NPTEL
NPTEL ONLINE COURSE
Transform Techniques for Engineers
Inverse Laplace Transform by Contour
Integration
With
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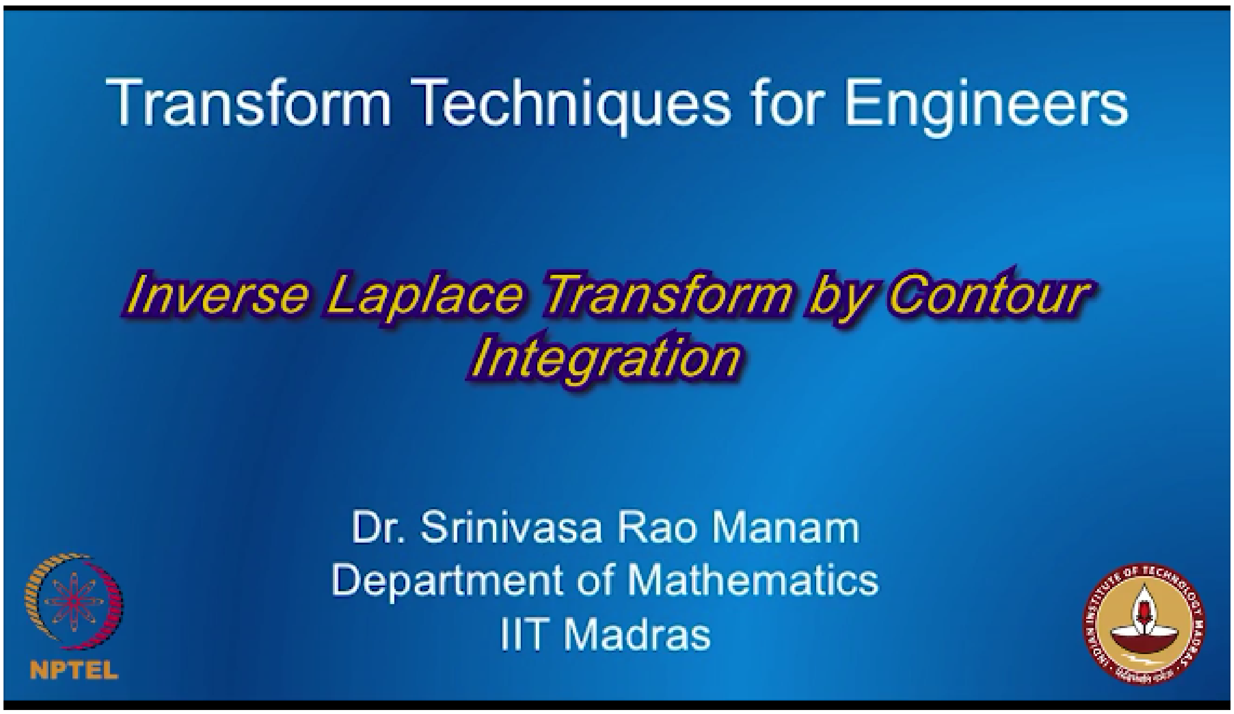
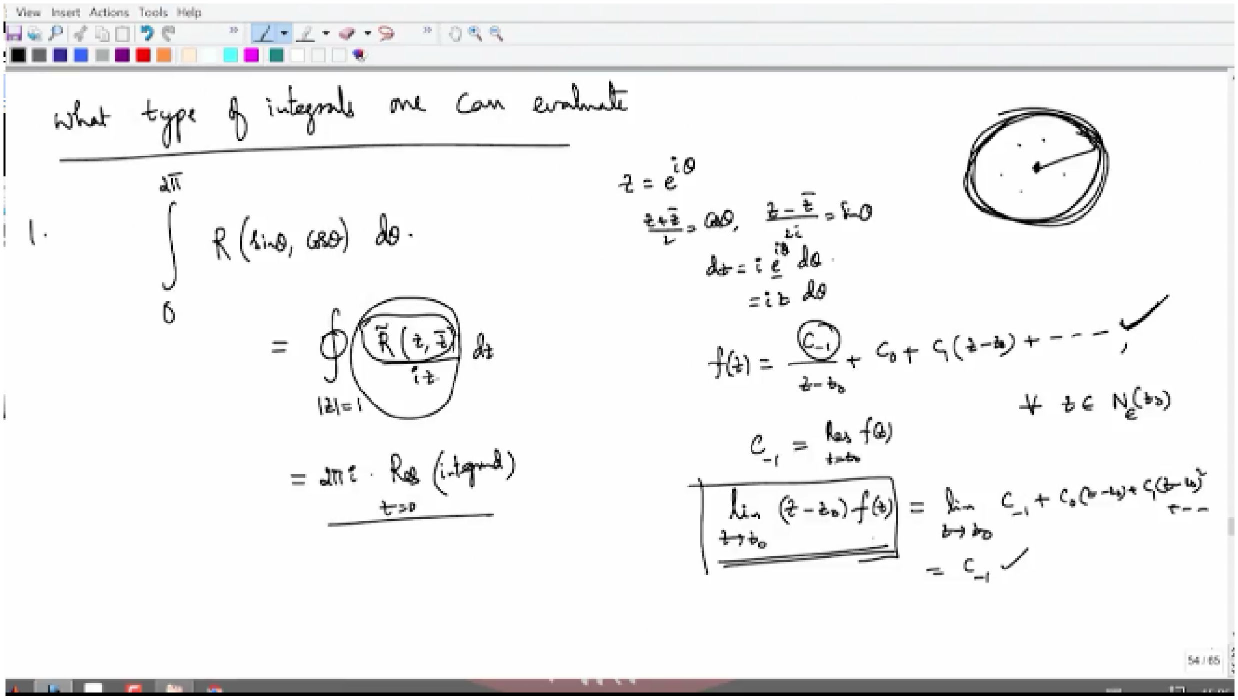


Welcome back, in the last video we have seen review of complex function theory up to the point where we have seen the Cauchy residue theorem, if you have analytical function in some domain D, except at some finite number of singular points, singular points can be poles of simple poles that means it's like 1/Z – Z naught or pole of order M that is like 1/Z-Z naught power M, so Z naught is, in this case Z naught is a pole of order M otherwise that's essential singularity that is where if you look at its Laurent series that means if you consider in the neighborhood of Z naught if you make a, suppose Z naught is the singular point, in the neighborhood of Z naught within that disk except that points Z naught you can have some series expansion that's a Laurent series, that's a theorem you can look into the book, complex function theory book for example Churchill where you can find you can have an expansion for F(z) in terms of Z- Z naught powers both negatives, negative powers and positive powers of Z-Z naught that means 1/Z-Z naught and Z naught power M are Z-Z naught power M, so where m is positive number, so positive integer.

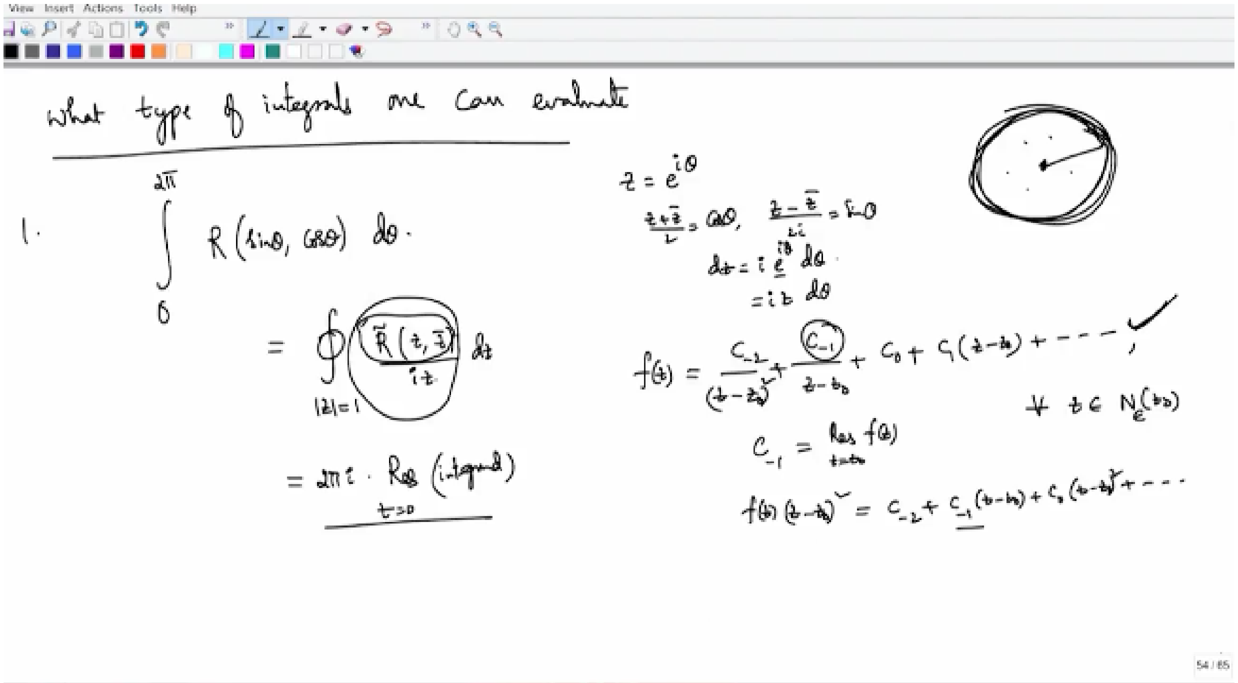
So that way you can have such a Laurent series expansion, and if you have a negative, you have infinite number of negative powers you say that, that singular point is essential singularity. For example E power -Z square 0 is you'll have 1/Z square E power -1/Z square or E power -1/Z if you look at it has all the negative power, so 0 is essential singularity okay E power 1/Z for

example, if F(z) is E power 1/Z then you have this infinite negative power, so 0 is essential singularity.
So we will see in this video how we can evaluate certain real integrals which are improper integrals or usual integrals, how do you make use of this Cauchy residue theorem to evaluate these integrals.
So let me start with this, what kind of integrals you can evaluate? What type of integrals one can evaluate using this complex function theory or Cauchy residue theorem, so for one simple thing is if you have anything from 0 to 2 pi integrals and some kind of rational function of both a sine theta and cos theta, and if you have D theta, if you have something like this what we do is so you have a rational function of sine and cos theta, so what I do is I consider Z as 0 to 2 pi, so I consider Z as E power I theta as a unit circle, unit circle I can represent like 0 E power I theta, theta is between 0 to 2 pi, so that also whatever the idea is to make this kind of integral as over a unit circle, for example unit circle is mod Z = 1, a closed curve so this is over this 0 to 2 pi I make this and real part of sine theta, so in this sine theta is Z + Z bar/2 is cos theta and then Z-Z bar divided by 2Y is sine theta, so you simply substitute R of sine theta as Z- some other function R tilde of Z and Z bar.
What is D theta? DZ is I times E power I theta D theta, so E power I theta D theta, E power I theta is Z so you have IZ D theta, so D theta you can replace with DZ/IZ, so once you have this you look at this function as your integrand over this closed interval, so you see Z = 0 is a singular point, 0 is a singular point so for which you calculate this is a 2 pi I times a residue of, if 0 is the only root you calculate this residue of whatever the integrand, residue of the integrand at Z = 0, if this is 0 is the only point if this rational function is having some other singular points which are inside this unit disk or unit circle, within the unit circle you have to find those residues sum of all the residues, if I assume that 0 is the only singular point because 1/Z are R tilde(Z and Z bar) if it doesn't have, if it's analytical function so you can have this residue, this is say this is the integral so this is the value of the integral for this, so this is one such type you can calculate.
So what is the meaning of residue, how do I find this residue, let me calculate, so if once you have this F(z) as a Laurent series expansion as I explained if you look into the textbook you will understand that if you have F(z), if there are three types of singularities, one is a removable singularity that is not a singular point is like sine Z/Z, if you look at this it is defined as a function of FZ is a function is defined for every Z naught = 0 okay which is analytic at every point which is Z naught = 0. At Z = 0 what happens? Z = 0 this limit of F(z) as Z goes to 0 is limit of sine Z/Z, just like usual limit if you calculate is 1, so if you define this as a function as 1 then this is analytical function, okay, so you can remove the singular, there is a 0 is not a singular point if you define it like this 0 is not a singular point, so this is such a, singular point is called are removable singular point that is not of our concern, what we have is if it is a pole you can get the Laurent series expansion like if it is Z naught is the pole for this function you can get this as some C-1, Z-Z naught + C naught + C1 Z-Z naught and so on, you get something like this, this is the expansion in the neighborhood of Z belongs to the neighborhood of Z naught, this is valid okay, this is how you get, you can get the expansion Laurent series.
And then C-1 is a residue, residue of F(z) at Z = Z naught, so in general without expanding this you can calculate what is that, from this suppose you have this form what is this C1? C1 is actually Z – Z naught into F(z) you consider this limit Z goes to Z naught, so this is exactly if you multiply this what you get is limit Z goes to Z naught, Z-Z naught into C-1 that is going to be C-1 + C naught times Z-Z naught + C1 times Z-Z naught square and so on, so as you see this

