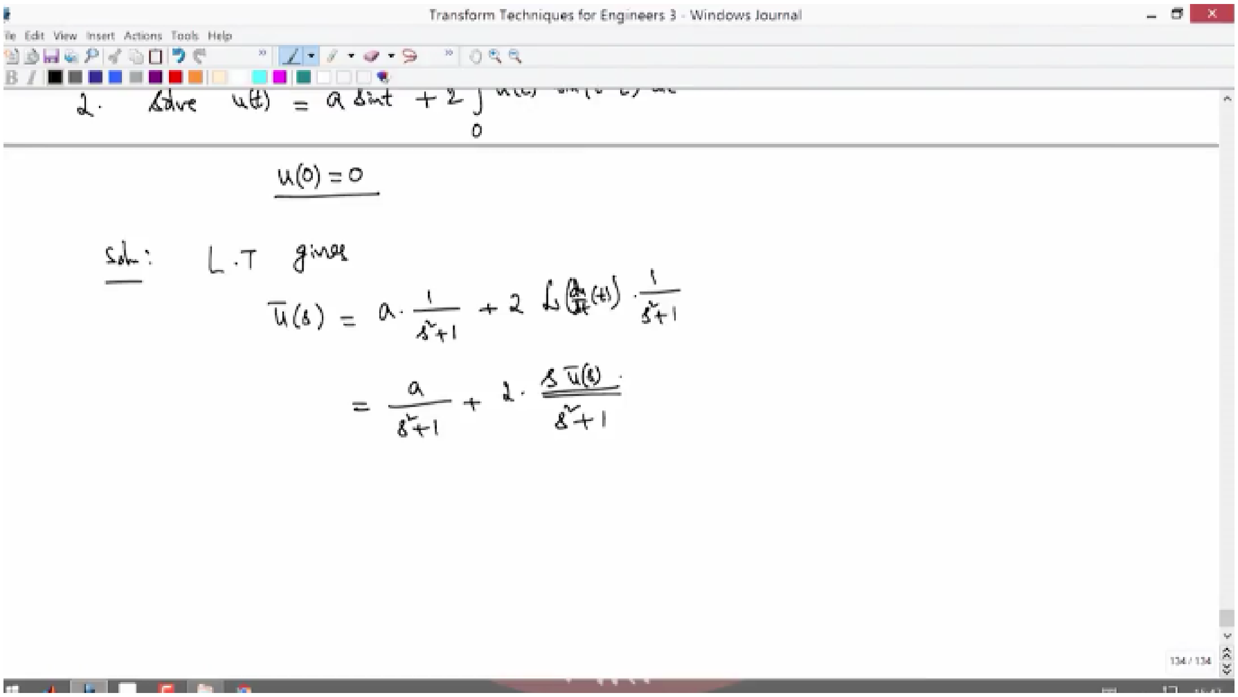
Let's say if this is your integral equation we can apply both sides Laplace transform so that you have this one plus lambda and because of this is your convolution you can get K bar(s) times U bar(s), you can easily see that this is what happens, so this makes it U bar(s) as F bar(s) divided by 1-lambda times K bar(s), right, so how do I get my solution? This is your solution because unknown is inside this integral that is what is called an integral equation, it's nothing to do only with involving, unknown is only U, not U square, U cube any square roots, so because of this it is a linear equation, so it's a linear so inversion will gives, so this is the general procedure to get your solution U(t) as Laplace inversion of this function F bar(s)/1 –lambda times K bar(s), it's Kernel as a function of T, so this is how you get your solution of the integral equation.



