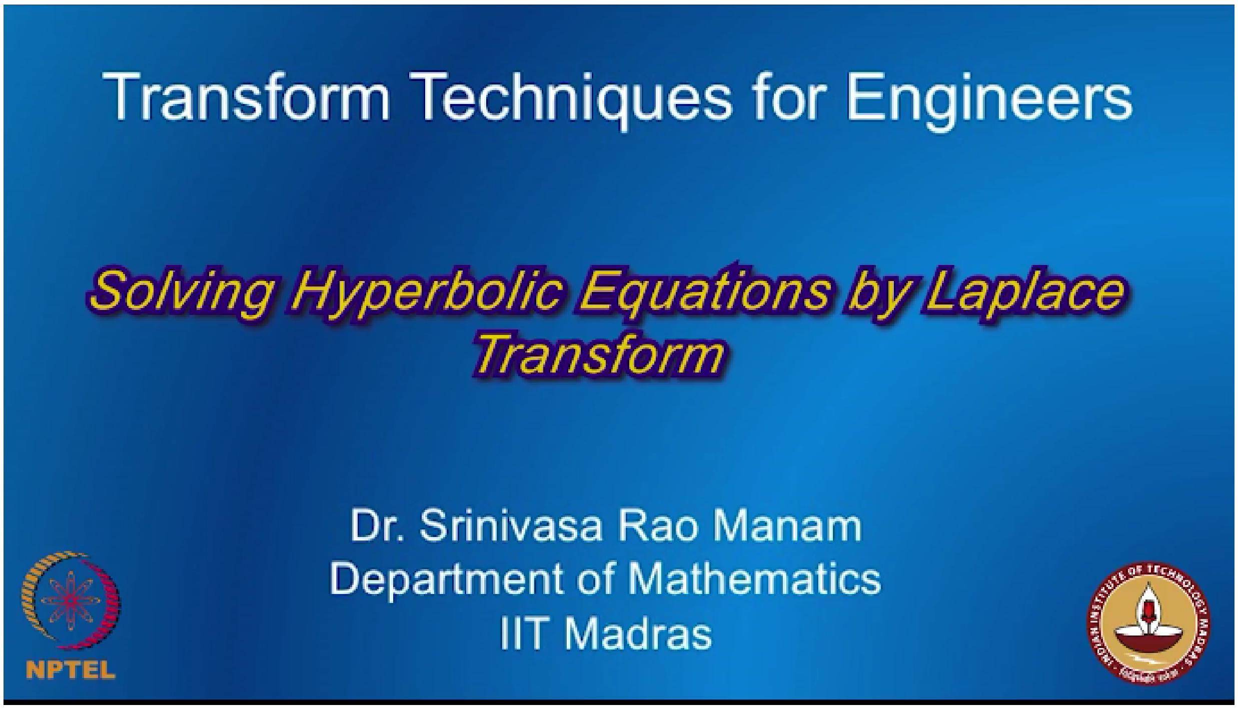
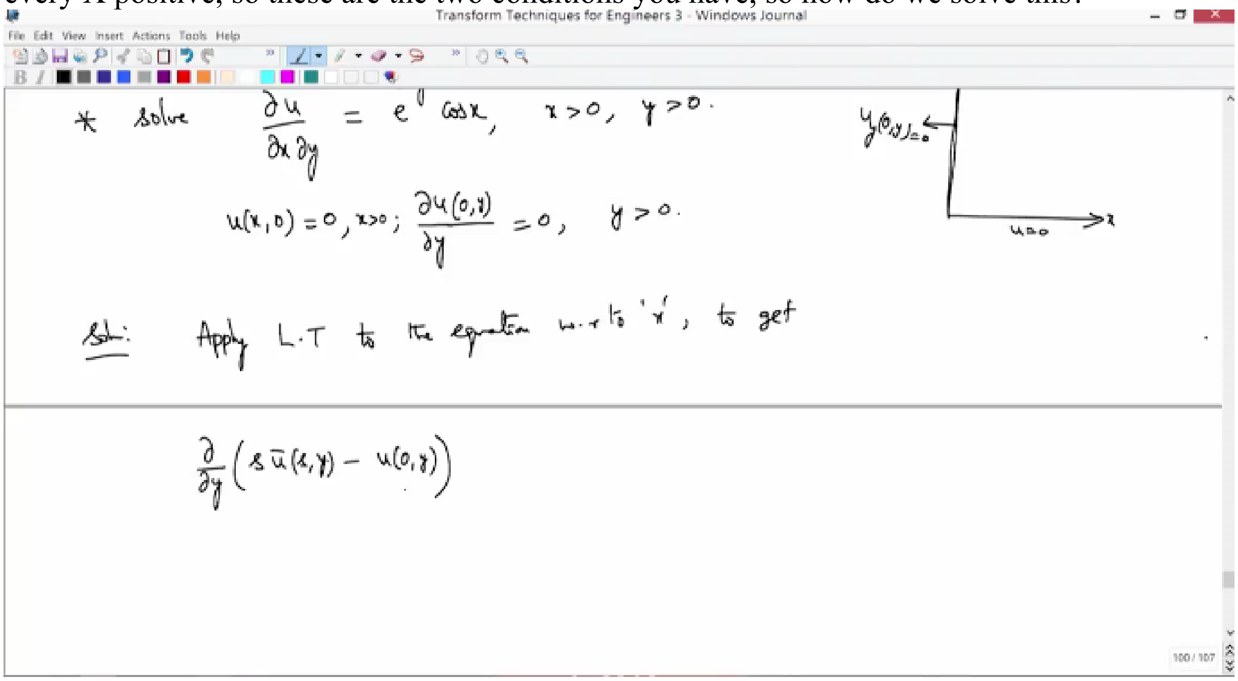
NPTEL
NPTEL ONLINE COURSE
Transform Techniques for Engineers
Solution Hyperbolic Equations by Laplace Transform
With
Dr. Srinivasa Rao Manam
Department of Mathematics
IIT Madras



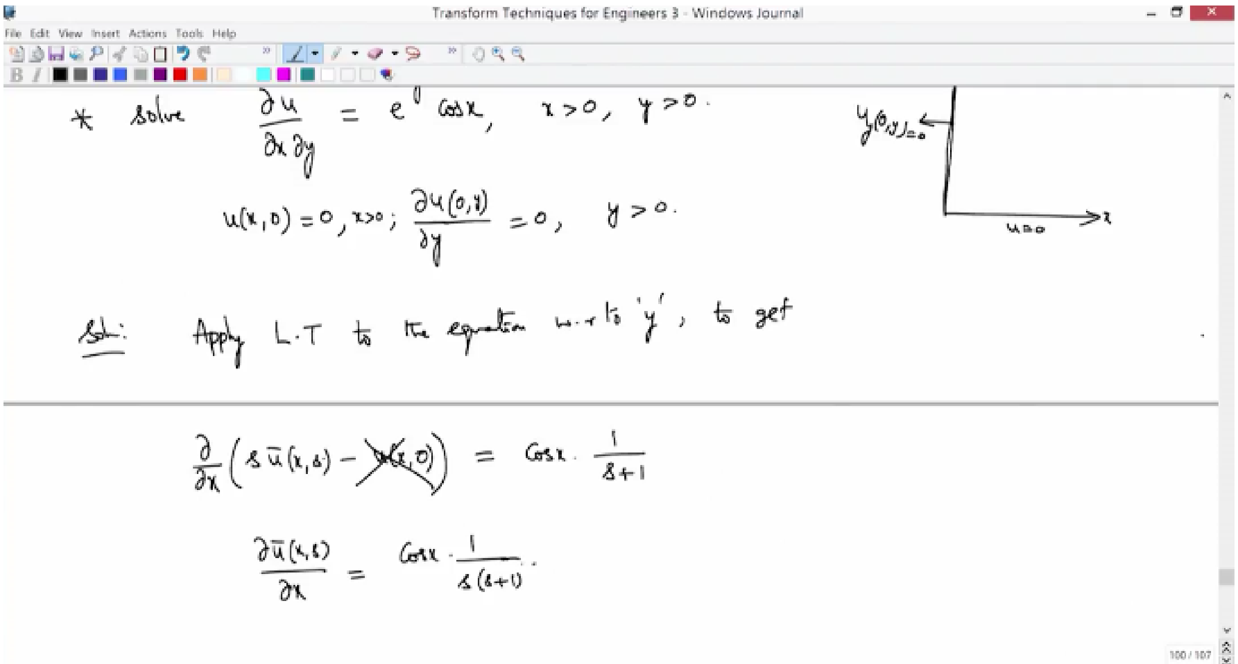

Welcome back on the discussion about the applications of Laplace transform. We were discussing about applications of wave equation or in general hyperbolic equations with initial and boundary conditions, we have done enough, many of the examples, we have done the examples few examples how to solve these boundary value problems, initial boundary value problems.

So before we move on to parabolic equations or specifically heat equation, initial boundary value problems for the heat equation, we'll do some more examples for hyperbolic type equation that is mixed derivative equal to some specific form if you have, if you have some specific equations but you have to use some kind of techniques, special technique to evaluate, let us just look at those examples before we move on to heat equation.

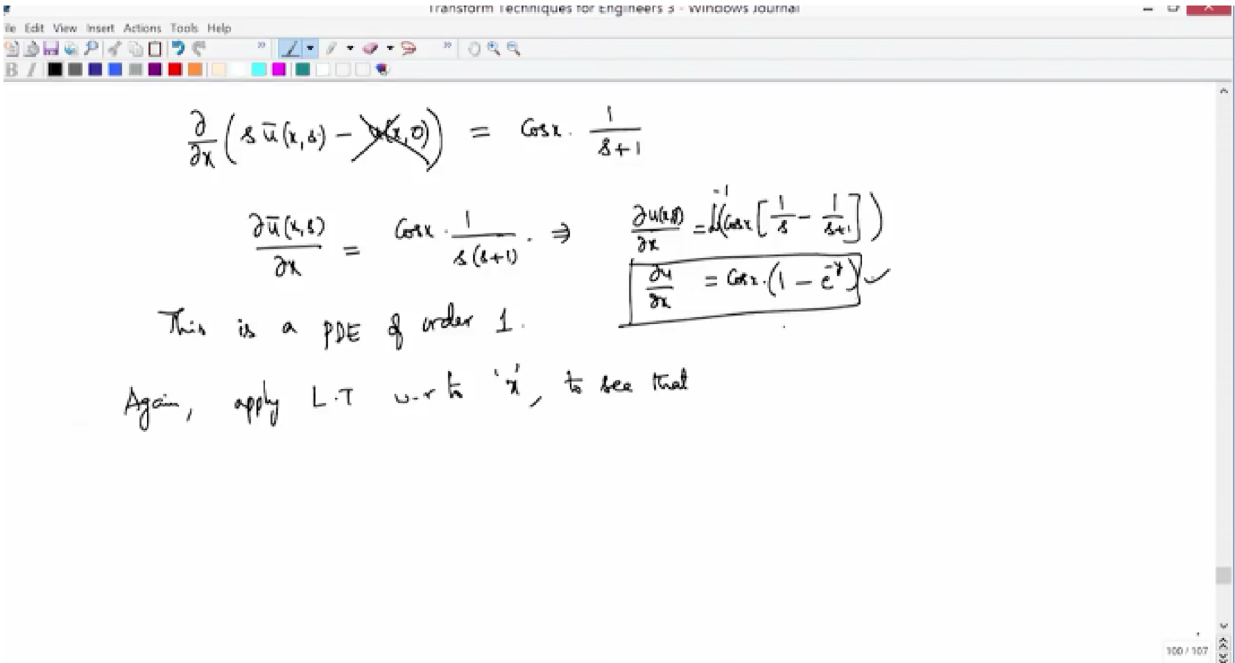
Let's solve PDE, that is a mixed derivative dou square U/dou X dou Y which is equal to E power -Y cos X, what is the special form that we can solve is actually some kind of things like this, when you have, when you have this mixed derivative we know that this is hyperbolic, this is already in this typical form of a hyperbolic equation, and you have a right hand side that is non-homogeneous term, and these boundary conditions are, so what is the domain, so this is basically without by default so you can think of X is positive, Y is positive, so that you can apply your Laplace transform, so the boundary conditions are our initial conditions whatever so conditions are U(x,0) is 0, so look at the domain this is your domain at X equal to, so this is X, Y, and you have X = 0 that is this line, and Y = 0 is this line, so Y = 0 and you have, so U is 0 here, another condition is the derivative here, so the normal derivative that is UY at X = 0,Y = 0, UY at X equal to, that is actually you say dou U/dou Y(0,y) = 0 for every Y positive, here for every X positive, so these are the two conditions you have, so how do we solve this?



