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Transform Techniques for Engineers
D'Alembert's Solution by Fourier
Transform
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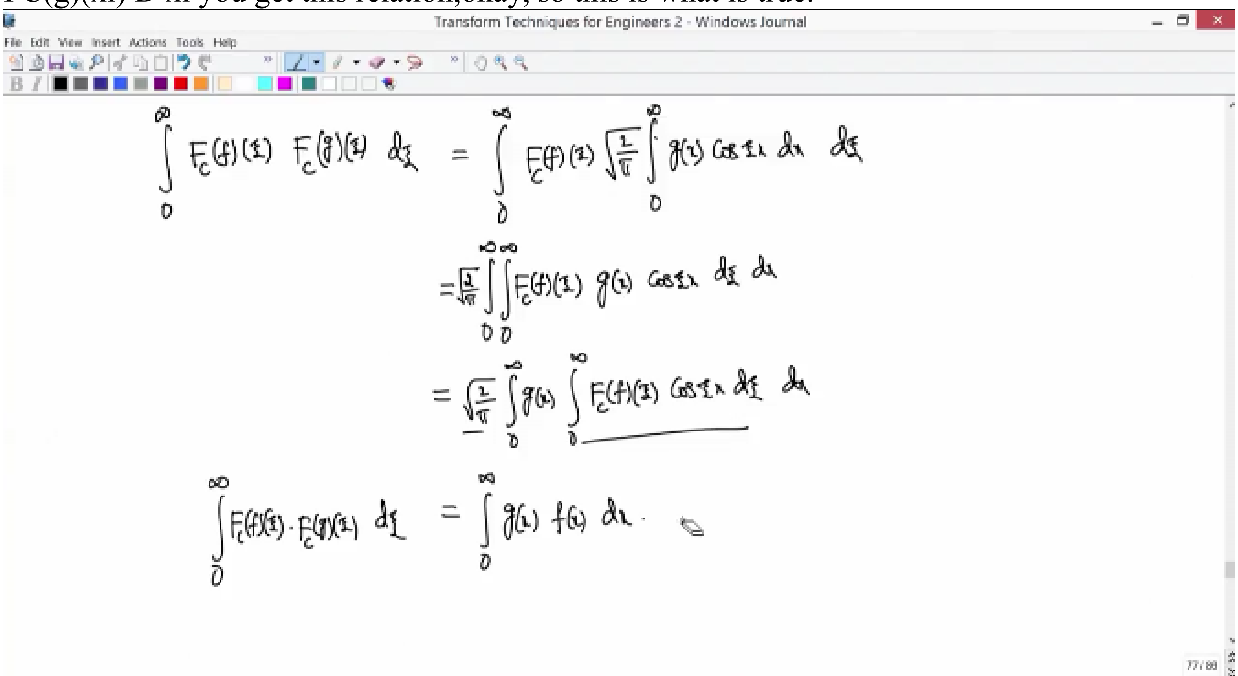
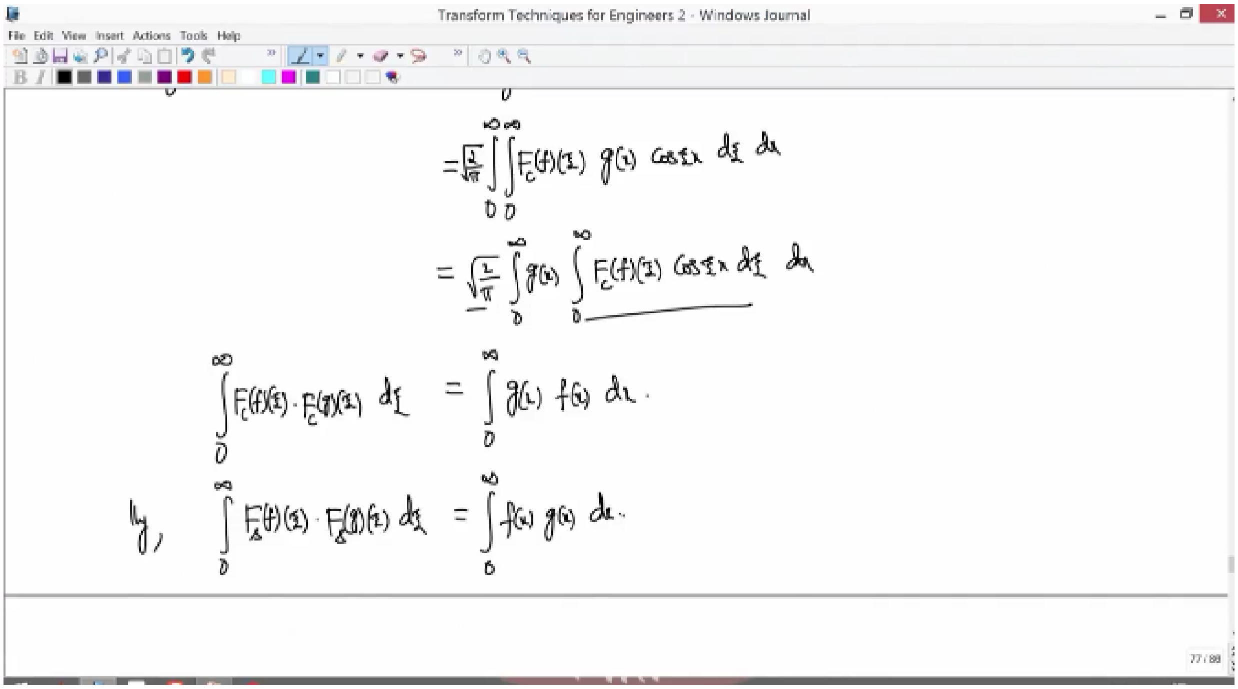
The last video we have seen theapplication of Fourier transform, one ofthe application of the Fourier transformthat is to evaluate certain integrals, wehave seen one integral that is fromminus infinity to infinity DX/Xsquare + A square into X square + B square, such integrals we canevaluate, so full Fourier transformswhich we use in the Parseval identity,so in that relation similar relation, similar such relationswe can derive for Fourier cosinetransform and Fourier sine transform, sobecause of the integral is from 0to infinity domain of that integrand isfrom 0 to infinity we can use Fouriercosine transform or Fourier sinetransform and then make use of suchrelation, so we have not seen suchrelation is a property of the Fouriertransformwe'll just derive such thing initially,and then we'll do that example.

So what we have is, we will try toevaluate this integral, integral 0 toinfinity so let me take someFourier transform, Fourier cosinetransform of F(xi) of Fourier cosinetransform of G, F and G are two functionswhich are functions of xi as a Fouriercosine transforms so functions of xi andyou have this such a, if you have thiswhat happens to this?This is with D xi this integral, so what exactly is this, sowe will try to see what we can see, so this is, what I do is I0 to infinity,and you have FC of function F of xi cosine transform and this you try toevaluate, so you write for cosinetransform definition you have root 2/pi,integral 0 to infinity, F cap of, sothis is for G of function, G(x) andcos xi X DX into D xi, so this is whatyou have, so I will try to change theorder of integration so if you do thatwhat you get isby the Fourier integral theorem you cansee that this double integral isactually a finite that means theintegrand is absolutely

integrable, inthat sense I can do the order ofintegration so if you do that, whatyou see is FC(f)(xi) root 2/picomes out and integral, so this integral0 to infinity can write here and youhave G(x), G(x) cos xi X instead of DXD xi you write D xi DX, so what is this? This if you rewrite, I justchange the order of integration so whatyou get is, you want DXso FC, so innerintegral is D xi, outer integral is G(x) soI write G(x) here, so inner integral 0 toinfinity FC(f)(xi) cos xi X D xi DX.
Sothis one is actually, this is byFourier inverse transform, inverse cosinetransform of the further, inverse cosinetransform for this Fourier cosine transform of F, if you usewhat you gain is 0 to infinityG(x) together, together with this constantthis integral will give me F(x) DXokay, so this is what you have, so if youhave a Fourier cosine transform of 2 functions FC(f)(xi) times FC(g)(xi) D xi you get this relation,okay, so this is what is true.

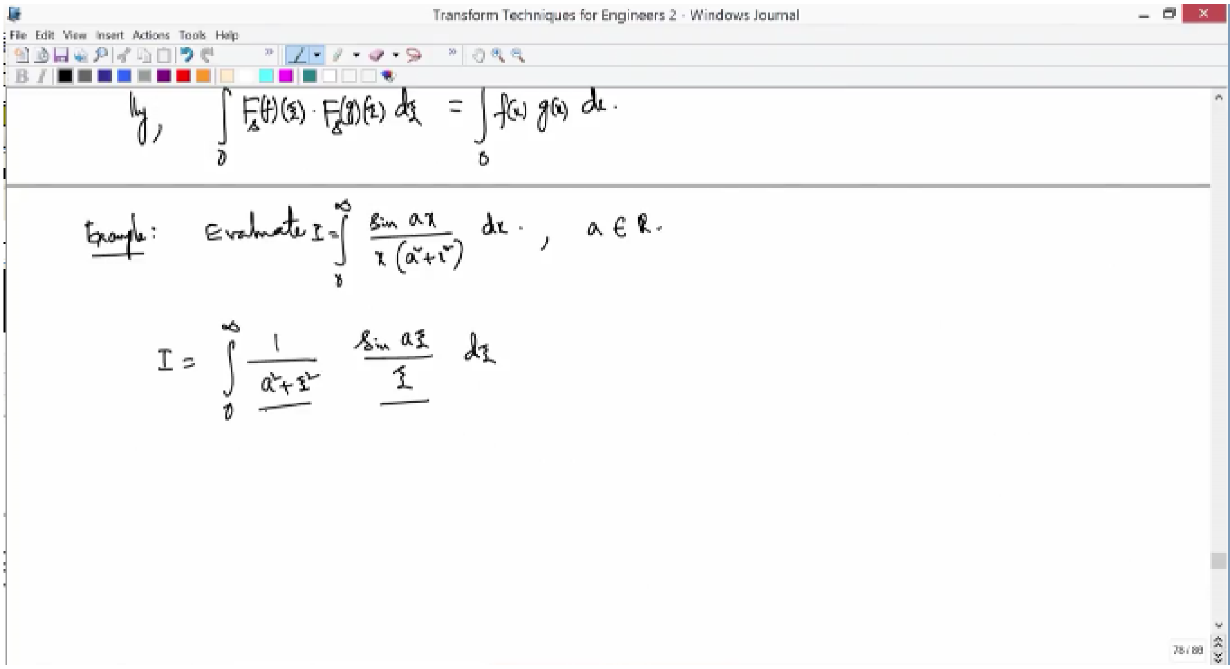



Similarlyyou can, instead of cosine if you usesinetransform you can alsoderive similar relation, so one can writethere is no change here so you can usethe same proof instead of C you have Shere, instead of cosine you have sine xi Xif you just follow the same procedureyou will end up seeing theserelations, so Fourier sine transform ofF which is function of xi into Fourier sine transform of G which is function ofxi, for this product you take theintegral and