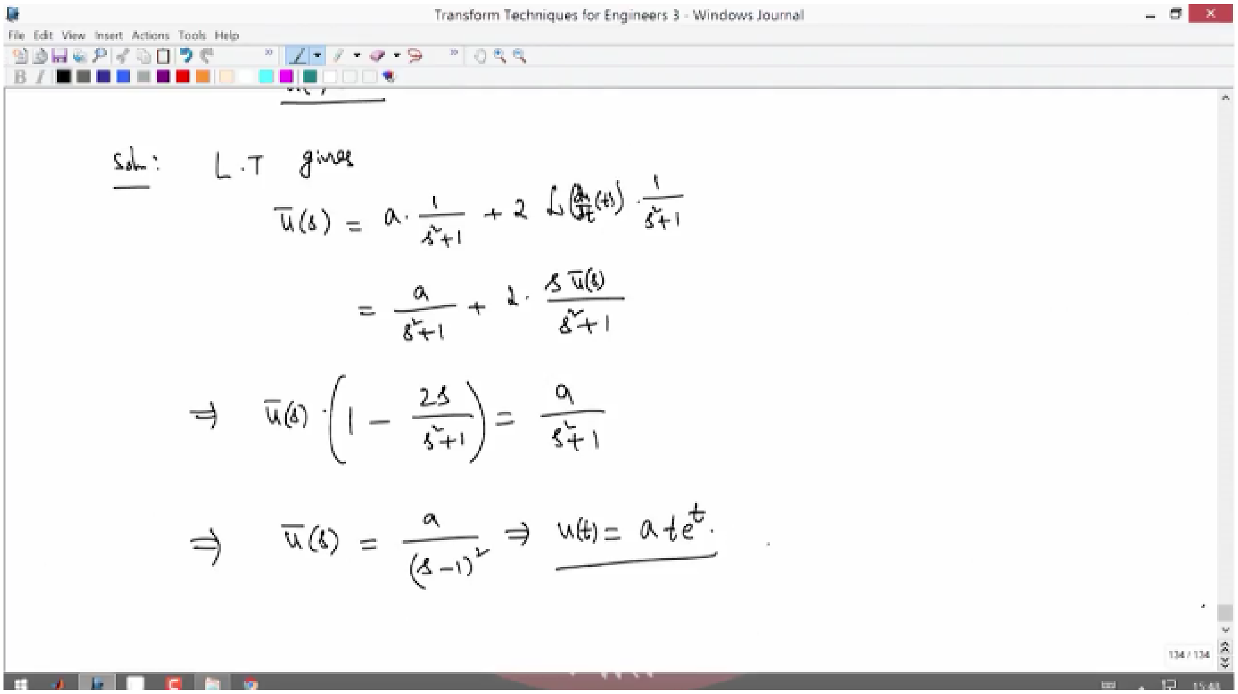
So let's give some examples, so let's solve some examples let me give only few examples one or two, so solve, if you solve U(t) = some constant A + lambda times 0 to T this is U tau D tau, so how do I solve this? Simple linear Volterra type integral equation you apply Laplace transform, Laplace transform gives, what is your Kernel? K(t-tau) equal to constant, okay, so this is in this form so you can apply the Laplace transform that gives U bar(s) = A, it's a constant so if you do this Laplace transform A/S + lambda this becomes K is 1, Laplace transform 1 is 1/S times U bar(s) so that makes it U bar(s) as you'll get coefficient as 1-lambda/S this is equal to A/S, so that makes it U bar(s) as, S S goes both sides you have A divided by S-lambda, so if you invert this, inversion gives the solution U(t) as A times because of this S-lambda, so you have E power lambda T as a solution, so this is your solution of your different integral equation, okay.



So let's do one more solution, one more problem, so one more example given, I'll close it here, if you have, not only that you have this linear Volterra type integral equation, if your unknown is having derivative that is called a integro differential equation, you also have a differential U as a derivative if you have here or outside you call this integro differential equation those things also you can solve, so let me give that example as U(t), solve U(t) which is equal to A sine T +2 times 0 to T, U dash(t) this is a derivative, and then sine T-, this is tau, okay, let me write it as tau, and T-tau this is your Kernel D tau, so this is one, and you see that U(0), U(0) if you substitute it has to be 0, okay, but you see here U(0) is simply A, so that is you need not give because of the differential equation you have to provide this condition, you may need this one, okay, so you may need this, this you can get it from here, if your derivative is here you can get, if your derivative is here for example then still you have to provide, otherwise you will get arbitrary constant so that you have to get it from your initial condition, so if you are derivatives is inside, inside the integral this is called integro differential equation okay, otherwise also if it is even here it's a integro differential equation, so let me give this, this you can directly see that when you put U0, A sine 0 is 0, when you put 0 to 0 whatever may be the derivative of the unknown function that is that may be the integrand, well-defined integrand this has to be 0, okay.

