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NPTEL ONLINE COURSE
Transform Techniques for Engineers
Methods of Finding Inverse Laplace Transform
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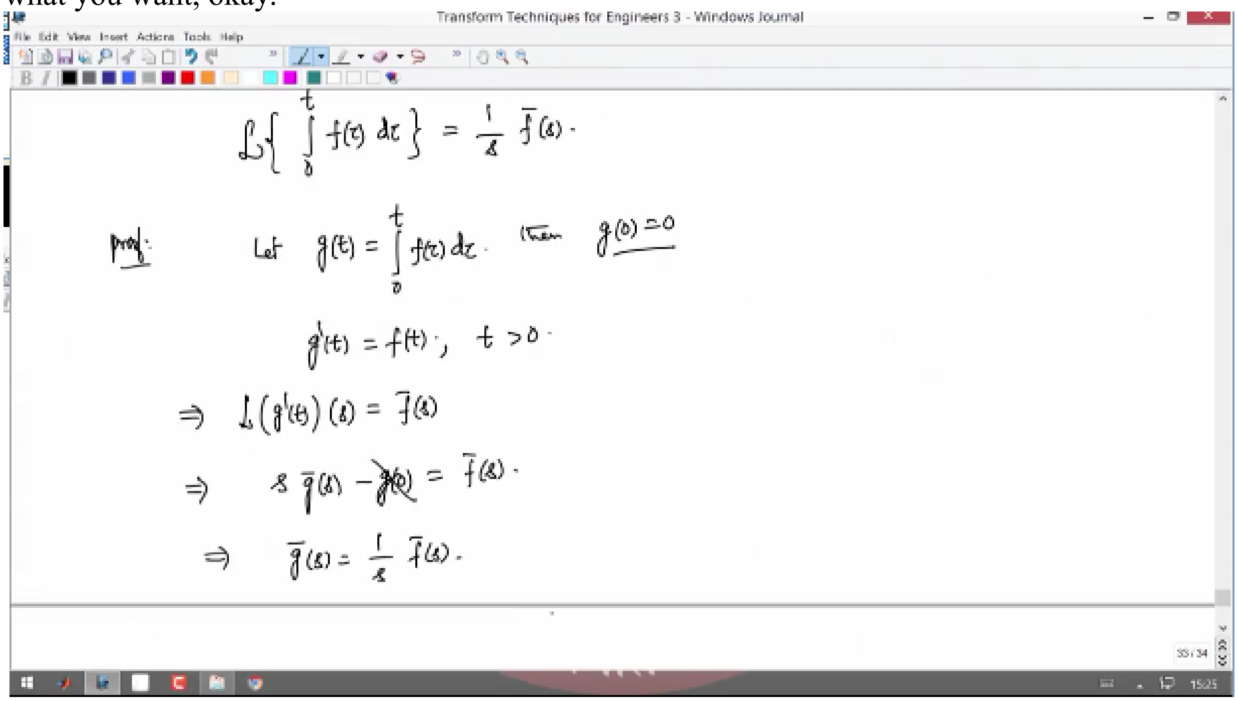
Welcome back, we were doing the problems, we were discussing about the properties of Laplace transform. In the last video we have seen what is the Laplace transform of a function, if you know a Laplace transform of a function you can easily see find Laplace transform of a function, that function multiplied with some T power N or sub T okay, so that's what you have seen earlier.

So we were discussing about the property that if you know already Laplace transform of certain function, this property tells you the Laplace transform of multiplication of T power N with such function is simply derivatives of Laplace transform with respect to S.

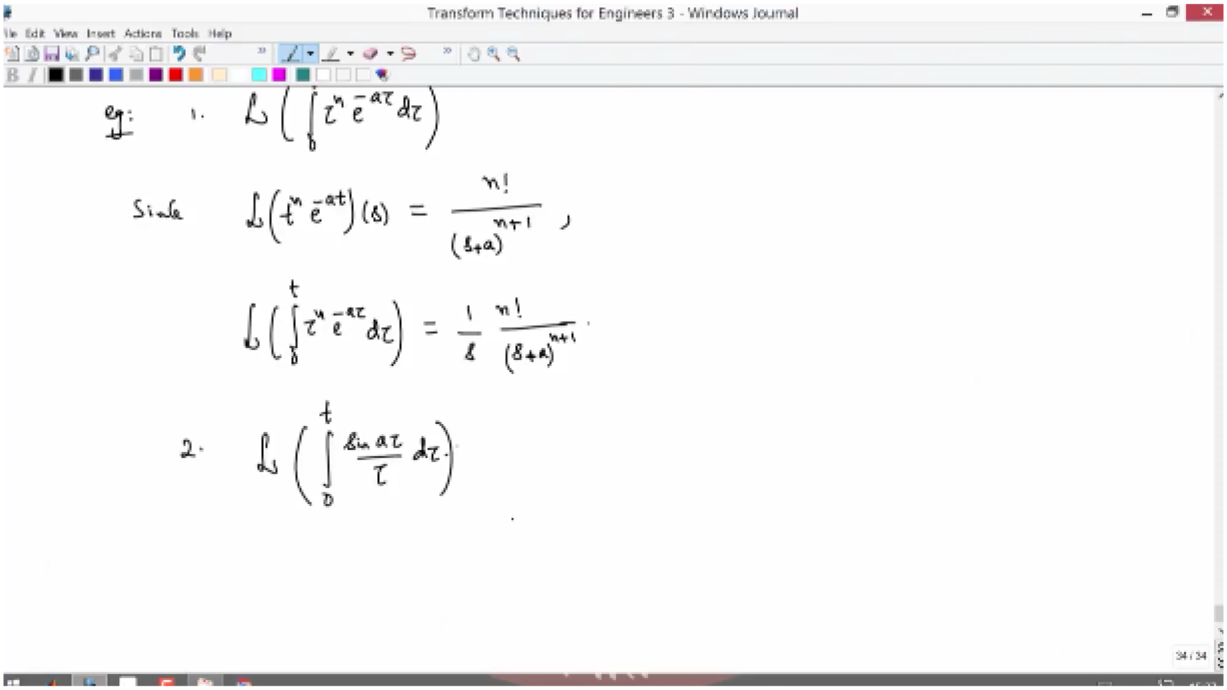
And similarly you have seen other transform, so if you know the Laplace transform certain function, if you want to find the Laplace transform of 1/T multiplied with that function that is also you have seen, so we will see one or more properties so you may also see when you try to solve integral equations you may have to find the Laplace transform of an integral, if your right hand side is some kind of integral so how do you aware, what is its Laplace transform, so let's evaluate that as a property 10, so this is a Laplace transform of an integral, so if F bar(s) is Laplace transform of F(t) which is function of S, then the Laplace transform of this integral 0 to T and you have F tau D tau, so this is your function, and what is this one? This is 1/S times F bar(s) Laplace transform itself.

So how do we prove this one? This is easy, if you call this as some G, okay, so let G(t) as 0 to T integral 0 to T F tau D tau, then G(0) is zero, it's clear, and then G dash(t) is F(t) and we already know what we want is Laplace transform of G(t), so you start with Laplace transform of F, so Laplace transform of, left hand side that is G dash(t) as a function of S is equal to Laplace transform of F(t) that is F bar(s), so this implies this, if you apply Laplace transform both sides, on applying both sides Laplace transform this is what you get this is obviously T positive such a function you have.

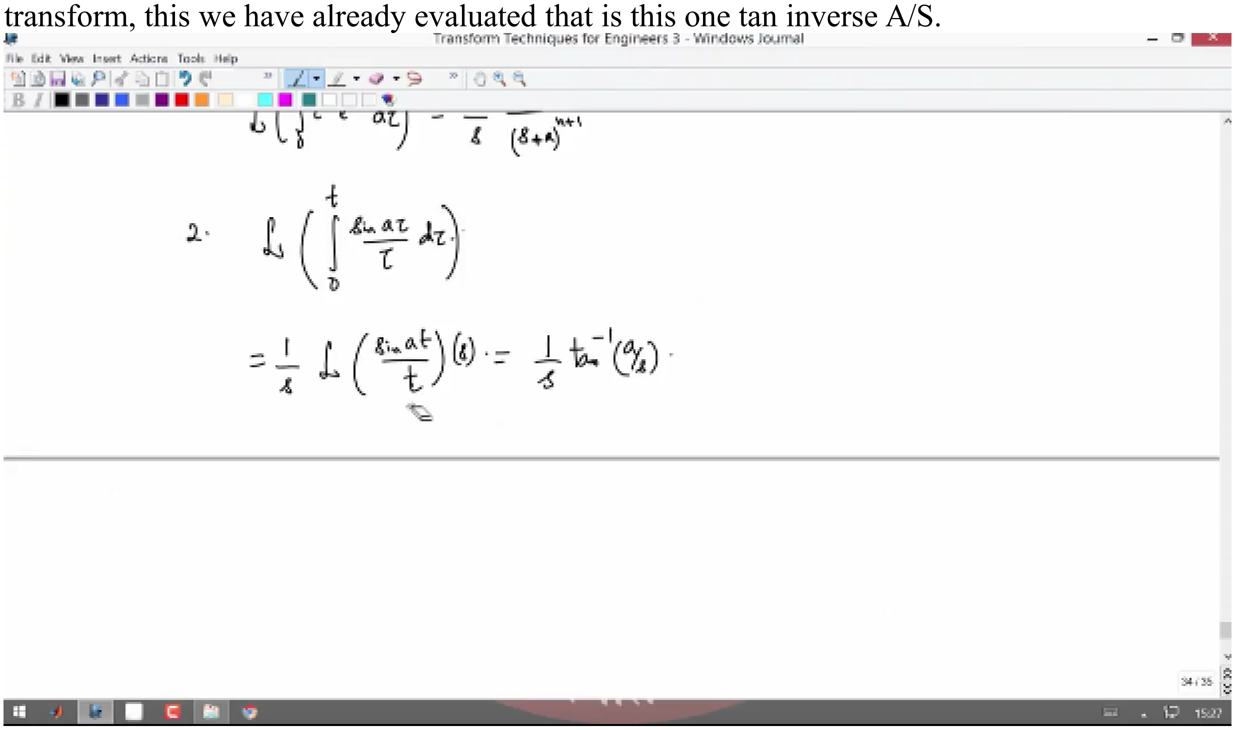
If F is having a Laplace transform you can expect integral if it exists, this is what it is okay, so integral of, F is exponential function if you integrate from 0 to T that is also function of exponential order, so in that sense this is also, if F is a function of exponential order a Laplace transform exists and this integral always exists for which Laplace transform also exists because integral of F(xi) 0 to T integral that is also exponential function of order, some order, rather same order rather, okay, so this implies we know the derivative a Laplace thing is S times G bar(s) – F(0) or rather –G(0) which is equal to F bar(s), so this implies because G(0) is 0 that is what is given here, so that implies G bar(s) which is equal to 1/S times F bar(s) this is exactly what you want, okay.