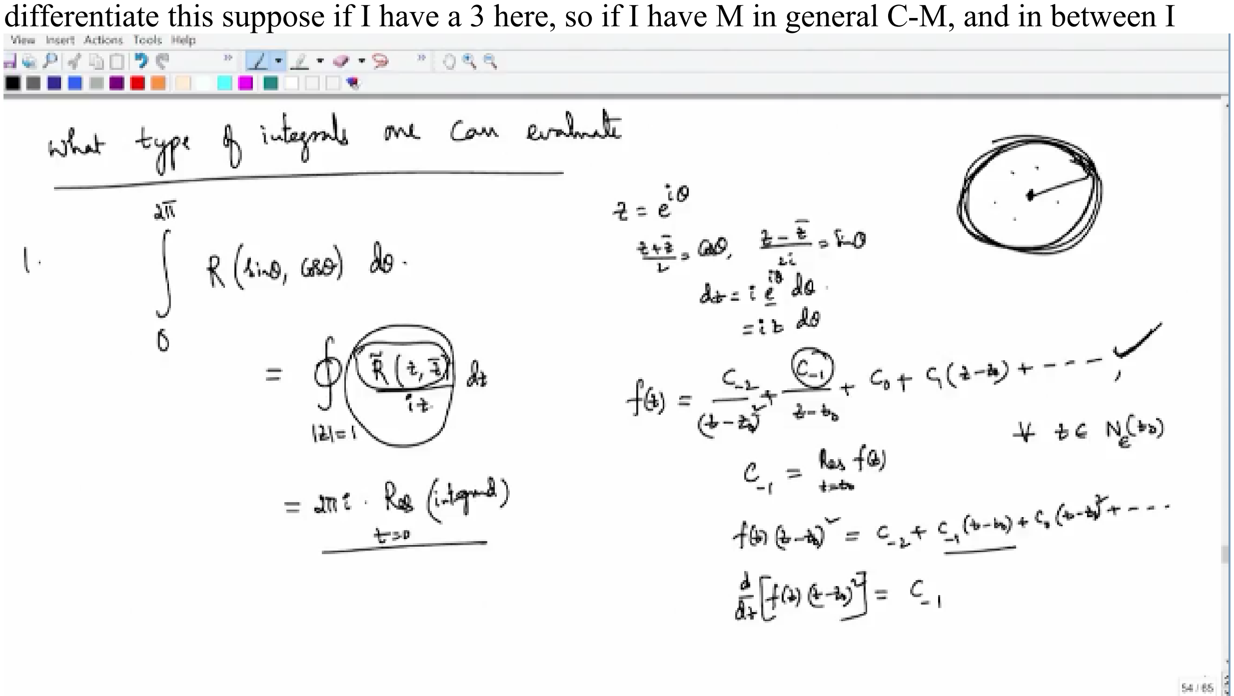
is going to be C-1, so this is exactly your residue, so if you calculate this, you can easily calculate this limit then expanding a function as a Laurent series and looking at its powers of Z-Z naught, okay, this is how you find if it is a simple for pole, that is a pole of order 1 if you have this is how you calculate the residue, so this is the expression so you use this one to calculate the residue.



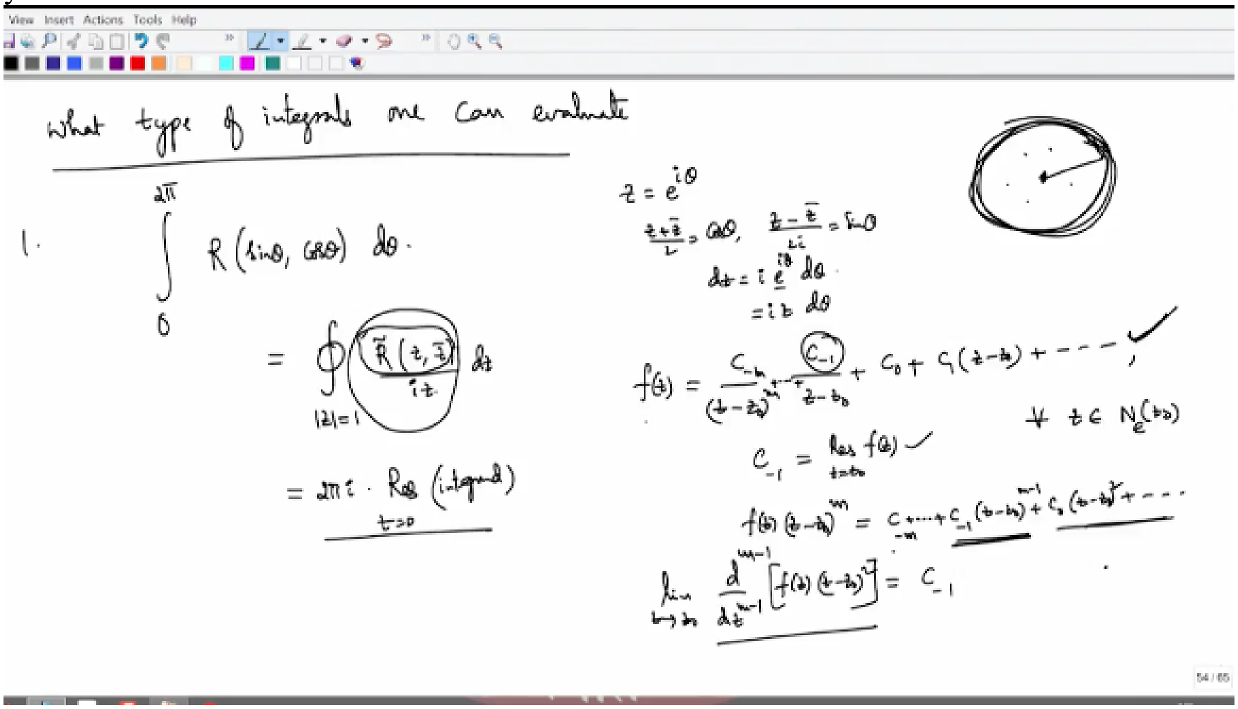
If it's, for example if you have a pole of order 2 of, in general pole of order 1/M, so what you have is suppose if it is a pole of order 2 let me do, so you have some constant times Laurent series will be like this, so but what you require is only C-1, so C-1 what you do here is so if you want to calculate C-1 to get that C-1, I'm just removing here and do, so we need we're looking for C-1, so what I do is I calculate F(z) times, I multiply Z – Z naught whole square okay if I do this C-2 + C-1 into Z-Z naught + C0 times Z-Z naught square and so on.



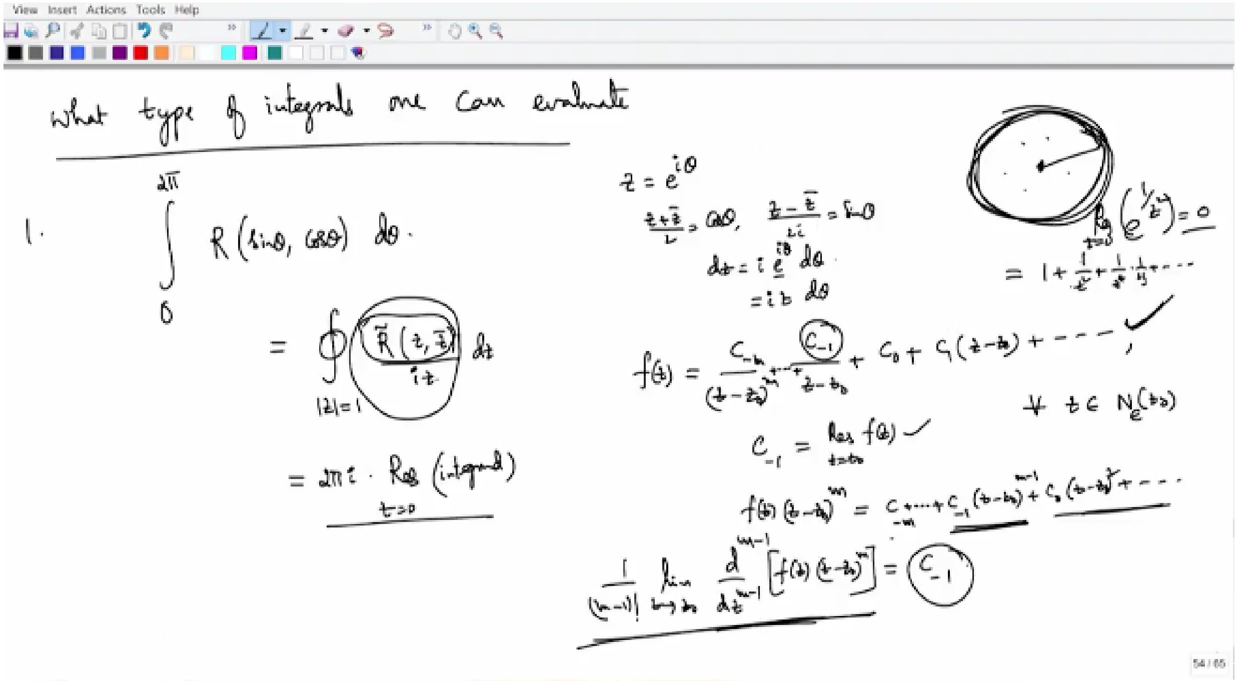
So how do I get my C-1 is simply a differentiate, D/DZ of F(z) times Z-Z naught whole square, if I do this if you are differentiate I get my C-1 okay, and then so this is exactly my C-1, I differentiate this suppose if I have a 3 here, so if I have M in general C-M, and in between I



have terms which are up to C-1, so I'm still I am looking for C-1 what I do is I multiply, I multiply Z - power M, so you have a C-M, C–, so you will have terms like this up to here and you have here C-M-1, okay, so you have to differentiate this up to M-1 terms D power M/D Z power M-1, so that is exactly your C1 right, so this one and then you take this limit, limit Z goes to Z naught, limit Z goes to Z naught if you take if you differentiate and take the limit Z goes to Z naught all other terms will become 0, you differentiate M-1 times and then you can get your C-1, other terms will simply become 0, okay, so bigger terms if you do still Z-Z naught terms will be there or 0, so Z-Z naught is there you, anyway taking the limits Z goes to Z naught those are all becoming 0, so except only this contribution will be C-1, so that is exactly your residue.

