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NPTEL ONLINE COURSE
Transform Techniques for Engineers
Initial Boundary value Problems for Heat Equations
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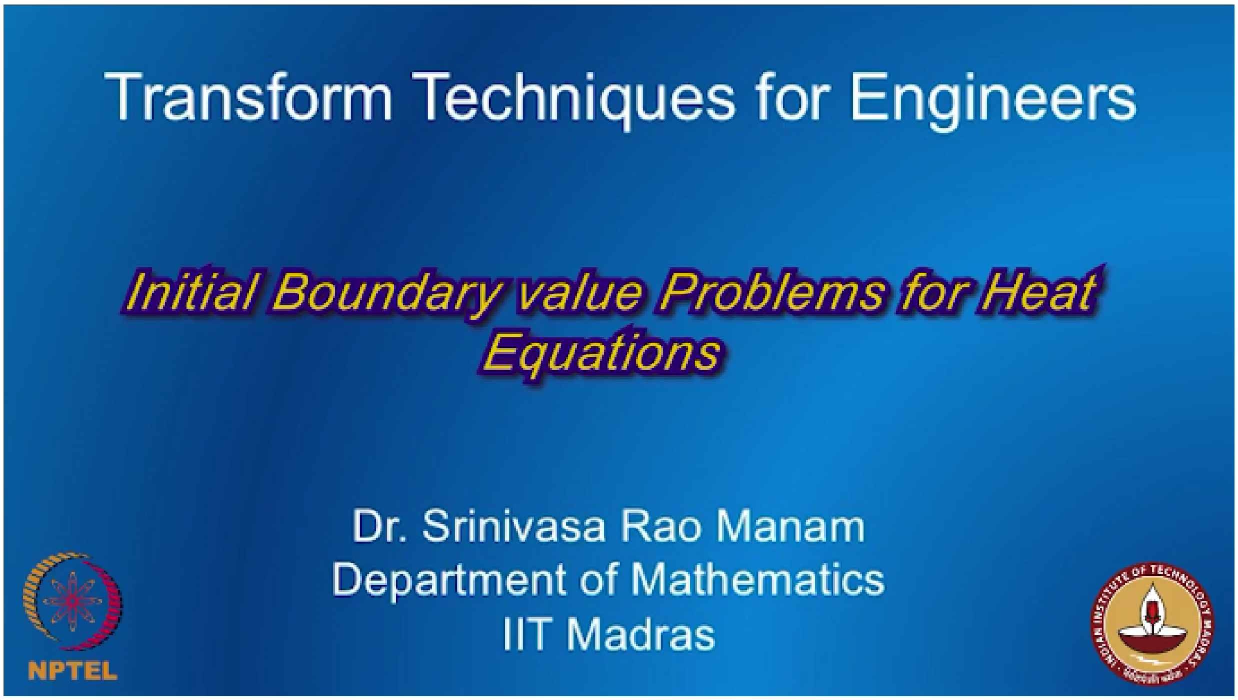
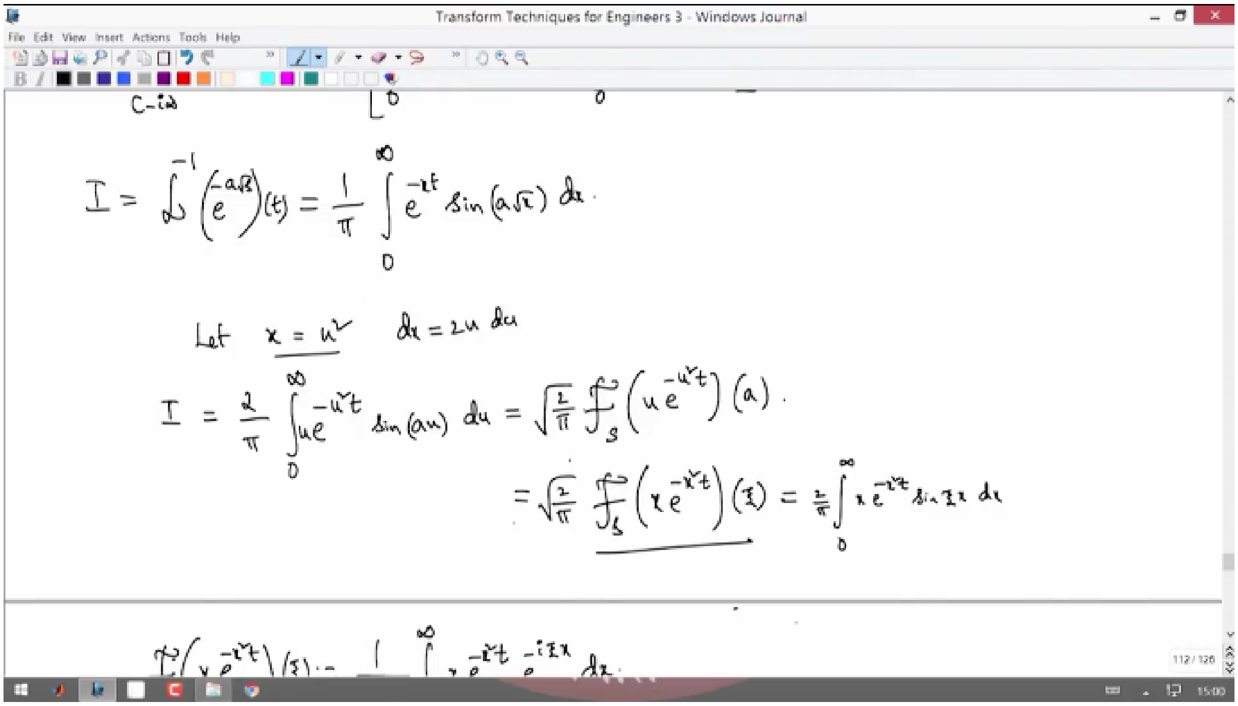
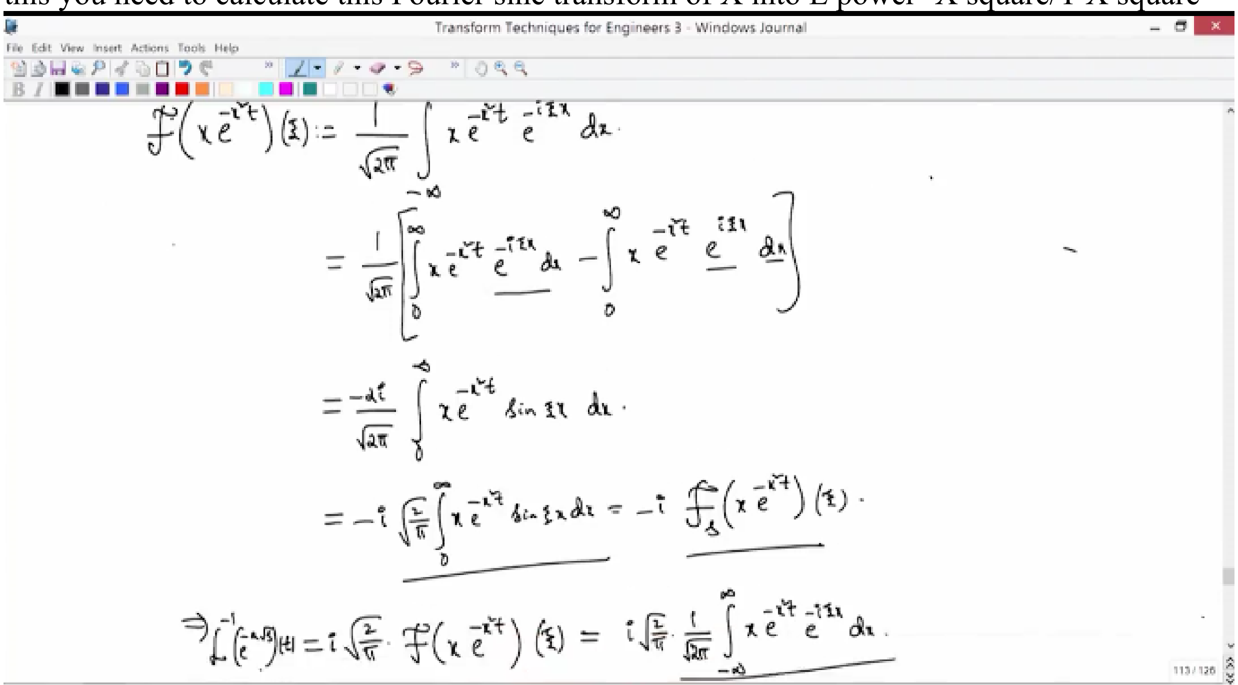


Welcome back, we're discussing about the Laplace inversion of a special function, the exponential of -A root S, where A is positive what we have done so far is I have seen that this

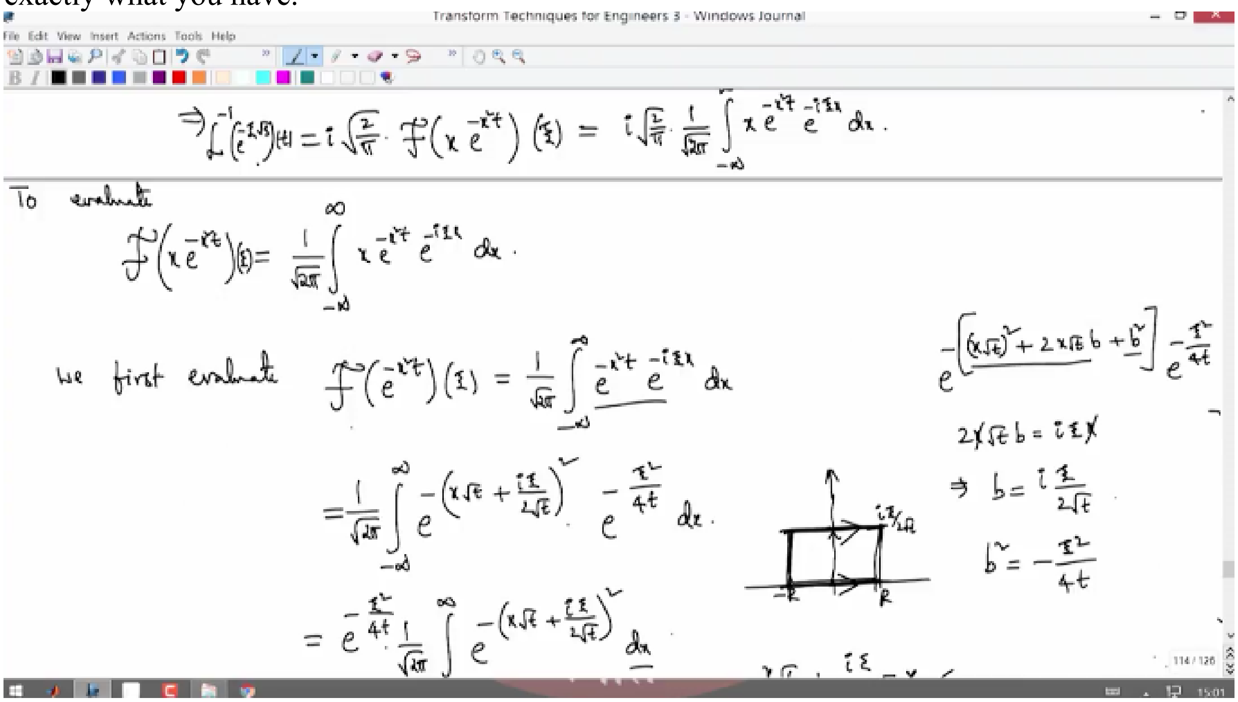


Laplace inversion as you see this is your I, I is I denote it as that, that you have seen that it is actually root 2/pi times Fourier sine transform of this function that is, if you write like this you can write this is equivalently because I changed A to xi, so what you have is L inversion of E power minus instead of A I write xi root S as a function of T, so this is what you have, so for this you need to calculate this Fourier sine transform of X into E power -X square/T X square