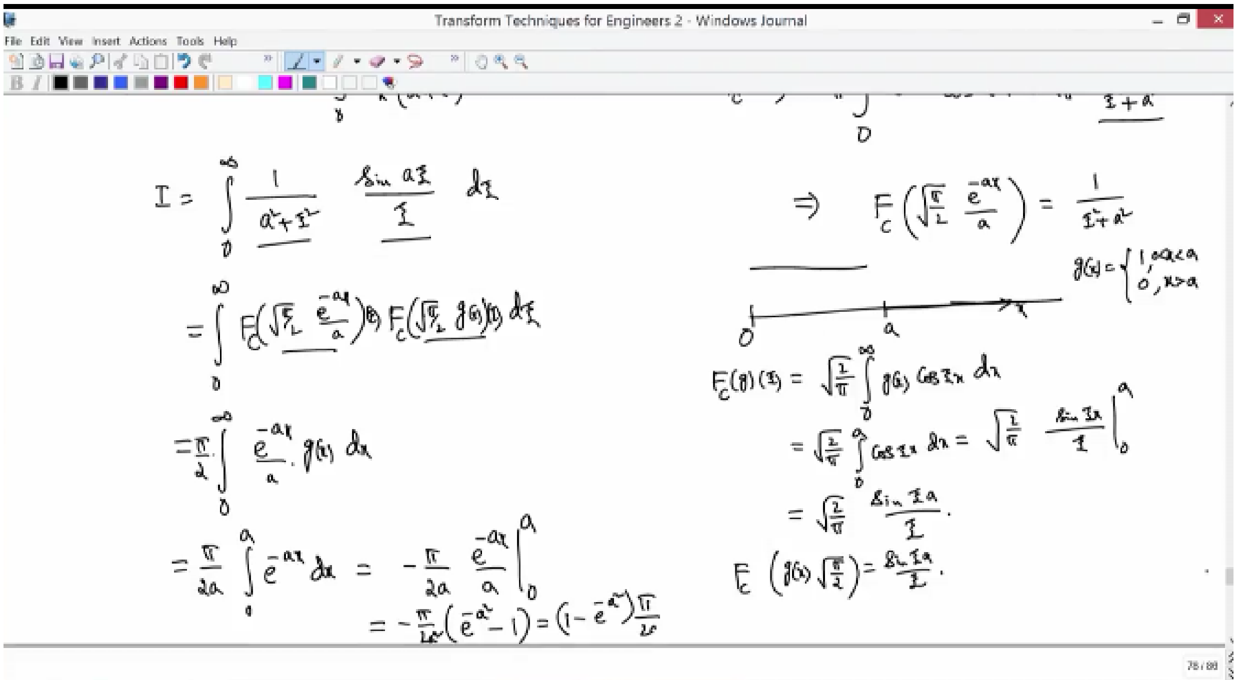

this will be again F(x) intoG(x) DX, so these are the two relationswhich you can use to evaluate certainintegrals, integrals from 0 toinfinity, so let's use example, one moreexample to apply these transforms tofind, to evaluate this integral soevaluate, if you want to evaluate 0 to integral infinity, 0 to infinity integral sine, let us say some AX divided by X times A square + X square,suppose you have this you want toevaluate, okay.

Then what happensso A is any real number,if A0 is anywayknown so if you fix any A which isnonzero so you can evaluate,you need to evaluate this integral, soif you call this I, so I will be,I will try to write like 0 to infinity, Iknow that I can write this with A square+ xi square let me put this as xi square as a dummy variable, and you havea sine A xi/xi D xi, I try towrite like this, so these two are Fourier transform, Fourier cosine transform of certain things thatyou need to know, okay.

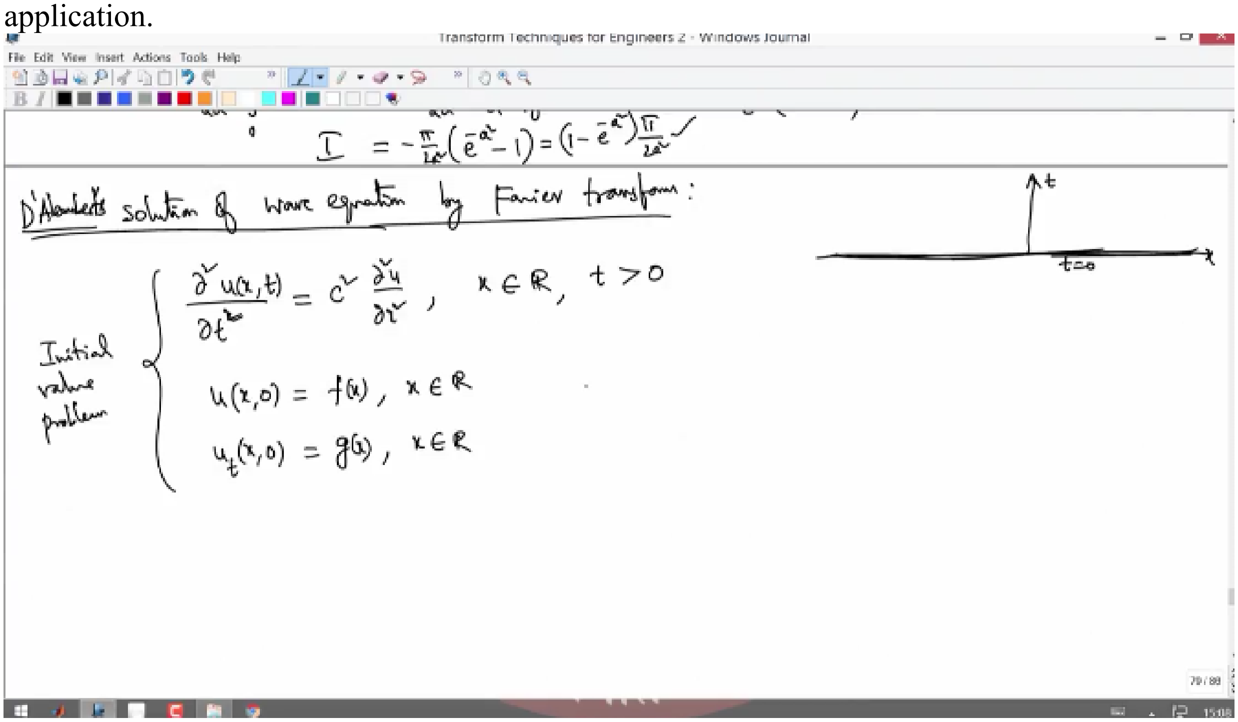



So what is thatexactly?If we see earlier thisis a Fourier cosine transform of E power-A modX, if you do you get similar suchexpression, this is full Fourier transform, so maybe if you remove this Xyou can get for Fourier cosine transform,if I take this Fourier cosine transformintegral from 0 to infinity root 2/pithis is exactly, what is this equal to?Root 2/pi A divided by xi square + A square, this you can justby integration by parts you can evaluatethis, okay, so if you use this so by makinguse of that so this will be exactlyequal to, this is what is thisleft-hand side?This is actually Fourier cosine transform of E power –AX,okay. So what is a Fourier cosine transformof root pi/2 times,E power –AX/A,if you pull this E inside and also pullthis constants inside this Fouriercosine transform then this will be equalto simply 1/xi square + A square. Similarly you can calculate somefunction like this, which is, this is Xaxis, it is a, function is 1,this is 1, if from A to 0 to let's say,let put this way this is from 0 toinfinity, 0 to A it's 1, otherwise it is 0,from 0 to infinity it is 0. So let'sdefine some G(x)as 1, if X is between0 to A otherwise 0, X greater thanA, if you have such a function what is itsFourier cosine transform of G, and G(x),which finally becomes afunction of xi is root 2/pi integral 0to infinity, if you apply G(x) here cos xi X DX,actually DX okay, this isnothing but 2/pi integral 0 to A, G(x)is 1 here cos xi X DX, so this isactually root 2/pi sine/xi, sine xi X/xi, just put apply limits 0 to A,so you end up getting root 2/pi sinexi A/xi, so you can see that if you wantto use this G, G times, G(x)times rootpi/2 is, so if you take the Fouriercosine transform that is exactly sine xi A/xi, so you write like this, this one has0 to infinityFourier cosine transform of thisfunction root pi/2 times E power–AX/A thentimes, this one is a Fourier transform,Fourier cosine transform of root pi/2times G(x) D xi, okay, which isfunction of xi, each of this.

And thiswe know from the earlier relations, thisone if you use this you can see thatthis integral 0 to infinity, thisfunction into this function that is pi/2 pi/2 that makes it pi/2 and E power –AX/A times G(x)DX,so this is pi/2A integral 0 to A, E power - AX DX this becomes E power,if you integrate so –pi/2A Epower –AX/A, this is from 0 to A, soyou end up getting so –pi/2 A square,E power -A square -1, or1- if you can absorb so you canmake it, you can rewrite like 1- E power - A square times pi/2 A square, so this is your answer, okay, so Ivalue this evolution of this integral isthat for any A belongs to R.