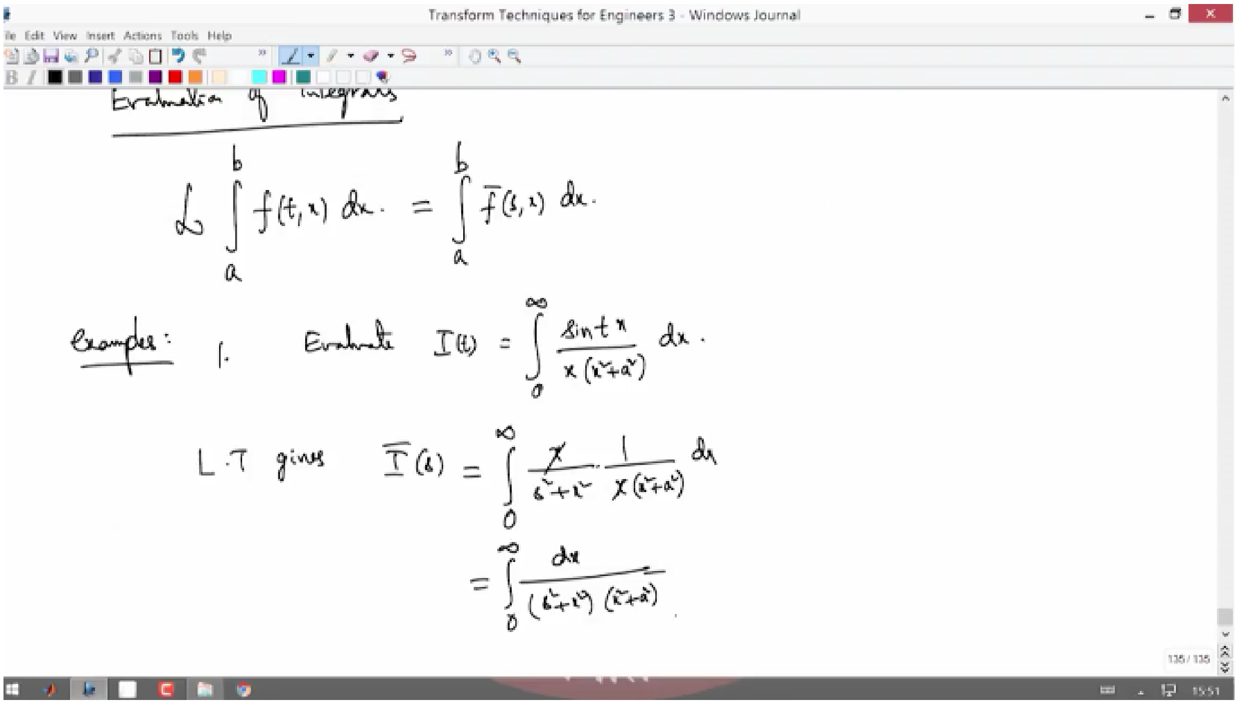


So solution, how do we, again Laplace transform gives U bar(s) and here A times sine T is 1/S square + 1 + 2 times, Laplace transform of U okay, U dash(t) that is right, so U dash(t) so that is DU/DT, okay, so DU/DT and here this will be Laplace transform of sine T, that is again 1/S square+1, so what is this one? This is equal to A/S square+1 +2 times Laplace transform of this is, this is S times U bar(s) – U(0), so that is exactly 0 is given so it goes divided by S square +1 so this is a Laplace transform of this part this one and you have this you write it here as a denominator, so if you see this one so you get U bar(s) which is 1 -2S/S square + 1 = A/S square+1, and if you write this as, simplify this as S square S square + 1 goes and you have S square + 1 – 2S is A over S-1 whole square, so inversion will give, so this gives me U(t), the inversion will give me A times T into E power T, so that is your solution, because T into E power T Laplace transform is S-1 whole square, so that is how you get your solution.

