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Transform Techniques for Engineers
Heavyside Expansion Theorem
With
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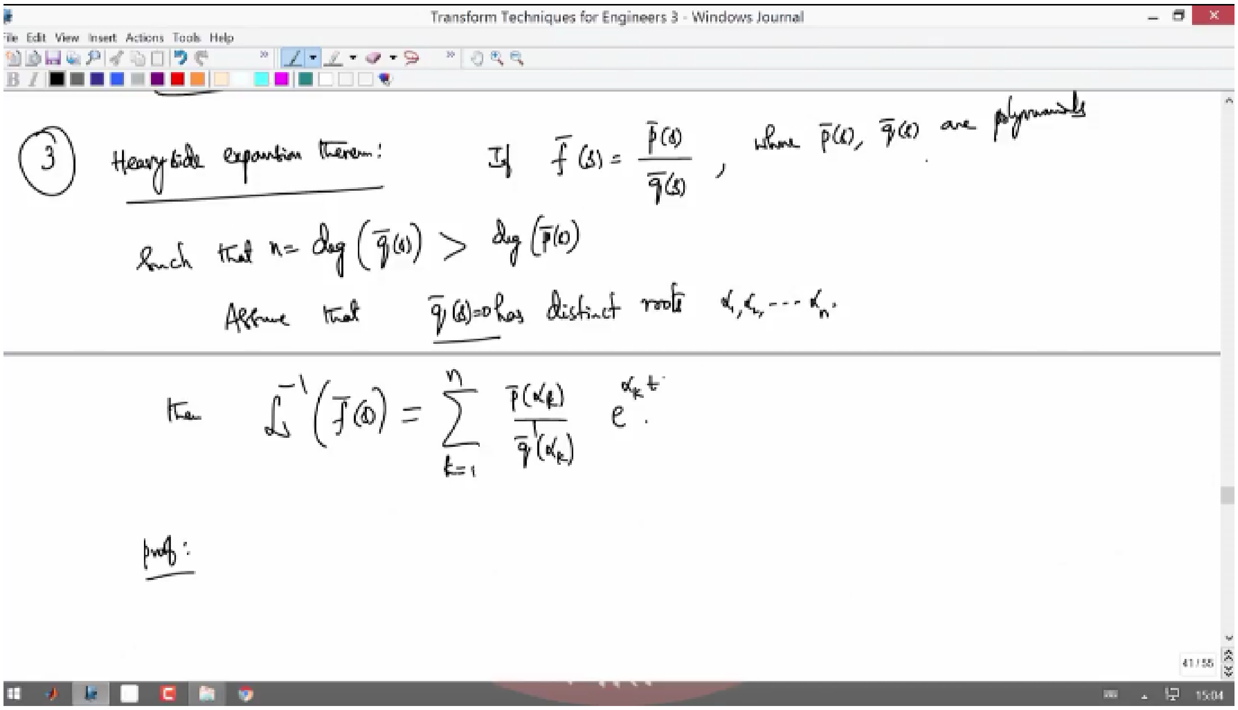
Welcome back, in the last video we have seen how to find the inverse Laplace transform, a ways to find inverse Laplace transforms started with finding using partial fractions, and then we also used convolution theorem to find the inverse, Laplace inverse.

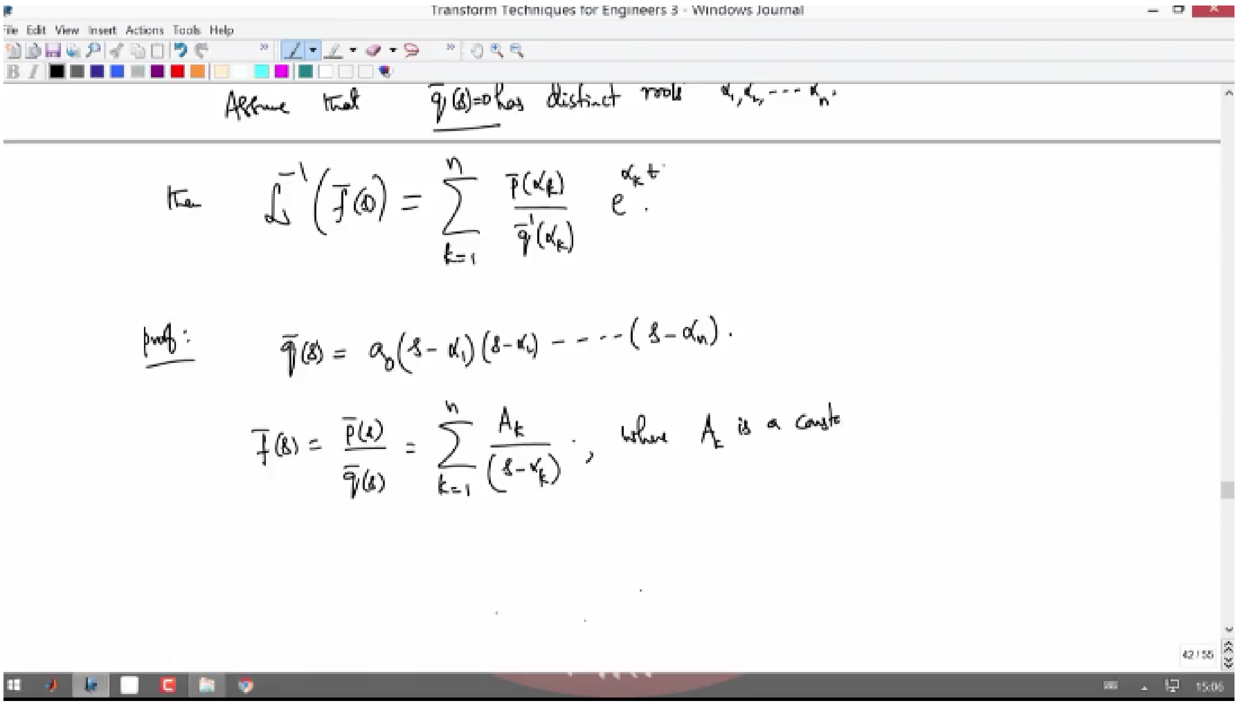
And the third way to do this is by a Heavyside expansion, so Heavyside expansion theorem tells you that if you have a rational kind of function of S which is the numerator is, degree of the numerator is less than the degree of the denominator, then if they're both are polynomials, polynomial degrees if it's rational expression then you can find, you can express the inverse transform, just using this also, the technique is basically using partial fractions but without finding the partial fractions just based on this P/Q which are the polynomials of the numerator and denominator you can find a Laplace inversion that is actually Heavyside expansion theorem.

Before I do this we can we can also see you can see that Heavyside expansion, let's first do this and then we'll see what is the generalization, it's not a generalization it's a kind of sometimes it may work, you may work with series, Heavyside expansion series, okay so before what we have seen is this is the Heavyside expansion theorem, this is the technique let me put it as 3, third way to get the Laplace inversion if your F bar(s) is this, then you have and where P bar(s)

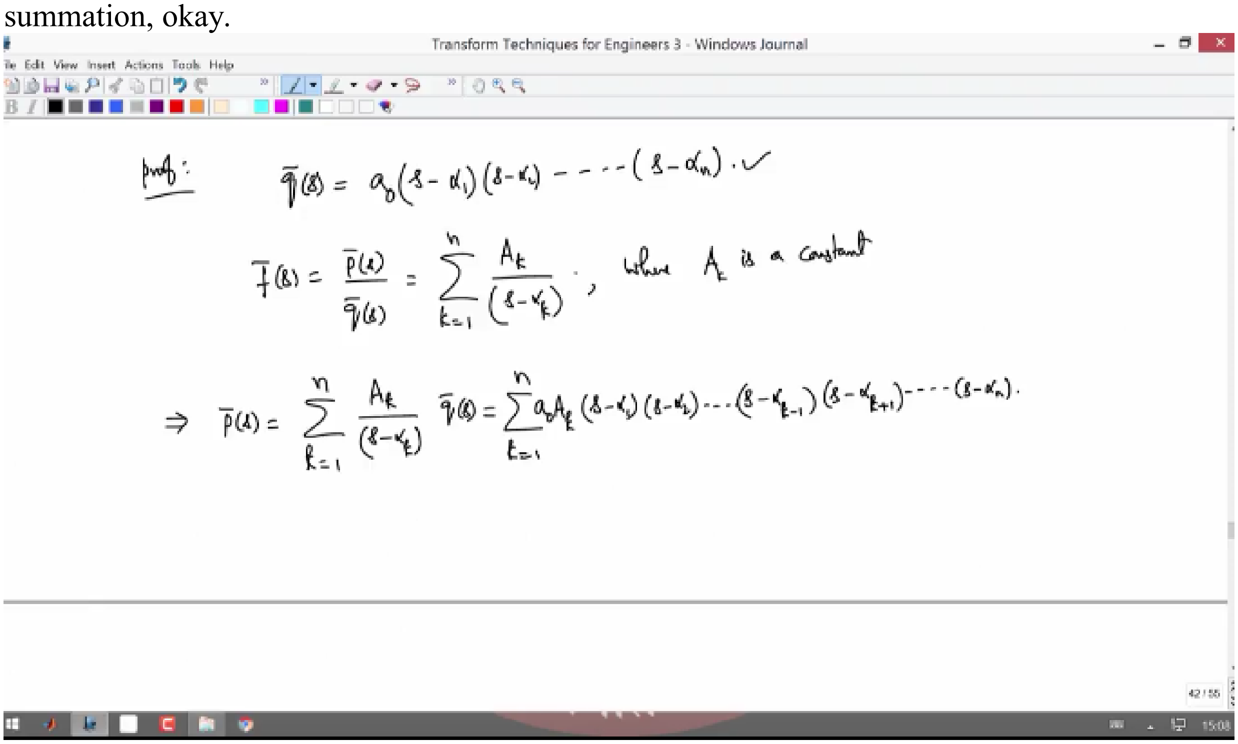


and Q bar(s) are polynomials such that degree of Q bar(f) is bigger than degree of PS, now let us say that a denominator degree is N, and you also assume that the denominator polynomial has distinct roots, then the Laplace inversion is simply will get like this, okay, so this is also actually similar to finding partial fractions but without finding the partial fractions you can just calculating, you can considering P bar and Q bar and we simply get this expression P bar/Q bar dash that's a derivative of Q bar, and you replace S/alpha K these are the roots of this Q bar(f) denominator, Q Bar(s) = 0.

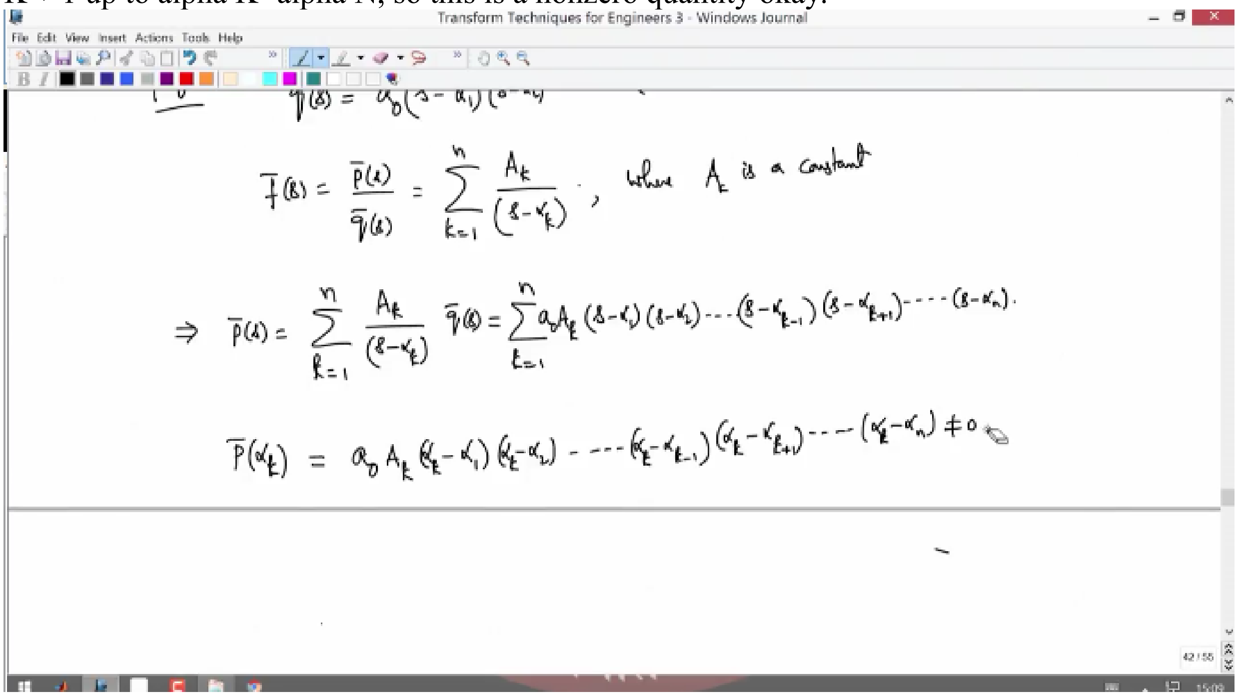
So let me prove this before I make some more remarks on this way of finding inversion, so you assume that Q bar of, what is given is Q bar(s) is basically having N distinct roots, so I can explain, I can write this as so Q bar is a polynomial, so if some constant times let's call this A naught times S - S naught, S - alpha 1, S - alpha 2 and so on, S - alpha N so you assume like this, then you have F bar(s) which is P bar(s) divided by Q bar(s), this is you can express this as a, because degree of P(s) is smaller than Q(s), and Q(s) has on all simple roots that is the distinct roots, you can write this using partial fractions I can write like some K is from 1 to N and simply some constant times AK divided by S - alpha K, so that's where AK are constants where AK is constant.



