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TRANSFORM TECHNIQUES FOR ENGINEERS

Complex Fourier Series

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Welcome back. So let's have a recap of what we have done so far. So you consider any function, a periodic function essentially piecewise a continuous periodic function which you think of as a times signal and it's a signal and so it's a function of time so we call time signal. If it's a real valued function so for example if you are just calculating, if you are measuring some voltage in this in some system so it takes some real numbers so between some finite so some set of numbers. So you can put those numbers in in some finite interval. You can think of a system repeats within this so you can extend this function. So range of this function is actually that particular will be falling into that particular interval and then -- so let us say its period is some L and you can so beyond 0 to L and L to 2L and so on you can go on extend the function as a real valued function over a full real line.

So such a time signal you can always represent as a linear combination of signals of discrete frequencies. So this is we call Fourier series. So what we have done is so far is we defined Fourier coefficients. So based on these-- so using these Fourier coefficients you just take the linear combination with these fundamental functions, fundamental signals are these discrete signals with discrete frequencies. You combine them and it will give you some series, function series and claim is that this function series is called Fourier series. This Fourier series converges point-wise to the original time signal. So this is – that's where we we just believe that it actually converges if it's piecewise continuous function and we just calculated with two examples Fourier series.

So if you look at this Fourier series so for a function f(x) of period L we have written like this a naught by 2 plus Sigma n is from 1 to infinity. An cos n Omega naught x plus bn sine n Omega naught x with Omega naught as 2 Pi by L where - so this is a Fourier series and these coefficients are 2 by L. Let's write an are same so an into minus L by 2 2L by 2 and then you have this f(x) cosine function dx and bns same thing with the sine function f(x) sine n Omega naught x, dx. So these are all running from n is from 0, 1, 2, 3, onwards this is running from n is from 1, 2, 3, onwards.

So this is not the only representation. So what we call this these are Fourier coefficients. This if you define like this, this if you take it as a definition and you take what you have is a function f that's time signal you can represent as a linear combination of ans, bns as coefficients linear combination of these fundamental signals. Okay. Discrete frequencies. Frequency is n Omega naught. So this is equivalently you can put it as a complex valued functions. These are all real valued functions; cosine function and the sine function. These coefficients an and bn if you observe these are all real functions, real constants. Okay. So you can use cosine and sine, cosine function and sine function if you write it as exponential functions by this you can -- so you can use the Euler representation of a complex number okay. So if cos theta I plus sine sine theta is if you can write it as E power I theta that is Euler representation of a complex number you can see that right hand side you can put you can make everything in terms of exponential functions and then coefficients will change accordingly but the left hand side – so f is the time signal which you started. If you are just measuring –measured quantity is a real number. So it's a real quantity, real function real valued function f(x) and that's a signal and we also mentioned that signal can be a complex quantity.

So you can think of a repeated system. So just position of a vector. So position of a circular system so you can see all the time that rotates about certain origin. So you can think of system taking some following a circular path. If you take it as that way R and theta. So R is a function of this is position vector xy you can write it as a complex number. Complex number z of t which is x of t plus i y of Twitter. So this you can write it as r of t every time it can change. This need not be in the circular shape. Circular shape if it is it is constant. For example rotation of satellites. Satellites if you say path of the satellite if we see that is actually it's elliptic path. So whatever may be the shape you can write it like this by Eulerian representation of a complex number that is e power i some theta of t . Okay so something like this. So theta is fixed. So this theta is between 0 to 2 Pi so theta so if it has certain frequency you can put it like Omega so let us write, so anyway this is a complex number so this is e power i theta. Theta is between 0 to 2 Pi. This is what you can write. So you can see that if you are measuring the position of a satellite then what you see is that this is a complex number for every time, each time you have its amplitude and its what is its argument, e power i theta. Okay so this is a complex number. So you can have so what I mean to say is this, that a signal can be a complex valued function of time. So if you have such a thing it's natural to represent the right-hand side as functions of complex valued function. Okay. Right hand side you can represent as complex valued function.