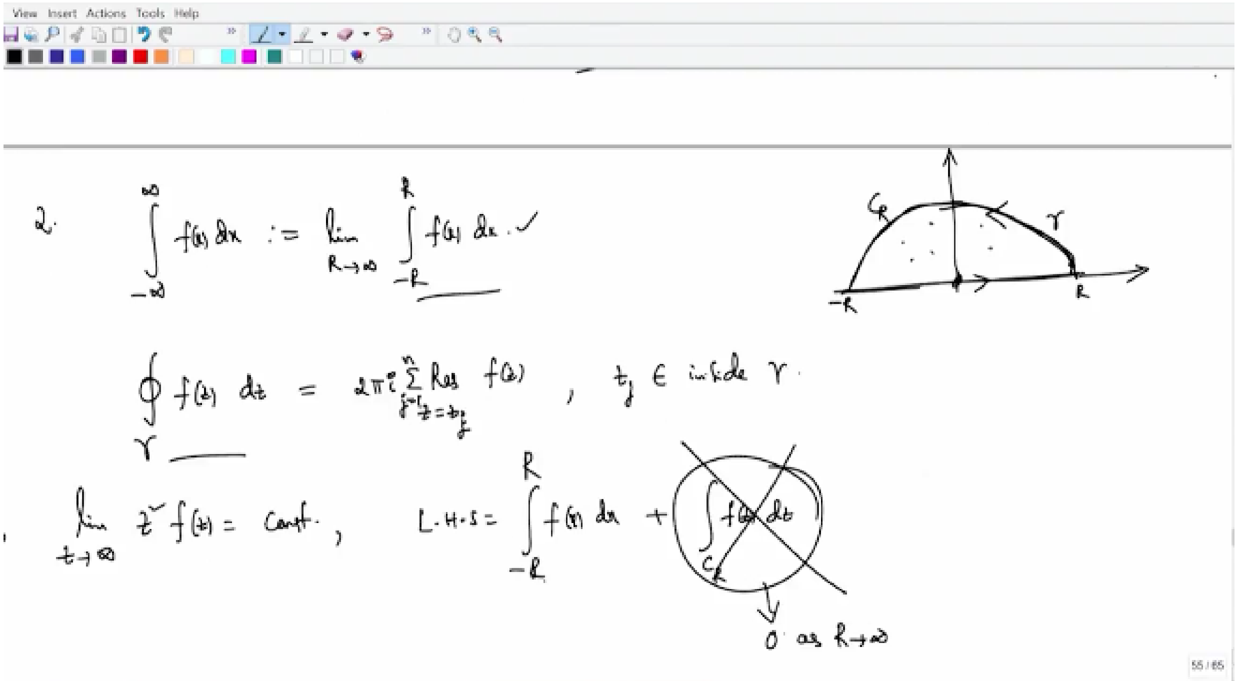


So if you have a function F(z) which is having a pole at Z naught of order M you calculate, so what you get is when if you differentiate M-1 times you get M-1 factorial here, so 1/M-1 factorial if I divide this side if I bring this side I get this one, okay, so C-1 is 1/M-1 factorial times you calculate this limit so here I have power M, okay, so you multiply, so F(z) times Z-Z naught power M you would for this, this you differentiate M-1 times and take the limits Z goes to Z naught that is exactly your residue, so this is how you calculate if it's a pole how do you find the residue, so if it is essential singularity if so happened that if your integrity is essential singularity something like 1/Z you don't have choice 1/Z or Z square you get the exponential expansion so 1 + 1/Z square by 1 factorial and so on, okay, 1/Z power 4 into 1/2 factorial and so on.



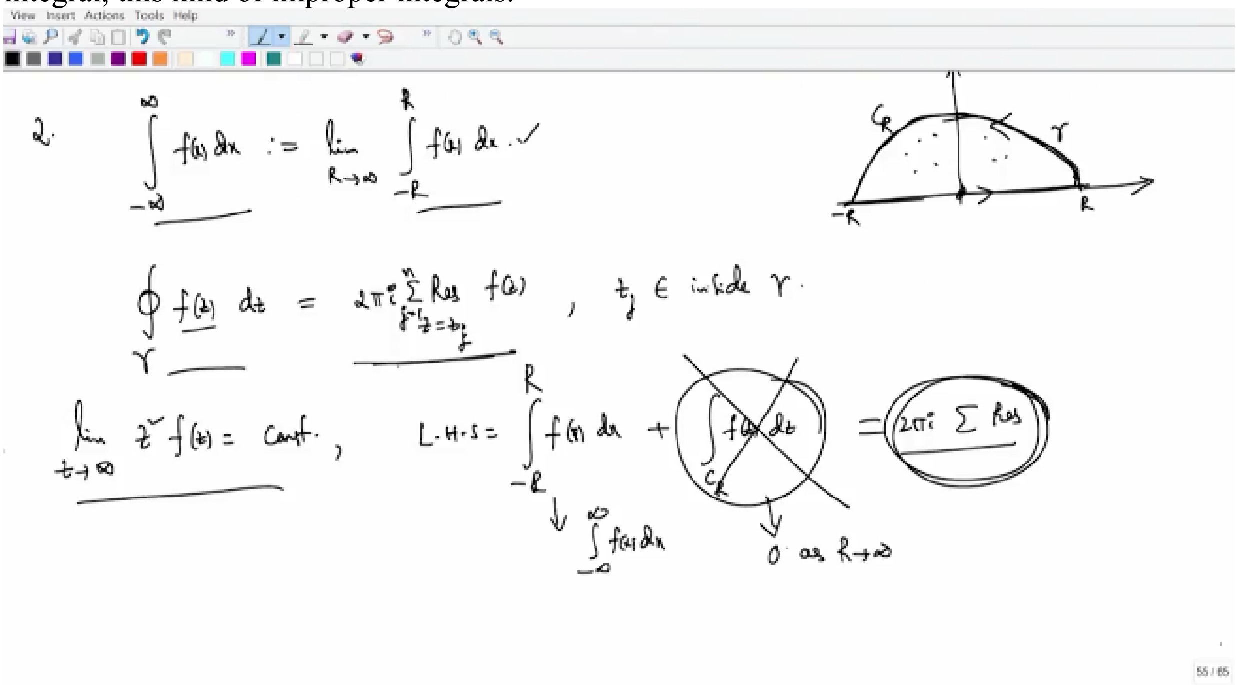
So what is the coefficient of 1/Z? That is 0, so there is no 1/Z coefficient so the residue of this function at Z = 0 is actually 0, because 1/Z coefficient is not there, if I have 1/Z this will be 1/Z so in this case we have 1/Z square and so on, so in this case 1/Z coefficient is 1 so you have residue is 1 for essential singularity at Z= 0, if your function is having essential singularity.

So basically I have three types of singularities, pole, simple pole, pole of order M or essential singularity, so in all these three cases you can find residue which you can use to evaluate this, to evaluate your integrals here, so if you this kind of simple real integrals okay, integral 0 to 2 pi rational function of sine theta cos theta if you have, you can evaluate and that is first type of simple integral, a second type is you can evaluate some improper integrals for example I am not giving, I'm not giving any example to do here, so you can look into the book and see the example how to do in this case.



Second type of integral that we can evaluate is improper integrals for example -infinity infinity F(x) DX so this as a definition we have seen that this is actually R goes to infinity -R to R F(x) DX this is an improper integral, so we can evaluate such integrals by considering F(z) DZ over a closed curve gamma okay, what is that gamma? Exactly to evaluate this type of integrals we can evaluate like this, if we consider something like this if you can write it like this so this is -R to +R so this is the integral, this is the contour you get this closed contour you try to get this is a semicircle like this if you can, if you use this semicircle as your contour gamma by considering this this is actually you have seen because if F(z) is analytic except at finite number of poles inside, then this is actually 2 pi I times residue of F(z) at all the poles at Z = ZI, so I is running from 1, 2, so let us say GJ, J is from 1 to N, so where ZJ are, ZJ belongs to inside gamma, inside gamma, okay, so this is so this way you can do, but then what happens to the left hand side where this is coming into picture, so if F(z) is of order of like 1/Z square, so that means you take this Z square multiply and take the limit Z goes to infinity and this is going to be constant, if this is a constant R for example F(z) is like 1/Z square +1 or Z/Z cube + or Z power 4 +1, so you see this is order of 1/Z cube, if it is Z cube this is order of Z square, if it is 1/Z square it is order of Z square okay, so such a functions if you consider you can easily show that by just considering the parameterization over this semicircle one can show that this integral, this integral left-hand side will be, if F(z) is such that this is the order of 1/Z square that means this, then this left-hand side we can write this as integral over, 1 integral is –R to R F(x), so here we are on the real line so you have F(x) DX so that is exactly what you want plus other integral that is over a semicircle, so semicircle if you call mod Z is, so semicircle so how do I put it? So if you call this semicircle as C, let me call this CR, CR is my semicircle only this part, upper semicircle if I write like this C/CR, now you are in the complex plane you have to write F(z) DZ.

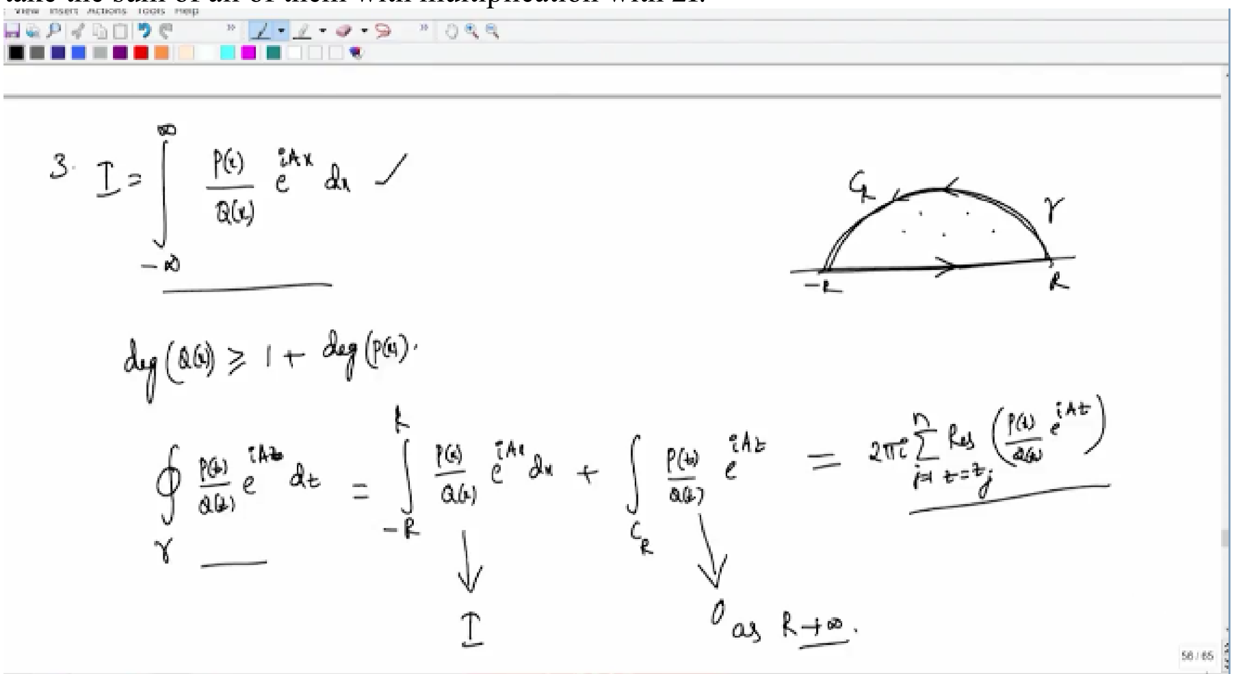
If F(z) is such that this I can show that this is actually this goes to C, C capital R here actually here, okay, is of radius R from 0 origin, so you can show that this goes to 0 as R goes to infinity, so this eventually this will become 0 as R goes to infinity this goes to -infinity infinity F(x) DX, and this is right hand side nothing to do with R so this is as it is 2 pi times sum of all the residues, okay, so this you calculate you can easily calculate for the given function F, if F is of this form and the left hand side becomes this, this is exactly what you require so this equal to this, so if you know all the residues of this function at the singularities inside this closed curve that means you know the value of this integral, so this is how you calculate this function, this integral, this kind of improper integrals.