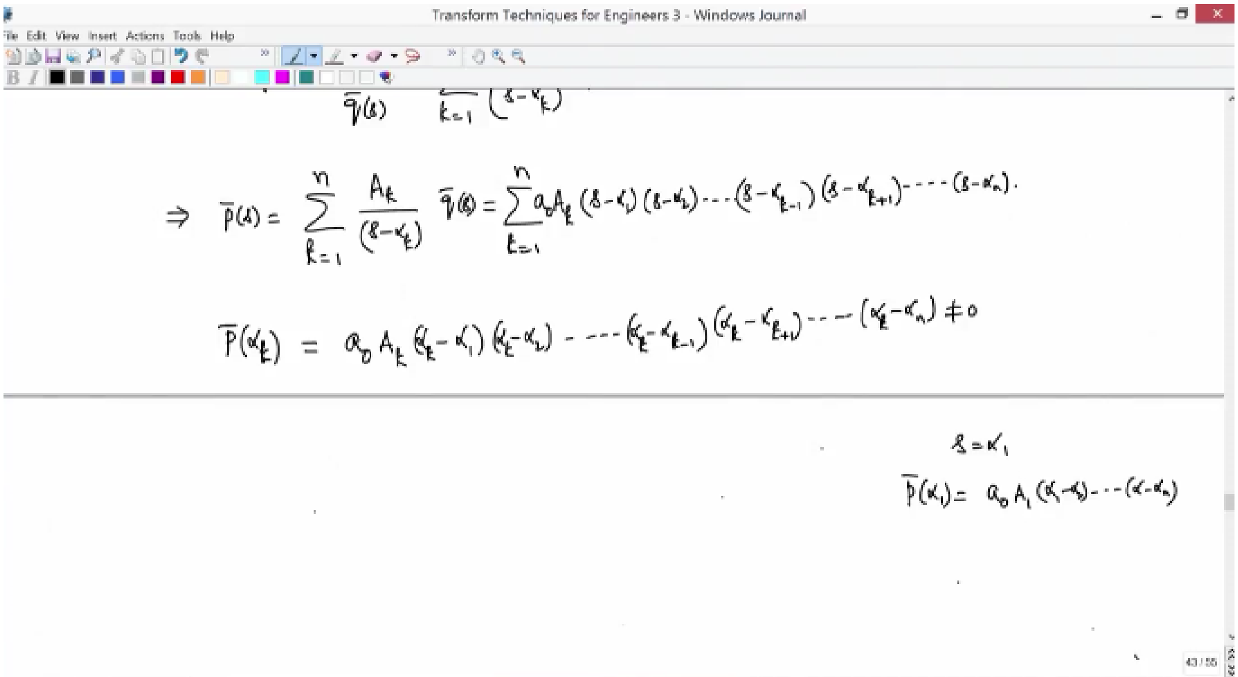
So if you do this, what is P bar(s)? If you write this, this implies, this will give me P bar(s) as Q bar(s) into this, so you have K is from 1 to N, AK/S -alpha K times Q bar(s) here, so this is simply I can write this as K is from 1 to N AK times, Q bar(s) is this one, this one if you use here S -alpha K will go so that you can write S - alpha 1, S - alpha 2 of course you have A naught here, A naught is a constant which is in there here, times, what you're missing is S - alpha K you can cancel it so you have S - alpha K – 1, and then next term is S - alpha K + 1 up to S - alpha N, this is what you will have as an expression, so where K is this one, so this is the summation, okay.



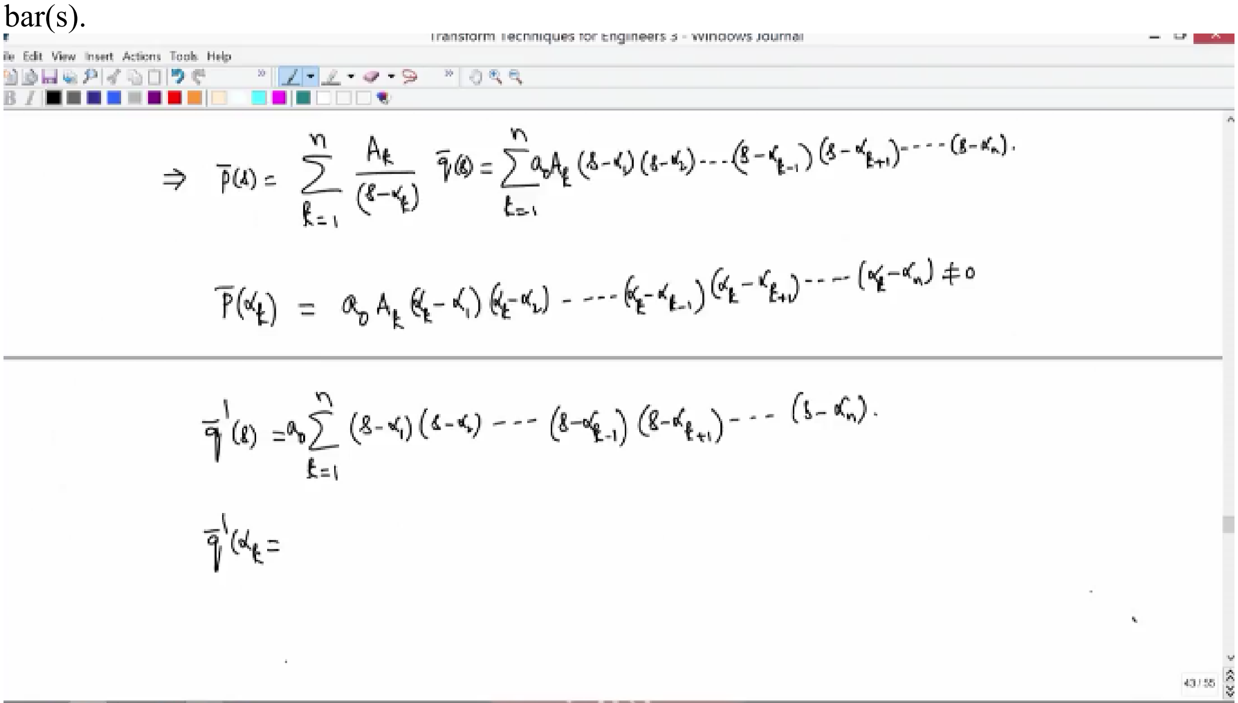
If I substitute P bar(alpha K) this is equal to only when K = alpha K when I substitute S = alpha K, S = alpha K only this term when K rather I think K is K -1K + 1, so okay, so all those terms will go except one term when K, for that S-K, S equal to when you put alpha K, so when you put alpha K so what is left is A0 AK S - alpha 1 S - alpha 2 up to S - alpha K-1 S minus, sorry this is alpha K, this is alpha K I'm placing in the place of S alpha K, so you have alpha K -alpha K + 1 up to alpha K -alpha N, so this is a nonzero quantity okay.



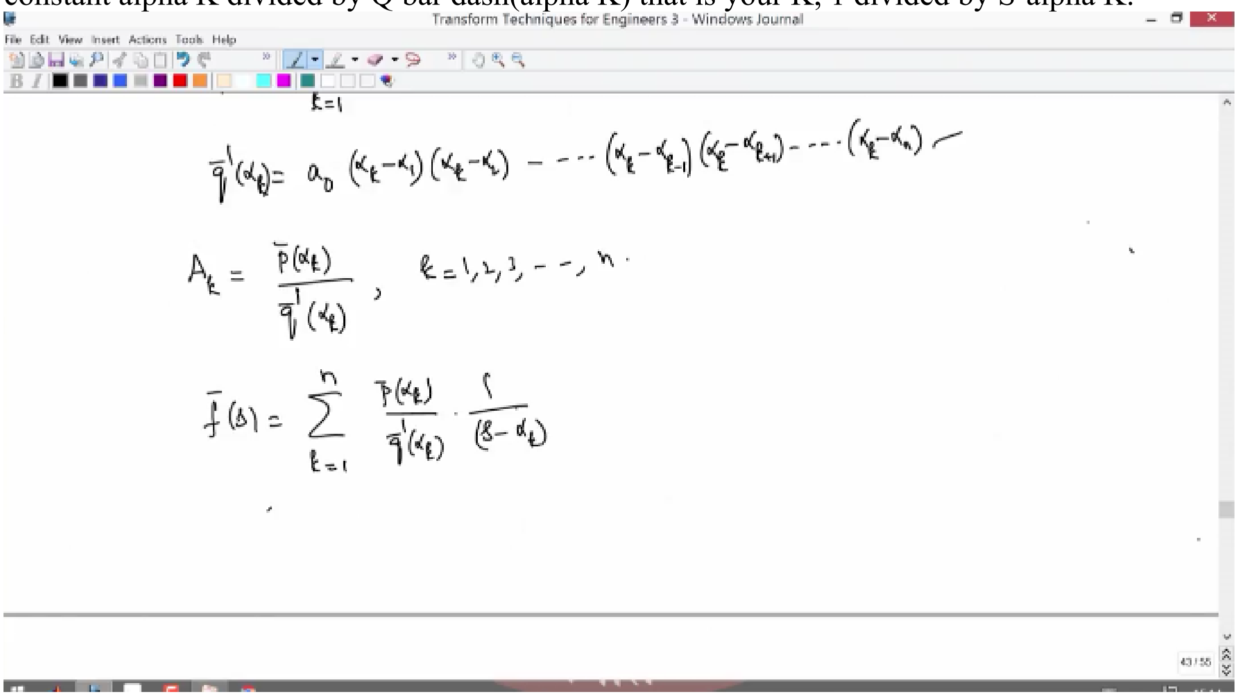
And then so only one term, so that is this general term when I replace X = alpha K, if I have S = alpha 1, when K = 1 will be having, so when K = 1 so you have, so when K equal to, when S = alpha 1 what you have is in this case first term won't be there, okay, so let us say if I use S = alpha 1 what exactly your P bar(alpha 1) this is going to be A naught, that first term which you will have what you lose is the first term so that is A naught times A1 S – alpha, that is alpha 1 - alpha 2 up to alpha - alpha N, so except that alpha 1 - alpha 1 that is 0 that divided with, when



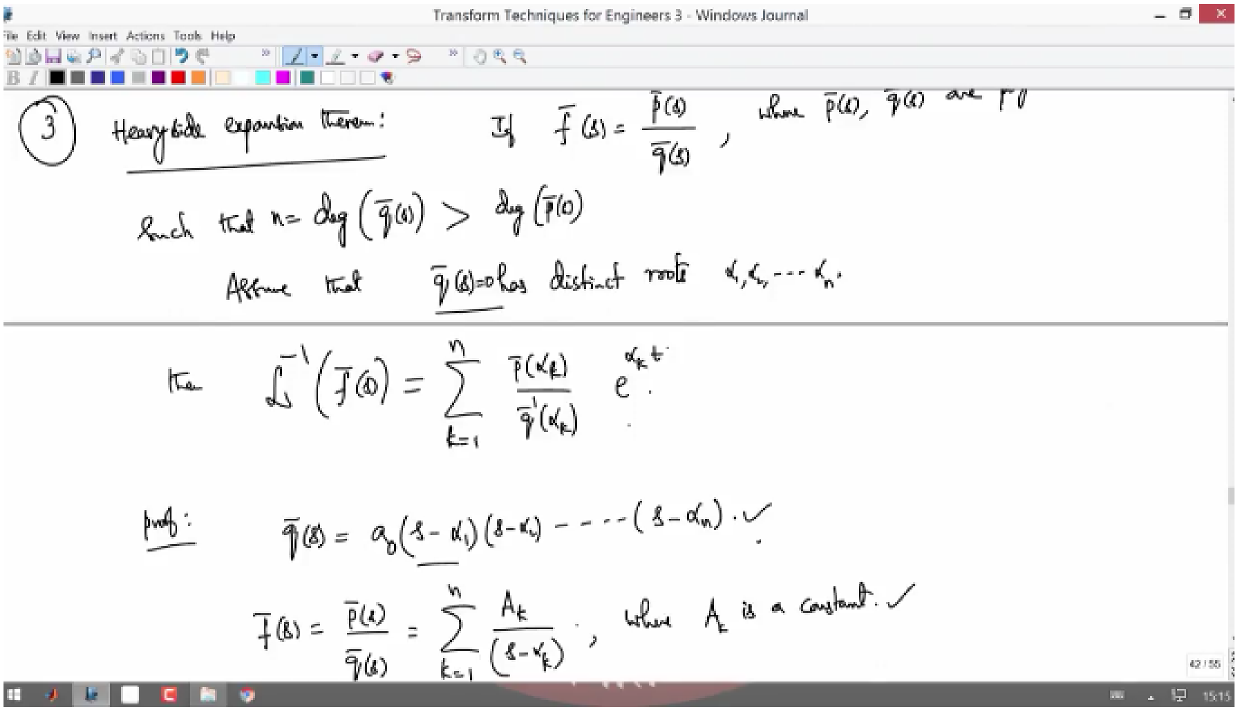
K = 1 that gets cancelled this is your first term, so first term will remain when S = alpha 1, when you take S = alpha K, only K-th term will remain, right K-th term is this general term, so that is what is your P bar(alpha K) you differentiate what is your Q bar, Q bar if you differentiate Q bar(s) Q bar dash(s) that is what is this one? Q bar(s) if you do this is also again you see that K is from 1 to N A naught times, if you differentiate so you have to differentiate first term and keeping everything as it is, so if you do that the only is, because of S - alpha 1, S - alpha 2, S - alpha N if you differentiate each term only thing is A naught is common that is comes out, and then and you end up N terms that is S - alpha 1, S - alpha 2, what you have only K-th will go that is S - alpha K-1 into S - K alpha K+1 up to S -alpha N so this is what is the Q bar(s).



Again Q bar(s) alpha if you replace Q bar(s)/alpha K this is going to be, what is Q bar(alpha K) this if you put alpha K, S = alpha K, S = alpha 1 if I put it, this is going to be a first term, when you have K = 1, now what you have is there is no alpha – 1, this term is not there so you have S - alpha 2, this is 2 onwards will be there, so whatever these are the terms won't be there, okay, this term won't be there because S- when K = 1 alpha 0 so there is no such term, so it's like K = 1, when S = alpha 1 and what you end, so you end up getting A naught times S alpha 1 - alpha 2 up to alpha 1 - alpha N, so you getting here you get the nth term again here, so something like alpha 1, alpha K - alpha 1 because I'm replacing S by alpha K and you have alpha K - alpha 2 up to alpha K - alpha K - 1 times alpha K - alpha K + 1, and then finally alpha K – alpha N, so if you compare this with this you can easily see that AK is actually equal to P bar(alpha K) divided by, what is remaining is, this Q Bar dash(alpha K) so this is true for every K running from 1, 2, 3 up to N, so if you go and substitute into this, these are unknown constant, F bar(s), F bar(s) is actually equal to, so you have K is from 1 to N, AK I'm replacing with P bar of this constant alpha K divided by Q bar dash(alpha K) that is your K, 1 divided by S-alpha K.