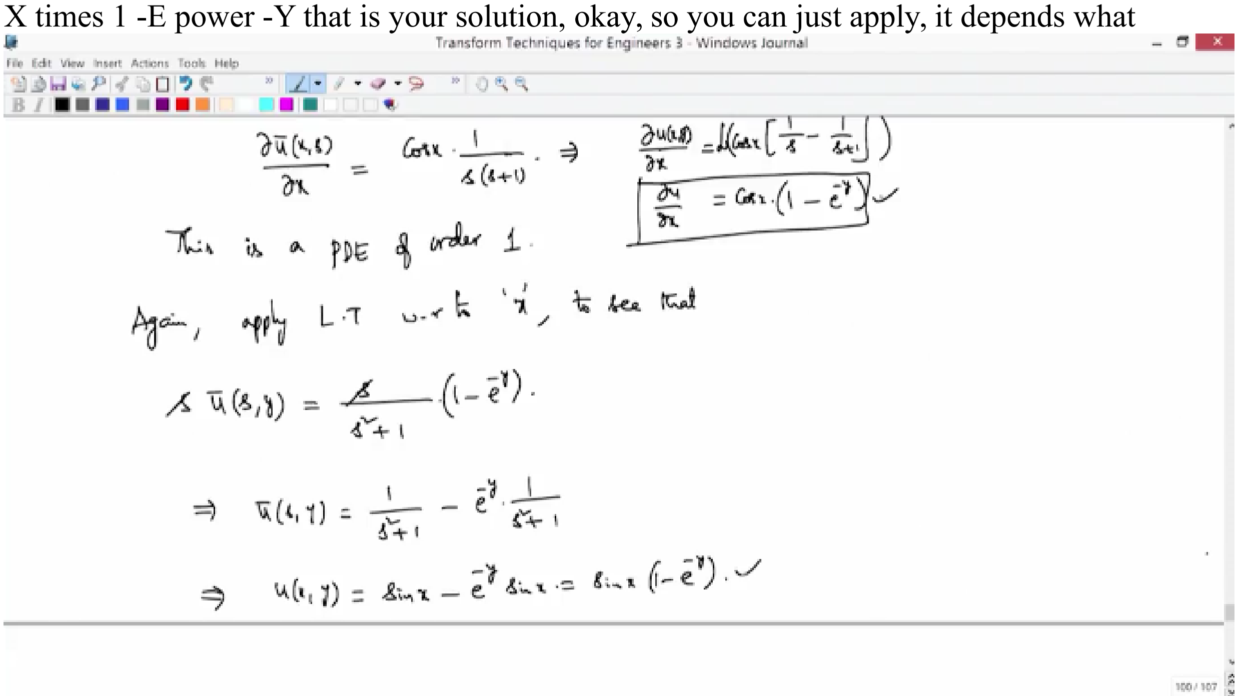

So let's use the same technique that we adopted earlier, so just apply a Laplace transform of U with respect to X variable let us say, so because of cos, cos we know what is its Laplace transform, so we can use this lets apply Laplace transform to the equation with respect to X to get, what we get is you apply when you apply you get dou/dou Y of S U bar of, you're doing with respect to X, so S, Y – U(x,0) so 0,Y so that is 0,Y we don't know really, right, this is not known, so okay let's have this, so if you directly apply Laplace transform with respect to X so you get this one but this is not known, so neither of these conditions we know, okay, but instead you can apply with respect to Y if you apply you see that is, what is the change? If you apply this is going to be X this is going to be X,S and this is going to be X,0 at Y = 0, so this we know so we can use this one, so this one I used and this is equal to E power -Y so if I'm doing with respect to Y this is going to be cos X times E power -Y is simply 1/S+1, so what is this one? So



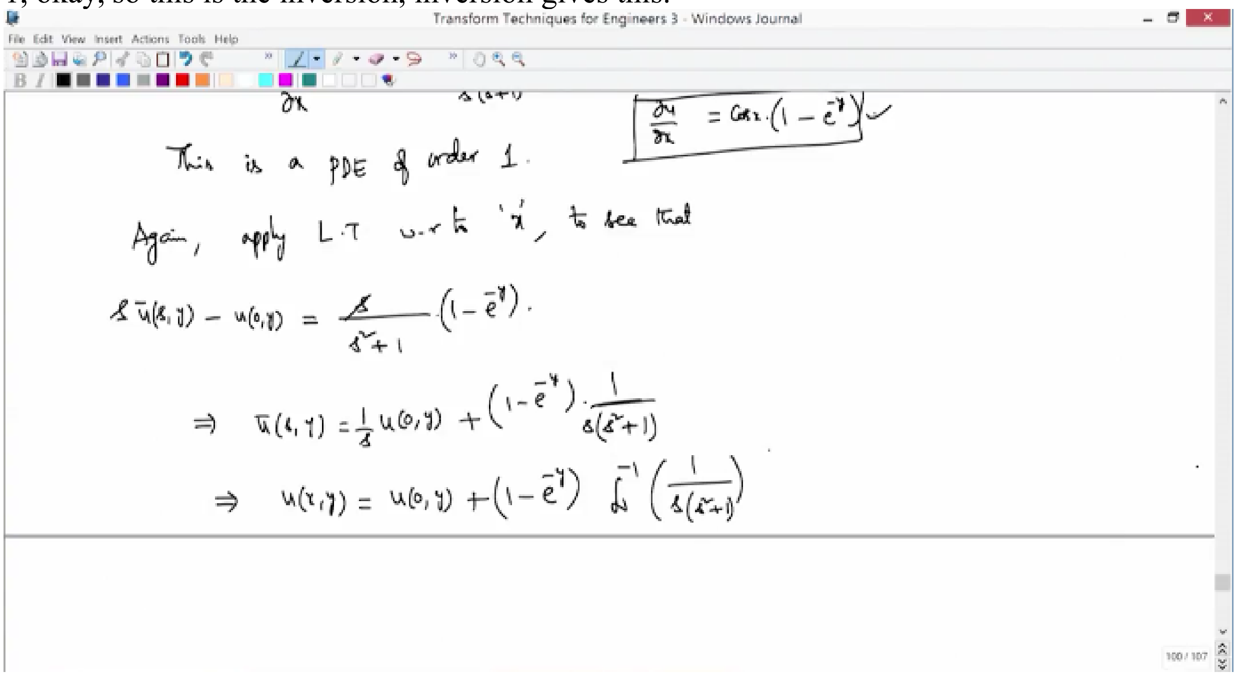
you have dou U bar(x,s)/dou X = cos X 1/S(s+1) this is what you have, how do we solve this? This one to solve this, this is a partial differential equation this was you can directly integrate or are once again you can apply now, so you can apply one more, so this is a PDE of order one, first order PDE, okay, so you apply again one more time, apply Laplace transform with respect to X now, with respect to X to see that, let's see what we can get, so if you do apply this you get, before you apply this Laplace transform we have to invert this, so if you get this inverse, so let's invert this if you invert this for example so dou, if you invert this and what you get is dou U/dou X (x,y) which is equal to, this is going to be inversion of this will be cos X times 1/S - 1/S+1, so this inverse Laplace inversion of this, this is going to be cos X times 1, cos X 1-E power –Y, because this is with respect to Y you have done so this is what you have, so you have dou U/dou X, so this is your first order PDE, so this is your first order PDE this you solve again with applying Laplace transform.




So if you apply this now with respect to X so you get US times U bar(s,y) this is equal to cos X will be S divided by S square + 1 times 1 -E power -1, so this implies so S S goes, so you get U bar(s,y) is 1/S square + 1 - E power -Y times 1/S square + 1, so you invert it, so inversion will give you have U(x,y) that is going to be sine X -E power -Y again sine X, so this is actually sine X times 1 -E power -Y that is your solution, okay, so you can just apply, it depends what

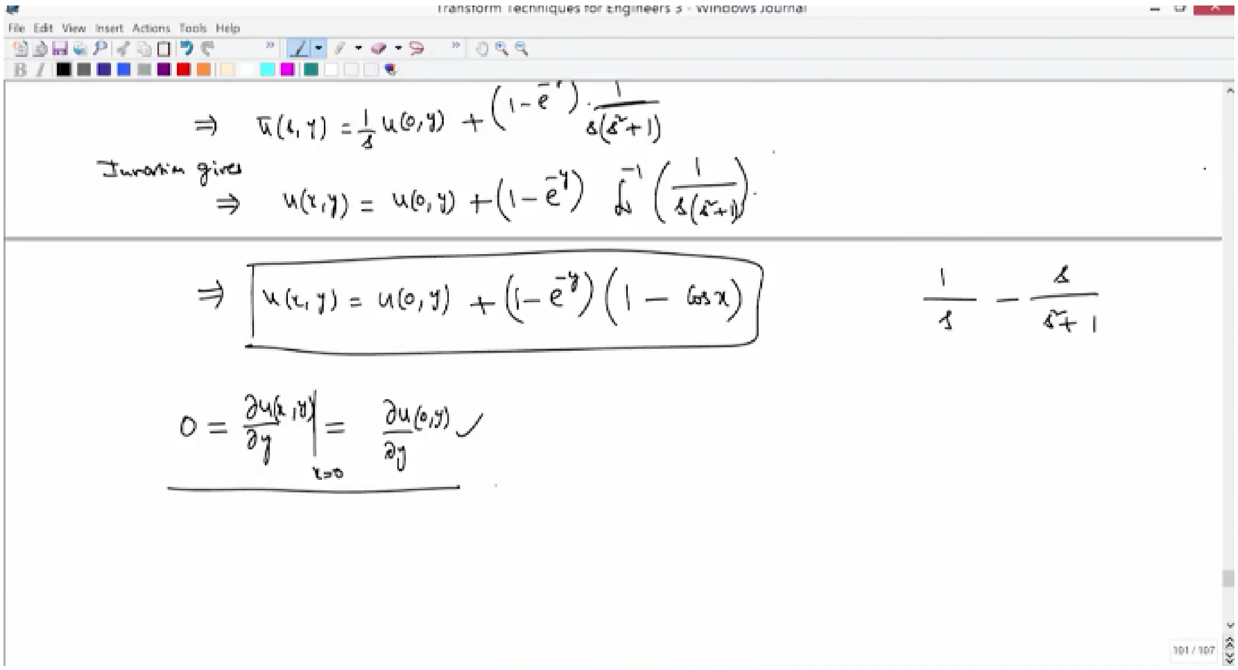


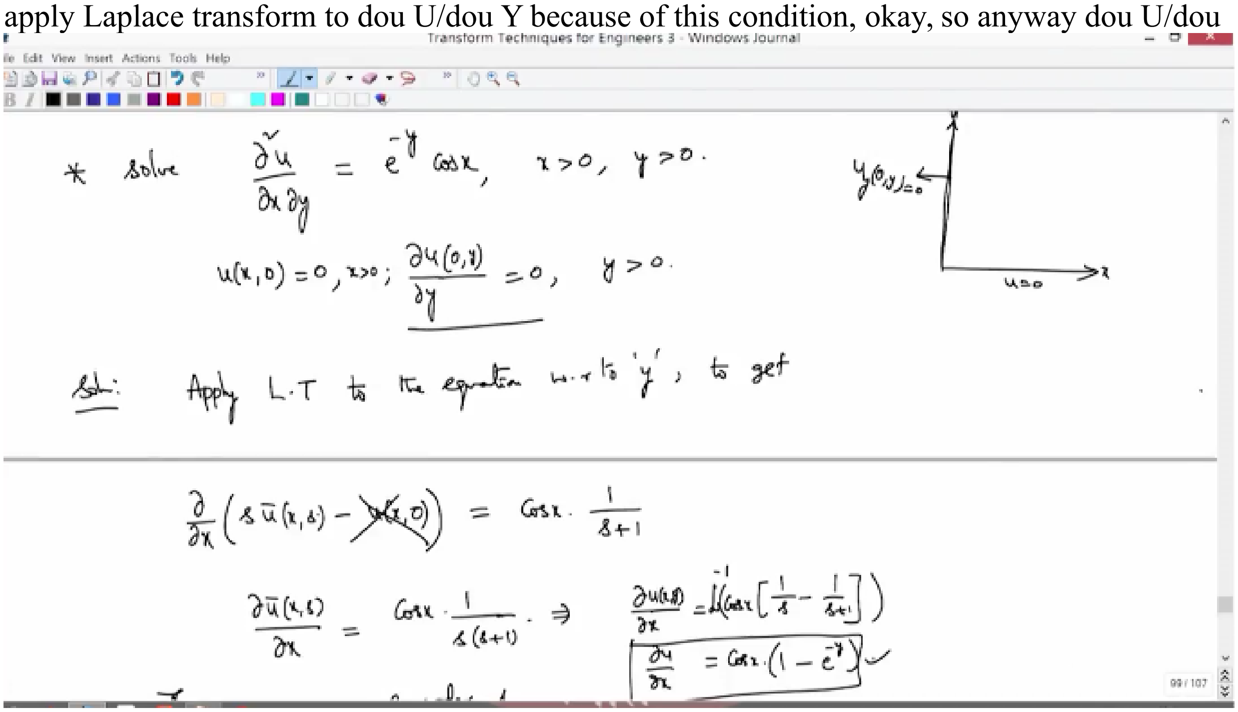

you so, we have not used this one right so far, we have not used this boundary condition, so what did we use? So we have not used this one dou U/dou X so we got this solution that satisfies this equation, and then but we used only this one and we have not used, no no I think we made a mistake, when we apply the Laplace transform and what you have is the left hand side is S times U(s,y) – U(0,y) so this is what we don't know, still we don't know, so we know only its derivative okay, so we actually got the answer right, but this is where we made wrong, so what happens to this? This we don't know, okay, this is what we don't know let's call this some function of Y, okay, so let's call this some function of Y so that you have S, this is going to be, so let's do this one, so U bar (s,y) is equal to let's call this U(0,y) this is still unknown + 1 - E power –Y times 1/1 + S square or S square + 1, so you divide this you have 1/S and this you have 1/S, so here you get Laplace inversion to see that U(x,y) = U(0,y) + this is going to be 1 - E power -Y times, this is going to be 1/S, so Laplace inversion of 1/S times 1 + S, or S square + 1, okay, so this is the inversion, inversion gives this.