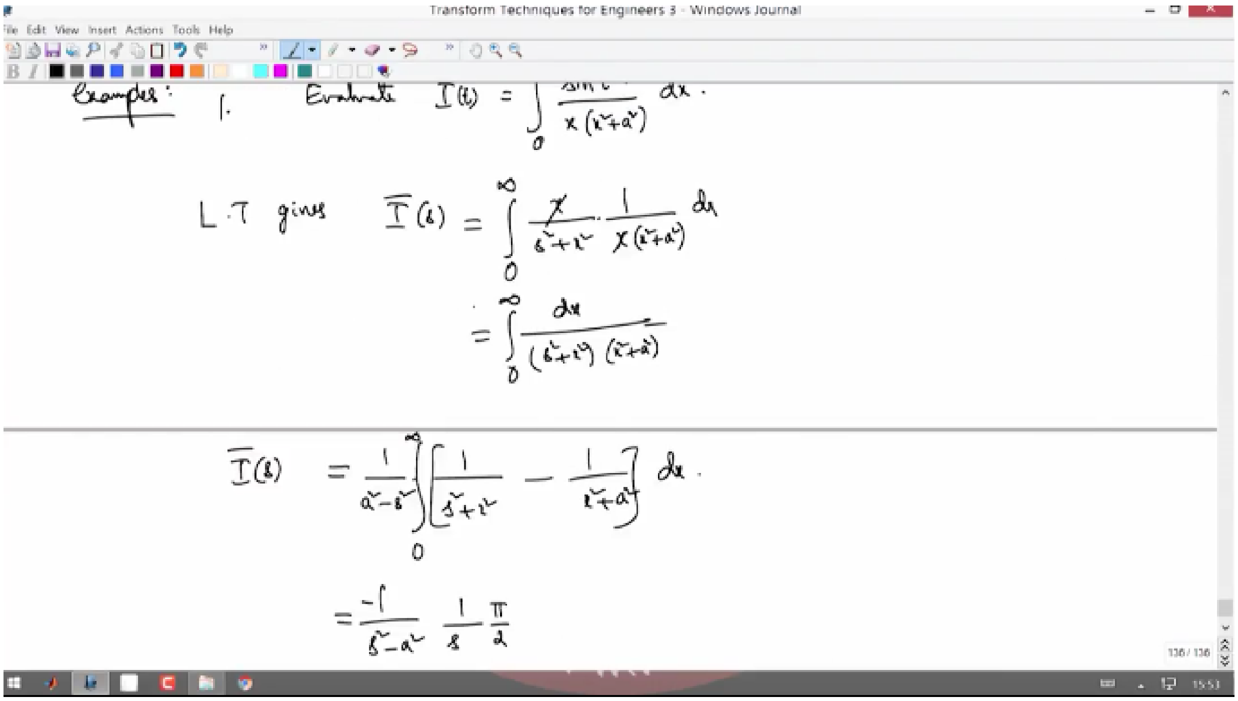


As you see U(0) is 0 still satisfied, so this is your solution of this integro differential equation, so you can say simply T positive, okay, so this is how we can solve some linear Volterra type of integral equations either linear integral equation or linear integro differential equations you can apply this Laplace transform and solve it, we can also solve some, you can evaluate certain integrals involving a parameter, okay, so we can also like Fourier transform, we can also evaluate certain integrals involving a parameter, so evaluation of integral let me give you one simple example, evaluation of integrals, integrals involving some integrals, okay, not all integrals you can, some integrals let me if you want, if you have A to B, F(t,x) DX, suppose you want to evaluate this where T is this you apply the Laplace transform both sides so because you, because of the Laplace transform you can apply you interchange this orders of integration, integration of Laplace and integration of this A to B, if you do this you have A to B Laplace transform of this, that becomes F bar(s,x) DX, okay, so this is the idea so once you use this because of this you can take this Laplace inside then you can do this only such integrals where you can take legitimately you can take this Laplace inversion, Laplace inside this integral you can change the order of integration, orders of integration if you can change then you get, then you have this form so once you have this form you can evaluate.