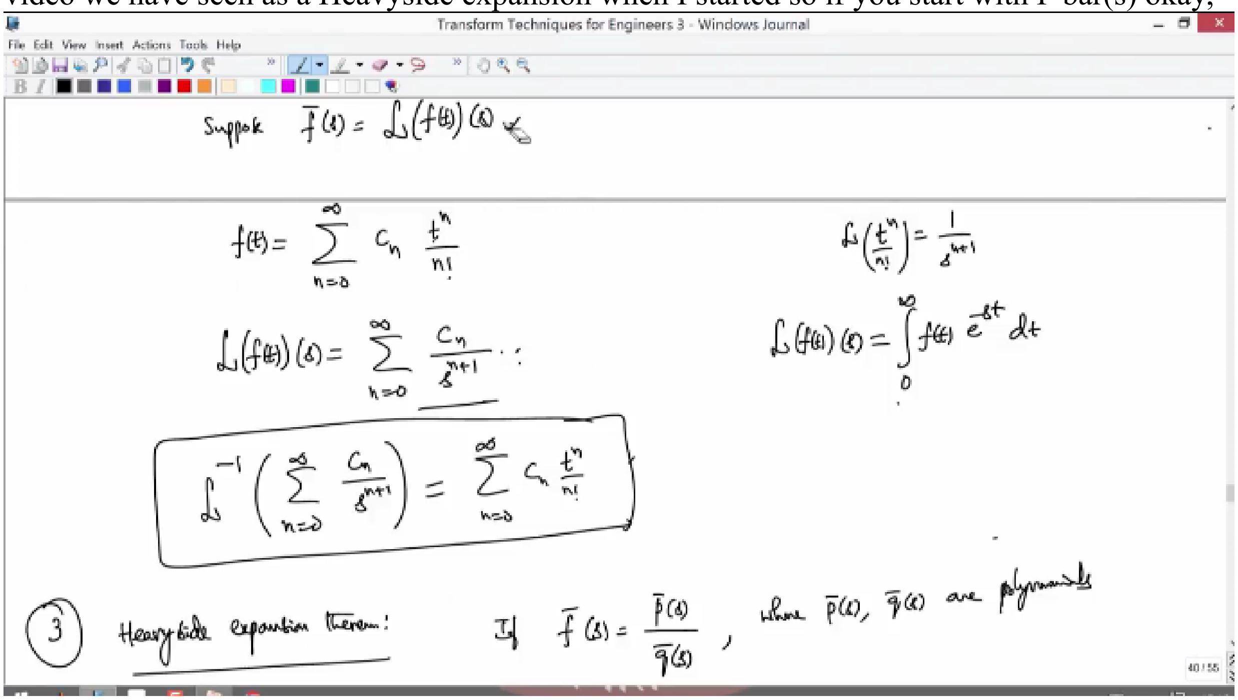


So from which you can get your, this is a finite sum, and I know what is the inverse transform of this so you have F(t) that is a Laplace inversion of F bar(s) which is a function of T is equal to this sum K is from 1 to N P bar(alpha)/Q bar dash(alpha K) times, this is going to be E power alpha KT, 1 over S - alpha K is whose Laplace inversion is E power alpha KT, so this is how you get this, Heavyside expansion so without making partial fractions if you have a distinct roots you can easily see that, if you have a, denominator is having distinct roots you simply get these derivatives and find those roots first of all and then get these roots, get the derivative and



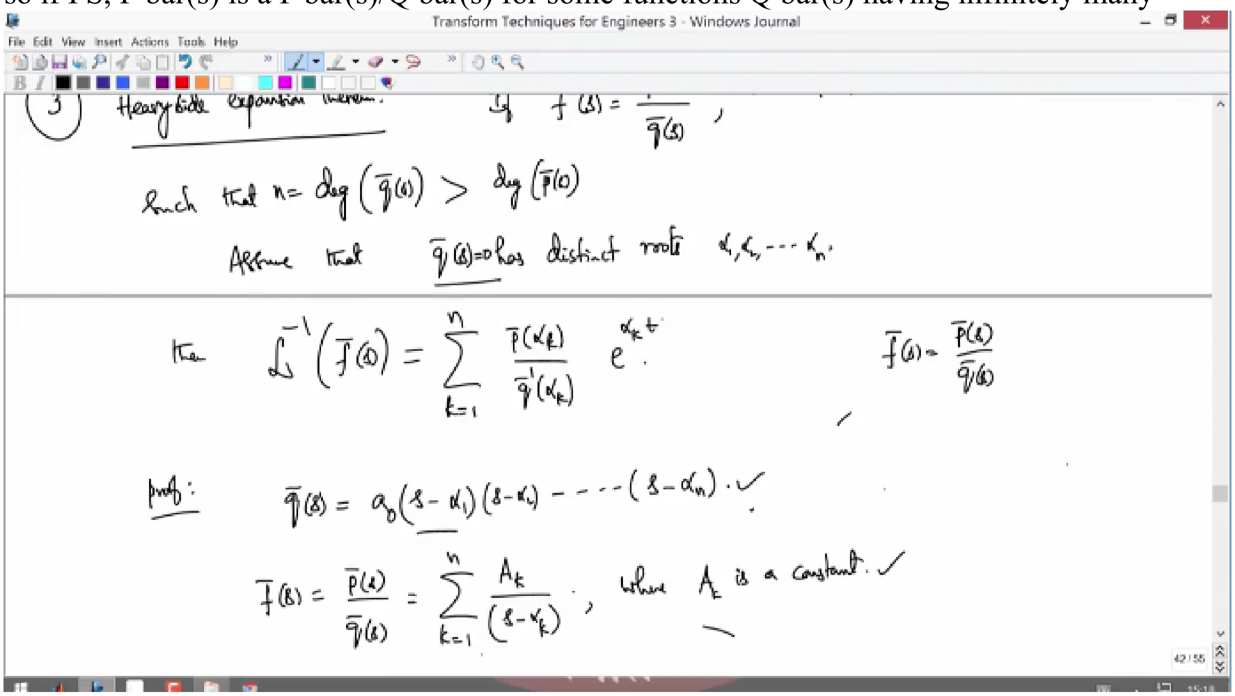
substitute so those coefficients are nothing but these, okay, coefficients of this partial fractions you can easily find this way, so this proof is really not that difficult okay.

So this is the Heavyside expansion this is the way you can get the inversion, and in the last video we have seen as a Heavyside expansion when I started so if you start with F bar(s) okay,

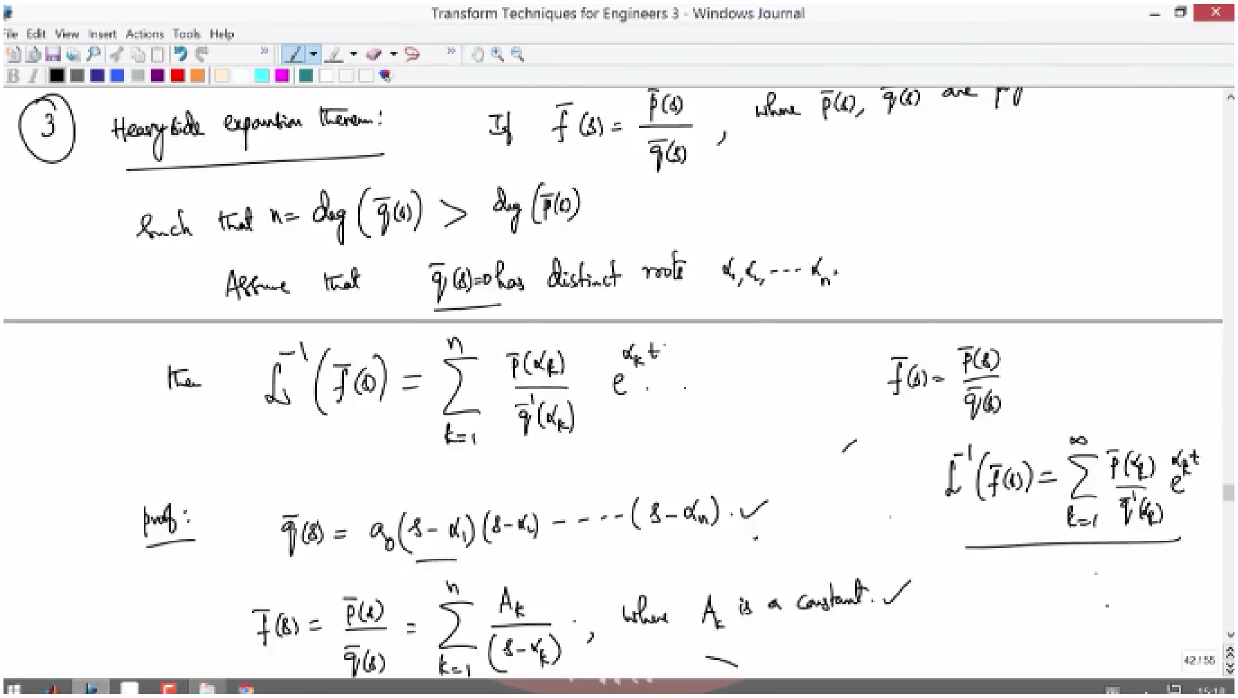


and suppose you have this and you started with the let us say you started with the Maclaurin series, so you have a Taylor series around 0, so for T positive you have already this expansion, so you have a validity once you say it's a Taylor expansion it's valid in the neighborhood of 0, so that means you started with the Taylor series in the neighborhood of 0 that means you have F(t) for T positive, in the neighborhood of 0 it's well defined you have for such a function, let us

say suppose that it is defined you have a Taylor series suppose it is valid for all T positive, you started with this type of Taylor series Maclaurin's, that is called Maclaurin series around 0, then if you find the Laplace transform for that you see that is going to be CN over SN + 1 with the summation this series, because you do the term by term because this is uniform convergence this Laplace inversion, Laplace transform is uniform convergent integral so you can bring this integral inside the series by after substituting FT as a series you can bring this integral inside, so for each of this integral is a Laplace transform of the each term here, so that makes it Laplace transform of this full series is this one, so that implies once you have this by inspection you can see if you have a series like this you can immediately write that you have a series, you have Laplace inversion is this function, okay.
So we can use this, this is a kind of, because we are seeing that this is a Heavyside expansion theorem so here of course we have only finitely many terms sometimes and this is another, this is not a Heavyside function, but it is kind of you have a series form you can do term by term Laplace inversion, so sometimes it may so happen so you can easily work out if you are having this Heavyside expansion for example this one, if you have some it's need not be polynomial if so if PS, F bar(s) is a P bar(s)/Q bar(s) for some functions Q bar(s) having infinitely many



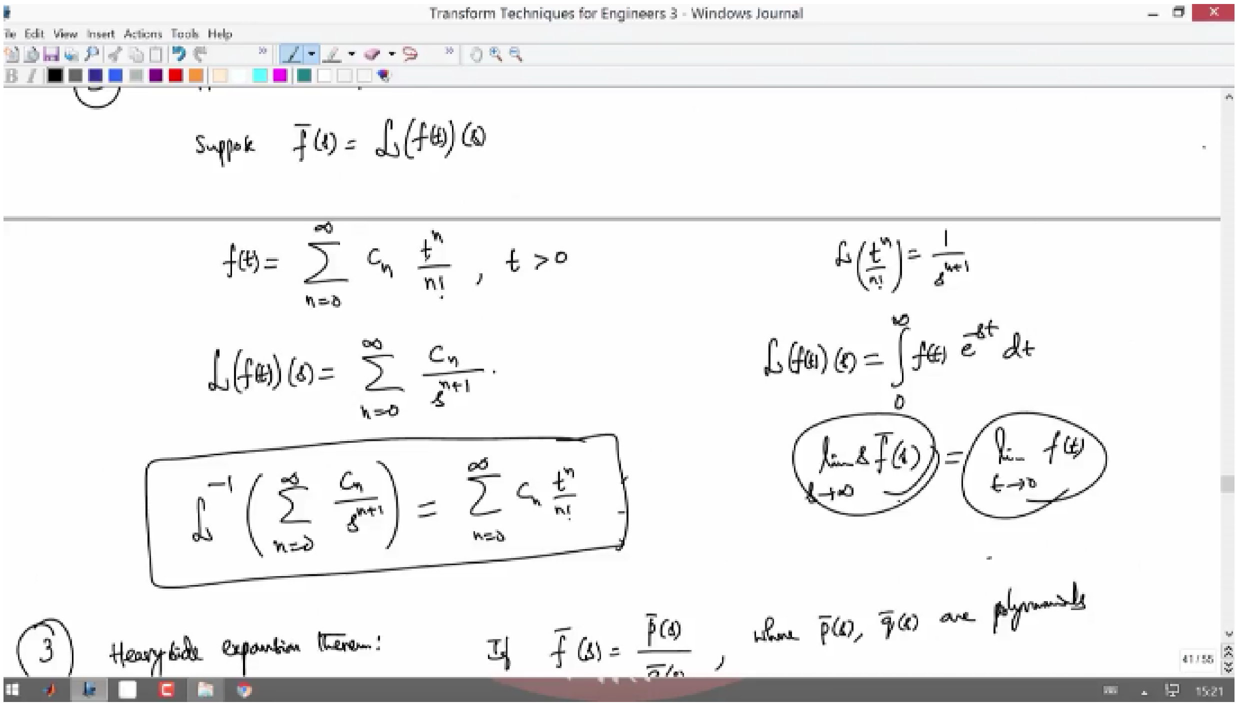
distinct roots for example then you can get this N in, Laplace inversion of F bar(s) not by this Heavyside expansion theorem so you can easily see the same way you can calculate which is from K is from 1 to infinity the same expression will be valid this is Q dash(alpha K) times E power alpha KT, so you see that as though you applied this heavyside expansion theorem even if you have a P bar(s) Q bar(s), Q bar(s) is having infinitely many roots, infinite roots, infinite