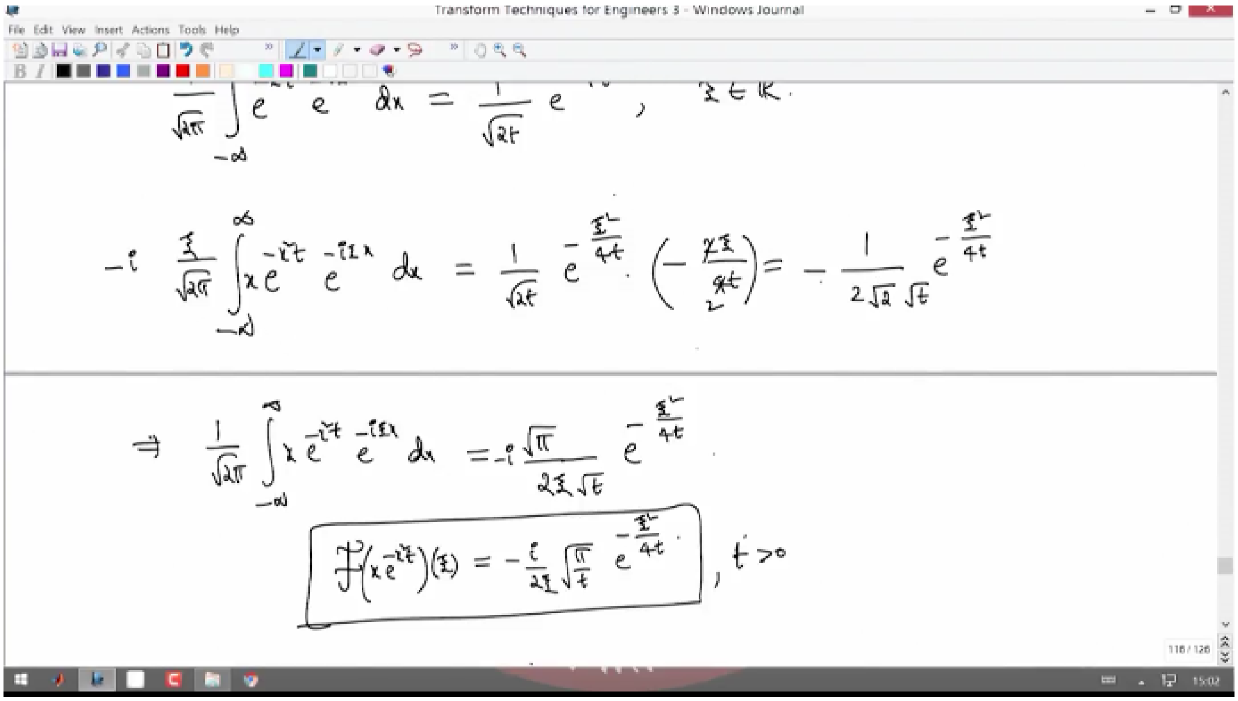


T, and to do that so what you have is this, this is related with Fourier sine transform of X into, Fourier sine transform is related with actually Fourier transform of X into E power -X square T, so you calculate Fourier transform so that what you have is E power –xi root S, xi positive will be this one, so this is where we are now, this is exactly where we are.

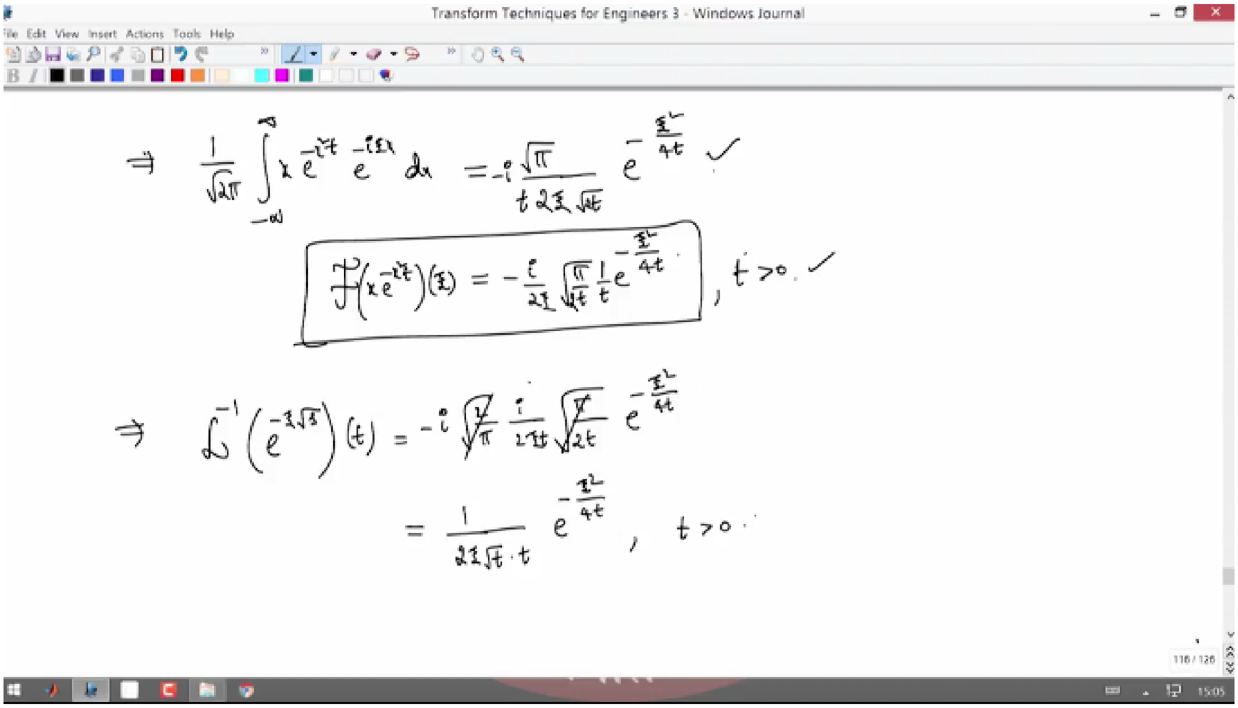
So we calculate this Fourier sine transform of, basically Fourier sine transform I use this Fourier transform of X into E power -X square, so ultimately what you see is the Laplace inversion of this function is now this integral you have to evaluate which is simply Laplace Fourier transform of that function, so this is exactly what we have so to evaluate this we can use direct technique, we just put it as E power minus something square so that is the idea, with that idea you end up this is your Fourier transform, so once you have this Fourier transform of, to do this, to get this Fourier transform what we consider actually we consider without X, this you look at the Fourier transform of this that we find that this is this, this is what we get and this is exactly what you have.



And now you differentiate with respect to xi both sides to see that, to see finally that Fourier transform of this X into E power -X square T you end up getting this one, so here I made a small mistake so 2 root 2 I missed, I put it, I removed root 2 because root, so what you should have is 1/2 root 2 T, so it should be 2 root T and this side – I xi you bring it to this side so that

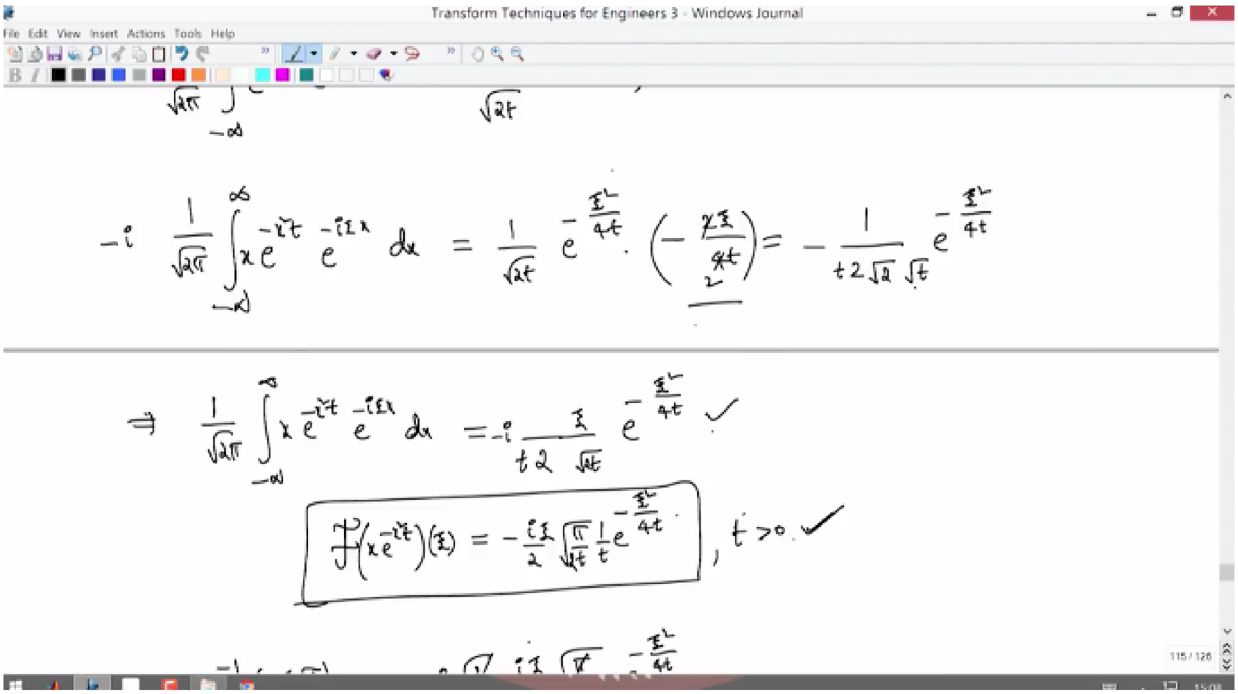


is become –I/xi okay, so 2 root 2 as it is and root T so this is exactly what you have, so you should have root 2 here so that is exactly your Fourier transform of this, now this gives me immediately once we know this you know from this Fourier transform of, Laplace inversion will be I times root 2/pi times whatever we have got, so let me write that, so this implies Laplace inversion of E power –xi root S as a function of T is simply I times exactly what you have is I times root 2/pi times this integral, I times root 2/pi times this Fourier transform that is exactly what you have is –I/2 xi root pi/2T E power -I square/4T, so this is exactly what you have, so if you simplify this - sign I will become +1, and root 2 root 2 goes here and what you have is 1/root pi, and now you have a pi xi comes out, and then root pi root pi goes, root pi/2 and root 2/pi goes, so you get 1, so you have 2 xi root T, so you end up getting xi/T so you have a 2T, so you have 2T, so I missed T outside, so from here so you have a 2T is actually 2T comes down, comes out, so there's a T here so that comes out, so you have 1/T and that is exactly your Fourier transform so if you use it here 2 xi T, so you have this times T okay, so you have root T and T is here and xi is here and 2, and root 2/pi this one and this one will go and you have -I is 1, so you have this into E power –xi square/4T, this is for T positive.