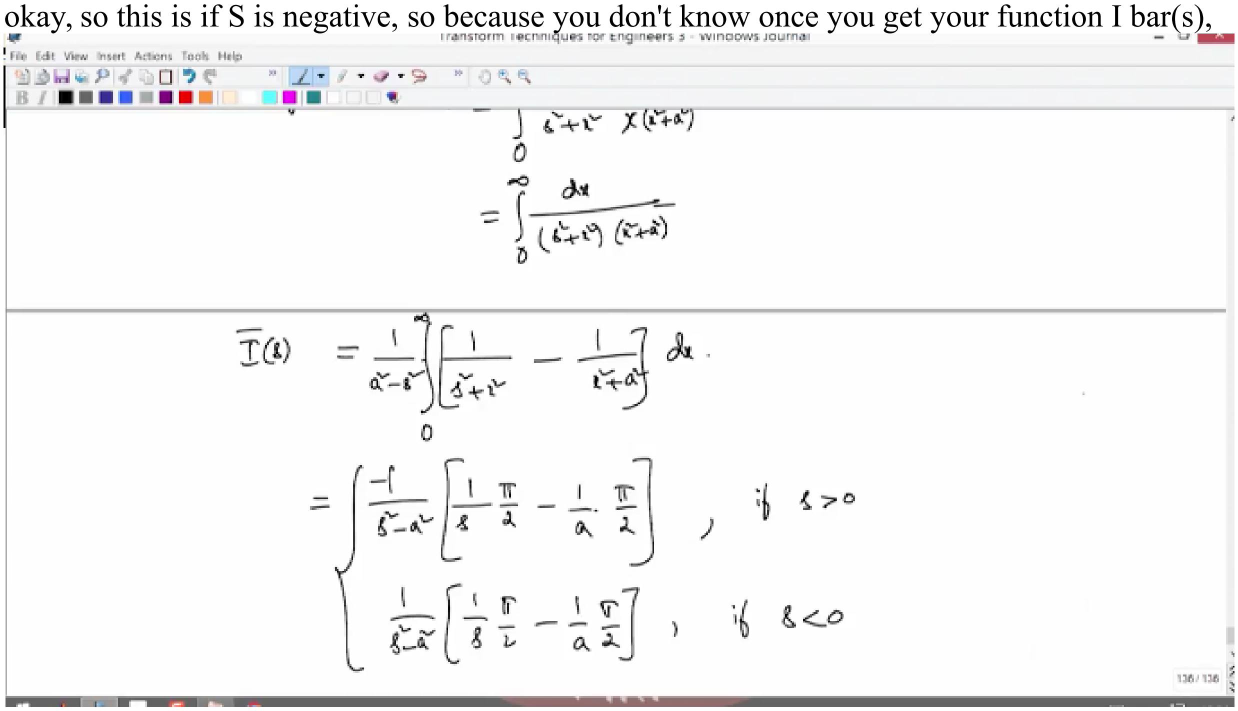
So let's see some examples given here, so let me give one, evaluate F(t), evaluate this integral F(t) as 0 to infinity integral, sine TX/X times X square + A square DX, so how do we get this F(t), let's see okay, evaluate this integral I(t), let's call this naught F, okay, so integral of this, so you apply a Laplace transform both sides, if you apply Laplace transform, Laplace transform again gives I bar(s) which is equal to, if you take this change the orders of integration 0 to infinity sine TX is X divided by S square + X square times 1/X into X square + A square DX, so X X goes, so what you have is 0 to infinity DX/S square + X square times X square + A