So what is the inversion? We have not seen this type of inversion right, so 1 by, if you use the Laplace, if you use this 1/S square + 1, S square, so you have S - S square and actually S into S square + 1 and you have S square + 1 – S square so you have this, this is what you have so you can replace this with this so that you have U(x,y) is U(0,y) +1 -E power -Y times, the L inversion will be 1/S that is 1-, this is going to be cos, you’re doing with respect to X so you have cos X. So this Laplace inversion is with respect to X, so when you invert it you will get back every X variable.

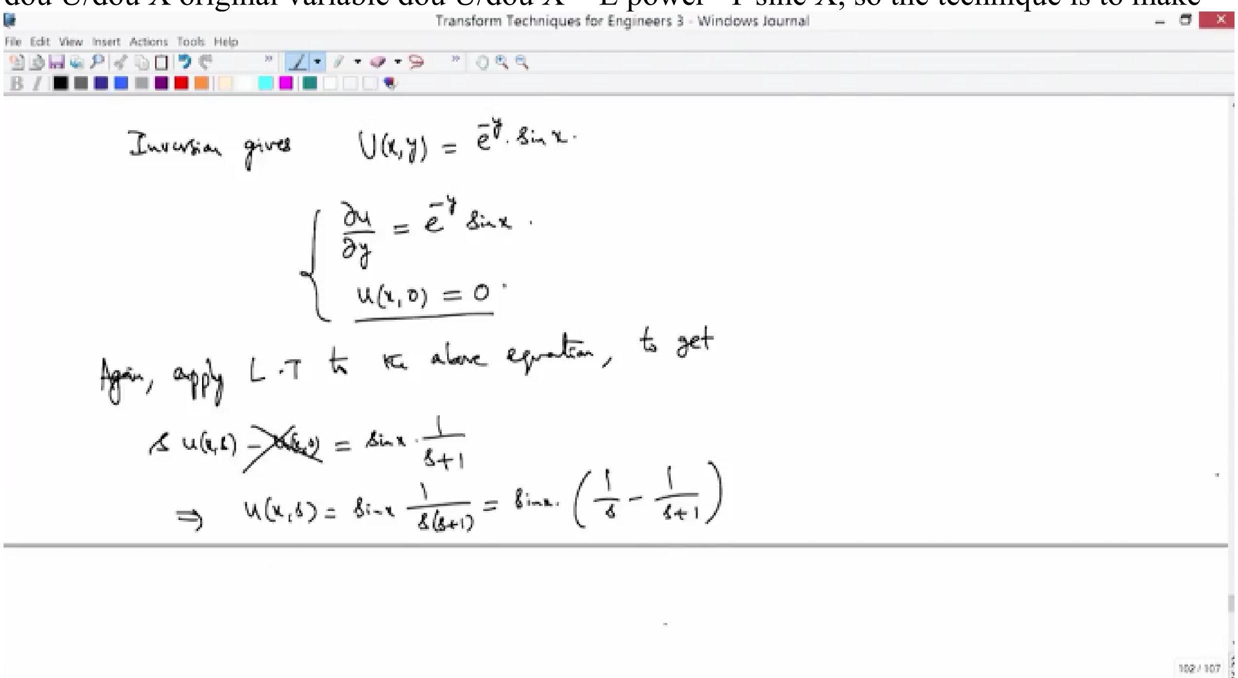
So what is now you apply your, what is the solution but with this still not known here, okay, so you apply, you have to apply this other condition dou U/dou Y(0,y) = 0, dou U/dou Y(0,y) so this is the condition I apply, so if you apply here so you get you have that dou U/dou Y(0,y), okay, so this is anyway 0 + when you apply X = 0 that is going to be 0 anyway, so this means dou U/dou Y = 0, okay, so this is at X = 0, and this is anyway right hand side here I applied, when you apply with respect to X dou U/dou Y and we simply, dou U/dou Y you have to apply, so when you apply the first part will be 0 other part will be +E power -Y times 1 -cos X, when you put X = 0 this is going to be 0 so it doesn't matter, so this part is this, we will not contribute finally you see that it's going to be, when you apply is actually 0, so both sides so that means the dou U/dou Y(0,y) you're not getting anything, so you should get back your this thing that



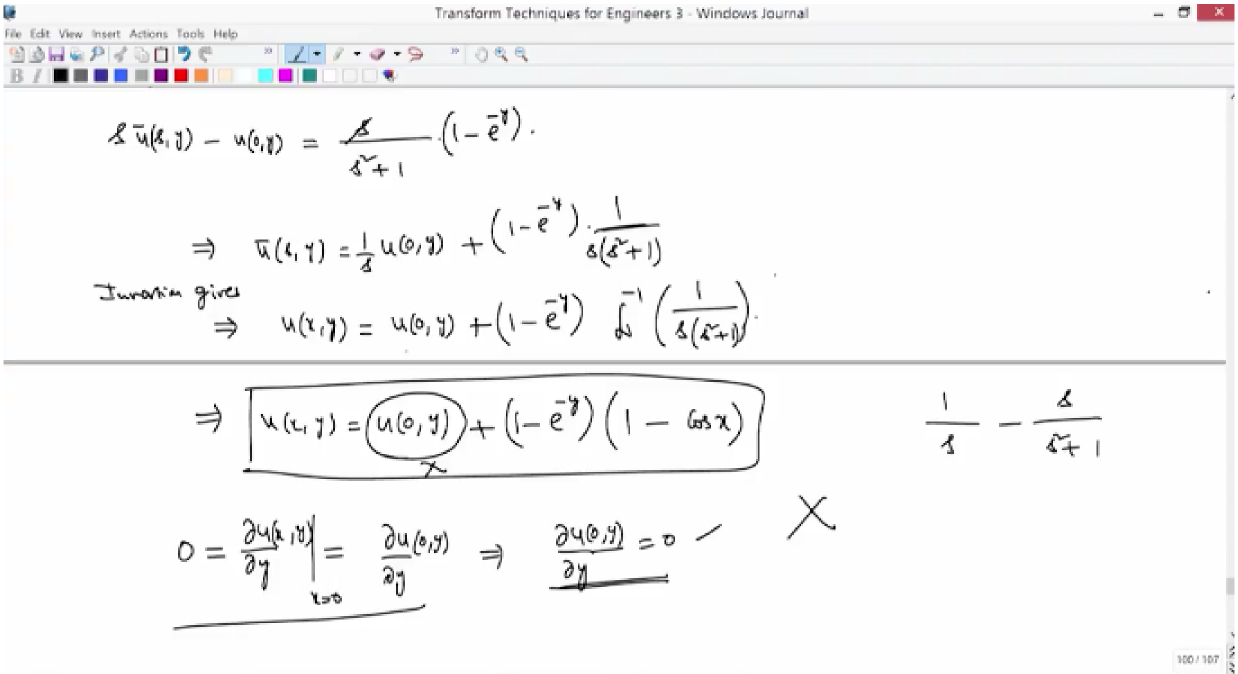
you will not get, dou U/dou Y from which if you apply U(0,y), so you're not gaining anything so okay if you apply this boundary condition to get back your U(0,y) I still don't get this, so this implies still you get dou U/dou Y(0,y) = 0 they're only the initial condition, not getting your, what is this unknown, this is what you need, this is not able to get, okay from this, so this is not, so this technique straightforward technique will not help so this reason I will give you special way of solving this, what we do is instead of applying Laplace transform to the variable Y, I apply Laplace transform to dou U/dou Y because of this condition, okay, so anyway dou U/dou


Y only here, so you don't have this side, okay. So this this usual way of applying Laplace transform to X and Y variable separately, and you are not able to get, you're not able to make use of these boundary conditions, both the boundary conditions, so it fails.

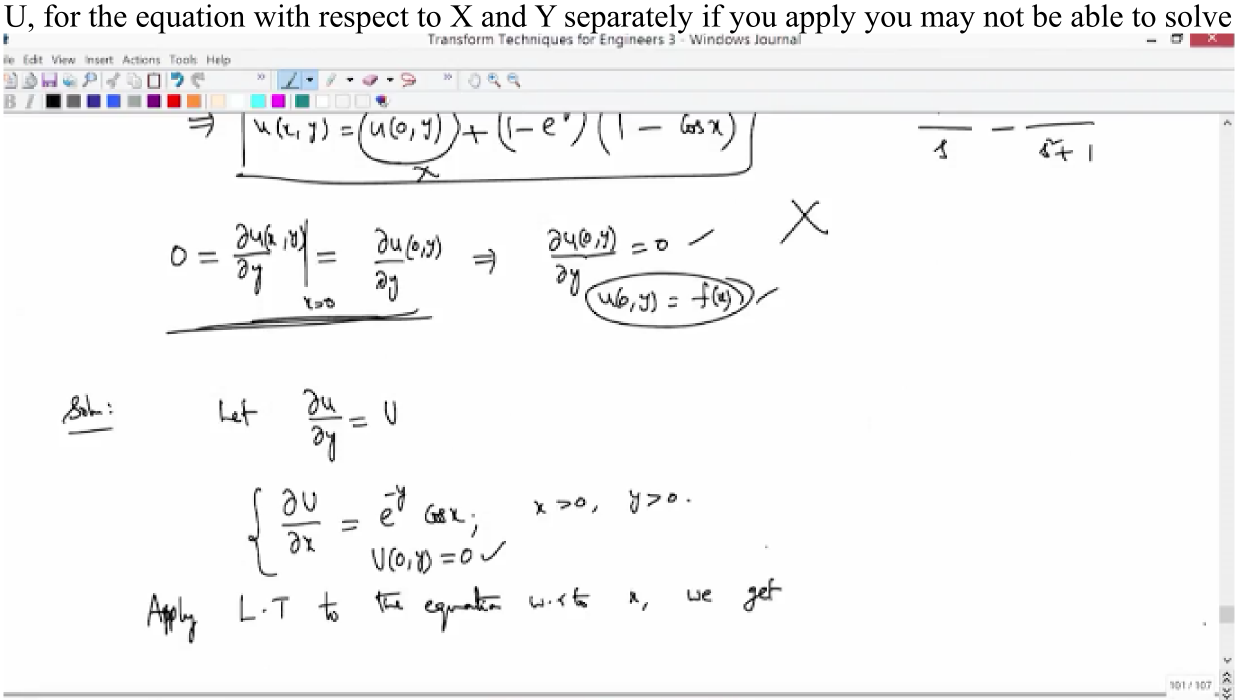
So let's solve by some other way, some other way is so let me use dou U/dou Y equal to some unknown function U, capital U, okay, so then your equation becomes dou U/dou X this is equal to E power -Y cos X.
So now you apply Laplace transform, apply the Laplace transform to the above equation, so anyway X is still positive Y is positive, and because of the boundary condition, so what is your boundary conditions? Dou U/dou Y(0,Y), so dou U/dou Y(0,Y) is U(0,y) that is dou U/dou Y(0,Y) is 0, so this is what you're going to solve first, okay.
So if you apply the Laplace transform to this equation with respect to X, we get SU capital U of, with respect to S, X becomes S and you keep Y = E power –Y cos X will be S divided by S square + 1, so this means S S goes what you have is, sorry you have, left hand side S times capital U(s,y) - capital U(0,y) so this is 0 because of this condition, so you have S S also goes you get finally U(s,y) = E power – Y/S square + 1 so this gives me, the inversion will give me, inversion gives a capital U(x,y) which is E power -Y times sine X, okay, but what is U? U is dou U/dou X original variable dou U/dou X = E power -Y sine X, so the technique is to make




the second-order equation into, a partial differential equation into first order partial differential equation, so now this is the condition this equation along with the first boundary condition that is U(0), U(x,0) is 0 so this is the condition we use now, U(x,0) is 0, capital U is dou U/dou Y so you have dou U/dou Y, so U(x) is so this you try to solve again by Laplace transform technique, so again apply Laplace transform to the equation, to the above equation to get U S times, now small u times with respect to, why you are doing this Laplace transform, so you have X,S, Y becomes S this is equal to sine X times 1/S+1, and then this will be U(x,s) – U(x,0) that is what is boundary condition that becomes 0, so you have U(x,s) will be sine X times 1/S(s+1), so this is sine X times 1/S – 1/S+1, so inversion will give me, inversion gives now actual solution U(x,y) which is equal to sine X times 1/S is 1 - E power –Y, so this is actually your solution, okay, so this you could get, you could make use of both the boundary conditions and so that, that is actual solution.