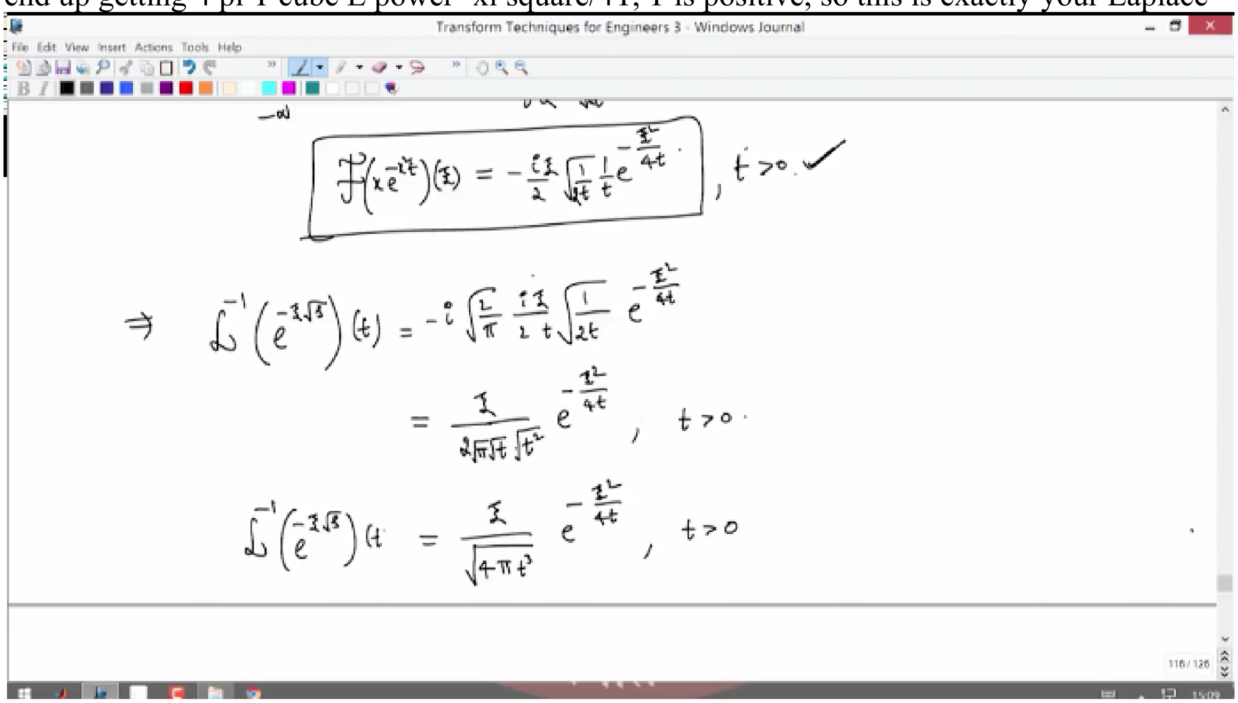


So if you write it, rewrite it one by, what I do is I put it as a T times, so T you can write it as square root of T square here, so when you differentiate this integral what you have is –IX, there is no xi here, so there's 1 and this side you have this, so you finally end up this you bring it –I so you have xi only in the top, so you have xi here from this xi/2T, so you have xi, xi it comes up actually, so that is what exactly your Fourier transform.

Now if you use this here so instead of in that down so you have in the numerator, instead of in the denominator you have in the numerator so you have xi divided by this, so you end up getting xi by square root of, this is 4 and this is T cube, T cube 4 then where is the pi? So some pi is also missing here, root pi/T well there is no pi here again that's again mistake here, so you see when you differentiate this side as this right-hand side is this, so from this you can write this there is no pi here, okay, so there's no pi here 2T, xi/2T and root 2T is here and that is xi/root T

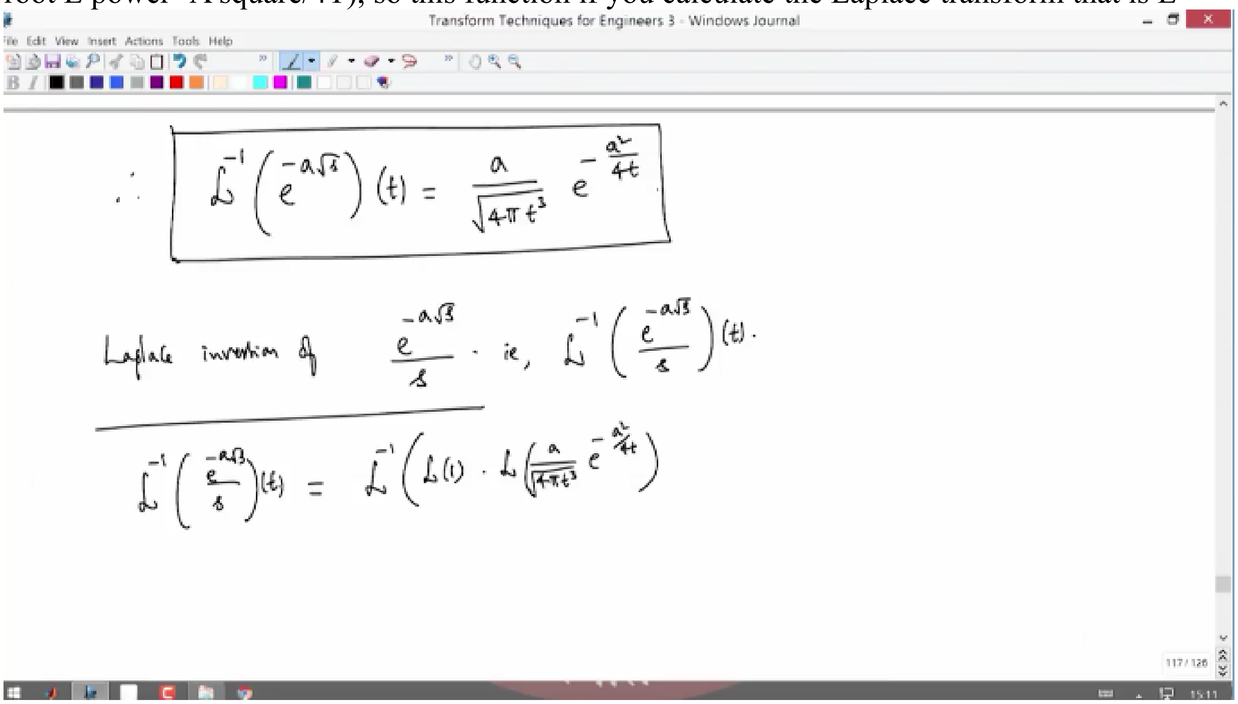


and E power -xi square by so, this is what you have to use so -I xi/2T and pi is not here so you have only one over this, so if you use this instead of, so I have this one times, so what I have is instead of pi I have 1/2T I xi/2T into this one, so this is exactly what you have.
So now if you do this -I I that is 1, and you have root 2 root 2 goes and you have 2 comes down and you also have root pi, root pi is here, and root T, T I can write it as T square so finally you end up getting 4 pi T cube E power -xi square/4T, T is positive, so this is exactly your Laplace



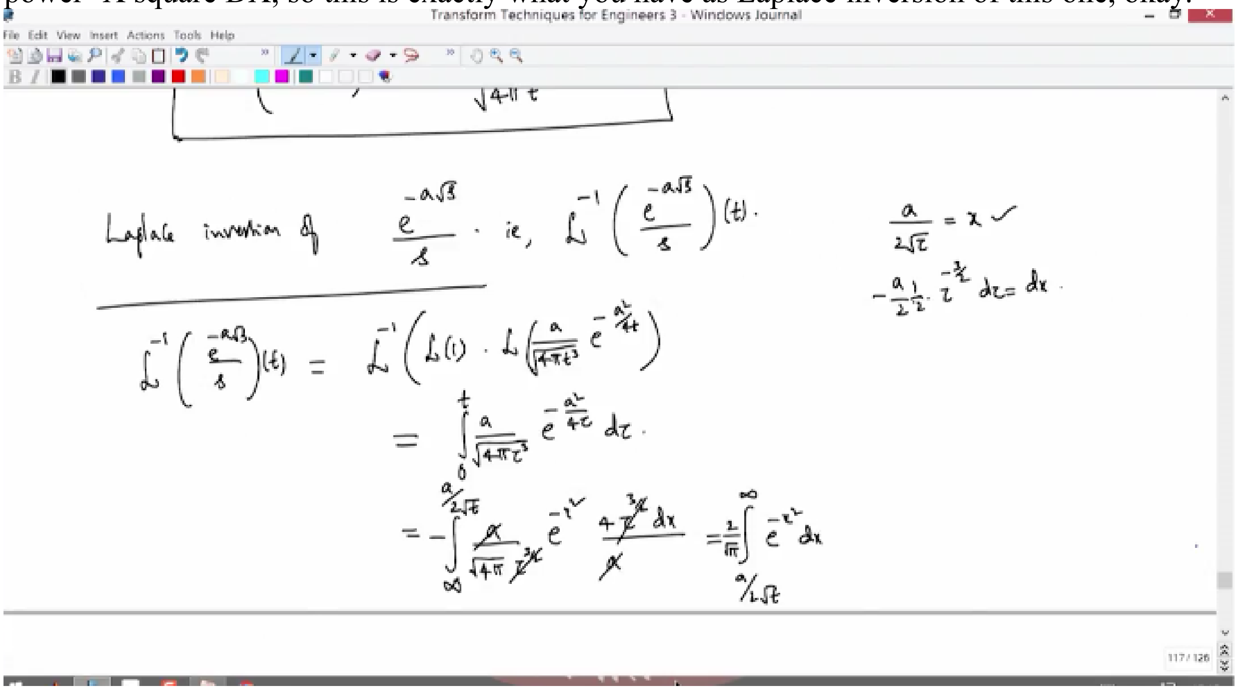
inversion of E power –xi root S(t), so if I replace xi by A, so let me write, so finally you end up getting, so therefore so you have Laplace inversion of E power -A root S that is where we started as a function of T this will be, instead of xi you put A by square root of 4 pi T cube times

E power -A square/4T, so this is exactly your Laplace inversion of the function you want, so not only that you need this, you can get this from, you can also get this from the Laplace transform table, inverse transform of E power -A root S from this you can get, you can also derive or two more related functions for example if you would look for, Laplace inversion of E power –A root S over S, okay, if you want this that is L inverse of E power -A root S/S as a function of T.
So what is this one? This is actually equal to E power -A root S/S as a function of T which is equal to you know that this is, you can write like L inverse of L(1) is 1/S times L of this function, L of E power –A S is L of this function so you can write this, L(A/4 pi T cube under root E power -A square/4T), so this function if you calculate the Laplace transform that is E

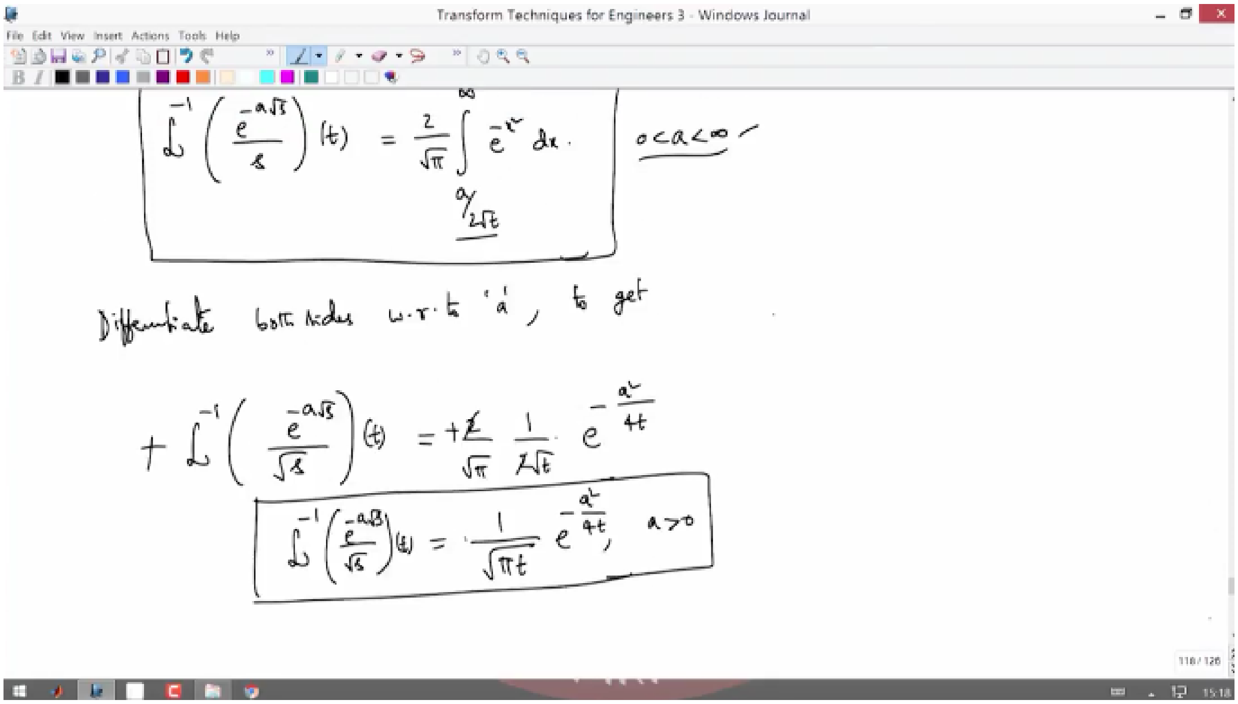


power -A root S from this, so if you write like this and this will be equal to L inverse of, L of this together is Laplace transform of this convolution function of this and this, that is A/square root of 4 pi tau cube and E power -A square/4 tau into, this I use as a F(tau) and this is a G, G is 1, so G(t-tau) that is simply 1 itself, so you have D tau, so this is exactly what you have.
So if you Laplace inversion, Laplace inversion will go so you have simply this is the one, so this is equal to this, so Laplace inversion of this function is this, so this you can simplify by writing, by making use of this change of variables A square/4 tau you take it as X square, so you have A/2 root T equal to, let us say X, then A times -1/2, so first write A/2 times T power -1/2 derivative is -1/2 times T power -3/2 DT = DX, so this you can use this, so let's say tau, this T is naught T so this is tau, so D tau, so if you use this what you get is 0 is, if you put 0 this is going to be infinity, when you put tau = 0, X is infinity, when you put this one this is going to be A divided by 2 root T that is it, so D tau is 4DX/A times tau power 3/2 DX, that is what you have, okay.
And this if you use A/4 that is 2, that is root 4 pi times tau power 3/2, so this gets cancelled, this gets cancelled, okay, so you end up getting and you have a - sign that is minus, D tau is minus of that so I put the minus here, so you can rewrite this as A/2 root T to infinity because of minus, minus I can interchange these limits so you have 4 times, so what happens to this? This is going to be you get this is root 2, root 4 is 2, 2 2 goes so you have 2, 2/root pi comes out, and

then, and you have E power -X square for this function, okay, because of this, so you have E power -X square DX, so this is exactly what you have as Laplace inversion of this one, okay.



