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NPTEL ONLINE COURSE
Transform Techniques for Engineers
Solution Heat Equation by Laplace Transform
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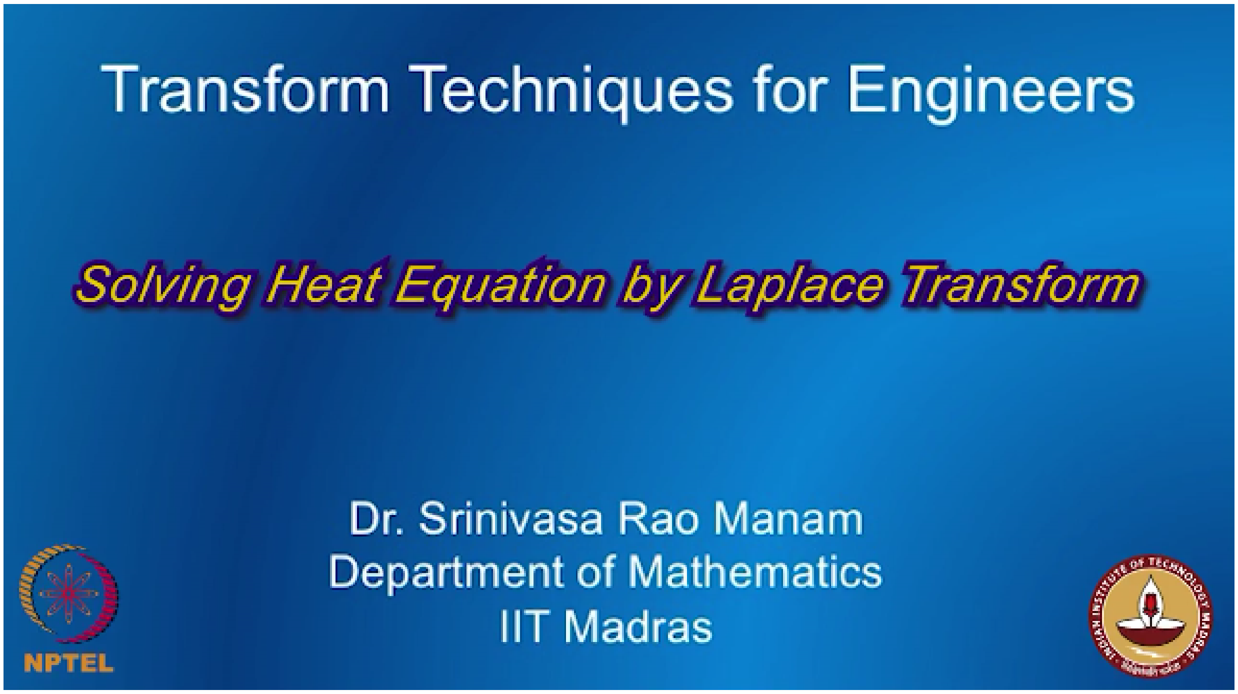
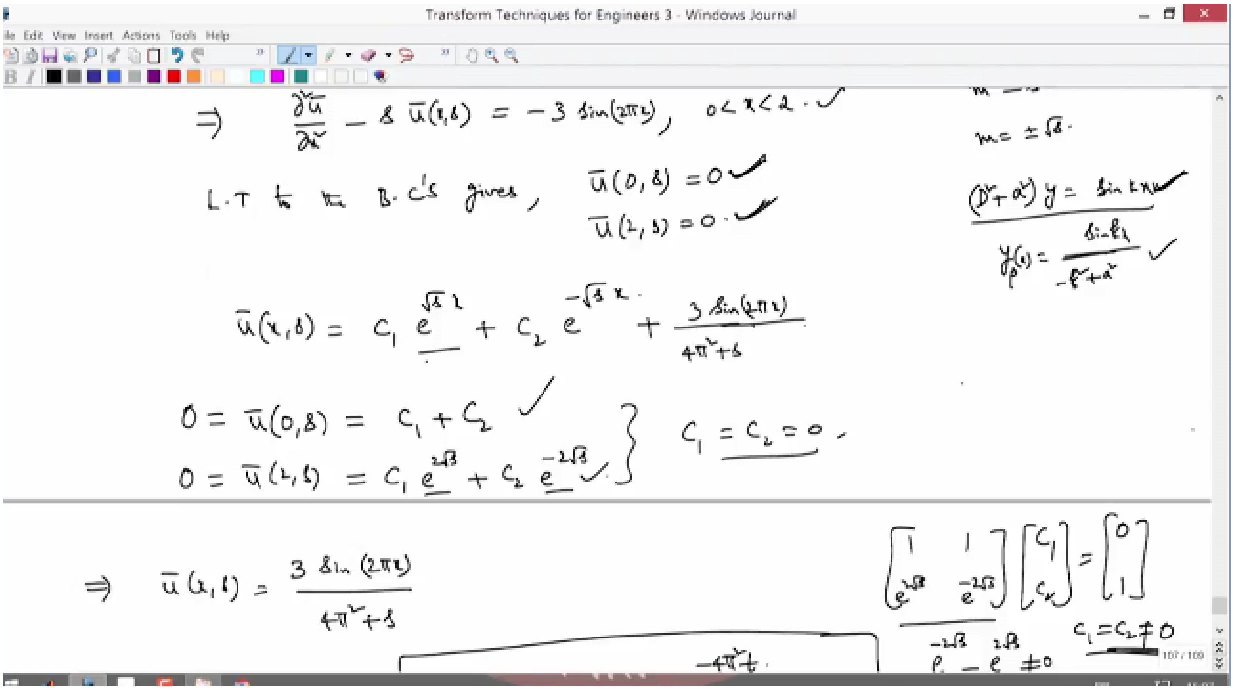


Welcome back to this video on the applications of Laplace transform. In the last video we have seen how to solve heat equation which is a parabolic equation in the domain special domain between 0 to 2, X between 0 to 2 and T positive, and that is actually you consider physically that is a rod of 2 meters with the boundary conditions we keep 0 temperature at both the ends that is U0, T0 and U2 T0, okay, and then at time T = 0 initially you keep the rod temperature is given that is 3 times sine 2 pi X as that is given so as a initial condition, so this problem we have solved by applying Laplace transform it so happened that when you apply the Laplace transform you see that U bar(x,s) Laplace transform of this temperature function U bar(x,t) that is, that becomes Laplace transform is U bar(x,s) which is 2 arbitrary constants involved in it in the complimentary function when you solve the ordinary differential equation, and C1 times E power root SX and +C2 times E power -root SX + some particular solution, okay.
And now you apply the boundary conditions and get your C1, C2, if C1 and C2 both are nonzero then you should be able to invert this function U bar(x,s). in the last video, in the last example C1, C2 becomes 0 so you have only that particular solution that you invert it and you get your solution, if C1 becomes 0 for example let us look at the equation, let us look at the solution U bar(x,s) is this if C1 is nonzero, if C1 is nonzero and you have to evaluate this

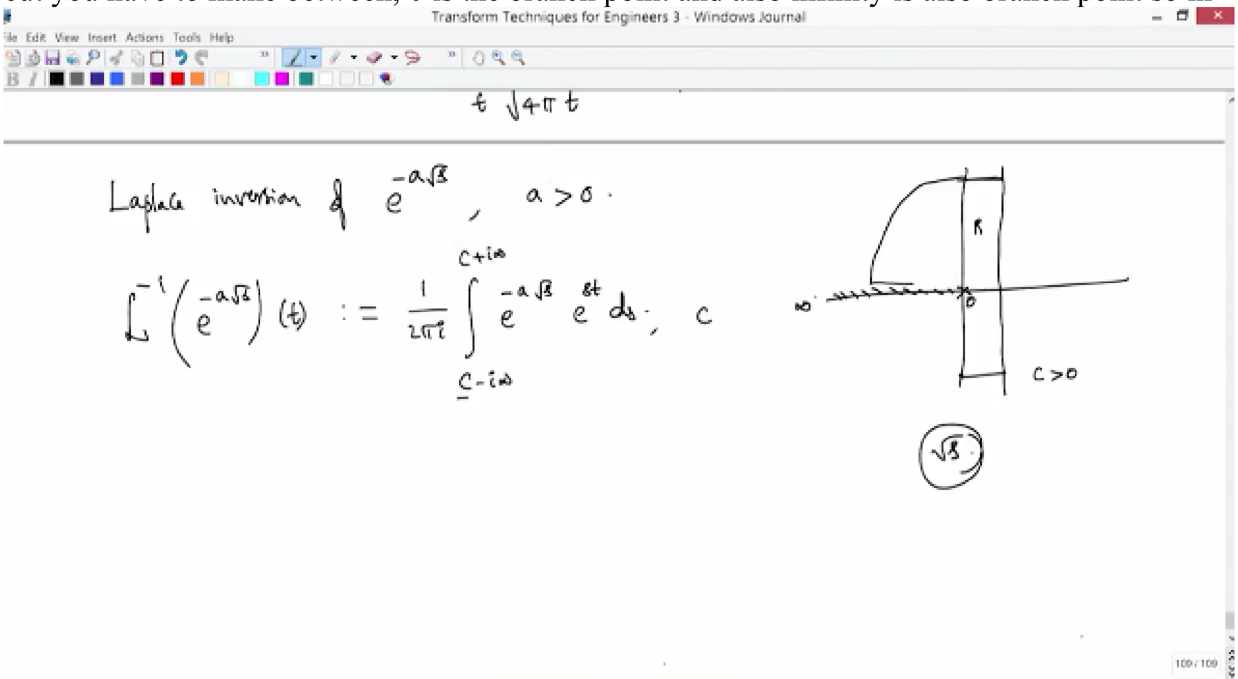


Laplace inversion of E power root SSX, okay X is positive so for which Laplace transform does not exist, okay, because if you look at that Laplace inversion integral for this one you don't, so you will not be able to, this is actually you contour integration technique may fail, this Bromwich contour may fail to evaluate this, otherwise the integral as such may exists, okay, Laplace inversion so you cannot simplify directly, that Laplace inversion you cannot, you may not be able to simplify with the usual techniques of contour integration, so but if C2 is nonzero and you have this exponential of -E square root S times X, X is positive for which inverse Laplace transform exists that's what we will show you how to evaluate this Laplace inversion for this, okay.