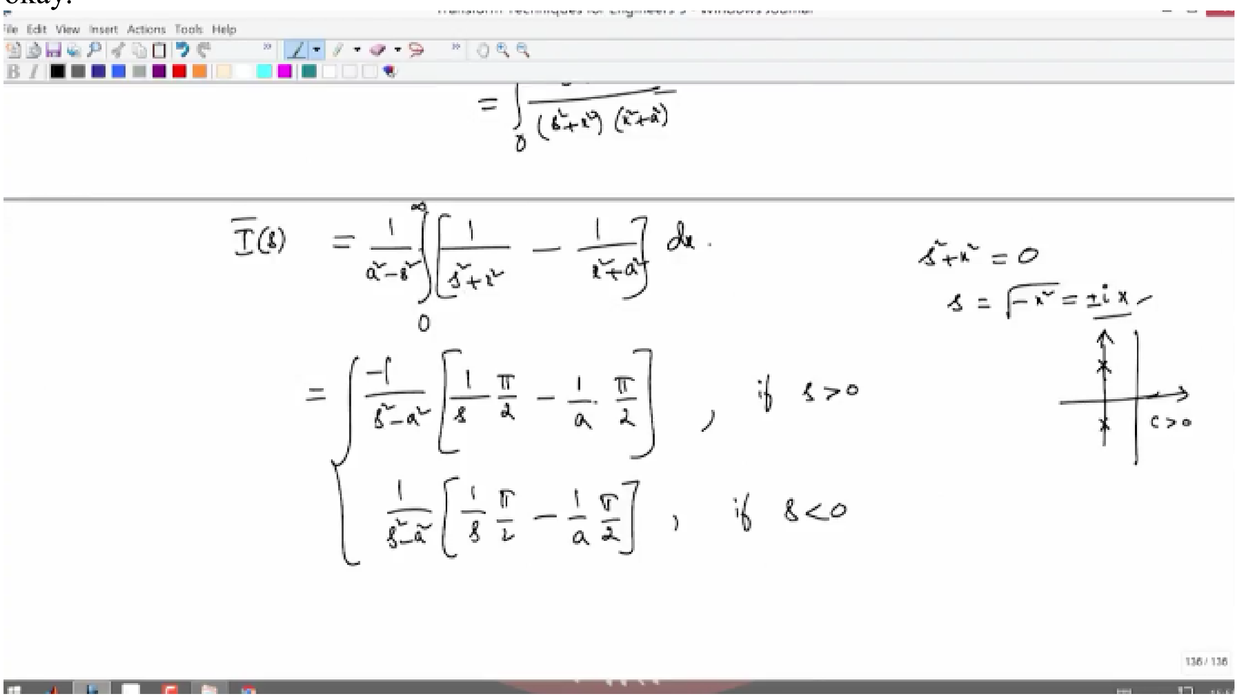
square, so this one you can evaluate easily which otherwise you are not able to do with directly, okay, after applying the Laplace transform you could evaluate this by rewriting as 0 to infinity rather let me put it here this as I bar(s) as integral 0 to infinity or rather if I do integrand as S square + X square -1/X square + A square, so X square + A square -S square so you have 1 over A square - S square, okay, so because of this integrand you can write like this as a product of this okay, so you integrate from 0 to infinity with respect to X, so this is easy to evaluate A square -X square, so you have -S square – A square you write and first part will give you this is like you bring it out, you get as 1/S X square +S square as though S is constant 1/S times tan inverse X/S, this is 1/S square comes out you have 1+X/S whole square okay, so you have this integral, so 1 by this or DX, D of, this is actually DX so you can write X/S, so you have 1/S times of that, so this is exactly this derivative, this integral is exactly tan inverse X/S and if you put 0 to infinity, 0 to infinity if you do you end up getting this as tan inverse infinity as pi/2, tan inverse 0 is 0, so you end up getting pi/2, okay.



