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NPTEL ONLINE COURSE
Transform Techniques for Engineers
Solution of Wave Equation by Laplace
Transform
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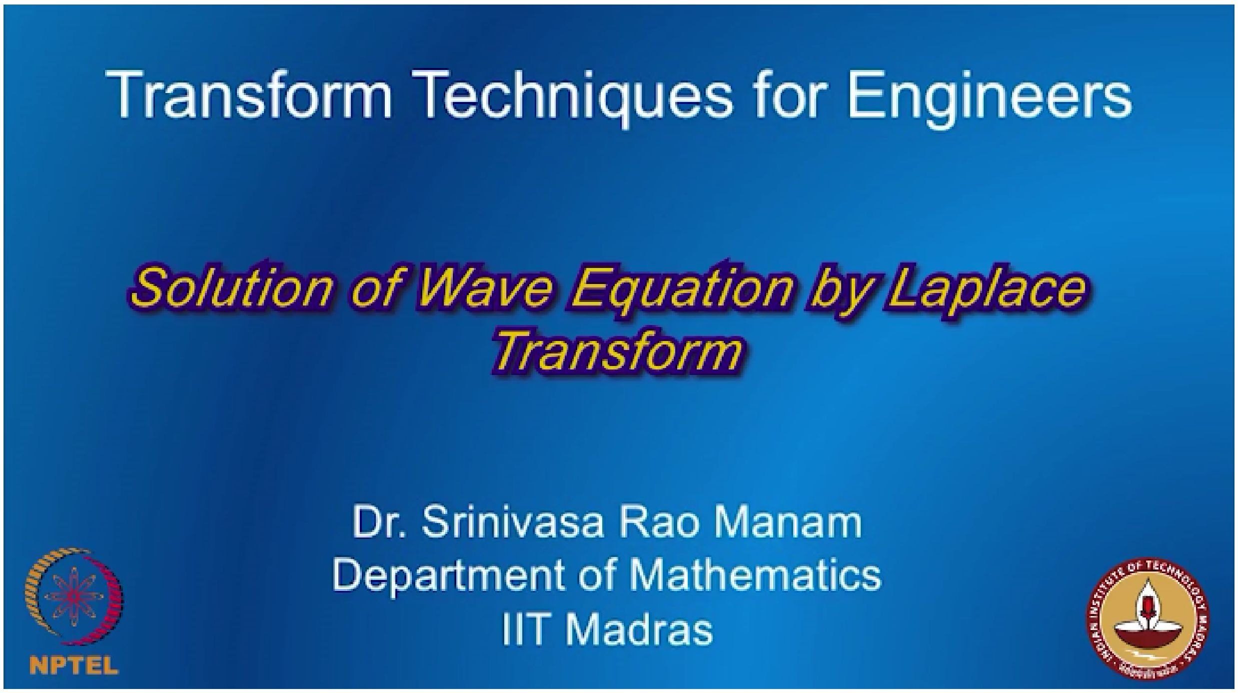
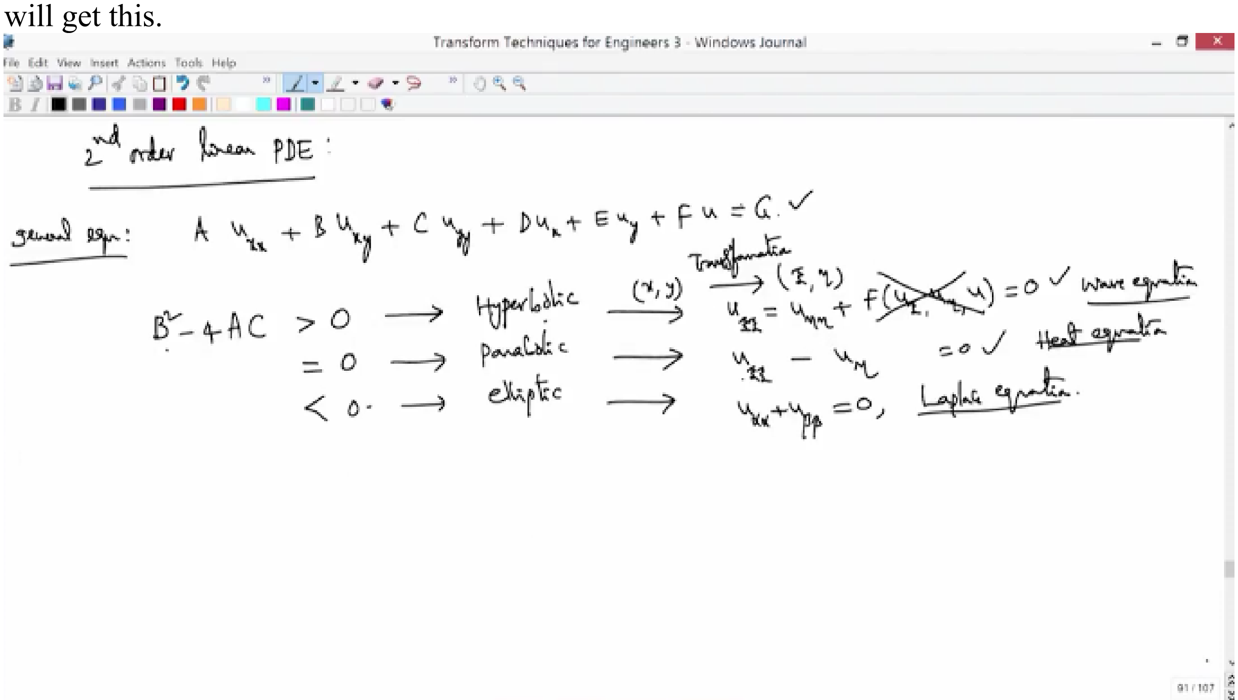


Welcome back, let us look at applications of Laplace transform. Let's continue looking at applications of Laplace transform, what we have seen in the last video is how to solve partial differential equation of first order with initial conditions, and we continue to do partial differential equations.

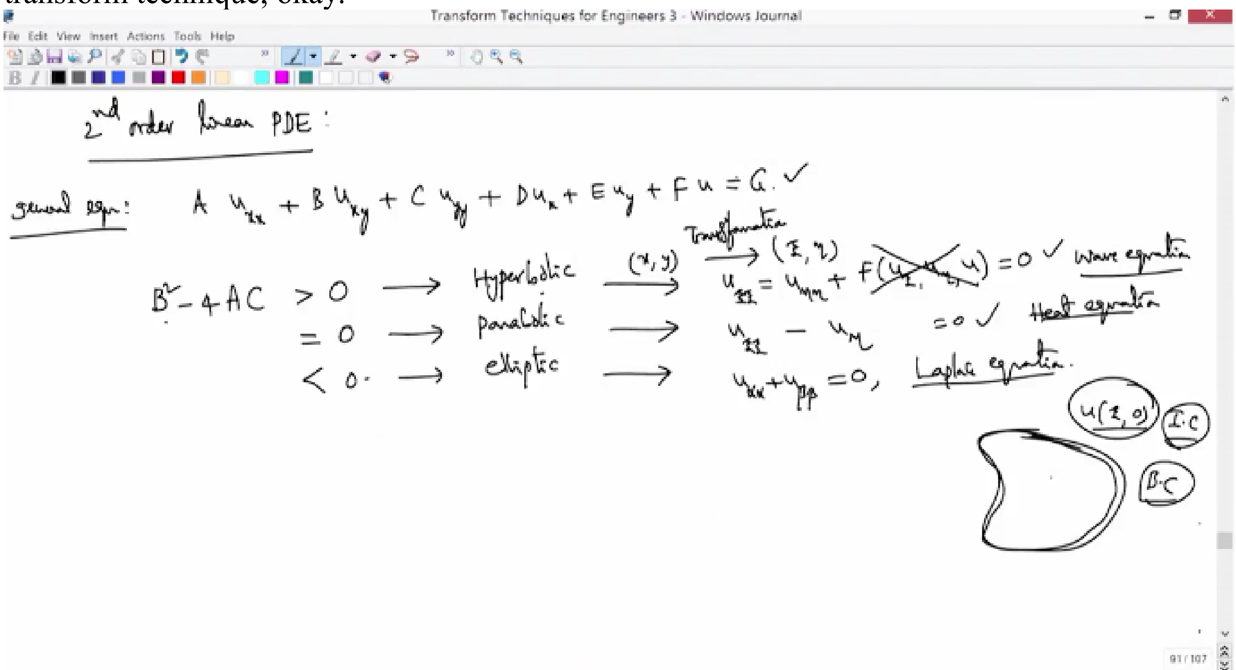
Let's look at the second order differential equations, second order linear partial differential equation, the general second order linear partial differential equation you can classify into one of the three types which is either hyperbolic, parabolic or elliptic, so you can second order term that means the second derivatives you can always put it in one of the forms either hyperbolic type that is looks like, typical equation looks like a wave equation, and parabolic equation typical equation looks like hyperbolic heat equation, and then elliptic equation for which typical equation is a Laplace equation.