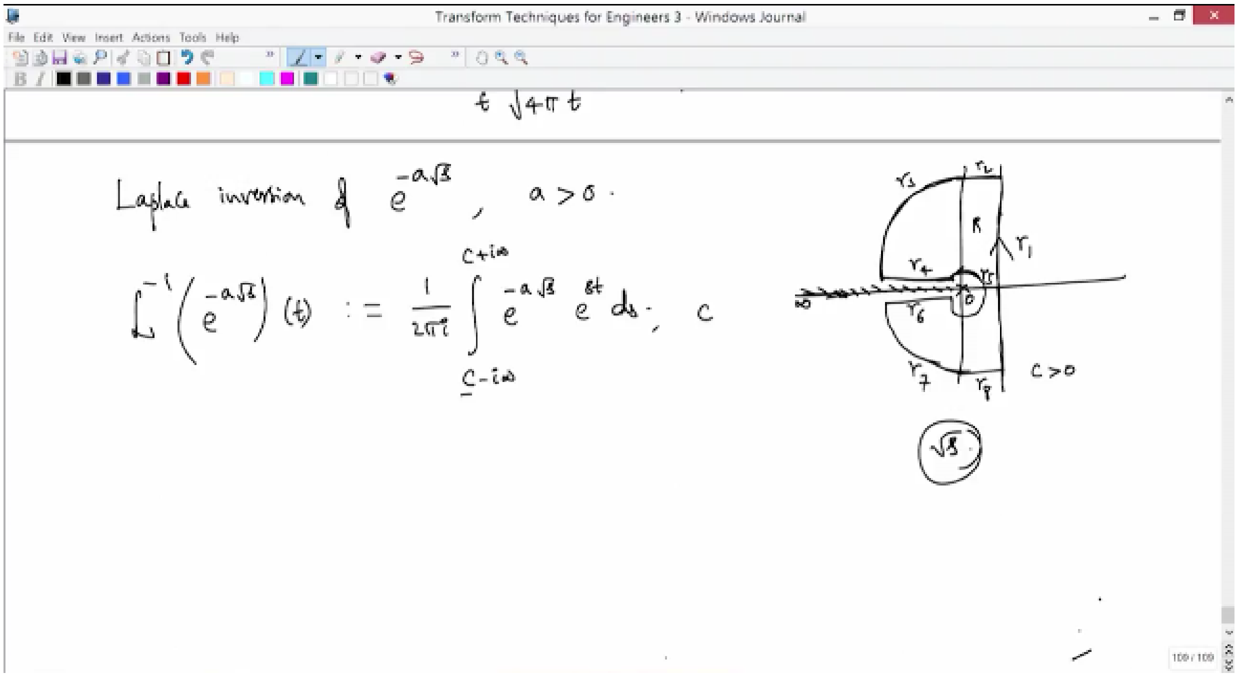
For other boundary conditions where C1 both, C1, C2 are nonzero in that case or C1 is 0 case you may have to, you may not be able to write explicitly as a function of T but as a solution will be in terms of an integral as a inverse integral, okay, inverse transform of this for which you may not be able to get it as an explicitly by the contour integration we just have to leave that, but if for some boundary condition if C1 is 0 and C2 is nonzero you can evaluate explicitly because of this, thanks to this contour integration technique.

So let’s look at this Laplace inversion of this function, Laplace inversion of E power -A root S where A is positive, so this is what we will see today, so this is a Laplace inversion of E power -A root S which is a function of T, so by definition this is 1/2 pi I times integral C-I infinity to C+I infinity E power -A root S, E power ST DS, so this is the integral you have to evaluate. If this is positive minus A if its positive then you have to leave this integral as such, a formally you have this is the solution as a function of T, if this is -A times root S for the exponential you can evaluate this as an in explicit form by the contour integration.

So let’s look at the Bromwich contour, so which is C, where C is a, because of this function having 0 is the branch point, 0 is the branch point, otherwise in this half plane this is analytic function, so you can consider C anything, anything positive okay, C is anything positive here and you look at this quantity this is a contour you look at it, okay, so if you look at this contour this is your R and this is your R, okay, so you look at this piece, and this piece, and this piece, okay, 0 is a branch point because of this root S is having multivalued functions, multivalued function so you have to cut this 0 from this complex plane so that you have only single valued function, so what you do is you try to remove this you make a branch cut from the, this is the cut you have to make between, 0 is the branch point and also infinity is also branch point so in

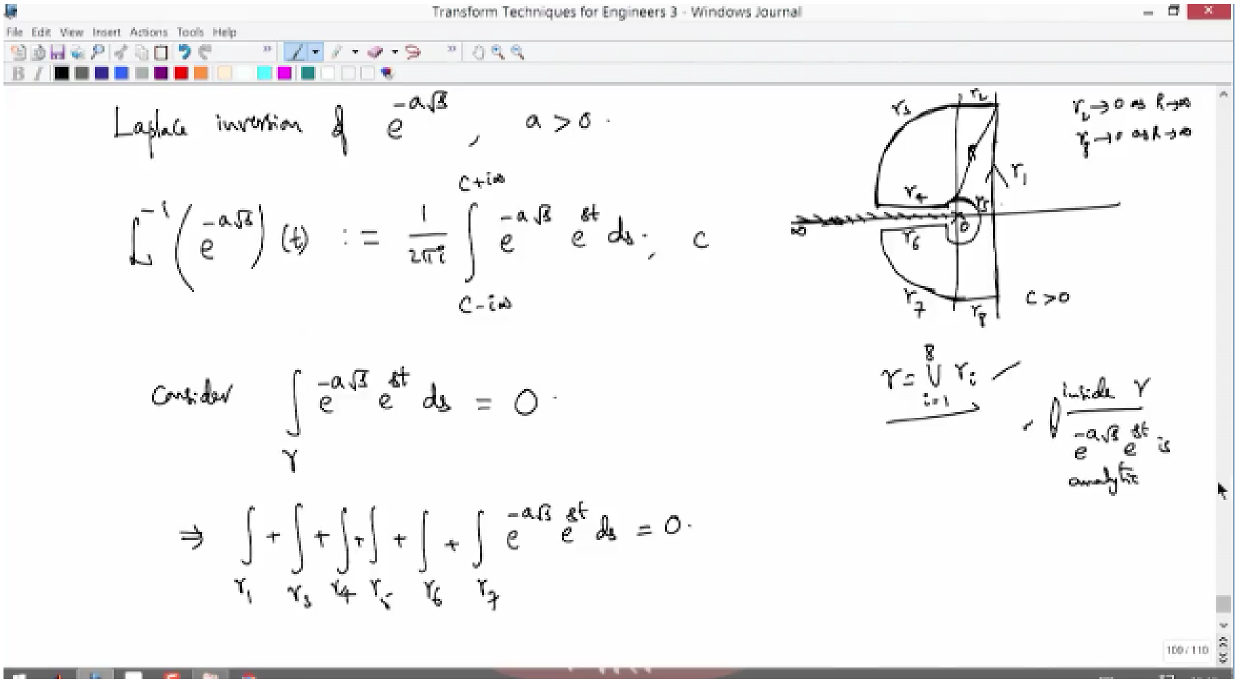


the complex plane infinity you can think of anything, anything which is not finite, there's no differentiation between -infinity +infinity here, so let's connect 0 infinity this way, if you remove from the complex plane you have this plane with this minus, this is a slit, this is removed, so you consider this contour is a circular arc and then you come like this and make a small circle around this, like this you make a small circle and then you come back here and then make again a circle of radius R and then attach it here, so this is the contour you have so if you do it from here C-I infinity + C+I infinity over this, and then this you call this gamma 1, and you call this gamma 2, and gamma 3 this piece between this to this, and this is between gamma 4, and this is gamma 5 the circular arc, circle and then this is gamma 6 and gamma 7 and gamma 8, so you have a gamma that is composed of union of all this gamma I’s, I is from 1 to 8, so that is your closed contour.



So what I do is I consider to evaluate this integral so that is over this gamma 1, okay, so what I consider is the integral E power -A root S E power ST DS over gamma, so if I consider this so this is actually equal to, now if I consider this integral, integral over this gamma because there is no pole for this function, this is analytic inside this closed, inside the domain of gamma, okay, gamma inside, inside gamma the function E power -A root S and E power ST both is analytic generally, because of this analyticity by contour Cauchy residue theorem you don't have 2 pi I times residue of, there is nothing so it has to be equal to 0, okay, so this implies, so let's make this 1/2 pi I here, now we can add it at the end, so this is 0, so that means over gamma 1 that is actually your, as R goes to infinity if you consider this as your R radius, as R goes to infinity gamma 1 becomes C-I infinity to C+I infinity, okay so gamma 1 + gamma 2 so what happens to gamma 2 let me not, as R goes to infinity gamma 2 becomes, gamma 2 goes to 0 as R goes to infinity, so that's what you'll see, initially you see that this piece as R becomes bigger and bigger this is vanishing so you will see that this R2 is going to 0, similarly R8 is going to 0 as R goes to infinity, so it doesn't really matter, okay, so you need not consider this gamma 2 gamma 8 so you have to worry about gamma 3, gamma 7, and gamma 5, gamma 4, comma 6, okay.

So gamma 3 over gamma 3 + gamma 4 + gamma 6, okay, let me plus I add it here and then that’s gamma 5, and then + gamma 6, + gamma 7 of this function E power -A root S times E power ST DS which is equal to 0, so let me calculate it over gamma 3, what happens to your