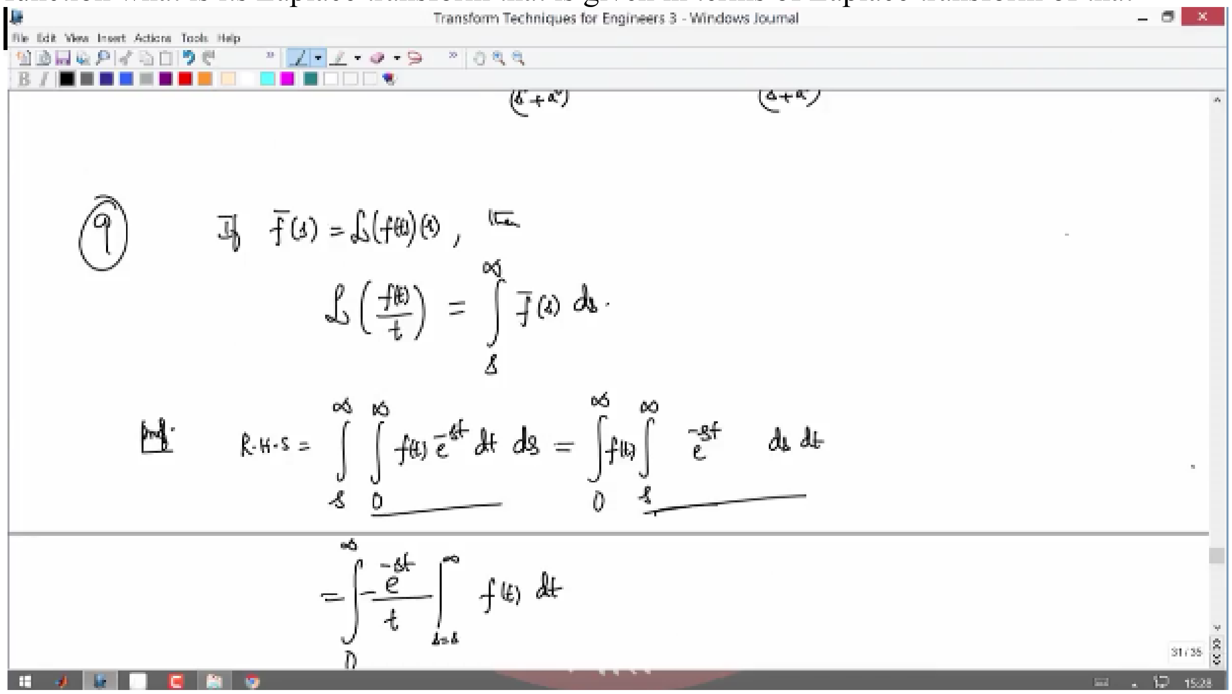
So using this property I can see some example, how do we find Laplace transform of, what is the Laplace transform of simply integral 0 to T tau power N E power -A tau D tau, so we already know that the Laplace transform of this is, what is the Laplace transform of T power N E power -A Tau AT as a function of S this is actually equal to, because of this exponential function we have 1 over, so T power N is N factorial divided by S power N+1 without this exponential, once you multiply this exponential now to replace S/S-A okay, so since this is the case we know this so the Laplace transform of this 0 to T tau power N E power -A tau D tau, integral thing is 1/S times N factorial this into, this Laplace transform of this function inside integral that is S-A power N+1, so this is –S so you have +A here, right, is actually S+A not S-A, it’s S+A, because it's –AT, so E power –AT you have addition, so this is one example, we



can do also some other example like what is the Laplace transform of some integral that is sine A tau/tau D tau, so we have seen we know already integrand what is the Laplace transform of this integral, so this is 1/S times Laplace transform of sine AT/T as a function of S, okay, by the property and this is exactly we have already seen this earlier, Laplace transform of this is 1/S times tan inverse A/S, so you just look at the earlier video and we can see this Laplace transform, this we have already evaluated that is this one tan inverse A/S.



So this is how we can use some of these properties, if there are different types for example here if you see integral type, integral with some integrand, integral is 0 to T times integrand is 1 then you can easily find what is the Laplace transform of it, earlier thing is if we have 1/T times of function what is its Laplace transform that is given in terms of Laplace transform of that



function F which is an integral, and if you multiply with T that is a derivative, okay, so this is how we can make use of these properties when we do the applications of Laplace transform, so that's what we will see.

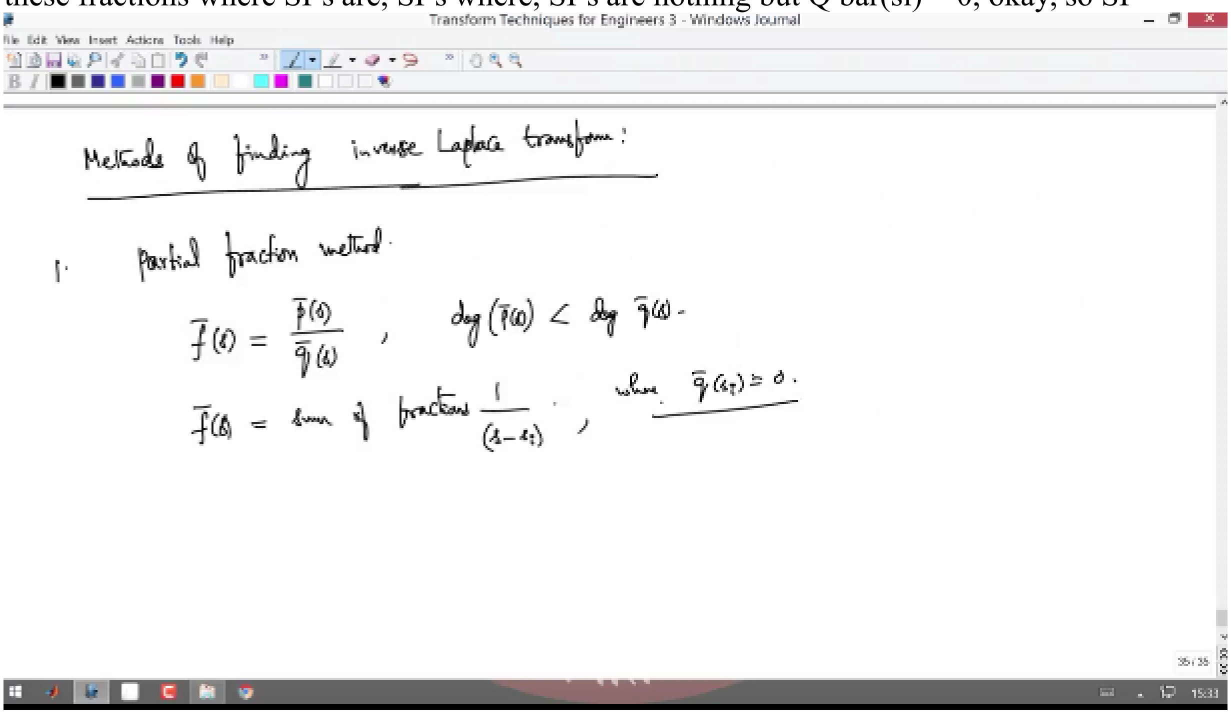
So far we're only seeing, we're discussing only about finding the Laplace transform, so you started with a known function and then you find the Laplace transform so that means, so inversion so whatever you get as a Laplace transform its inversion is the function itself assuming that the function is continuous given function is started where this is a continuous, and function of exponential order, then Laplace transform is exists and it is unique that is what we have seen earlier, so in that sense you have the uniqueness so both inversion possible from the Laplace transform functions here to transform functions that side, so from there to you can come back here, so this is what you can see.

So otherwise if you don't know what is your starting function and so, but you know as a function, as a it's transform but you really don't know what that function F(t) as a function of T what is its inverse, so what are the ways to find this inverse, if you can recognize this function of S this Laplace transform which is one of these forms which you have seen so far, then you can guess it, guess its inverse transform just by having a tabular form put them all together in the tabular form and you can see sometimes if you just looking at it you can see what is its inverse transform that is one way using these elementary Laplace transform or its inversions you can also find Laplace transform of certain fractions, so use partial fractions and then make use of these elementary properties of inversions to find the partial fractions you can find the Laplace inversion that is another one.

And then the more general way is, and there is also something called, that is called one more technique, let me explain what it is. So there is another way that is called heavyside expansion theorem, so if you have a some finite expansion, finite expansions we can so, that is also something similar so if you have a kind of partial fractions you can write and then you can get the inversion possible, these are the, and more generally if you are having only F bar(s) as a

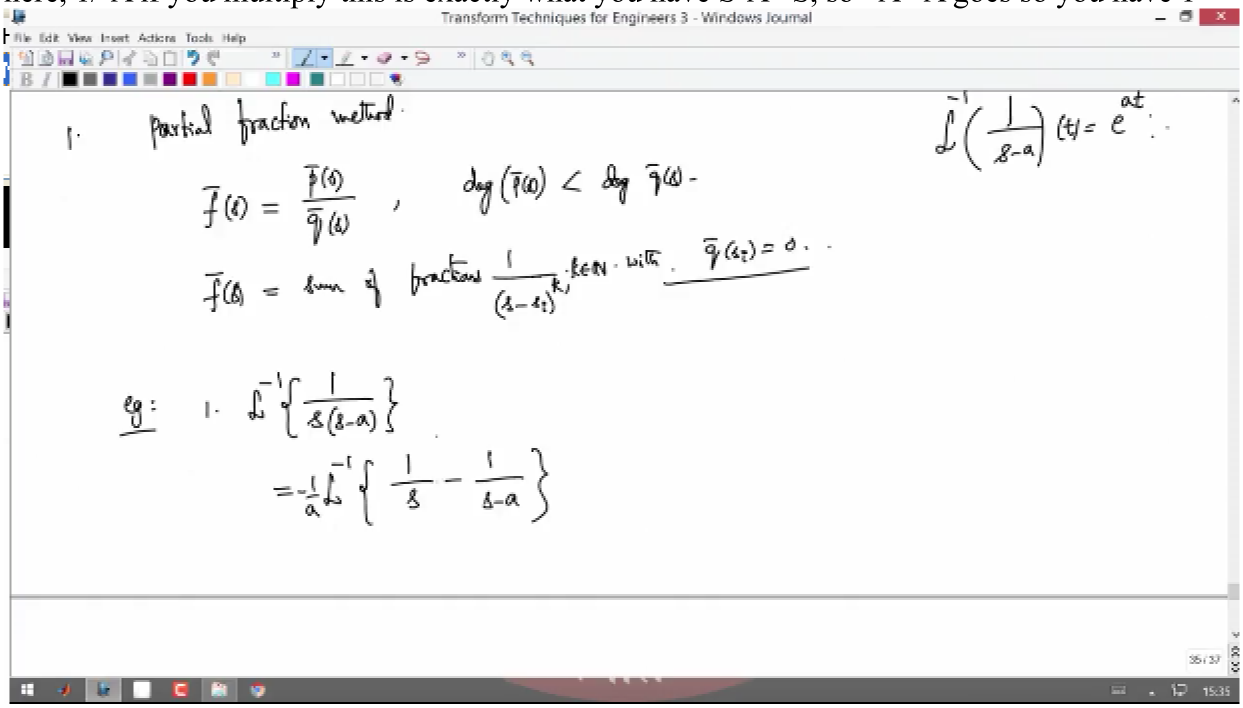
Laplace transform you don't know, you don't have any other way to find the Laplace inversion, so you don't have choice but to go for the actual inversion that you have seen earlier and the definition of the inverse Laplace transform which is integral, line integral over in the complex plane C-I infinity to C+I infinity, so that is actually over a Bromwich contour that I will explain you later on how to evaluate that integral, so that inverse integral to find the inverse transform of given Laplace transform F bar(s), so these are the four techniques, so let me start one by one, so starting point is partial fractions method, so one is the usual way is if you know already if you can recognize from whatever you have seen the transforms and it's inversions you can guess, and the first methods of finding, let me write these methods of finding inversion, inverse Laplace transform.