So how do we do this. So the way is cosine and sine function you can think of n Omega naught x you can rewrite it as exponential once you know that it is z of t is or rather cos theta plus i sine theta as e power i theta. So this is what is Euler representation of complex number cos theta plus i sin theta. So if you use this one exponential it's a cosine and sine is the sum of e power i n Omega naught x plus e power minus i n Omega naught x divided by 2. This is for cosine and sine n Omega naught x also you can write e power i n Omega naught x minus e power minus i n Omega naught x divided by 2i. If you write like this and use them in the Fourier series. Here. What you see is that directly become – right-hand side becomes complex valued functions in terms of complex valued functions. These are exponential functions. So we'll see, we'll just replace cosine and sine there and see what happens to your signal. So therefore f(x) it's a function which is a0 by 2 as of now I will just leave constants as it is plus Sigma n is from 1 to infinity you have an as our constants. We replace cosine with this exponential i n w naught x plus e power minus i n w naught x divided by 2 plus bn e power i n w naught x minus e power minus i n w naught x divided by 2i. So if you simplify this take -- you can write a0 by 2 plus the Sigma n is from 1 to infinity you take simply what is the coefficient of e power i n w naught of X that you collected so you see that an by 2 minus. So you can bring this i up so you have a minus i times bn 2. Okay. Replace EBITDA power collect the coefficient of e power minus i n Omega naught x that will become an by 2 and this minus minus plus here becomes we get i bn by 2. Okay. So this is what you get.

So this is finally what you get is clearly you can see that bo equal to 0 if we actually see bn put n equal to 0 because of sine, sine function it is 0 so clearly b0 is 0 okay. See that b0 is zero so if you add this a0 plus b0 okay so you can write n equal to 0 so you can write a0 minus i b0 by 2 is same as a0 by b, a0 by 2 plus this first part so you can this one combine this with this e power i n Omega naught x an minus ib n by 2 plus e power minus i n Omega naught x an plus i bn by 2. This is what for this full summation.

And you can add put this in terms in the first term so that will make it n is from 0 to infinity e power i n n equal to 0, once you put n equal to 0 i n Omega naught is simply 1 so that is 1 into a0 minus i bn so I have added everything. So an minus i bn divided by 2. So n is from 1 to infinity is this term n equal to 0 term is this one and you can also have n is from 1 to infinity that is for the second term e power - i n Omega naught X an plus i bn divided by 2 . This is what you have. So let's define. So this constants has some cn. So that these are your Fourier coefficients cn complex-valued Fourier coefficients and these are cns are an minus i bn by 2 running from n is from 0, 1, 2, 3, and so on. Okay. So then what happens here so this is zn is the coefficient of exponential e power i n Omega naught x here this side if that is the case if you define like this then c minus n will be here from the second sum you can see that it's going to be a an plus i bn by 2. Okay.

So this is running from n is from so this is how I defined. Cns, n it's from 0 to infinity I define as this one this term and this coefficient I define it as c minus n which are running from 1, 2, 3, and so on. So if you define like this then what happens to your f of x? f of x will be now you have this summation n is from 0 to infinity. cn e power i n Omega naught x plus another Sigma n is from 1 to infinity c minus n e power minus i n Omega naught x. Okay. So this is combined together so what you get is – so this you can rewrite. So you can rewrite this as n is from 0 to infinity this you as it is you write e power i n Omega naught x. The other one you can change the index with n replace, you replace n by minus n so you get minus n equal to 1 that is same as n equal to minus 1 to infinity, minus infinity you have a cn e power the place of n I'm putting minus n so you get this will become place and i n Omega naught x.

Now you can combine this together so you can write it put it together so that is going to be minus infinity to 1 minus 1 and then to 0 to infinity. So finally it's going to be minus infinity infinity cn e power i n Omega naught x, this is what it has become a Fourier series as we know okay where the coefficients are an plus what is this cn, an minus i bn by 2 so what is this an, bn you've already defined. So if you replace what happens to cn, cns become an is 2 by L so 1 by 2 times an is 2 by L minus L by 2 2L by 2 f(x) cos n Omega naught x, dx then minus i bn is also so 2 by L minus L by 2 2L by 2 f(x) sine n Omega naught x, dx. So this is what it has become. Cn equal to so 2-2 goes and what you have is 1 by L comes out. This is from minus L by 2 2L by 2 f(x) is common you can combine this two together, f(x) what you have is cos n Omega naught x minus i sine and Omega naught x this into dx, and this is nothing but 1 by L minus L by 2 2L by 2 f(x) and this is nothing but e power minus i n Omega naught x, dx. So this is my coefficient it has become in terms of this fundamental function instead of writing cosines and sines separately we put together and write it as an exponential function. So what you get is this one. So this if you say that this is your Fourier coefficient this is your if you say this is your definition of Fourier coefficient what you get is the and that your time signal you can write it as this is your Fourier series. The Fourier series has become a complex Fourier series. Okay so if you have a time signal that is a complex valued so you can represent – always represent this way. Okay. It makes it makes sense to represent as a complex Fourier series.

