NPTEL
NPTEL ONLINE COURSE
Transform Techniques for Engineers
Evaluation of Integrals by Fourier
Transforms
With
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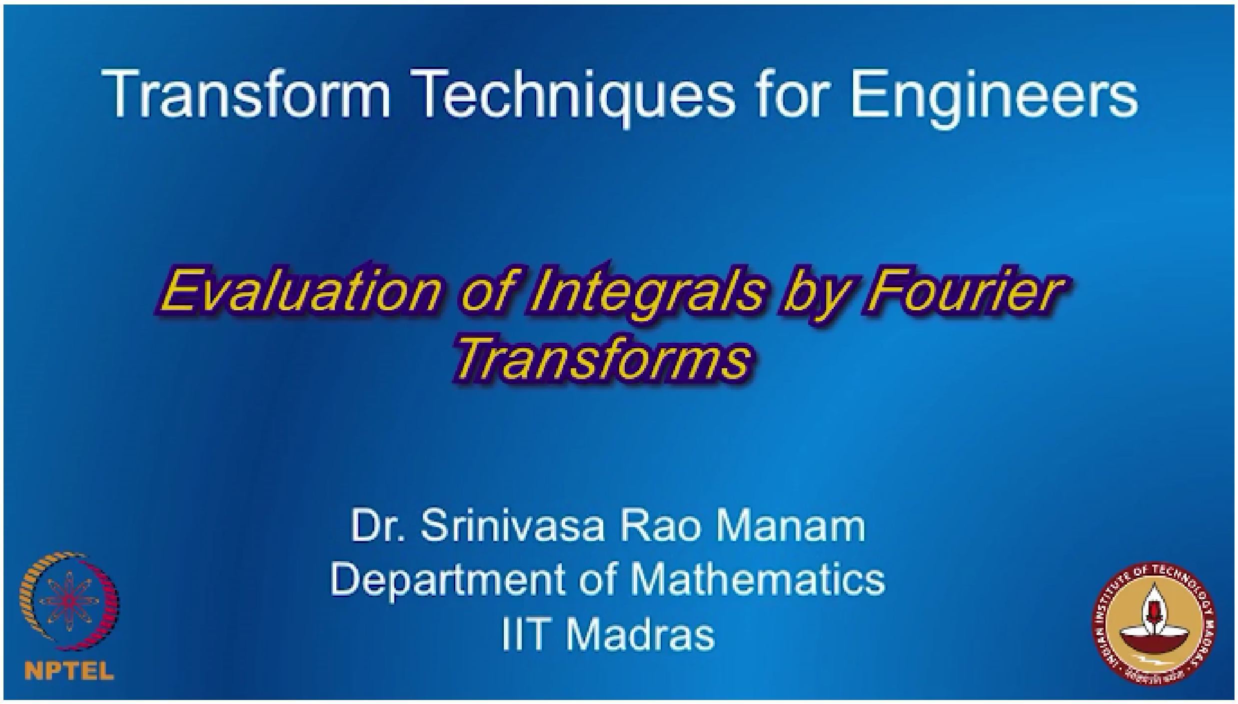
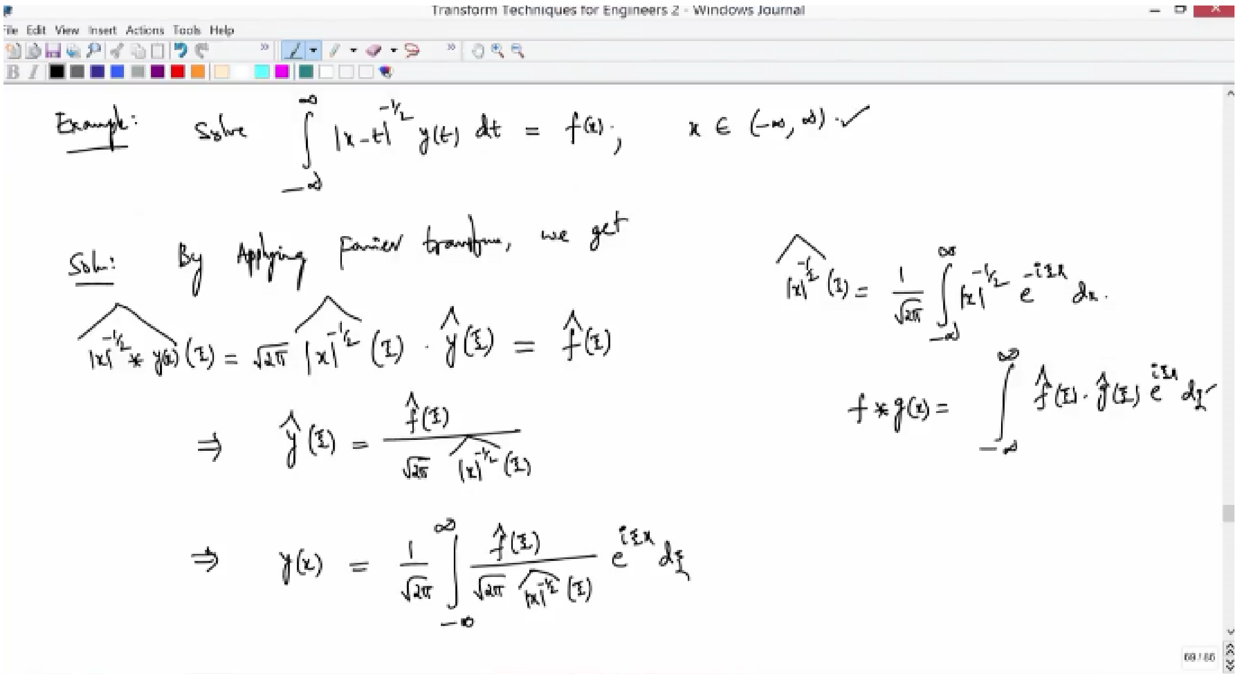


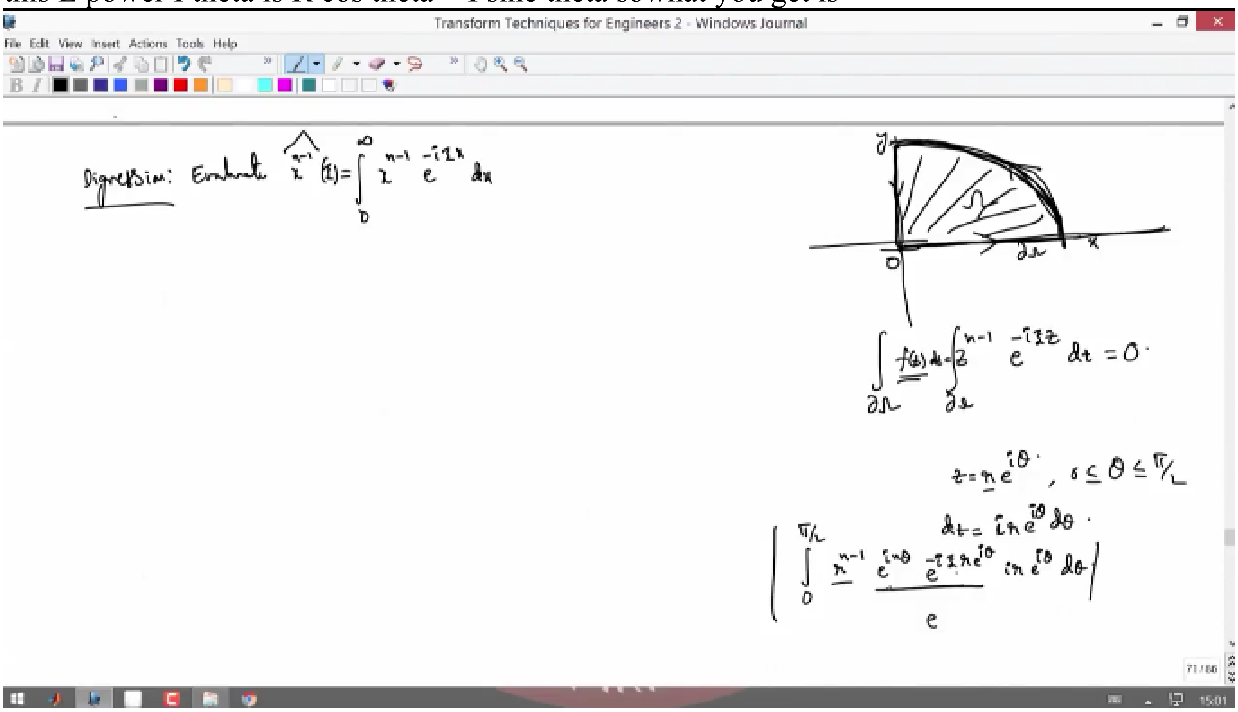
Welcome back, in the last video we haveseen how to solve some special linearintegral equations using Fouriertransform, we have done some examplehow to find the solution of linearintegral equation, and we did there, wehave not finished the problems, so we’ll