So that'show you can apply Fouriereither Fourier cosine transform or sine transform, and soif you know Apriori few functions whatis its Fourier transform if you have inmind and if you can see such functionsFourier transforms in your integrand asa product you can use these relations toevaluate the product integration of thatproduct, okay, this is one applicationsimple application.



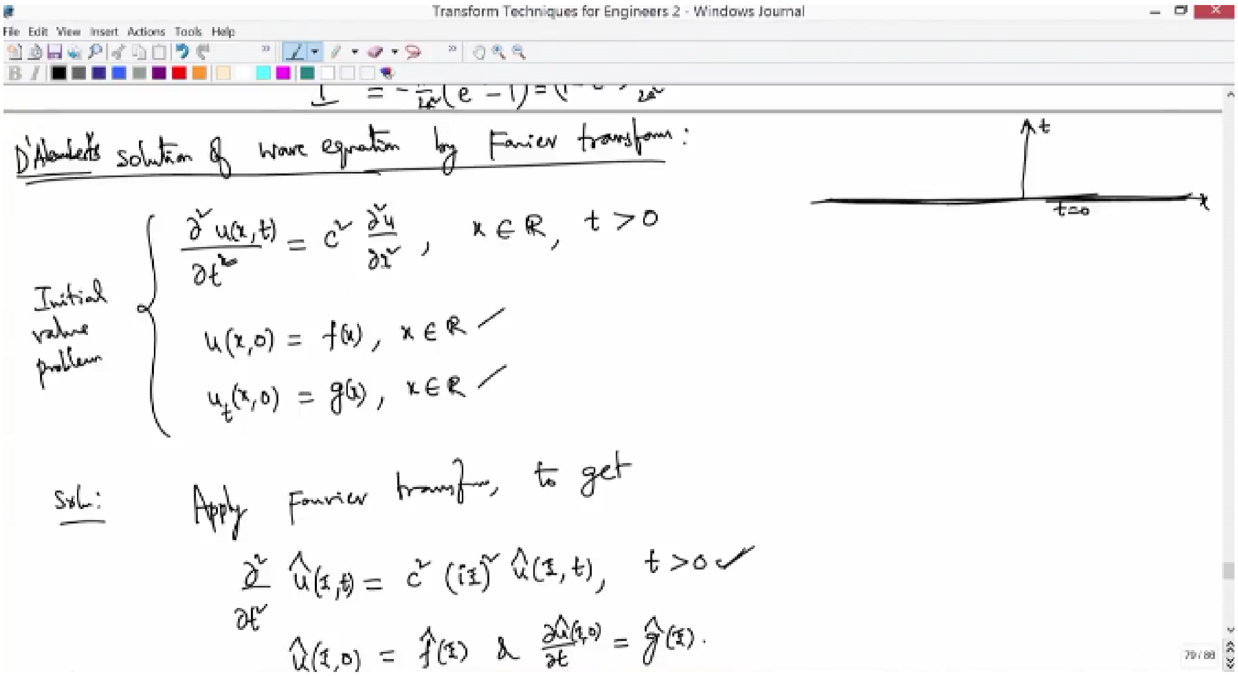

Let's move on toevaluate, let's move on to apply Fouriertransforms to solve some boundary valueproblems for, boundary value problemsinvolving partial differential equations,as we know partial differentialequations or second-order linearpartial differential equations you canput it in two variables, you canalways put it in by classification,you can put it in one of the forms, ontypical each group you have atypical equation if it's a hyperbolicequation you have wave equation, so this equation, partial differential equationis actually typical equation is waveequation, and for parabolic typeequations you have a heat equation andfor elliptic equations you have aLaplace equation, so each of this are, wewill just try to pick up any each ofthese wave, heat, and Laplace equations,and over a domain if they aredefined over a domain will give theboundary conditions and we'll try toapply the Fourier transform and try toderive some boundary value problem we'lltry to get the solutions.

We'llstart withwave equation, so we already know how to, so in differential equations course ifyou have done you have seen how to find D'Alembert's solution for wave equation, let's start doingthat,  let's do a wave equation solutionof wave equationby, so D'Alembert's solution, D'Alembert'ssolution of wave equation byFourier transforms.

So let's define theproblem, so problem is to find this twovariable function XU,T, this is adisplacement of an infinite stringlet us say X belongs to full real line,so you have infinite string and this isgoverned by the wave equation so thedisplacement of the spring, the displacement of the string is governed by this wave equation, so you have a dousquare U/dou T square, and then equalto the speed of this propagating wave, if it's vibrating the wavesare propagating with the speed C, C is thespeed and then some constant and thenyou have dou square U/dou X square,Uis a function of XT, so what is T?T istime initiallyso you have a time T andthis is a T and this is your X, okay, sothis is your T axis and at T = 0it's only here, so T = 0 so youshould provide because the second orderequation you should provide two initialconditions so you have a U at X0, it's adisplacement of the initial,initial displacement of the string if you can giveanything you can give F(x), and if yougive velocity of the string initially atT = 0 if you give the stringvelocity that is G(x) let us say, Xbelongs to R, so this is well-definedproblem there is no boundary,boundary isonly at infinity so we will try to getthe solution of this initial valueproblem, initial value problem.

So how do find thesolution?Using Fourier transform, so wehave already seen, we have seen in theearlier course if you have done thedifferential equations for engineers, you have seen how to find this solutionby separation of variables,actually by classification you candirectly get the solution, today we applyFourier transform because the domain isa, full domain is X belongs to minusinfinity to infinity so that we can applyFourier transform for the variable X, soapply solution is to find apply Fourier transform, full Fourier transformto get, so if you see thisyou get dou square, if you apply if dousquare U(x,t)/dou T square, for thisif you apply Fourier transform as afunction of xi, dou T which is nothing to do with the functions of X, sothis dou square/dou T square comesout of the Fourier integral and you endup getting dou square/dou T squarecomes out and you simply get this is Ucap(xi), okay, so that is this is equal to C square if youapply Fourier transform for derivative youget I xi 1 derivative, you have 2 derivatives, you have I xi square and thatis into U cap(xi), U cap(xi,t), X becomes xi after transform butT is as it is, so you have now U cap(xi,t) is a function Tis now positive.

Now you can also applyfor the initial conditions because it'sa function of X, you get U cap(xi,0) at T = 0 this is F cap(xi) andsimilarly U cap(t) so this is a timederivative dou U by, and dou U cap/dou T is a function of xi 0, which is Gcap(xi)this is what you have, so if you look atthis equation we'll try to solve this,this becomes aODE now, so theadvantage of the

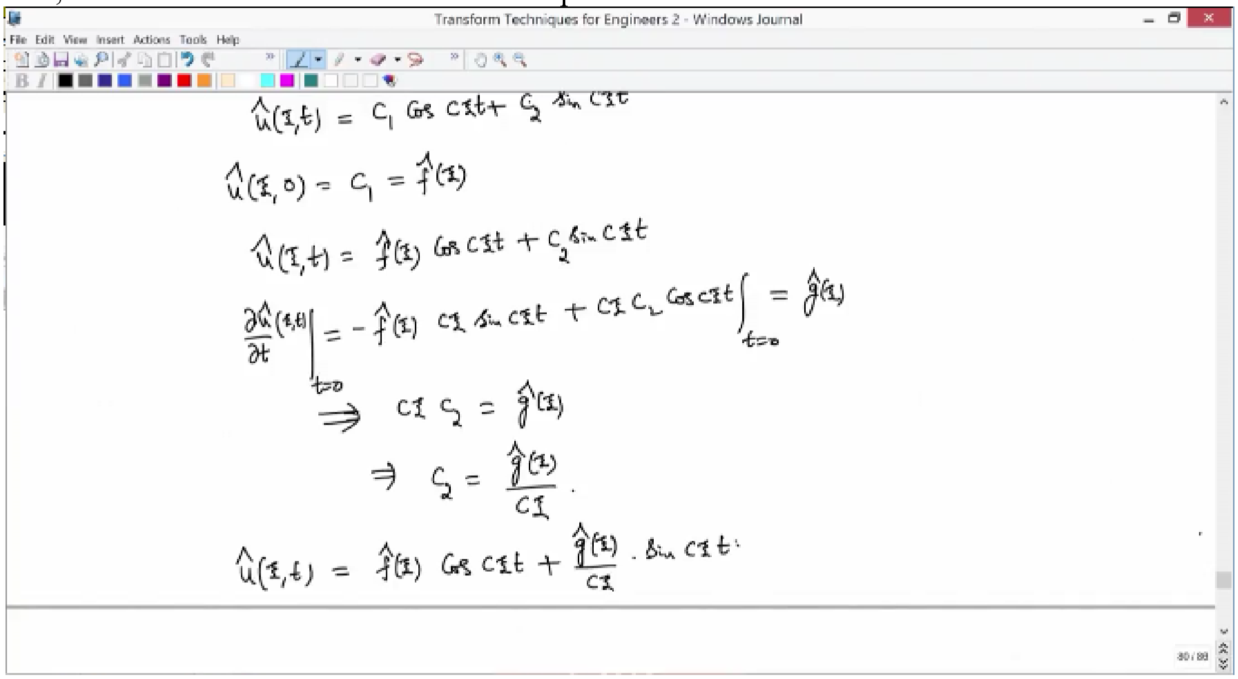



Fourier transform is ifyou apply, if you apply to thederivatives some partial derivativeswith respect to X that is removed,so it's no more derivative, it's a simplealgebraic equation, so once you removethe X derivative so what you end up isbecause it's only 2 derivatives T andX derivatives only you end up in that,now this is ordinarydifferential equation because now thisfunction is, this dependent variable is U cap(xi,t) for which you have independent variablesonly T, so you have a differentialequation in one variable so it's a kindof ordinary differential equation.