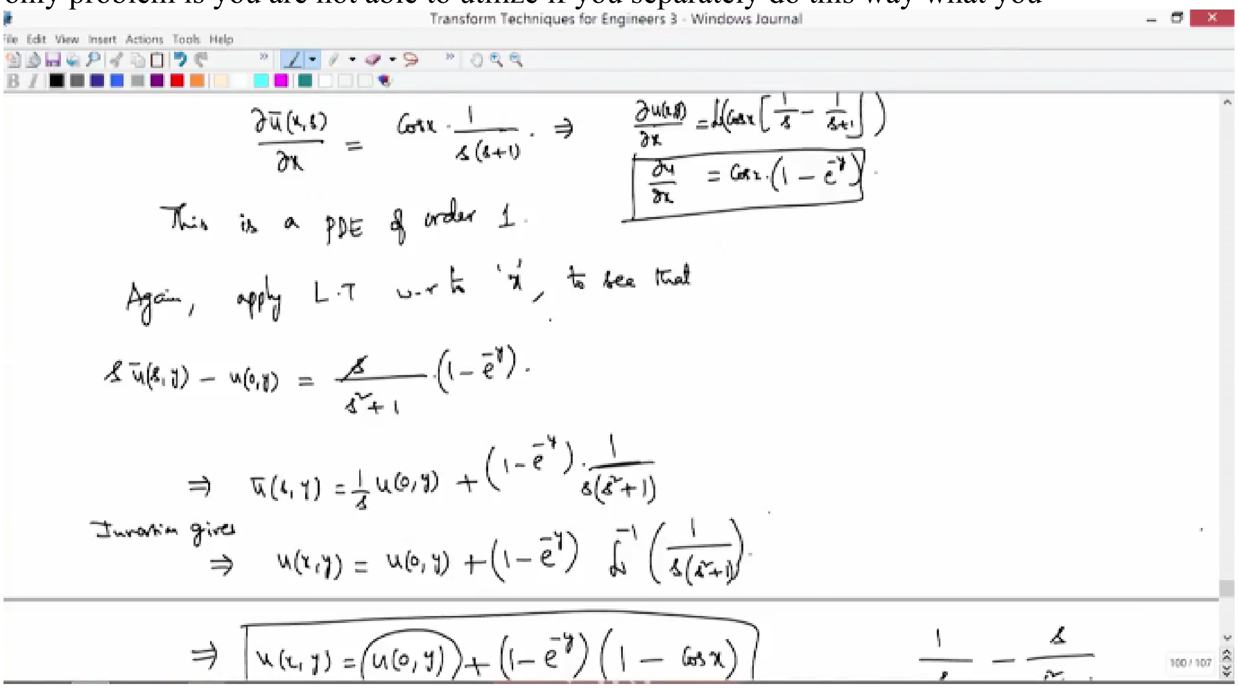



So here we are not able to get rid of this unknown function, so U(x,y) we got as an unknown function U(0,y) that even after applying this boundary condition we are not able to get rid of it, we're not able to find that U(0,y), so from this if you actually integrate you may get your 0 of Y as simply some function of X that you still don't know what is this function, F is arbitrary function, okay, so this method fails so if you directly use the Laplace transform for that variable U, for the equation with respect to X and Y separately if you apply you may not be able to solve

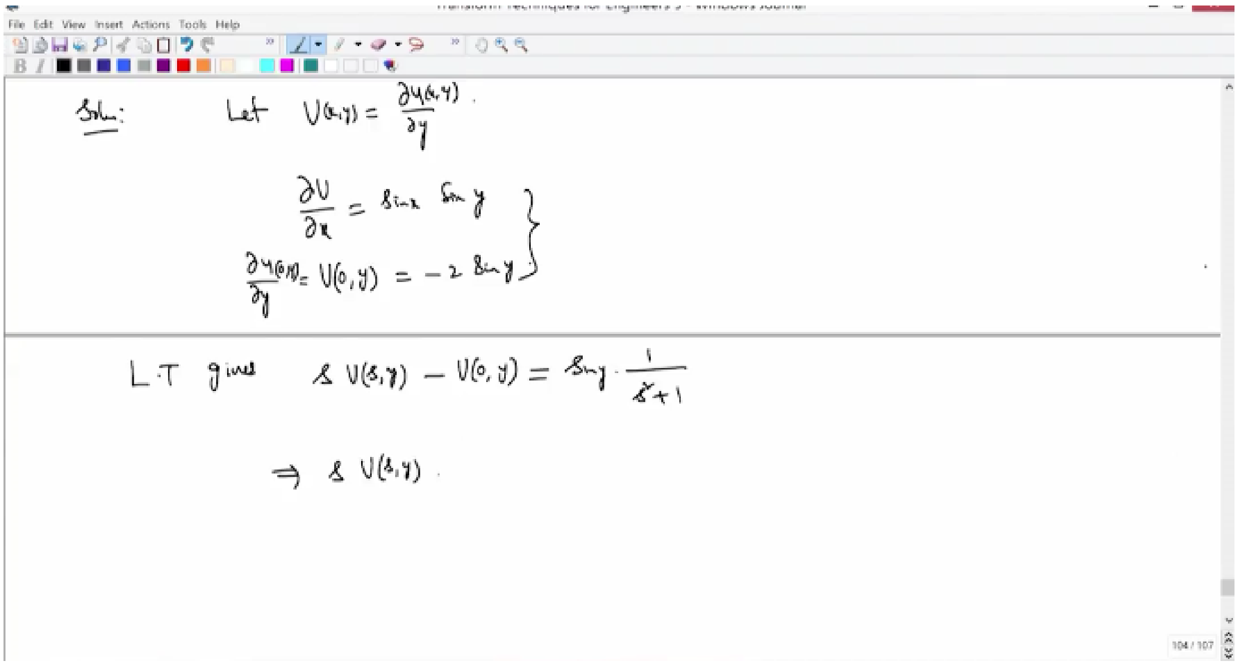



the problem because of, because you're not able to utilize the boundary conditions, given boundary conditions, so because of the given boundary conditions you may have to use this small technique, make this second-order equation into first order equations, and with one boundary condition you solve that. And then once you know this one, once you solve this you

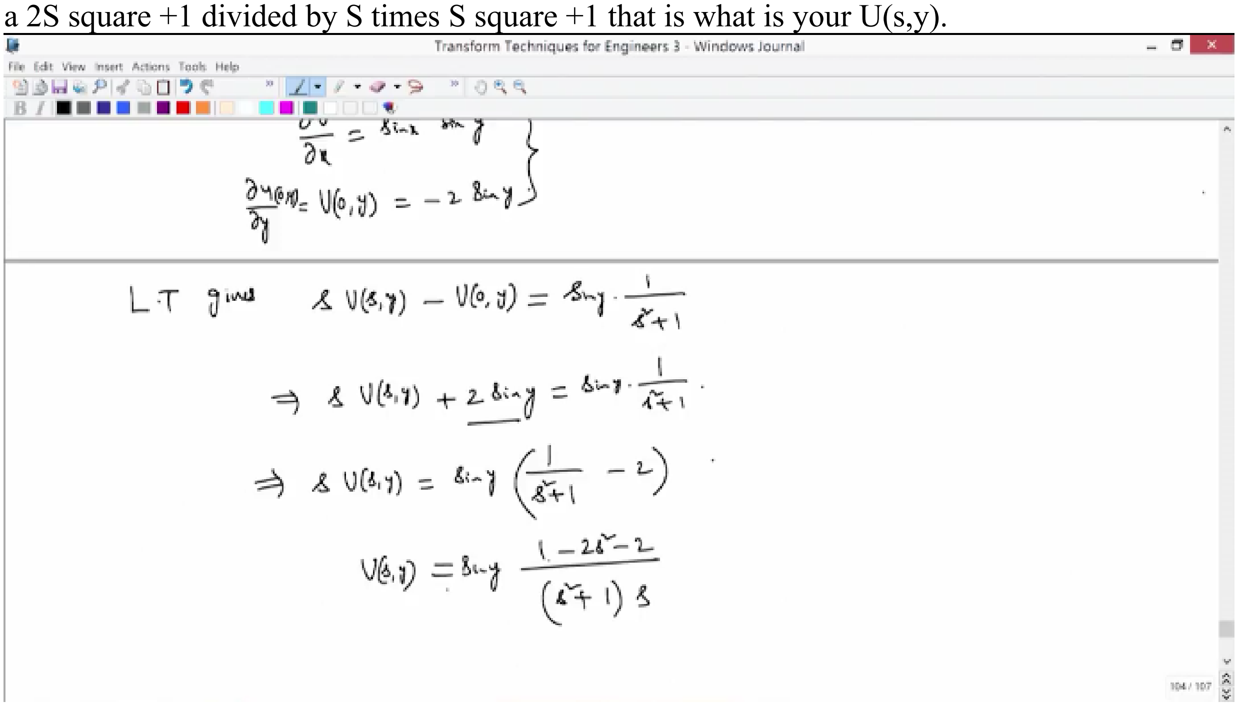
can get other first-order PDE with the other boundary condition, so with this technique you can solve this equation, directly you will not be able to do.
So let's solve one more problem, let's solve dou square U/dou X dou Y = sine X sine Y with the boundary conditions U(x,0) is 1 + cos X, UY(0,y) = -2 sine Y, again because of the boundary conditions so if you have U(x,0) and U(0,y) then you will be able to solve directly like earlier technique, so apply first with respect to X, so how do we solve this one? With respect to Y we calculated, we apply the Laplace transform and we use the first condition and once you solve this you get your PDE as this, first order PDE, even here you are getting the first order PDE, only problem is you are not able to utilize if you separately do this way what you




got here once you get this you apply the Laplace transform with respect to X, you'll get 0,Y, so you don't have UY(0,Y), so it's not coming by applying the Laplace transform, so you will not be able to utilize that boundary condition that will still remain there.
So if you have boundary conditions are not U(x,0) and U(0,y) if you are given then you can apply the Laplace transform of U with respect to X first and then apply directly, okay, and then you utilize, and again whatever the resulting the solution, once you invert it you apply the Laplace transform then you can utilize this one, otherwise if you have Y derivative here then this technique you have to follow so that you have to use, you have to convert this into right away convert this into first order PDE by allowing some U(x,y) as capital U(x,y) as dou U/dou Y(x,y), by choosing this you get, your equation becomes dou U/dou X = sine X sine Y, okay, so this one with U(0,Y) is now -2 sine Y because of this second condition, so this you solve it first by applying  Laplace transform, so Laplace transform gives S times capital U of, with respect to X I'm doing so you have S,Y – U(0,y), U,0 is nothing but dou U/dou Y(0,y) okay, so that is this, equal to with respect to X you are doing so you have sine Y times, sine X will be 1/1+S square + 1 Laplace transform of sine X is this, so you have this, this condition goes, okay, sorry this is S times capital U(s,y) minus minus plus so we have +2 sine Y = sine Y times 1/S square

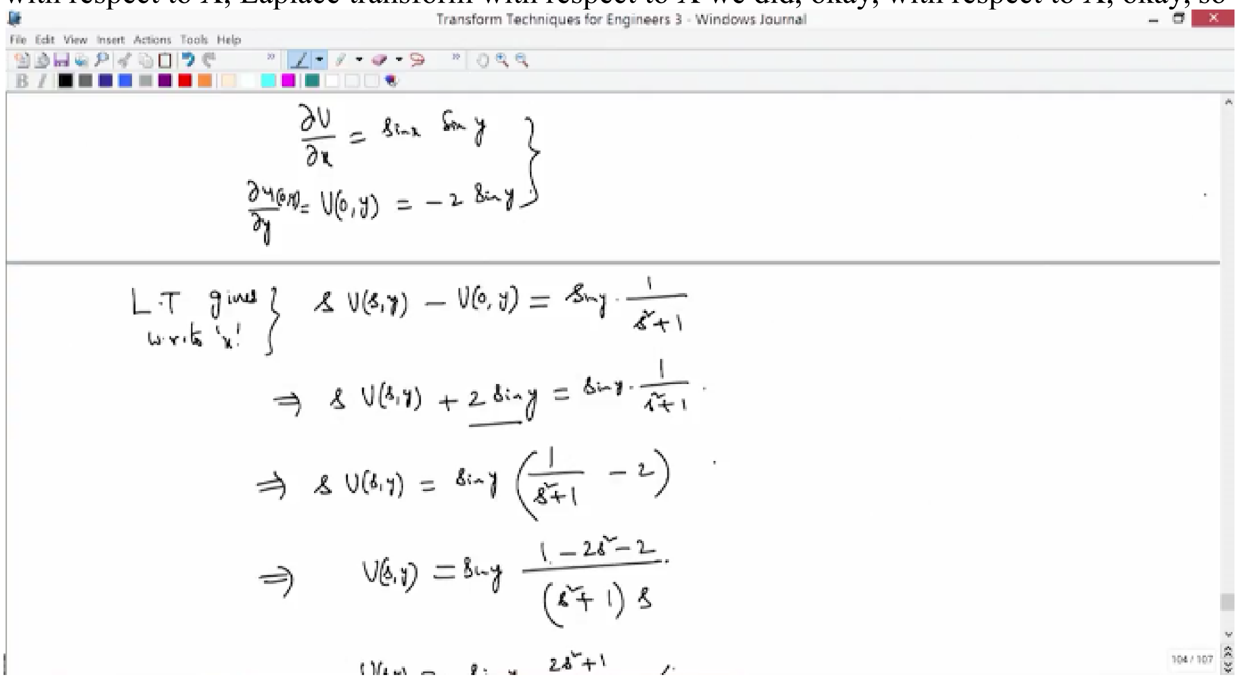


+ 1, so if you apply now S times U(s,y) you bring this that side you'll get sine Y is common, 1/S square + 1 – 2, so you have a -2 S square - 2 divided by this, so you have finally sine Y times, we’ll do it carefully so you have 1/S square + 1 -2 sine Y that is 2, so this is sine Y, sine square+1 1-2S square -2 and you have division with S, so this will give me U(s,y), S I divided both sides, okay, so this is equal to sine Y times minus, minus you can bring it out and you have a 2S square +1 divided by S times S square +1 that is what is your U(s,y).