So we start with this method that is partial fractions method, partial fractions, in this what I do is if you are given F bar(s) which is of this form some polynomial type P(s)/Q bar(s) this you write this polynomial assume that degree of P bar(s) is less than degree of Q bar(s), okay, then you can write this F bar(s) as a partial fractions, sum of fractions let me write, what do you mean by factions? Fractions are one over some S- some known roots, okay, S- SI's, some of these fractions where SI's are, SI's where, SI's are nothing but Q bar(si) = 0, okay, so SI

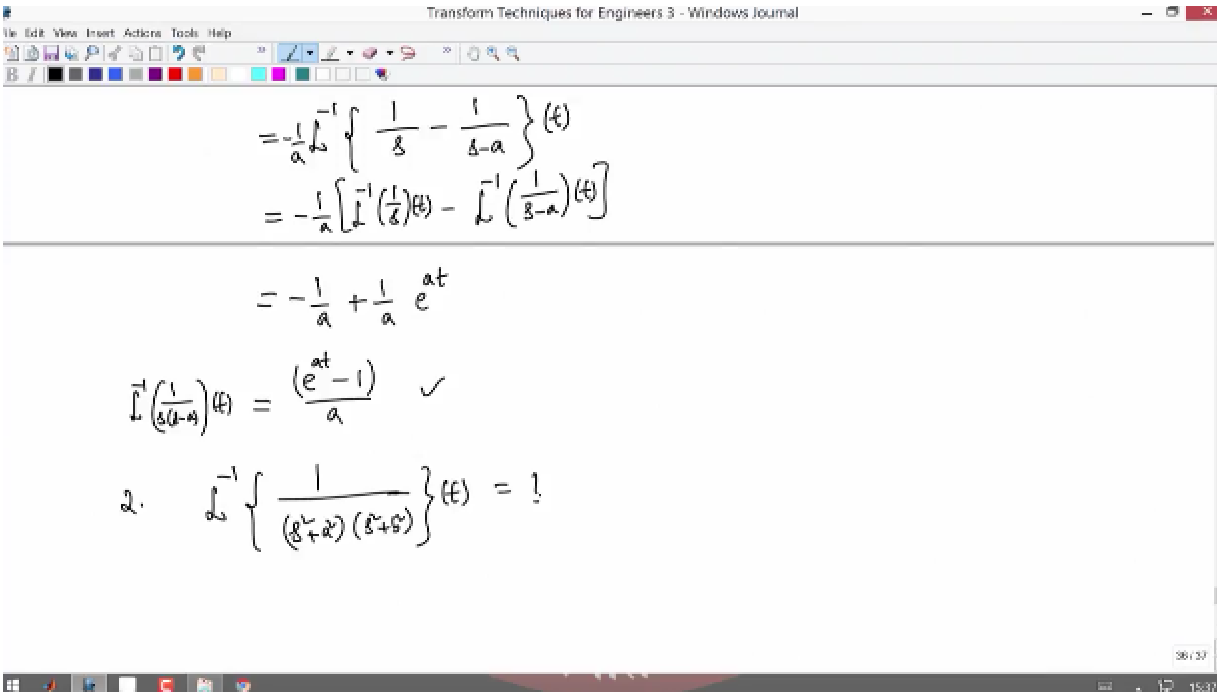


fractions of this or some it's square or whatever, okay, so this can be even square, cube, or what anything so for some K, okay, fractions are like this K belongs to natural numbers with Q bar(si) is 0, okay, if such is the case we can easily evaluate because we know that 1/S-A is a Laplace inversion of this is E power as a function of T, once you write this inverse is a function of T this is simply E power AT, because you find the Laplace transform for this you will see that 1/s-a, okay.

So let's do some examples, I'll start with the examples find the Laplace transform of Laplace inversion of 1/s times S-A if you have like this we don't see, can if you recognize you cannot recognize this as, you cannot use any of these properties, okay, actually you can use one property that 1/T times F(t) if you have a Laplace transform this is different, okay, this you have a Laplace transform 1/S this no other method will give you this whatever the property you can use you cannot find this inverse, so what we do is this is because the L inverse is a linear operator, integral operator, this Laplace transform and inverse transform are linear operators, so you can write this L inverse as 1 some A divided by S + B divided by S-A with some constants A+A, so what is that constant S-A -B -1, -1 for example if I put here S–A –S so you have -A here, 1/-A if you multiply this is exactly what you have S-A –S, so –A –A goes so you have 1

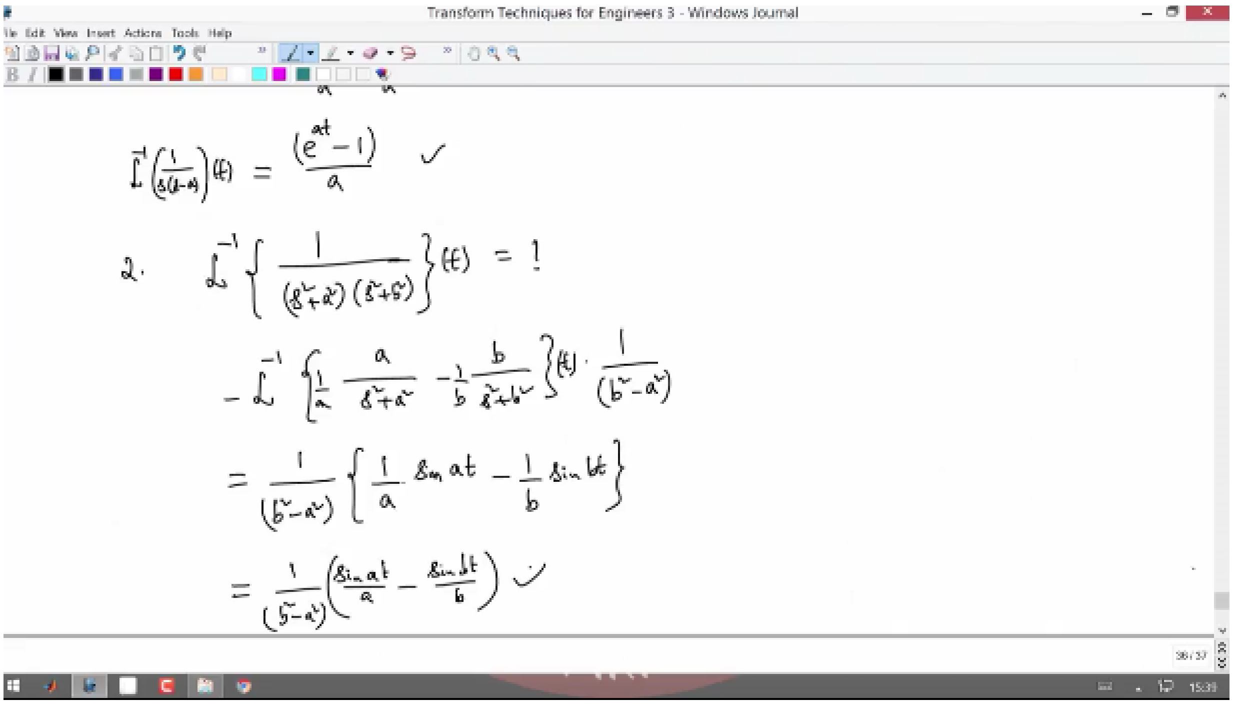


over S/S-A that is by partial fractions, and this is a function of T final you will get, this is equal to -1/A times, earlier inversion of 1/S as a function of T -1 over S- or rather L inverse of 1/S-A, there's a function of T, so this is 1/A times, this is just 1, a simply 1, and then you have minus and E here, L inversion of this one will give you, so you have this one -1/A and this is going to be +1/A times minus minus plus 1/A this is E power AT, so this is nothing but E power AT- 1 over A is the its inverse, okay, so this is inversion of 1/S times S-A as a function of T this is what it is, you can look at the other examples as such as what is the L inversion of 1 over S square + A square times S square + B square as a function of T, what is this one? So this also



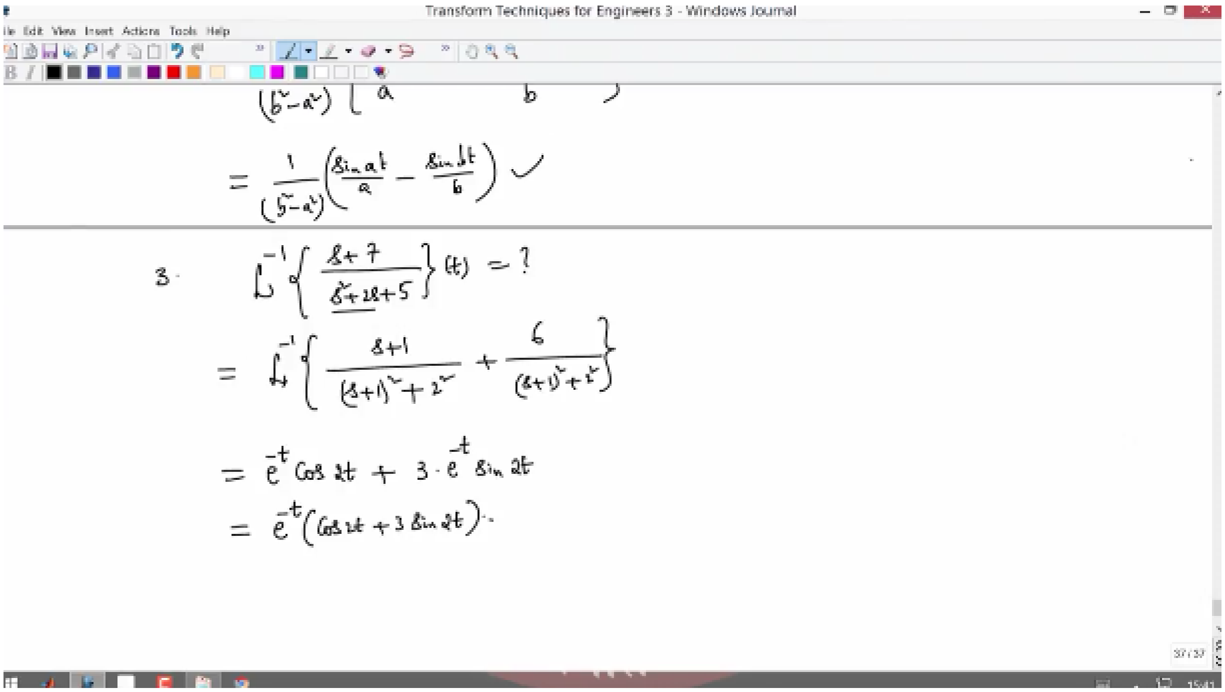
you can calculate, you can evaluate, how do we do this? I try to write this as L inversion of 1/S square + A square, and then -1/S square + B square to get this S square + B square -S square -A square, so B square -A square which you have so that you multiplied with, we divide with the B square -A square, so if you do like this which is function of T times this, this is equal to 1/B square –A square that you can write here.