So with this definition as your Fourier coefficient this is easy to remember okay so and you can put it this way. So this is in a better form than the Fourier coefficients in terms of cosines and sines and then writing that Fourier series okay in terms of them. This looks much compact and it's nice one. So it's natural that a signal can be a complex signal in that case you have a complex Fourier series that means a signal complex signal you're writing as a linear combination of all these fundamental complex signals of discrete frequencies. Those are exponential functions e power i n Omega naught x. So this is how you get this Fourier series. So as of so far this is still have not answered the question that whether the series is converging to f(x) or not so you are only believing that. So you define this as a Fourier coefficient you get a Fourier series. This is again is actually so we are not done anything new so far so we just combined the ans and bns together and put it as a complex Fourier series. So the signal you are writing as a Fourier series and it actually again if the signal complex signal is a piecewise continuous function, you can believe still believe that this Fourier series based on the cns and exponential functions put it together that series converges, that complex number series converges to the signal, time signal f(x) once you fix x that's what it happens. So you can see we will prove that eventually.

So this is your actually a crux of the Fourier series what you to calculate the Fourier series you just have to define your Fourier coefficients and calculate the Fourier series, so but then you have to remember every time you are what is your cosine function. So where these cosines infractions are coming from? Cos n Omega naught x and sine n Omega naught x when you have the fundamental frequency is 2 Pi by L that means your signal is of time signal with period L you have this is your fundamental frequency, these are fundamental signals and you just have to remember these Fourier coefficients an which you define as some integrals ans and bns. So where are they coming from? Actually if you know the differential equations they are actually coming from differential equation. So some particular differential equation. So that is a Sturm–Liouville problem you can get all these as a solutions of a Sturm–Liouville system so which are nonzero solutions of a Sturm–Liouville system there is where and you can see that these cosine functions sine functions are actually coming from a differential solution of a differential equation. So if you if you know the differential equation if you have done the earlier course differential equations for engineers so you can see that we have done Sturm–Liouville system if I write like this y double dash plus a lambda y equal to 0 so this is in the Sturm -- self adjoint form that is p equal to 1 p of x equal to 1 into y for which you have a derivative y dash and you have a derivative that is nothing but y double dash okay plus q okay plus lambda times Omega is 1, Omega of x is 1 that is a weight function and y equal to 0. So this is your differential equation. y is a function of x so x is between 0 to L if you define it like this and what happens to if it's a periodic Sturm–Liouville system, system if you know the differential equations you just understand this otherwise you don't need okay. So you just have to remember these cosines and sines and this is your Sturm–Liouville form so the self adjoint form your differential equation is a self-adjoint form with the boundary conditions the domain is 0 to L and you have to give the boundary conditions. Those are periodic bound because it is a periodic Sturm–Liouville system lambda is the parameter. So lambda is a parameter if it's self-adjoint form lambda is always real so it's a real parameter because it's in the self-adjoint form and once you see this is a self-adjoint form the dot product, the dot product between the solutions always, how to identify that that is always in this form. So integral this between 0 to L that is the domain and f(x) g(x) into w(x). The w(x) is a weight function that is 1.