So let's look into this, any general second order linear PDE, linear PDE, so let’s how does it look? So you have some constant times A(x,y) times UXX + B times UXY plus+ C times, so all A, B, C, D all these coefficients are functions of XY, UYY and then + DUX + EUY + FU and then is equal to some G which is function of XY, so this is the typical general equation, general equation looks like this, this if you classify it based on B square -4AC either it is 0 positive and this is equal to 0 or negative, so depending on this you classify this equation into here hyperbolic, and then just this is as you see conics based on the, motivated by the conics, similar

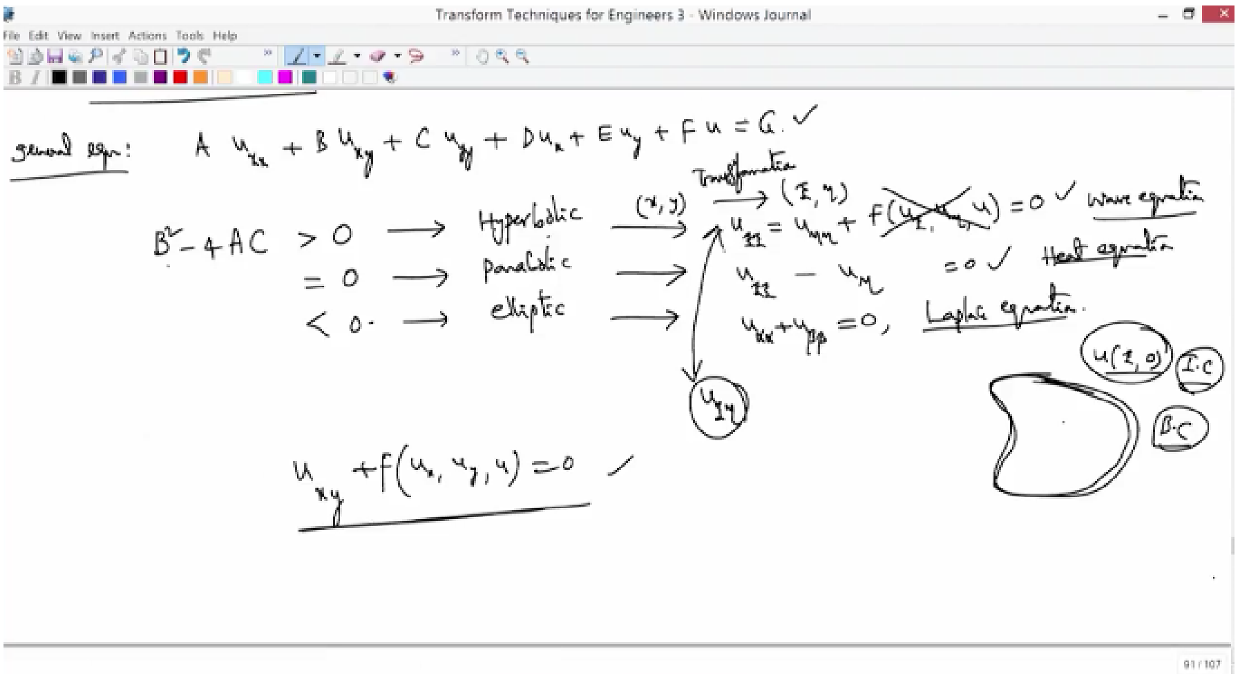
to the conics if this is positive, you can call them as hyperbolic type, and this is a parabolic, and this is a elliptic.
So once you have this form, so once you choose this you change this XY to new variables xi and eta, so there is a transformation, a transformation involved that is from XY to xi eta, so that is xi(x,y) and eta(x,y), so xi = xi(x,y), eta = eta(x,y), such a transformation if you use or either transformation from this variable to new variables, so what you have is in this case, typical case, in the hyperbolic case what you get is U xi xi = U eta eta + lower order terms, that is function of, that's function of U xi, U eta and U, now U is a function of xi and eta this is equal to 0, so this is how it looks this typical hyperbolic equation, if you consider this equal to 0 this is exactly your wave equation, okay, so typical equation you can remove this what you end up is this heat equation, what is that? Wave equation, so now you have, in the parabolic case when this is 0 you have a transformation from X, Y to xi eta so that what you get is you can get this U xi xi or U eta eta equal to lower order terms, or function of U xi U eta and U = 0, so typical equation here looks like, so this is the parabolic type of equation, this is the canonical form, and if you remove this and then typical equation will be U eta, and this is your heat equation, okay. So in third case, this is similar to this hyperbolic case only difference is xi and eta will be a complex value would transformations, okay, so if you again one more transformation you consider as a real part of xi, and imaginary part of xi as new variable alpha beta this will become U alpha alpha + U beta beta + lower order terms F(U alpha, U beta, U xi, U) = 0, now U is function of alpha beta, so alpha beta are real part of xi, beta is imaginary part of xi, so xi and eta are complex valued, xi = eta bar in that case, okay, if they are complex that's how you will get this.



So in any case a typical equation will be, so remove this, so this is your typical equation that is your Laplace equation, so the domain is basically in the plane if you use xi eta variables you have a plane alpha beta, so you have a plane some region or which you have one of these equations is satisfied, and you're depending on this boundary and you have to provide, it’s your boundary data and also if it is a heat equation, if it is eta if we consider as a time that you call if you give U(xi) eta = 0 that is called initial value, okay, initial value, initial condition, if you provide this that is initially condition, if you provide on the boundary that is boundary condition, so you provide this that you make it initial boundary value problems, you make a initial boundary value problem for one of these typical equations and we'll try to solve this by other techniques, you can solve by other techniques but in this course we will solve by Laplace transform technique, okay.



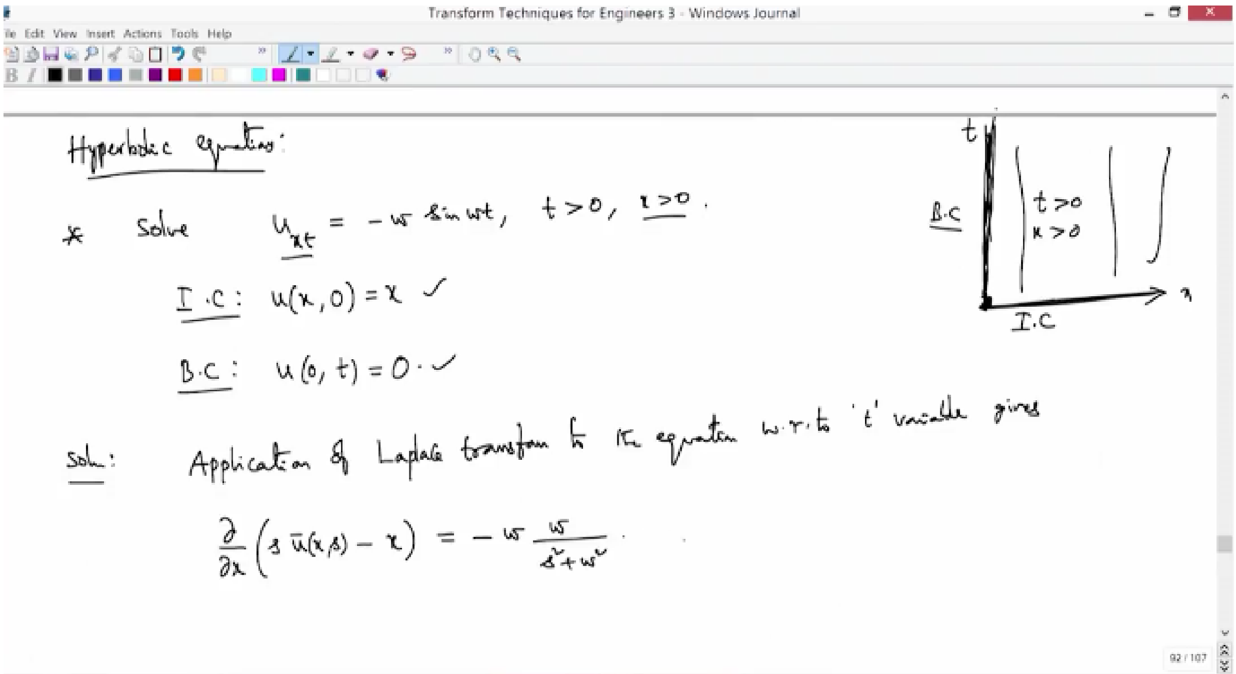
So let me start with this first hyperbolic that is a wave equation, so let me write directly what kind of, so you have an example wave equation, wave equation type, so before I do this wave equation so let me solve some general equation, so if you cannot put, so you can put this in this form, without putting canonical form you may end up something like, some typical equations, some simpler equations even though they are like XY, so for example XY here, XY if you have a lower order terms U or rather functions of UX UY U = 0,this is actually hyperbolic type, okay, so you first you get U xi eta, and then once you have this, this is also another canonical form, so these two are equivalent, so this one and this one, you can go from this + this, okay, so rather this two, these two are equivalent because this is also kind of hyperbolic, okay.