Soadvantage of the Fourier transform is itreduces partial differential equationinto any, partial differential equationof order N into partial differentialequation of order N-1 it reducesone level down as when you apply transform.

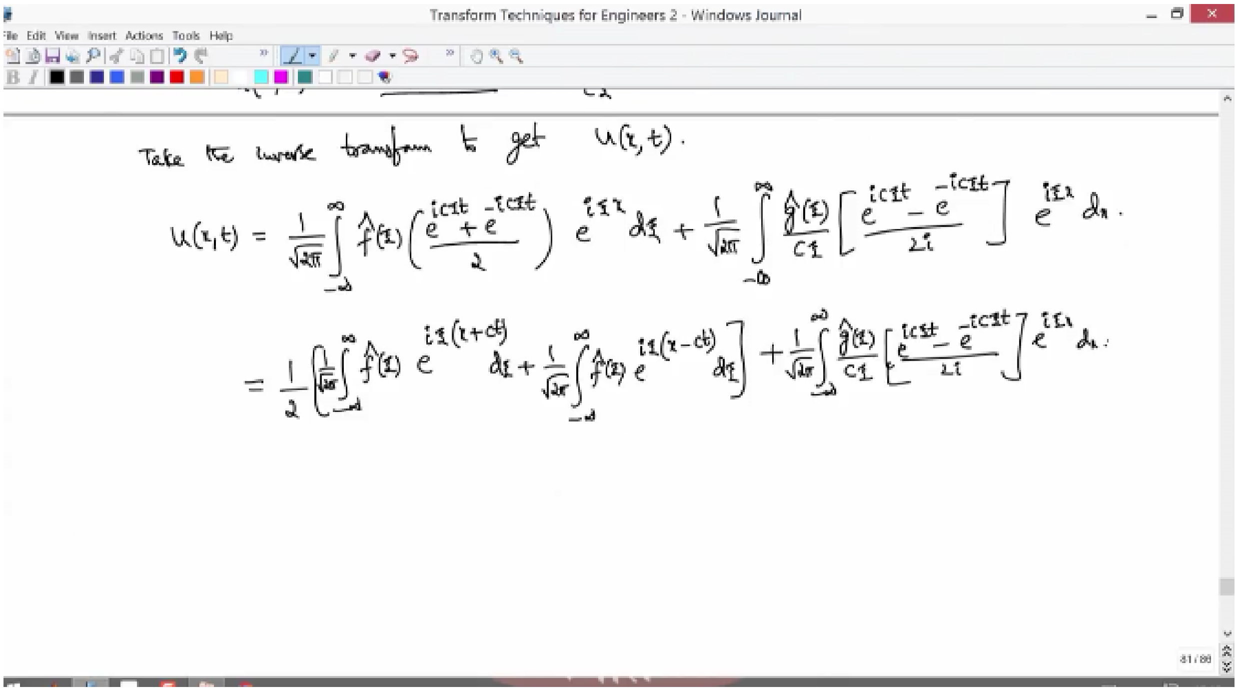
So we will solve it, so if you solve directly this ordinarydifferential equation as a function of Tthis becomes, so this is like if I removethis U cap(tt)- C square xi square U cap = 0, so thisis the equation, if you see this one unknown is Ucap, two derivatives dousquare U/dou T square so this is ifyou apply solution is, this is like youcan see the complementary typeof, complementary equation is M square-C square xi square = 0, soyou have M is C xi + or -, so U cap(xi,t) is actually equal to someconstant times E power – C xi T andthen +C2Epower + C xi T,okay.Sorry this equation becomes plus right, this is plus, so you have a plus, so you have I times, so you have I, if it is I you can write here I, I, so this is also sameas saying,if you take the real part and imaginarypart this will be C1 times, you can rewrite in terms of cosines and sines okay,so general solution of this equationis C1 times cos C xi + C2 timesanother arbitrary constant times sine C xi T, okay, C is the constant involvedin the equation.

So now you try toapply your you know initial conditionshere, these are the 2 conditions if you use, you end up, you try to apply here, soxi is 0, this is C1 + C2 into 0 sothat is equal to F cap(xi) that isgiven, so that makes it C1 as so whathappens to your general solution now seeU xi T as F cap(xi) cos xi C xi T + C2 so that makes it C1, so C2 is stillarbitrary sine C xi T, now you try toapply one more condition so you find thederivative dou U cap/dou T (xi,t), this is F cap(xi) C xi – C xi times sine C xi T + C xi C2 cos C xi T, now you apply T = 0, if youput T= 0 for full expression soyou end up getting this is 0 and you getC xi, C2, is cos 1 this is equal towe’vegiven that this is G cap(xi), okay, sothis is actually G cap(xi) from thissecond this one, left hand side isthis up to here, right hand side is this,so this is this if you calculate thisone left hand side,C xi, C2 and that implies C2 is G cap(xi)/C xi. So what happens to your generalsolution now?U cap(xi,t) = F cap(xi) cos C xi T + C2is now G cap(xi)/C xi times sine C xi T, so this isthe solution that satisfies this problem.

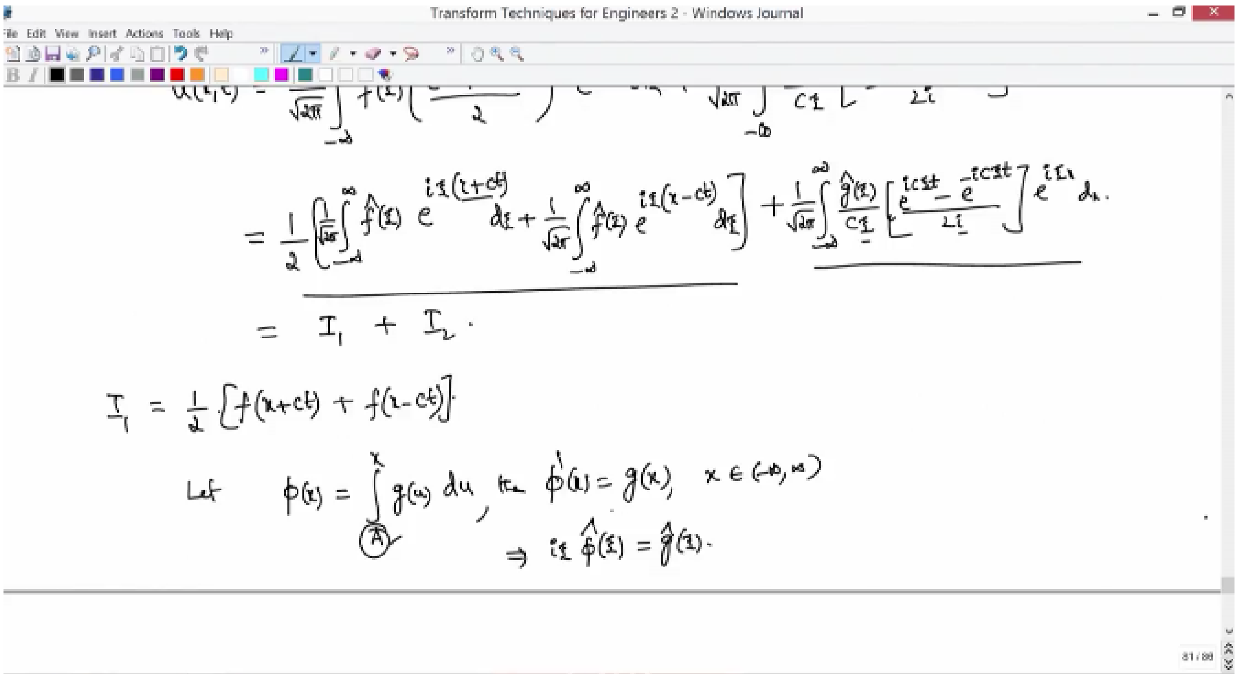



Now,so everything is known here, so thisis known because F is known, this isknown because G is known, so ifyou can invert this you can get your, ifyou invert this you just inverse,take the inverse transform to getU of, if you take the inversetransform for this Fourier transformwith respect to X variable you get backyour X variable and this is this, okay, sohow do I do?To do this U(x,t) so lefthand side it will become U(x,t), righthand side you take the inverse transformso that is 1/root 2 pi the definitionminus infinity to infinity, F cap(xi)this function whole function I try towrite this as exponential cos in termsof, exponential is E powerIC xi T + E power – IC xi T divided by 2 times this is the function,E power I xi X D xi, so this isfor the first term.

The second term againyou can apply the Fourier inversion,this is from minus infinity toinfinity, you have G cap(xi) divided byC xi sine, in the place of sine Iwill put that as E power I C xi T – E power - IC xi T divided by 2I into E power I xi X DX, this is by inversion formula.So if you see this one1/2 you take it out and you write likeintegral 1/root 2 pi minusinfinity to infinity,F cap(xi) E power if you combine this exponentialand this exponential and you end upgetting E power I times xi is common andyou get X+CT, and if you do theother one and then that is D xi, again ifyou write this part this is minusinfinity to infinity, F cap(xi) E powerI times xi common X–CT into D xi, this is what it becomes this term,first term.