justlook into that and relevant some moreproblems on that, and we'll see, you alsohave seen how to find, so youcan see that, we have seen thatcosine Fourier transform and sineFourier transform and usual full Fouriertransform or some kind of integralequations if the right-hand side isgiven so inversion will give thesolution, okay, that's what we have seen, we will see these kind of examplesconcerning these integral equations.



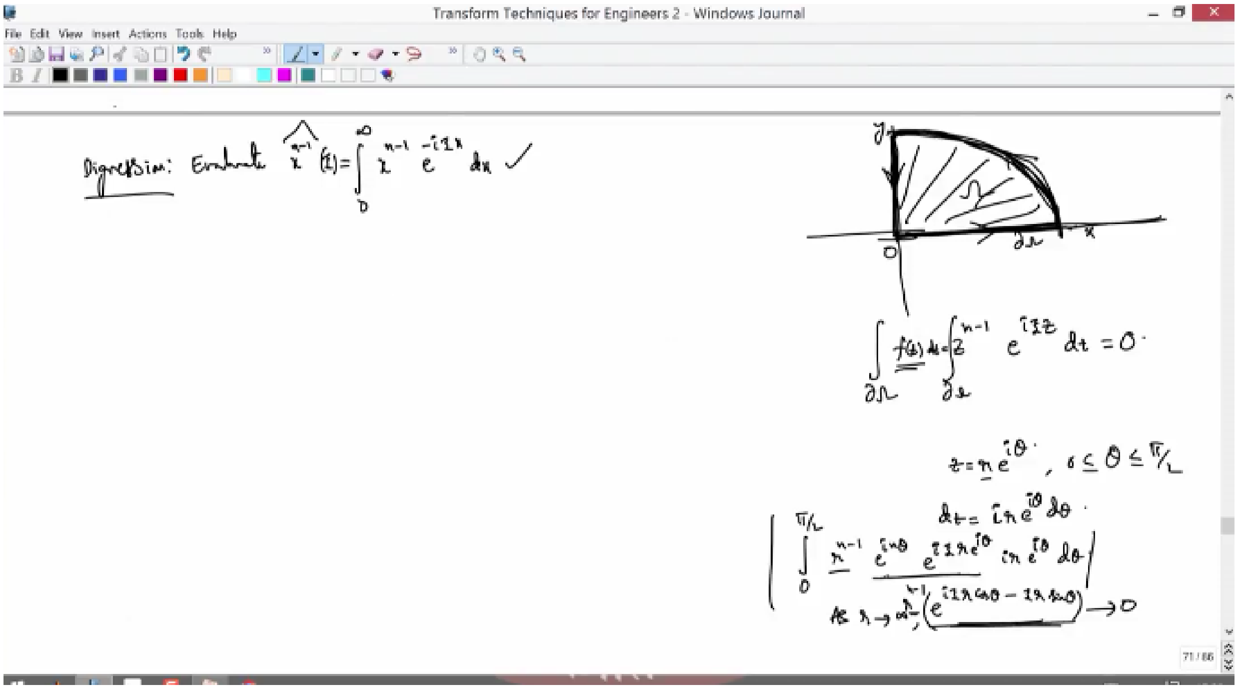
Soto start with the example that we havestarted yesterday so this is theequation to solve, and then by applying,we applied the Fourier transform bothsides so given integral, this integralequation, the integral term is aconvolution product of mod X power -1/2 with Y, X for which if you take theFourier transform this is by theproperty of the Fourier transform.aproduct of these two, so once you havethis you can get your Y cap, that is aFourier transform of 1 on function Y, andwhich you see that this is this, F is givenfunction, so F cap is the Fouriertransform of F, and if you do theinversion you will get this.