I'm just giving you briefly idea how to, how you can getting it, so if you really want a complete picture you can look at the technicality of this in the textbook.

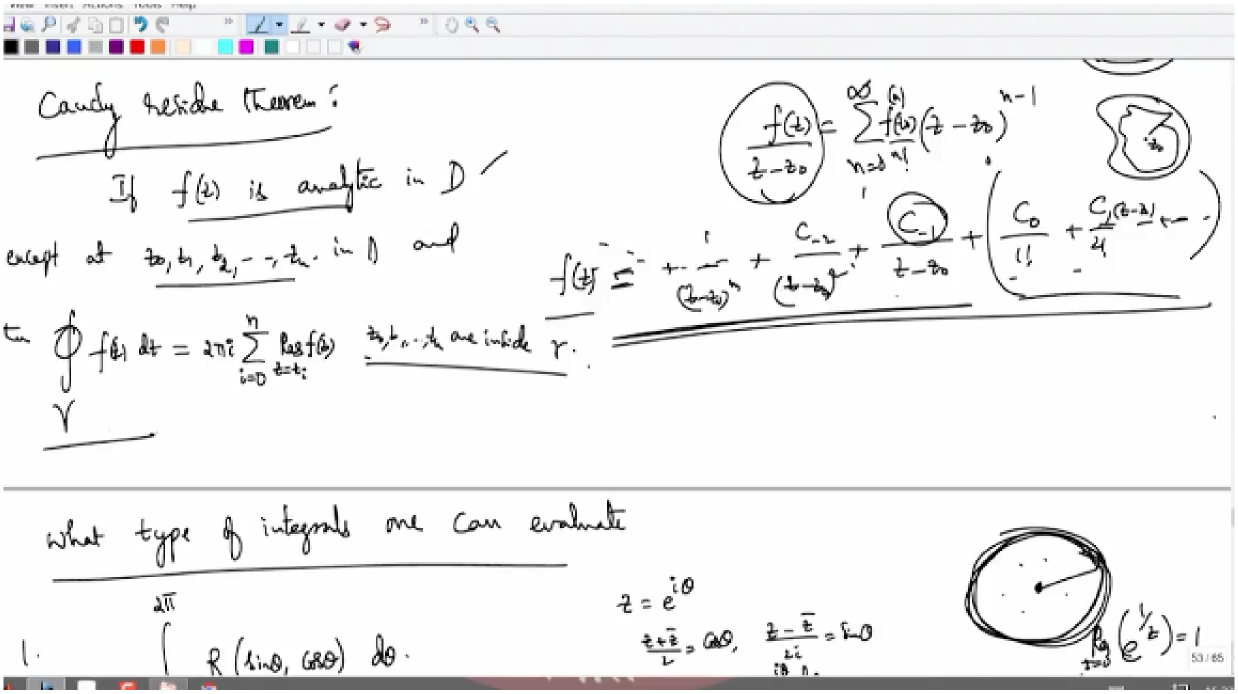
So let me see the other type, other type you can easily see is using if you have -infinity to infinity, and if you have a PX/QX these are the polynomials, let's call this these are the polynomials of minimum, so here I have chosen F(x), F(x) is such that order of F(x) is like 1/Z square, okay, that means numerator is what exactly this mean is? F(z) is P(z) divided by Q(z), where P(z) Q(z) are polynomials but minimum degree of Q is more than or equal to 2, so minimum degree of Q is actually equal to degree of P+ or greater than 2, greater than or equal to 2, so degree of Q-P or rather how do I put it, so degree of P is smaller than, degree of Q is more than or equal to 2, more than 2 of degree P, okay, so if it is Z here you should have Z cubes here, okay, 2Z+ this is the order of 1/Z square, this is the form, this is the meaning of this, okay, but if it is not like this if you have some oscillatory functions like E power if you have E power I for example cos X or sine X, cos TX or sine TX okay, or let me put it like E power ITX, I some, let's say AX, A is a constant AX DX, if you want to evaluate this, if you want to use the same kind of technique same contour what you need is a degree of Q(x) has to be equal to, it's more than is equal to 1+ a degree of P, degree of P(x) this should be one more than, a degree of P if you have then you can apply the same technique using this what I do is in this case to evaluate this integral I consider over this contour -R to R, and this is like gamma so same way you can calculate, gamma is the closed curve and if you call this as C capital R as a semicircular arc, so you consider this over gamma the closed curve this P(z)/Q(z) times E power IAZ DZ.

So you consider this integral and this is a -R to R P(x)/Q(x) times E power IAX DX if it is on this plus the other integral that is over CR P(z)/Q(z) and E power IZ, so over this semicircular arc one can show that this goes to 0 as R goes to infinity, okay, so this goes to what you require so as a integral what you want, and this is actually equal to this and which is by Cauchy residue theorem you have 2 pi I times all the residues again of this P(z)/Q(z) times E power IAZ for this function at all ZJ, all these ZJ's are inside and you take the sum, J is from if they are N and you take the sum of all of them with multiplication with 2I.

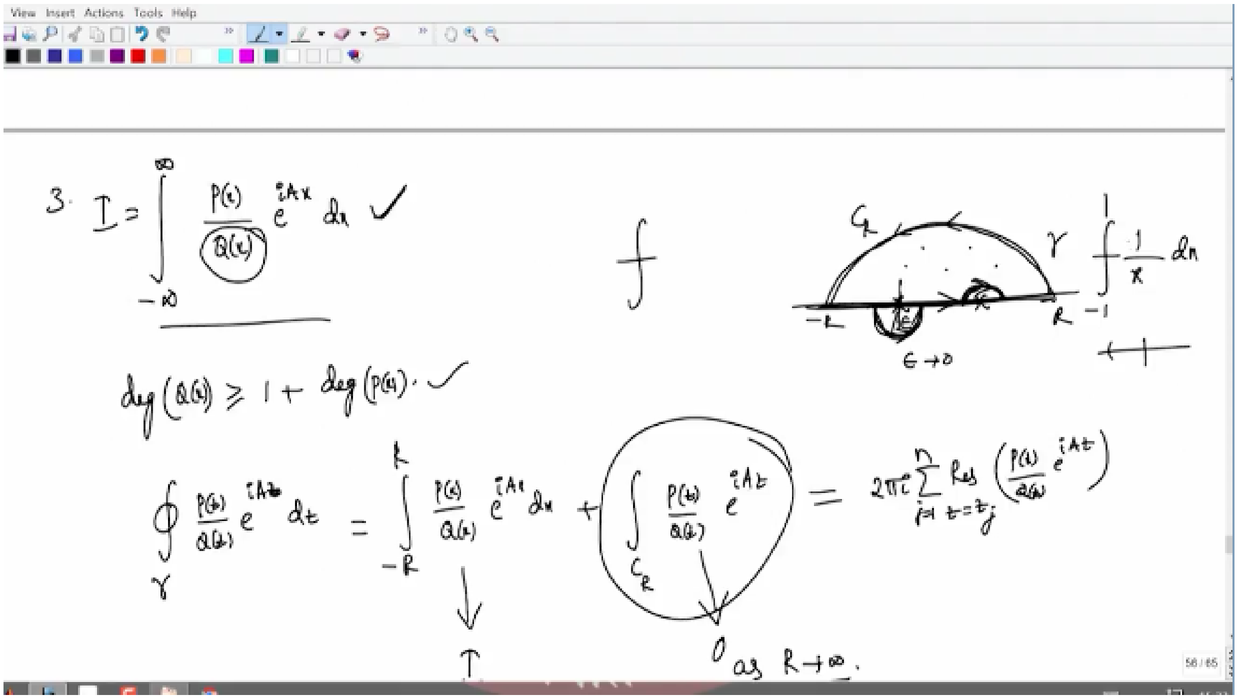


And if Q(z) is having a singular point here you indent it, you indent it with contour like this so that you contour like this, you take so you look at this one so this contribution here will give you some other things, so that is going to be this one + pi I times or I think minus or minus or plus, so there will be contribution here, okay, so you have to consider this contour and then look at the same way on the small piece it is a contribution, this integral will be 0 but here this is going to be, that's going to be so in that case if you call this, it's a cut you are actually removing this point, so equally you're going from 0, if it is a 0 so such a thing is called principle value of the integral, okay.

So let's not bother those kind of things are not required here, so if you look into this kind of techniques, this kind of, this is how you pick up proper contour, closed contour and look at the proper integrand as a complex function, and so that your integrand comes into the picture over real line, and other part you should be able to make it 0 as R goes to infinity, if your integrand is sufficiently behaves, if your integrand behaves sufficiently sufficient conditions, for example here is this one earlier you have this, this is the condition, okay, so if this is satisfied you can actually get this integral using this Cauchy residue theorem, so this is how you can get, there are many other things for example when you have roots Q(z) is having a real roots here you cannot allow this contour because this is, on the contour you have a singular point, as you see Cauchy residue theorem in the earlier video you have to see that if F(z) is analytic in D except at this few points in D and gamma is inside D, so gamma, so if you look at Z naught, Z1, ZN

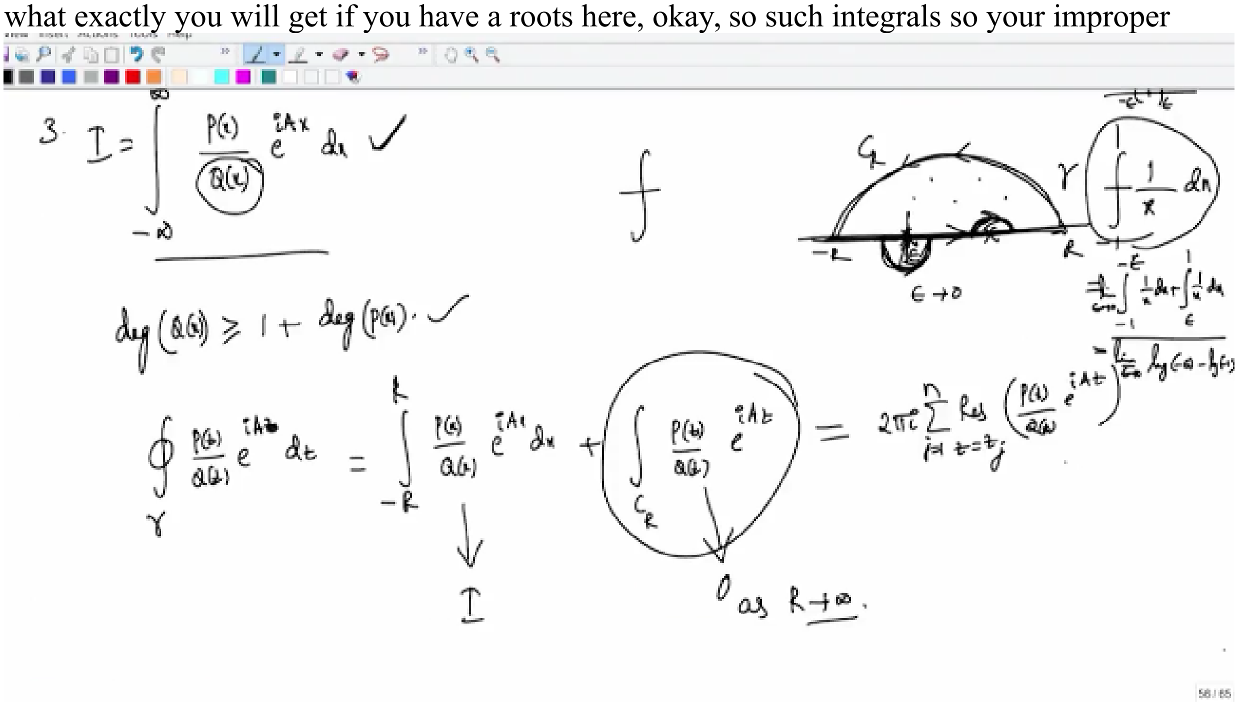


are the singular points inside gamma but not on gamma, okay, so it's like inside gamma and all these singular points and you are looking at the integral over this contour gamma, so here also if you can, but if you have a Q(x) is having a root here you cannot do that, in that case you take the indentation like this or like this whichever way you can take, and then evaluate over this so that you are, at finally this is let us say epsilon, you allow epsilon goes to 0 so over this you calculate as epsilon goes to 0 the contribution of this that may be 0 and the contribution here that makes it here, you're actually removing this point equally so that is a kind of a singular, that is in that case what you get this integral here is as R goes to infinity epsilon goes to 0 you have a cut integral, that is a Cauchy principle value of the integral, so you remove this point you look at the integral so Cauchy principle value of the integral is if you want to calculate Cauchy residue value of let us say 1/X around 0 you consider integral from -1 to 1 if you want DX this as such it is not integrable because 1/0 is the singular point, if you cut it then you have a Cauchy integral value this is, how do I do it? This is like 1-1- epsilon 1/X DX + epsilon to 1