This you invert, if you invert this inversion will give dou U by, so inversion is dou U/dou Y is equal to, so -sine Y this you can rewrite like 1/S – S/1+S square, so what is this one? 1+S square – S square, so you have 1, 1 by that 2S squares, so let me write this 2S square/S(S square+1) is 1 + 1/S(S square+1) so this is a Laplace inversion gives Laplace inversion of this, so this is going to be -sine Y times Laplace inversion of, so the first term 2S square by, so S S goes so this is going to be Laplace inversion of 2/S square+1, so 2 you can bring it out anyway so + Laplace inversion of here is 1/S – 1/S square+1 –S, so that you have this, this term is actually sum of these two.

So this is nothing but -sine Y, 2 comes out because of this and you have sine X you're doing with respect to X, Laplace transform with respect to X we did, okay, with respect to X, okay, so



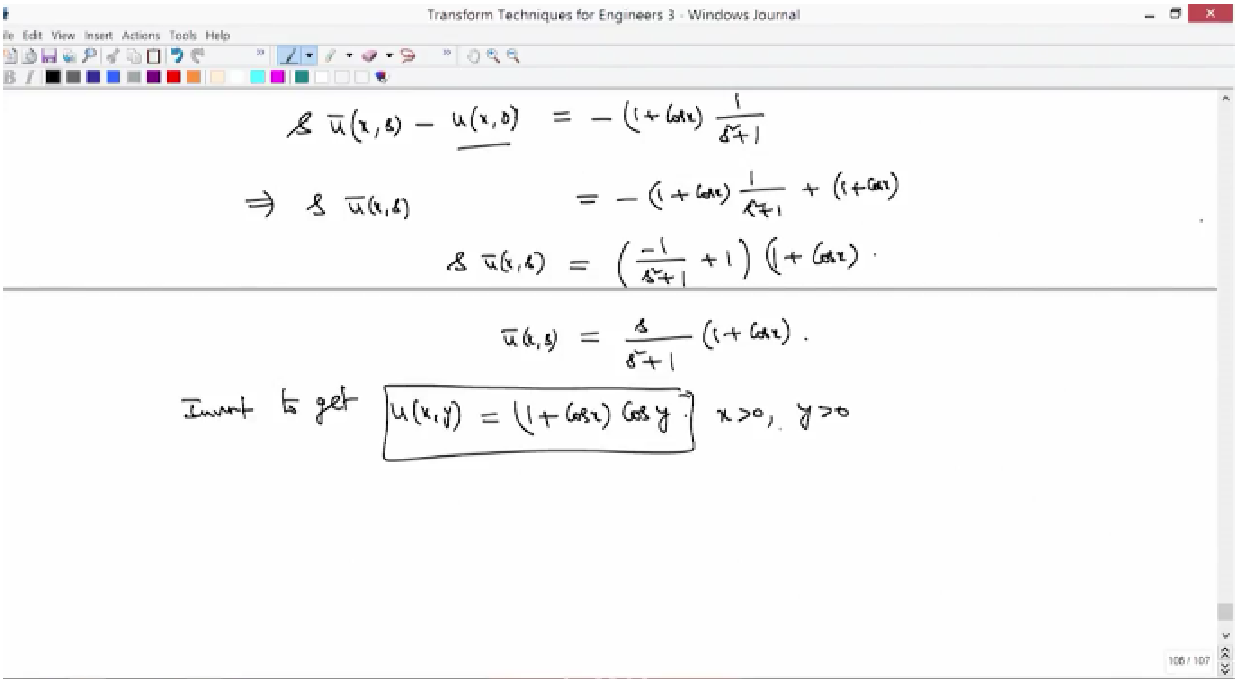

because of that you have sine X, inversion will give you sine X, this is sine X and okay the first term, first term is this, second term will be -sine Y times 1 - cos X, cos X inversion is S/S square+1 so this is what is your equation, so dou U/dou Y which is equal to -2 sine Y sine X - sine Y + sine Y cos X, we’re making a mistake here, so when we choose -2S square -2, so this is going to be -1, minus minus plus yeah fine, 2S square by this S S goes, so you have 2S by this, oh you have S here, you have S here, so 2 comes out so 2S, this is 2S so let me change this, so you have –sine Y, so this is 2 times S/S square is cos X and then here plus and this is this term with –sine Y, Laplace inversion of 1/S is –sine Y, this is going to be + sine Y the last term, this is going to be cos X, so this is 2 and this is 1 so that makes it sine Y cos X – sine Y, okay, so there's no term like this, so you have finally -2 sine Y S square + 1 - S square, that is going to be 1, 1 over that that is okay, yeah this is your first order PDE.

$$= -\sin y \, \mathcal{L}^{-1}\left[\frac{2s}{s^2+1} + \frac{1}{s} - \frac{s}{s^2+1}\right]$$

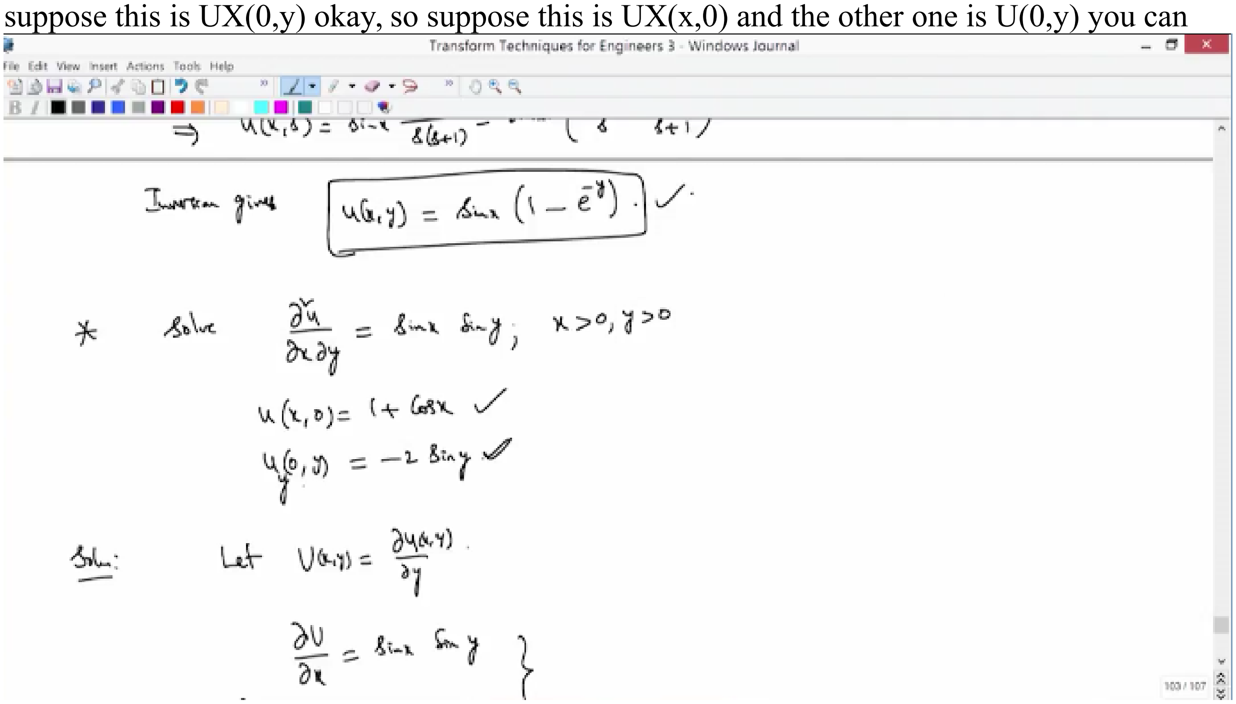
$$= -\underline{\sin y \cdot 2 \cdot \cos x} - \sin y + \underline{\sin y \cos x}$$

$$\frac{\partial u}{\partial y} = -\sin y \cos x - \sin y \checkmark$$

Now you can utilize other boundary condition that is U(x,0), if you apply with respect to X, so application of Laplace transform with respect to Y gives S times U bar of, did I use U bar here I think I have not done, so you write U bar, this is U bar, so inversion will give you this, this is U bar inversion actually gives U(x,y) that is actually dou U/dou Y. So when you write this U bar of, with respect to Y you are doing so you have X as it is, Y becomes S –U(x,0) and this is what is given as about first boundary condition that is 1+cos X, and this is going to be 1+cos X and this is equal to, now this is with respect to Y if you do -cos X, with respect to sine Y 1/S square+1, and you have a sine Y that is anyway, so let me use minus with respect to Y you are doing, so you have 1+cos X times sine Y, so sine Y if you do a Laplace transform that gives me 1/S square+1, so this implies S times U bar(x,s) this is going to be 1-cos X okay, this is equal to -1 + cos X/1+S square + 1, bring this the other side so that you have 1+ cos X here, so you have finally S U bar(x,s) simply S square+1, 1/S square+1 - this +1 times 1+cos X, so this is going to be S square/S square+1 times 1 + cos X is my S U bar(x,s), again S S goes, so if I remove S 1 goes, so you have this so this gives me, now inversion invert, invert to get U(x,y) this actual solution as 1+cos X times this is actually cos Y, because you're doing with respect to Y, Laplace transform with respect to Y so when I take the inversion that is a function of Y, so this is the actual solution that you're looking for X positive, Y positive.

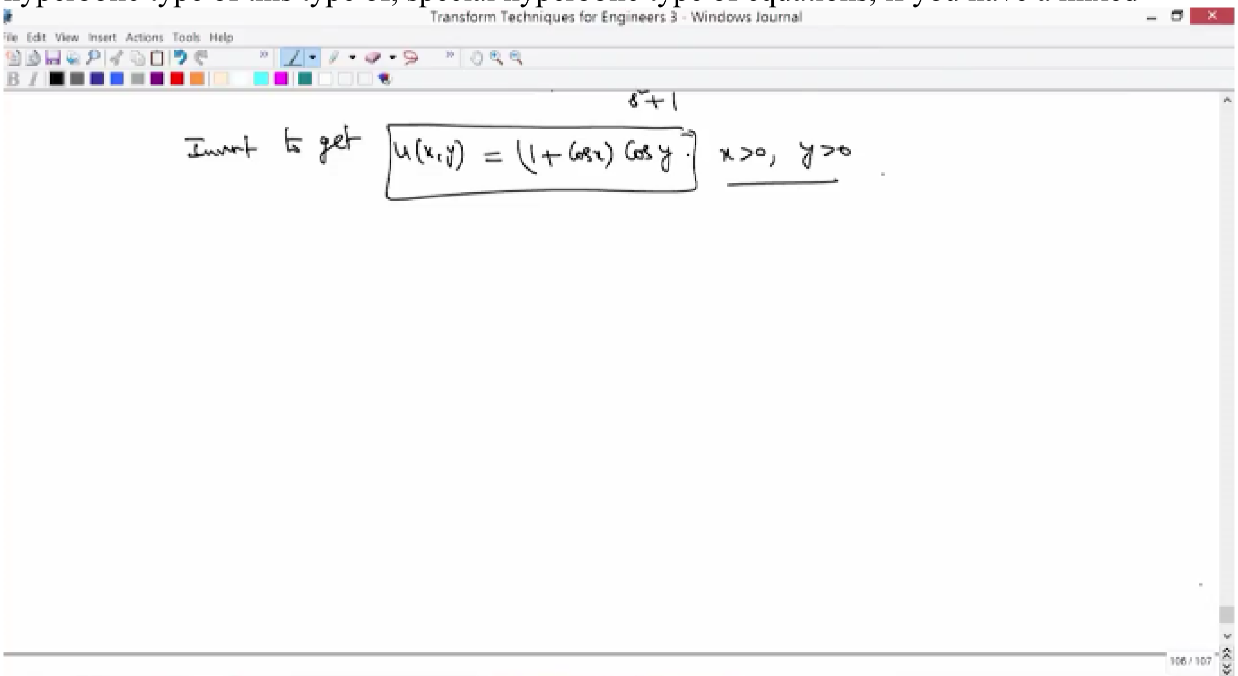



So this is the boundary value problem that is given as this, for in this domain X positive Y positive, but your boundary conditions are 1 is UY, other one is U, if 1 is UX other one is U, suppose this is UX(0,y) okay, so suppose this is UX(x,0) and the other one is U(0,y) you can