So in orderto find, evaluate this integral we needto calculate thisterm, wewill see how we do this, we need to calculate this Fourier transformof mod X power minus off, so to dothis we will just do the general, someother Fourier transform of some certainfunction and using the gammafunction we will be able to find thisFourier transform of this, okay, solet's do that, and then come back to this and we'll be substitute and try to get the inversion, so what I do is I try tofind, so this little digression todo this, to find that thing we need toevaluate sine, so we'll consider, soevaluate this integral if youdo this integral if you can evaluate Xpower N-1 E power -I xi X DX that is Fourier transform of Xpower N-1, okay, evaluate Fouriertransform of X power N-1 for thecap which is a function of xi, so thishow do we do this?What I do is this is,you can see that this is actually thisis X, X is from 0 to infinity, so this isthe integral over with this line, this isfrom 0 to infinity, and if you considerthis as X-axis and Y-axis and youthink of this quantity or youconsider Fourier integral,sorry you consider this counterintegration we will try to evaluate overthis closed interval withinfor the integral F(z) is Zpower N-1 times E power -I xi Z.

If you want to evaluate this DZ, because you are in the complex plane,this is your dou omega,so this isyour omega inside, in that omega withinthis domain your function is analyticwhich is differentiable function becauseit's exponential and a polynomial, soyou need to find this dou omegabecause this is analytic including thisline and this circular curve, and thisline so this has to be, this is integralover this dou omega by Cauchy theorem this is actually equal to 0, okay. Then because you see that this, if you calculate this one youparameterize this curve,you parameterize this curve and you parameterize this curve, and if you do the parameterization