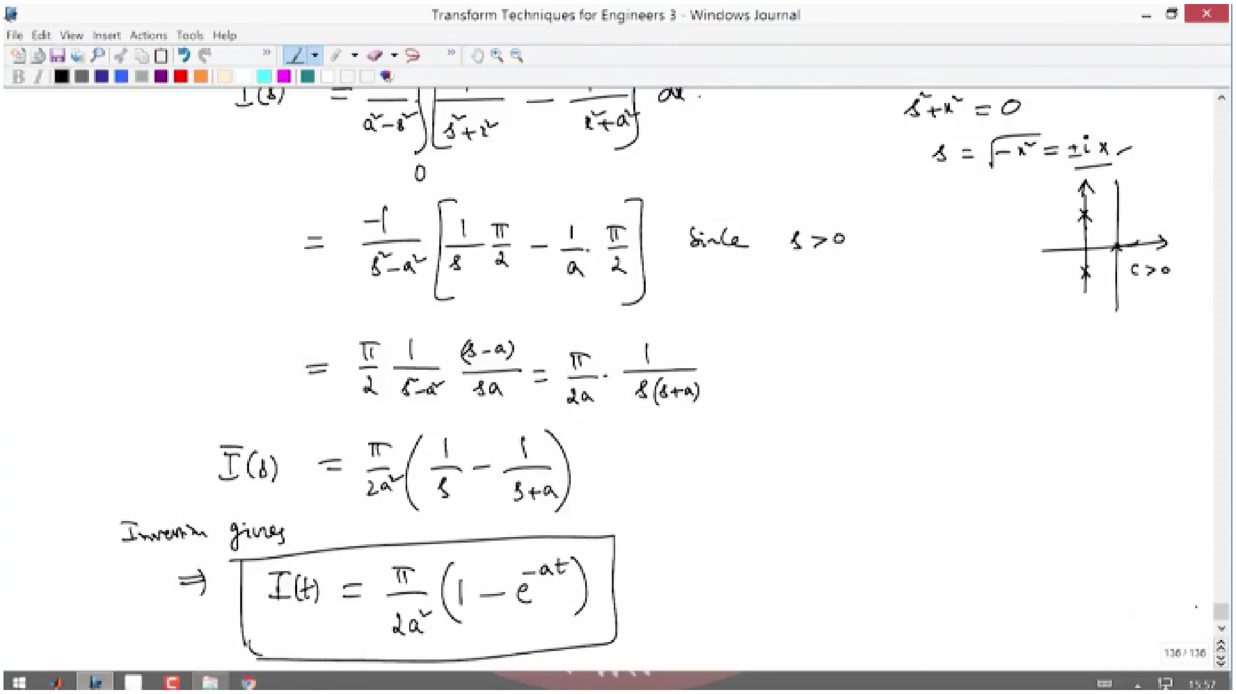
Depending on X, what is your X? X is actually positive so you have pi/2, okay and then other part I have, so other part this one will be again 1/S times pi/2 depending on the sine of A, okay, so here depending on the sine of S, sine of S if S is positive, if S is positive you have + or +, okay, if S is positive this is plus, and here pi/2 times here, here this is going to be 1/A square, 1/A here, 1/A depending on what is your A? A is positive, if A is positive so you don't know whether S is positive or negative, okay, if S is positive is this, if S is negative you still have the same one with positive sign, so this is going to be 1/S you have –pi/2 that makes it plus, and here also you have –pi/2 that becomes, minus you can take it out you may get plus together, okay, so this is if S is negative, so because you don't know once you get your function I bar(s),



if you look at this S square is, so what is your S square + X square, you look at its equal to 0, we look at it singular points S = -X square S square, this is square root of this, S is the square root of this that is actually I times + or – X, so you have the singular points are over here depending on your X, X is anyway positive, so your singular points are here, S is actually you have singular points on the imaginary axis, this is your S complex plane, so if S is positive that means if you consider your Laplace inversion as over this C positive, S is anyway positive, okay.



So this, as though if your S line is here and you are evaluating this integral as though S is here that means S is here, S is positive, so let's take it that way so you have S is positive, okay, so S is positive since S is positive you have this part, so what you get is pi/2 is common, 1/S square --A square and this becomes A-S, S-A you can write with this -sign that becomes S-A and you have here SA, AS okay, so you end up getting this gets cancelled and you end up getting pi/2A times 1/S (S+A) so this is pi/2A this you write it as S - S+A, S+A –S though you have A comes up so you have A square, so this is your I bar(s), so you take its inverse, inversion will give, inversion gives I(t) as is, so that is exactly your integral you want to evaluate as pi/2A square, pi/2A square and 1/S as 1, and this is E power –AT, so this is exactly your integral value as a function of T, okay.



So we'll try to give one more example and we also look at, we'll get back to ordinary differential equation when you have impulse that means a delta function as your right hand side, forcing function is your impulse for example you may have a beams, when you have a load at one point is concentrated load you can use this as a right hand side the applications it'll come as a ordinary differential equation you may end up and the right hand side as a non-homogeneous term forcing function which is a delta function for which if you have such a thing how do you find the solutions using the Laplace transform, to do this you need a Laplace transform of these generalized function that is a delta function as though as we have done for since we have done Fourier transform for those special function, special generalized function as a delta function here also we'll try to give the definition of Laplace transform for this delta function, we will try to solve one or two problems on problems of ordinary differential equation with non-homogeneous term being as a forcing function being a delta function, okay.

So with that we close these applications of this Laplace transform maybe in the next video and then we start Z transforms okay, so we'll see in the next video. Thank you very much.

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