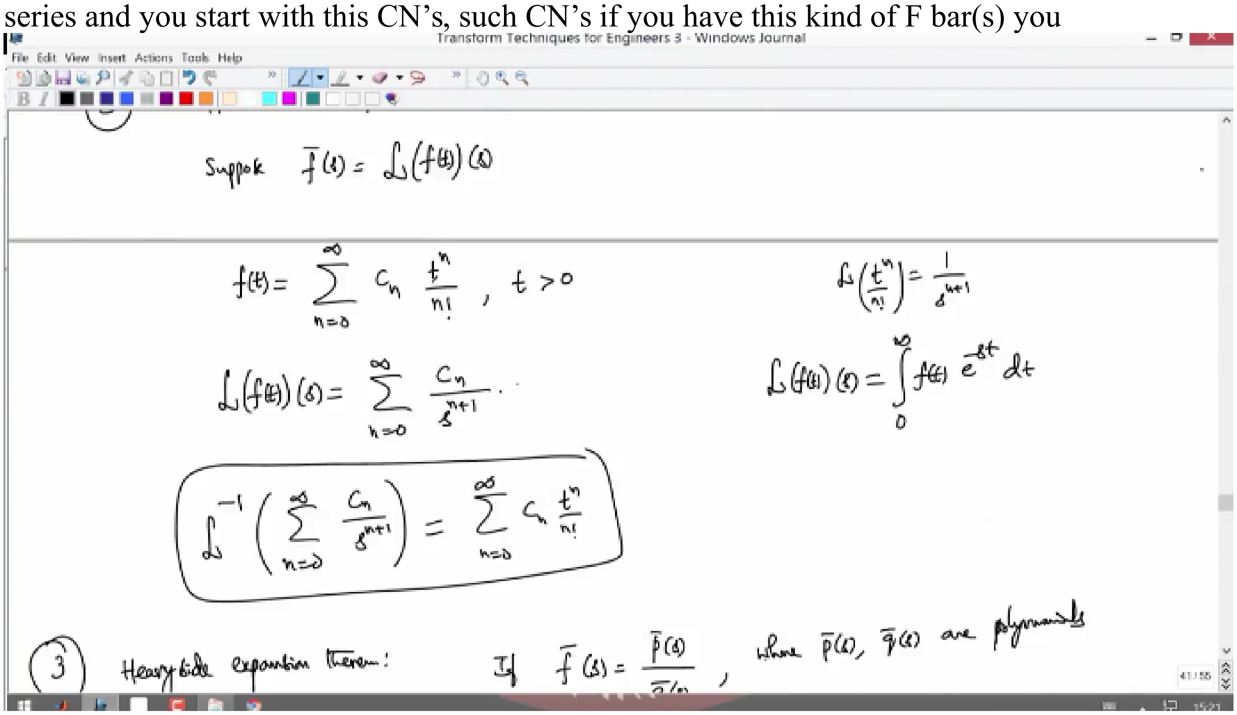
zeros, okay, even if you have infinite zeros what you have is as a polynomial if you have infinitely distinct roots you can still get this kind of distinct roots, infinite distinct roots, distinct zeros you can show that this is actually this by a general form of, general way of getting the Laplace inversion that is by actual definition, inverse integral inverse transform of the, inverse Laplace transform of F bar(s) that is by contour integration, integration of over 1/2 pi times integral over C-I infinity to C+I infinity the definition, the F bar(s) into E power ST DS that is a complex integral we may have to evaluate, and then using the contour integration technique you can actually see that you can get the inversion, that we will see in the next way of finding the Laplace inversion using that you see that you can see that the term by term you can easily almost like a heavyside expansion you can get it, this is a finite sum you can easily, you prove this heavyside expansion theorem, but if you actually use the basic definition of the inversion by contour integration technique sometimes if you have a F bar(s) is with P bar/Q bar(s) and Q bar(s) is having an infinitely distinct root zeros.

The inversion of this Laplace transform is you can simply do the term by term thing with, work with the same way like you have done for heavyside expansion theorem, okay, so that we will see when you do the complex integration technique okay.

So let’s and this thing will help you this is like a straightforward, if you start with F(t) you end up getting the Laplace transform is this, so your Laplace inversion is only this one, okay, so this way it is well-defined for T positive so that you can see you have for T positive such, over such class of functions you can easily see that if you have expansion and you can at least see that you have some kind of theorems like if your F(t) as T goes to 0 is, what is this one? As S goes to, what is the limit of F bar(s)? As S goes to infinity so you will know what is in terms of limit of F(t) as T goes to infinity, so these two things can be related maybe with S power S into this,

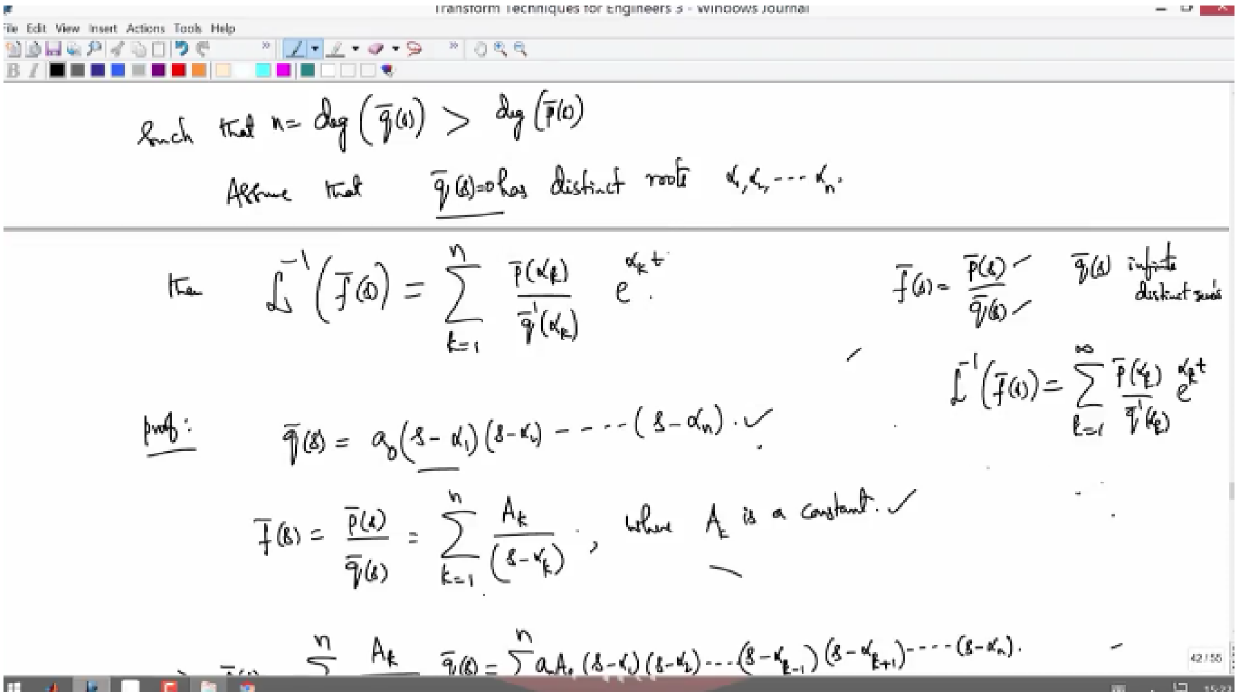


okay, these are some, it's a kind of asymptotic analysis these are useful so that we will see when as an when it comes, we will explain this, okay, so this part for such class of functions F(t) if you have series like this and you can easily see that, inversion will be like this provided this series is convergent, this series is a kind of convergence if your right hand side is convergent series and you start with this CN's, such CN's if you have this kind of F bar(s) you



can get the Laplace inversion in this form provided this is, the function you started with is convergent integral for all T positive, okay.

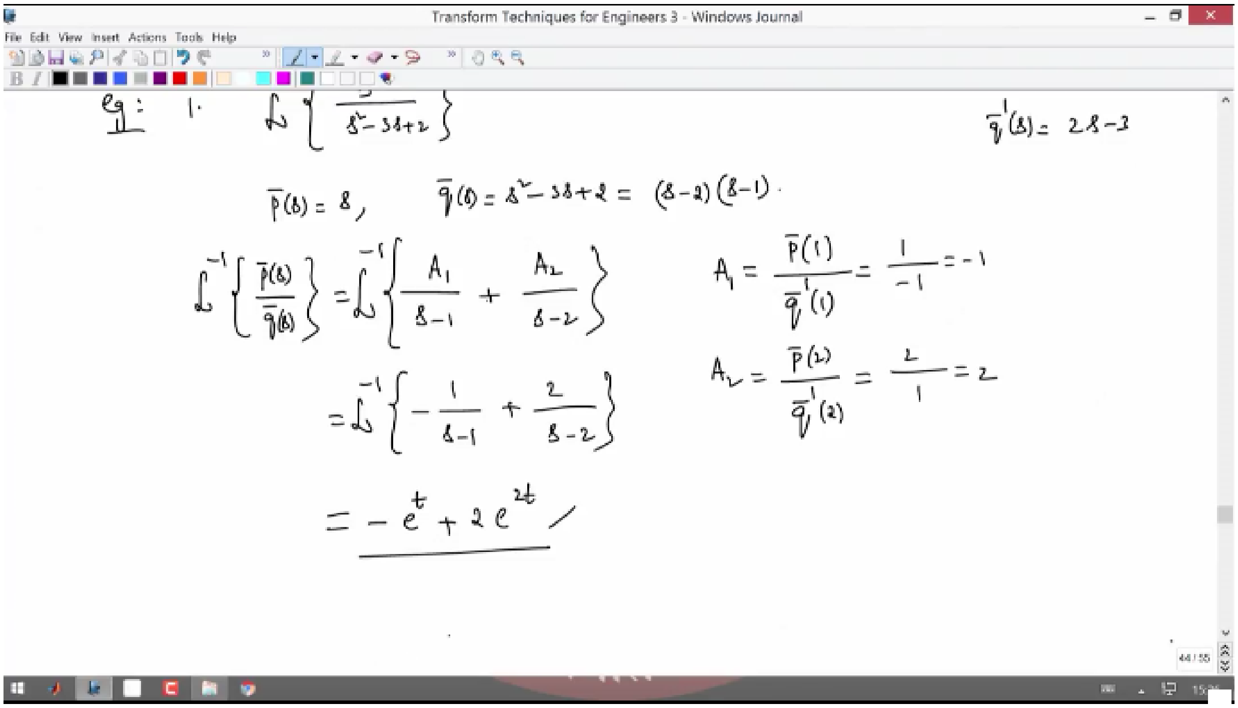
So this is actually this is a degress, this is not relevant to this kind of heavyside expansion, any case this is the third way of finding the inverse transform that is by heavyside expansion which



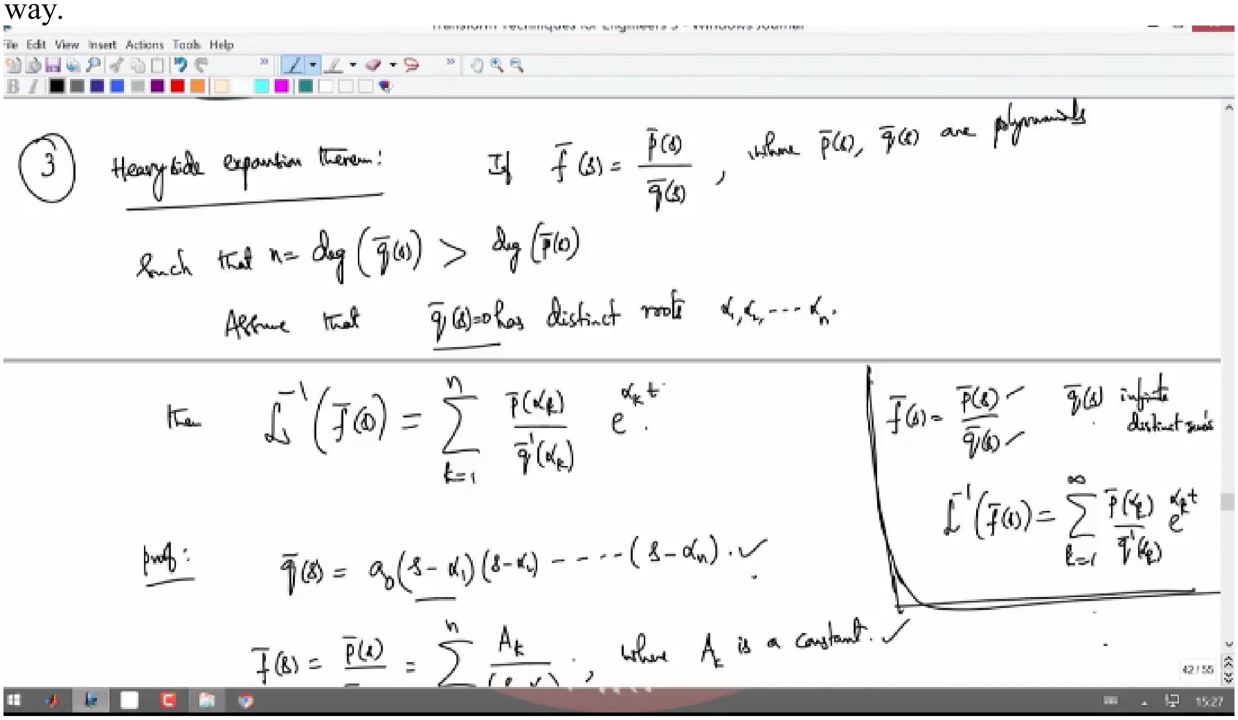
you, the same way you can work out for in general even if P bar, Q bar are not polynomials, but some other functions with infinitely distinct zeros you still get similar kind of expression that we will prove this you can actually show that by, we will not show but you can show for with some example using contour integration technique you can see that these constants are nothing but 0, they are the residues of roots, that is called contributes this numbers, okay, so this is the same form as in the heavyside expansion theorem, which is also, which you can use even if P bar or Q bar are not polynomials, okay.