over thiscurve what happens is that Z is Rtimes E power I theta, R is fixed thetais between 0 to pi/2, 0 to pi/2, soif you do that DZ will be IR E power I theta D theta, okay, so if youevaluate this over this circular arc CRthe integral contribution will beintegral this is from theta will be 0 topi/2, F(z) that is Z power N-1 that is R power N-1 Epower IN theta, okay, and then E power-I xi, R is, xi is anyway real number, soZ is R times E power I theta, DZ is again IR E power I theta, D theta, so this is the integral on thiscontribution, so you can easily see thisquantity is a bounded thing if you lookat this quantity what happens to this, asR goes to infinity, okay, so as R goes toinfinity this is the polynomial this isexponential function Epower this is, ifyou look at this E power I theta is R cos theta + I sine theta sowhat you get is



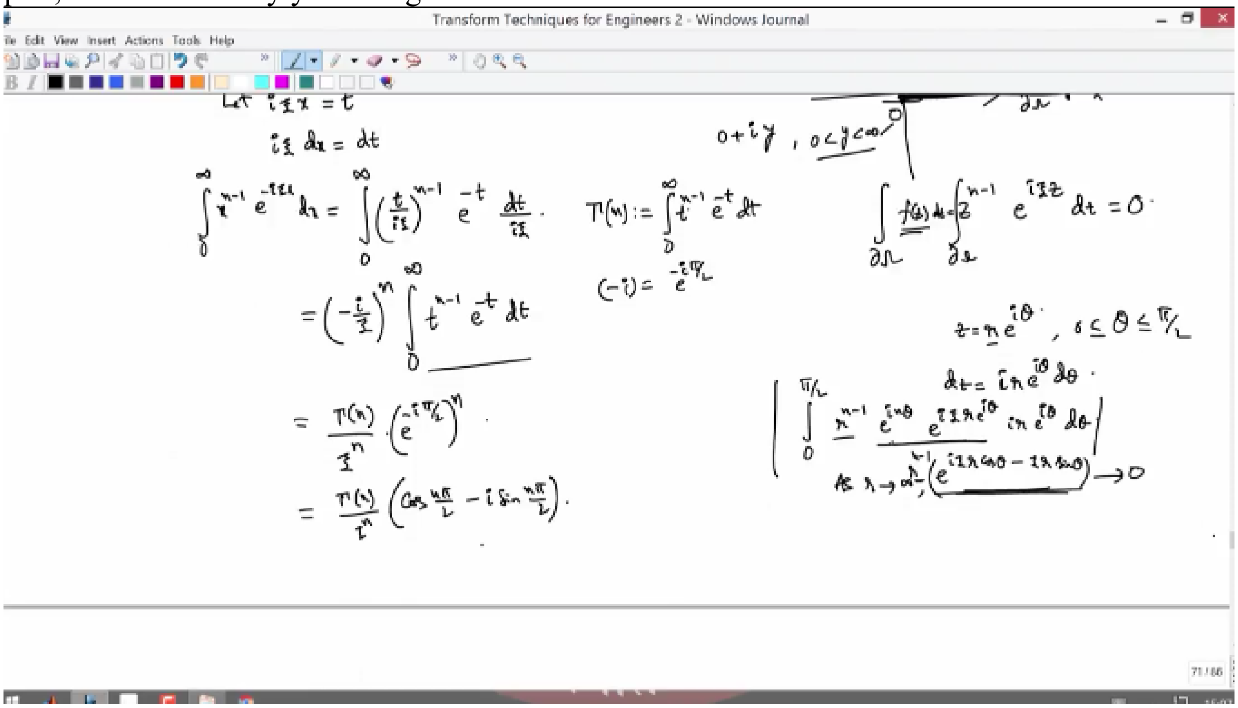
that I and this – Ibecomes, so what you have to do is thisone E power I theta, I theta is this hereyou look at cos theta into E power, youneed to get I square is -1 that isgoing to be plus, so you considerhereis plus so that I get E power -Ixi R cos theta,so you have I here that is plus, and E power I this, and then –xi R sine theta, okay, theta is between 0 to pi/2, soas R goes to infinity this quantity isgoing to 0, even whatever you multiplywith R power N-1 doesn't matterbecause R power, this is a polynomialinto exponential function always goes to0, so that way the contribution overthis is 0, so if you look atthis one, if you look at theparameterization for this that is thisintegral, and if you look at this onewhatever is left that means this isequal to, so its integral overthis to 0 to I infinity, okay, so if youactually redo it so this is, so that ishow



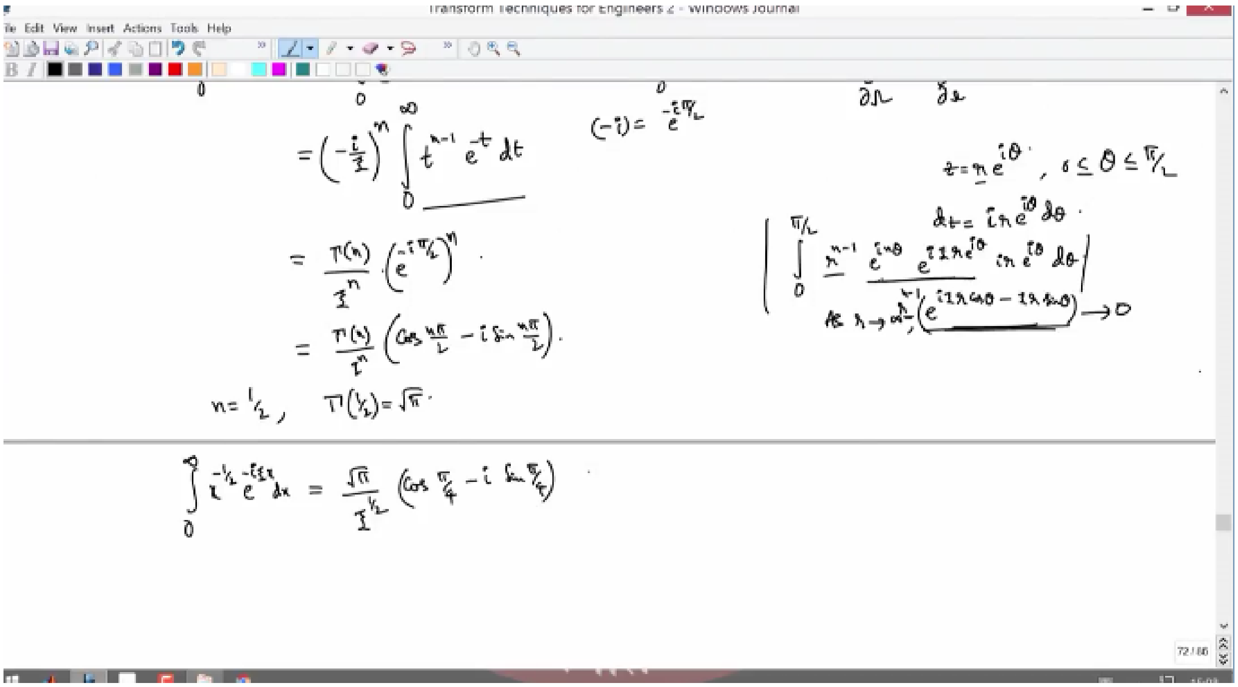