So you have a usual dot product. So this is the dot product with respect to which the solutions of this will satisfy. They I actually form complete orthogonal set. The orthogonal means complete set of functions any function you can represent in terms of the solutions of this with respect to this dot product. So let's see the boundary conditions. So boundary conditions what we give in this periodic system is y at 0 is same as y at L and y dash at 0 is same as y dash at L. So these take this as a boundary conditions and this is your equation. If you solve it solutions are solutions, if you see if you actually solve this for different term because it's a real parameter, is real, a real parameter because it's in the self-adjoint form lambda can be positive, lambda can be negative and lambda zero in all cases if you calculate what you see is these are cos so what are the solutions, so let me put these solutions in a different form. So what I do is because we have not taken the period zero – we have taken the period L but domain is not 0 to L earlier when we have the Fourier series we have taken from minus L by 2 to L by 2. So here also let me take that way so that your integral will be from minus L by 2 to L by 2. This is the dot product between the functions which are the solutions of this equation satisfying this boundary condition. So 0 yL by 2 or rather so minus L by 2, 2 L by 2 and here minus L by 2 y derivative is also will have plus L by 2. So these are your boundary condition. So periodic boundary conditions. Now your domain only we have changed.

So what are the solutions here? So if you have such a thing your solutions will be what you – if you actually see that the solutions are nonzero solutions will be if you actually see they are cos n 2 Pi 2 Pi by L x and sine and 2 Pi by L into x. So n is running from 0, 1, 2, 3, and so on. This is what we will see okay. If you just go through how to – we just find this solutions of this periodic Sturm–Liouville system, you will see that these are your solutions. These are exactly our cos n Omega naught fundamental frequency x and sine n Omega naught x, n is from 0, 1, 2, 3, onwards. So for the n, for cosine function this is n is running from 0 to 4 –for sine function when you put n equal to 0 that is anyway zero. So you're looking for nonzero solution that doesn't count okay.

So you say it's from one so you have to one cos n equal to 0 that is one so you say these are your solutions, nonzero solutions. So in terms of these are your fundamental signals in terms of this you are writing piecewise continuous function. You're writing in terms of these fundamental functions. Okay. So that's how these cosine functions and sine functions are coming into your Fourier series okay. So actually what is giving is a Sturm–Liouville system with periodic boundary conditions is giving you actual Fourier series. If you change this differential equation you may get a different Fourier series okay. So it depends on – so if we want to remember this cosine sine function with exact this fundamental frequency you just write this y double dash plus lambda some parameter into y. So this one equal to 0 over the domain, over which over the time or the domain of the time signal that is minus L by 2 to L by 2 and then you have there you give the boundary conditions at the endpoints as always like this, endpoint that minus L minus L by 2 is one endpoint a value of the function y at that point it should be same as value at L by 2 because it is repeating. It is a periodic and its derivative also should satisfy the same conditions. So if you give this you will get what you want those fundamental solutions. The solutions of the system is actually those fundamental solutions. They're actually orthogonal. So with respect to this dot product they are all actually orthogonal that means integral minus L by 2 to L by 2 instead of f of g you give cosine and sine, you give anything different from this that is cos n Omega naught x into sine n Omega naught x, dx is always 0 okay for every n. n equal to 0 anyway it's true so this is true. And if you calculate L by 2 2L by 2 same function for example cos square itself you take cos square n Omega naught x, dx this will give me what is this value so 1 + cos 2 n Omega naught by x divided by 2 so that will give you so L by 2 okay. So that will give you L by 2, so this is the number which is showing up in your coefficients an, bn you have this coefficient that is integral so 2 by L times minus L by 2 2L by 2 f(x) cos and sine both n Omega naught x, dx. Okay. So this is actually both these together so an, bn or can be represented this way one is an cosine you consider for an, and sine you consider for bn.