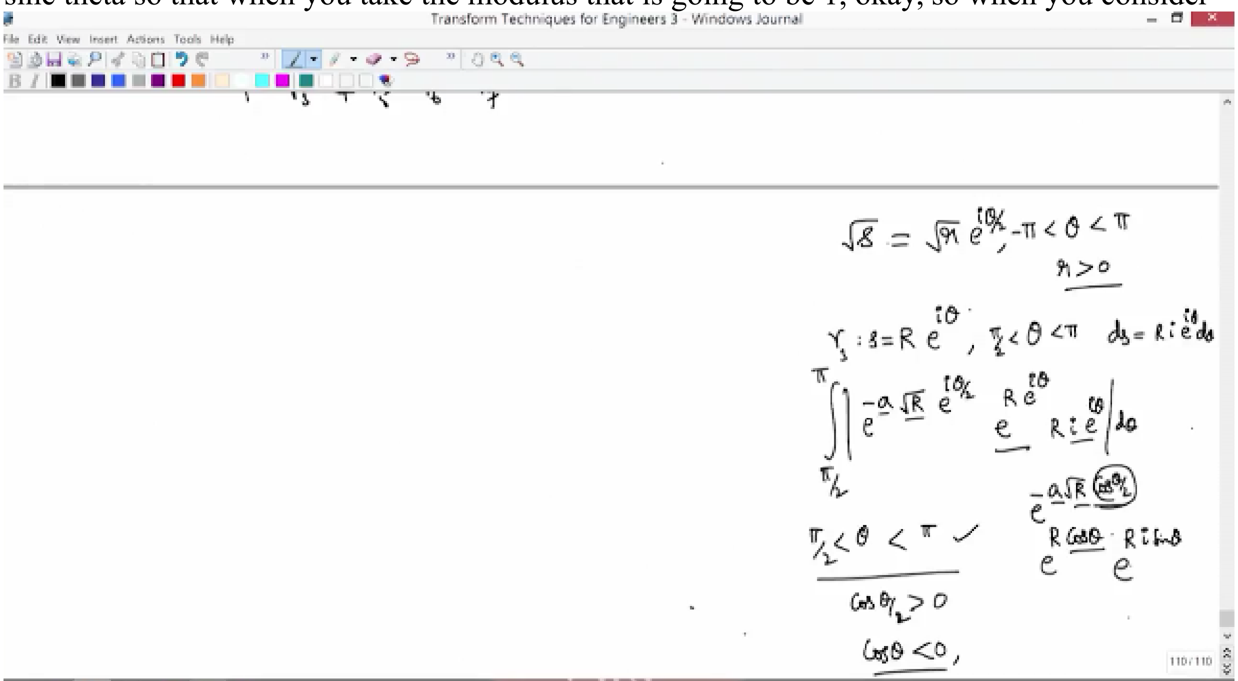
gamma 3? Let's look at in this gap I will write everything what happens to gamma 3, integral over gamma 3, what is gamma 3? Gamma 3 is a curve, let me write it here, so if you look at the gamma 3, so gamma 3 if you look at so this is E power, okay, first of all once you make a cut S is the complex plane once you make a cut in polar coordinates this is R into E power I theta, theta is between minus, this is pi to or -pi to pi, so this is how you have once you cut this pi thing you cut, right, this is 0, this is theta = 0 and you come back here that is pi, when you combine here this is –pi, so you have this is your theta, theta between -pi to pi, this function is root S is a single valued function, okay, that is this divided by this, root S which is equal to this, where theta is this and R is positive so this is the domain of the complex plane, so full S plane I write it as in this, this is how you represent in the polar coordinates.

Now what happens if you look at this gamma 3 this is a circular arc, gamma 3 if you parameterize gamma 3 what happens if you parameterize, that is R times that is fixed R, so R times capital R is that distance E power I theta, what is theta? Theta is between that is between straight, that is pi/2 to pi, so if you look at this theta, theta is pi/2 here and when you come here this is theta 2 is, this is pi/4, okay, this is up to here pi by, this is 0, pi/2 and then this is PI, so we have this.

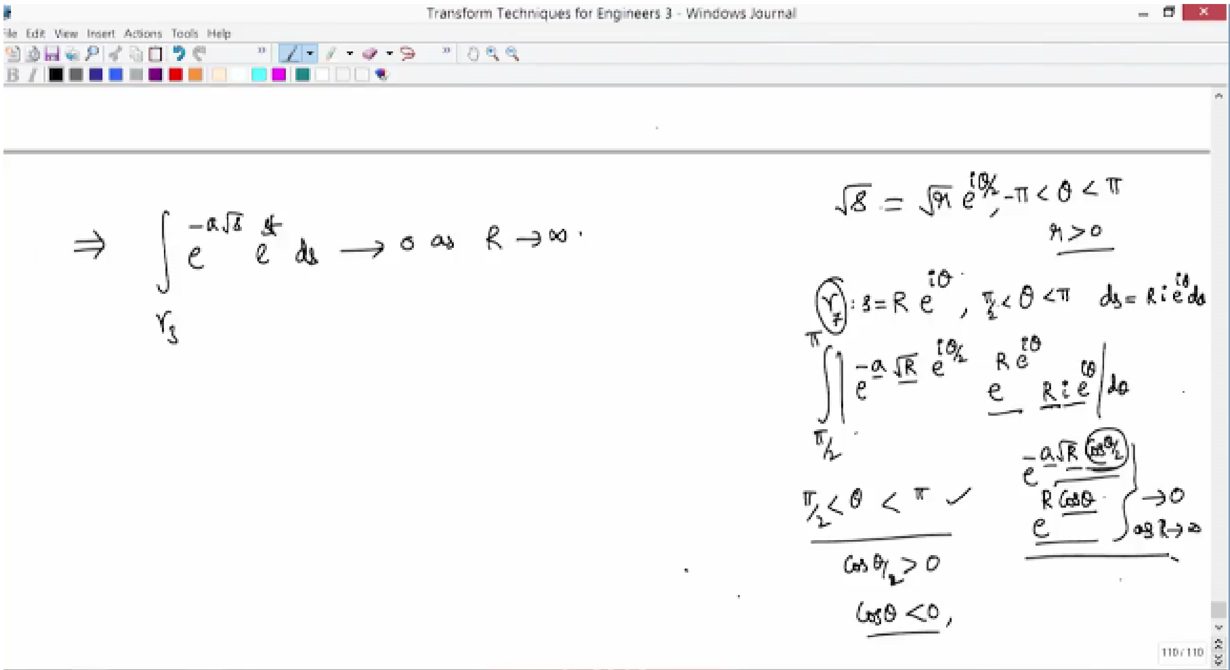
Now if you look at this E power, so what is your domain, so what is your integrand is E power -A times root S, S is this, okay, root S is a root of R times E power I theta/2, okay, and then, and what happens to E power ST, S is E power R times E power I theta and then DS is, S is this, once you parameterize DS is over that curve gamma 3, so S is actually this, this a parameterization for this, S equal to this, okay, S equal to this, you have DS will be R times I E power I theta D theta so you can write this, so integral over minus pi/2 to pi/pi and you have this one times.

Now DS I write R times I times E power I theta D theta, so this quantity if you look at because theta is, theta/2 is between pi/4 to pi/2, okay, so within this this quantity is E power I theta/2 is real part is always positive, cos theta, cos theta/2 is positive okay, because this is positive and you already have E power -times A is positive, root R is positive, this quantity real part is positive, so what is left is E power I times something, okay, E power I sine theta that part when you consider this modulus that goes inside that is less than or equal to that inside that will be 0 that will be 1, E power I times sine theta/2 part that is actually becomes rather E power, what do you have is E power -A root R I sine theta/2, so this part is actually 1 when you consider the modulus, okay, so with that this is the only contribution cos theta/2, so E power -A root R E power and you have cos theta/2 that is positive, positive quantity and you have positive quantity, positive quantity.

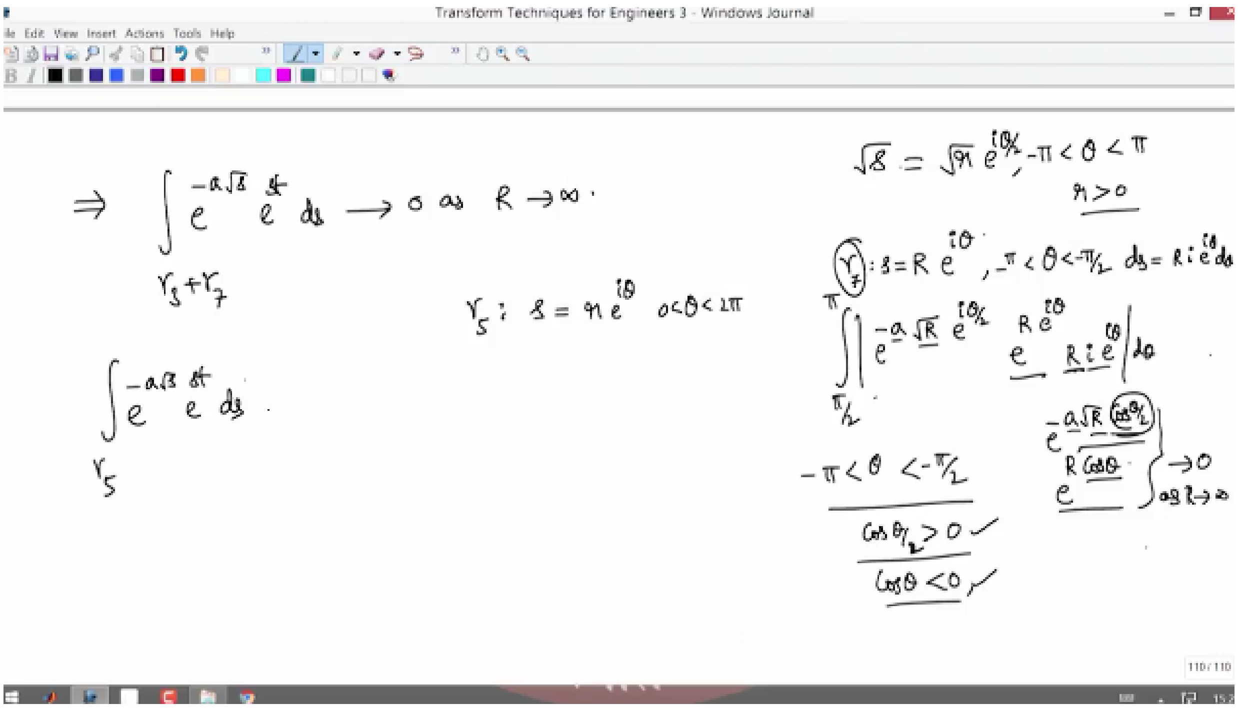
And then now what happens to this E power R theta? E power R cos theta, when theta is between, theta is between pi/2 to pi this quantity, cos theta is negative between this theta, okay, so when theta is this, cos theta is negative, so this quantity is negative times minus, so E power negative quantity power R, and of course you have another quantity this is E power R times I sine theta so that when you take the modulus that is going to be 1, okay, so when you consider