you can see that integralover 0 to infinity is same asintegral over 0 to I infinity or ratherover this line, or over the Y-axis yourintegral value, so that is same as sayingif I consider I xi X as some T, some variable T if we considerlike this and what you get is ID xi DX is DT, so both if you, by complex variable method so both are sameso you will see that if you calculateover this so this integral over thisthat means this integral is same asintegral over this line is exactly whatI am going to write here, so by justsubstituting this variable I xi X with a new variable T if you do this,this integral value if you call this Ithe integral value X power N-1 Epower -I xi X D xi, DX this isequal to, so this is and you substitutethis I xi, so what you get in the place of X, you get T/I xi power N-1, and E power –T, DT is so youneed to put DX that is DT/I xi, okay, and this is 0 to infinity.

So thisis exactly what you get if you directlyput it, so 0 to infinity along Y-axis,okay, so if you do this one so here alsoif you do the parameterization this is Xis 0 + Y is I times Y, Yis between 0to infinity something like this, so youdo the same thing, ifyou do this substitution also you'll getthe same thing, along the Y axis or along this Y axis we actually have towrite I times 0 into I times infinity,in the complex plane I infinity or -I infinity anything is actually only one infinity, so anything, there's nothinglike -infinity or +infinityin a complex plane, you write onlyas infinity, so in that sense or if youdo directly here you will get the samething so you will see that integralalong this Y axis is actually this, oryou simply put it in this way you willsee that but you have to compromisesomewhere that you see when you put X= 0, T is 0 when you put X = infinity, and this is I xi timesinfinity you cannot write, you don’t write thisis I infinity this is in complex plane,I infinity is actually there's one infinity so that's going to be onlyinfinity, so if you do this one this isyour integral.