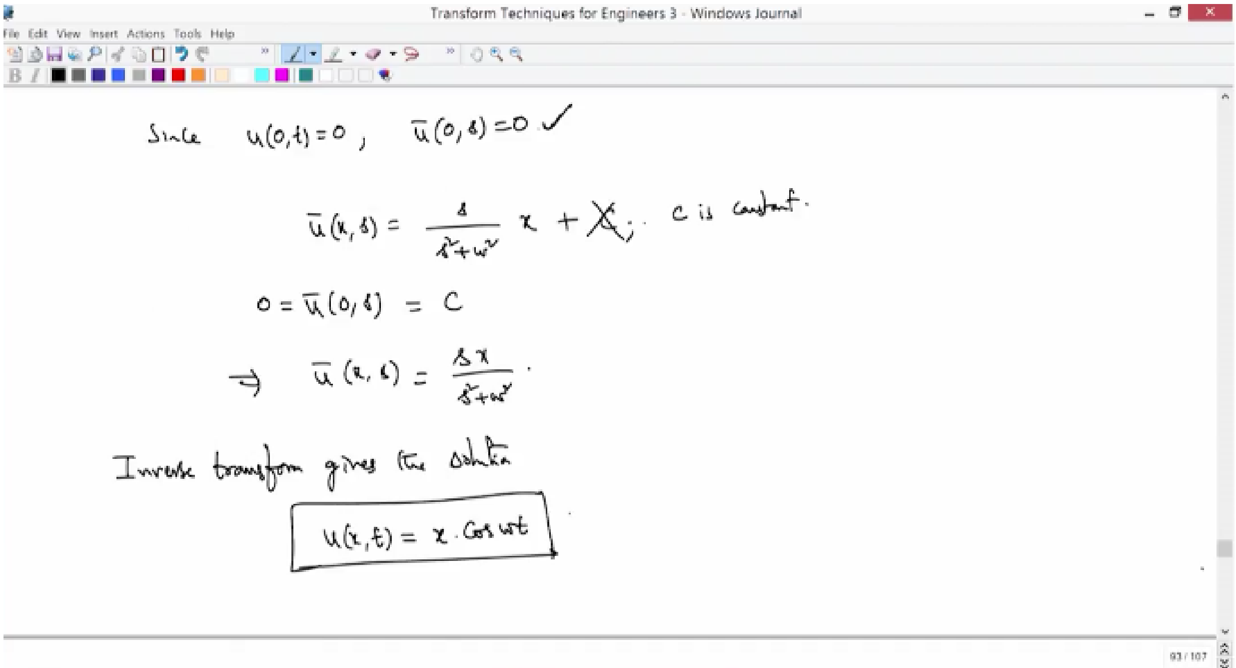
So let me give, if you are given equation in a simpler form like this with some known UX, UY its simpler form for this F you will, let us see how we solve this hyperbolic type, so start with let me write as a hyperbolic equations, how do you solve this? Let me write the problem, so solve this UXT, X and T are independent variables, U is a dependent variable, and this is -omega sine omega T, T is positive, so you need to apply Laplace transform so I consider one of the variables as T, so T is this, and X is any full real line, so this is upper half plane is your domain T positive X belongs to full R, so this is what it is, this is X and this is T.

So that is where in this upper half plane this is the equation is satisfied, and what you provide the data is you have a mixed derivative so you have to provide U at initial data, so T derivative is there so you consider at T = 0 for all X you make it equal to some X that is an initial condition, this is the initial condition, okay, and then and this is your boundary data so let me provide boundary condition, so on this you have this, okay, XT = 0, so this is your initial data, initial data here, initial condition at T = 0 on the boundary U(0,t) equal to let us say 0, so what is that boundary? X = 0 that is at X = 0 here and then all along T, for all T you have this boundary, so on this we have these are the conditions you have got, okay, so if this is your domain, initial condition is this and you have X derivative, because of X derivative let me provide a data U at X = 0, so X = 0 basically my domain should not be this full thing, so otherwise this is not a domain so this is your domain, so let's take this as X, X positive so then only this makes sense.

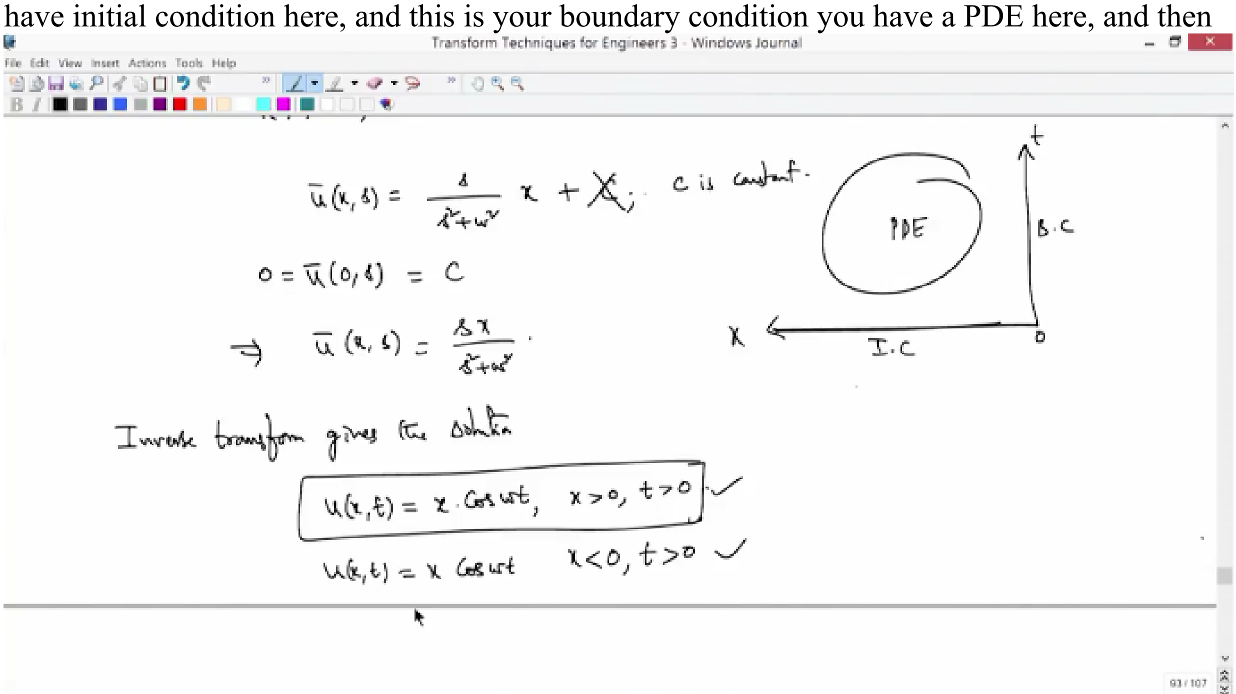
So then you see that this is one boundary and this is another boundary, so on this boundary you provide a boundary condition so that is what is this, and this is your initial condition okay, so how do we solve this? So solution simply as usual you apply a Laplace transform for T variable, application of Laplace transform to the equation with respect to the T variable gives, what it gives is you apply this you end up getting D, so what you get is those dou/dou X (dou u/ dou T) for which you apply the Laplace transform that is S U bar(x,s) – U(x,0), so U (x,0) is X, so let me use this as X that is your left hand side which is equal to - omega that is constant, sine omega T that we know that this is actually omega divided by S square + omega square that is the Laplace transform of sine omega T, so here X is still positive.



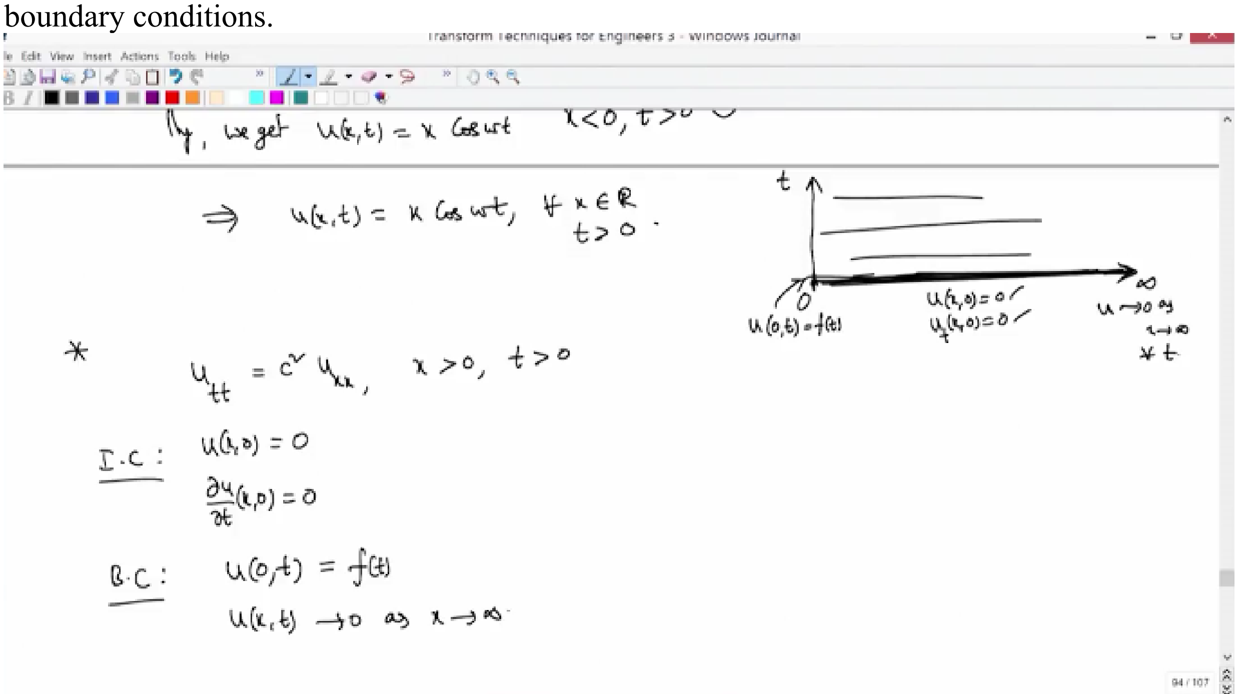
And then you consider this, so you have S dou U bar(x,s)/dou X -1 and that is going to be this side if you bring it that is going to be here, so we have 1 - omega square/S square + omega square, this is S square/S square + omega square, S square + omega square - omega square, simplify this you get this, so S S cancel both sides so you end up getting dou U bar/dou X = S divided by S square + omega square, so you apply this if you solve this and use this boundary condition, boundary condition is if you apply boundary condition, since the boundary condition is since U(0,t) is 0, U bar(0,s) is also 0, because for which you can take Laplace transform and this is this, so you solve this equation with this condition at X = 0 so you get U bar(x,s) is simply integral, integral of this that is XS/S square + omega square, that is constant and you have X + some constant, okay, so there C is constant, you apply this condition, boundary condition that makes it 0 which is U bar(0,s) so this is typically C, if you put X = 0 this is 0 and you have C, so C is 0 that means C is 0, so that implies U bar (x,s) the solution is SX/S square + omega square.