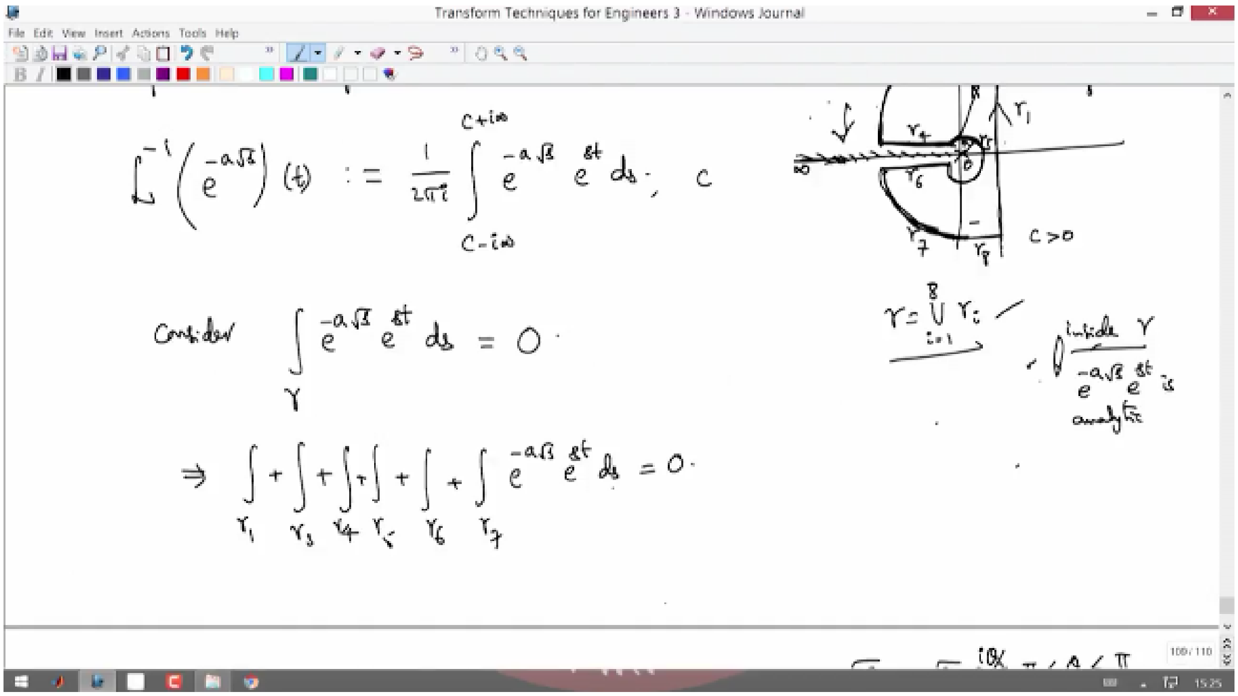
this modulus which is less than or equal to this modulus inside, okay, and of course you have to put it here so that this quantity goes I times mod I, mod I is 1 E power I theta mod is 1 and you have this quantity that is actually the product of these two, product of these two this 1 into this, okay, so this quantity because this is the exponential of negative and this is also exponential negative that goes to together this goes to 0 as R goes to infinity, this is exponential growth and this R doesn't matter so you have R divided by exponential positive R whatever, okay, so as R goes to infinity that goes to 0, so the contribution over gamma 3 as R goes to infinity is also goes to 0, so that means a contribution over this means this implies integral over gamma 3 that E power -A root S E power ST DS which is that goes to 0 as R goes to infinity, so you have seen as R goes to infinity gamma 2 anyway goes to 0 this piece the straight-line piece and this circular arc as R over gamma 3 that is also goes to 0 as R goes to infinity, by the same technique over this, this circular piece which is R which is actually between –pi/2, so in this case what you have to consider is same parameterization gamma, what is that? Gamma 7, okay, that is gamma 7, if you consider gamma 7 here so parameterization is –pi/2 to –pi.



So even in this case you see that –pi/2 to -pi or -pi to –pi/2 so smaller you have to consider, so in this case -pi to –pi/2 if you consider you see that cos theta/2 this is also positive, this is also negative, okay so again this goes to 0 as R goes to infinity, the same technique so you see that gamma 3 + gamma 7 both are going to 0 as R goes to infinity, so this quantity is not contributing and this quantity is not contributing, this quantity is not contributing integral over this part is not contributing, so you have to look at gamma 4 and gamma 6 and gamma 5.
Let us look at gamma 5 now, gamma 5 is also is going to 0 so integral over gamma 5 E power -A root S times E power ST DS, so for which if you consider now gamma 5 if you consider gamma 5 as your arc, circular arc if you look at that as a circle, whatever it is parameterization? S equal to, let us look at this small r, you consider a small circle here that is small r let us say some, that is some small r, this is equal, so parameterization is R times E power I theta, theta is between 0 to 2 pi, okay.

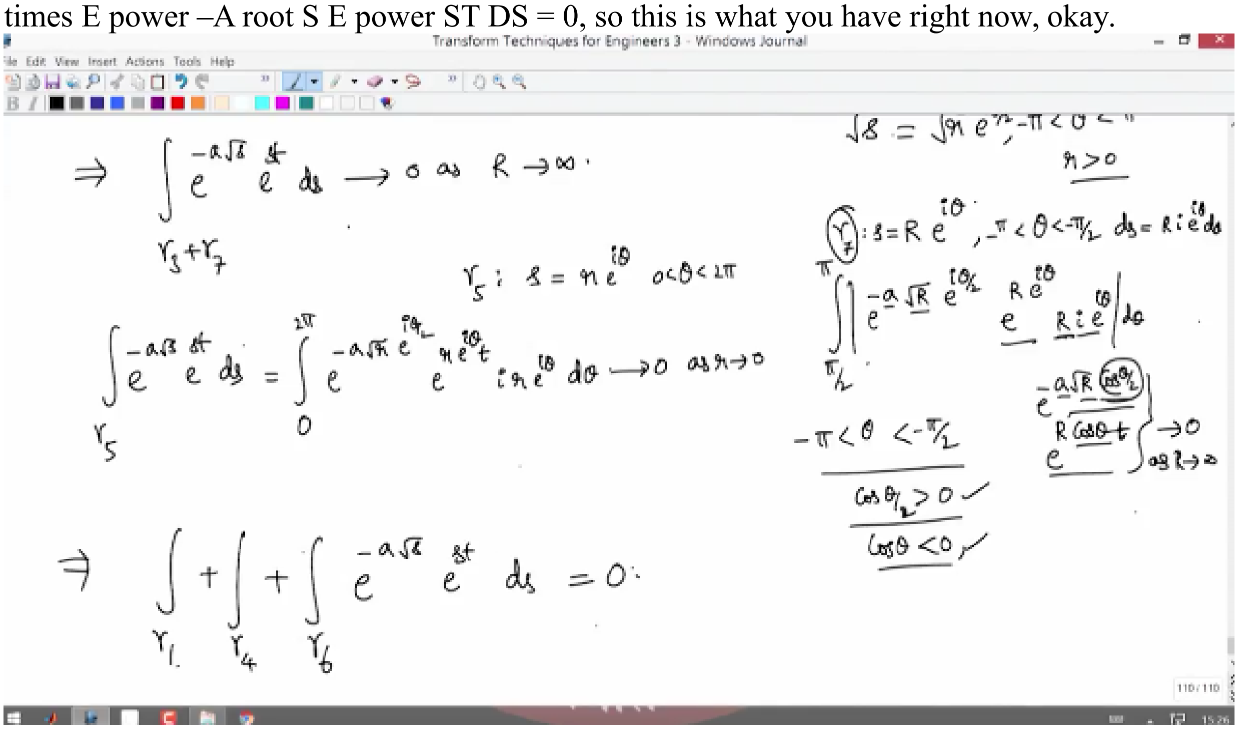


So let us look at what happens here integral over gamma 5 becomes integral over 0 to 2 pi times E power -A times root S is root R times E power I theta/2 times E power R into E power I theta times T, of course here also we have T, okay T is anyway positive so it doesn't matter, so this E power T, DS is I times R E power I theta D theta, so in this case whatever maybe, as R goes to 0, as R goes to 0 this is the bounded quantity this goes to 1, and this goes to 0, so this quantity is anyway R, as R goes to 0 that means you diminish this, as R goes to infinity this small r goes to 0 and this gamma 4 becomes lying on the line here, on the negative X axis, and gamma 6 also coming from the bottom, the limiting limit this piece will become the 0 to infinity which is going to match with negative X axis from the bottom, and gamma 4 is a positive to, it's a piece from 0 to infinity that is lying from, its coming from a top from this part it's lying, it's going to

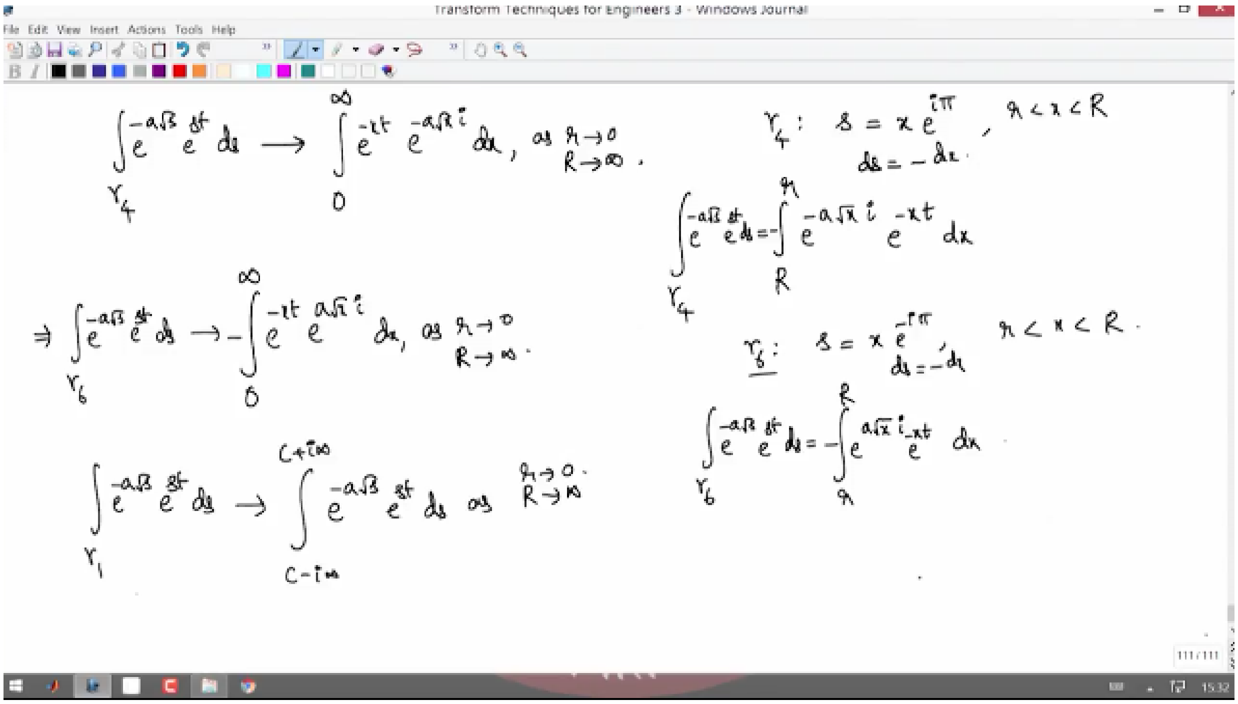


map with negative X-axis, okay, it becomes a negative X-axis from the top, and the gamma 6 is the negative X-axis from the bottom, so as the limiting value of that as R goes to infinity small r goes to infinity.