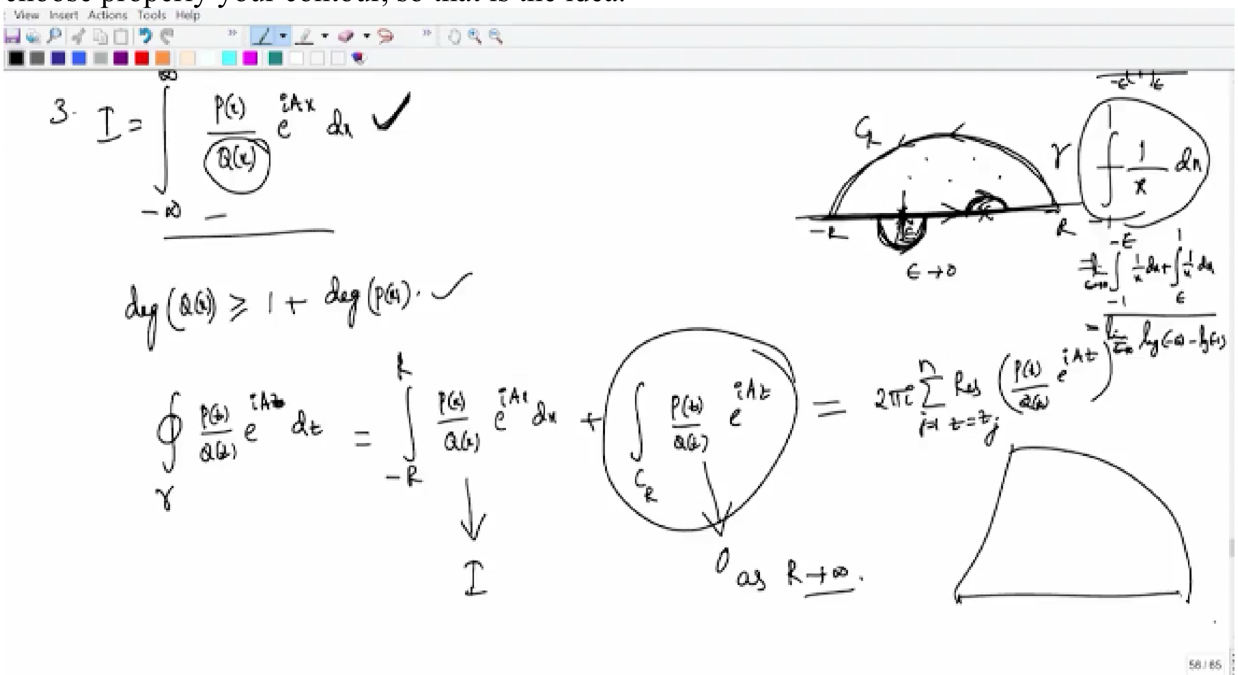


1/X DX you look at this way, so you remove this point 0 you cut it as from - epsilon to epsilon and finally you allow epsilon goes to 0, so this limit as such may exist.

For example this is log -epsilon -log -1 + log 1 is 0 -log epsilon, this -log epsilon, so log epsilon log epsilon if you cancel it with of course log-, that contribution E I pi - log -1 something like this, so this limit will exist as a, if this limit exist as an epsilon goes to 0, okay, I'm not doing the calculations here, if this limit exists you say that principle value of this integral exists that is what exactly you will get if you have a roots here, okay, so such integrals so your improper



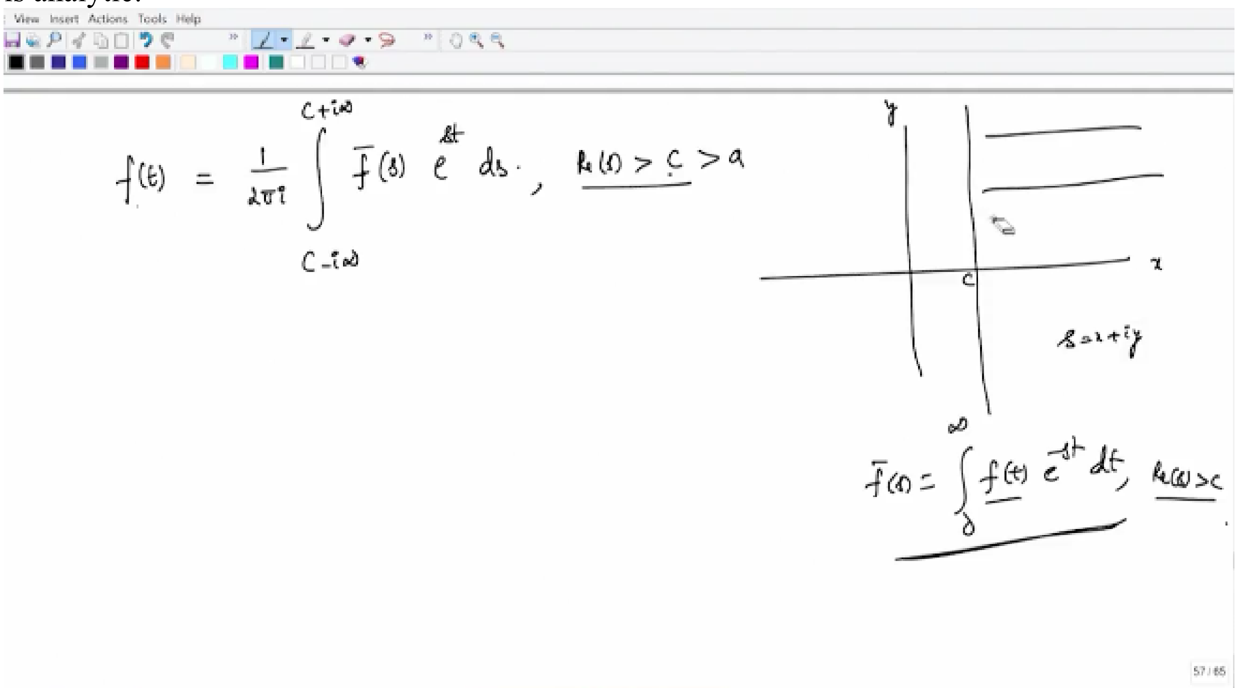
integrals will be Cauchy principle value of the integrals, if you have a singular point on the real line, so those integrals you will be evaluating by the same technique if you have a singular point here, so in that case this I becomes Cauchy principle value of integral if Q(x) is having a real root, but the technique is same, so this is how, this is just one of the glimpse of how you apply Cauchy residue calculus, Cauchy residue theorem to evaluate some real integrals which are, your simple integrals or finite integrals that is some 0 to 2 pi type of integrals are improper integrals where the integral from -infinity infinity, also R from 0 to infinity also you can do but you have to properly choose your contour, if you want 0 to infinity, 0 to infinity part should be there on this and you close this with some other curve for example like this, so but only thing is you have to make sure that wherever complex plane you have that maybe, so you how to choose depending on what is your integrand of your integral you want to evaluate, you have to choose properly your contour, so that is the idea.



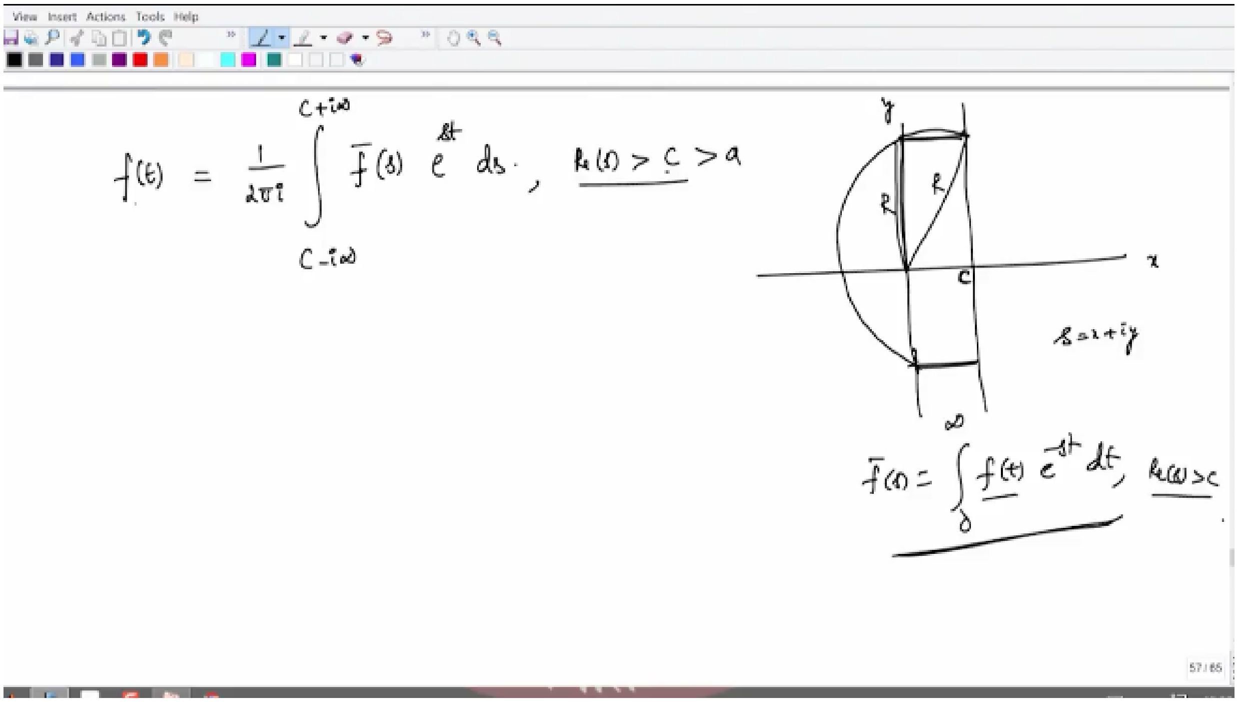
So if you want to evaluate certain integrals, certain integral using contour integration you have to smartly you have to properly choose your contour in the complex plane and make use of Cauchy residue theorem to get the integral. And remaining part of the contour so the one particular piece of the contour you get what you want, and the other particular pieces you will be able to evaluate or you will be able to show that that goes to 0, okay, so that is the idea, basic idea of how to evaluate the contour integration, the complex function theory.