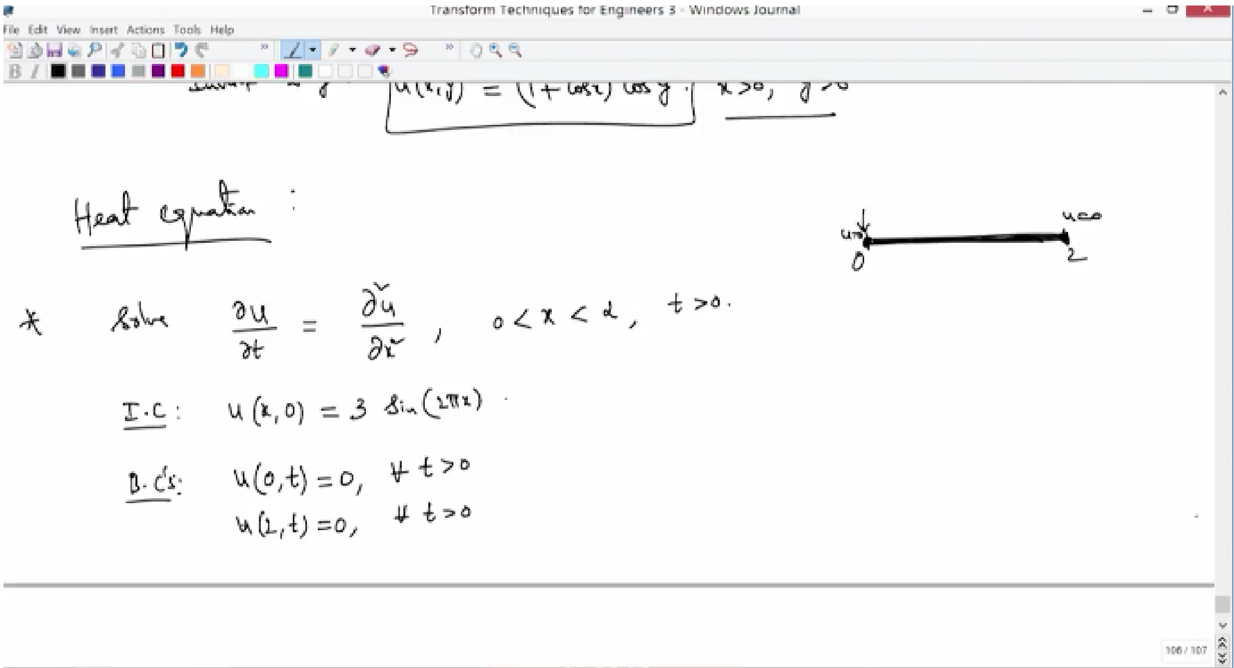

follow the same technique, okay, if you have a mixed boundary conditions not, this is not mixed you don't say mixed, one is this Dirichlet is just U given, other one is a derivative, okay, so let me not say anything, so boundary conditions are if you are given like this you can use this substitution technique, we call this as an unknown, new unknown and solve this for start-up PDE with one condition, and then once you solve this you get your, you will get a first order another first order PDE.

Now using the other boundary condition you can solve it by Laplace transform technique, and finally to get the actual solution of the boundary value problem, so this is how you solve a hyperbolic type of this type of, special hyperbolic type of equations, if you have a mixed



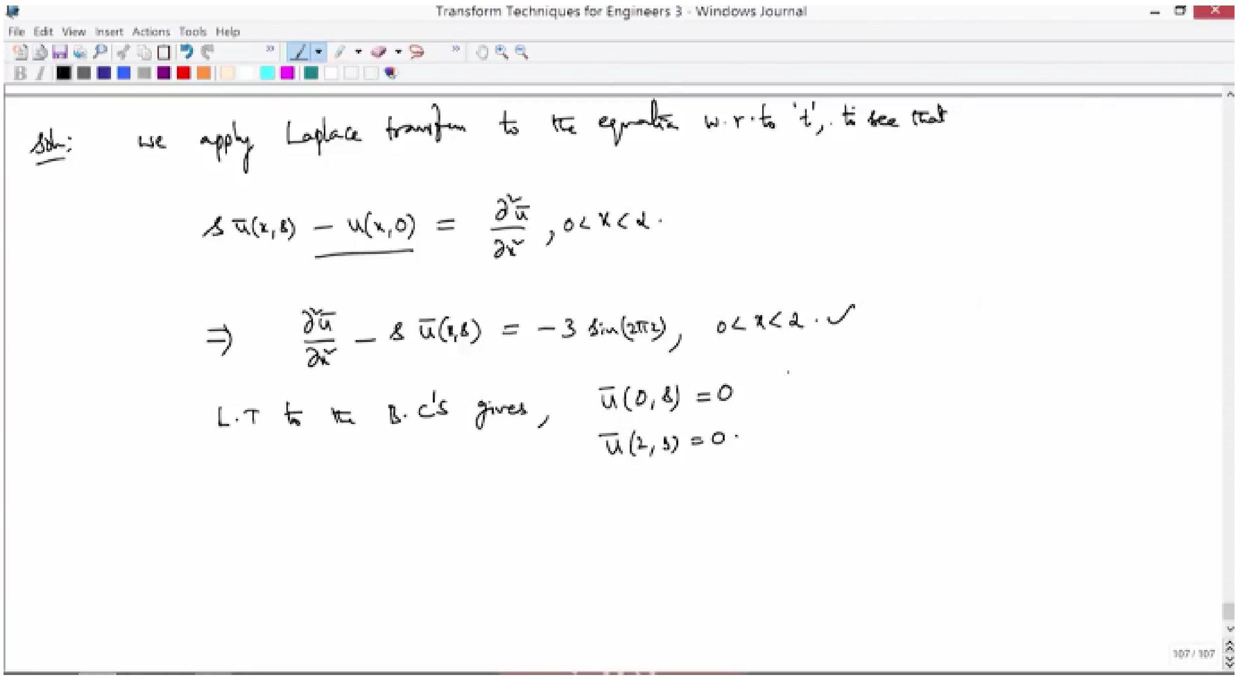

derivative right side is known function, function of X, Y then you can apply this technique and get your solutions, so these are very limited so not every equation you can solve by this technique, some equation like this can be utilized, you can utilize your Laplace transform and get your solutions, okay.

Let's apply this technique to heat equation, okay, so let's solve heat equation problems using this Laplace transform, so let's solve the heat equation that is a UT, dou U/dou T = dou square U/dou X square(x,t) this is let me use this domain as 0 between X and 2, so your domain is, this is have a rod of length 2, so you have a thin rod of length 2 this is 0 and this is 2 and that is the heat is, this temperature, U is the temperature of this rod governed by this heat equation, if you know PDE okay, so T is the time variable and you have U initially at X = 0, initially you have this rod is having this temperature 3 times sine 2 pi X, so this is the boundary condition, this is the initial condition, initial condition because the one time one variable, one derivative so you give only one condition that is as initial condition at T = 0, boundary conditions so let's use you have 2 boundaries now 0 and 2, so U(0,t) for all times this is 0 for every T positive that is the boundary, at this point I keep this rod always I maintain the temperature at 0, U(0) here, okay,

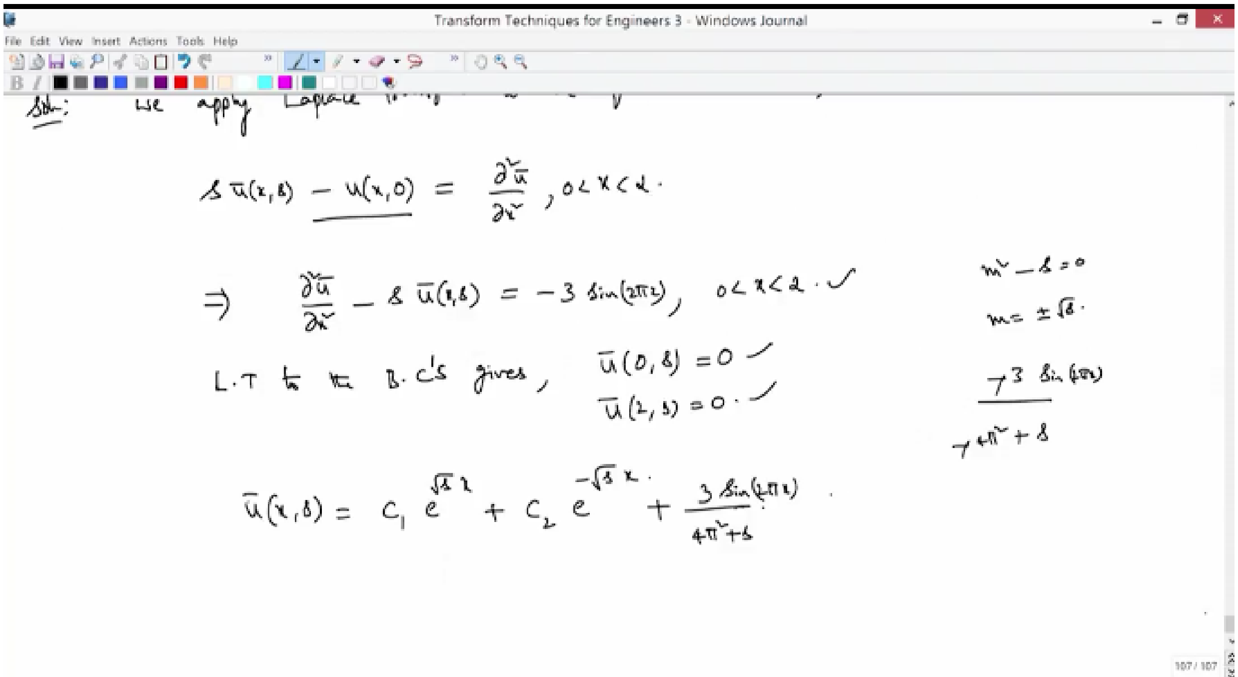



at this here U(0) other place also I maintain both the ends as 0 temperatures, okay, even here I maintain U = 0, then how do we solve this heat equations, so what is the temperature of this rod for all other times, initially you know what is this temperature and all the times I keep my rod both the ends of the rod at 0 temperature, I maintain at 0 temperature then what should be the temperature of this rod, because the temperature governed by this, heat is governed by this heat equation that's a parabolic type of equation.

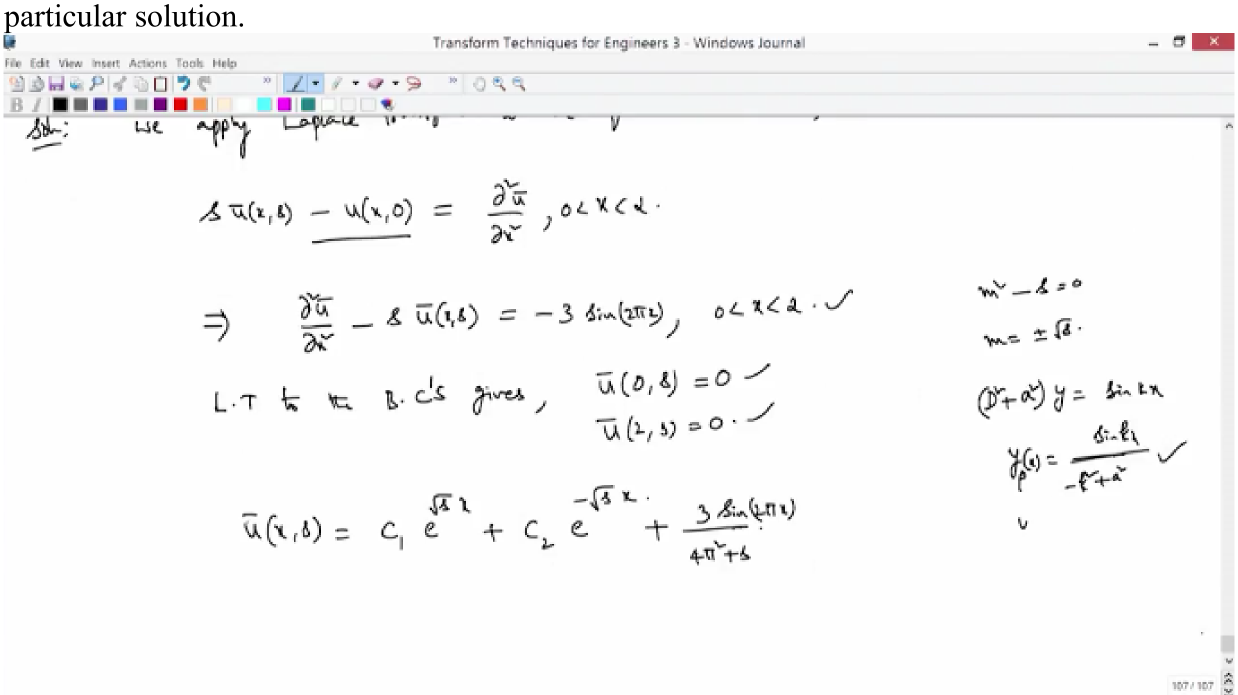
So let's apply Laplace transform as usual, now we apply Laplace transform to the equation with respect to 2 because this is the domain over which you can apply because this is a finite domain, the domain of the Laplace transform 0 to infinity only with respect to T we can apply, okay, then to see that what we see is that dou U/dou T so you have S U bar(x,s) – U(x,0), okay, so this is equal to, now you can apply this dou square/dou X square that is simply U bar (x,s) okay, so this is the equation now this X between 0 to 2, I already applied this initial condition you can apply now, so you have dou square U bar/dou X square –S times U bar(x,s) = -U(x,0) that is U(x,0) is that is going to be minus, so if you bring it this side if you keep so far –U(x,0) you can write, you can use this initial condition that is -3 times sine 2 pi X, so this is ordinary differential equation for the domain between 0 to 2, T is removed now it's become S, okay, so how do we solve this? We can apply because it's a T variable these boundary conditions you can apply Laplace transform, Laplace transform to the boundary conditions gives U(0,t) that T becomes S U bar that is 0, and U bar(2,s) is also 0, so this is the equation and these are the