So a small r goes to 0 this quantity is going to, so integral over gamma 5 the contribution of this integration over this circle, small circle is also 0, so only thing is what is left is gamma 1, gamma 4 and gamma 6 together is 0, so what we have is this 7 is gone, 0, 3 is 0 and you have gamma 5 is 0, so you have what you have left is gamma 1, gamma 5, gamma 6, that together is 0, so this implies integral over gamma 1 + integral over gamma 4 + integral over gamma 6 times E power –A root S E power ST DS = 0, so this is what you have right now, okay.

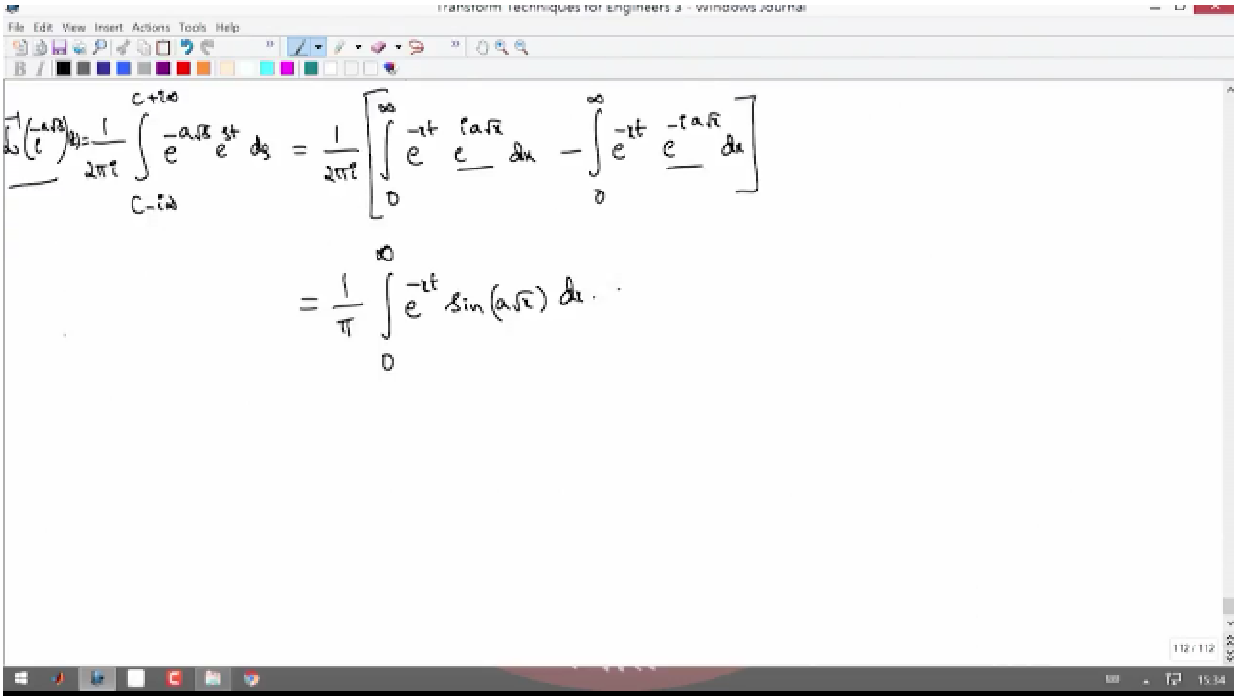


So what is gamma 1? As R goes to, so let us look at what happens to this gamma 4, gamma 4 so for this you look at gamma 4 what is its parameterization, so you look at the gamma 4 it is S is there that piece, that piece over the negative X-axis is, that is actually, that is becomes a real number times E power I pi so this is the parameterization for the negative X-axis from the top because it's a pi E power I theta, theta is pi, if you come if you take it -I pi this is coming from the negative piece, if this is your X axis and this becomes that if you come from the bottom side that is the parameterization for gamma 6, gamma 6 you have to consider with –pi, so this is let’s look at only gamma 4 is this, so what is DS? DS is simply X is constant, X is actually varying so you have a DX times E power -I pi is a minus, right now it's I pi for gamma 4, and DX times cos pi that is –DX, sine pi is 0, so this is what you have.
So what happens to their integrand? Integrand is 0 to infinity, so X is between 0 to infinity, right, so if you look at gamma 4, gamma 4 is actually coming from this side so you look at this direction gamma 3 is gone, gamma 4 is actually this way coming from this side and like this it goes so gamma 4 is coming from 0 to infinity, so 0 to infinity means it's minus, minus of gamma 4 so you have to look at gamma 4 as minus of gamma 4 is this one, okay, between 0 to infinity, if actual gamma 4 is infinity to 0, okay, so you have to write infinity to 0, let me write it as infinity to 0, okay, so X is between infinity to 0, so is actually 0 to infinity because of the direction I am writing integral from 0 to infinity, and infinity to 0, okay.
Actually it's between, so you don't write limits this is actually r, small r this is actually capital R, so let us write from R to R, so let's use this strictly speaking this is between small r to capital R, so you have integral is actually gamma R that is from big R to small r, that is our gamma so integral over gamma 4 this function E power -A root S times E power ST DS is actually equal to this integral R to R and E power -A times, so root S is a root X times E power I pi/2, so E power I pi/2 is cos pi/2 is 0 + I sine pi/2, so this you have I, and you have E power S is X times –X, so you have -X here, cos pi is -1 + I sine pi 0 so you have E power -X of course you have ST, T is that you have DS is –DX, so you have put minus you have DX so this is the integral that it has become, so as R goes to infinity E power -A root S E power ST DS goes to integral this is from as this goes to 0 to infinity of E power -XT times E power -A root X I DX as R goes to 0 and small r goes to, big R goes to infinity because of this, okay, if you look at this gamma 6, if you look at the gamma 6 same way you have X is X times E power -I pi is your this thing and this is your parameterization for this, integral over gamma 6 now E power -A root S E power ST DS this is equal to, and you have this is now because of the direction coming from this to this, so you have R to R, so integral from small r to big R times, DX is -DX okay, so what is DS? DS is cos pi that is still –DX, and you have, what happens to E power -A times root X, and you have root S and E power -I pi/2 that is cos pi/2 is 0 –I, so you have a -I that is going to be +I.
And here exponential S is E power X, cos E power -I pi is –XT, so this is what is the integral, so this implies you have integral over gamma 6 –E power -A root S E power ST DS, this goes to -0 to infinity, E power –XT, E power +A root X times I DX as R goes to 0 or capital R goes to infinity.
So what happens to the last one which is gamma 1? E power -A root S, E power ST DS this goes to C-I infinity to C+I infinity times E power –A root S E power ST DS as R goes to

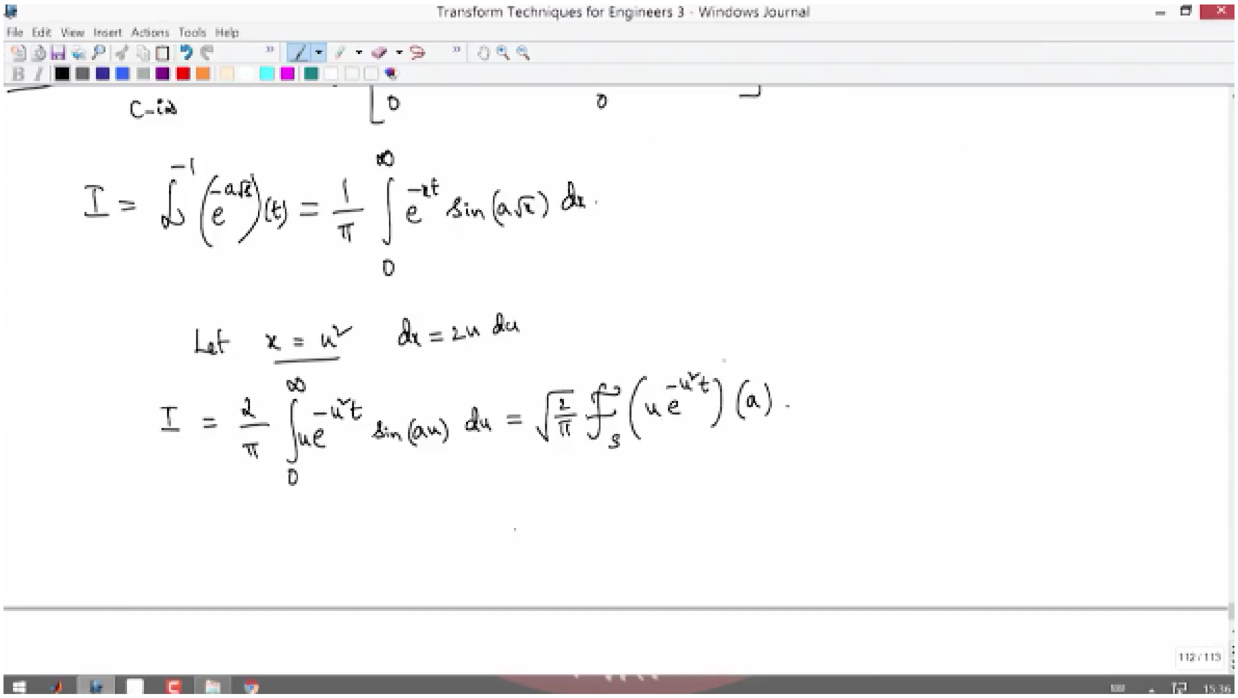


infinity, small r goes to 0, so what you have right now, so because all these three together is 0 this is what you want, so you can simply multiply this 1/2 pi I times over gamma 1 that is C-I infinity to C+I infinity E power -A root S E power ST DS this is equal to minus times, so you bring this minus, if you take this minus this becomes plus side, this take it to the other side so that is going to be 1/2 pi I times integral over gamma 6 that is 0 to infinity E power -XT times E power IA root X DX and this becomes, this goes to the other side that is this and you have the other one is minus that is plus because this is going to be plus and when it bring to the other side that is minus, -1/2 pi I common because I multiplied all the three quantities so you have here also 0 to infinity E power -XT E power -IA root X DX.