We'll do some examples, so let me do some example using this heavyside expansion so one is how do you find this inversion with simple partial fractions S square -3S+2, so this is equal to so what is your P bar(s), if you actually consider that is S, Q bar(s) is a second degree polynomial S square -3S+2 this is S-2 and S-1, okay, so this Laplace inversion of P bar(s)/Q bar(s) is exactly what you have, so this is L inversion of, so this we already know that this is sigma so what you have, the first term is a summation that is you have only two terms S-1 so you have 1 over L inversion of 1 over S-1 with A1 + A2 divided by S-2, so without finding this partial fractions you know what is exactly A1 is actually P bar (alpha 1), so let us use which is 1 divided by Q bar dash(1) okay that is exactly your number A1.

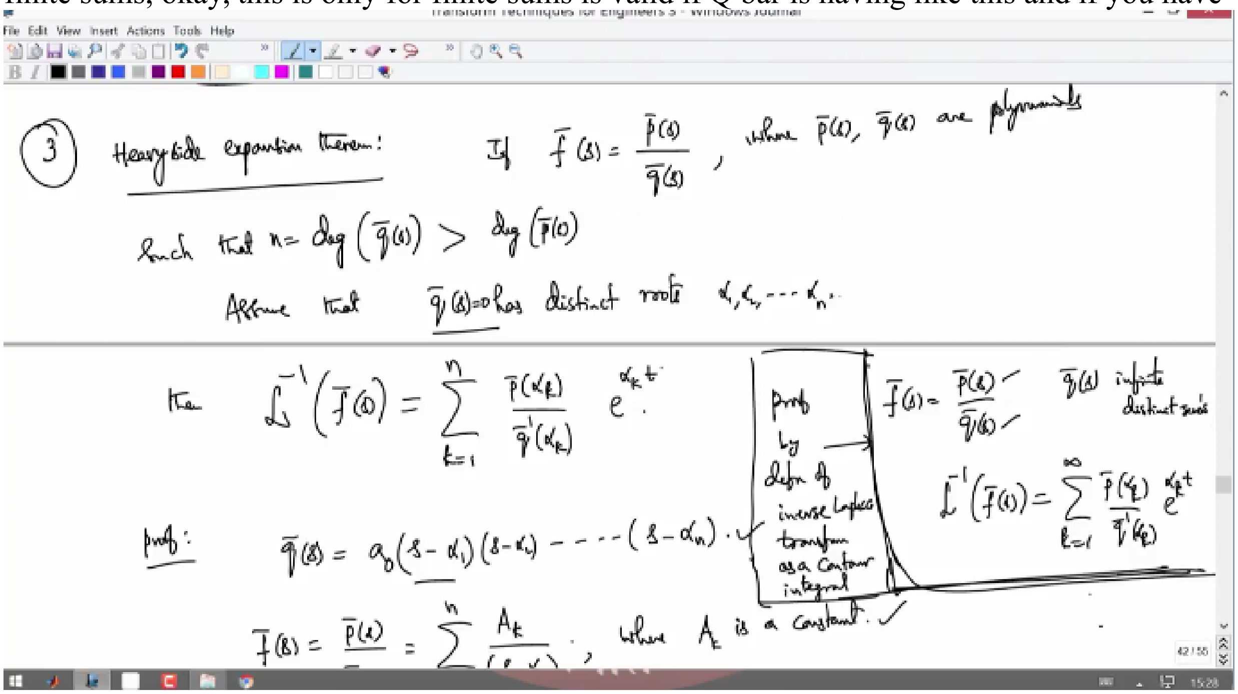
So similarly A2 is P bar(2) divided by Q bar dash(2), so if you calculate this P bar(1) is which is 1, so this is simply 1, as P bar(s) is S so it is 1 divided by Q, derivative is Q bar dash(s) is 2S-3 when you put S = 1 that is going to be -1, so you have -1 here, so you have L inverse of -1/S-1, and here this will be 2 divided by 2 into 4-3 that is 1, so you have 2, so you have +2 divided by S-2, so you have -E power T and then +2 times E power -2T this is what is the Laplace inversion okay.



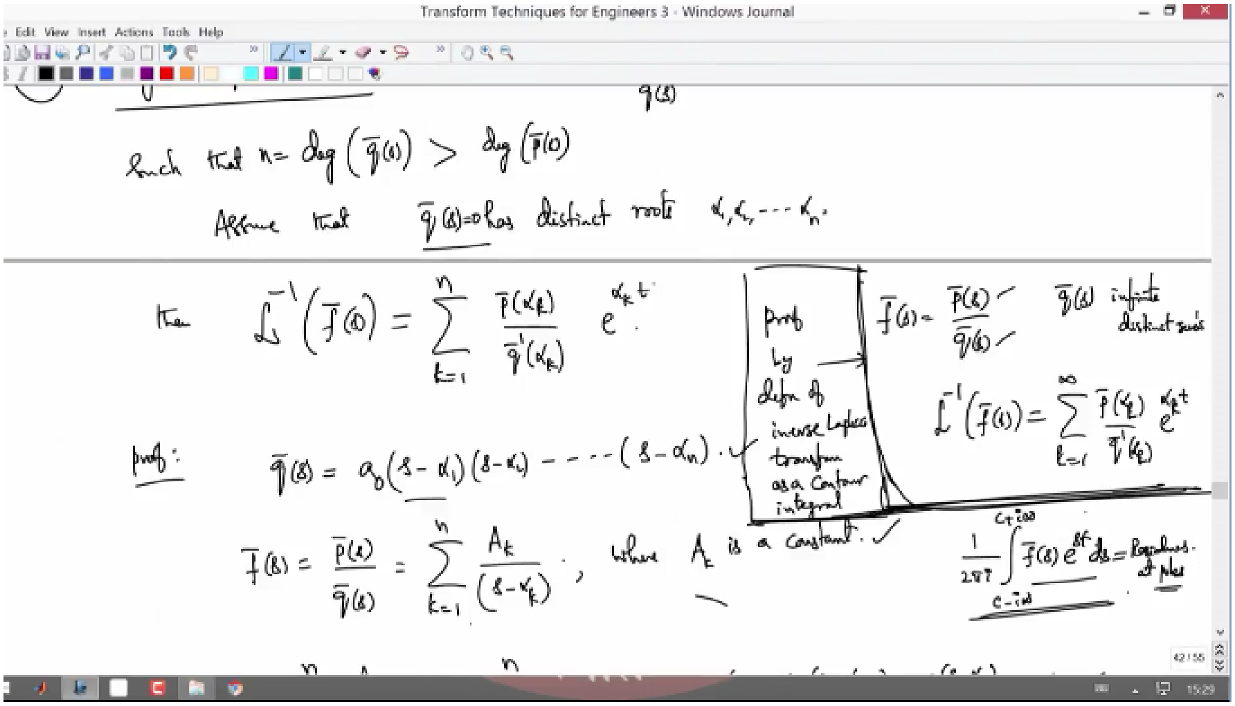
So I just use directly this expression, so you have to calculate its derivative and you have to look at the alpha 1 is 1, alpha 2 is 2, okay, so this coefficient of A1 is the coefficient of S-1 then you have to choose A1 as P bar(alpha 1) that is 1, okay, this corresponding to this one, if you use A to, whatever say I need not call A1 here, so I can write A by 1/S-2, 1/S-1, those coefficients, corresponding coefficients you have to consider this P and Q dash values at that value S- whatever that root, okay, so this is how you find given any polynomial by polynomial if you have a distinct roots in the denominator you can just get the Laplace inversion by this way.



I will try to give you some, I will try to see what is some example where this heavyside expansion is so happen that it works here, it works because it's not by this theorem, I'm just giving you some example where P and Q are not polynomials, but something else I give you but I use this expression because of assuming that this theorem is valid, okay, even if you have infinite, if you have distinct roots F bar is having like this P bar/Q bar, Q bar is having infinitely many distinct roots then this Laplace inversion is this, I'm using directly this one this is proof is by proof by contour integration, okay, a definition of inverse transform, inverse Laplace transform as a contour integration, as a contour integral, that we will see as a way to get the Laplace inversion in the next, as part of four, this is third which is using heavyside, is only for finite sums, okay, this is only for finite sums is valid if Q bar is having like this and if you have

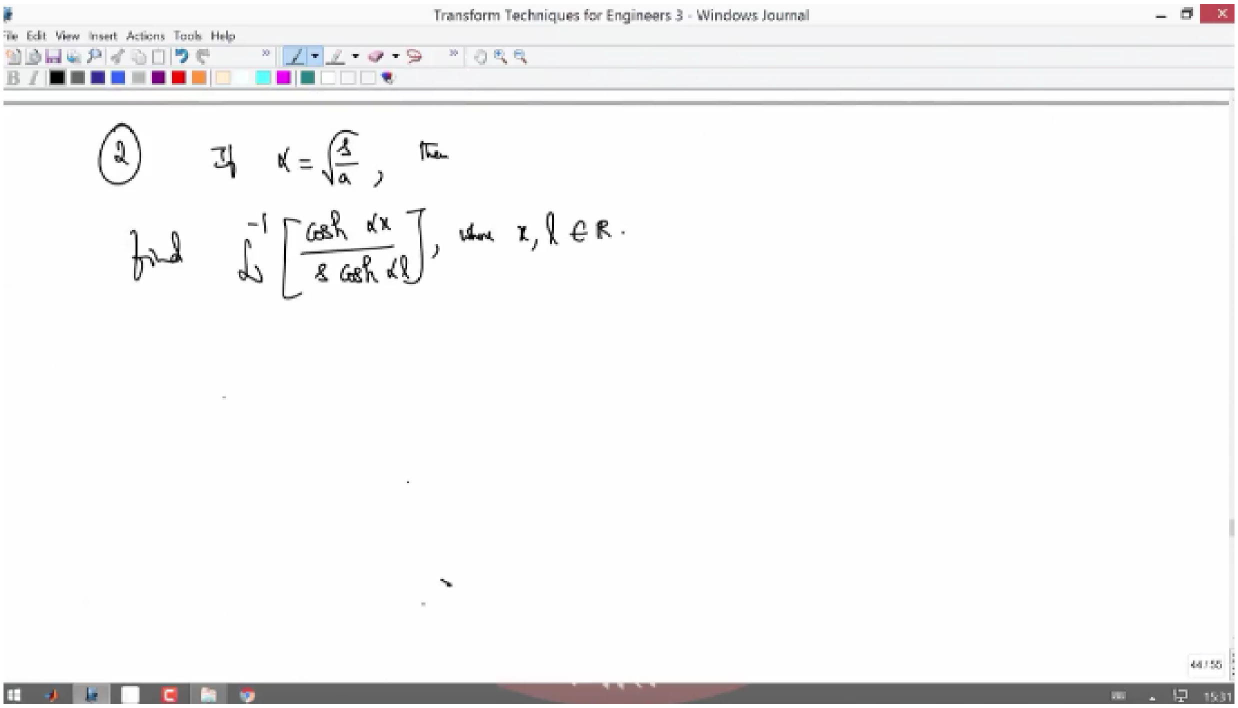


only distinctly many, if only finitely many zeros this is works, but this also happens to work the same way like it works for heavyside expansion, but the proof is not because if the proof is valid for, you can prove this if you consider this as P/Q where Q is having infinitely many roots, distinct roots then this is true, not this way but basic definition of front contour, inverse contour integration, inverse Laplace transform as a contour integration that contour integral, integral over that actually integral is 1/2 pi times C-I infinity to C+I infinity, this is the contour integration, using contour integration you can evaluate this as F bar(s) E power ST DS, this if you calculate, if you having a distinct roots, infinitely many distinct roots whatever you have and you can see that this is actually you see that this integral value is 2 pi is 2 pi goes, 2 pi times all the residues, all the residues at the poles, residues at poles those poles are these roots, okay, those poles residues actually will see that this one into, so when you calculate this



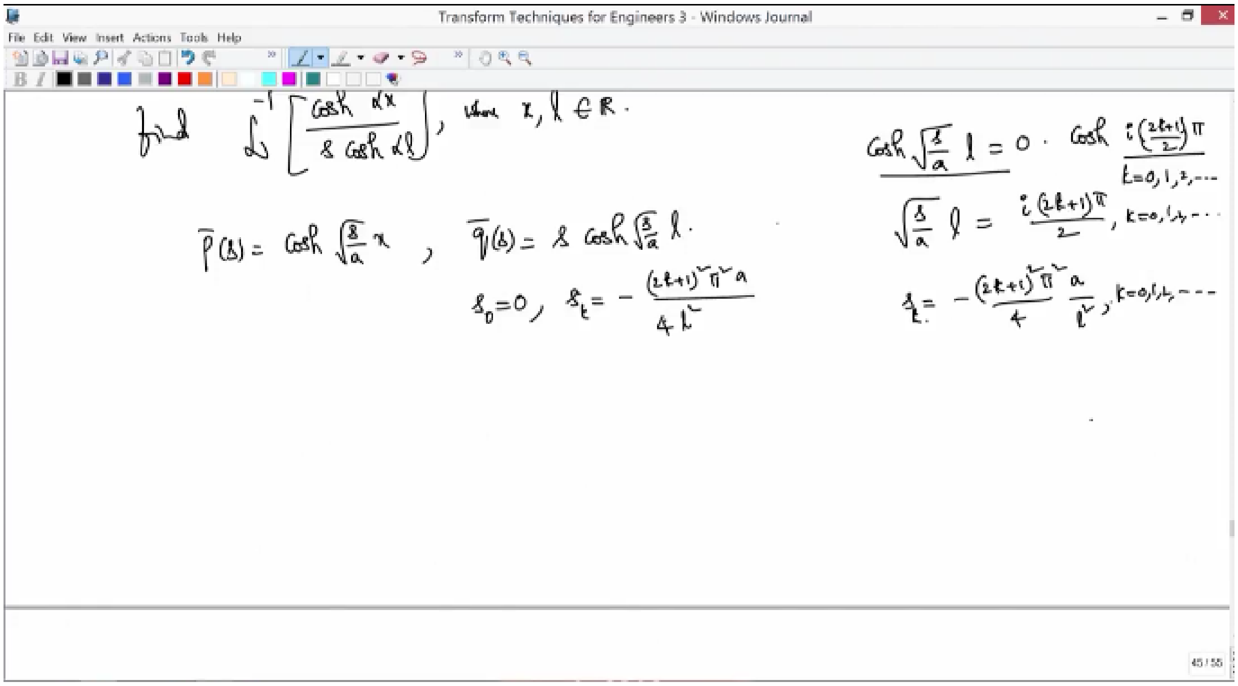
residues it happens that this comes because of E power S, those poles are alpha K, E power alpha K times T, F bar(s) you will see that residue is actually this part, simple poles, at simple poles the residue value is this, so that is because of this inverse integral, inverse transform definition you get this kind of form which you can use for some examples, but that is looks like a version of heavyside expansion, generalization of heavyside expansion theorem, but the proof the what whatever that we can prove, we cannot prove the way this is not the proof for that, okay, the proof that is given for the heavyside expansion theorem is not a proof for this generalization, whatever you are doing for the series, okay.