So let me not go into details, many details of this what we need is we are going to evaluate this integral Bromwich contour to evaluate our Laplace transform F(t) which is 1/2 pi I times and what you have is C-I infinity to C+I infinity, this is F bar(s) times E power ST DS, S is the complex thing so to evaluate this what we do is, S is the complex plane X and Y are the, X + IY is S, S = X +IY this is the complex plane, S is a complex variable, so and you have C is somewhere here so that all your F bar(s) is analytic this side, this side F bar(s) is analytic where if you have C, that is how you've got this, okay, where C is bigger than, real part of C is or rather real part of S is bigger than C, this is how you have seen, right, C which is bigger than A, A is the exponential, order of exponential of this function F(t) okay, F is a function of exponential of order A, if F is a function of exponential of order A you have to choose C such that, C is bigger than A, and B and C you can see that real part of B and C, F(s) is analytic because there is nothing singular, everything is, there's no singular point on the right hand side.

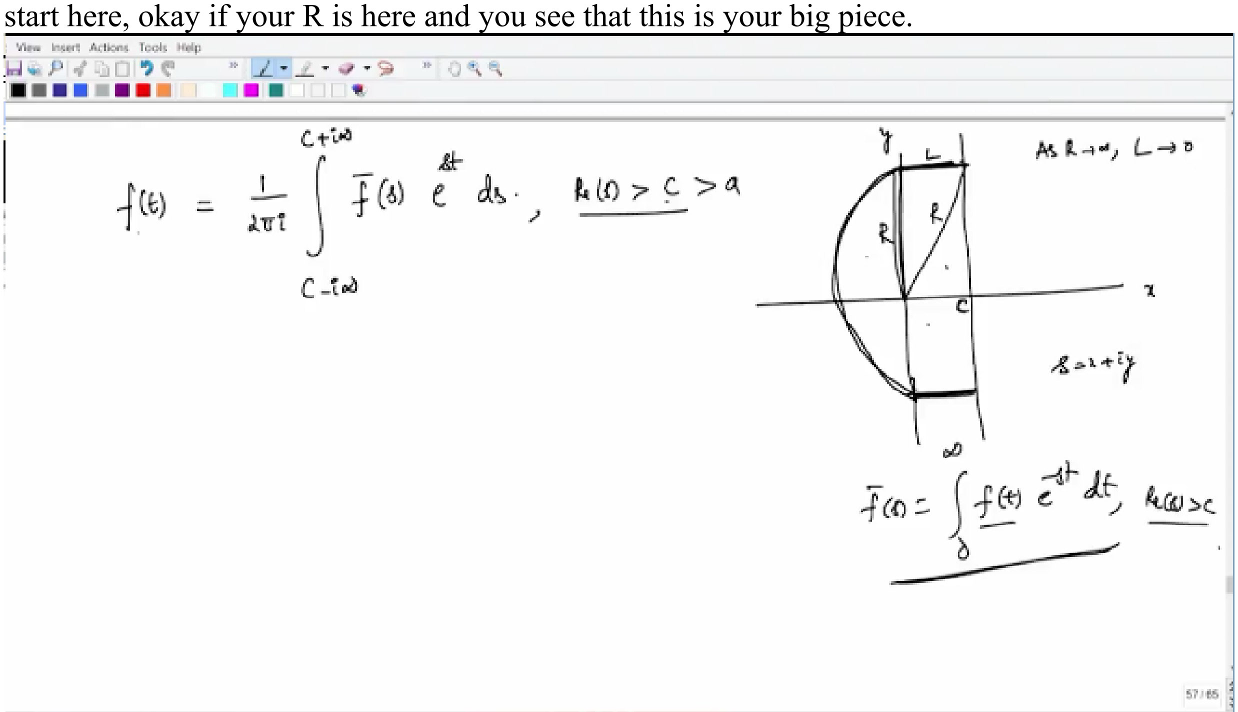
So F bar(s) is analytic you can easily see by looking at that integral, F bar(s) by definition that 0 to infinity F(t) E power –ST DT, so one can show that this is analytic by a complex function theory that this is in the, when you have, when you choose real part of S is bigger than C, so where C is bigger than the function exponential order of F(t), A is exponential of order, A is order of exponential of F(t) and if you choose C, any C you can choose and beyond this C that is the real part of S is greater than C that means this plane, this half plane this function F bar(s) is analytic.



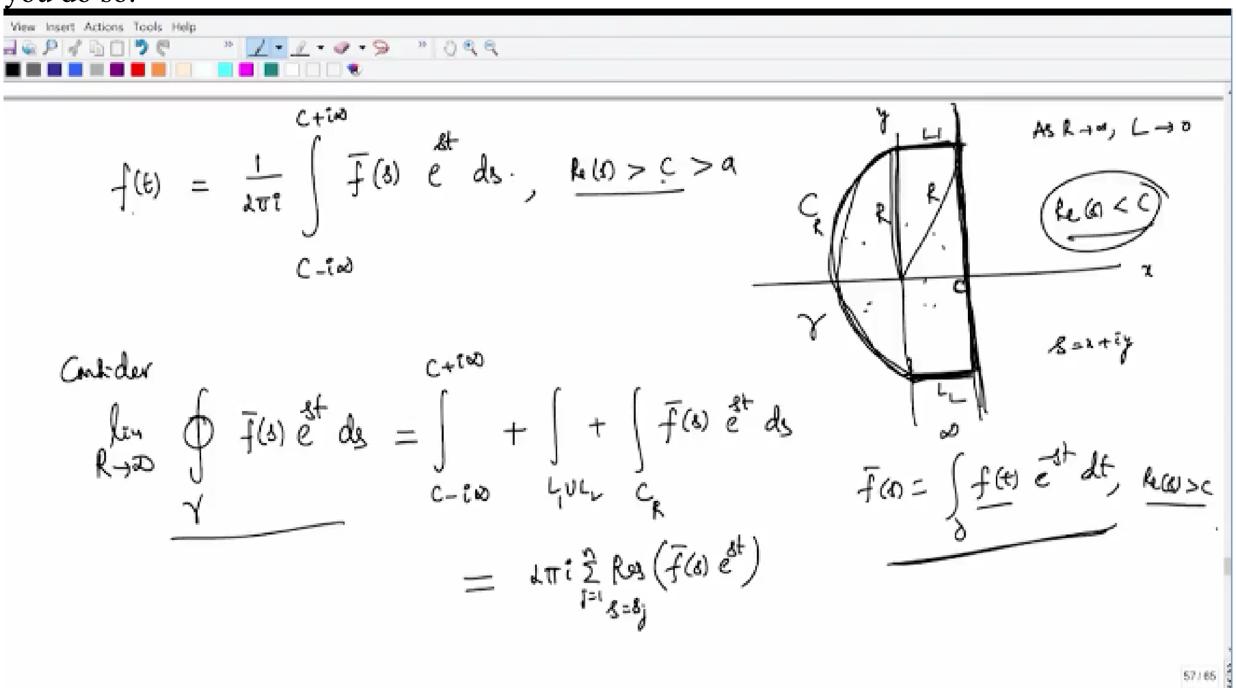
Now if you are given this F bar(s) you can easily recognize what are its singular points, if you have a singular points you cannot have many singular points for example, so if it is a proper Laplace transform it will have only some singular points in the finite plane, so beyond which you will have there is no singular point, so you can by looking at this function you can choose properly see where this is, this may be some hundred or thousand, if thousand is if you have thousand is a singular point you take to see something bigger than thousand line, okay, so once you choose this my contour is, I choose my contour like this, so I break this as I choose, I make a semicircle here this is of radius R, this is of radius R, okay, so if I take actually radius R for example here I go like this, I go like this semicircle so but from at S = 0, okay at X = 0 I choose, I instead of choosing this piece which is curvy piece contour, okay, instead of this what I choose is I simply take this line, this is radius R, this is also radius R, but this is not radius R



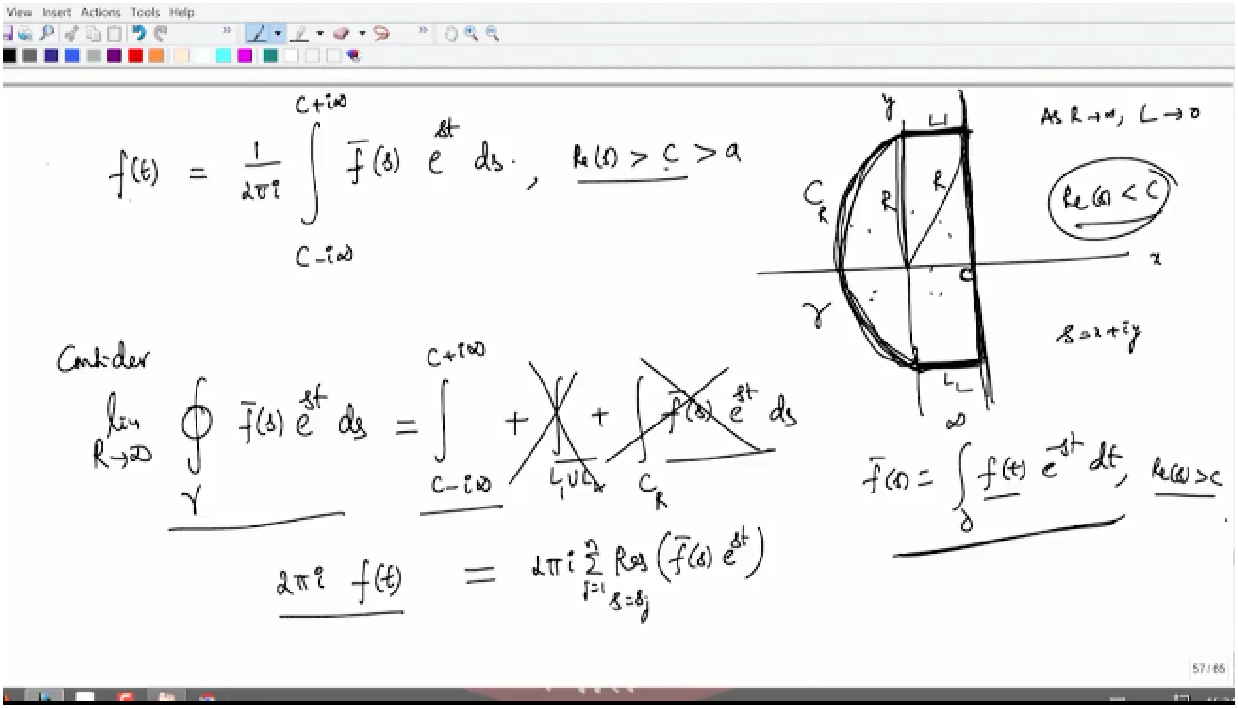
because actual radius is this, this is comes like this, okay, that I am not using so I just make a straight pieces here, both sides as R goes to infinity this small piece is going to, let us say if I call this some L as R goes to infinity this L goes to 0, as you can easily see as R becoming steep and steep, this is going to be this L is going to be 0, okay, all right, so if your R is here, if you'll start here, okay if your R is here and you see that this is your big piece.



If your R is very steep, if you take as R goes to infinity so eventually this is not contribution, so this we don't care but we are only looking at this part and also inside here any singular points this side, left hand side, because anyway you're allowing R goes to infinity you have to see all the singular points on the left hand side of this line, okay, so if you do that so this is your curve, so gamma is the curve which is this piece the semicircular and this piece and this line, so you consider like the contour integration we consider this integral so closed part of gamma this will be F bar(s) E power ST DS this is what I am considering, so this is equal to over this is, this I break this into actually this piece that is going to be what exactly that is actually –C, C+I what exactly is this, this is not IR, right this is IR, IR C-IR to C+IR as R goes to infinity and I take this R goes to infinity, finally I allow R goes to infinity this is going to be C-I infinity to C+I infinity so over this piece, okay, this piece + over L1 and L2 for example, L1 and L2, L1 union L2 this one, as R goes to infinity this is going to be 0 so this contribution will be 0, okay and then plus over the semicircle let me call this CR like earlier if I call this CR F bar(s) E power ST DS, so this definition, by definition this is actually 2 pi I times all the residues of F bar(s) E power ST for this function, at S = SJ summation from J is from 1 to N which are, SJ's are all residue point, singular points on the left hand side of this S equal to real part, real part of S is less than C, so if this means this left hand side of this straight line, okay, so this is exactly what you do so.