So what is this value? This value is actually, this integral value what you are looking for is the Laplace inversion of the function of E power -A root S as a function of T is this quantity, so this is equal to 1/pi integral 0 to infinity E power -XT and this minus this will give me 2I, 2I 2I goes 2I times sine A root X DX, so this is the integral you still need to evaluate, if you evaluate this

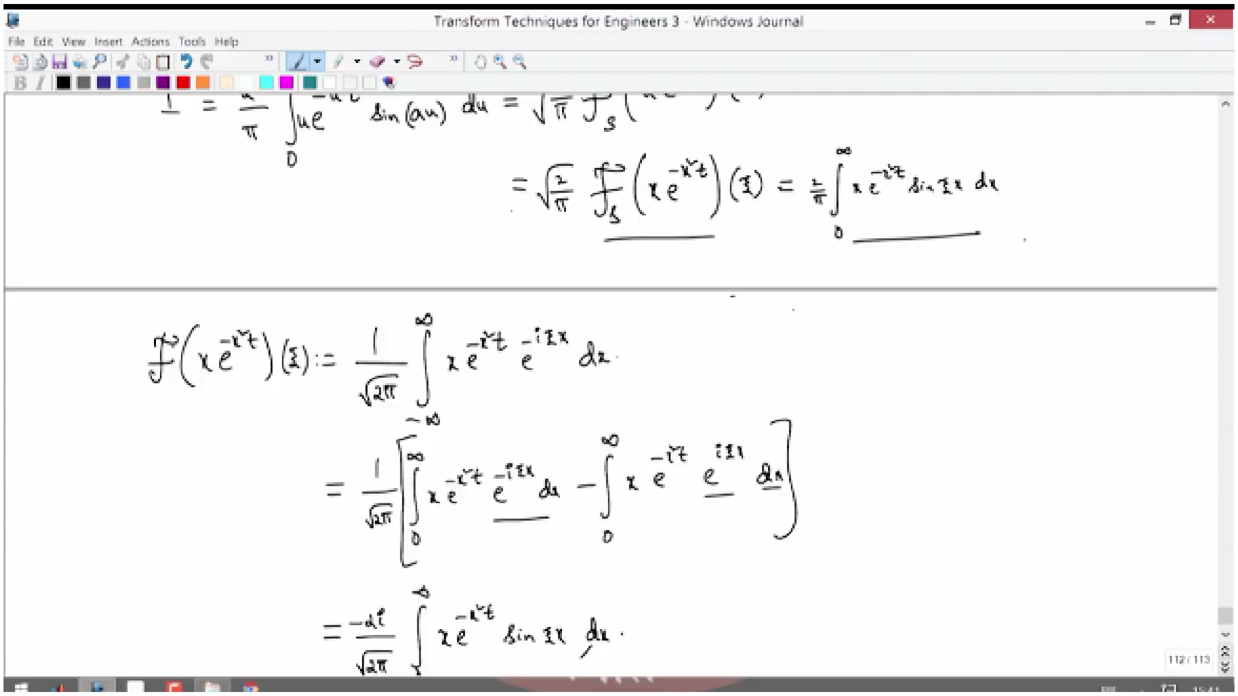


one I know what is the Laplace inversion of this, okay, so to evaluate this let me use the Fourier transform this is actually Fourier, this is actually what Fourier sine transform if I just use the sum, this is actually your Laplace inversion of E power –A root S this is a function of T, I need to find this, I need to evaluate this integral, to do this let X equal to, you call some U square, some new variable if I use, if I use the new variable what happens to your DX? DX will be 2 U DU, so this I let us call this I so you have I = 1/pi times integral because of this 0 to infinity only, and E power -X is U square so you have U square T times sine A times, root X is AU D, DX is 2U DU, so you have U here, comes here 2U and you have DU, so this is exactly equal to root 2/pi times 1 root 2/pi with this is actually Fourier transform, so how do I write Fourier sine transform FS of U times E power or if I use as a function of X, U you can use, you can write it as a function of, so let me write this as same way, U times E power -U square T as a function of

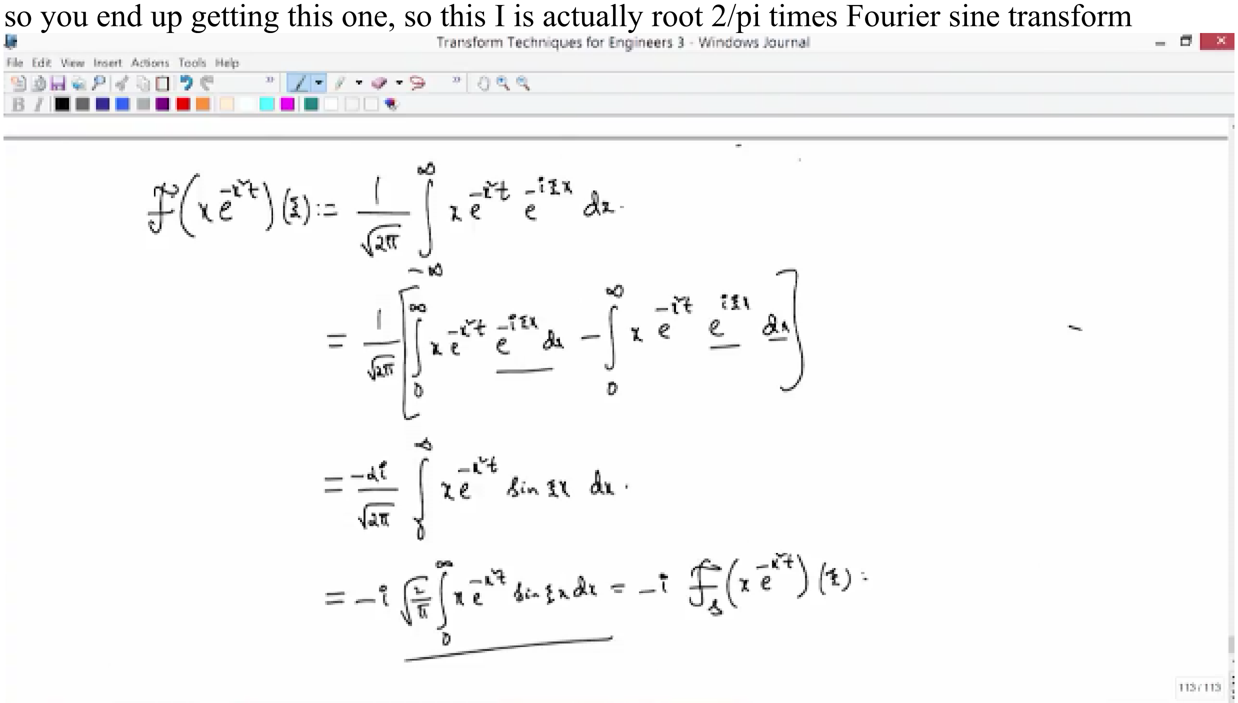


A okay, function of A, because this at the end you get a function of A, T is a constant, given constant, A is the variable, so A is, because of A is a variable so you have, this is a Fourier sine transform of this function, so if you write this as a Fourier transform with new variables if F(s) as a function is X times E power -X square times some T, this is a constant which is a function of xi, so this is what you have, okay, Fourier sine transform of that.

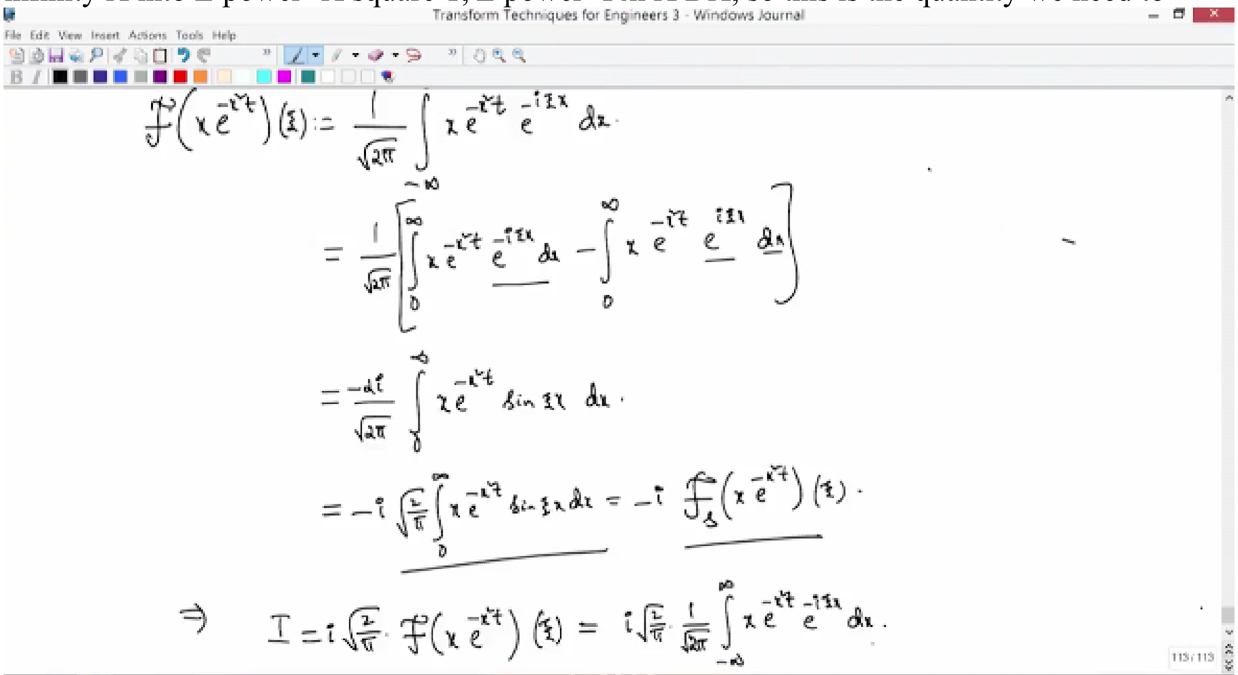
And if you actually consider Fourier transform of X into E power -X square T as a function of xi if you actually write what is the definition? 1/root 2 pi integral -infinity to infinity X into E power -X square T E power -I xi X DX, okay, so this is your definition of Fourier transform, this is Fourier sine transform, and let me look at the Fourier transform and I relate this I calculate this one, so this is equal to if you actually split this as 2 pi times 0 to infinity X into E power -X square T, E power -I xi X DX + 1/root 2 pi, so let me use this as factor and then plus -infinity to 0 X times E power -X square T, E power –I xi X DX, so here if you try to replace X/-X so you have X/-X this and DX will become plus, and this becomes plus and this is as it is, and because of X/-X this is going to be infinity to 0, and that makes it 0 to infinity with negative sign, okay, so this is equal to 1/root 2 pi and this becomes integral 0 to infinity X E power -X square T and this quantity is this minus this, so this is the definition we followed for the Fourier transform so this is this, and you have this is going to be -2I times sine xi X or DX, right, so this sum is this difference, this minus this is -2I times of that.