And then L inversion of this is actually same as A divided by if you put so that I can write 1/A here, okay, 1/A and you can write B here so that you can write 1/B, so what you get is 1/A times this is sine AT, sine AT if you look at sine AT Laplace transform we have seen that this is A divided by A square + S square so that is what it is and you have other one is a -1/B times sine BT, okay, so what you finally get is 1/B square –A square sine AT/A - sine BT/B, so this is the required inversion.

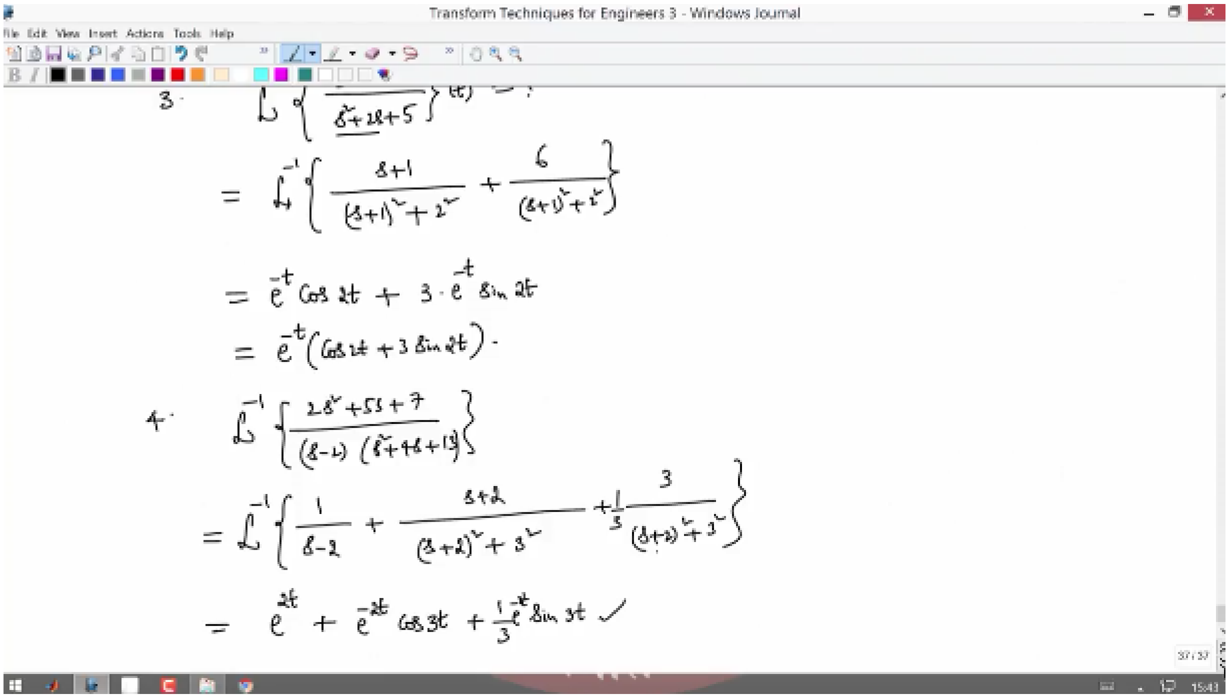


Let me do one more example, for example if you simply consider some fraction such as S+7 divided by S square +2S + 5, as a function of T is what, that is the question, so left hand side is actually equal to rather what is given, so this is equal to L inversion of, now if you do the partial fractions for this that I'm directly writing, what you do is the denominator you can put like S+1 so this put it together S+1 whole square if you do that already one is accounted so you have 4, so you have a 2 square here, S+1 whole square so you know that is S square + A square so you have S here with S+1 so you need S+1 here +6 okay, so that +6 divided by the whole thing you write separately +2 square this is what, so if you do, if you apply Laplace transform for each of this, this is like S divided by S square + A square that is cos A is 2, cos 2T times because of S+1 this is going to be E power, this is E power S+1, so because of E power -T, you have to multiply with this is the property that we are using here, E power -T if you multiply with cosine, so cosine thing what you have is S divided by S square + A square, A is 2 so you have 4 here if you multiply this transform then you have to replace S+1, if it is E power T you have to replace S-1 okay, S- wherever S is there S-1 so you have S+1 here, so I put –T, so the inversion of this is actually this one.

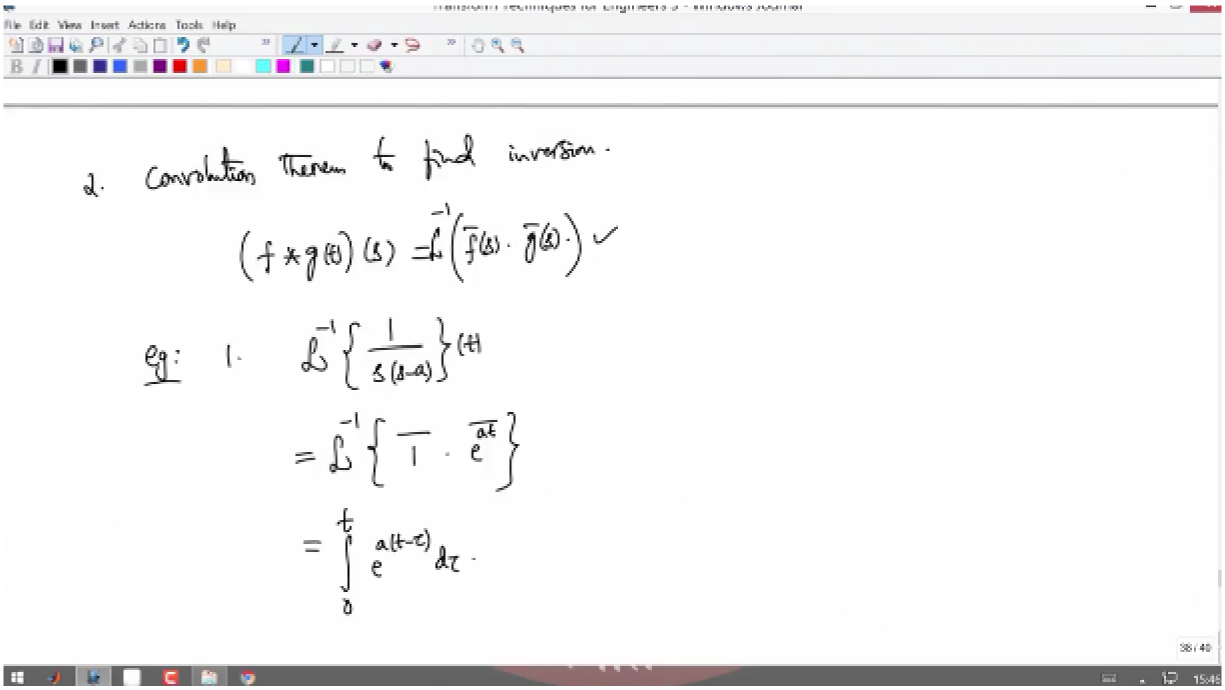
Similarly this one, so you can write 2, A is 2, so if you write 3 times, 2 divided by S square + 2 square that is sine 2T and you have S+1 this is E power –T, so this is exactly what you have, this is E power -T into cos 2T + 3 sine 2T, so this is your inverse function.



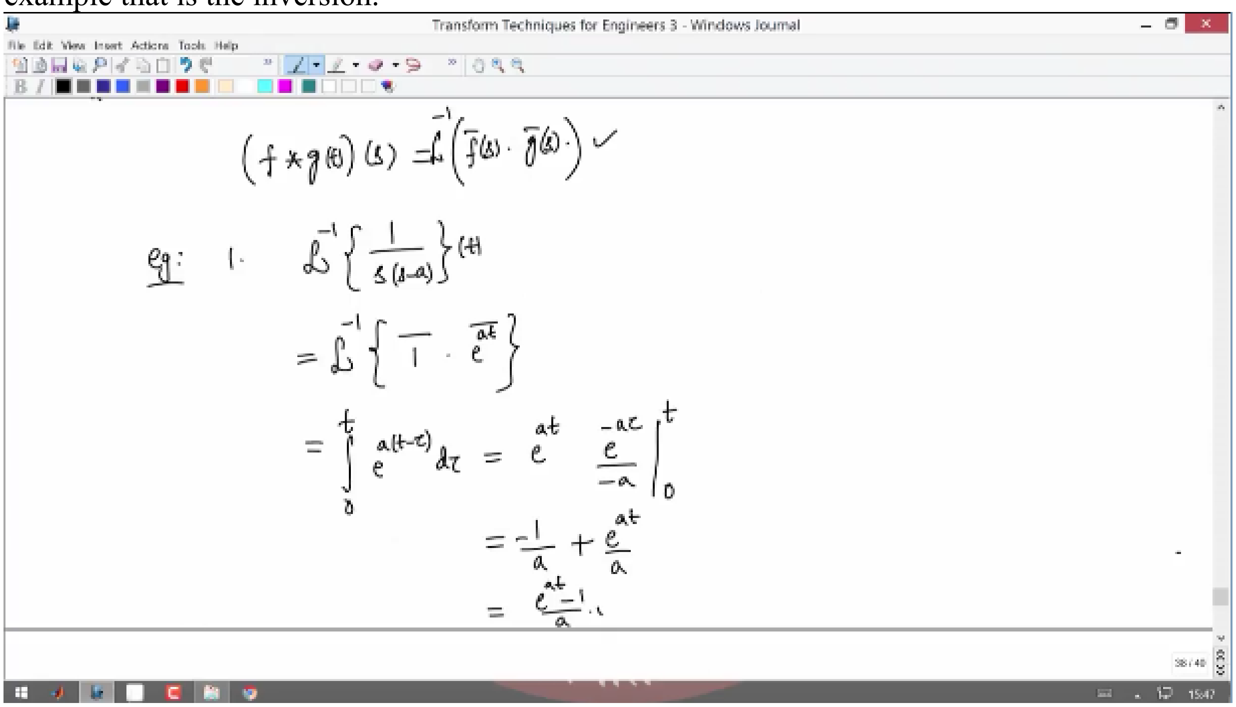
The same way you can do one more example that is, what is a Laplace inversion of 2S square + 5S +7/S-2 S square + 4S+13, as a function of T what is this one? This is actually equal to, I'll write directly these partial fractions that is S-2 and this one, this denominator whatever you have here you try to put this way, so I write S+2S, S+2 whole square will give me S square + 4S + 4 that removes +9 that is 3 square, so if you do the partial fractions for this, in the numerator you may end up something like S+3 1 if you see here A, B S+C this is what you have you have to do because this is, you're considering this here square, this is how you calculate use the partial fractions you see that it is going to be 1 here and B is you will see that is 1, and C is 3, so because of S+2 I write this S+3 as S+2 + 1 over 1, so that one over this one I am writing separately, so this is actually equal to this will give me E power 2T for which you have S/2 1 by S/2 and this will give me E power -2 T this is cos 3T, this A is 3 here cos AT okay. And here this is, you need 3 here so I multiply 1/3 and divide 3, so you have 1/3 times sine 3T times E power -2T because of S+2, so this is your inverse transform.