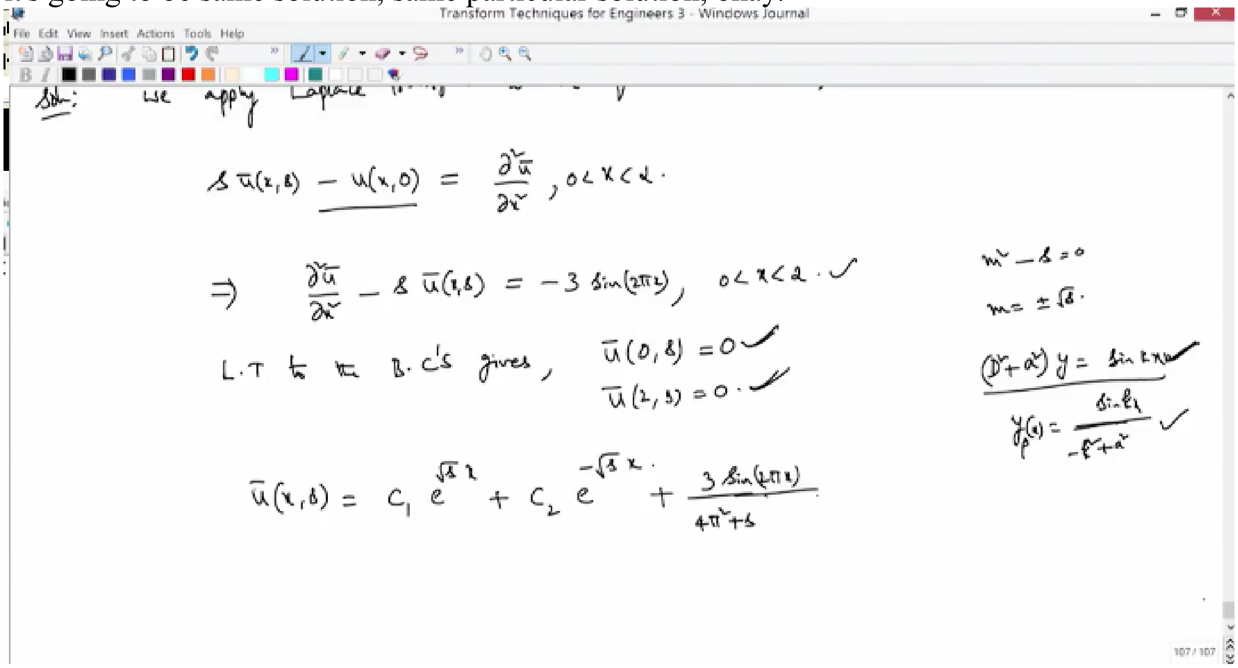

boundary conditions now for this ordinary differential equation, this you solve, to solve this ODE you solve first homogeneous equation this is going to be M square, this complementary equation M square - S = 0 so you have M is + or – root S, as well as C is a constant, so you have the solution complimentary function is U bar(x,s) is C1 E power root S times T, sorry with respect to X you are doing, right, this ODE is with respect to X, so root X + C2 times E power -root S times X, so this is your complementary function plus particular solution of this if you apply, so if you apply this method 1 by, this is operated at D square, so shortcut method I'm using so –S, okay, this is the operator left hand side differential operator right hand side -3 times sine 2 pi X, so in the place of D you put - 4 pi square, so minus minus goes this becomes plus, so you have 3/4 pi square, so 4 pi square + S times that is into sine 2 pi X is your particular solution, so you can solve this and see.



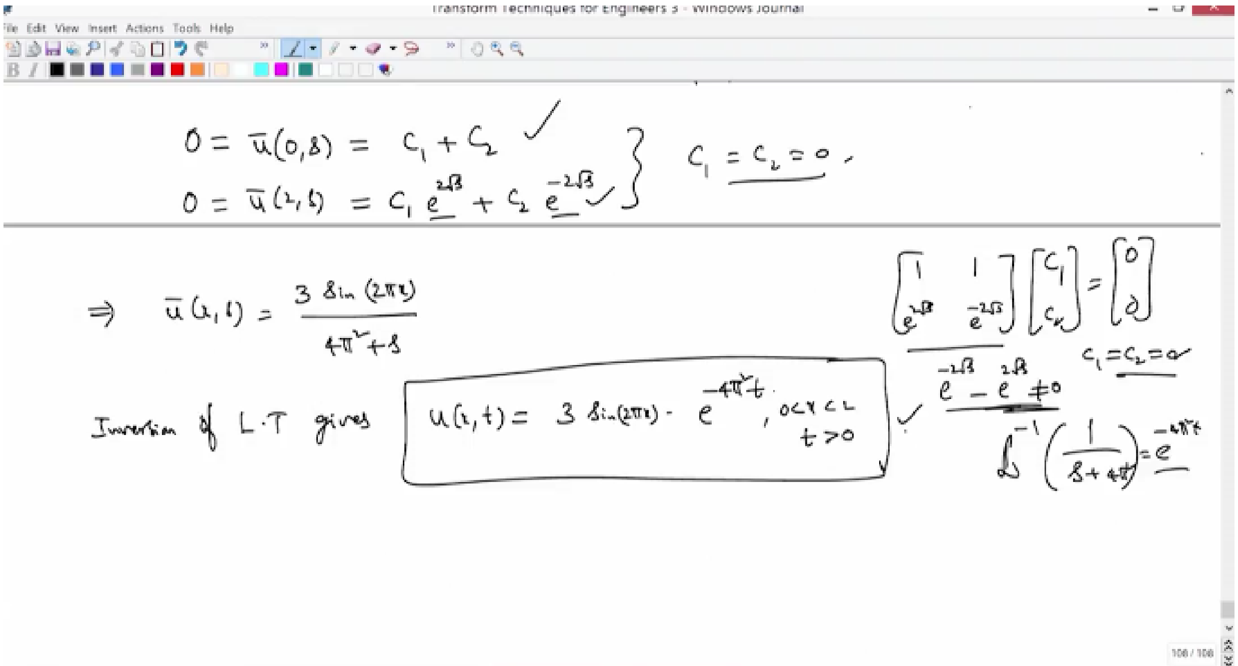

If you apply you have -3/4 pi square S+S, and if you differentiate you have 4 pi square okay -S times 3 divided by 4 pi square + S, so sine 2 pi X is common here, okay, so is this going to be -3? So yes it is right this is going to be -3, right hand side also that sine 2 pi X cancel, so this is your solution particular solution is this that I use the shortcut technique as the operator kind of techniques if you have D square + A square Y = sine some KX particular solution of this YP(x) is 1/D square + A square into sine KX, in the place of D I have to put -K whole square, rather -K square, you have minus of this K square, and you have this one, okay, so this is your particular solution.



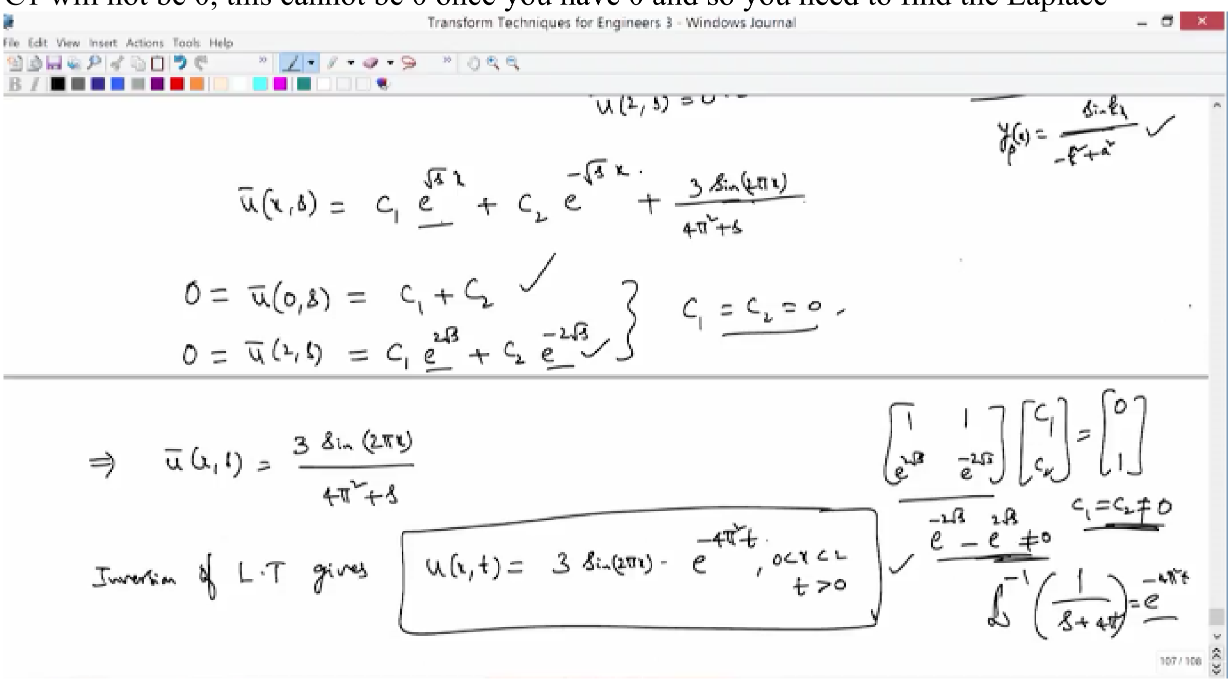

You can also apply if you don't like this a particular solution you look for as A sine KX + B sine KX, this you substitute into the equation and get your unknowns A, B this is a method of undetermined coefficients, these are undetermined coefficients A, B you put it here and get your solution by comparing the coefficients you can get your A, B and you will end up seeing that it's going to be same solution, same particular solution, okay.




So once you get back the solution you apply these conditions U bar(0,s) is going to be C1 0 that is 0 which is equal to C1+C2 + this is 0, so this is 1, and the other boundary condition is also U bar(2,s) which is C1 E power 2 root S + C2 E power -2 root S + 3 times sine 2 pi, sine 4 pi that is also 0 so this is 0, so these two if you solve will give me C1 and C2 has to be 0 because of nonzero coefficients, if you look at this as a matrix 1 1 E power 2 root S, E power -2 root S times C1 C2 = 0 0, so this system because of this determinant of this matrix is nonzero, this is what is the determinate? E power -2 root S - E power 2 root S this is always non-zero so that means C1 C2 has to be 0, okay, so once this so what you see is U bar(x,s) now, the general solution this because of the arbitrary constant you apply these conditions to remove the C1 C2 is 0, so you have 3 times sine 2 pi X/4 pi square + S, so now you invert this, inversion of Laplace transform gives U(x,t) = 3 sine 2 pi X times, we need 4 pi so you divide by 4 pi so you have 4 pi/4 pi square or rather 2 pi, so we have put 2 pi here so you 2 pi here, so you have 4 pi you can write it as 2 pi square + S, +S only right so +S, now I don't need anything, so let's use 3, so if you use 3 sine this and this is going to be E power -4 pi square S- of that right? So that is your solution, okay, so Laplace inversion of 1/S+4 pi square is actually E power -4 pi square T, okay, so this is a solution that you are looking for, for X is between 0 to 2, between 0 to 2 for all times this is what is your solution okay.