So what is this one? If you write like this, this is root 2/pi integral 0 to infinity or rather 2/pi 2/pi goes comes out because of this you have 1/root 2 pi, 2/pi that is X into E power -X square T times sine xi X DX, so this is the quantity for Laplace transform, so sine transform, Fourier sine transform is this, so what you get is a same here, so 2 2 goes so what you end up is -I root 2/pi times integral 0 to infinity X times E power -X square T sine xi X DX, this is nothing but -I times this is actually Fourier sine transform of X, E power -X square T as a function of xi, okay, so you end up getting this one, so this I is actually root 2/pi times Fourier sine transform



of this one, so which is, so this implies I is root 2/pi times, I is this, and you have a root 2/pi times Fourier sine transform I replace Fourier sine transform from this you have, when you bring it this side so you have I times Fourier transform of X into E power -X square T as a function of xi, which is finally is I root 2/pi times 1/root 2 pi, Fourier transform is -infinity infinity X into E power -X square T, E power -I xi X DX, so this is the quantity we need to

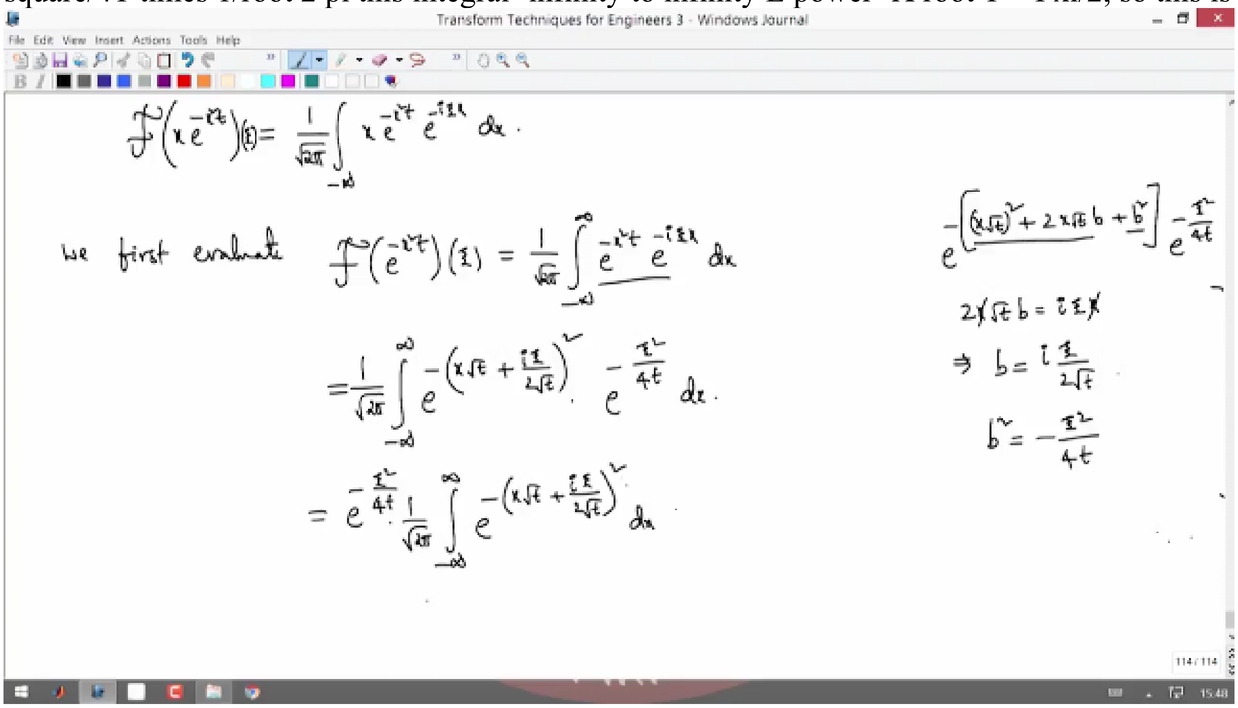


evaluate, so this I will be able to do by evaluating this integral, if I know this integral then I know what is the I, that is Laplace inversion of E power minus A root S as a function of T, okay, so this quantity is.

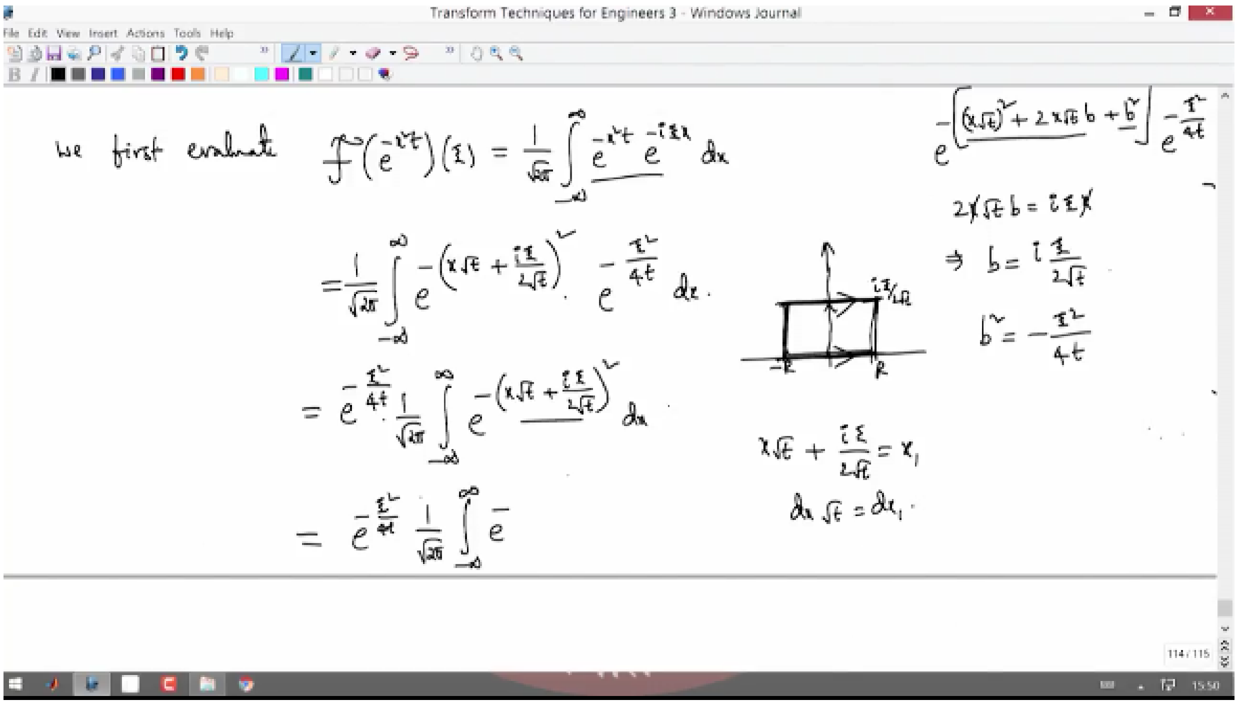
So let’s look at Fourier transform of X into E power -X square T, which is 1/root 2 by pi times -infinity infinity, X into E power -X square T, E power -I xi X DX, this is straight forward, directly we can evaluate by rewriting so to evaluate this we evaluate this now, we evaluate this so to evaluate this what we do is, we cannot evaluate this directly so in a different way to get this, to evaluate this, we evaluate, we first evaluate Fourier transform of E power minus, let's say X square times T as a function of xi this is a function of xi finally, okay, this is 1/root 2 pi times -infinity infinity E power -X square T times E power -I xi X DX, this quantity we first evaluate and we make use of this to evaluate this that we will finally put it here and get your Laplace inversion that is the idea.

So let's look at this one first, so how do I do this now? This I try to write, I put this in the form of E power -X square type, okay, some E power -something square, so how do I put this there like that? You have E power -X root T this is square and you have +xi X, to get xi X I already have if I say A square + B square so + some B square 2BX, 2BX root T already X is there, X root T + B square so that is what you have, so what is this 2B root X? 2B X root T is actually is your xi X, so X X goes so what is my B is xi/2 root T, so you have xi/2 root T, so 2 2 goes here, so this also will go, so what is your B? B is xi, B square that is xi square/4T, so this together is nothing but E power this, so you have this quantity is already here and, of course there is I missing right so you have, let me write again so this is X square + 2X root T times B + B square, so what is my B? So 2X root TB is actually what I have is I xi X, so this gives me X X goes and you have what is B? B is I xi/2 root T, so if you replace there as B, in the place of B if 2 root T you remove you end up getting that.

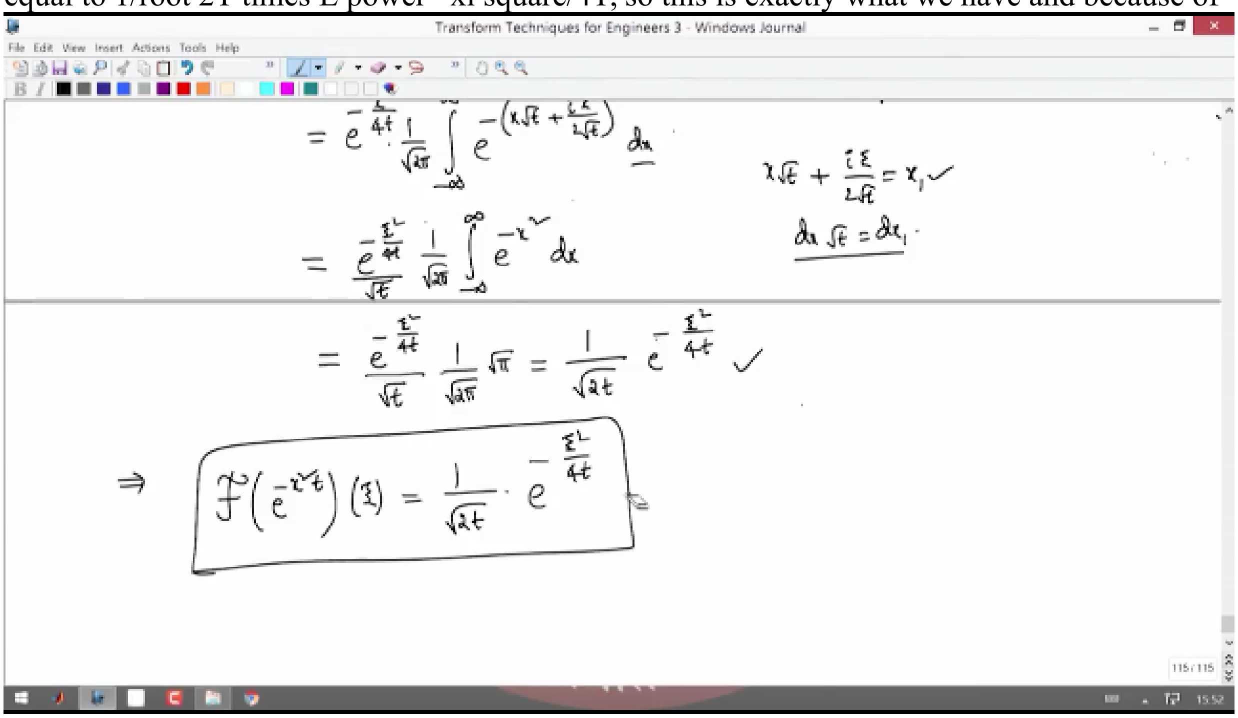
And B square, what happens to your B square? You have a –xi square/4T, and this quantity is actually what you have here, so this additional thing you have to multiply E power B square, that is -B square so you have +B square, so this is going to be –xi square/4T, okay.
So let me write that so you have 1/root 2 pi and you have -infinity to infinity I rewrote this part as E power -X root T + B, B is I xi/2 root T whole square, so if whole square if I want this integrand I have to multiply E power –xi square/4T this is DX as it is, so this is nothing to do with the integrand, that integral so you can write this outside so you have E power –xi square/4T times 1/root 2 pi this integral -infinity to infinity E power -X root T + I xi/2, so this is