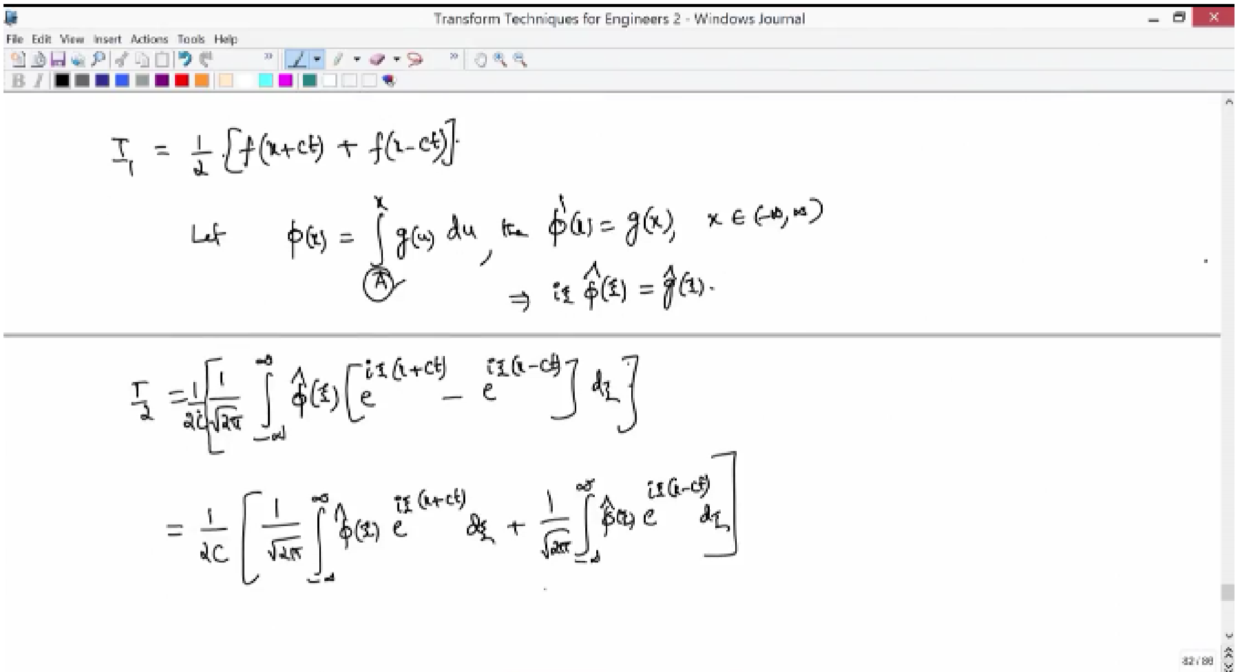



And then second term, secondterms will deal it separately, so wewill write as it is,by 2I times E power I xi X DX,okay, so we can see that this first term,so we call this first term so this is I1 + I2 let us say, this is I1 andthis is I2, so what happens to I1? I1is 1/2 times if you look at the firstterm in this, this is Fourier inversion, sothis is the inversion formula for F atthis I xi X, instead of X now I have X+CT, X+CT+ the secondone becomes F(x-ct), Fourier inversion will give me this one, this isfirst one.So I2, to get I2 welook at the, to get this I2 I willdefine a new function, so that thisintegral this will be inversion ofsuch function, so let me define what thatis. If I define this new function PhiX as from constant, some constant to Xlet us say, so G(u) let us say if I useG(u) DU, this is a function of X, if Itake the Fourier if its derivativePhi dash(x) is actually equal to G(x) right, because this is right, this iswhat you call this, if you define likethis then it's derivative is this simpleby direct differentiation you can getthis, now if you apply the Fouriertransform here for this because thefunction X belongs to minus infinity toinfinity, so you can apply this one,A isany fixed constant, okay, so if A canbe any fixed constant you can fixanything 0, 2, anything, anything you canfix but finite real number.

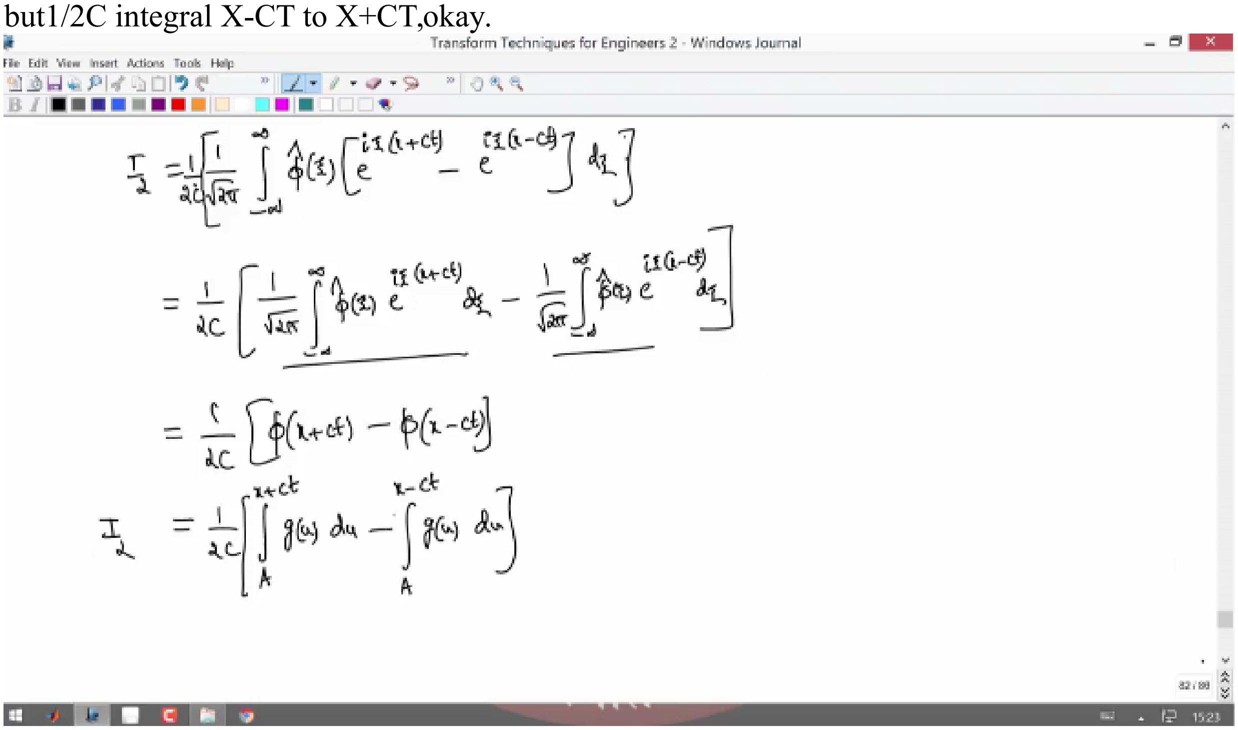



If youapply Fourier transform I xi times Phi cap(xi) = G cap(xi) that's whatis this, so what happens to your G cap(xi)? So G cap(xi)/I xi which wehave here, G cap of by this I and thisxi you can replace with this Phi, okay, so I2 becomes 1/root 2 pi integralminus infinity to infinity G cap of by,G cap(xi)/I xi I replace withthe Phi cap(xi) times, so you canwrite remaining 1 by C comes out, 1/2C comes out rather, this C and this 2comes out 1/2C times this one Phi cap(xi), again this is again E power Itimes, xi is common, this isX+CT,and then this is X-CT Epower – I xi comes out X-CT this together, so denominator isaccounted and you have this DX, this isexactly what you have.

So this isactually equal to 1/2C and you canrewrite this as minusinfinity to infinity,Phi cap(xi) into Epower I xi X+CT DX + similarterm,okay,and minus infinity to infinity, Phi cap(xi) E power I xi X-CTDX, DX or D xi, this is D xi right, so this is D xi so it should be,doing inverse transform that is withrespect to D xi so you have this isD xi, so you have D xi,




this isD xi, by inverse transformation this isnothing but 1/2C, this is Phi(x+ct) + this part will be Phi(x-ct), so you know thedefinition of Phi, you substitute here to seewhat is I2, I2 is 1/2C this is aPhi 2 integral A2, X is X+CT,G(u) DU + integral X-A2, X-CT G(u) DU, this is minus right, so I've missed a minus it's notplus its minus, so you have a minus here andyou have a minus here, so this is whatyou have.So if you take the minus, so this put it together this is nothing but1/2C integral X-CT to X+CT,okay.



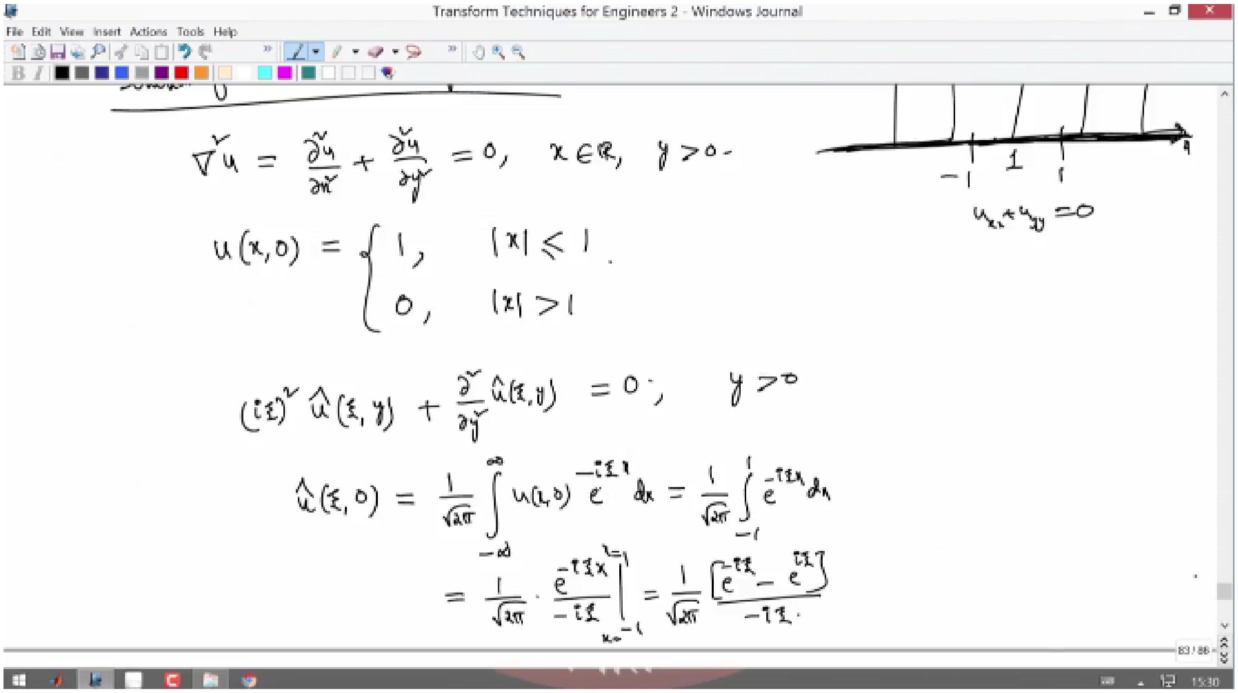

X-CT, T is positive,X–CT is smaller, so you have this minus signmakes it X-CT to X+CT this issmaller,A2 bigger so if you remove itsomewhere X-CT will be insidehere, so finally you end up getting X-CT to X+CT, okay, so this is G(u) DU, so put it together I1 + I2 if you put it together here U(x,t) becomes the D'Alembert's solution, whichis well known 1/2 of initialdisplacement of the string that is at X+CT +F(x-ct) + 1/2Cintegral X-CT to X+CT of G(u) DU, so Xbelongs to –R, T is positive, so thisis the D'Alembert's solution, well-known D'Alembert's solution, this is how you canapply Fourier transform to get the D'Alembert's solution.