So this is how you can calculate, you can calculate by partial fractions given function if it is a polynomial by polynomial with a numerator degree is smaller than the denominator degree you can make a partial, make use of partial fractions and make use of elementary Laplace inversions to find the inversion of the Laplace transform, so we will see other method that is we can also use a convolution theorem for example okay, we have seen certain property that convolution theorem, convolution theorem to find inversion, so let's see how it is done, convolution theorem what we had is Laplace transform of, what is the Laplace transform of your convolution, F convolving with G(t) as a function of S this is equal to F bar(s) times G bar(s), if you can write this L inversion, if you can write so this if you take the inversion here so inversion comes here, so if you can write your given Laplace transform as a function, as a product of two functions each function you can recognize its Laplace transform, what is its inversion transform so that that is F(t) for this, and F(t) for this and you convolve them that is exactly we'll give you the inversion, so if you use this property we can find you can make use of this convolution, that is same as using these convolution theorem.
So for example so let me use this example, let me use the same example that I have used, what is the inversion of, we have already seen that what is the result for this, that is E power AT -1 over A so this is what is the inversion, we already know this one okay, so if I use this, this property this is like L inversion of, this is Laplace transform of 1, okay, and then multiplied with Laplace transform of E power AT Laplace transform okay I'm putting bar for the full function, so this is what you have.

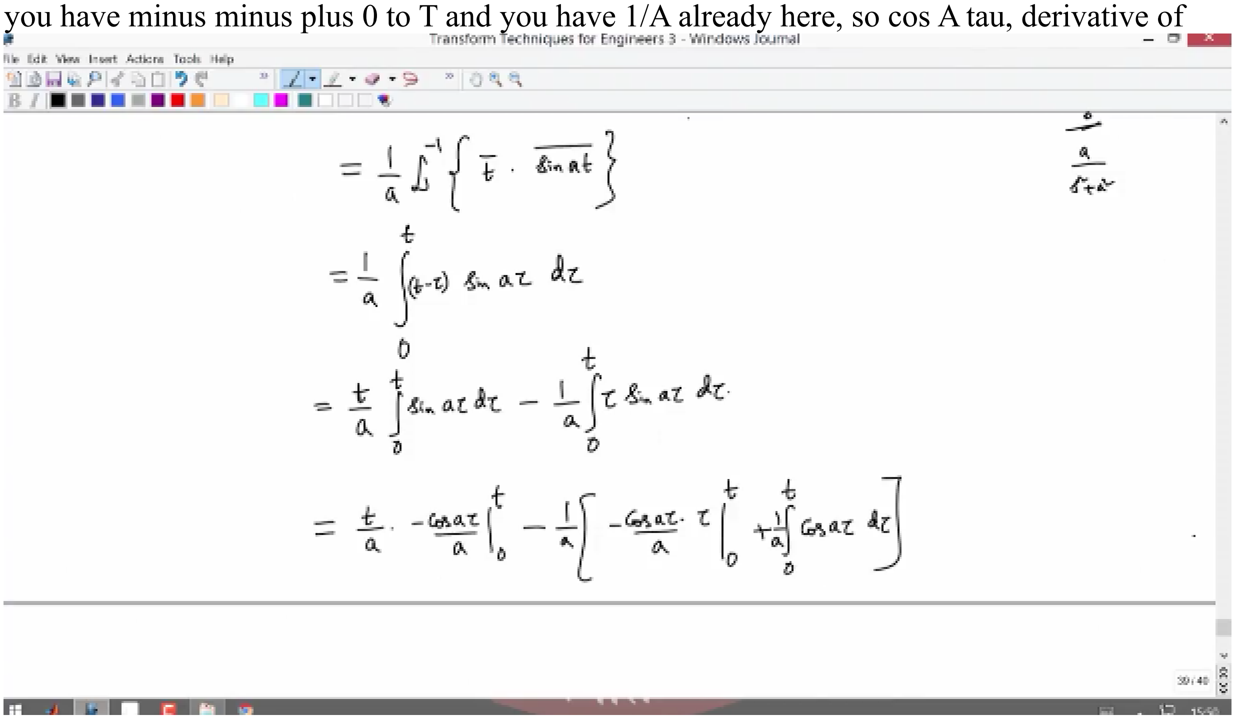


So what is this one? By this if you make use of this one simply 0 to T, 1 is 1 E power A times T-tau D tau this is exactly your Fourier, your Laplace transform. So what is this one? So this is actually equal to E power AT, E power AT what is the inversion? So E power -A tau/-A that is the anti-derivative and put 0 to T so you get 1 over -1/A minus minus plus you have E power AT/A, so this is exactly what you need to get, E power AT-1 over A, okay, so and this second example that is the inversion.

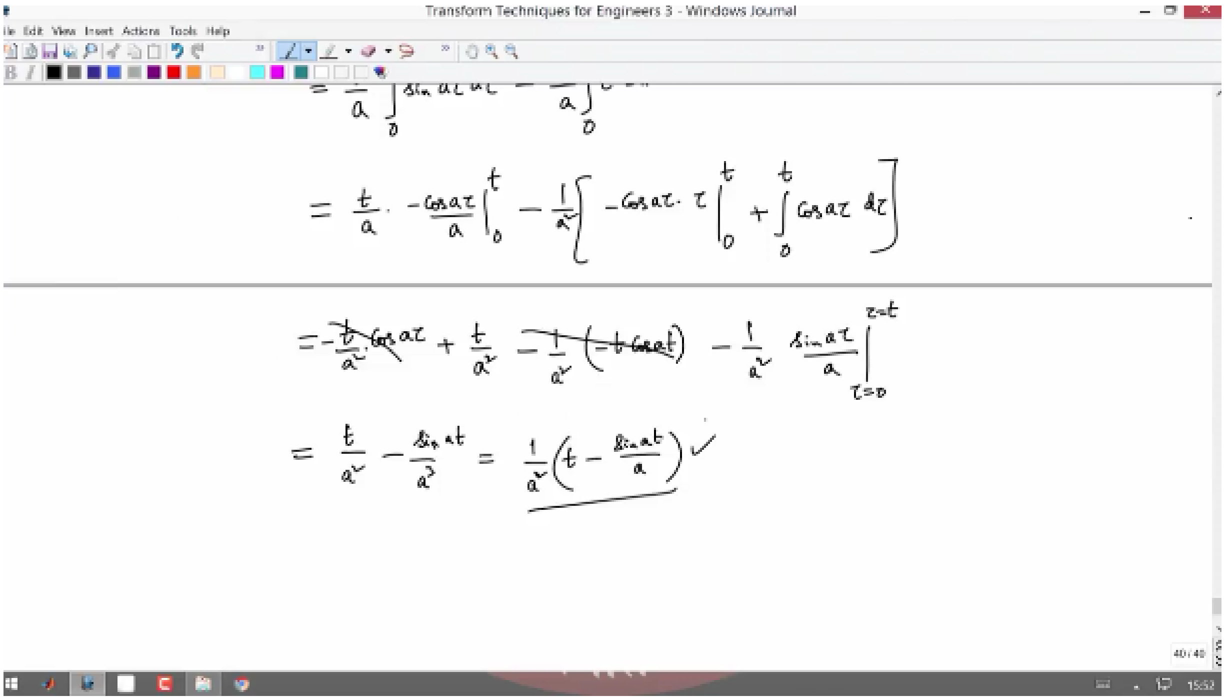


So what is the inversion of, if I use 1 over S square and S square + A square, what is this one? So this is a Laplace inversion of 1/S square if you know, if you calculate, so what is the Laplace transform of T for example? Laplace transform of T is actually 0 to infinity T times E power

-ST DT if you do the integration by parts this is going to be 1/S square, this is what you'll get, so you have T bar multiplied with and 1/S square + A square, so A/S square + A square I can multiply A and divide with A this is what you have, okay, so I have division, so when I write like A divided by let me write this as 1/S square times A square by, or rather A/S square + A square, so because you have to account for A so I divide, so I multiply 1/A, so this is actually equal to 1/A times earlier inversion of 1/S square is Laplace transform of T multiplied with this is the Laplace transform of sine AT bar okay, so this is nothing but 1/A times this convolution of T and sine AT, 0 to t this is tau sine A times T-tau D tau, okay.
You can also write, because convolution of T and this is same as convolution of sine AT convolving with T, so you can replace here for this function you can write also T-tau this function you can replace with tau, so that your calculations will be simplified, it's easy to calculate this one, so you have 1/A times so first integral is 0 to T, T comes out sine A tau D tau, other one is -1/A integral 0 to T tau sine A tau D tau, you evaluate this what you see is T/A times this is – cos A tau/A so you multiply this one you substitute these limits and here you can do this integration by parts that –cos A tau into tau/A this for which you apply the limits and you have minus minus plus 0 to T and you have 1/A already here, so cos A tau, derivative of



tau is this function that is 1 so D tau, so this whole thing I can write, anyway you can do one more time, so you have T/A times cos A tau - cos A tau A square, cos 0 is 1, okay, and this is minus and this is going to be plus T/A square this is what you have, and here -1/A square and this if you put this here A square, so I remove this A square, I'll put it outside, so I have A square, and for this you put these limits you have T times -cos AT and this is going to be + 0 when you put tau = 0 that is 0, so this is what it is, so this one plus and this is minus this gets cancelled, okay, and then what you have? So if you remove this one, this is what it is and I have -1/A square this integral if you write this is sine A tau/A 0 to T, so what you get is T/A square – 1/A cube sine AT minus when you put tau = 0 that is 0, this is tau = 0 to tau = T, these are the limits if you do, put this is what it is, so you have A square comes out 1/A square T-sine AT/A, so this is what is your inversion function, okay.

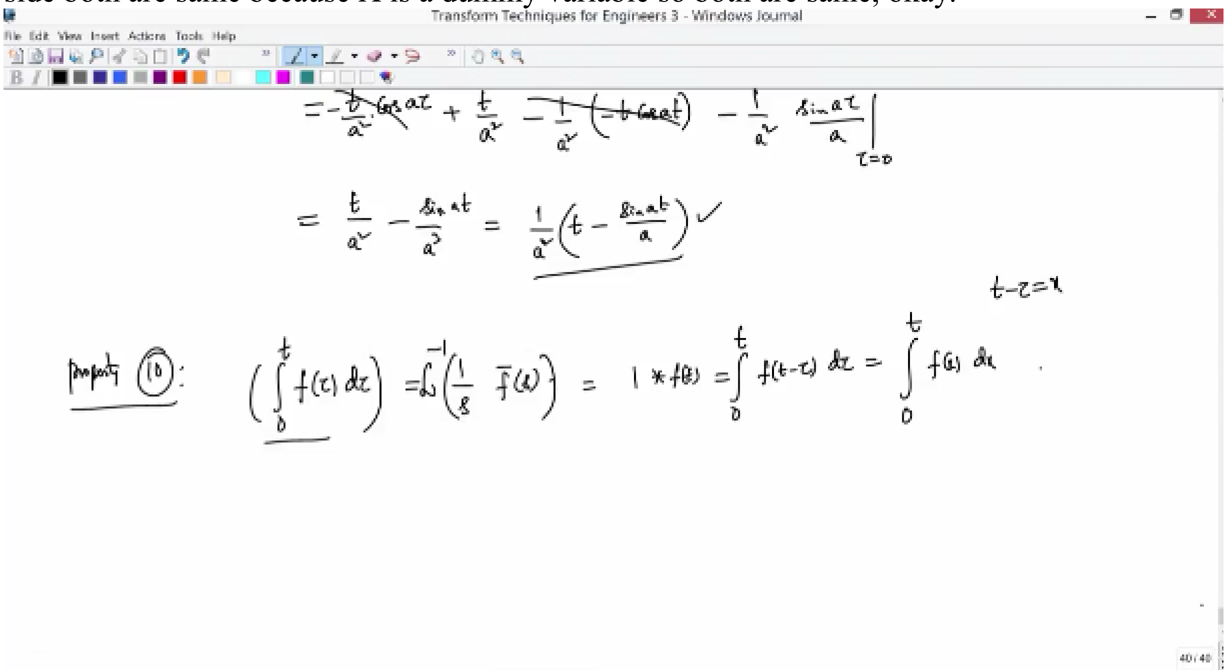


Like this you can go on doing it, so you can make use of known elementary functions whichever you can, if you can recognize as a product of two functions for which if you know the inverse transform that those two inverse transforms you convolve them as an integral you evaluate that integral, that will be the inversion of this product, so this is how you can do, you can also do other complicated functions such as error functions, so far I have not defined what is error function, error function is kind of 0 to infinity E-X square DX, this is a pi/2, okay, this is actually root pi/2 or something okay, so this is a value, and if you just do it up to, error function is up to say let us say some T, so this is your normally error function ERF(t) as defined as this, okay, ERF at infinity is actually this one this value, otherwise this is a definition of error function, if you can do like this, this may come in the applications for this function as a function of T, this is error function, you can apply Laplace transform, so that as an when it is required we will do the Laplace transform and it's inversion when it comes in the application, so we will see later, otherwise so far we'll do, otherwise we'll do only simple elementary functions for which you find Laplace transform and inverse transform, okay.