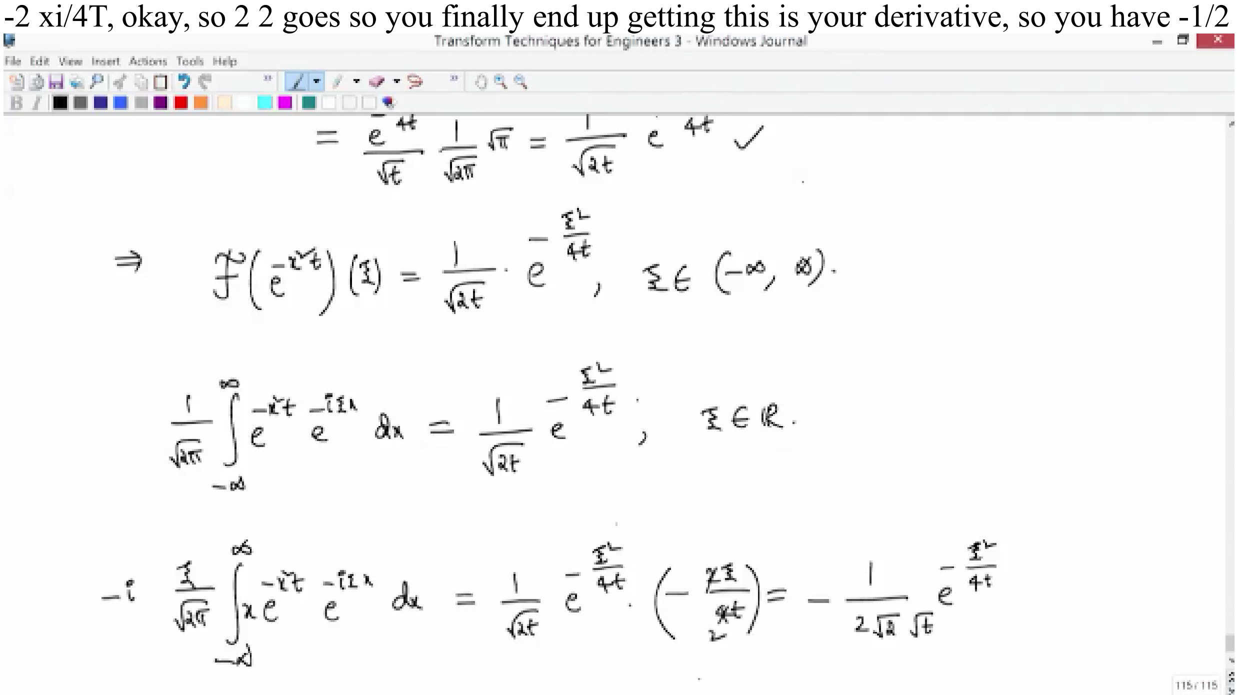
a constant square DX, so I think we will not be able to finish today, so let's wind up, so before okay, so this is let me rewrite this as E power –xi square/4T and 1/root 2 pi and this one -infinity infinity as we have seen earlier if you consider this contour, this is your X-axis contour, and this is your Y-axis, okay, this is your X-axis and you have 0 here -R to R and this is going to be xi/2 root T this is a constant, this is what you have that is the point here, this point is this value of this is I times this, okay, so that is on the imaginary axis you have this you consider that much another rectangle like this you will see that if you consider this integrand you will see that integral over this contribution + this contribution is 0, as R goes to infinity, so this is equal to, this if you want this one, this over this as R goes to infinity you finally end up this one, so that is same as saying this is equal to this one, so you will see that this will be same as E power minus this over this is actually or rather you should put it this way, over this actually this integral and that should be same as this part, okay, this plus this is 0 that means this is same as this, over this, so that is E power minus simply X, this whole thing you can put it as some X root T + I xi/2 root T as some new variable let us call this X1 okay, so you have DX is same as DX into root T = DX1.
So even by contour integration you have to justify by contour integration this by considering this contour, or in a formal way which is not legitimate but formal way if you do also you get



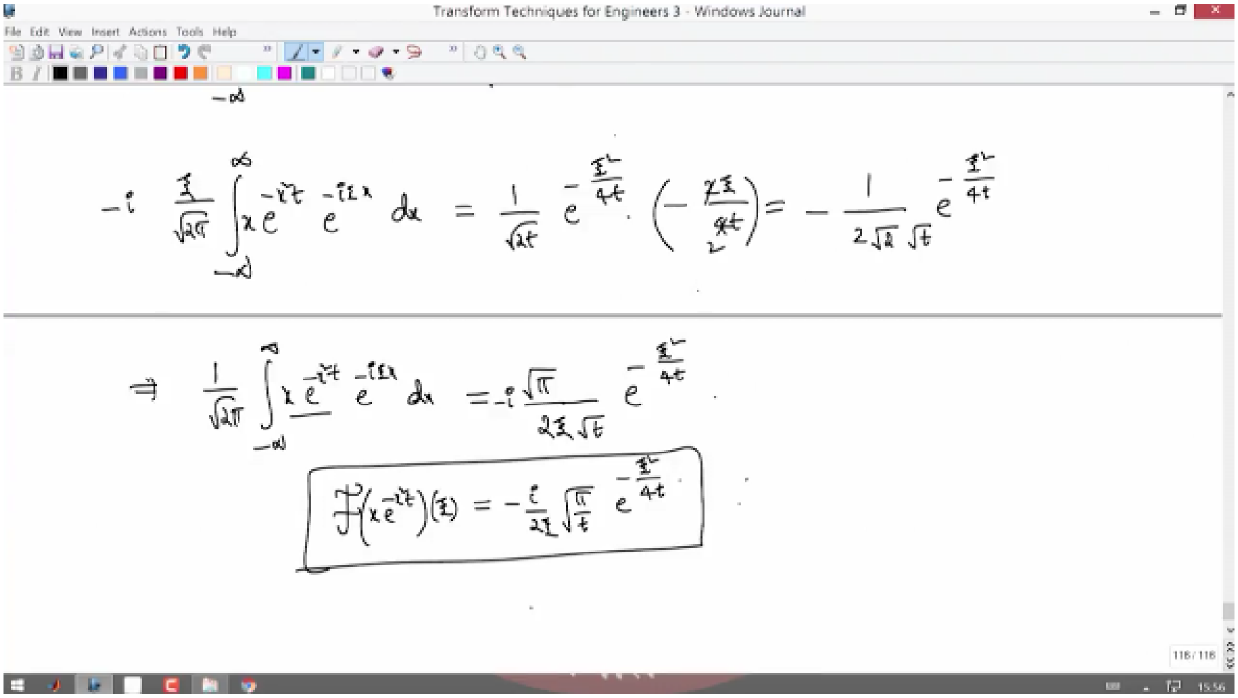
the same thing, so if I get this one DX I have to replace by DX1/root T okay, that is as DX E power minus this whole thing as X1, so you have X1 square, so you divided by root T, root T times and so that I removed here I write here so as DX1, so X1 dummy variable lets put it DX square okay, so this is same as this one, so this is your Fourier transform of this one as this, this we already know that this is E power -xi square/4T divided by root T times 1/root 2 pi this quantity is root pi, okay, let me write it carefully, this is root pi, so root pi root pi goes so you finally end up getting 1/root 2T and you have E power –xi square/4T, so this is your Fourier transform of this, so implies Fourier transform of E power -X square T as a function of xi is equal to 1/root 2T times E power –xi square/4T, so this is exactly what we have and because of



this Fourier transform xi is positive, xi is between, xi belongs to 0 to infinity Fourier transform, right, so it's actually minus infinity infinity okay.
So I can differentiate this with respect to xi, if you do you will end up getting or rather so okay, let’s look at this part, let me look at this Fourier transform of this, let me write what is this, 1/root 2 pi integral -infinity infinity E power -X square T E power -I xi X DX this is equal to 1/root 2T E power –xi/4T, so this is what we have.
Now T is also positive both sides so T belongs to 0 to infinity, I differentiate with respect to T if I do, or rather I think I should do with respect to xi, so xi is belongs to full R, so if I differentiate with respect to xi, if I differentiate with respect to xi for example what you get is, if you differentiate this with respect to xi you get 1/root 2 pi derivative you can take it inside that is by Leibniz rule this doesn't matter so E power minus X square T and this you differentiate as it is you get I xi X if you differentiate -I xi X, -I xi X comes out, I xi X, X will be here as xi will be here, so this is DX as it is this is equal to if you differentiate this with respect to xi here 1/root 2T times E power –xi square/4T, now you differentiate this you have a -2 xi/4T, okay, so 2 2 goes so you finally end up getting this is your derivative, so you have -1/2



root 2 times root T E power, that is going to be E power –xi square/4T, so from this you can see that Fourier transform of that is 1/root 2 pi you can get -infinity infinity X E power -X square T E power minus I xi X DX that is a Fourier transform of this part is equal to, you bring it to that side so root 2 root 2 goes so you have a root pi comes out this side divided by 2, and divided by xi comes down and you bring this divided by –I, minus minus goes you have –I, okay, so bring it this side is with this minus, so you have –I, root T is here this part and you have E power – xi square/4T, so this is exactly what you have, so you get –I/2 xi times root pi/T times E power –xi square/4T as your Fourier transform of X into E power -X square T as a function of xi, okay, as always T is positive, so what we do is, we use this here this Fourier transform of this, this is



your Fourier transform, so in the place of this if you put it you will see, you will end up what exactly your Laplace inversion, okay.

So I'll stop here, we will look at this, we will complete this in the next video along with other results, so that we can use application of, we will try to solve, we can use the Laplace transform to solve the initial boundary value problems for the heat equation in other domains, okay. We'll look into this boundary value problems in the next video. Thank you very much.

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