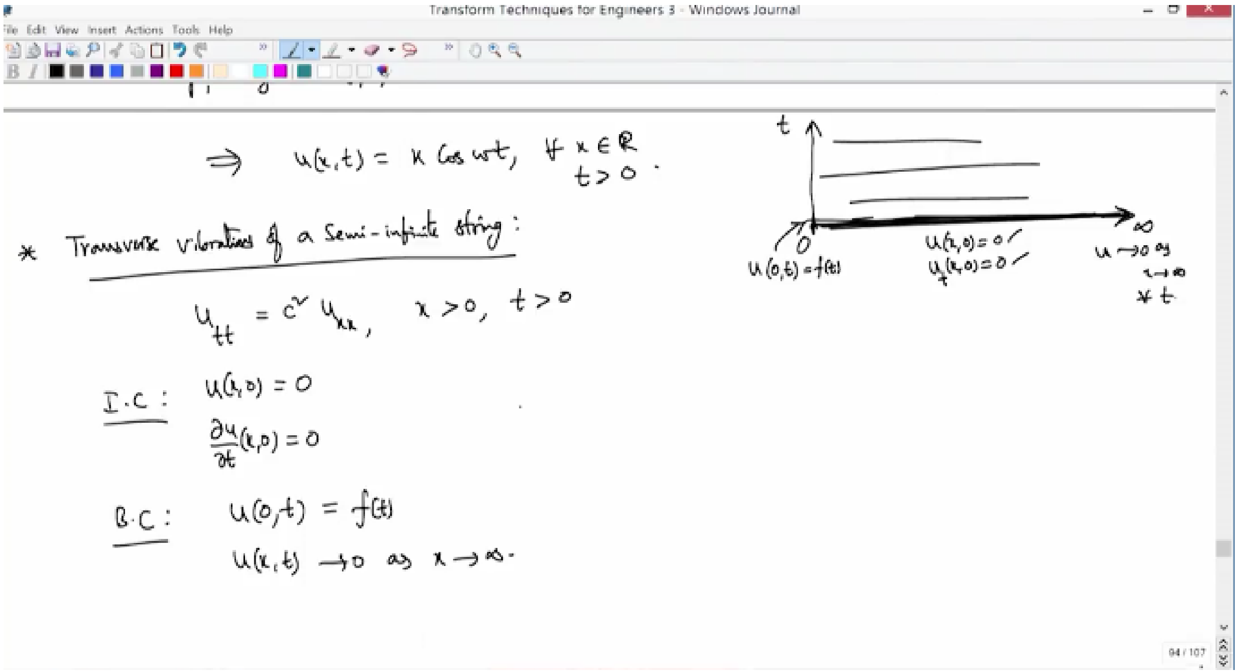
Now you consider its inverse transform, inverse transform gives a solution U(x,t) = X times, you are doing with respect to S right, S divided by S square + omega square is cos omega T is your solution, is your inverse transform, so this is your typical solution, this is your solution for X positive and T positive, so this is your solution you can also consider now this way, so separately you can also consider this way, as though this is your domain you can also consider as T = 0, so this initial condition you consider also here and here separately you get the same equation, so eventually if you consider that problem you will get that, that is also valid, same solution you will get for this 3 positive and X negative, so you consider the domain as left-hand side for example this one, this is your domain this is your T and this is your X, X is 0 and you have initial condition here, and this is your boundary condition you have a PDE here, and then



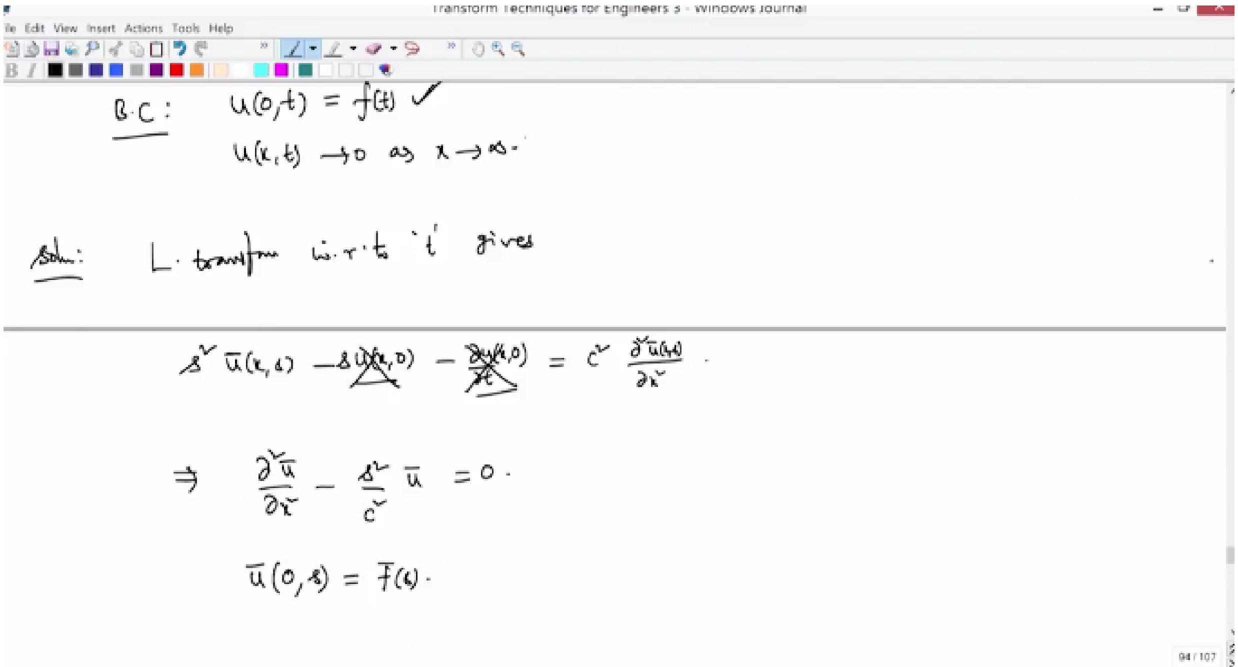
if you do this is your domain, within this domain you see that you will get the same solution there is no change, so you end up getting, so the final solution, so you can write both the cases if I remove that X belongs to R, X belongs to R you can, what I worked with is initially X positive side, X negative side also you can get, similarly we get X negative side in place U(x,t) is X cos omega T for every X belongs to R and T positive so this is your solution, okay.
So let's look at another example, in this case we will try to solve some wave equation, let's consider a wave equation for transverse vibration of a semi-infinite string, I consider some semi- infinite string this is 0 to infinity, okay, if I have initially this string is at rest that means U(x,0) is 0, UT(x,0) its velocity and its displacement both are 0 initially at rest, at T = 0, and what I do is here I provide some boundary conditions, so there's some boundary conditions if I provide U(x) that is X(0), so U(0,t) I give you some function of X, function of T, at this point and you assume that if you do that, and what happens at infinity you don't have any displacement, because whatever you are giving here it's going to be propagating, because your equation is wave equation, so U has to be, U has to go to 0 as X goes to infinity, okay for all times, so that is at infinity, so those are the two boundary conditions these are the initial condition, so how do I put this as a boundary value problem, so because of this, this is transverse vibrations in a semi-infinite string so you have a wave equation is satisfied, so UTT = C square UXX, X is positive, and T is also positive, so this is your domain, and so if you look at this way this is your T, so this is your domain, semi quarter plane you have this is your domain, and you have initial data, initial conditions are U(x,0) is 0, dou U/dou T (X,0) is 0, and you have the boundary conditions those are U(0,t) let me give as some constant times F(t) okay, so let me give only F(t) okay, F(t) and then U(x,t) goes to 0 as X goes to infinity, so this is the boundary conditions.