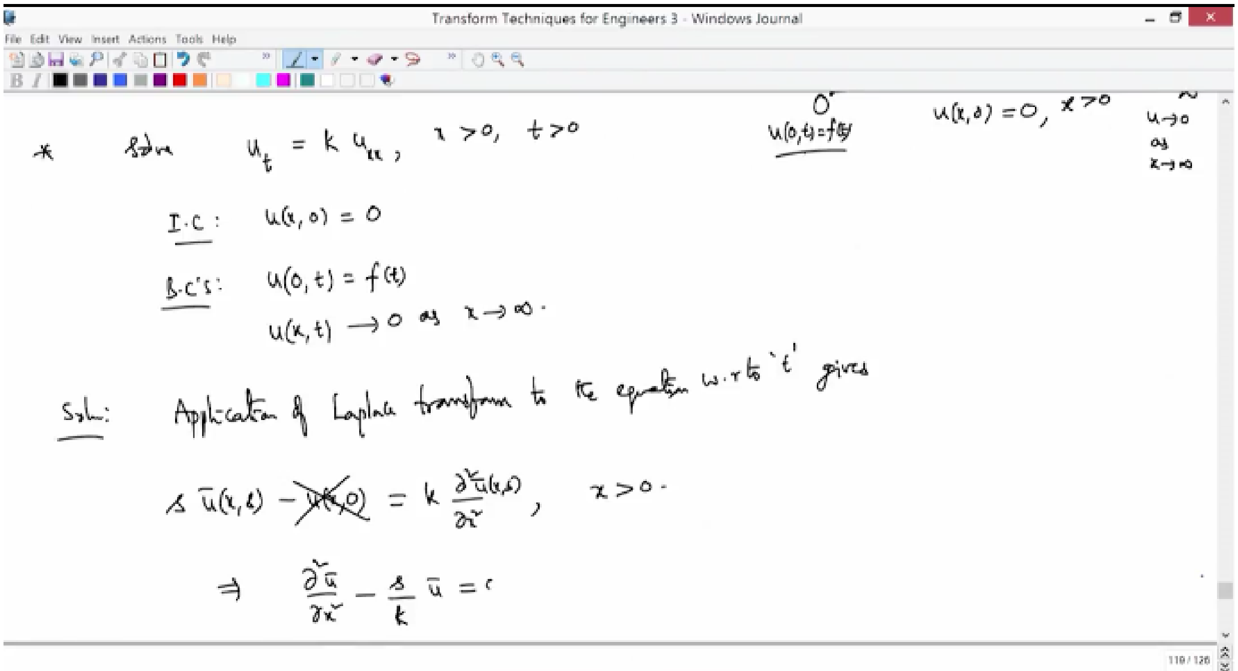
So Laplace inversion is simply this function, okay, so Laplace inversion of E power -A root S/S a function of T = 2/root pi times A/2 root T goes to infinity E power -X square DX so this is one, you can get again from this we can also make use another Laplace inversion, that is Laplace inversion of, we can also get Laplace inversion of, you simply differentiate this quantity, differentiate since A is positive, A is between 0 to infinity so A is positive, so you can differentiate both sides, both sides with respect to A to get, what you get is a Laplace inversion that is an integral, whatever the integral, so if you integrate so you end up getting, in the integrand you have this one, if you differentiate with respect to A it's going to be -root S comes out as an integrand, so that's the only different, so you -Laplace inversion of root S E power –A root S/S, so this gets cancelled and become here, so this is what you have on the left hand side once you differentiate with respect to A, and so that you have here 2/root pi and this one if you differentiate with respect to A and it's going to be - sign and you have 1/2 root T times, wherever X is there you just put E power -A square/4T, okay, so this is what you have.
So if you simplify this, 2 2 goes, minus minus goes, and you have 1/root pi T E power -A square/4T this as your Laplace inversion of E power –A root S/root S, this is what you get as



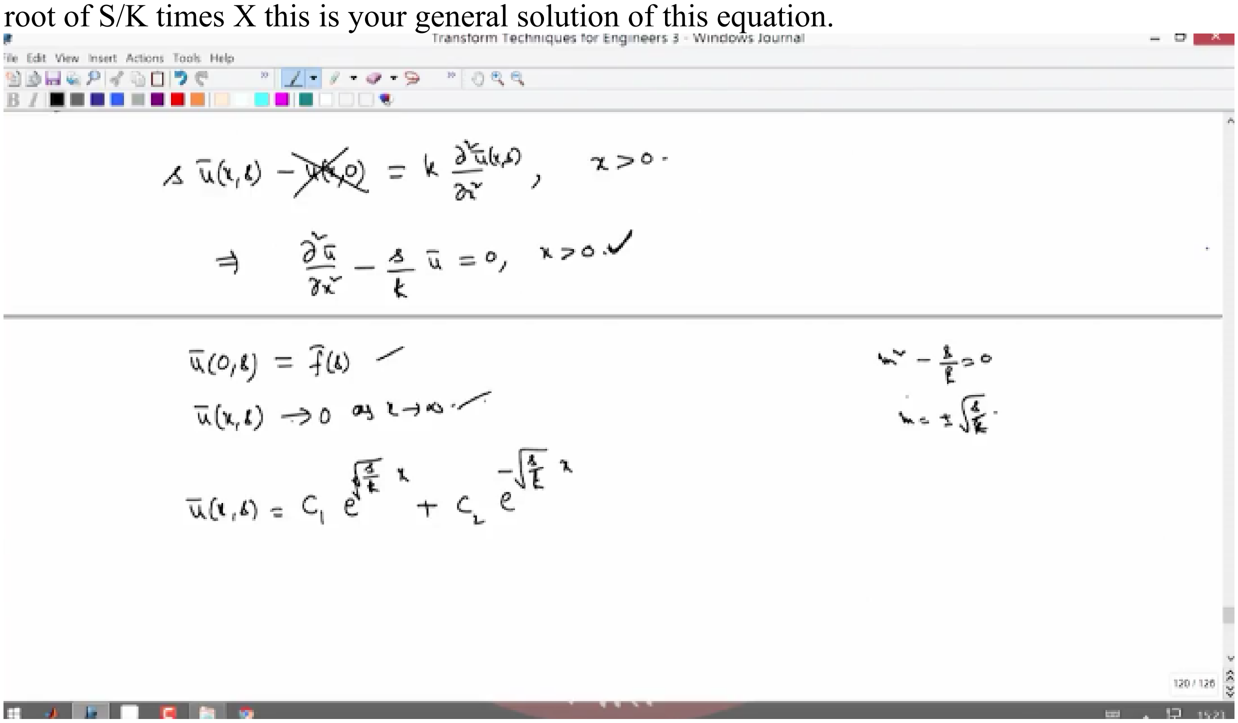
transform so you have A is positive, so same thing here so A is positive, so you have now so derived Laplace inversion of three special functions these are the two, and you have one more here, so this is where you have done originally all the work is positive again here, so this is how you can get this Laplace inversion.

Now we make use of these three special Laplace inversions to look into the initial boundary value problem for the heat equation, so let's look into some other problem, let's solve this problem, problem of heat equation in a semi-infinite domain, so let's consider semi-infinite rod that is you have a semi-infinite rod, so you have a rod which is at 0 to infinity, this is at 0 and this goes to infinity, up to infinity you have this rod and a temperature in this rod initially U(x,0) you keep it temperature 0, initially the rod is at 0 temperature and you have U(0) at this end I try to give the temperature, I freed for all times as F(t), some general function I can give, at infinity I expect this heat to go to 0 as X goes to infinity, so these are the boundary conditions for the heat equation, because this models this temperature and a rod is modeled by the heat equation, so if I write mathematically a problem of heat conduction, heat conduction in a semi-infinite rod let's say, okay, so the problem is initial boundary value problem is to find solution of, this is to solve UT equal to some K times UXX, X is positive and T is also positive, X is from 0 to infinity, T is time 0 to infinity, and you write these boundary initial conditions, so initial condition is U(x,0) is 0 and boundary conditions, or because two boundaries 0 and infinity, so you have U(0,t) as F(t) and U(x,t) goes to 0, as X goes to infinity, so this is the second boundary condition.

So if you, so to get the solution out of this as usual we apply Laplace transform for the T variable, application of Laplace transform to the equation with respect to T gives, what it gives is you're doing with respect to T so this becomes S times U bar(x,s) – U(x,0), so U(x,0) is 0 so that goes, that’s the left-hand side = K times UXX is dou square/dou X square of U bar, that is simply U bar(x,s), so this is for X positive. T variable is gone to S variable that is actually complex variable, so that is what it is, this is the equation it has become, so this implies dou square U bar/dou X square –S/K times U bar = 0 this is for X positive, so this is your equation.

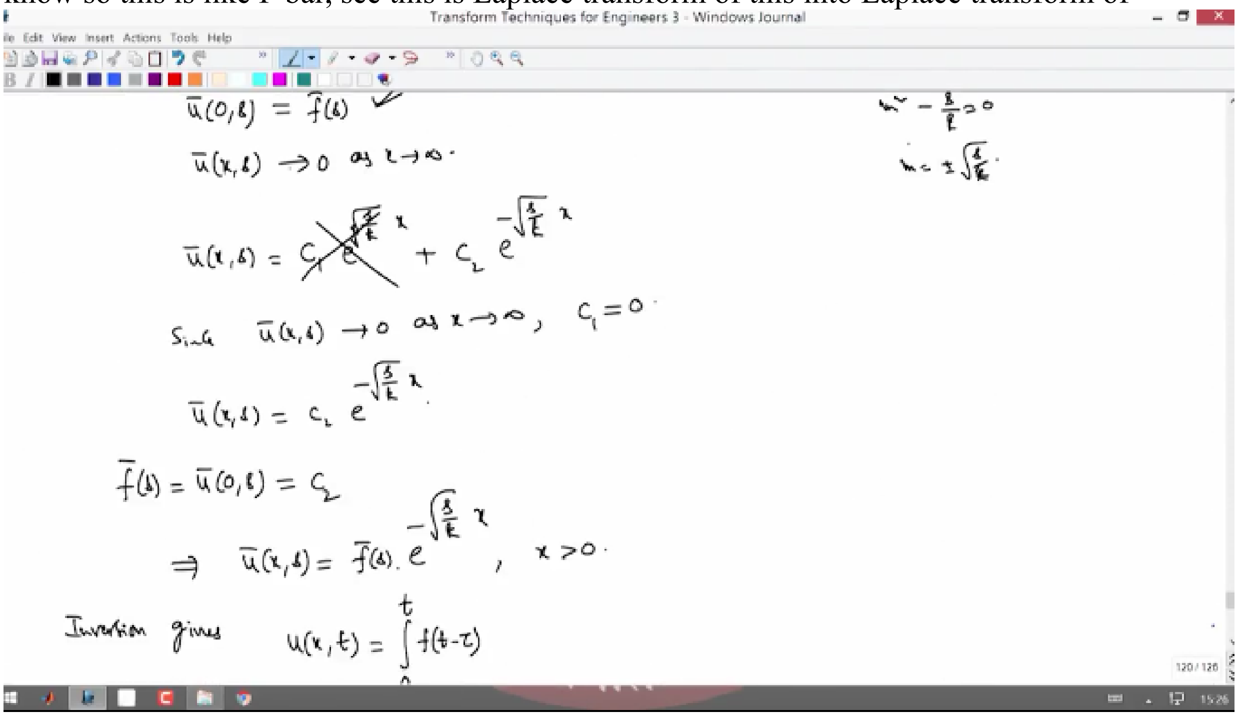


And now if you apply the boundary condition, initial condition I already applied so if you use the boundary condition, boundary conditions gives U bar(0,s) after applying Laplace transform for this condition this becomes F bar(s) and U bar(x,s) goes to 0 as X goes to infinity, so these are the two boundary conditions and we solve this ordinary differential equation, so if you solve this, so what is the solution of this problem U bar(x,s) this is M square, this is a complementary function is M square –S/K is 0 so that makes it M equal to plus or minus square root of S/K, so you have C1 times E power root S/K that is a constant times X and +C2 times E power -square root of S/K times X this is your general solution of this equation.