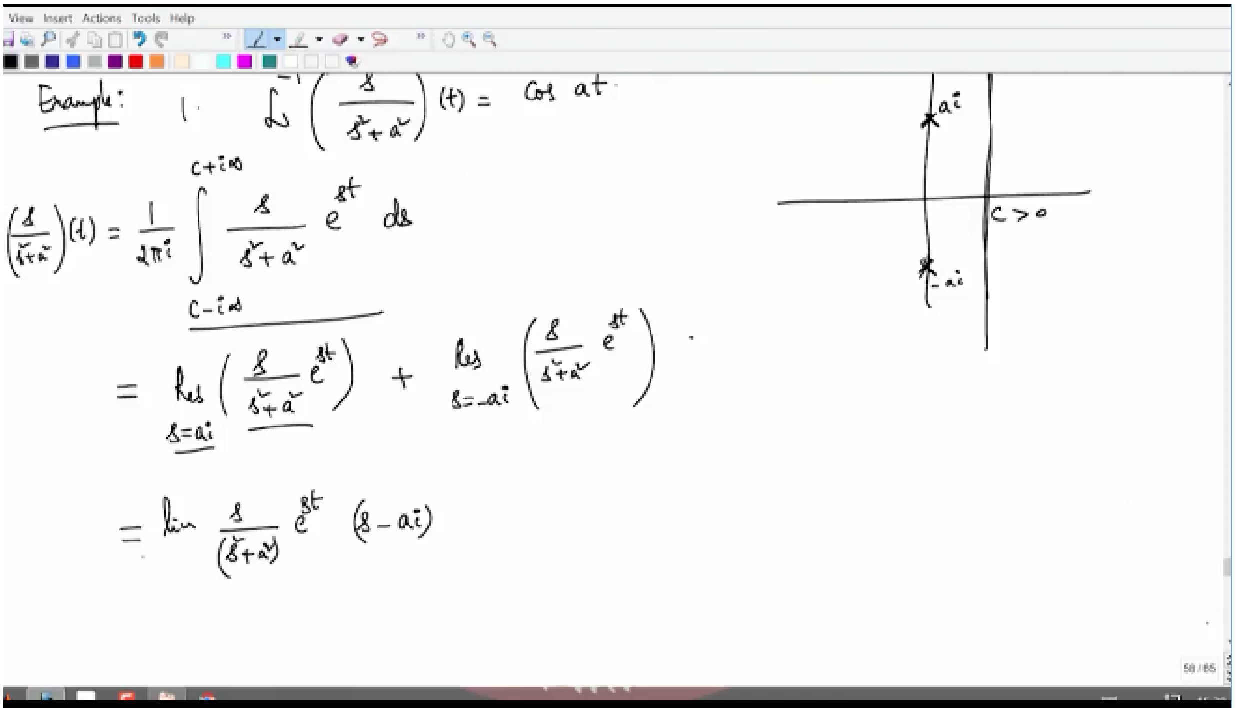
And eventually as you can easily see as R goes to infinity over this line and you can easily see that E power ST, E power ST, S is a real part is a negative so this side negative part is real as R goes to infinity this whole thing is going to be, because of exponential function whatever may be F bar(s) this is, this integral as R goes to infinity is going to be 0, and this contribution this as such this won't be there, integral over a null set because L1 and L2 becomes 0, empty set okay eventually this becomes a point okay, so this that way this contribution is 0, this contribution I can show because of this E power ST, S is a real part is a negative side so as R goes to infinity, so R cos theta, R cos theta is E to -R cos theta T+I times, so because of that E power -R cos theta T as R goes to infinity this is going to be 0 as R goes to infinity, okay so this whole thing is going to be 0, this part is going to 0, this part is going to 0, what you're left with is only this part that is exactly your 2 pi times F(t), this is 2 pi I times F(t) by your definition of your



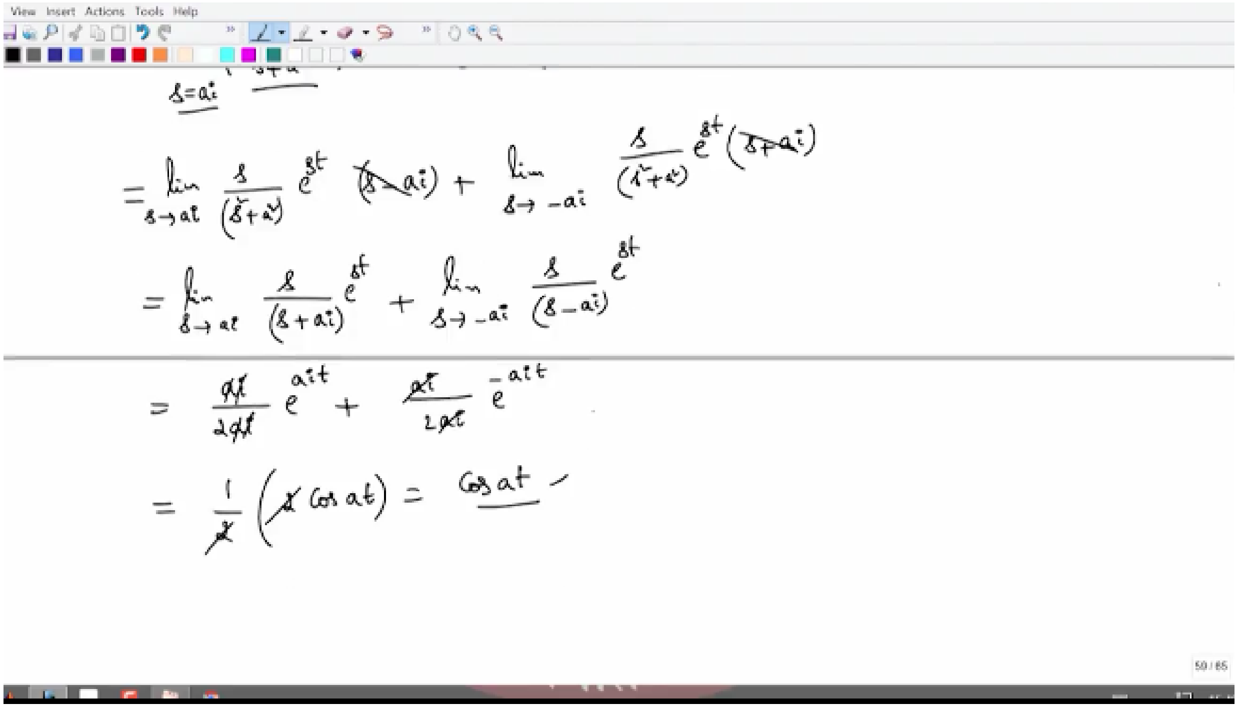
inverse Laplace transform, 2 pi 2 pi goes, okay so what you left is F(t) is nothing but, F(t) is simply defined all the residues for this function in the left hand side of this line X = C.

So let's do some example, so example how to find Laplace inversion okay, so let me choose the usual which you already know some example, for example you will already know that what is the Laplace inversion of S/S square + A square so this as a function of T we know that this is actually cos AT, so let me show by this general procedure of how do you find this Laplace inversion, so general procedure is 1/2 pi I if you calculate so you know what exactly what you get 1/2 pi I C-I infinity to C+I infinity F bar (s) is S square + A square this is having if you look at its singular points S-, S = + or – AI, so AI and –AI, -AI and +AI are the singular points for this function, so I can choose anything positive C as, any C positive as my line because cos AT is exponential function of order 0, okay, so for that reason I am choosing C as bigger than that that is 0 so you can choose any line like here, and E power ST DS, if I calculate this that is exactly what I need to find that is X actually what is my inverse Laplace transform of S/S square + A square(t) okay, this is by definition, but then how do I calculate this? To do this I have already seen that this is actually 2 pi I times and this integral is nothing but we have already seen that this is actually equal to residue of S/S square + A square E power ST where at S equal to, what are the poles, what are the singular points? Which are at AI+ residue at S = -AI of the same function S square + A square E power ST, okay.

So how do I calculate this residue? We have seen this is your function S/S square + A square E power ST if this is your singular point you write S-AI Z-Z naught, this is like S is Z, singular point is Z naught is AI so Z-Z naught, now you take because it's a simple pole you can easily see that it is not a pole of order, there's only simple pole only single pole, single pole so you have a limit S goes to AI.

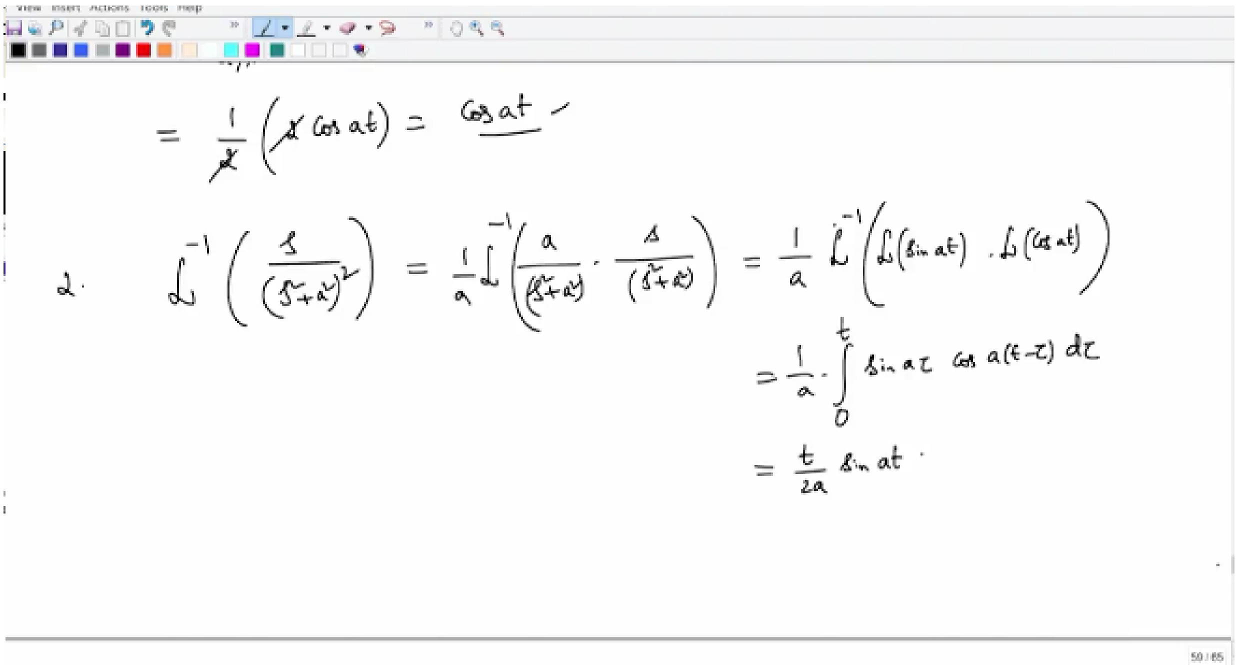


And similarly here this is a limit S goes to -AI S/S square + A square times E power ST times S- S+ AI, S- of A-AI that is S+AI, so what is this one? This is actually limit S goes to AI and this gets cancelled here some part so you have S - S + AI, S +AI and S-AI is S square + A square so this gets cancelled here so you have left with this, again here limit S goes to -AI here also this gets cancelled here so you have S –AI times E power ST, you apply now the limit and you see that is AI/2AI times E power AIT + this limit will be -AI divided by -2AI, so minus minus goes E power –AIT, so AI/AI, I I goes, A A goes here so you have 1/2 comes out, sum of this, half of, this is actually 2 cos AT, so 2 2 goes this is exactly cos AT, so this is exactly what we know already that this is cos AT okay.