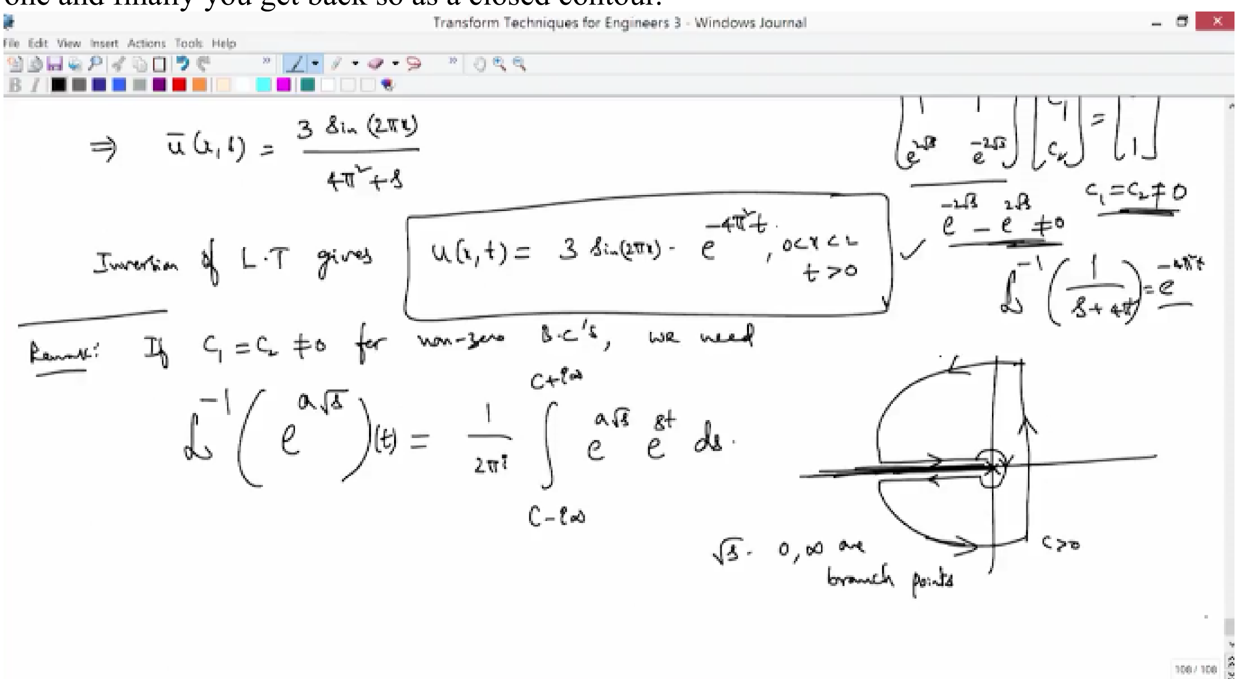
You can also apply, if you cannot solve, if you suppose if you have boundary conditions or nonzero boundary conditions let us say 0 1 if you have, if these are 0 1, so then you will see that this is going to be 0 1 if you have these boundary conditions become this, and you will see that C1 will not be 0, this cannot be 0 once you have 0 and so you need to find the Laplace



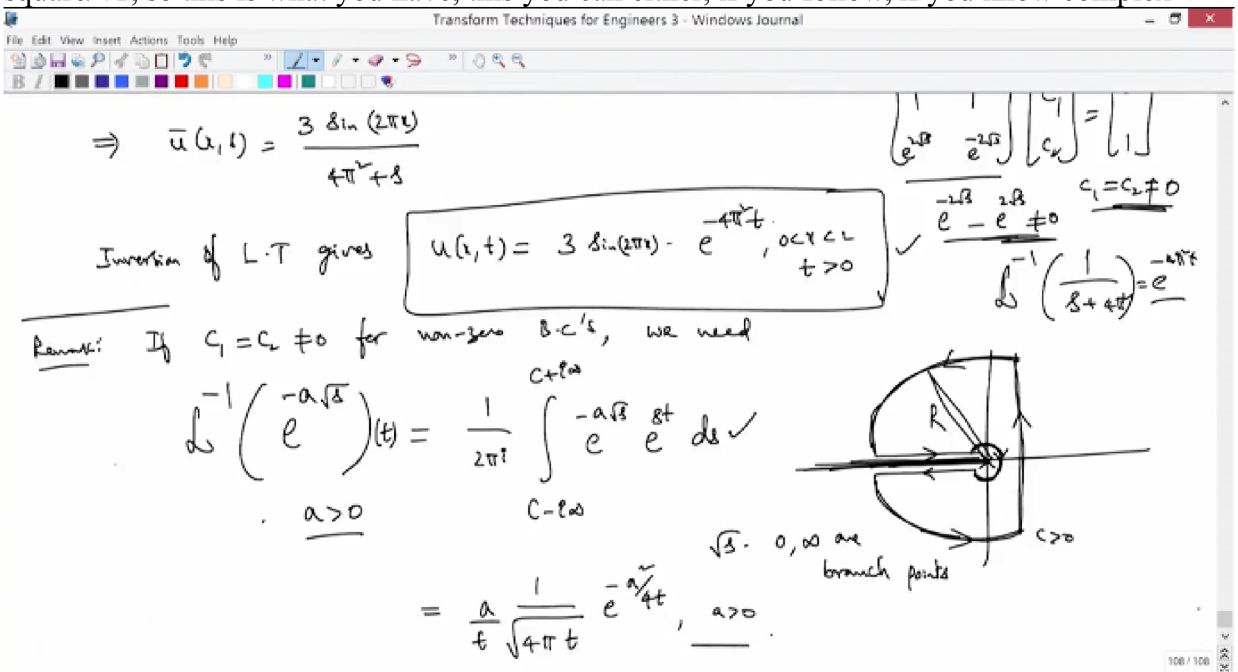

inversion of E power root S okay, when C1 and C2 both are non-zero and you have ,you need to find, once you write that, once you find some constant C1, C2 will be determined as a nonzero constant you put it here, and you can invert this easily like this, but this here you may need Laplace inversion of this, when you invert this Laplace inversion you need to know what is the Laplace inversion of E power some say, let us call A root S, so this is what you may require, okay, so this is a remark if C1 and C2 nonzero for nonzero boundary conditions, we need this

Laplace inversion of this, okay, because of this general kind of, general solution okay, so because of this you have to invert this get back the U(x,t) you need to know this one, okay, so this is actually a value of this I'll write directly, but we'll try to give you in the next video what is this Laplace inversion of this.

We use the direct technique, how do we find this 1/2 pi times, inversion of C-I infinity to C+I infinity E power A root S times E power ST DS, if you do this you will get back as a function of T, so what is this one? Because of this function root S is having a, if you look at, if you take its contour this is Laplace, so E power A root S it's all positive here so it's analytic function in this side as long as S is positive, so I use this C as anything C positive, any C positive but the problem with here is 0 is branch point, so it's not defined for root S, for root S 0 is the branch point, and branch points are comes as a pairs so infinity is also 0 and infinity or branch points, so you need to cut this in the complex plane and S plane you need to cut this, so if you cut this how I take this my contour? As like this, you make a small loop here so this is the contour you have to choose if you want, if you want to get this if you want to evaluate this inverse transform when you have 0 as a branch point you need to cut this complex plane, okay, you need to cut this complex plane and take this, this is how you enter and you have this small loop and this one and finally you get back so as a closed contour.

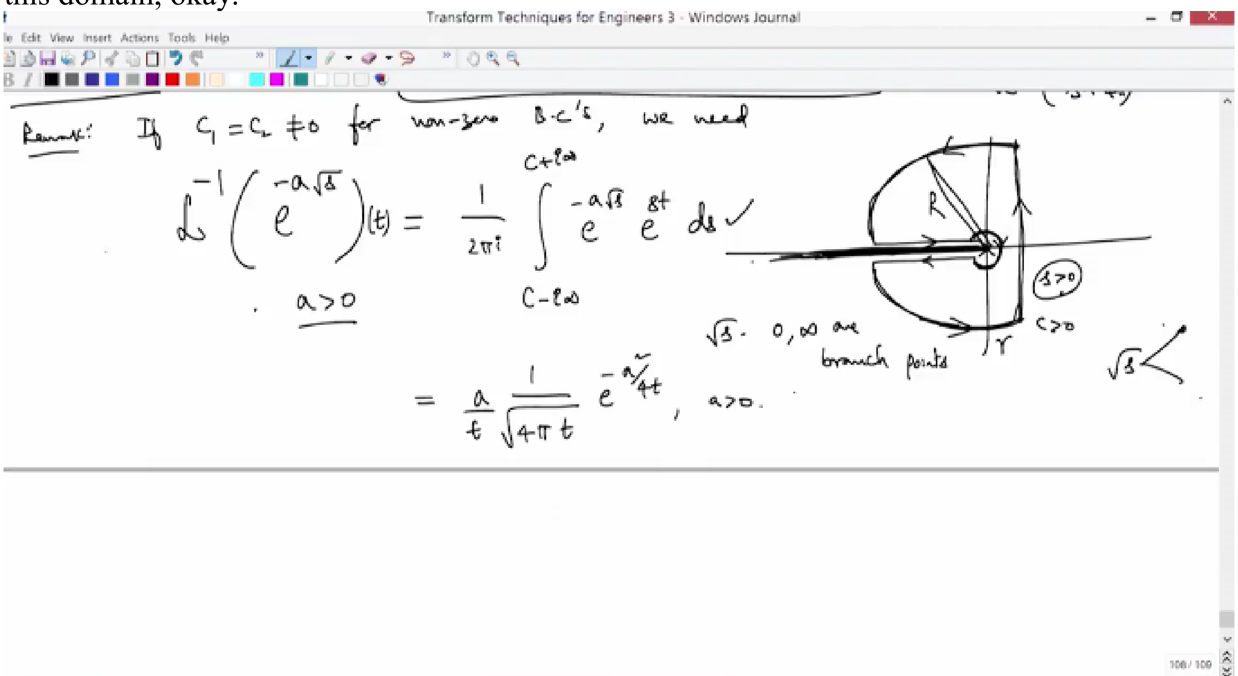



So if you evaluate over this you will get this, over this is what you want, and over these and over these two lines gets cancelled and but this contribution over this and contribution over these two pieces you can show this is 0, and so it's going to be contribution over this will be something or contribution over this whatever comes out as your value of this integral, value of this integral that is actually Laplace inversion, so let me write what that, we may need A, A is positive or A is negative, okay whatever we can choose, so let me so here we can solve this, we can get this thing if A is negative so let me put - sign here so that this integral makes sense just to make sure that over this piece when you calculate, and finally allow R goes to infinity so this is the radius of this circle so this R, R goes to infinity this has to, this integral has to be finite, so in order to do that, to make it 0 we use this with A positive, okay A is positive, so A is positive we need E power –A root S, so this integral I give you directly, so we will try to do it in the next video how do we find, how do you evaluate this Laplace inversion, this is required if you need, if you get your C1 C2 constants arbitrary constants of this ODE, this complimentary function when you apply this boundary conditions if your C1, C2 are nonzero you may have to, you need these two to evaluate its inversion, okay, so to get that inversion you need this Laplace inversion of this type of function E power -A root S, I write directly what it is, this is going to be A times 1/square root of 4 pi T into A/T times square root of 4 pi T into E power -A square/4T, so this is what you have, this you can either, if you follow, if you know complex




variable technique, if you know contour integration technique you can apply this technique you can follow this technique and get your Laplace inversion of this E power -A root S as this one, you can understand and see that this is actually this thing, otherwise you can get it from that table, so you need only this special Laplace inversion that you can assume that it will be in a table of Laplace transform you can put it in the table and make use of it whenever you see this kind of functions for which you need a Laplace inversion, okay, so you can use, you can put some special type of, this type of techniques, this type of integrals, these type of functions for which you want Laplace inversion, you can look into the books and see you can put it as a table of Laplace inversion, and use it whenever in your applications or if you want to understand you need to know what is the branch point, and branch point is the point if you revolve around this you will get back to, you will go to different branch of this root S, root S as a function it's not a single valued function it has a multi valued function, so every time you revolve around this 0 point, the point branch point when you come back to this you will go to the next branch, you have two branches at some point, you have to multi valued function, multi valued means at some point you have 2 values.

As long as you start here and you revolve around, you go back to other value, other branch, okay, so other branch when you revolve around a branch point your function value will go to the next branch, other branch, okay so such points are branch points, so root Z or root S, 0 is the branch point, root branch points always come in the pairs, so infinity is also branch point because the infinity can put it as 1/Z, Z goes to 0, okay, or 1/S S goes to 0 as infinity, okay this is what it is, so if you write it as a root of 1/S that is actually like 1/root S this also will have the same behavior, so S equal to infinity is also a branch point, so if you connect these two points 0 and infinity wherever you like and you remove from that S complex plane and then the function root S will be the single valued function, okay, so root S this function root S will be single valued function that means E power -A root S is also single valued function that is why we cut this line along this Z axis negative axis we cut because this can be cut anywhere, even this way you can cut but if you cut this you may have to work with your calculations will be very difficult because of cut, need not be cut this 0 and infinity you can connect it to even with the curve, but we don't do like that because you need simpler calculations that's why we always say that choose this, this line or if you have this line, this side we don't connect it because it's not there, we need, we have our domain is only this side because S is positive, okay, for which E power –A root S is analytic function all positive side, okay, so that is the reason you take this cut 0 and infinity this side so that your function E power –A root S is single valued function in this domain, okay.




Domain of this omega if it's your curve, if omega if is your curve inside this omega your E power –A root S is a single valued function, so if we evaluate this contour integration technique you will get the same this thing, or you can just accept this as a table, you can table this have it use it, use it whenever you see this type of, when you need this inversion you can use from this table, okay. So I'll try to give in next video, our next one or two videos I'll try to give how to evaluate this Laplace inversion, before we move on to do one or two other applications of Laplace transform, okay, we'll see in the next video. Thank you very much.

**Online Editing and Post Production**

Karthik

Ravichandran

Mohanarangan

Sribalaji

Komathi

Vignesh

Mahesh Kumar

**Web-Studio Team**
Anitha
Bharathi
Catherine
Clifford
Deepthi
Dhivya
Divya
Gayathri
Gokulsekar
Halid
Hemavathy
Jagadeeshwaran
Jayanthi
Kamala
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Poornika
Premkumar
Ragavi
Renuka
Saravanan
Sathya
Shirley
Sorna
Subhash
Suriyaprakash
Vinothini

**Executive Producer**
Kannan Krishnamurty

**NPTEL Coordinator**
Prof. Andrew Thangaraj
Prof. Prathap Haridoss

**IIT Madras Production**

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