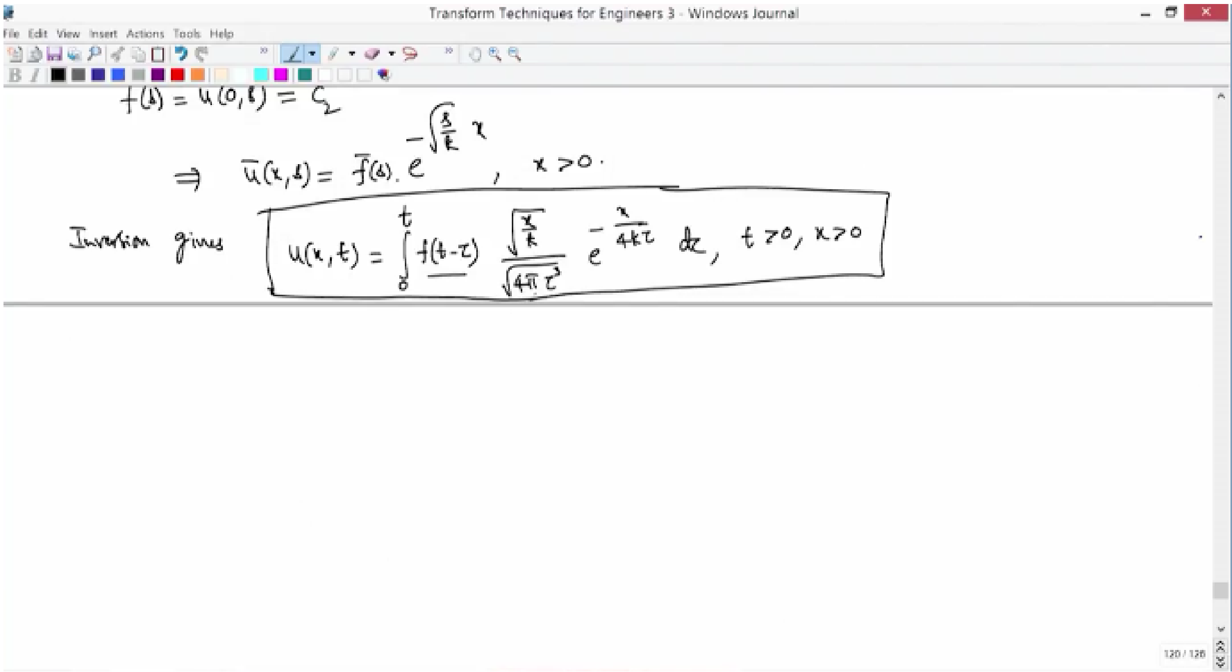


Now you apply these two boundary conditions to get rid of this C1 and C2, so as X1, XS goes to 0 as X goes to infinity C1 has to be 0 because otherwise if C1 is nonzero this cannot go to 0,

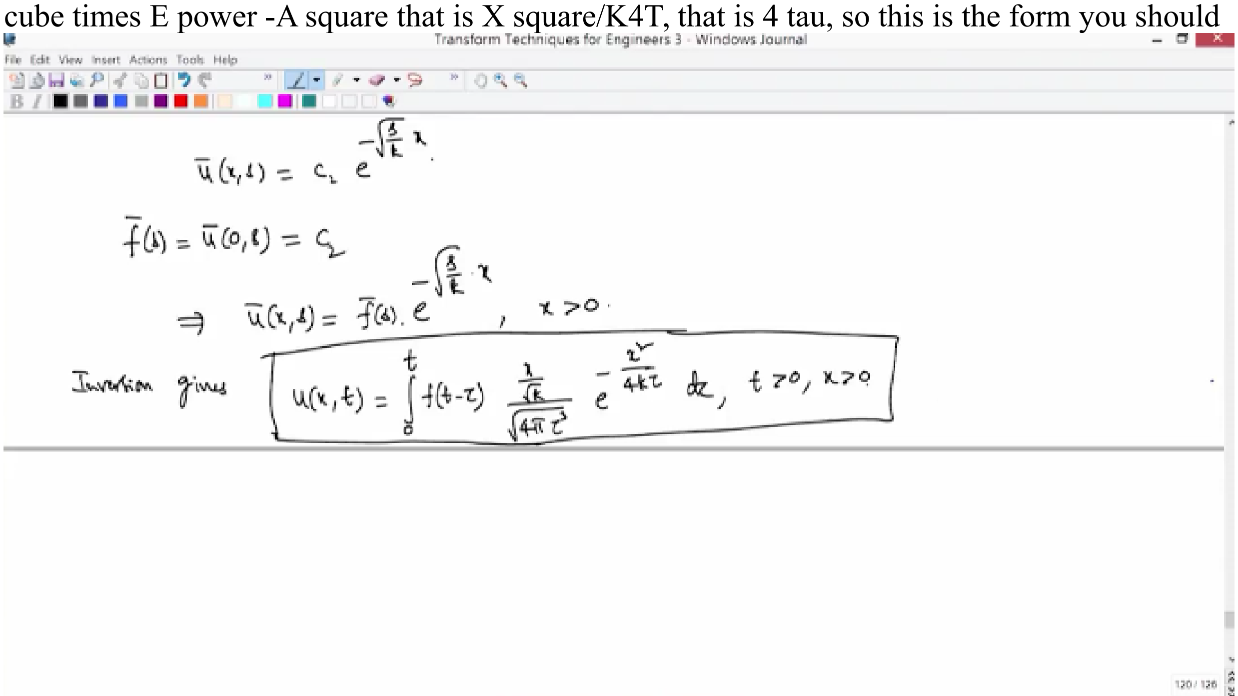
okay, because of this condition C1 has to be 0, okay since U bar(x,s) goes to 0 as X goes to infinity, C1 must be 0, so this is gone, so you have U bar(x,s) is C2 times E power -root S/K times X, so now you apply the first condition that is this, if you apply this and you get U bar(0,s) which is equal to F bar(s), this if you put X = 0 here from this equation C2 times E power, when you put X = 0 this is simply 1, so we have C2 is simply F bar(s) so that implies U bar(x) is the solution it becomes F bar(s) times E power -square root of S/K times X, and this is for X positive.
Now you invert this, inversion gives U(x,t), so what is the inversion of this? This we already know so this is like F bar, see this is Laplace transform of this into Laplace transform of



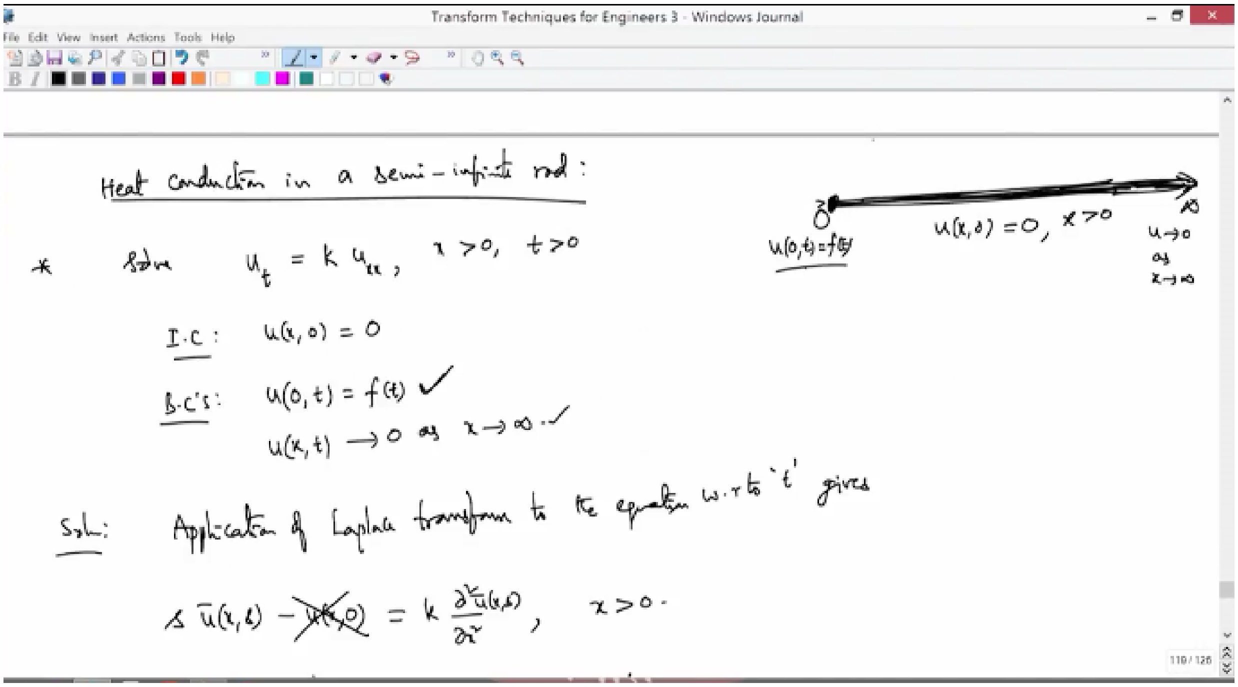
something, some function for this, so that is a convolution between 0 to T, this is F(t), let's write T-tau here, so that this function let's put tau, so what is that one? Now you already have seen just now Laplace inversion of this function, A is this A/square root of 4 pi T cube, okay, what is A here? Square root of X/K that is your A, A/square root of 4 pi or tau cube I have to write as it tau here, and then E power - tau square or rather, what is exactly you have? A/square root of 4 pi T cube times exponential -A square/4T –A square, A square is X/4KT that means, okay, T is tau so I put tau, for this you, so you had the convolution for this function and this function, so you have this is a final form, so this is your solution of this differential that initial boundary value problem whatever you have, as this is for T positive and X positive, so this is your solution for the boundary value problem, so that we have, okay.