So but any case and let me give you an example and then anyway I'm going to give the last way how, general way how to find a Laplace inversion, so let me give you this example, if I give you, if I choose alpha as a root S/A okay, and then what is the Laplace inversion of cos hyperbolic alpha X divided by S times cos hyperbolic alpha L for some X and L, okay, X and L are, L belongs to real numbers let us say, okay, where this is, then find this, find this Laplace inversion.



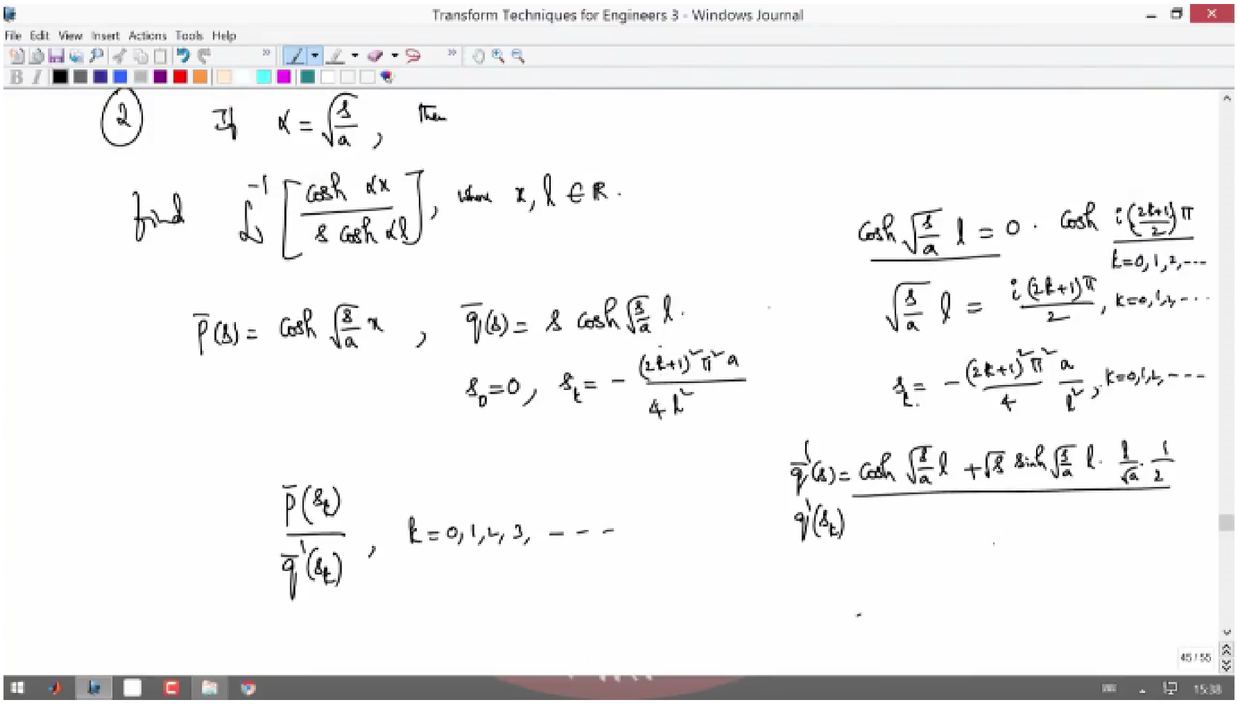
So what I do is, how do I, you can easily see that this for this example if you actually see what is my P bar or Q bar, if you write like that way P bar(s) is cos hyperbolic alpha X, alpha is root S/A, okay, root S/A into X. Q bar(s) is S times cos hyperbolic root S/A times L, and you see that cos hyperbolic this has Q bar(s) has all the roots, these are the roots 0 is the one root, so let's call this S alpha 1, so I'm using alpha here so let me call those roots as S1, S1 as 0, so let me call this S0 is 0, because of this S, S into this cos hyperbolic S root S/A = L, so that is cos hyperbolic X = 0, when we have these roots, roots of this or what are the roots of these roots of, what are these roots of cos hyperbolic S/A L = 0, cos hyperbolic root S/L is root S/A times L equal to, if you use cos hyperbolic you see that cos hyperbolic I times K+1/2, any K+1/2, now that means 2K+1 pi, 2K+1 so that is odd, 2K+1/2 times pi with this, this is going to be cos 2K+1/2 by pi, so that is cos, that is actually, so these are cos things which are 0, okay, so that is K = 0, if I take K = 0 is going to be cos pi/2 cos that is 0, cos K = 1, 3 pi/2, okay, 3 pi/2, 5 pi/2 and so on, you can get like this.

So root of S/A L if I choose like 2K+1 pi/2 and K is from 0, 1, 2, and so on, and I have I know that this value equal to 0, so that makes it S = I square that is minus, if I square both sides you see that -2K+1 whole square pi square/4 times A square, so that is A because the root A when you square it is A divided by L square, so this is what exactly your SK, so for all these SK's K is from 0, 1, 2, 3, onwards for which this is Q bar(s) is 0, okay, so S these are the roots, SK is a -2K+1 whole square pi square A/L square, 4L square. So these are all distinct, these are all distinct negative roots, clearly they're all negative side they are all, because they are negative roots as you see, as you will see in the next technique to find the Laplace inversion which is

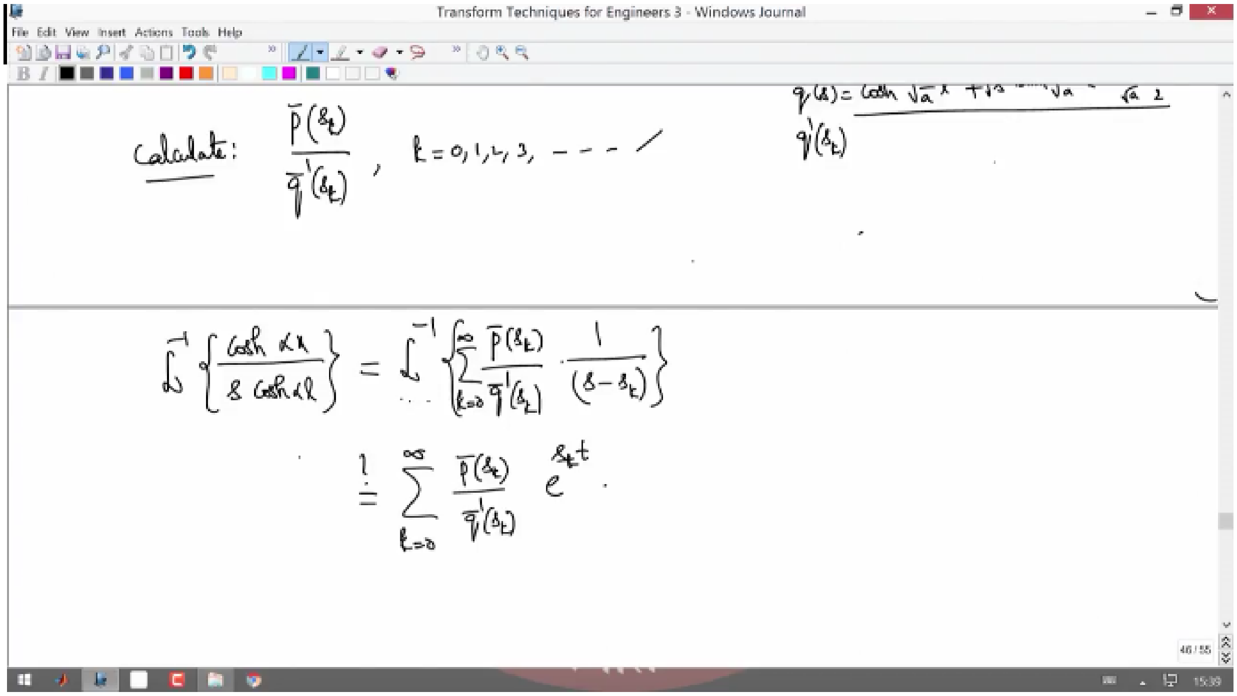


basically using the Laplace actual inversion to find the, using the contour integration technique to find, using the Bromwich contour general way of finding the Laplace inversion you will see that these things if you apply, these things will contribute as a residue set these poles, okay, so we will see that.

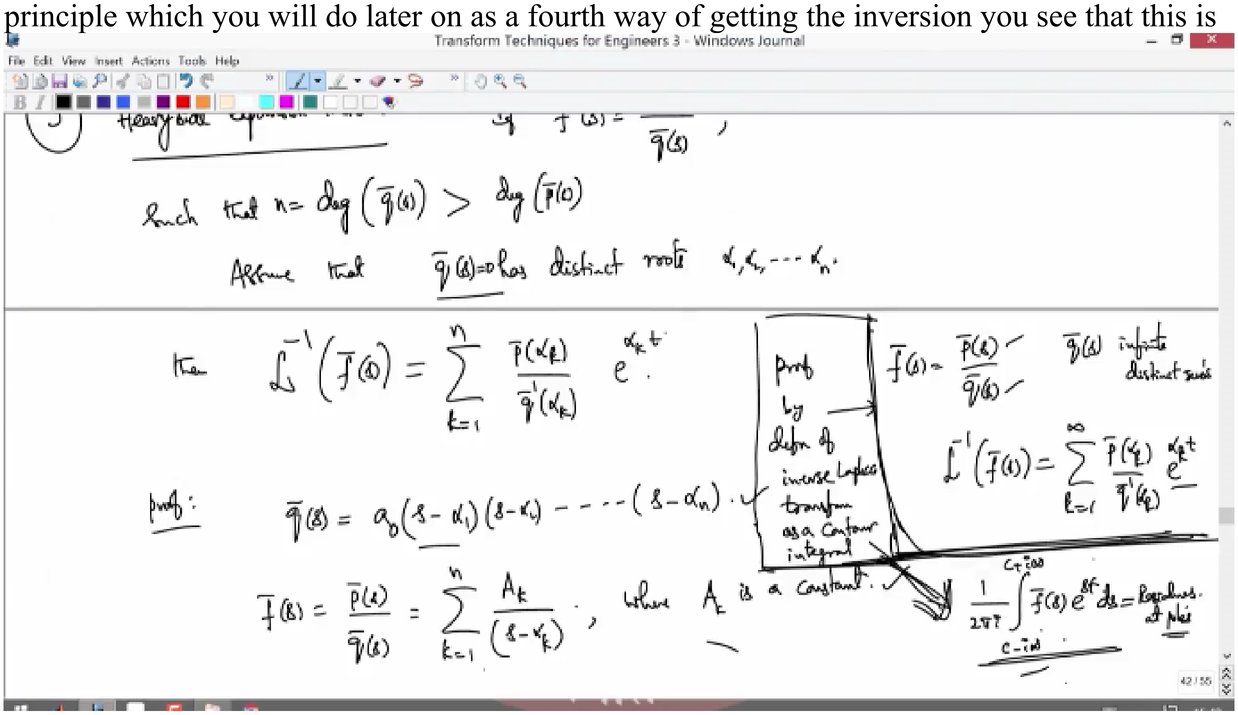
In any case here if you simply think that this is true, using the proof is by technique of this contour integration and when you see that this is going to be same as, this is on the similar lines to the heavyside expansion theorem, if it's valid you just see that it's going to be P bar, if you actually see that Q bar, and Q bar is this these are the roots, now what you do is as though the theorem is valid you calculate a P bar(sk) divided by Q bar dash, Q bar dash(sk) if you calculate, okay, what is Q bar dash? If you actually see Q bar dash(s) is cos hyperbolic root S/A L + S times sine hyperbolic root S/A into L times, L/root A 1/2 root S, okay that is what is a derivative, so this root S S goes so you have this, so this is exactly what you have as a Q bar(s). So now calculate Q bar(sk) you put it here in this, and you substitute here so if you substitute here I'm just writing directly you can work out, you will see that you will end up getting, so for each K so this you calculate for each K, K is from 0, 1, 2, 3, onwards, if you actually calculate