Let's move on to get the other type ofsolution, so we will start, we'll consider,this is anyway so youcan, this problem can besolved even directly by other methodsyou can solve, and certain equations are,for example if you define Fouriertransform if you define a problem forthe Laplace equation let us find asolution of, solution of a boundary valueproblem for Laplace equation, solution of2 dimensional Laplace equation, Laplaceequation is basically kind of a steadystate, heat equation, steady stateheat equation means you have a hot platewhich is at steady state, afterwards once,steady state means no heat is no,no heat transfer between any fromthis plate this one, okay, that is whatis the steady state, there is no more itover a time there is no change in thetemperature of this plate, so when itreaches such a thing that iscalled the steady state heat equation, soyour volume 2-dimensional Laplaceequation means it's a 2 dimensionalspacethat means you have a plate, and plate isa 2 dimensional thing and then T is time, so you have parabolic equationthat is heat equation,when it is steadystate that T derivatives gone what youare left with is UXX+ UYY = 0, okay, this K times actually you have UT equalto this one, this is a 2 dimensionalheat equation that is for plate, plate isa 2 dimensional spatial domain, so if what we consider is upper half, upperhalf space, this is X and this is Y.This upper of space you consider steadystate means this is gone, this is 0 that means this is equal to 0, so letme write gradient square of let's say, instead of U let me use, let meuse the same U = UXX that is dou square U/dou Xsquare +  dou square U/dou Y square= 0.

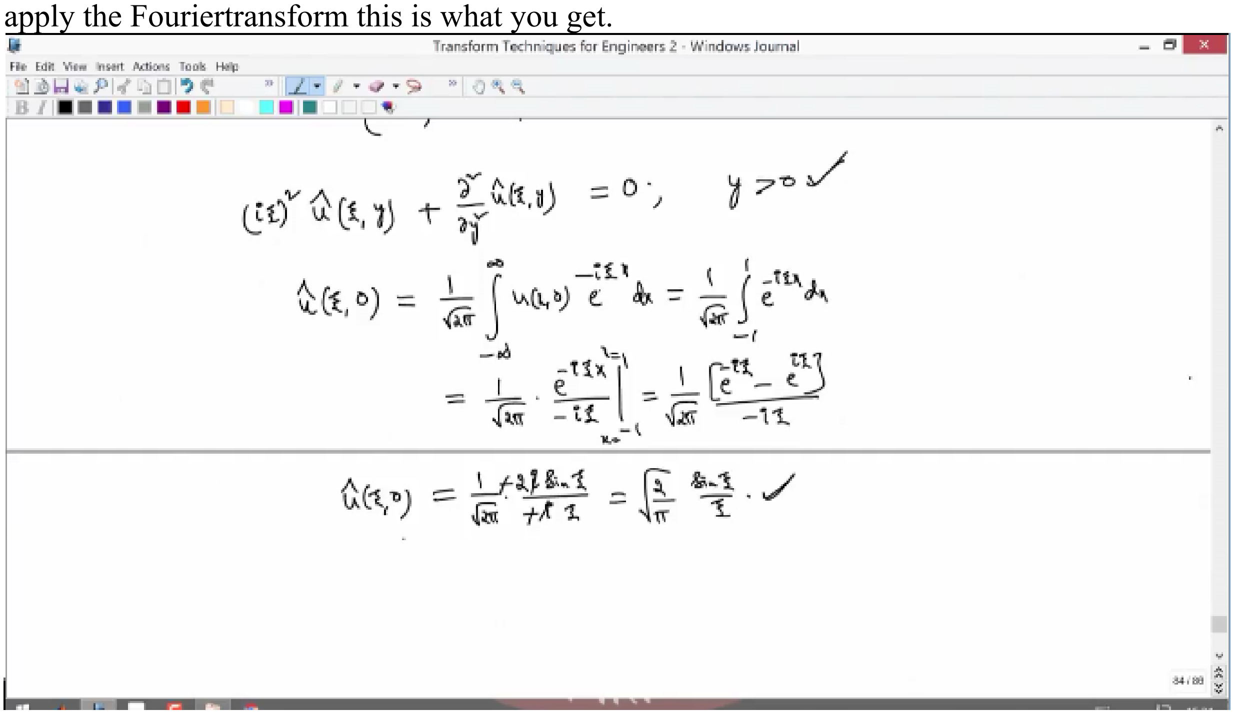
What is the domain? X isfull real line, and Y is positive that isone. And what is the domain boundary?Boundary is this, and there is no timederivative so what you have to give is, you have to give the boundary conditions, this is the only boundary so I providethe boundary as Phi at XU at X0 as Igive you such a way this one, so Xis between -1 to 1I keep, give the temperature, I maintainthe temperature as 1 here, okay, so U(x) is 1 when X is between, mod Xis less than 1 or equal to 1, sootherwise outside I may think the temperature is 0 all the timeokay.

Then what happens to this plate?Whatis the steady state?The temperature inthe plate, in this plate of infiniteplate, semi-infinite plate we can view ita problem, so this is mod X is greaterthan 1, so this is the problem you cansolve by Fourier transform because X isfull real line, you can apply for the Xvariable Fourier transform to see thatlike earlier and how I xi square U cap(xi) Y + here dou square Y/dou Ysquare so nothing to do with Fouriertransform variable X, so you have U cap(xi,y) = 0, so you end upgetting, so this means you have this isagain now xi is belongs to full R thatwe don't care, and you have Y is positive.

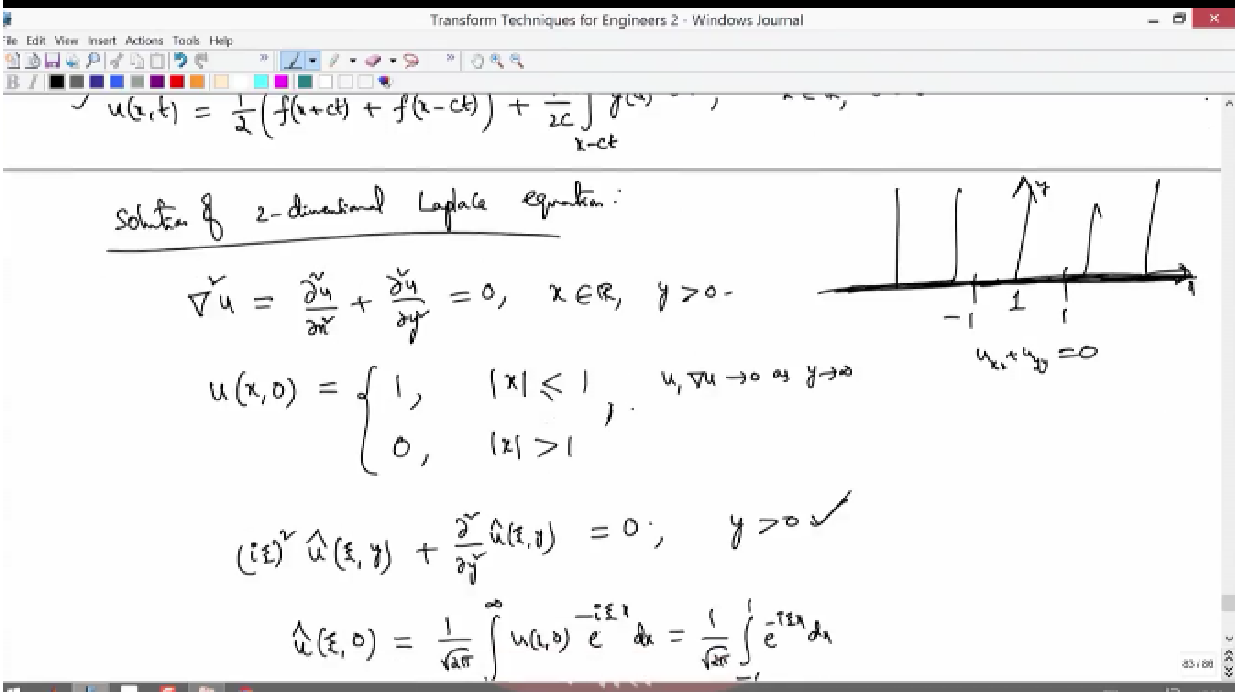
And what happens to this U cap(xi) is 0 at Y = 0, if you apply Fouriertransform 1/root 2 pi you have aminus infinity to infinity U(x,0) E power I xi X – I xi X D xi, DX right, this is DX this is atransformation, so you end up getting 1/root 2 pi -1 to 1 you have thatis 1U is 1, and you have E power -Ixi X DX this is nothing but 1/root 2pi timesE power -I xi X divided by -I xi and put -1 to 1limitsto see finally that this is E power -I xi X = -1and X = 1, I'm substituting