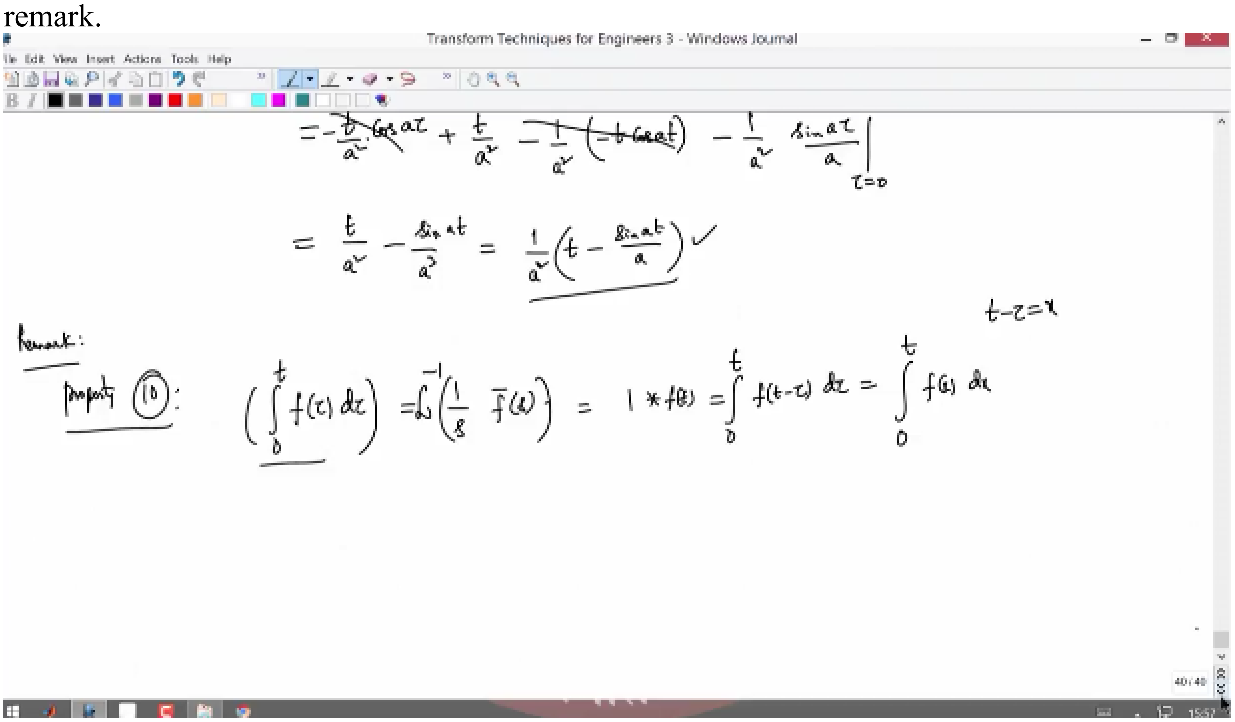
Otherwise technique is same, we may use one of these techniques to find in Laplace transform of the error function and inverse transform of it, whatever comes out of it, okay, whatever the, if you want error function as a Laplace inversion you have to choose properly, you have to recognize what is exactly your Laplace transform of, what is you should know what is the Laplace transform of error function, okay, so anyway these complicated functions we need not worry we will see later, as and when they come in the applications we may have to evaluate that time we can evaluate.

So as a final example I will use, to just as a remark we have used that last property of the Laplace transform of integral 0 to T F(tau) D tau for example, if you have this this is actually equal to 1/S times F bar(s) we have seen this one, so this is the property number 10, okay, so property number 10 that is what we have seen, if you take the inversion here L inversion of this, so you can easily see that this you start with here, this is actually equal to 1/S we can recognize that this is 1, the Laplace transform of 1 is 1/S and other function is F(t), so for which if you take the convolution as a function this is what it should be this, what is this one? This is actually

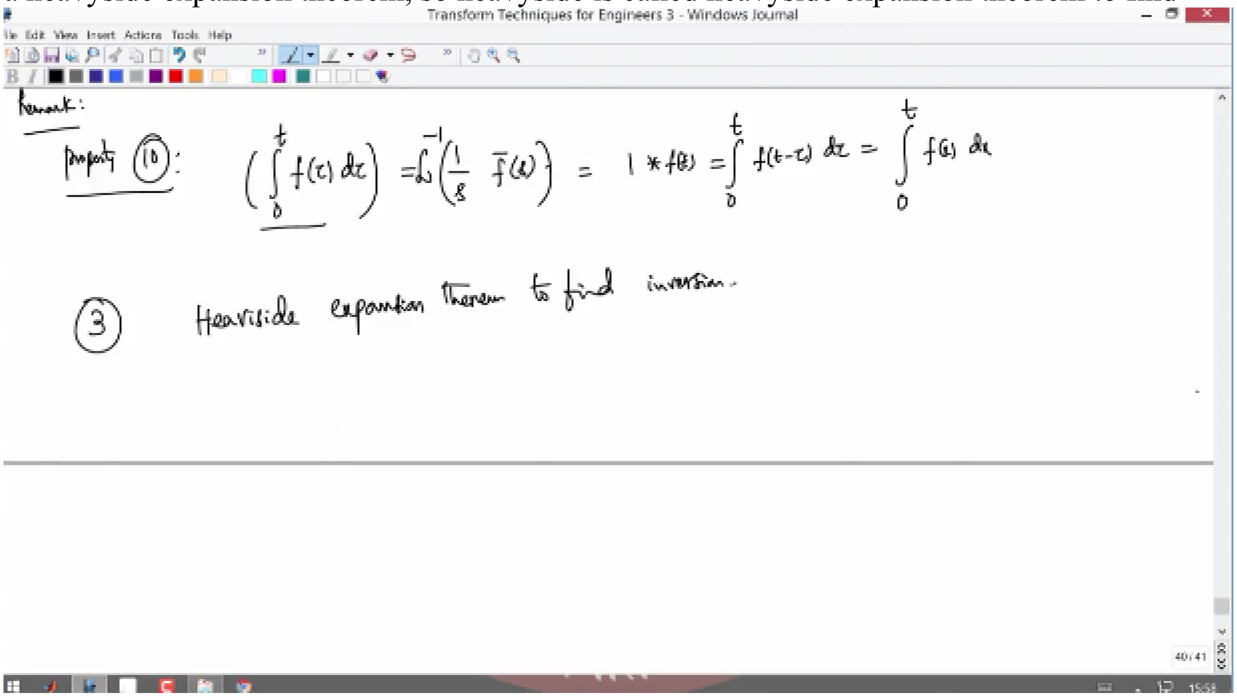
0 to T F(t-tau), 1 is function of tau so either this D tau, this is same as if you use T-tau as new variable X, you have integral X F(x) D tau is –D tau is DX that is D tau is –DX when you put tau = 0 that is T = X, okay, so sorry when you put tau = 0, X = T and when you put tau = T, X is 0, this minus sign makes it 0 to T, so both are same, so see that left hand side and right hand side both are same because X is a dummy variable so both are same, okay.



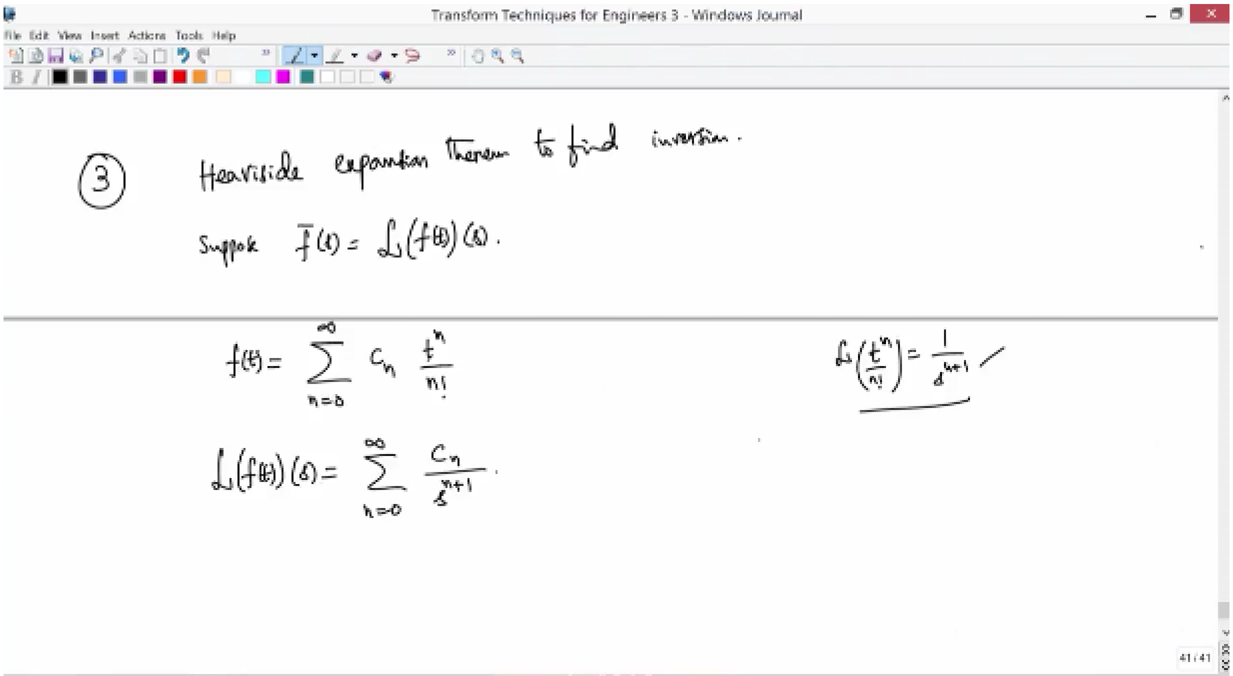
So as you can see the property number 10 is not actually anything new, you can use a convolution theorem to see that property number 10 that is as a remark, you can take it as a remark.