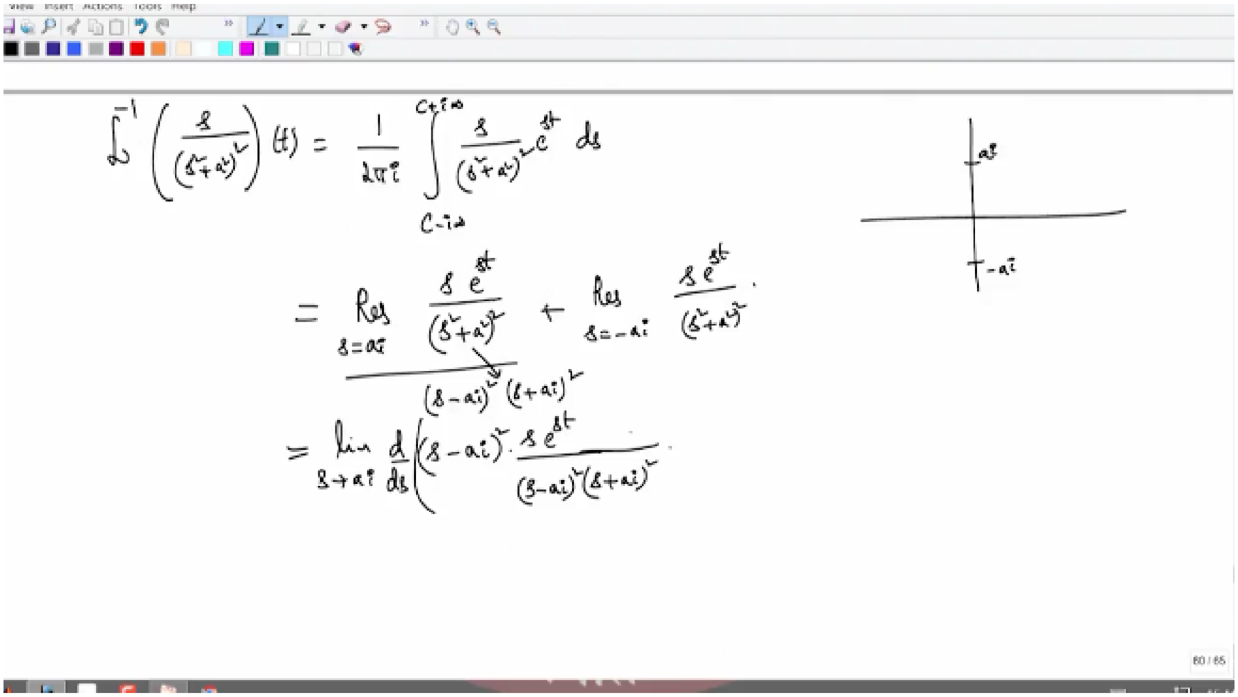


We'll do some more, again one more example before we wrap up this, wrap up this of this finding inverse Laplace transform, so for any general Laplace transform of F bar(s) if it is given you look at it where you have to put this line C depending on by looking at all its singular points, put them all on the left hand side of this line and choose this contour you know that this is actually the residues you consider, find the residues of all the poles or singular points of this F bar(s) into E power ST and then you add them up that is actually your F(t) okay.

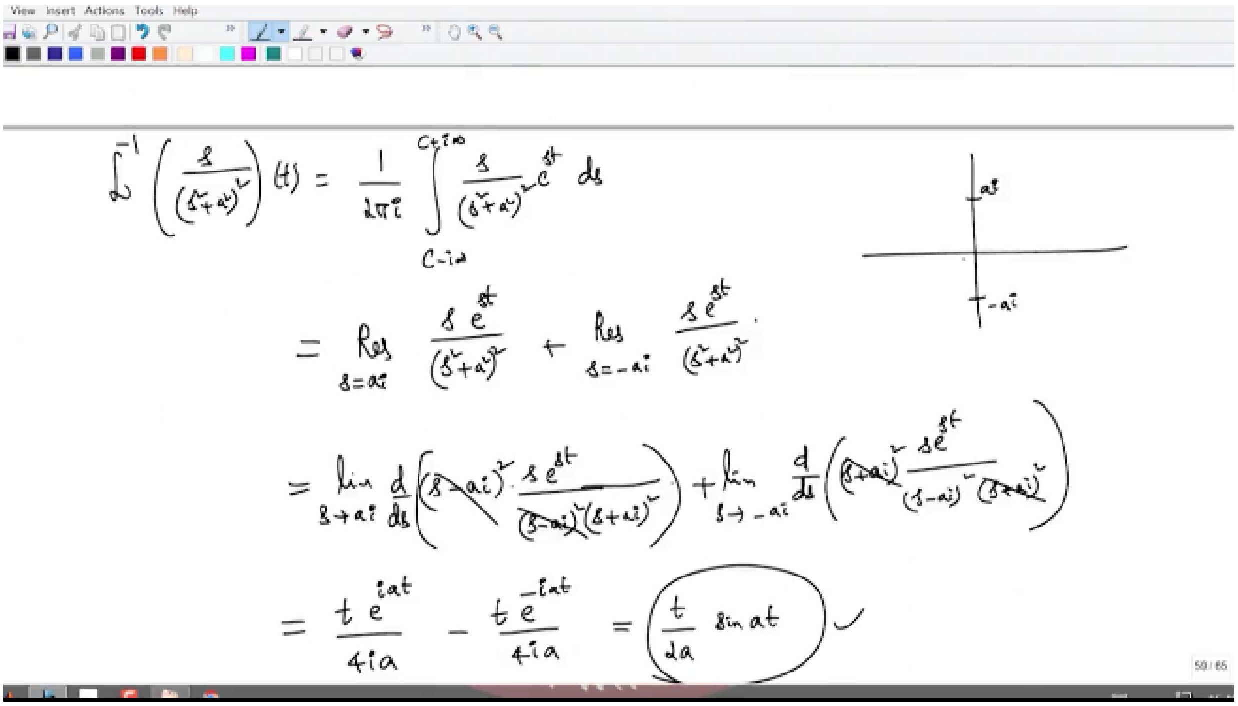
So let me do one more example, the second example is what is the L inversion of S/S square + A square whole square, if I choose this if you actually calculate this you will see that it's by some other means, but if you calculate like 1/S square + A square times S/S square + A square, okay, so you have a let me put A here so you have 1/A, so L inversion of this, this is same as this so you get 1/A times L inversion of, this is Laplace transform of, Laplace transform of sine AT and this is Laplace transform of cos AT, so 1/A times integral 0 to T sine AT, sine A tau cos A times T-tau D tau, so this is exactly using convolution theorem I calculated this you will get as you will see that is going to be T/2A sine AT, so I'm not calculating I am just writing directly, this is the result you know already by earlier method.



Now let me use by the general method of finding this, so what is this one general method of finding this inverse, we have seen that the L inversion of this S/S square + A square whole square as a function of T we know that this is actually 1/2 pi I times the C-I infinity to C+I infinity F bar(s) that is S/S square + A square whole square E power ST DS, this we know that this is actually all the residues, you look at what are the poles, what are the singular points of this, again AI and –AI, but it's a pole of order 2 because of square okay, it's a pole of order 2 so you calculate the residue, some of other residues, so we know that this is actually equal to the residue of S/S square + A square whole square times E power ST at S = AI + the residue of S = -AI, but for E power ST/S square + A square whole square, and as a pole of order 2 this means a limit S goes to AI times, how do you find the residue? If it is a AI is having a pole of order 2 so I can write like this like S-AI whole square S+AI whole square this part S square + A square whole square I can write like this, because of this AI is having a pole of order 2, so if it is like this you have to multiply S-AI square to the function that is S into E power ST/S-AI whole square times S +AI whole square, okay, I multiplied and I have to differentiate this with respect to DS, there DZ okay, and I explained that is DZ so this is what exactly you have to do.



And of course you have one derivatives so you have one factorial, so if you are doing, if this is M and if you have a M, pole of M, order M you have to multiply M and you have to differentiate M-1 times, okay with respect to S, let me not do this so M is 1 here, so because M is 2 here so the derivative only 1, so 1/M-1 factorial I have this, I'm using this formula, so this is 1, so same way you can get S goes to -AI D/DS of, now instead of S- you have S+AI whole square, and you have SE power ST/S – AI whole square times S+AI whole square, so this gets cancelled both the places, and what is this limit? Limit you differentiate, if you differentiate this you want to calculate directly, so let me write directly I'm not, I'm not calculating you can do it yourself, and you see that is going to be T times E power IAT/4IA, for this is you will see that is going to be -T times E power –IAT/4IA, so if you calculate, you put it together you have T/2A sine AT, so this is exactly what you get if you also do it directly, directly also you'll get the same answer.



So this is the general procedure of finding a inverse Laplace transform, sorry, so this is a general way of finding inverse Laplace transform of a function F bar(s) which is the Laplace transform of function F(t), general way is through contour integration so you need a little bit of complex function theory and it's applications, how do you find there, how do you evaluate integrals, so here you don't need really that contour integration technique, so only thing is how you apply the contour integration technique for this Bromwich contour, if you apply this Bromwich contour here so you know exactly how to find this what you want this integral, and on the curve part on the circular part as R goes to infinity you can show that is going to be 0 because of a negative side, it is negative real part, okay, S is a negative real part so E power ST and exponential function of a negative real part, so that is E power minus real part of S into T, so as R goes to infinity that is going to be, as R goes to infinity negative part is becoming bigger and bigger so that makes it that integral 0 so you end up seeing that finally the required integral is simply sum of all the residues on the left side of this line C- I infinity to C+I infinity, so you just need to calculate the residues, how do you find the residue value of, residue of a function which are at poles are poles of order M, usually you see if it is required if we look at some functions you how you can expand, you should know how to find it's a Laurent series expansion or a Taylor series expansion, if it is usually you end up getting some elementary functions whatever may be you're working with some function it will be composition of elementary function, so you can expand all of them together and you see that you can get Laurent series expansion, and in that coefficient of 1/Z - Z naught, if Z naught is a singular point that is your residue, so if it is a pole, pole of order you have the formulas to find the residue, otherwise you have to expand its Laurent’s expansion, Laurent series expansion and look at its coefficient of 1/Z-Z naught okay.

So that's how you find this, only find the residues and you add them up you end up getting a inverse Laplace transform of a function okay, F bar(s) that is F(t), so this is how you find these inverse Laplace transforms you have seen so far, how to calculate inverse Laplace transform of in a different ways, if it's a simpler functions you can use partial fractions, convolution theorem you can apply, you can apply a heavyside expansion theorem, and also it looks like heavyside expansion, but if it is a kind of series it's not because of the heavyside expansion but by general form that is a fourth type, that is finding, using the inverse transform definition that is using Bromwich contour complex function theory, contour integration you can apply and get the sum of, get the inverse transform as sum of all the residues that will give you kind of as though you are applying heavyside expansion theorem for thus heavyside series, okay, we have seen some example like when you have instead of taking polynomials you have cos hyperbolic function and sine hyperbolic function, cos hyperbolic functions you have chosen, and for which you have seen what are its poles, there are simple poles that works only if it is a simple pole okay, it looks like heavyside expansion theorem as though you applied without bothering a finite or not, in that example that we have seen in the earlier video, actually that's it looks like a heavyside expansion as though you have to simply apply only if your function is having a poles which are simple poles, if you have poles of order some other order higher than one, then that will not work so it will be looking different.

So in any case a general way of finding the inverse transform that is by contour integration is finding all the residues, residues of the function F bar(s) into E power ST that is your function complex function for which you look at its residues for which at all the poles of that function and then you calculate is residue and sum it up you can end up getting inverse transform, so we will see Laplace transform you know now how to find the Laplace transform and inverse transform, we will see the applications of this Laplace transforms, for example start with ordinary differential equations and ending up solving some boundary value problems for the partial differential equations, and also for integral equation, how do I solve certain type of integral equations, we will see that in the next one or two videos. Thank you very much.

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