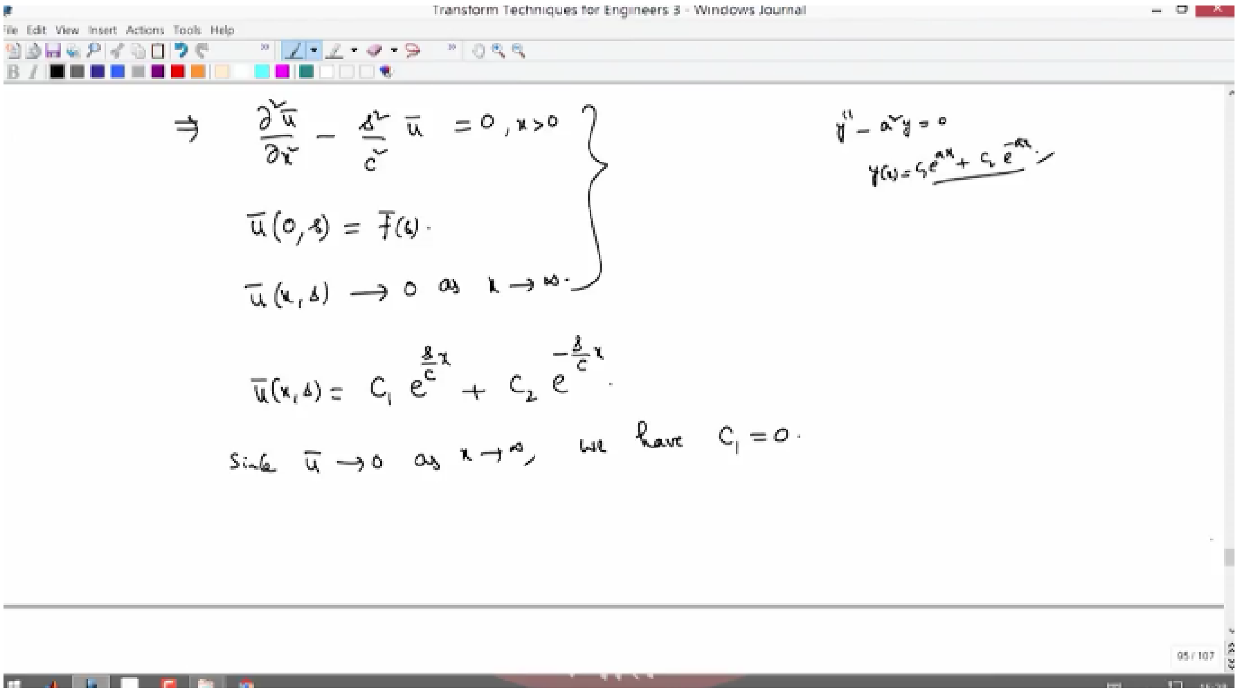
So this is your problem so let me solve this, this is a wave equation, non-homogeneous wave equation, non-homogeneous I'm sorry this is a wave equation, so wave equation so or rather how do I put it, vibrations of, transverse vibrations of a semi-infinite string, so this is a typical boundary value problem for this vibration, if you want to look at the vibrations or the



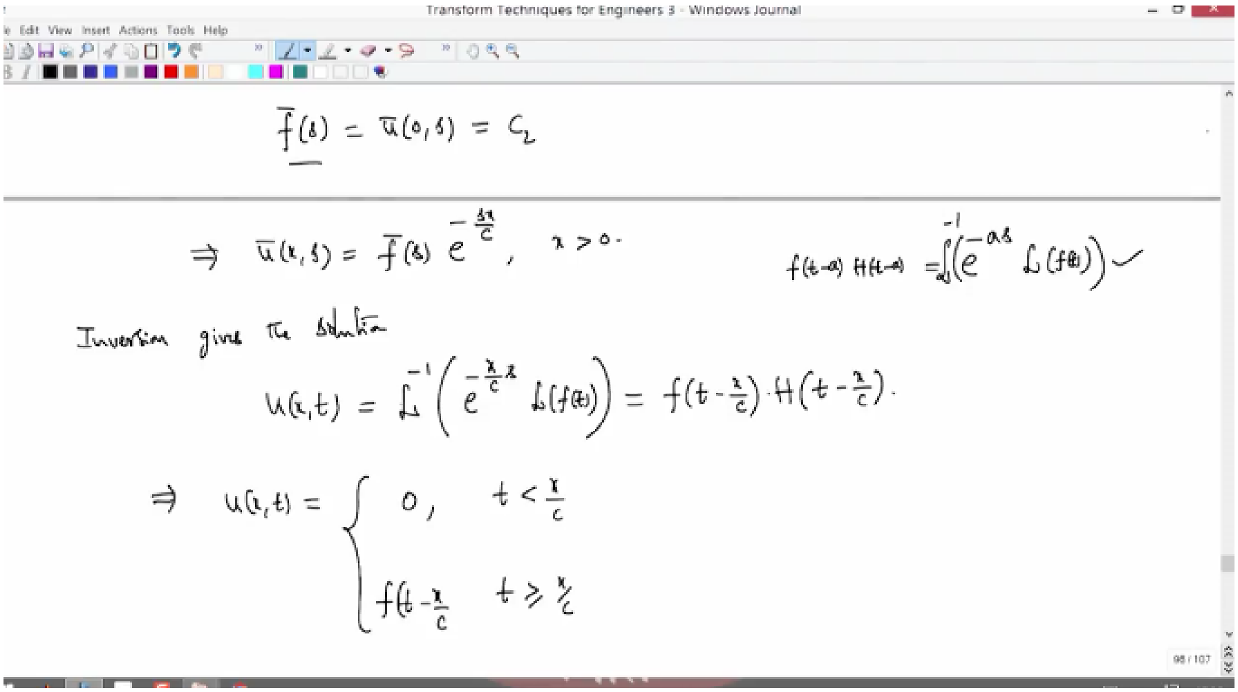
displacement in the string and you have, I will provide at this end, disturbance you provide at this end otherwise which is at rest initially, you disturb that at this end all the time with F(t) as time progresses you have F(t), at time T you will give a disturbance of F(t) how it propagates all along this as a displacement in the string, so the solution as usual it's because in the quarter plane problem we apply Laplace transform, Laplace transform gives, the equation becomes, what is this equation becoming? You apply this here, UTT if you apply S square U bar(x,s) with respect to T, okay, with respect to T gives this S square U bar - U bar at, sorry U(x,0) - S times this and U here dou U/dou T (x,0) so this is your UTT, this is equal to C square if you apply this here simply dou square U bar(x,s)/dou X square, so U(x,0) this is 0 and this is also 0, so these are by initial conditions, use the initial conditions to make this 0 so this implies dou square U bar/dou X square - S square/C square, I divide with C square, because C is non-zero which is the speed of wave, okay, C square is positive or C is non-zero times U bar which is equal to 0, and you have this boundary conditions will provide U bar(0,s) that is equal to F bar(s) Fourier Laplace transform of this if you consider that is U bar(0,s) which is equal to F bar(s).



And then U here this becomes, this becomes U bar(x,s) goes to 0 as X goes to infinity, so these are the, this is the problem now, X becomes ordinary differential equation here, the domain is X positive, S is something which you don't know still S is, S is a Laplace transform variable, there is a complex variable and you have this, so how do we solve this? You solve this ODE with typically you have solution of this is U bar(x,s) as, this is M so if you use a complementary, this is like Y double dash - A square Y = 0, so Y(x) is E power AX + C1 times this, C2 times E power -AX so that is your solution, so if you use this same thing here you have C1 times because this is the derivative so your C1 of, yeah this is a constant, so this is C1 times, some constant times E power A is S/C times X + C2 times E power –S/C times X, now you apply these initial conditions and then if you apply this initial condition let's see, if you apply first and the last one as X goes to infinity this has to go to 0 that is possible only if C1 = 0, okay, so this implies C1 has to be C, C1 has to be function of S okay, so any case so let's use since U bar has to go to 0 as X goes to infinity we must have C1 = 0, so this gets cancelled, so the general solution is the only this one.

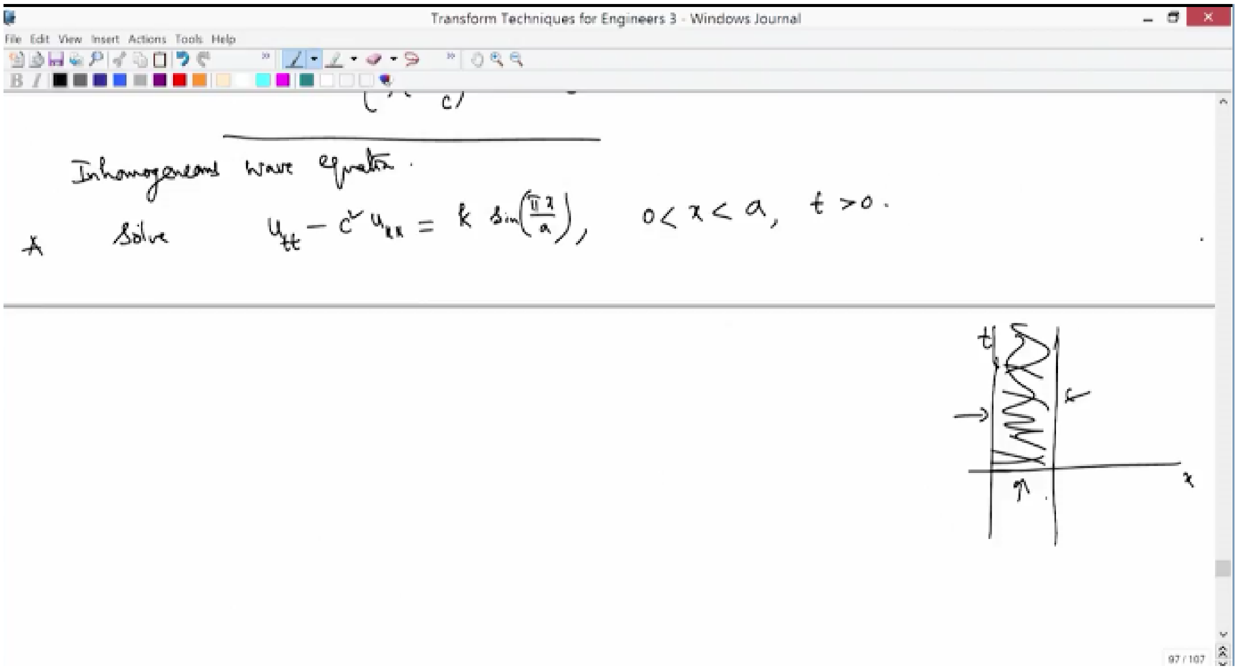


Now we apply the first initial condition that is U bar(0,s) that is F bar(s) which is U bar(0,s) which is, from here you have C2 times you put X = 0 that is C2, so this implies U bar(x,s), now I know what is my C2, that is F bar(s) times E power –SX/C, X positive, so you can invert now, so inversion now gives the solution U(x,t) which is equal to, so what is its solution? And you have already seen that earlier E power -AS times Laplace transform of F(t), I'm using this property which is equal to Laplace transform of F(t-a) times H(t-a), okay, so this makes L inverse here, L inverse of this is equal to this, okay, so here we need L inverse of E power -A is S/C times X, sorry X/C is your A times S Laplace transform of F(t), so if you use this property you'll end up getting F(t-x/c) times heavyside function of T-X/C, so if you write this you get, if you write the definition of the heavyside function you end up getting, if T is less than XC that is 0, so you have 0 if T is less than X/C, if T is greater than or equal to X/C you get F(t-x/c), so this is your solution of your boundary value problem, so this is how it behaves, this is your