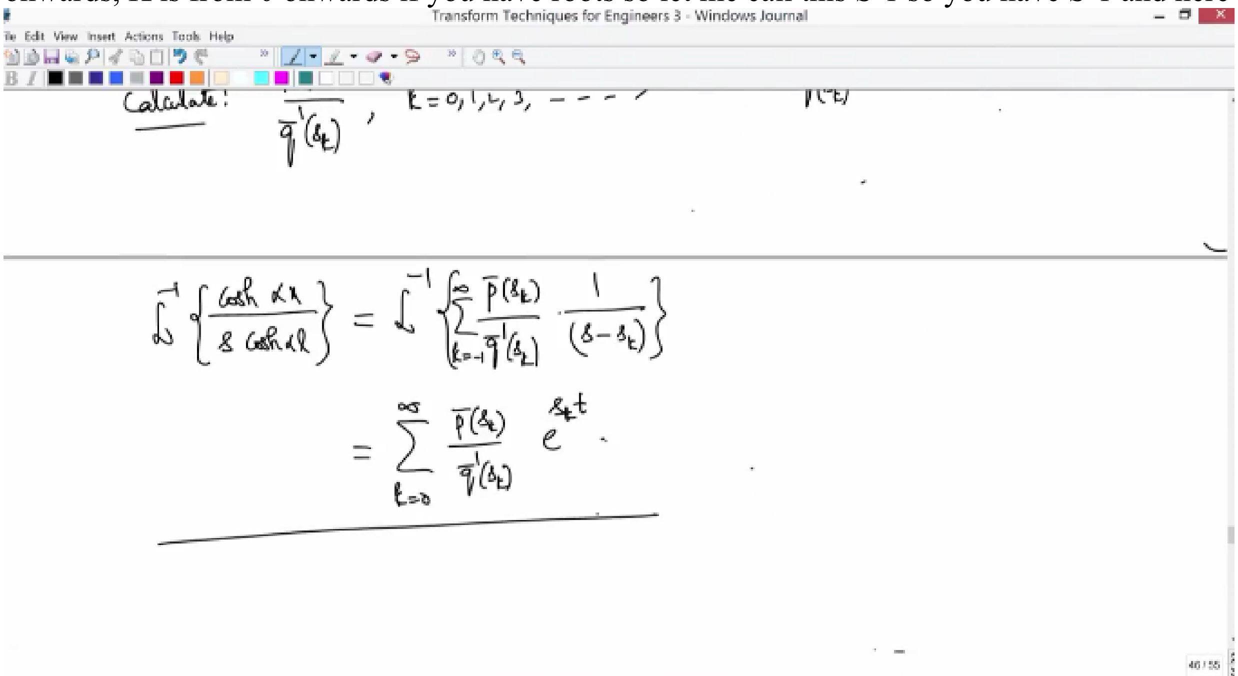
so what you see is, so what you have is this times once you calculate, calculate all these things calculate, now you know what is SK's and you know what is Q bar(s) and Q bar(s)? Q bar(s) is this, P bar(s) is this, Q bar(s) is this, Q dash is this one, you calculate and you calculate these terms first and then what you see is L inversion of cos hyperbolic alpha X divided by S times cos hyperbolic alpha L this is, as though you can write it, you can get the partial fractions for each of this, you try to write L inversion of this P as the partial fractions this number is, whatever you calculated PK/Q bar dash(sk), of course with summation K is from 0 to infinity because this is of such that many number into 1 over S-SK, okay, so this is equal to so L inversion for this as though you can apply, you can apply this inversion as though you can take this Laplace inversion, as though it's a linear operator though, though it's not, we're only doing formally here you take this inversion inside this summation you have this K is from 0 to infinity P bar(sk)/Q bar dash(sk) times L inversion of, this is a constant so 1/S-SK, this inversion is we know that E power SKT.



So this is actually questionable we don't know whether we can take this inversion inside this infinite sum, but as such so we are not using, we're not using this the proof is we're not using that, we're not saying that we can take this L inversion inside but when you actually using this principle which you will do later on as a fourth way of getting the inversion you see that this is

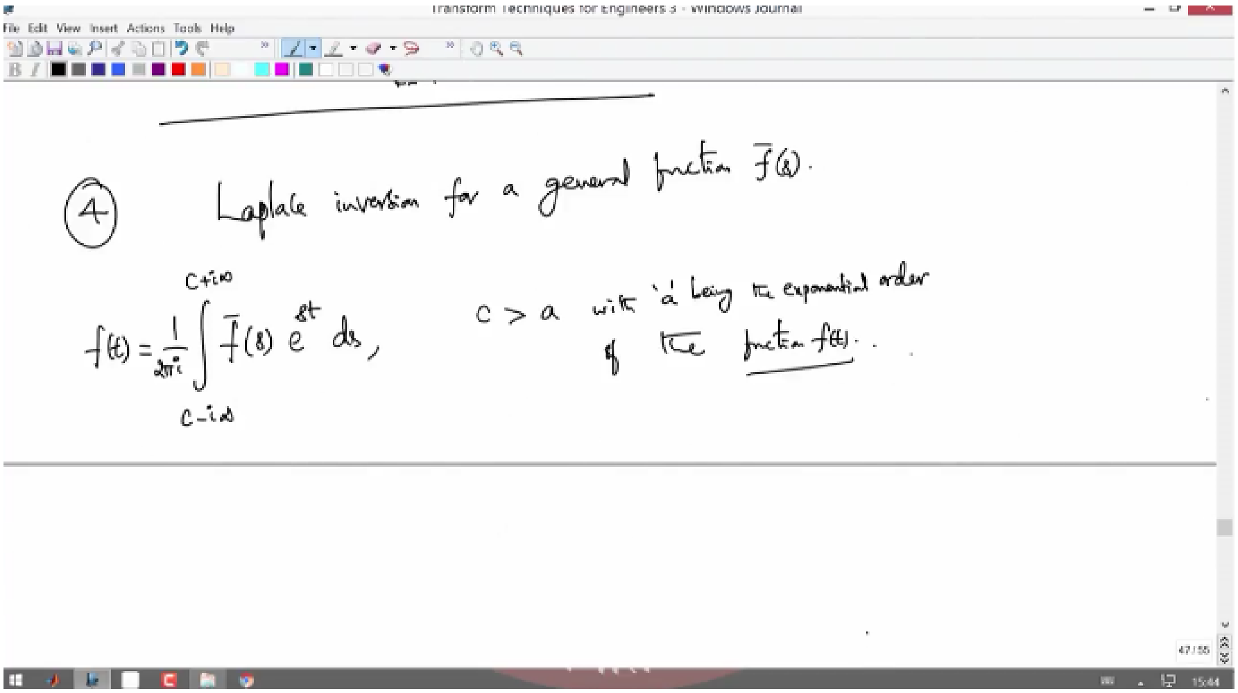


always true if you have Q bar(s) is having infinitely distinct roots, okay, for this this is the example for which this is true, so these are the residues, these coefficients has a residues so that way once you have that residues L inversion as a residues if you can calculate it come naturally like this, the same expression you are getting it here by just formally I am allowing this L inverse going inside this infinite sum you get the same expression, so it looks like heavyside expansion theorem but it is questionable okay, but it is actually by some other means but the general way of finding the Laplace inversion you can find this and finally you end up seeing that this is going to be some kind of, for K = 0 you end up getting 1 and you see the other terms are like, no, not 1, so let me not write this okay, as such I'll leave it here, let's not do any calculations here, okay, so if alpha is this if SK's are the roots, S0 is 1 root and SK, K is from 0 onwards, K is from 0 onwards if you have roots so let me call this S-1 so you have S-1 and here



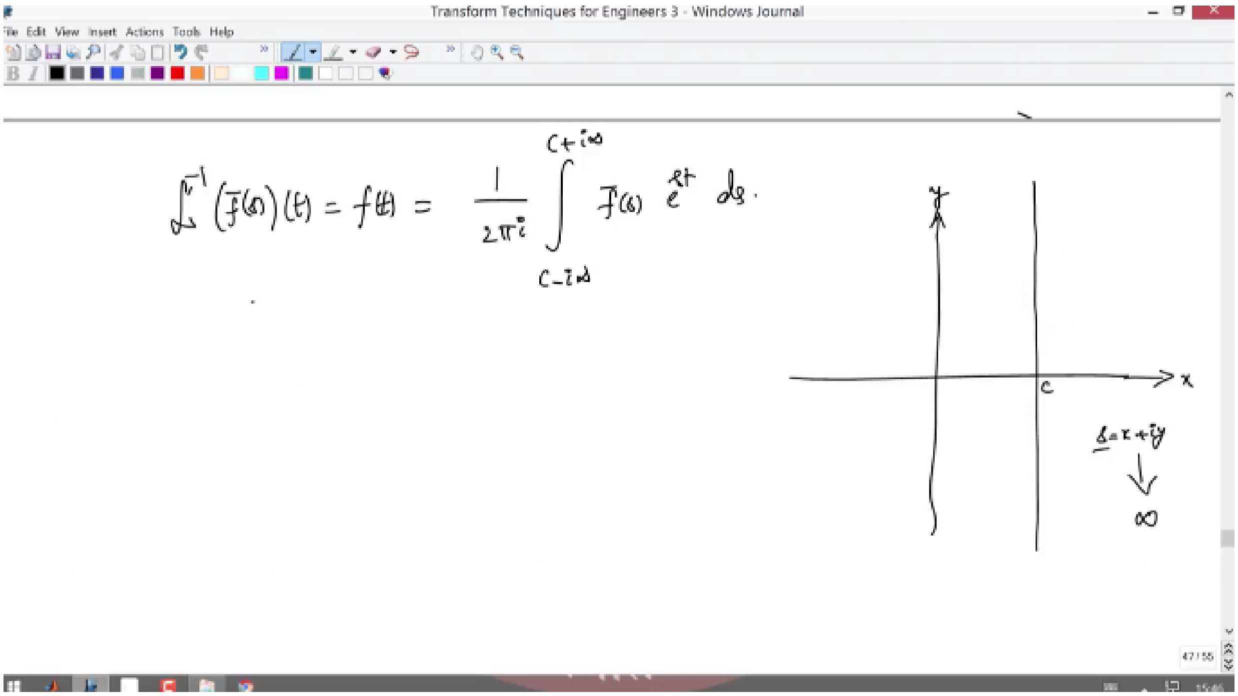
SK is from K is from 0 onwards, so you have a K is from -1 onwards, okay, K-1 is S-1 you can write this, S-1 is 0 okay, 1 over S, so if you do like that so formally you have K-1 from -1 to infinity you have this expression that is same as that looks like you have formally if you apply this heavyside expansion theorem this is what you get, but actually if you use the general way of finding that I'm going to do next, general way of finding the Laplace inversion using the contour integration technique if you do you'll get this expression naturally okay, so that is the reason I give you this expression.

So let us use the next way of finding the Laplace transform inverse Laplace transform Laplace inversion for a general function F bar(s) okay, so how do I do this? If you have a rational function by polynomial you can do partial fractions, if it is a product of 2 Laplace transform you can use a convolution, if it is kind of again, again like polynomial by polynomial with distinct roots you can use the Heavyside expansion theorem, and also yeah these are the three types you have seen so far. And the fourth type is a general Laplace inversion for a general function, even in the 3 cases are, you can use it here so all the three cases or three ways of finding the Laplace inversion they are all ad hoc methods all the three which I've seen, this is the general method given F(s), F bar(s) given F(s) if it is a Laplace transform F bar of, F(t) is the inverse function is 1/2 pi times, this is C-I infinity to C+I infinity, F bar(s) E power ST DS, where C is bigger than A being the order of exponential, order of exponential function F(t), exponential order rather being the exponential order of the function F(t), so if you have this, this is what we have defined, that's how we had defined the Laplace inversion.

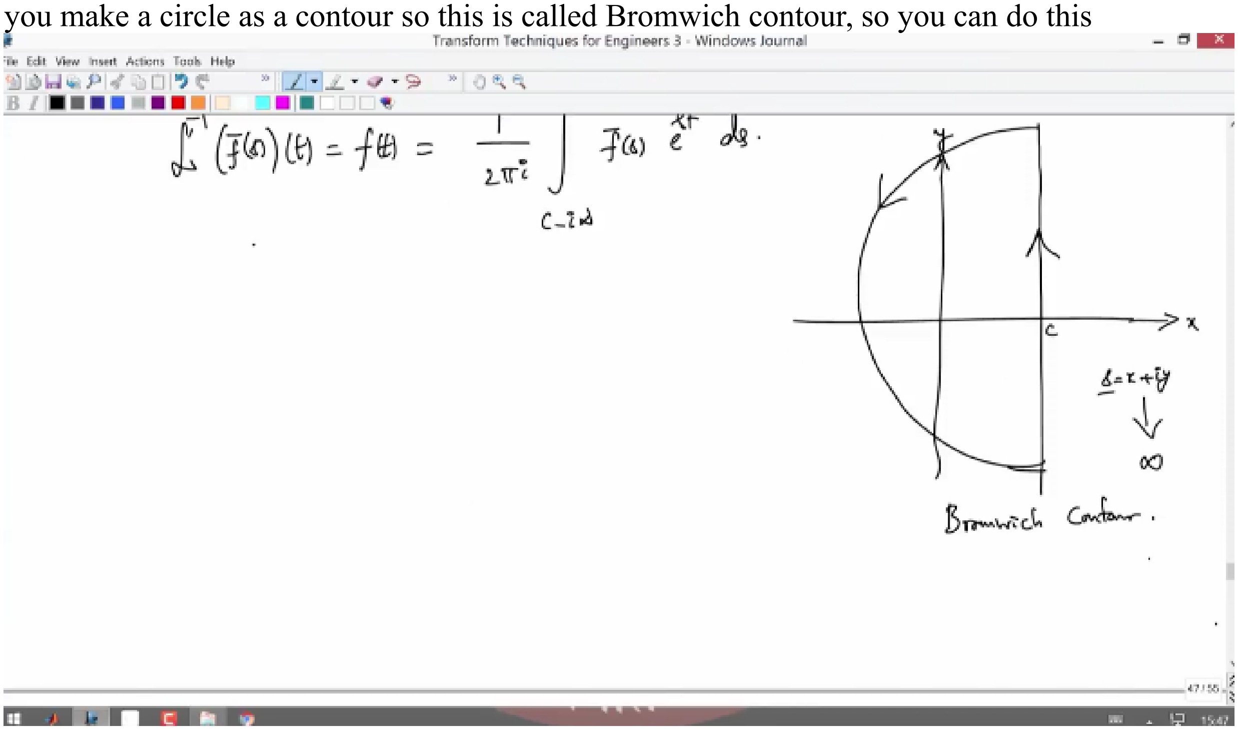


So I can directly find this Laplace inversion of F bar(s), that is as a function of T which is function of T as this 2 pi I times integral C-I infinity to C+I infinity F bar(s) E power ST DS, so how do I evaluate this in general? So you have a complex integration you need, you need complex integration that I think I will take one or two hours to explain how we do this integration in the complex variable, so let me use this XY variable, S is this, S is actually X+IY, S is a complex variable, how do I do this? Actually this is your C, this is your C beyond which this is having exponential order, this is actually going to 0 as X goes to infinity, this is a valid okay, as valid as X goes to infinity in the complex planes infinity means you need all directions if it goes to, if it goes to infinity this goes to infinity as X, suppose Y = 0, X goes to infinity that is also infinity.