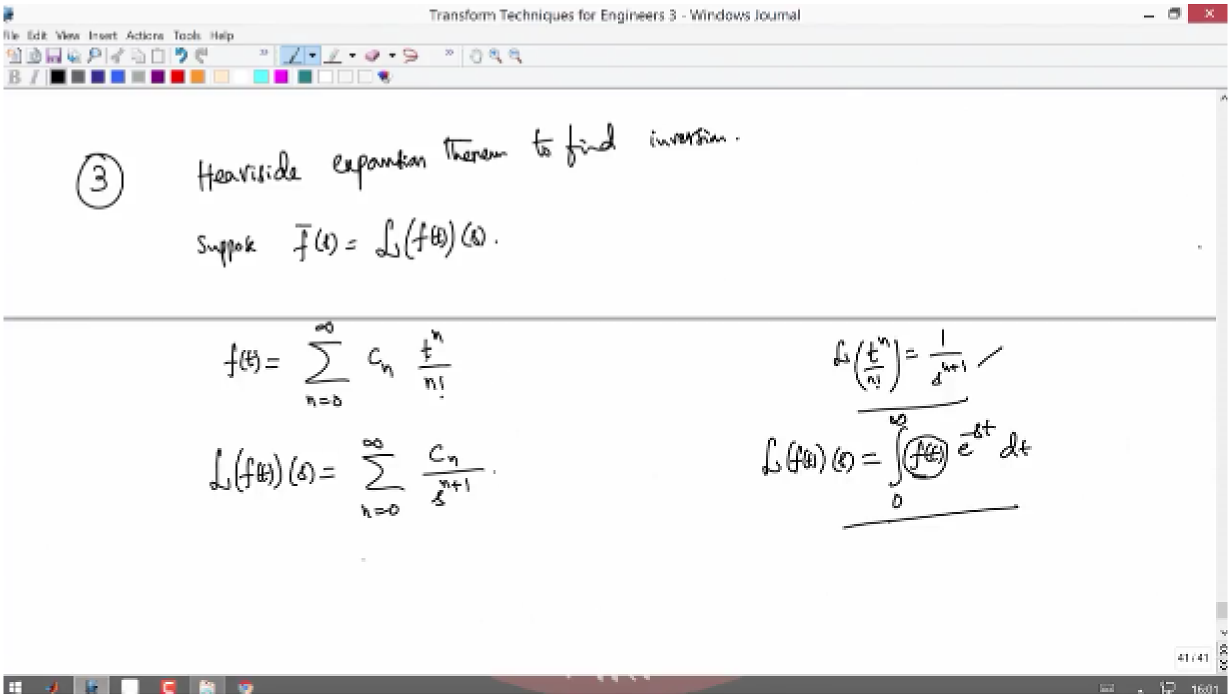
Next technique to find the inversion is Fourier, actually we will see the other technique, there is another simpler technique that we will see before we go for the general technique, how many minutes ma? 35 so far, 5 minutes I'll say, okay this technique also I'll explain. So we will see some other techniques, so we will see other technique that is a third technique to evaluate that is a heavyside expansion theorem, so heavyside is called heavyside expansion theorem to find



inversion, before we go for the general way of finding inversion that is basically directly using the inverse Laplace transform, we can also use this theorem this heavyside expansion theorem this tells you that if you have suppose F bar(s) is a Laplace transform of F(t) okay, then and F(t) has a Maclaurin series, okay, such as a series like this some CN, N is from 0 to infinity so these are the coefficients just like our Taylor series coefficients, CN times T power N divided by N factorial, suppose you have like this, okay, what is its Laplace transform? If I find the Laplace transform of this F(t) as a function of S, then you can see that this is going to be N is from 0 to infinity, CN times this is because you know the Laplace transform of T power N is N factorial divided by S power, N factorial N factorial goes so this is going to 1/S power N+1, T power N Laplace transform is N factorial divided by S power N+1, so bring this thing here so we have N factorial, so this will be this one, 1 over this, that's what I used here.

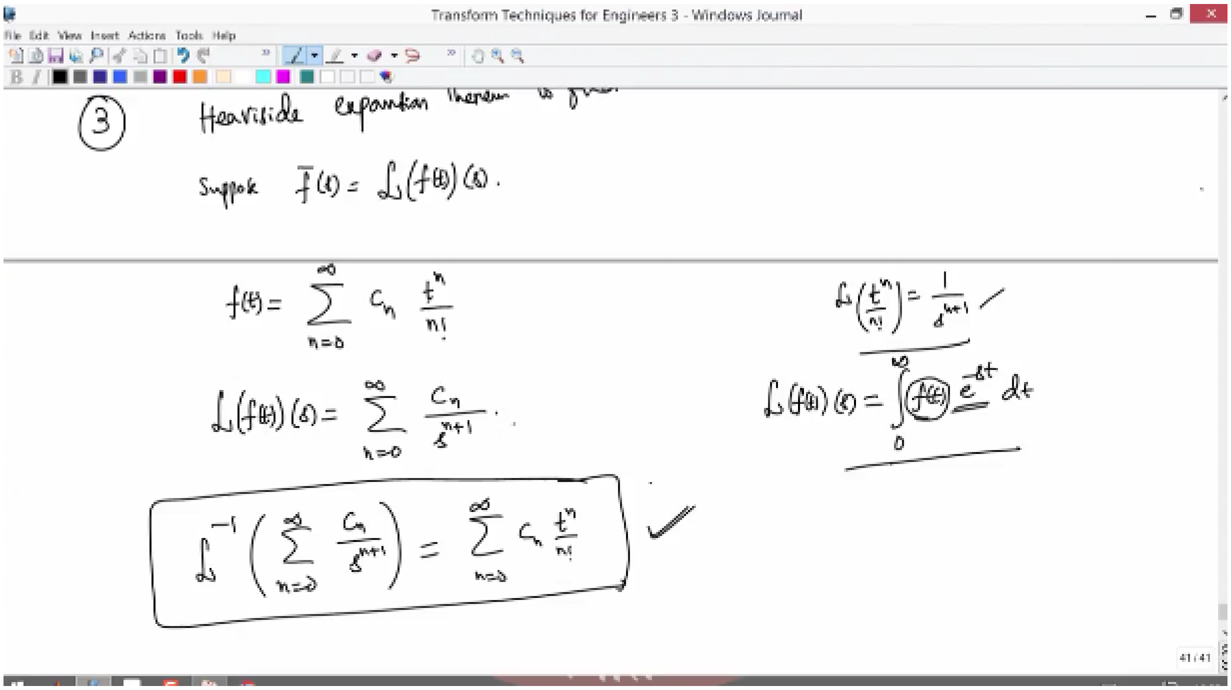


So you have CN, CN divided by S power N+1, so this is what you see, if this function is exponential order, so this is a series that means this is an elementary function, okay, so if you can find its Laplace transform, so what is its Laplace transform? Once you have an Maclaurin series this is well-defined function, and you have 0 to infinity F(t) E power –ST, so for a such a nice function F(t) which has a Maclaurin series this DT, this integral will exist because of exponential - ST for some S values, okay, assume that it exists then you have this Laplace transform, so existence of this integral is actually is something which you have to work out because if it's not straightforward assume that you have such a nice first of all you have to get this Taylor series around 0 that is it should be a nice function, this is an elementary function something good function for which you have this, and for such function assume that Laplace transform is, it exists, Laplace transform exists, okay, if you have that if you apply then it is, because if it exists the Laplace transform is uniformly convergent, that is a convergent integral Laplace transform of F is uniformly convergent integral that is what we have seen yesterday in the last video, so implies I can even if this is of F(t) is replaced with this series I can take this series is uniformly convergent this is because the Taylor series which is a power series, power series is uniformly convergent on any, on every closed set, okay, so in that sense this you can



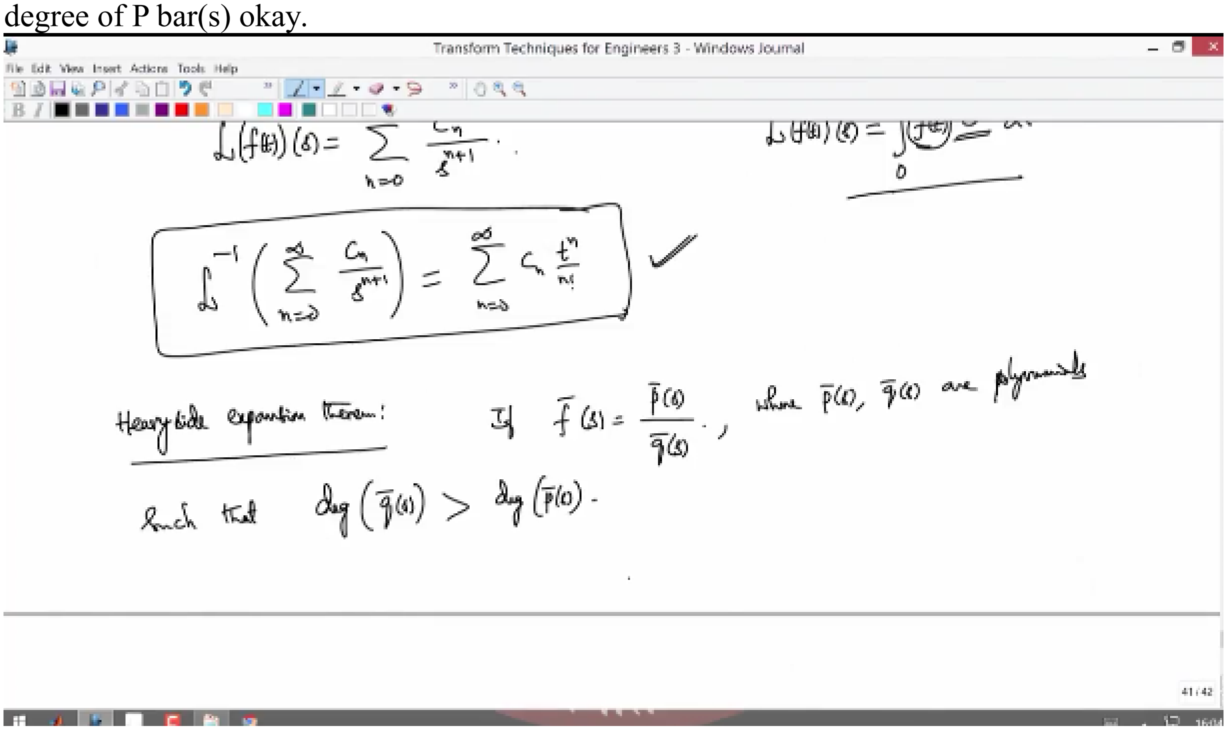
take Laplace transformation, assume that this is uniformly convergent you apply this inversion, this integral multiplication with this so you can apply, assume that Laplace transform you can apply for this series, so you see the term by term if you can do for such functions for example, okay, its justification is not straightforward so we're assuming that the function F(t) is having a Taylor series around 0, and it's uniformly convergent series you get, so that you can apply when you apply multiplied with E power -ST if you integrate you can take the integration into the term by term integration so that is what you get, okay.