solution, so this is how you get the vibrations as that is X is less than or X you can put it this X is less than T times C, as time progresses less than this you will always find this non-zero, so as you see after some time T times multiplied with this up to that point you have the disturbance, it propagates with the speed C up to time T, TC times, TC is this shows that T times this is the velocity that is X/T that T T goes so it's some kind of distance, okay, so this is a distance up to which you have the disturbance that is traveling, okay so you solve this, so this is how you solve a wave equation, a wave equation or hyperbolic type equation simpler equations with initial and boundary conditions you can make use of them and use the Laplace transform to find the solution of this initial boundary value problems.

Let's look at one more example, one more example of wave equation, so let's solve, so this is about inhomogeneous or non-homogeneous wave equation, how do you solve this? So problem is about solving, so solve this problem, so you have UTT = C square UXX, again X is between so this is U- of this is equal to, I give you some constant times K times sine pi X/A so this is a kind of forcing, okay, so outside force is this so this you have typically I give between 0, domain is strip let us say T positive so you have only strip here so you have a strip that is 0 to or rather let me write this way, so if you have this is X and this is T so you have this strip so your equation is valid here, okay, or T is positive so you have only up to here so this is your strip, so these are your boundaries.



So let me provide the boundary data that is initial conditions at T = 0 I have to provide because you have, this is your boundary so at this T = 0 so you have to provide boundary so let us say it is at rest, so you have a string of length A, okay, which is at rest, which is at rest initially, so U(x,0) = 0 and dou U/dou T (x,0), T = 0 this is 0, these are your initial conditions, a boundary is these are the two boundary conditions that is at X = 0 and X = A these two, okay, so U(0,t) for all times I provide, this is also 0 let me use this also 0, U(a,t) is also 0, so let's attach the string which I fix it whose displacement is 0, so I fixed these edge so that you have these boundary conditions, the only thing is because of this forcing outside force how this string is, how this behaves? String is behaving, how the string gets the vibrations in this string, okay, we'll see how it is by looking at these solutions, solutions for what we do is again now because T is a variable that is typically, so C is here as speed, speed of wave, C is the speed, and K is constant, K and A are constants, okay, so solution will be as usual T is positive, T is valid for all 0 to infinity you can apply Laplace transform to T variable, Laplace transform to the T variable gives what you get is again so you have S square U bar(x,s) – S times U(x,0) - dou U/dou T(x,0), so both are 0 from this initial conditions, so 0-C square dou square U bar(x,s) divided by dou X square = K times sine of, sine pi way this is a constant I am doing with respect to T, so sine pi x/a times K, is a constant comes out, so it's only Laplace transform of 1 that is 1/S, I use the initial condition so you finally get dou square U bar/dou X square, I divide with C so you have, you bring it to the other side.

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Inhomogeneous wave equation.

Solve $u_{tt} - c^2 u_{xx} = k \sin\left(\frac{\pi x}{a}\right)$, $0 < x < a$, $t > 0$.

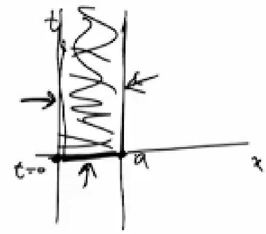
c - speed, k, a are constants.

I.C: $u(x,0) = 0$

$\frac{\partial u}{\partial t}(x,0) = 0$

B.C's: $u(0,t) = 0$

$u(a,t) = 0$



Soln: L.T to the 't' variable gives

$$s^2 \bar{u}(x,s) - \cancel{s\, u(x,0)} - \cancel{\frac{\partial u}{\partial t}(x,0)} - c^2 \frac{\partial^2 \bar{u}(x,s)}{\partial x^2} = k \sin\left(\frac{\pi x}{a}\right) \frac{1}{s}$$

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And then so you have a -S square/C square U bar is equal to, you bring it so you bring this one to this side so you have –K/S, and K/SC square sine pi X/A, so this is your solution between X to A, and you have a boundary data so that you can have now U bar(0,s) is 0, and also U bar(a,s) is also 0 from the boundary conditions, if you apply the Laplace transform of these two boundary conditions you get this, so this is the problem you have to solve, and finally before you take the inverse transform.

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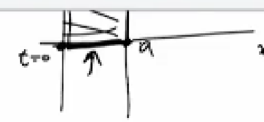
B.C's: $u(0,t) = 0$

$u(a,t) = 0$



Soln: L.T to the 't' variable gives

$$s^2 \bar{u}(x,s) - \cancel{s\, u(x,0)} - \cancel{\frac{\partial u}{\partial t}(x,0)} - c^2 \frac{\partial^2 \bar{u}(x,s)}{\partial x^2} = k \sin\left(\frac{\pi x}{a}\right) \frac{1}{s}.$$

$$\Rightarrow \begin{cases} \dfrac{\partial^2 \bar{u}}{\partial x^2} - \dfrac{s^2}{c^2} \bar{u} = -\dfrac{k}{sc^2} \sin\left(\dfrac{\pi x}{a}\right), & 0 < x < a \\ \bar{u}(0,s) = 0, \quad \bar{u}(a,s) = 0 \end{cases}$$