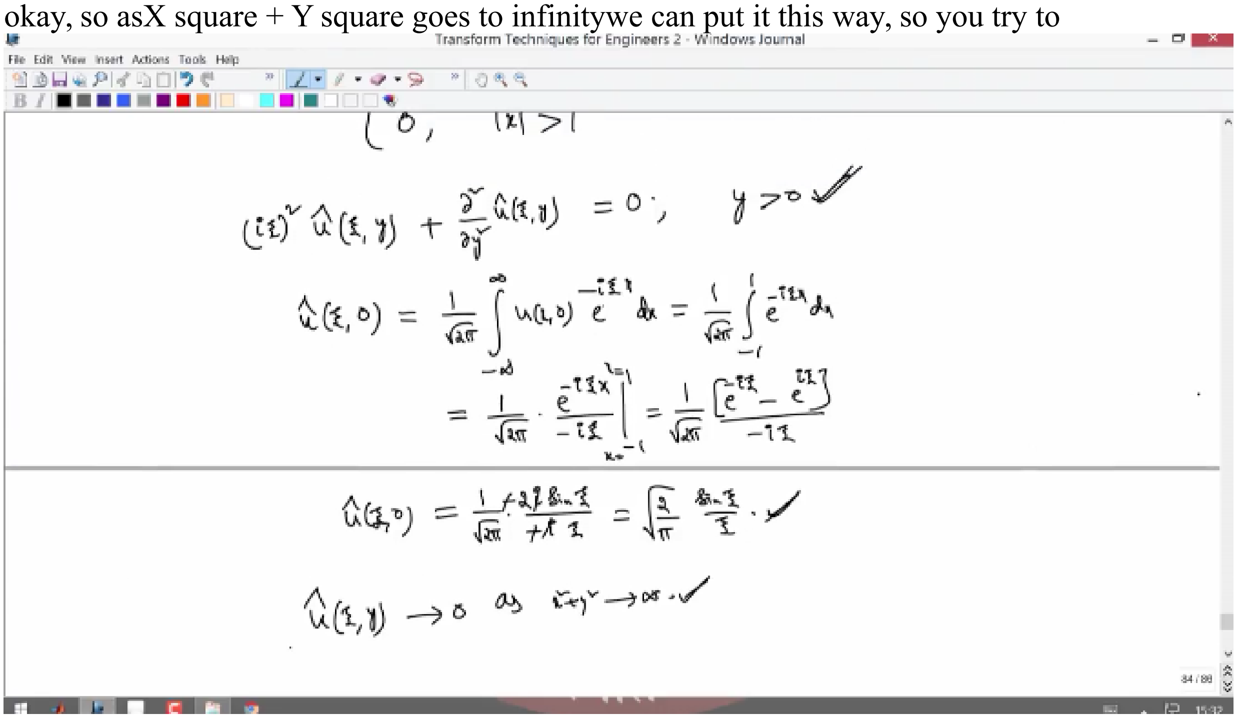

so E power -I - E power I xi divided by –I xi, okay, so if you finally see thatthis is nothing but 1/root 2 pidivided by, so this is nothing but 2times,-2 times sine,- 2Itimes sine xi divided by –I xi, so this minus minusI goes this 2 if you cancel it you willend up getting root 2/pi times sine xi/xi, this is the Fourier transform ofthis U the function,the initial, boundarycondition if you apply the Fouriertransform this is what you get.




So youtry to solve this condition now, U cap(xi) this ordinary differentialequation now, this is ordinarydifferential equation with the initialcondition, this is the second orderequation and you have the, you have onlyone condition, so what happens the otherthing?At Y= infinity you wantyour temperature to be a finite, to be 0okay, because it is infinite plate youshould give as a second order equation,so you should provide one boundary, otherboundary means that is at infinity inall directions, so that is U and thegradient of U has to go to 0 as Y goesto

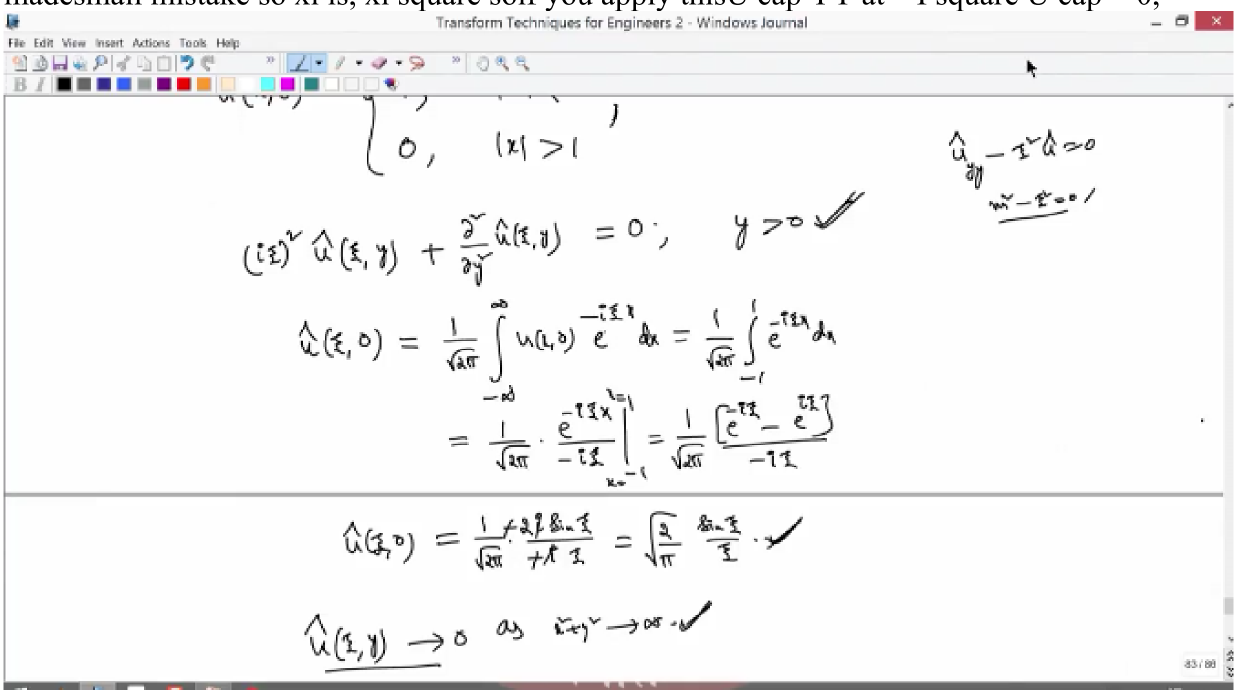



infinity,so that is another condition of theoriginal problem, so if you apply that, soif you apply this U cap(xi,y) hasto go to 0 as Y goes to infinity, okay,naught Y goes to infinity meansis only one direction, so to put alldirections you can write X square + Ysquare should go to infinity, okay, so asX square + Y square goes to infinitywe can put it this way, so you try to

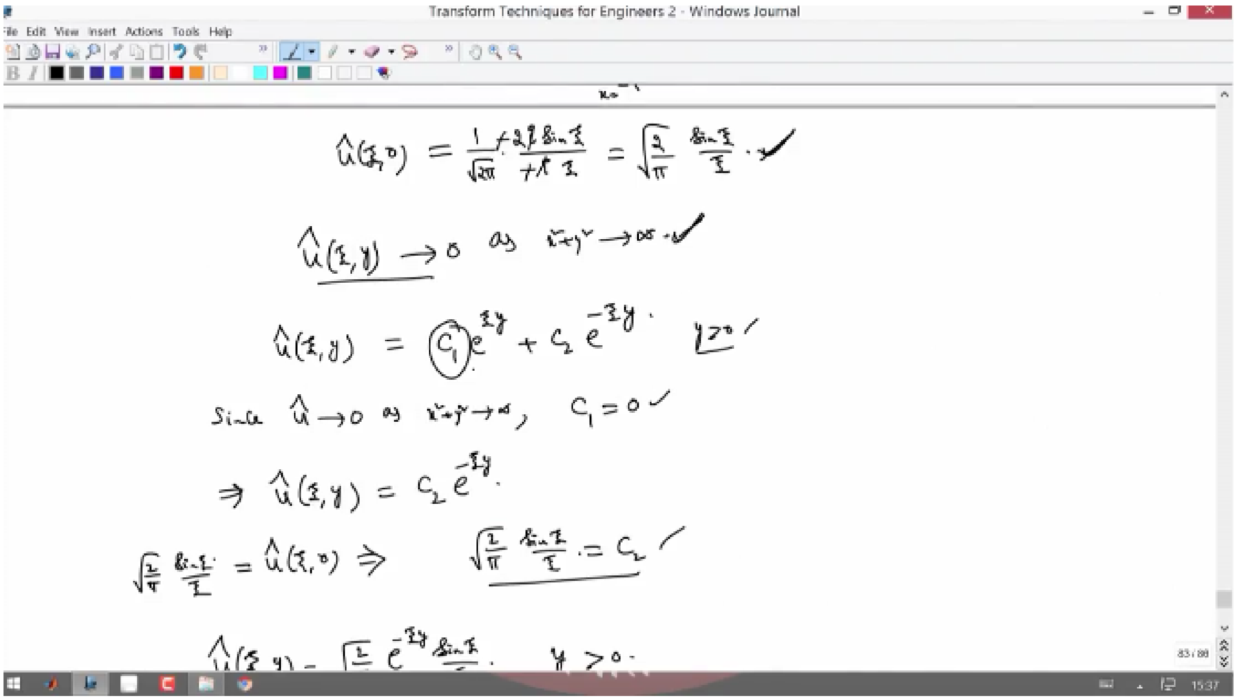



apply these3, if you try to solve this ODEwith these conditions what happens to,what is the general solution of this ODE if you calculate, you see that is goingto be Ucap(xi,y) is equal to, ending up getting – xi squarehere, so U cap(xi) is C1 E power xi Y + C2 times E power –xi, y.