So because you give the F so you can simplify this integral again by rewriting this as E power minus, this I think A square right so you have X square, so X square becomes, A square is, A is this so you have X/K, fine, so this is how I can solve this, by the way just a minute, so X is not within the root, X by this is actually X/A is X/root K, sorry X/root K so you have X, so this becomes X square/K so you have, this is the form you get, okay, so this is X is outside this root so you have X/root K is your A, A divided by, A is X/root K divided by square root of 4 pi tau cube times E power -A square that is X square/K4T, that is 4 tau, so this is the form you should

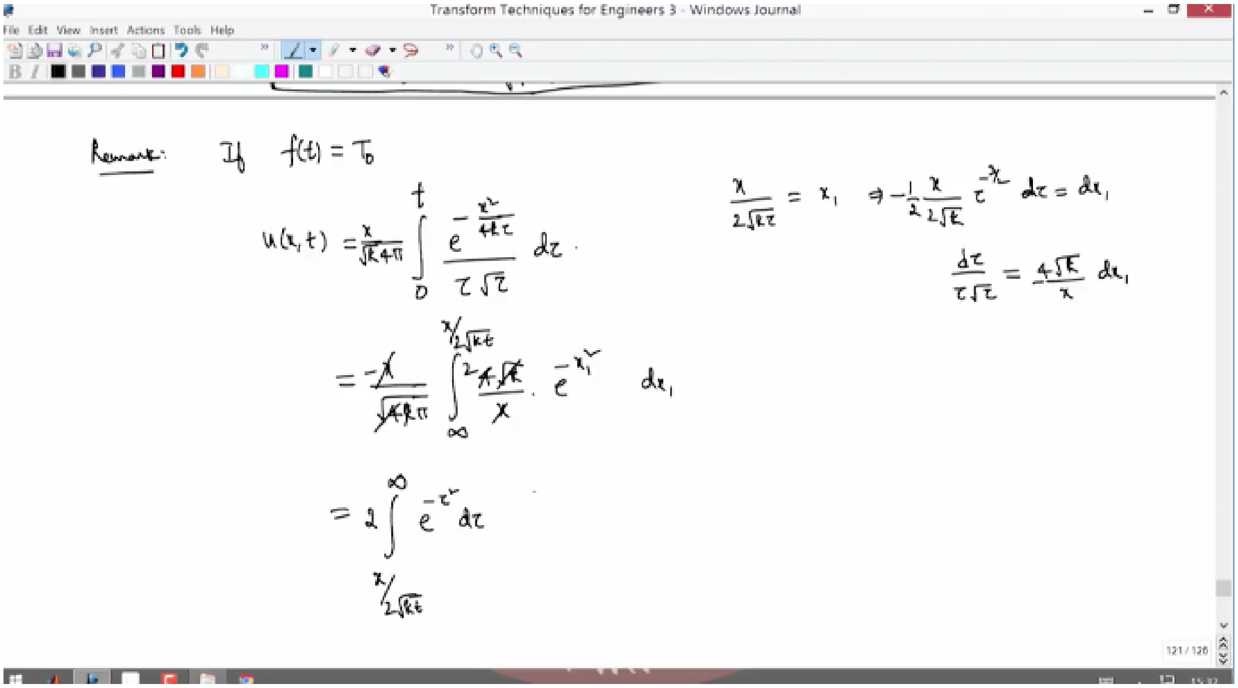


get, okay, so you could solve this boundary value problem, an initial boundary value problem for a temperature of a rod, semi-infinite rod which is at rest, and you keep pumping the heat here for all times so eventually as you see at infinity its you always expect this to be 0



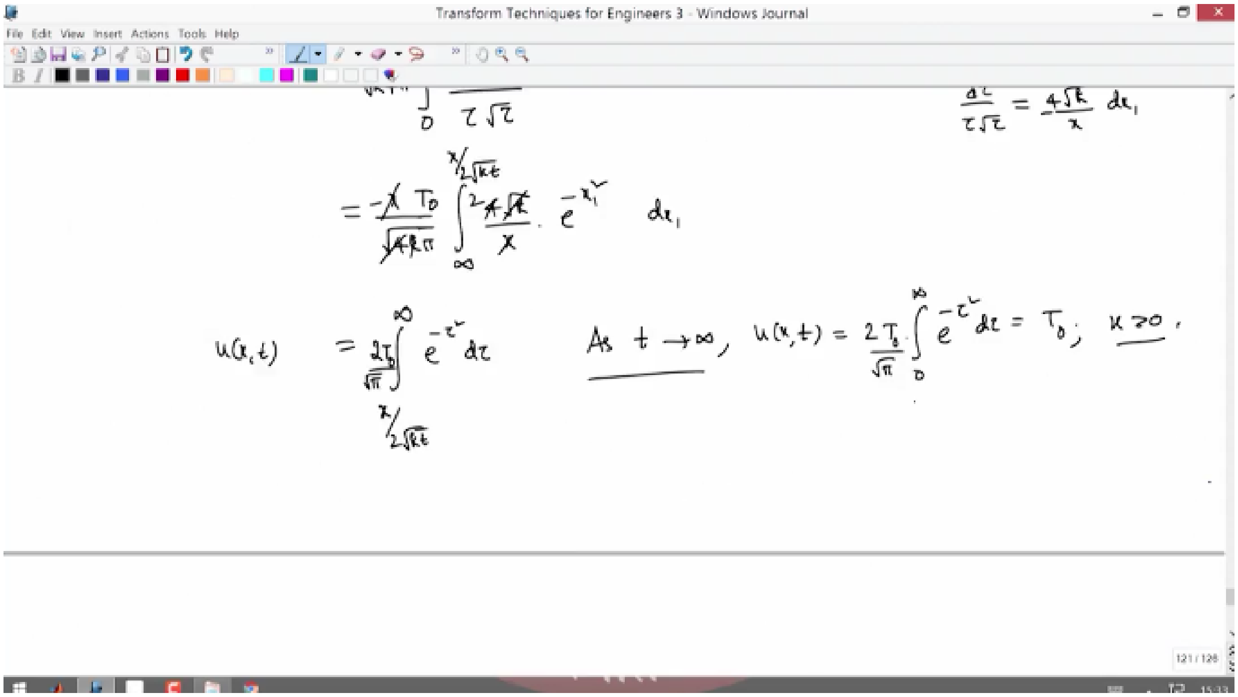
temperature as T goes to infinity what you end up is finally T goes to infinity, what you expect is if F is constant, if you keep pumping some constant temperature eventually as T goes to infinity U(x,t) becomes a constant all along the temperature, right, all along the rod you can expect this.

So as a remark you can write, as a remark, so if F(t) is a constant let us say T naught some fixed temperature you keep pumping at the end, end of the rod exactly at X = 0 what you expect is for all times your temperature is integral 0 to T, X comes out X/root K comes out so you have a root K is 4 pi K and you have integral 0 to T times E power -X square /4KT, 4K tau/T root T, root tau cube I am writing, tau root tau as D tau, so again you write this as, you consider X/2 root K tau as a new variable let us say some X1 then this implies DX, sorry, you need let's use a new variable X1, okay, so X/2 root K that is a constant and tau power -1/2 that gives me -1/2 times tau power -3/2 which is into D tau = DX1, so what you have is 1/tau root tau D tau = 4 root K/X with -sign with DX1, so that's what you have to write, so this becomes X/square root of 4K pi, 4 root K pi times integral, now when you put tau = 0 see it's going to be infinity here when you put here, this is going to be X/2 root KT and you have this -sign for DX, you have 4 root K/X times DX1, and you have this will become E power -X1 square, so this is how this integral, so X is a constant this goes 4K goes, sorry 4K will not go, so root K will go and you have this, this goes finally make it 2 so you have minus you can interchange the limits that is X/2 root KT to infinity, you have 2 comes out E power -X square rather we can put it as some



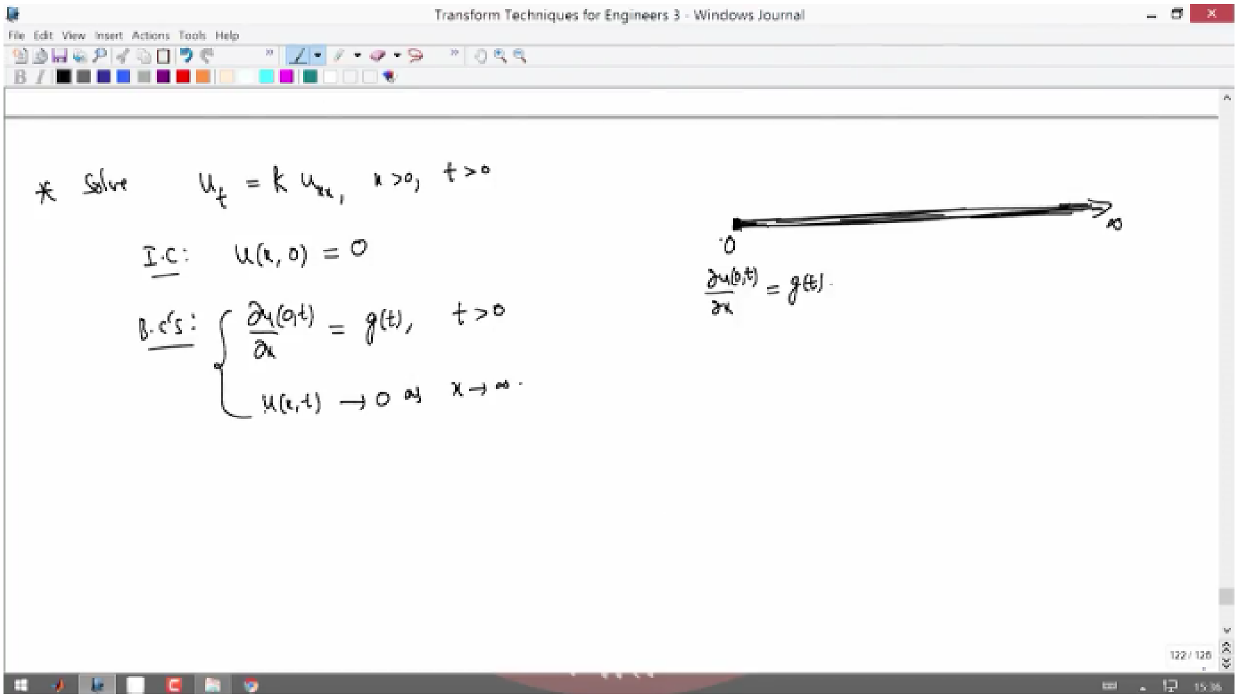
tau square D tau, X1 is a dummy variable I can put it like this, so what you expect is as T goes to infinity, as X goes to infinity as for times, okay, this is what is your U(x,t), so as X goes to infinity, as T goes to infinity eventually what happens?

As T goes to infinity you end up getting U(x,t) as, so I didn't write this F(t), in the place of F(t) you have to write T naught, so you have T naught which has a constant all the times 2T naught, so you have a 2T naught times integral as T goes to infinity this becomes 0 to infinity E power -tau square D tau this is actually root pi by, there's a root pi which I missed, so divided by root pi, so root pi is not cancelled, so you have by root pi this one so this quantity is root pi/2 so this root pi/2 and 2/root pi cancel, so you end up getting T0, so that is expected as you see in this rod, infinite rod if you keep pumping the constant temperature here, so let's say if we say T naught, constant temperature if we keep on doing it eventually as T goes to infinity you pick up any point, at any point X eventually as T goes to infinity at bigger times it will reach that same temperature will reach this point, that's what it means, okay, as you see as T goes to infinity this temperature at every point X it becomes T naught, okay, so that is what it is, so X is positive.