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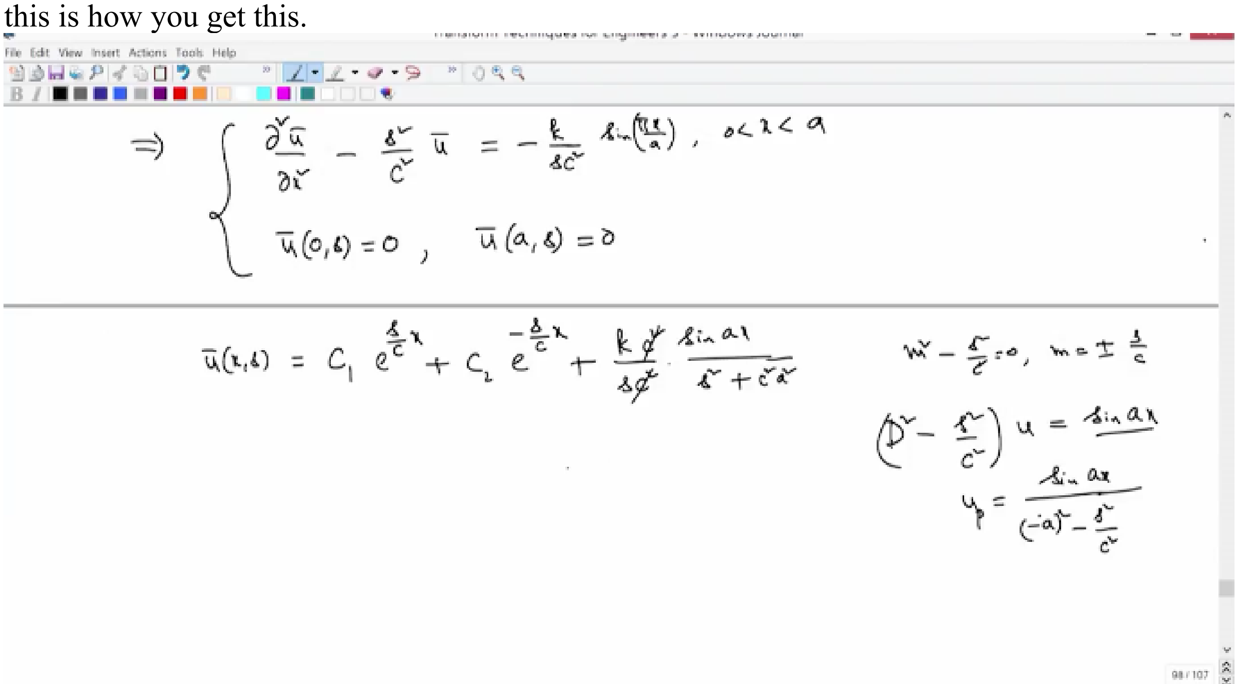
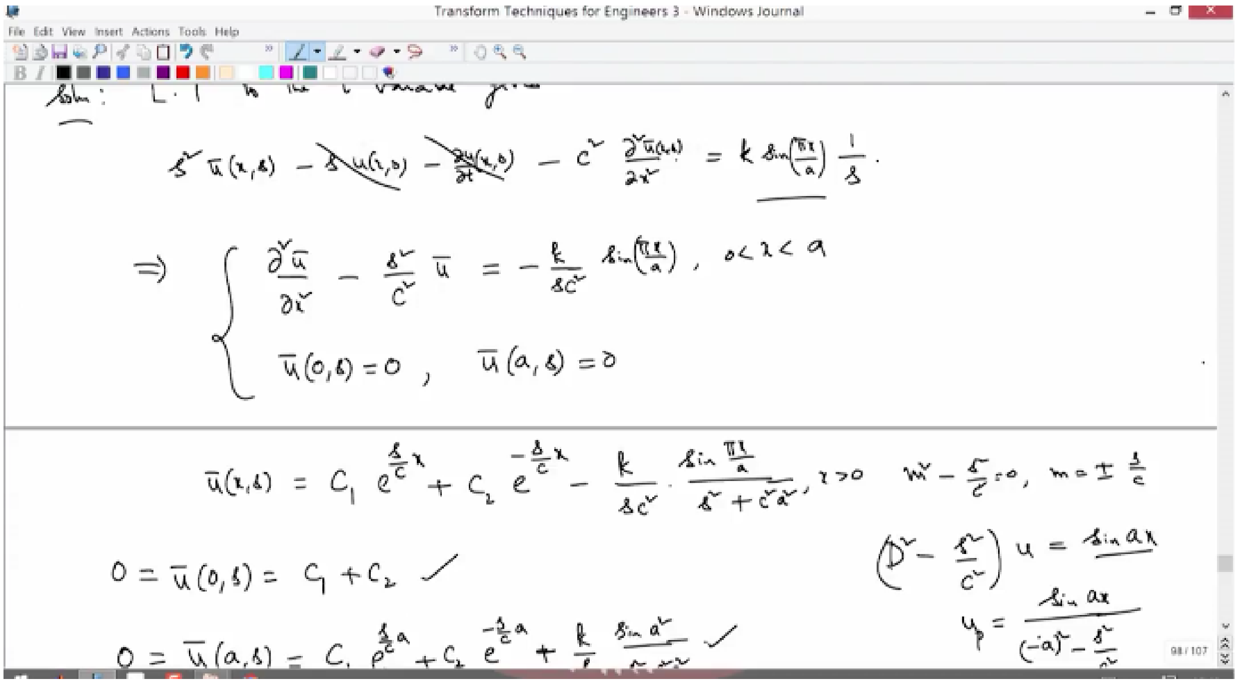
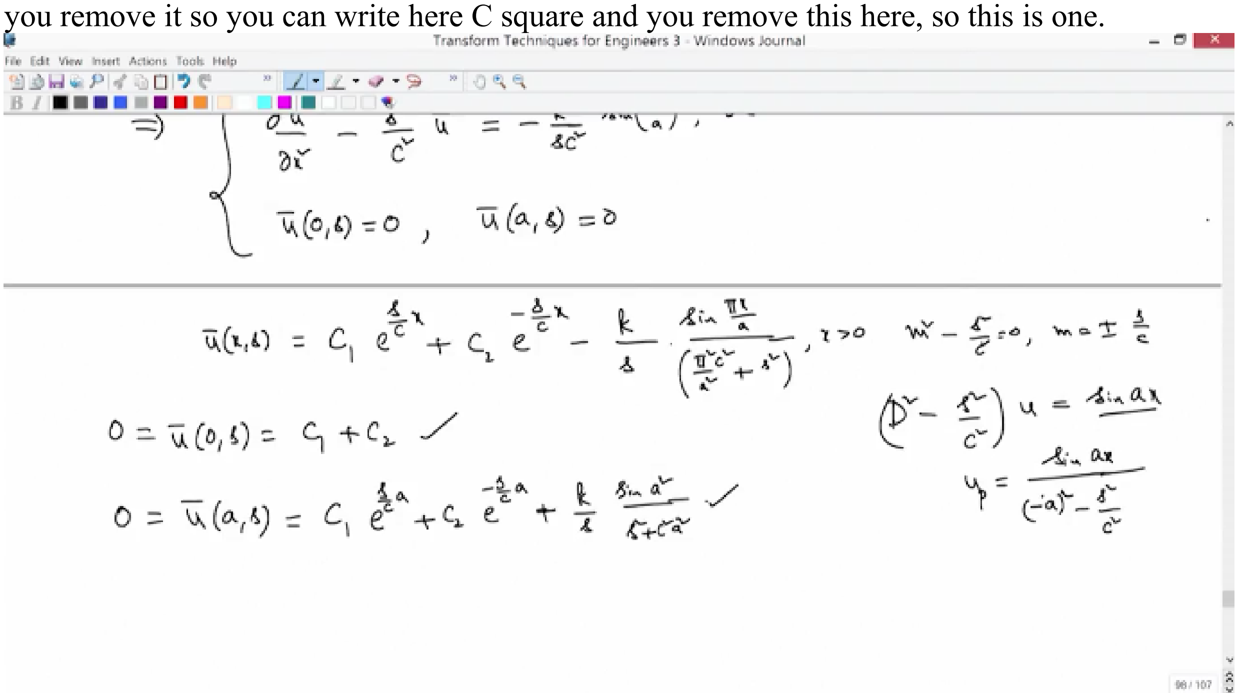
So what is its solution? So solution is so U bar(x,s) if you solve this ordinary differential equation, complimentary function is M square - S square/C square = 0, so M equal to + or – S/C so your typical solution is looking like C1 times E power S/CX + C2 times E power –S/C times X, then plus a particular integral so this particular integral you can get it because if you have a sine you write the equation as D square – S square/C square operating on U equal to some constant times whatever, if you have a sine, sine AX then what happens the particular solution is UP is 1 over, wherever D is there you put -A whole square -S square/C square this is your particular solution of course with sine AX, so this is a usual way to get operator form, so this is the shortcut way to find the solution, particular solution if you are familiar with the differential equations or otherwise you look for a particular solution of the form YP of here UP, UP(x), UP bar(x,s) you look for something like A times sine pi X/A + B times cos X pi/A, if you look for a substitute into the equation and you can get your A and B so that you see that A will be this one, okay, you'll end up C getting B = 0, and you'll finally get what is this value? This value will see that this is going to be something like this, okay, of course for this you have this, so here I have -K divided by SC square times and this you have a sine AX by this minus minus plus and here you get S square + C square A square, C square goes up so you have C square cancels okay, so this is how you get this.



So let me remove this, this is what you get, now this is X positive this is your solution, now I have not applied this initial boundary conditions, these conditions, so if you use these conditions U bar(0,s) = 0 that is C1 + C2 sine AX that is 0, so C1 = -C2, and what happens to other solution U bar (a,s) that is C1 times E power S/CA + C2 times E power –S/CA + K/S times sine A square which is constant, so S square + C square A square, so if C1 = C2 if you put it here so you see that C1 = -C2, so if this has to be, so if you solve this to you see that you consider these two equations this is one equation this is another equation you try to solve for C1 and C2 you end up getting, you may not have any solution at all, okay, this is inconsistent C1 + C2 cannot be same as you may get, determinant is non-zero, oh I think I made a small mistake over here this is not AX this is pi X/A, so K/SC square this is actually, you start with looking at this is a particular solution let me write I have written wrongly SC square sine pi X/A and S

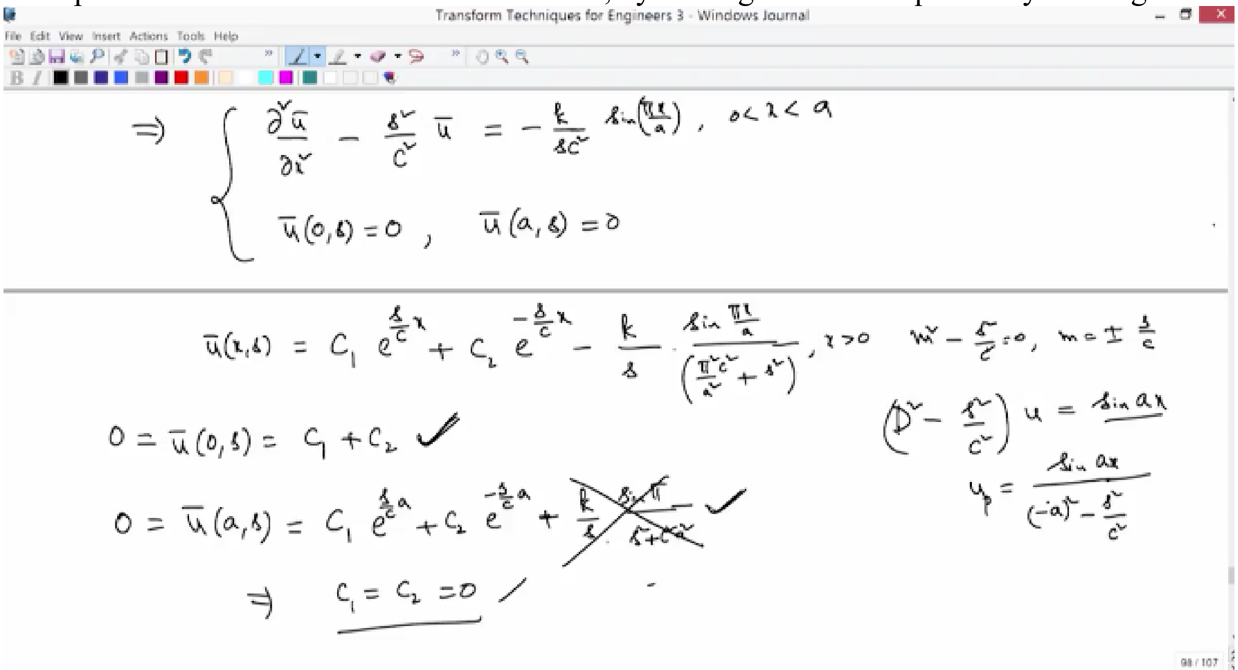


square, S square by minus, so you have here this is going to be A square, A is pi/A here, pi square/A square + S square/C square that is exactly what you have, because here A, this A is different, so what you have is pi/A is your constant, so pi/A whole square + S square/C square, so here you have C square A square comes out okay or rather I multiply C square, C square if you remove it so you can write here C square and you remove this here, so this is one.