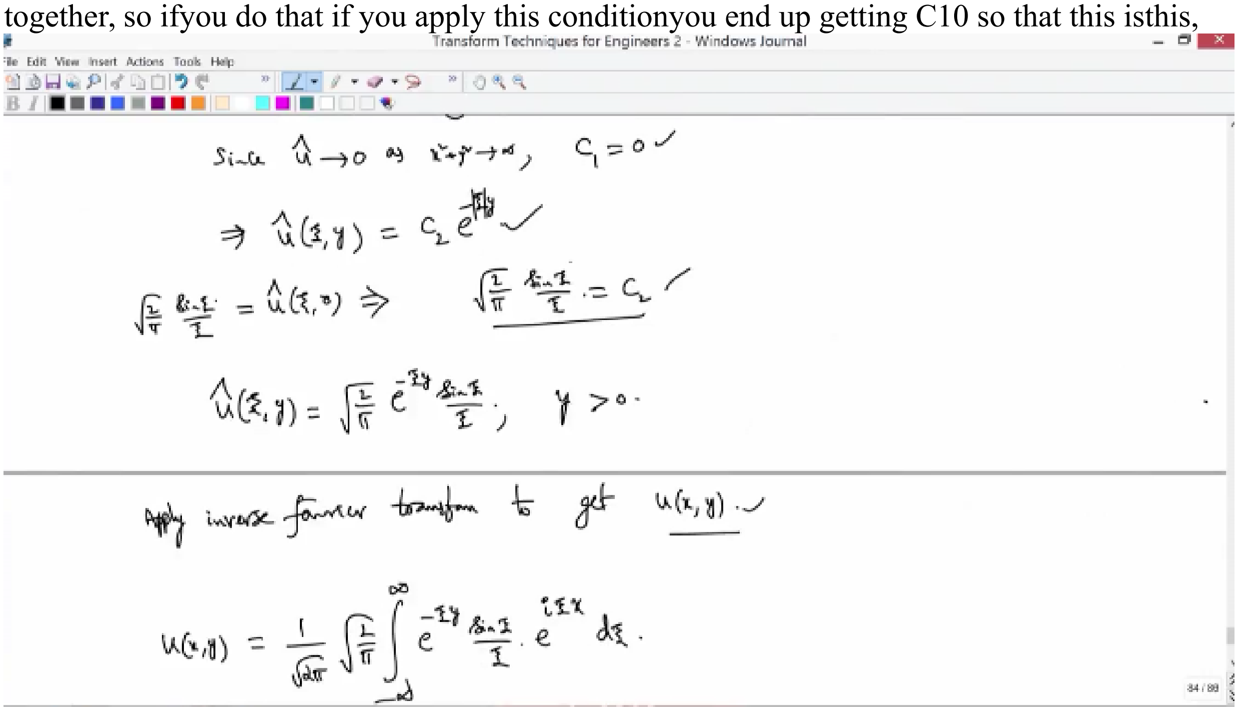
Ifyou apply this one this last one C1 hasto be 0 because this is not, as Y goes toinfinity this has to go to 0 for thatthere is no other choice C1 has to be 0,okay, if I apply since U cap goes to 0 asX square + Y square that goes toinfinity that is possible even with Ygoes to infinity, okay, then C1 has to be 0 that implies U cap(xi,y) is C2 times E power –xi, y forthis you apply this condition, condition number one, so U cap(xi) is 0 to get C2,so C2 times this is equal to root 2/pi sine xi/xi, if we apply this is equal to U cap(xi) = root 2/pi sine xi/xi this implies you end up getting, this isequal to if you apply here U cap(xi,0) is C2, so this is what you get.
So yoursolution U(xi,y) now  is C2, now is known now so 2/pi E power –xi,y sine xi/xi, now this iswhat you get for Y positive.
Now youcan find the inversion, so applyinverse Fourier transform, inverseFourier transform to get the solution,the steady state heat equation solutionthat is temperature of infinite plate,semi-infinite plate at steady state thatis U(x,y), okay, if you invert this xi, so xi becomes X, because that isthe Fourier transform of X variable,so you get left-hand side U(x,y) andyou end up getting 1/root 2 pi intoroot 2/pi integral actually minusinfinity to infinity, I'll write thisintegral outside, so you have minus infinityto infinity, and E power – xi, y sine xi, y, sine xi/xi into this is now function U cap(xi,y) into E power I xi X D xi, this is theinverse, okay, root 2 root 2 goes what you end upgetting 1/pi integral minus infinityto infinity, wait, wait, so I think we madesmall mistake so xi is, xi square soif you apply thisU cap YY at – I square U cap = 0,



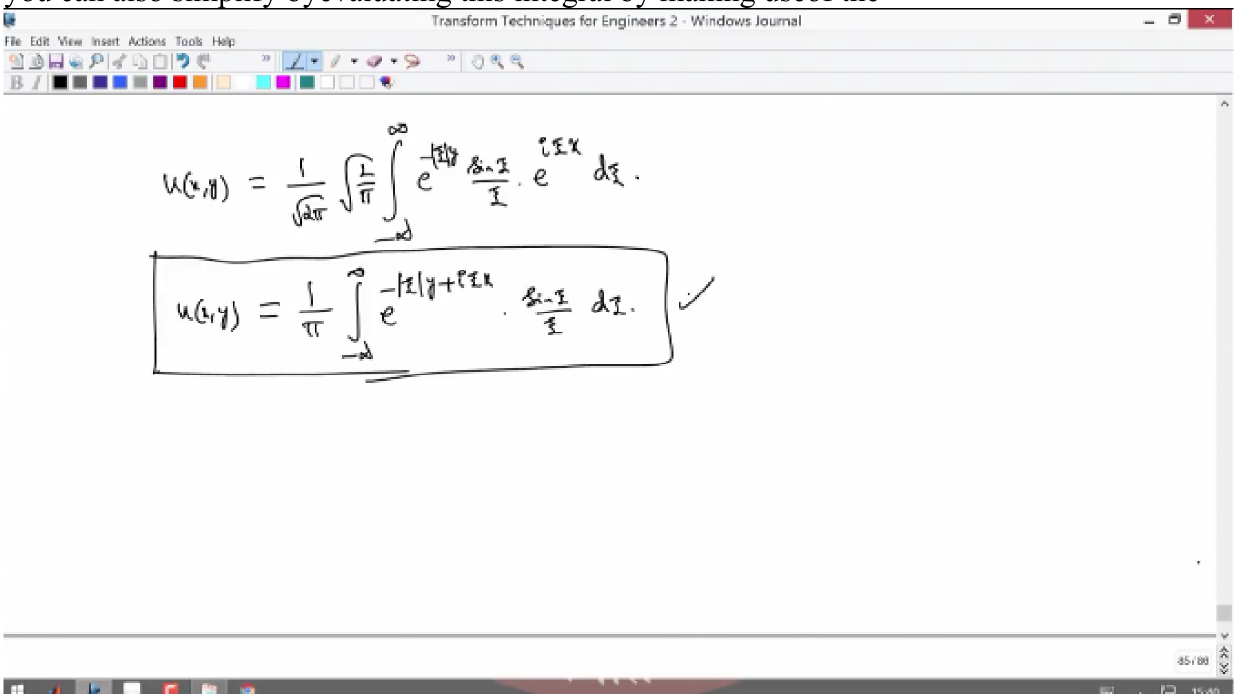

so what is this,complementary equation is xi square= 0, you have both, there's amistake here, so M is + or – xi, okay, so you have a solution like this, Y is always positive, right, Y is always positive,xi can be, xi is actually full real line, xi belongsto full real line, if xi is positivethen C1 is 0, so thisis all true up to xi is positive.



If xi isnegative then C2 has to be 0, rightbecause in that case as Y goes toinfinity even xi is negative this is –xi is positive, and then as Y goes toinfinity this has to go to 0,but that makes C2 = 0, so toavoid this both cases we can put it C1of mod xi of Y, C2 of mod xi(y), Y positive,now that works for any xi positive, if xi is positive its mod xi is xi, if xi is negative only thing is the roles of C1and C2 are changed, okay, interchanged, sothat way you can put it together, so ifyou do that if you apply this conditionyou end up getting C10 so that this isthis,



so now our general solution is C2times E power -mod xi, y, so you have – mod xi, y times this is thesolution after applying second thing, first condition, thiscondition if you apply you end upgetting what is C2, that is okay, so youfinally get this, for this if you invertso not mod xi, y without mod xi this is onlyvalid for xi positive, for xi negativethis will become E power, if you actuallydo the same thing with xi negative, if you take the xi negative thing you finallyending up getting Epower xi, y, soput it together it's like mod xi, y, that is also you can do, so now boththe cases you get the same answer sothis is the one, you have mod xi, y, okay, so you have E power - mod xi, y timesthis you can combine with exponentialfunction, xi is common, xi, y, let me putit like this E power I xi X into sine xi/xi D xi, so this we canevaluate this U(x,y) you can evaluateby the earlier methods, so let me see how we can do this, if I choose tosee this, to evaluate this inversion thisis actually now xi variable goesbecause of theintegration what you endup getting functions of X, Y this is thesolution, this is a function of X, Y thisis one solution, you can also simplify byevaluating this integral by making useof the




application that to evaluatecertain integrals you can also do ithere, if you do that you may end up, wecan get this proper solution may be thatI will see in the next video.
So we willend up, so this is one solution without,this is one form of the solution that wehave derived for the steady state heatequation of a semi-infinite plate, thatmeans upper half plane and you haveone form of solution, if you simplify bymaking use of application of the Fouriertransform, so that is a Parseval identity we can rewrite thissolution in a nice form, that we willsee later, along with that we will alsodo how to solve the heat equation in thenext video.Thank you very much.[Music]

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