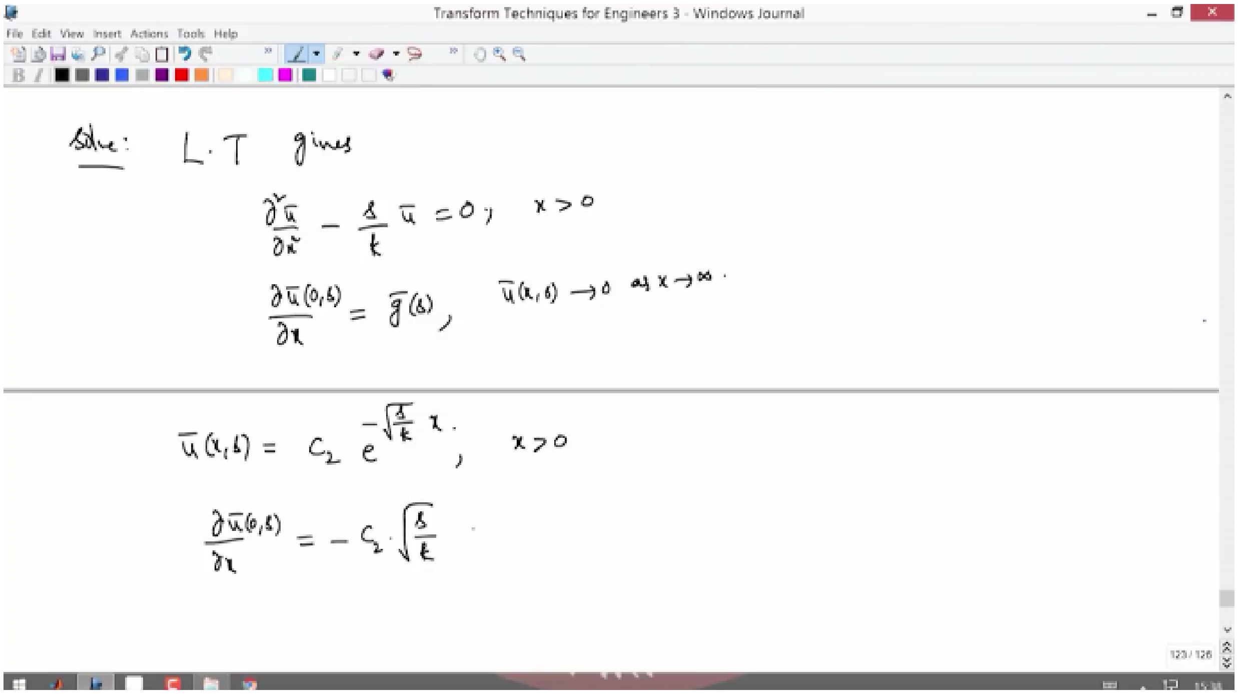
So this time you solve this initial boundary value problem, let's look into some things, next very similar to this problem that is if you change the boundary condition at X = 0 instead of pumping the temperature here, instead of pumping the heat at this end 0 you can just insulate it for example, are you allow the flux, okay, you allow the heat to go inside, okay, you give the rate of flow of heat, okay, not a constant rate so at X, at this point you allow some heat is passing from this end to this end, so that is a flux, that is a heat flux you allow, so the rate of that is called rate of flow of heat, you prescribed, so at 0 that means dou U/dou X at X = 0 and T for all times that is 0,T you give it as some function of T, instead of U you give the temperature what you give is dou U/dou X you provide that is the rate of change of heat, the rate of heat flow, the rate of heat flow you allow this heat to pass from this, from outside to this or inside, inside in anyway 0 temperature, so if you allow as a minus so that is going to be, you're allowing this to go inside, okay, so let's give like this –GT, or just think of G(t) depending on the sign of G, so you see that is either going outside or inside, okay, so that's how your condition here, all other things are same.

So how do I write this as a mathematical problem, solve UT = K times UXX, X positive, T positive and this initial condition is as it is so you have U(x,t) = 0 is 0, initially temperature is 0 and boundary conditions, only boundary condition is this, that is dou U/dou X(0,t) = G(t), T positive and U(x,t) goes to 0 as X goes to infinity, so this is other boundary condition, okay,

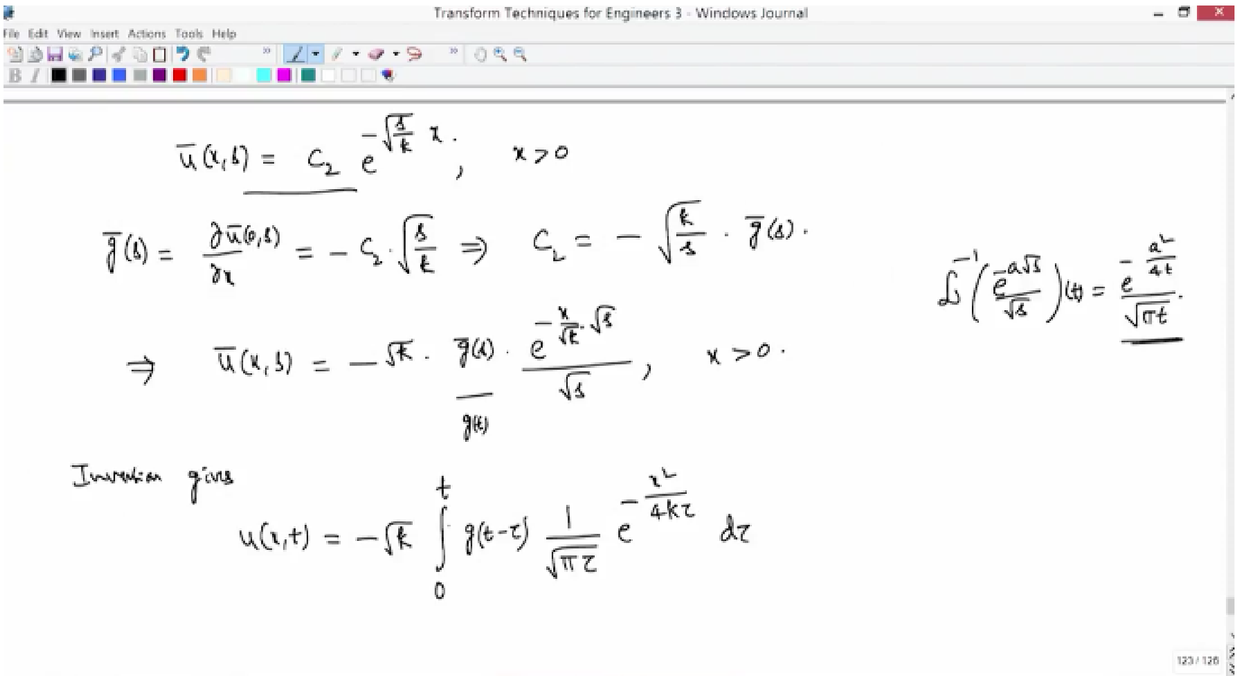


these are the boundary conditions so you solve, how do we solve this? Everything else is same except the equation when you apply, when you apply the Laplace transform you get the same thing, Laplace transform gives the ODE that I write directly as dou square U bar/dou X square –S/K U bar = 0, for X positive and your boundary condition is dou U bar so this is with respect to T you are doing so you get dou U bar(0,s)/dou X = G bar(s) for at X = 0, okay, and this is one boundary condition.

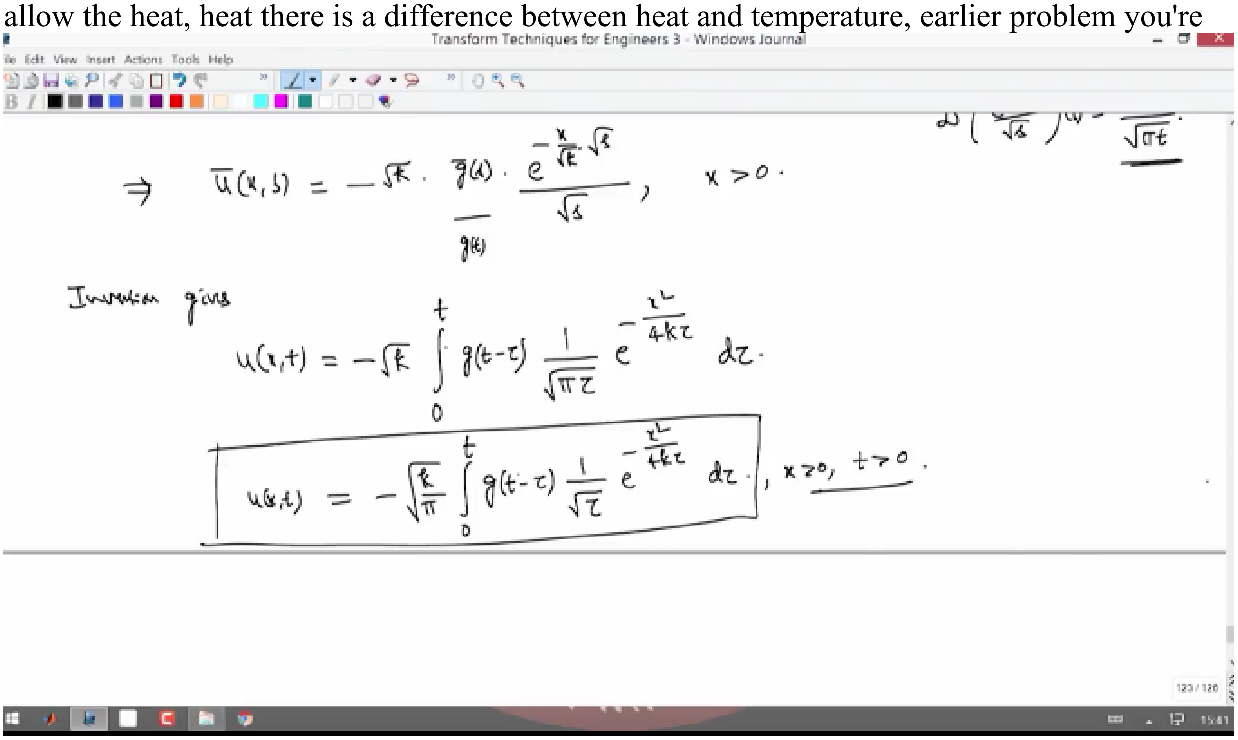
Other boundary condition is U bar(x,s) goes to 0 as X goes to infinity, so again the solution for this becomes, the C1 has to, because of this C1 has to be 0, C1 is the coefficient of exponential of root S/KX, okay, and what you end up is C2 times E power -square root of S/K times X, for X positive and you have by applying this you lose the other solution that is C1 times E power + root X/K times X, because of this that C1 has to be 0, so C2 is this you apply this boundary condition now dou U bar/dou X at X = 0,S is actually equal to C2 times, you differentiate this



and put X = 0, so if you differentiate with respect to X you have -root S/K times E power that one, so that becomes simply 1 so you have this, so this is equal to G bar(s) so this makes it, that gives me, what is my C2? C2 = -root K/S times G bar(s) this you substitute into the general solution, so you get U bar(x,s) as -root K times G bar(s) times E power -root X/root K times root S divided by root S, okay, so this is what you have for X positive.
So now you get the inversion, so inversion will give you, now you are going to use Laplace inversion of E power -A root S, by root S this is as a function of T if you recall this is actually is E power -A square/4T by root pi T, so this is what you have to use, so you get inversion gives now, inversion gives U(x,t) as -root K is common, is the constant comes out for this Laplace transform of G and Laplace transform of this, so this Laplace transform of this is Laplace transform of this function is this one, so you write that so you have that is a convolution of G, Laplace transform of G for which you have a G bar(s), Laplace transform of G(t) is G bar(s), Laplace transform of this function is this function, so with A of course, A is X/root K, X is positive root K, K is a constant that is also positive, so you have this convolution is 0 to T times G(t-tau) and this one I'll put it, this function I'll put it as T, so A is 1/root pi times tau times E power -A is A square is X, X/root K is A, so X square is X square/K4T, 4KT you can write this

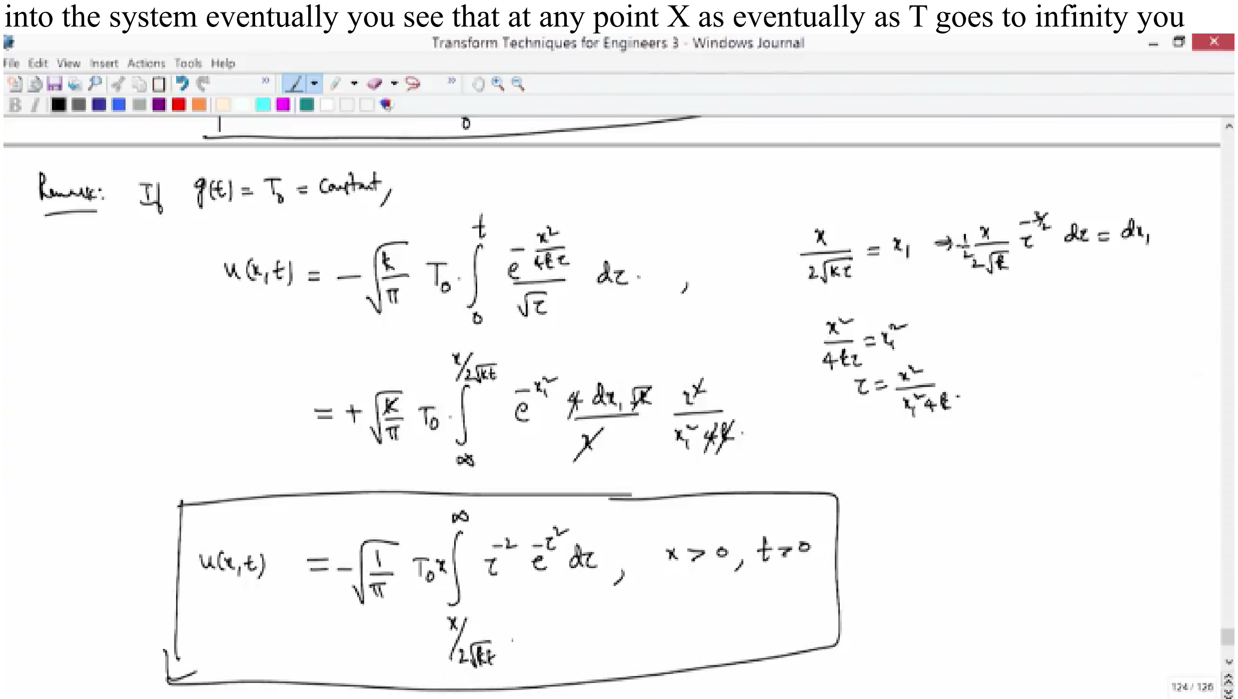


is exactly your 4K tau D tau, so this is the convolution you have to apply, so this is equal to -root K/pi integral 0 to T, G(t-tau) times 1 over root tau times E power -X square/4K tau D tau, so this is your solution for this problem, of course X positive this is X positive and T positive, so again the same way you can, if you give constant temperature, constant flux you allow, you allow the heat, heat there is a difference between heat and temperature, earlier problem you're



providing the temperature you're keeping that end with the constant temperature, here you allow the heat to go inside from outside to inside the rod at X = 0 or you allow the heat from the rod to go out, so that means you’re not allow the heat to go out, okay, so that is what is the second problem, so for which this is the solution.