So where is this L by 2 so 1 this so this over minus L by 2 2L by 2 cos square n Omega naught x or sine square n Omega naught that is dx that value is actually L by 2. So that is what you will see, you are seeing here okay. So either this cos square or sine square. So together you will have only the same value and if it is n so if you simply take this one even that includes n equal to 0, 0 also n equal to 0 you have this function constant function 1 for which if you take this limit dot product itself that value is actually L by 2. So it's valid from n is from 0, 1, 2, 3, onwards. So this is how you get this cosine function, sine functions. The same way you can see so if you combine them what you are seeing is this complex Fourier series anyway okay. So complex thing you don't get it from the Sturm–Liouville system and this is how you get this fundamental signals cos n Omega naught x and sine n Omega naught x. So given a time signal you need not represent always say in terms of cosine and sine functions okay. So you consider some Sturm–Liouville differential equation with the periodic boundary conditions, you may end up by different solutions, nonzero solutions. They are linearly independent solutions and they form complete orthogonal set. So that complete orthogonal you can get it from here. So this is that means they are actually perpendicular to each other. If you take any two different fundamental functions they are always orthogonal that means the dot product is zero and if they are and also they form complete set, complete means complete set, complete set means any signal f of x every function f of x I can represent in terms of these functions. So that is exactly your Fourier series. That is start with a0 some arbitrary constant with this one plus Sigma, n is from 1 to infinity so you can write an cos n Omega naught x plus bn sine n Omega naught x, dx. Yeah this is your complete set that means any function if any signal I can write in terms of these complete set. That is exactly what Fourier series. A naught is a naught b naught where an a naught is equal to so how do you get this a naught so a naught if you have this represent complete set means this what is this a naught? A naught is actually you take a dot product with both sides with a naught to a naught, the dot product both sides and because they are orthogonal this other everything will else will go only this right-hand side you have this, left hand side you have f(x) dot product with a naught. So what is this one? This is same as so you have a naught so dot product is L by 2 a naught just comes out okay. So what is this one? This is actually minus L by 2 2L by 2 a naught square dx right. It's not a naught square dx yeah that is true or you just take the dot product with a fundamental function. So that is one. So you have only a naught dx. This is equal to a naught L by 2 which is equal to minus L by 2 2L by 2 f(x) a naught dx. So what is this one? So a naught-a naught goes what you have left with is, I'm sorry, so I have taken a naught as one so you have this there is no a naught here now. So this will give me a naught, the constant arbitrary constant as 2 by L minus L by 2 2L by 2 f(x) dx. So instead of 1 if you take the dot product with f(x) times cos n Omega naught x if you take the dot product so first term a naught cos n Omega naught x that is one a naught into the dot product with cosine that is 0 so what you have the right-hand side only a naught contribution will be there. So you will get a naught dot product with cos or rather an, only an will be there. So an cos n Omega naught x dot product with cos n Omega naught x. This symbol represents the dot product which is – which you should recognize from as soon as you have this Sturm–Liouville system okay. So this is exactly L by 2 an we have seen already this dot product. So that's again so you see that an equal to 2 by L integral minus L by 2 2L by 2 f(x) cos n Omega naught x, dx these are exactly your Fourier coefficients, an and similarly you can do if you consider dot product with sines you have what you get is a bn so instead of ans you get bns and here instead of cosine you write sines okay and what you end up is L by 2 bn. So you get bn as again 2 by L minus L by 2 2L by 2 f(x) sine n Omega naught x, dx. So these are exactly your Fourier coefficients. Okay so you change this differential equation. This periodic Sturm–Liouville system with each time you change the differential equation you may end up these non zero solutions they form complete orthogonal. This is the orthogonal complete sets that is exactly or Fourier series okay. So you will have a different representation but every time you will have only discrete number of such functions solutions are only discrete such functions. So that means every signal you need not represent in terms of discrete signals in terms of cosines and sines. It can be different functions okay, but they are all discrete countable number of solutions, fundamental solutions. Okay. So that's how you get this Fourier series.

So we have not only the an, bn we have seen that this in terms of complex numbers complex Fourier series you have seen so now we are in a position to to see, we will just take up some other example. We'll take some example and try to give you -- try to represent that in the complex Fourier series, for example the one we have done. So let's do some example. Example you consider the function f(x) which is mod X. let's take only a real valued function okay. Still you can represent in terms of exponential function, complex Fourier series you can. We have seen what this as in terms of an, bns the real Fourier series you have seen now we can write in terms of complex Fourier series, that is between 0 to let us take between x is between 0 to L okay. So what does this look? This looks like 0 minus so let's say 0 to L and this looks like, so let's do it from minus L by 2 2L by 2. Then how does this look? L by 2 2L by 2 this is the period of period L, this is a function with the period L. So you have this is always positive. So this looks like a v-shape and you extend it over L 3 by 2 so again it looks like this is your L and this looks a V like this you can go on extend it as periodic function or full real line. So if you take this one ans, bns you have calculated. Now your cn. Cn is a Fourier coefficients or Fourier coefficient complex Fourier coefficients you can calculate now. Cns both are same. Okay. So you have cns what is our cn? You have so the period is L okay and the fundamental frequency is the Omega naught which is equal to 2 Pi by a period L that is what it is. So you have cn is 1 by L minus L by 2 2L by 2 so f(x) e power minus i n Omega naught is 2 Pi by L x, dx. So let's calculate for this and see what it is. So 1 by L minus L by 2 2L by 2 f(x) is when it is negative side and it is negative side minus L by 2 to 0 so this is simply minus x so you have minus x e power minus i n to Pi by L x, dx plus 1 by L or a different domain 0 to L by 2 mod x will be simply x e power minus i n 2 Pi by L x, dx. You call this work – you work with w naught. That's better. So for simplicity you can do that and now you can see you can represent minus 1 by L put x equal to minus x if you do that minus minus plus x minus x and dx is minus dx that is minus minus plus and this will become L by 2 to 0 and then this will become e power i n Omega naught x, dx. Right.