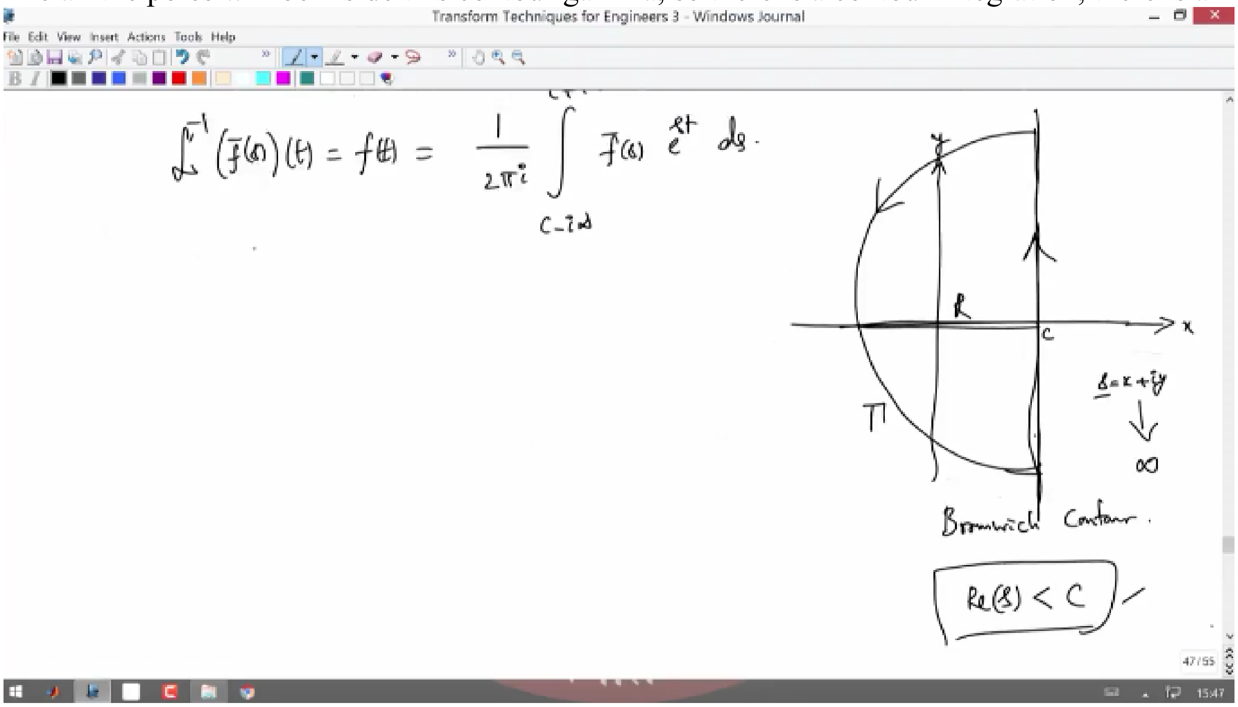
Suppose Y = 0, X goes to minus infinity that is also infinity, so in a complex plane as you go in the extremes that is actually called infinity, all are same -infinity +infinity there is no distinction this is infinity means it's an imaginary quantity, there's nothing like actually, there is nothing like infinite thing that is a notion which is not something which is not finite, okay, so in the



complex plane you can go in all directions so the real line you can go to two directions that is negative side and positive side, that's why we are making a distinction between those two extremes, -infinity and +infinity, but in the complex plane you can, which is not a finite complex number means you can go to in all directions that's the reason the notation wise we put it as 1 infinity, okay, so -infinity +infinity or any infinity, infinity + I, plus some constant times I they are all same as infinity okay which is not finite in the complex plane, so if we do that this function is having, so what I do is you try to take this, you make a circle out of this like this, you make a circle as a contour so this is called Bromwich contour, so you can do this



Bromwich contour while doing this if I call this gamma, gamma as this plus this and you have, this is your radius of the circle, this is R, as R goes to infinity this covers entirely this side you know all the poles which are inside this side, which are, here this this side whatever all the poles as, how do I root it, real part of RS, S is real part of S less than C side, this is the imaginary, okay, this is this side left side, this side of the plane, this is your line, left side of this line all the poles will be inside this contour gamma, so there is a contour integration, there is a

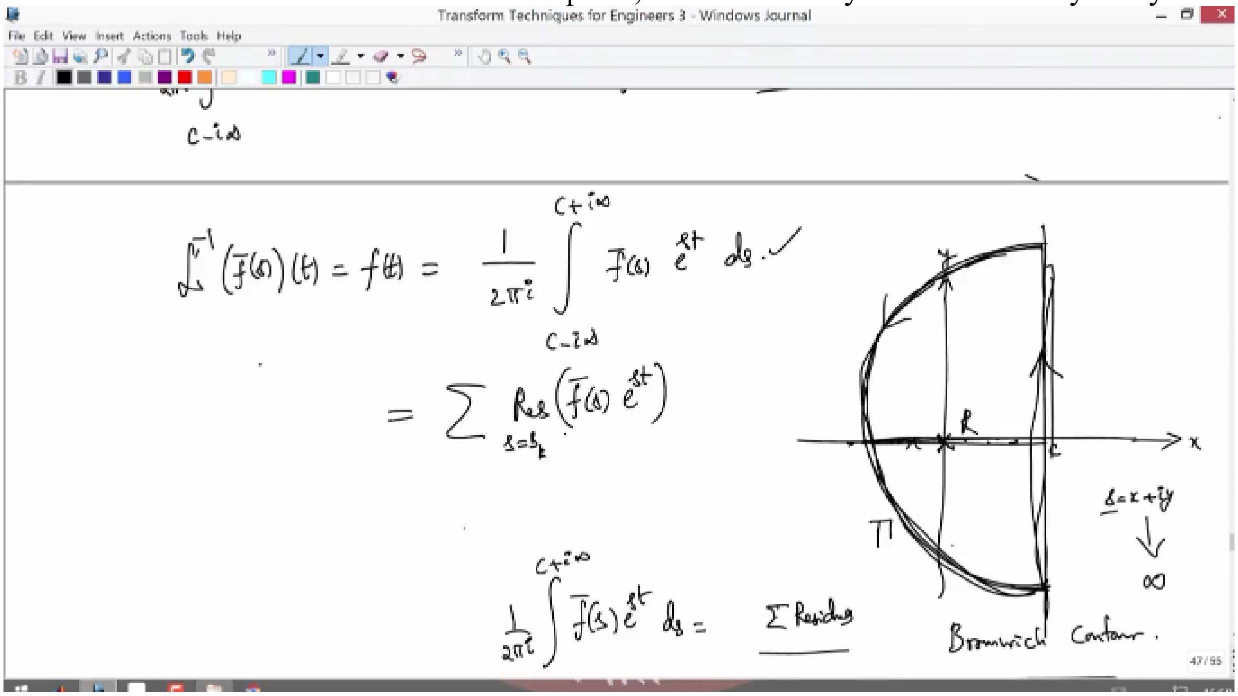


residue, Cauchy residue theorem that tells you that contour integration over this plus this is equal to 2 pi times, okay, that means this into 2 pi I times that integral over gamma, let us call this F bar(s) E power ST as your complex valued function, DS is actually equal to 2 pi I times all the residues, sum of all residues.

So 1/2 pi if we bring it here so you have to calculate this integral over this, over this is actually the your integral, so your actual integral plus over this integral because of this nature of this function the integral over this contour will be vanishing, so that way what remains on the left hand side is what you need that is the integral over not F bar(s), F bar(s) integral over C-I infinity to C+I infinity will be a summation of all the residues.

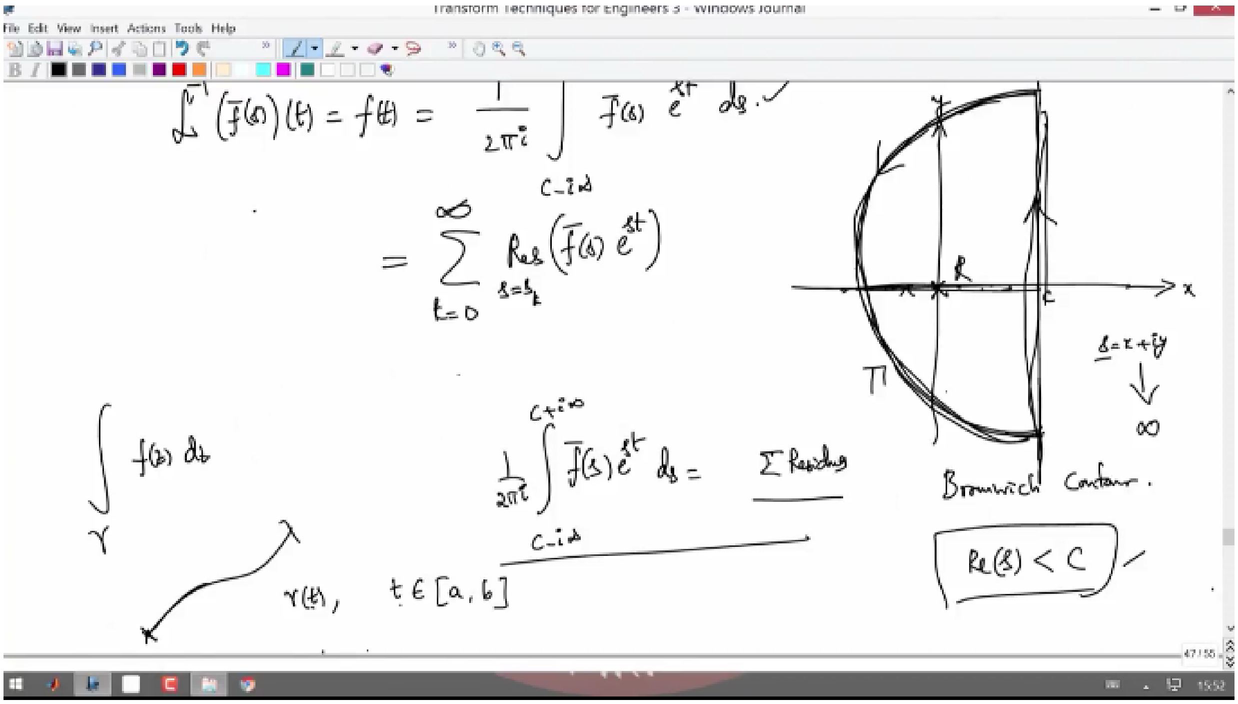
Now if you have infinitely many roots as in this example, if you consider this as a function, as a complex valued function cos hyperbolic alpha X/S into cos hyperbolic alpha L, where alpha is root S/L that involves S, okay, so using that you see that all the poles will be inside, all the poles on the negative X axis as you see the roots of that Q bar(s) all are like this, they're all negative sign off, so they're all on the negative side of the X axis, they’re all here, okay, everything is here so when you expand as R goes to infinity you see that this is going to be 2 pi times residues at those points K = 0 to infinity.

So X = 0 is also here, so this one and all the negative signs are here, so you see that in that way if you use this principle this theorem or rather this contour integration technique you see that you end up getting some of all the residues that is exactly that is exactly how you calculate this F(t) as inverse Laplace transform of F bar(s), so this is actually equal to 2 pi I times, so 2 pi 2 pi goes you see that sum of all residues, residues of F bar(s) E power ST, this function is a function of A, and S belongs to this complex plane, real part of, for at all residues, at all poles or rather let me call S = SK where SK's are poles, so K is from if you have infinitely many K is



from 0 to infinity you call this, so this is how you calculate, so you should know how to find the residues of a complex valued function, okay, defined over a complex plane.

So this is what I will explain on the next few videos how to find the, how to make integration in the complex variable we start with what is actually integration in the real line, so over an integral what is the meaning of real valued function what is the integral of it, and then we'll move on getting what is the integral over a real, on the real line what is a complex valued function integral and then we will go to integral over a complex plane over a line in the complex plane, what is the integration of a complex valued function, so these things I'll try to make a distinction and finally we end up seeing, finally what you see is integral over F(z), F(z) is a complex valued function that is U(x,y) + I times V(x,y) okay, it's a complex that is a complex valued function of complex plane, XY belongs to the plane, and this is over some curve gamma, so let me call this gamma here, if I put gamma this has your F(z) DZ, so this is the integral you are interested, so this you can easily calculate if you can parameterize this gamma, okay, gamma if you call like this in your plane you should know how to parameterize as gamma(t), T belongs to some real line, so real interval that is called this AB, so if you take this piece gamma(a) is actually this point as you move along T from A to B you move along gamma T moves along from this point to this point on the curve, so integral over that curve



means what? That is exactly we will see, okay.

And then so those are such integrals these are the integrals, those are such integrals what you have, what you need, this is one such integral, so how do I find this using contour residue, Cauchy residue theorem, so this in order to get these theorems we will briefly explain what is our integration and then finally briefly about those results in the complex variables how to find the contour integration technique I will try to spend one or two hours on this and then come back to this general way of finding the Laplace inversion of a given function F bar(s), and then we'll do some example how to find the inversion, so this is the general way of finding the inversion if you don't know how do you, if you don't know already from the list, if you don't know, if you cannot use the list of Laplace transform and it's inversion, this is the way to find the elementary functions you know, if you have a complicated function so there is no other way to use this general inversion formula to find the Laplace inversion which is a function of T, but then this formula is actually contour integration, it's an integral, complex valued integral, so complex integration over some contour in the complex plane of a complex valued function, so you need to know how to find a contour integration to evaluate this.

So that is what exactly we evaluate using this Bromwich contour method, and so that you see that is all residues is that is going to be 2 pi, that is going to be sum of all the residues, okay. So we will explain how these things in the complex variables, how do you find this integration complex variable technique, residue technique, residue calculus technique in the complex integration, we'll spend one or two hours in this and then come back to this Laplace inversion using this contour integration technique. Thank you very much, we'll see in the next few videos all such things. Thank you.

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