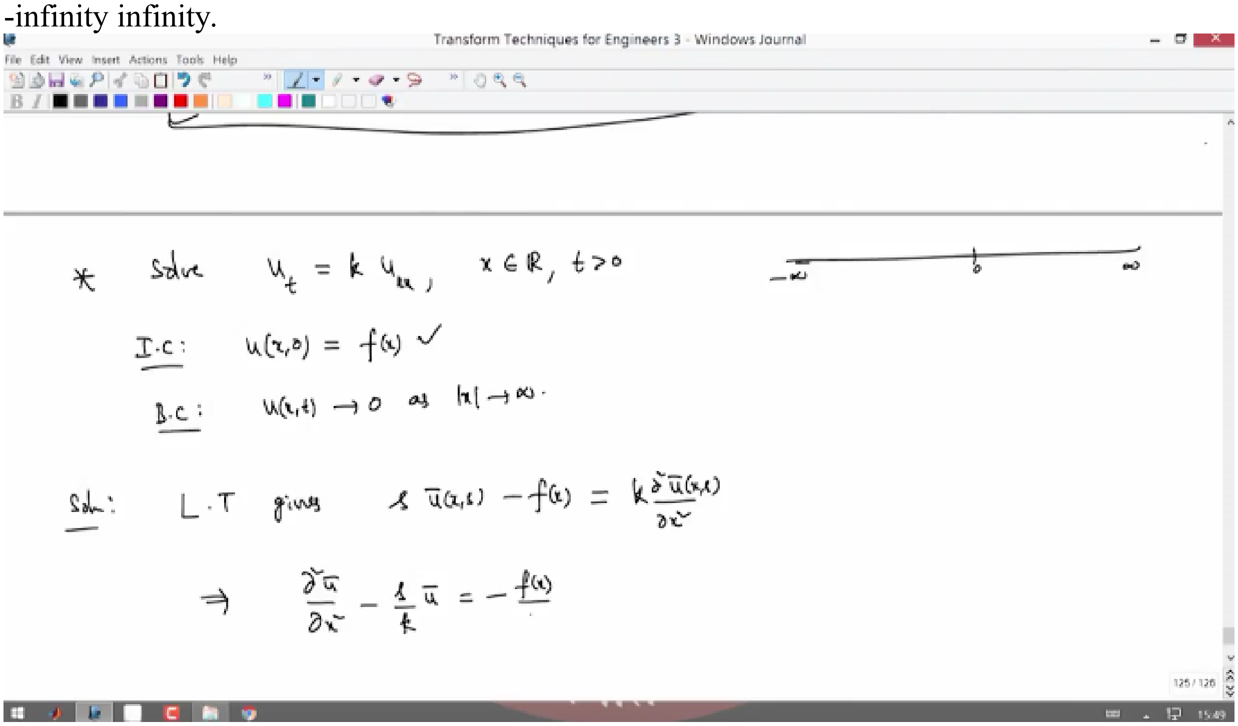
So if you use a constant G as a constant, you can see that if you just take, if as a remark you can get this if G(t) as a constant, so this flux is a constant rate if you take it a constant what happens is that U(x,t) simply become -root K/pi times T naught times integral 0 to T, and you have E power -X square/4K tau divided by root tau D tau, so again you can use, make use of this X/2 root K tau as a new variable X1, so you get X/root 2K and you have -1/2 times tau power -3/2 D tau = DX1, so if you use that -root K/pi times T naught this is going to be, at 0 this is going to be infinity this is going to be T which is X/2 root KT, and you have this becomes E power -X1 square D tau/root 2, D tau/root tau is DX, so what happens 4DX1/root K of course, root K comes out/X, and then this is going to be of course you have tau, so tau is up, okay, so what is tau? Tau is from this you can see, you can get this tau as X square/4K tau as X1 square, so you have what tau is X square/X1 square 4K, so tau you can write X square/X1 square 4K, so X X goes, so you end up finally getting of course there is a -here so there -sign makes it +, and now if you, K/pi T naught this you make it this you change the limits so you have X/2 root KT to infinity, and you have 4 4 goes here, K root K so this K K goes here, this K and this root K goes here, and you have X here X comes out T naught X, and then that's all, so you have X and this is X if you use, X1 you use as a dummy variable you have tau power -2 times E power -tau square D tau, so this is your X, U(x,t), so this is exactly what you have, that’s all right so you have X square is this, okay, so this is your solution what you get as for X positive, T positive, so again as X goes to infinity you can think of, if you keep pumping the constant flux you allow into the system eventually you see that at any point X as eventually as T goes to infinity you



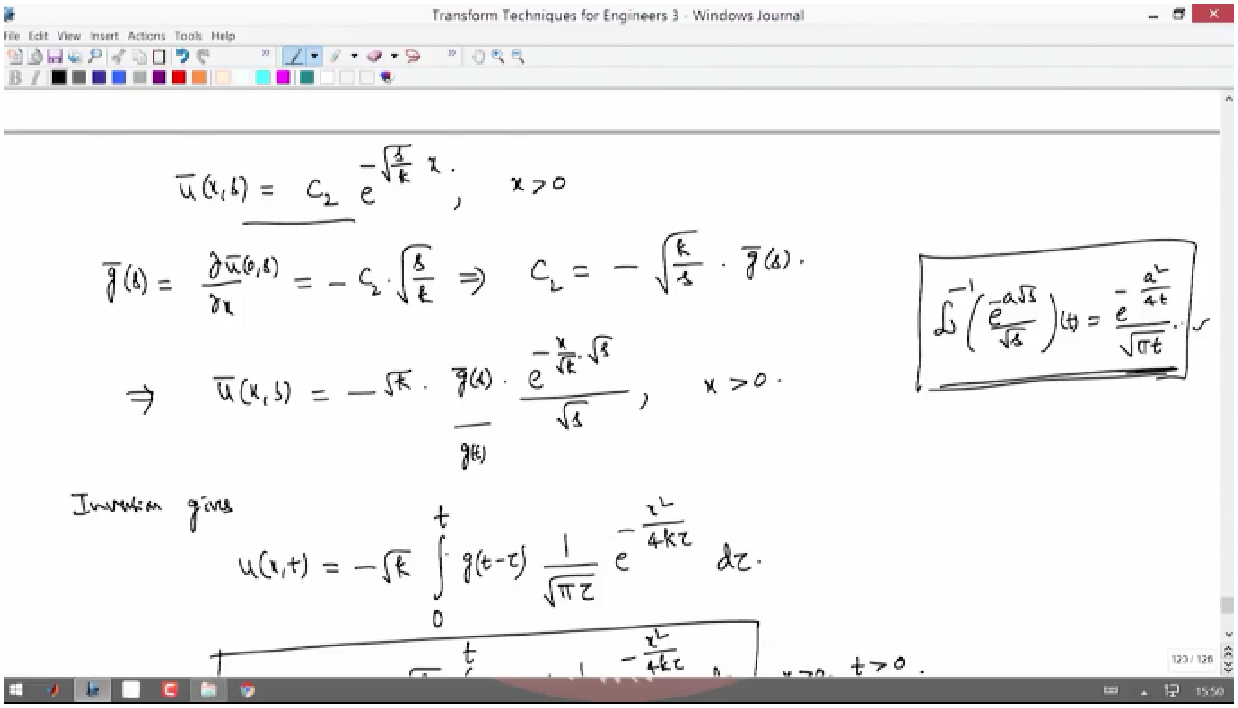
end up getting some kind of, as T goes to infinity this is going to be 0 to infinity this integral you have to get this integral, and as whatever be the, that is your temperature, okay, temperature in the rod.

So you look at some other relevant problem, we look at the relevant initial boundary value problem for a infinite rod from -infinity infinity, where you end up similar getting E-power one of these Laplace inversions, we'll just write that problem.

So let's look at the relevant problem that if you solve an infinite rod that is UT = K times UTT, UXX that is X, X is now full infinite rod that you have 0, -infinity to infinity if I have infinite rod X belongs to full R and T positive, suppose you have this and initially you have a temperature of this infinite rod is having, you provide some F(x) as your initial condition, there is no boundary so what you do is at infinity it has to be bounded, we cannot allow it to be 0, okay, so you just give 0 it physically, it infinity there is nothing like infinity, so at infinity you can expect the temperature to be 0, so U(x,t) goes to 0 as mod X goes to infinity, so both the cases, both the infinities it has to go to 0, so let's use this to, these are, you have two boundary conditions here, okay, this involves and you have this is the initial condition, so what is the solution if you follow the same technique, you end up getting Laplace transform gives UT that is S dou U bar/dou, sorry, this is U bar(x,s) SU XSS – U(x,0) that is –F(x) that is the initial condition I used that is a left-hand side, K you have U bar(x,s) for which you take two derivatives with respect to X, so you could get dou square U bar/dou X square S/K – S/K times U bar equal to you are dividing with K so that you have –F(x)/K, okay, so this is X belongs to -infinity infinity.



And if you apply these boundary conditions what you get is U bar(x,s) goes to 0, as X goes to infinity and also U bar(x,s) goes to 0 as X goes to –infinity, so these are the two conditions you have to use, and to solve this ordinary differential equation, so we will solve this and you end up getting finally again you can end up using this Laplace inversion of this, you end up using Laplace inversion, you will see that you need to find this Laplace, you need a Laplace inversion



of E power -A root S/root S, similar type of Laplace inversion you need that I think we will see in the next video, and along with the other boundary value problems where you can use, you can get other type of, other than these special Laplace inversions you may get other type of Laplace inversion you may need, okay, so we look at some more examples of initial boundary value problems for a heat equation so that you can utilize your Laplace transform and get your solutions, okay.

So if you cannot use the Laplace transform maybe other methods may work, the advantage of this Laplace transform is when you, when other, for certain problems in some domains when you are solving the heat equation or the wave equation you may not be able to solve by other methods maybe or maybe because of the difficulty with the other methods if you use the Laplace transform technique it is much simpler way you can find the solutions, if you are not able to find a solution by the Laplace transform that doesn't mean that you cannot solve by other method, so other methods may be easier where other methods failed, for example separation of variables technique in the differential equations or some other Fourier transform technique fails you may sometimes Laplace transform procedure may be simpler to derive the solution.

So we will see these other examples for heat equation and we'll look at the other applications of solving integral equations and you can also solve, you can evaluate certain integrals involving a parameter these applications we will see in the next video. Thank you very much.

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Funded by
Department of Higher Education
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Government of India