If you do the term by term integration left hand side is this integral, right hand side that means the F(t) I replace with this series, so that now I integral I take it inside this sum for each term if you calculate this is what you have, assume that such class of functions, for certain class of functions it's always possible to get this thing, so that you can make use of this, once you find this inversion is like you know what is this function, you know its inversion, so you know its Laplace transformation, so the inversion of such a series N is from 0 to infinity CN/S power N+1 is actually this series TN/N factorial, N is from 0 to infinity, okay, so this is a kind of

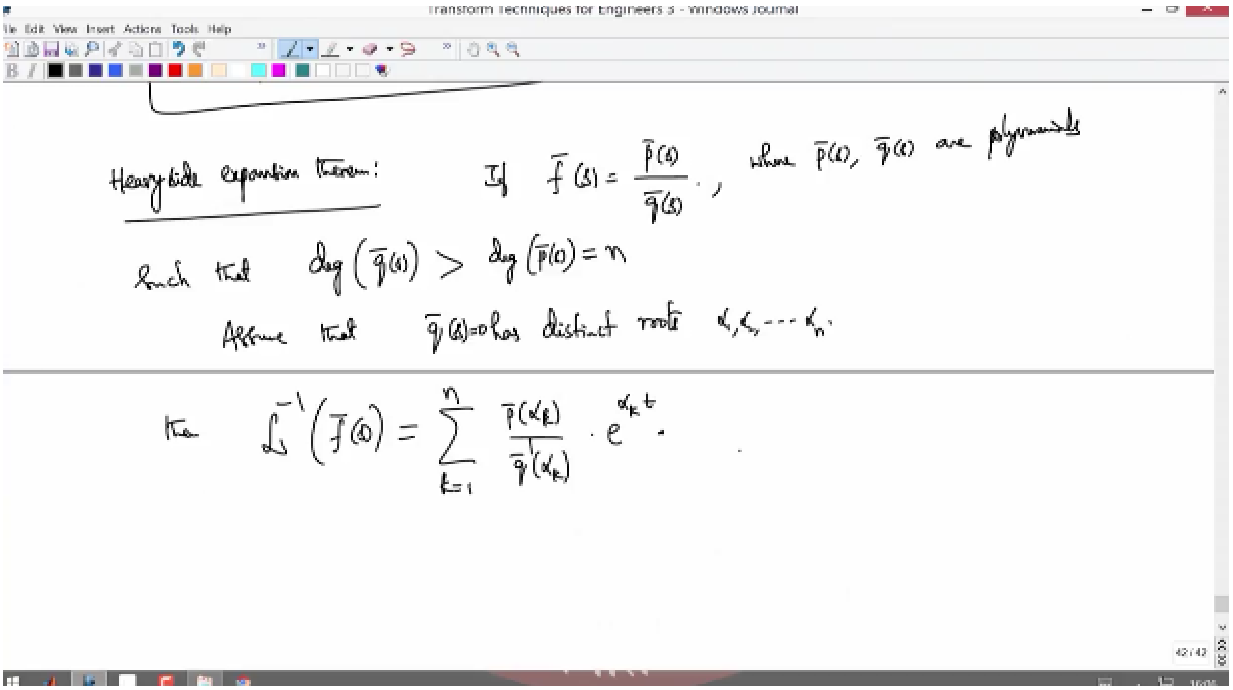


straightforward like you started with some function just like elementary functions we started with the function, we calculated the Laplace transform from which you can easily see what is its inversion, okay, this is one, if it is a series form, if it is a finite sum that is expansion theorem that is actually theorem, so let me state that.

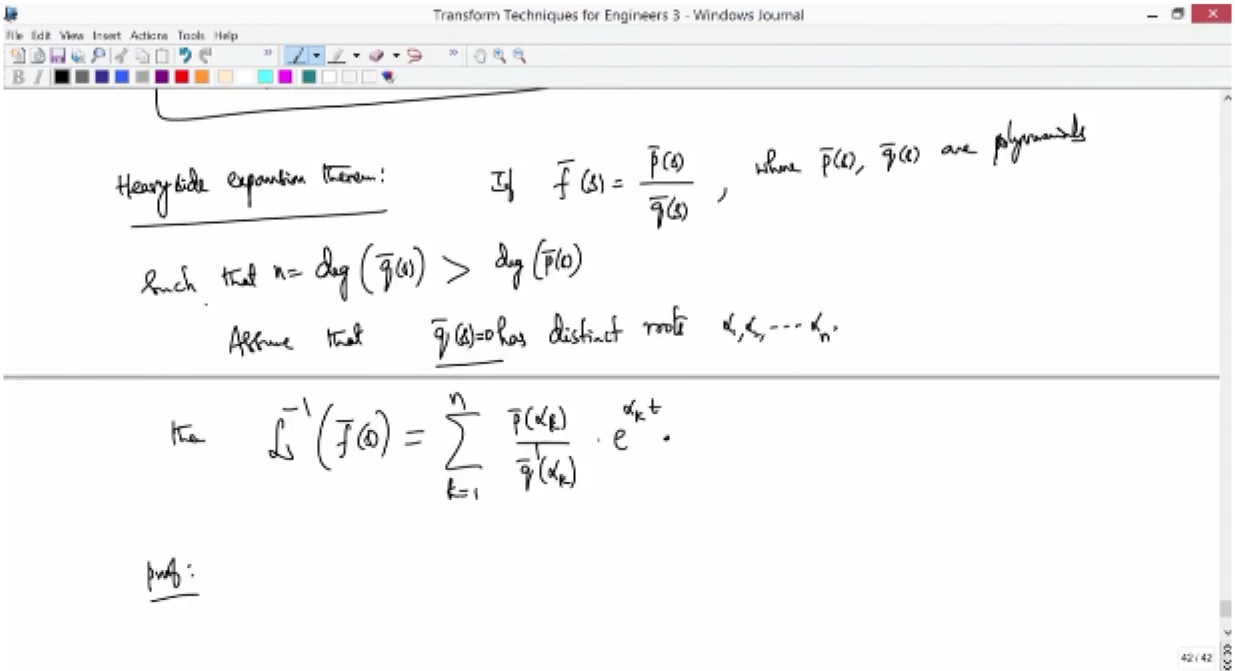
And then so heavyside expansion theorem, a finite expansion theorem okay this is what we can use directly to find the inversion, so if F bar(s) is like earlier you have P bar(s)/Q bar(s) okay, there's such a thing there are polynomials and Q bar(s) is having higher order, higher degree, where P bar(s) Q bar(s) are polynomials such that degree of Q bar(s) is always bigger than degree of P bar(s) okay.



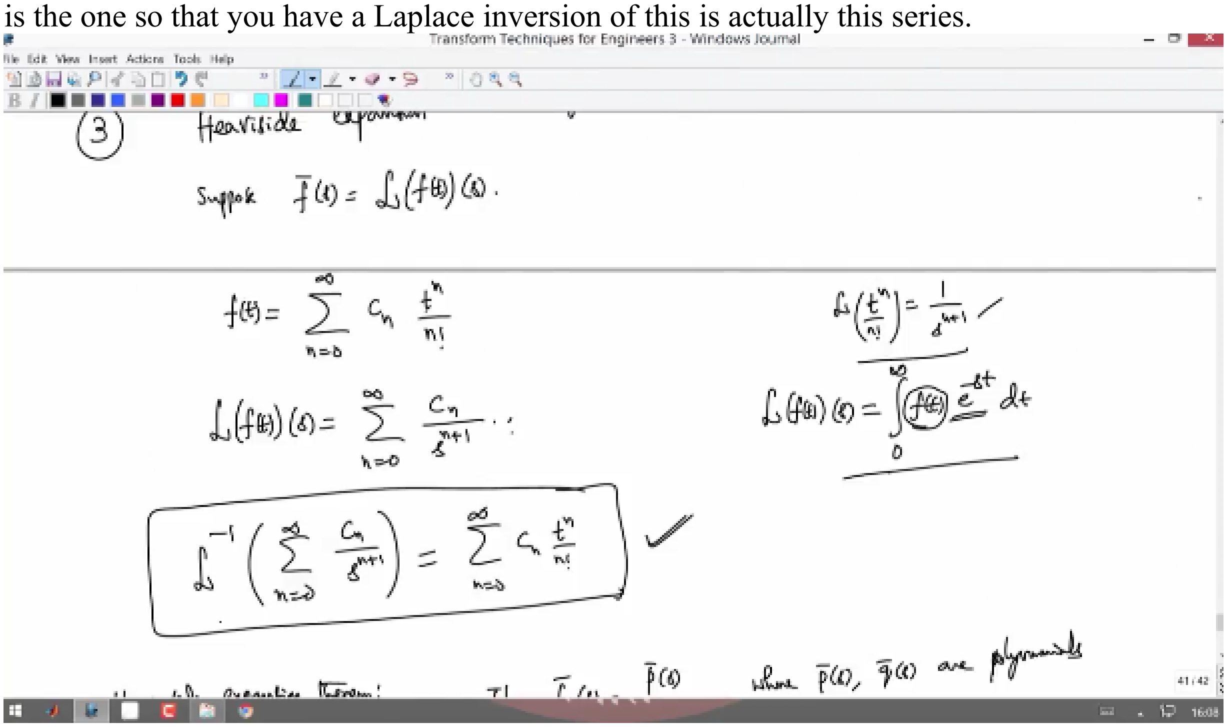
And then assume that, also assume that Q bar(s) has distinct zeros, call them alpha 1, alpha 2, alpha N, so degree of this for example N if you call this N so you have alpha 1, alpha 2, alpha N, so they have distinct zeros for this that means this equation has distinct roots, okay, that is the meaning if you assume that then Laplace inversion of F bar(s) is actually this, idea is already there, once you have this, this you can write it as partial fractions, once you write the partial fractions you know you can write what it is, what is its inversion okay, so let me write exactly what is the form you see this is going to be K from 1 to N because you have N factors so you have index running from 1 to N, and what you get is a P bar(alpha K) divided by Q bar dash(alpha K) into E power alpha K times T, so this you can easily see by partial fractions this is the inversion, the coefficient you can directly looking at the, in the Laplace transform F bar(s), okay, so you don't have to look at really find the coefficients in the partial fractions and then see its inversion.



So if you use this heavyside expansion you can easily calculate Laplace inversion for certain fractions, rational functions, okay, such rational functions where polynomial degree of the numerator is smaller than degree of the denominator, okay. Q(s) is N, this is N, not this okay, so the denominator degree is N if you assume that N then you have Q bar(s) has N distinct roots then this is the results, so proof you can easily this is not difficult to prove maybe this proof I will do in the next video.



So using this helps you to find if you have a Laplace transform which is rational function with big degrees for example numerator is smaller than denominator degree but it's difficult to find the partial fractions, but if you have a distinct roots in the denominator is having distinct roots you can actually find this inversion very easily, okay, by this formula that is a heavyside expansion theorem, otherwise if you already know that this series form, heavyside expansion this is also called expansion theorem but this is not a theorem actually, this is your expansion to find inversion, so it's not a theorem, heavyside expansion is if you have a Maclaurin series and if you can find its Laplace transform assume that it always exists, it exists and you see that this is the one so that you have a Laplace inversion of this is actually this series.



So this is a kind of ad hoc technique to find the Laplace inversion, so this proof and examples we will see in the next video, and then we will see the general method of finding if your Laplace transform F bar(s) is given which is in general, general thing which you cannot recognize what is any of these methods don't work you can find its inversion just by inverse Laplace transform that is a contour integration, that we will see, we will discuss about this contour integration in the next video that is using Bromwich contour, you have to choose properly Bromwich contour, a contour in the complex plane so that you can evaluate that inverse integral, okay, so this is what we will see in the next video along with examples we'll see. Thank you very much.

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