So this minus will become 0 to L by 2. So this 1 plus 1 by L 0 to L by 2 of course x is there and x into e power minus i n Omega naught x, dx. So you see that both are same so we get 1 by L so actually this is some of it if you combine them together this is actually 2 by L integral 0 to L by 2 x cos n Omega naught x, dx. So this you can do because it's a sum you can calculate this cn. Okay. So here you don't have to do for separately for an and bn so you have this is running from minus infinity to infinity. So this is like saying 0 plus or minus 1 plus or minus 2 and so on. So like this you can put so you can you're actually calculating for every n.

So now this you can do the integration by parts 2 by L so if you do the 2 by L integration by parts sine n Omega naught x divided by n Omega naught so the derivative is this x now you apply this limits, minus 2 by L into n Omega naught integral 0 to L by 2 and you simply have sine n Omega naught x into the derivative of x is 1 so you have dx.

So in this one n Omega naught is n into 2 Pi by L so L by 2 when you substitute is n Pi that is 0 and when you put x equal to 0 this is anyway 0. So contribution is nil here. So you are left with minus 2 by L into so n 2 Pi by L, so 2 Pi n so simply Pi n 2-2 goes here, L n Omega naught that is L n 2 Pi by L. So 2 n Pi. So divided by 2 and Pi this 2-2 goes numerator denominator. So this will be minus minus plus cos n Omega naught x by n Omega naught this is running from L by 2.

So this is what is this value so finally n Omega naught is nothing but n 2 Pi by L right. So you have 2 Pi L this and have this yeah. N Omega naught is is L divided by n square Pi square by 2 right. So what is this n, n Omega naught so n Omega naught is this one so 2 n Pi by L, 2 n Pi by L so L by 2 n Pi so this is what it is and if you apply this cos limit cos n 2 Pi by L into L by 2, 2-2 goes so cos n Pi minus cos 0 that is 1. So this is what it is. So you have L divided by 2 n square Pi square and here cos n Pi is -1 power n minus 1. This is exactly because you see that it has to be real value cns because f is a function and everything has to be real value. So cns are real so you get it as this function cns are real so this implies that function mod X in the interval minus L by 2 to L by 2 this function at mod X you can write it like, so linear combination n is from minus infinity infinity, the cn that is L by 2 n square Pi square minus 1 power n minus 1 times e power minus this is no place what is your Fourier series? Fourier series, complex Fourier series you can see that it's going to be but plus sign so you see that this is Fourier series complex Fourier series with plus an e power i n Omega naught x, e power i n Omega naught and Omega naught is 2 Pi by L x this is exactly x is between minus L by 2 2L by 2. So this is the Fourier series for the time signal mod X whose period is L okay.

Still this convergence of this complex numbers this complex number series whether it converges to mod X once you fix x still questionable. So this is what exactly we will see in the next video. We will try to develop a relevant things we will do and then we will try to prove the Fourier series of a time signal. It converges to the time signal at some particular time always. Whenever there is – the minimal condition is f is piecewise smooth function or piecewise differentiable function. Okay. It is also true so that means there are sufficient conditions. If time signal is piecewise differentiable function I can immediately prove, easily prove that this Fourier series converges to the time signal that function f(x) okay. We will see in the next video. Thank you very much.

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