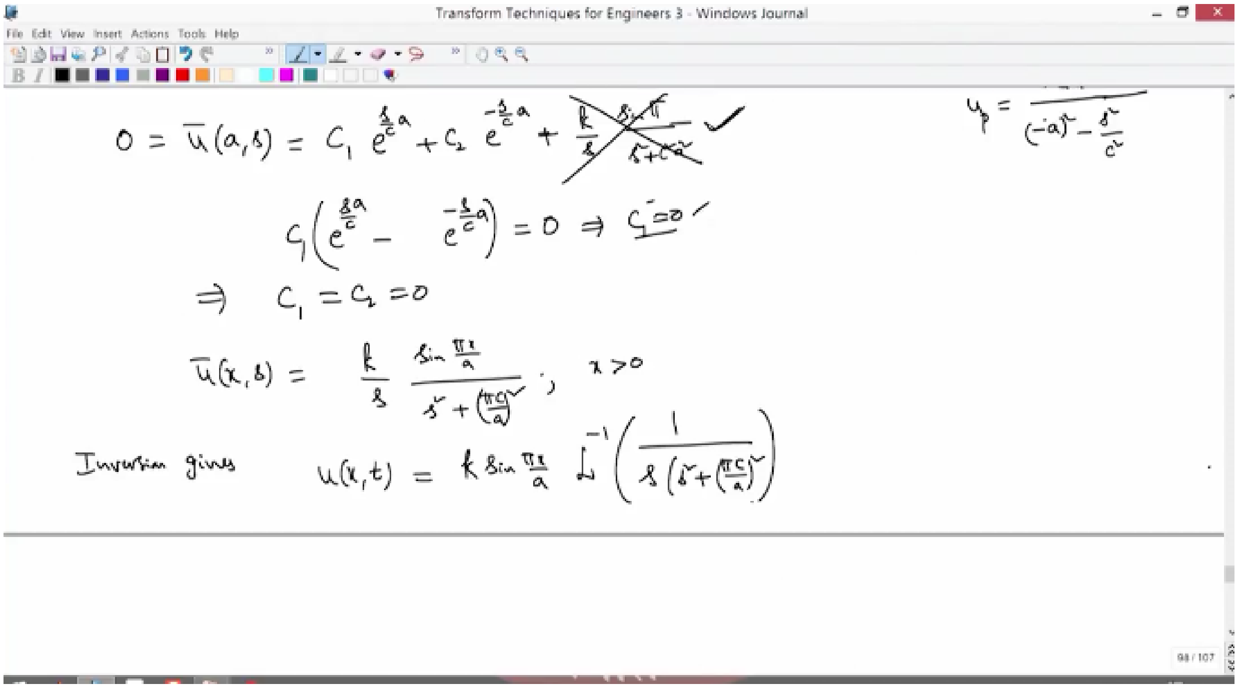
And if you now use this sine, it's a sine pi A/A that is one so sine pi, so this is 0 because of this and you can see that C1 has to be C2 has to be 0, okay, if you don't have this sine pi X/A pi and then you will end up getting C1 and C2 something for which then you will have to worry about how do you invert it, okay, you can also do that is a little complicated, okay, instead of sine pi

X/A if you take sine X/A then you have sine one you will get so this is a non-constant, you will end up some constants we will be able to solve, by solving these two equations you can get

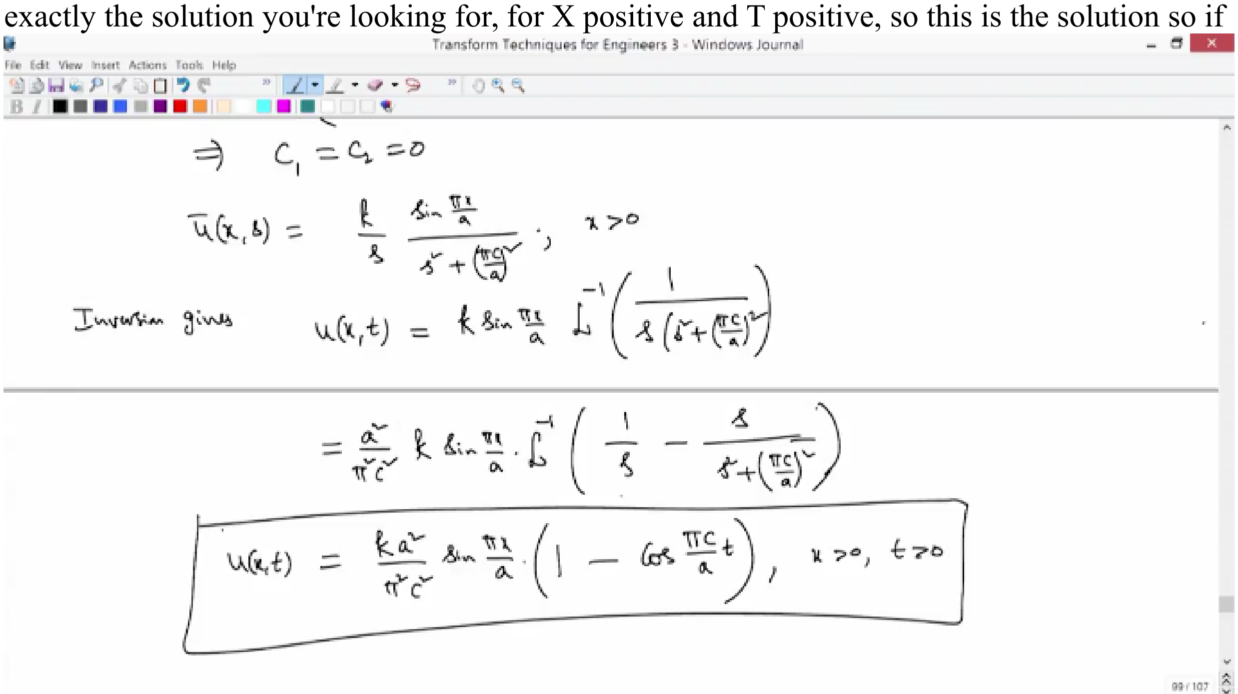


your C1 and C2 and you put it here, and now you have to invert it that is so you should know what exactly that Laplace inversion of E power AS, okay, what this is actually Laplace is so some heavyside function of T–A something you will get, okay, T+A whatever you get so you can use that and get this, so you can use, still you can use, let me use because I choose this pi X/A so things are simplified here.

So if you use this you have C1, C2 both are 0 because if you put C1 = -C2 here, so if you put -C2 so you end up getting, if you put it here so C1 and then E power S/CA, and this C1 E power –S/CA, because I put C2 = -C1 by the first equation, so C1 is common has to be 0, and this cannot be 0 so this implies C1 = 0, once C1 is 0 C2 is 0 from the first equation, so this implies C1, C2 both are 0, so that means your solution of that equation is of X is –K/S of course this becomes plus okay, and you apply this minus and that minus becomes plus I missed it, so K/S sine pi X/A divided by pi S square A square + pi C/A whole square, so for which this is your X positive and you have, you can invert it now, Laplace inversion gives U(x,t) this is your solution.



Now this is K sine pi X/A that is nothing to do with S variable, you have Laplace inversion of this you can write 1 over S times S square + pi C/A whole square, okay, this you do the partial fractions here, if you use the partial fractions K times sine pi X/A times L inversion of 1/S, let's say -1/S square + pi C/A whole square, so this 1 -S if I do so you end up getting this whole thing into you get pi C square/A square so if you multiply here A square/pi square C square if you do, then this is same as this one, okay, so you have KA square/pi square C square sine pi X/A, Laplace inversion of 1/S is 1 and then remaining this is cos pi C/AT okay, so this is exactly the solution you're looking for, for X positive and T positive, so this is the solution so if



you have a string of finite thing which is fixed at both ends and initially at rest, and by the external force, that force is maybe wind or something but it is in this form for all at, at every

point X you have that much pressure on the string, then for all times if you keep that thing the string vibrates in this fashion, okay, so this is how you solve this wave equation or hyperbolic equation with the initial boundary conditions, okay, if you have a wave equation or any hyperbolic type of equation with initial boundary conditions you can apply a Laplace transform and finally inverting it use all these initial and boundary conditions you get your U bar(x,s) and finally you invert this to get your solution U(x,t) okay, so this is how we solve this typical hyperbolic equations.
Similar such equations can be solved but there's a limitation, not all boundary conditions, all boundary value problems you can solve for this any equation rather not just hyperbolic, even other equations not all equations, not all boundary value problems you can solve by this technique, certain simpler some equations which you cannot solve by other, which are difficult to solve by other methods this Laplace transform will give you nice and easy solution, so it has its limitations, only limitation is you may end up getting U bar(x,s) as something a little complicated, and for which if you to find the inversion you may not get that easily so you have to apply your contour integration technique to find that Laplace inversion, and finally to see its explicit solution.
So in principle you can apply this Laplace transform and it's inversion to get the solution for this boundary value problem, initial and boundary value problems for this hyperbolic type of equations, okay. We'll see next, in the next video we will see how to solve heat equation with initial and boundary conditions, and different problems we'll try to work out using Laplace transform, we'll see next video. Thank you so much.

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