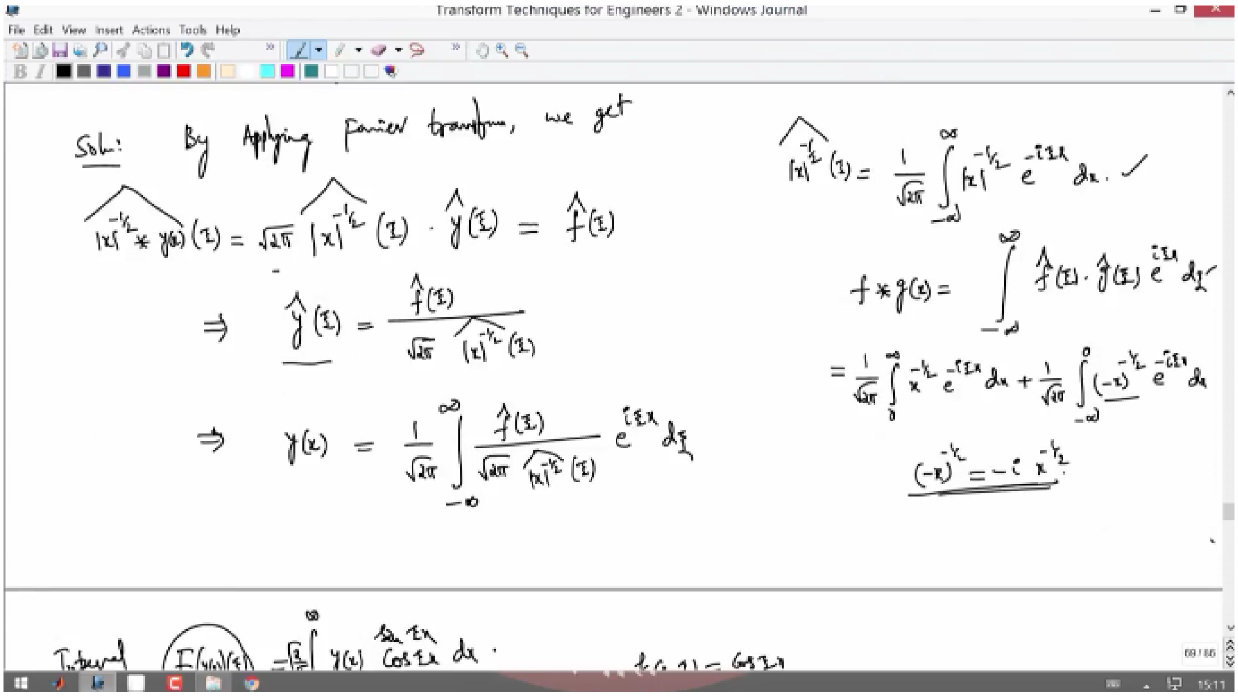
Now I use gamma functionby rewriting, I just take –I, if I bringthis I up –I/xi power N-1 Ihave one more here, so that comesout and 0 to infinity what you have is Tpower N-1 into E power -T DTthis is exactly the definition of gammafunction, gamma function is, gamma ofN is actually definition is 0 toinfinity, T power N-1 into E power-T DT, so if you do that so what Iget is gamma N divided by xi power Ntimes –I, -I I can write like E power I pi/2, I sine pi/2 that is-E power -I pi/2 is –I,so if you use this E power -I pi/2 power N so that is N pi/2, okay, sothis power N into this is a gamma anywayI’ve written, so this is gamma power, gamma of N divided by xi power N, this is E power-INpi/2 that is a cos I canrewrite like cos N pi/2 and then-I sine N pi/2, so this isactually your integral.



So if I put N =1/2 we know that gamma 1/2 isroot pi okay, so root pi, so this isactually if you look at this Fourier transform, Fourier transform of, this isnot Fourier transform this is actually kindof it's not, we're not doing it fromminus infinity to infinity as a Fouriertransform, so I don't write this as a Fourier transform, I’m just simply evaluatingthis integral so then you get this gamma1/2 is this, so if you do this if we putN equal to this what you have is 0 toinfinity X power N-1/2 is X power-1/2 power E -I xi X DXvalue = gamma 1/2 that isroot pi/xi power 1/2 into cos N is 1/2, so pi/4 -I sine pi/4.

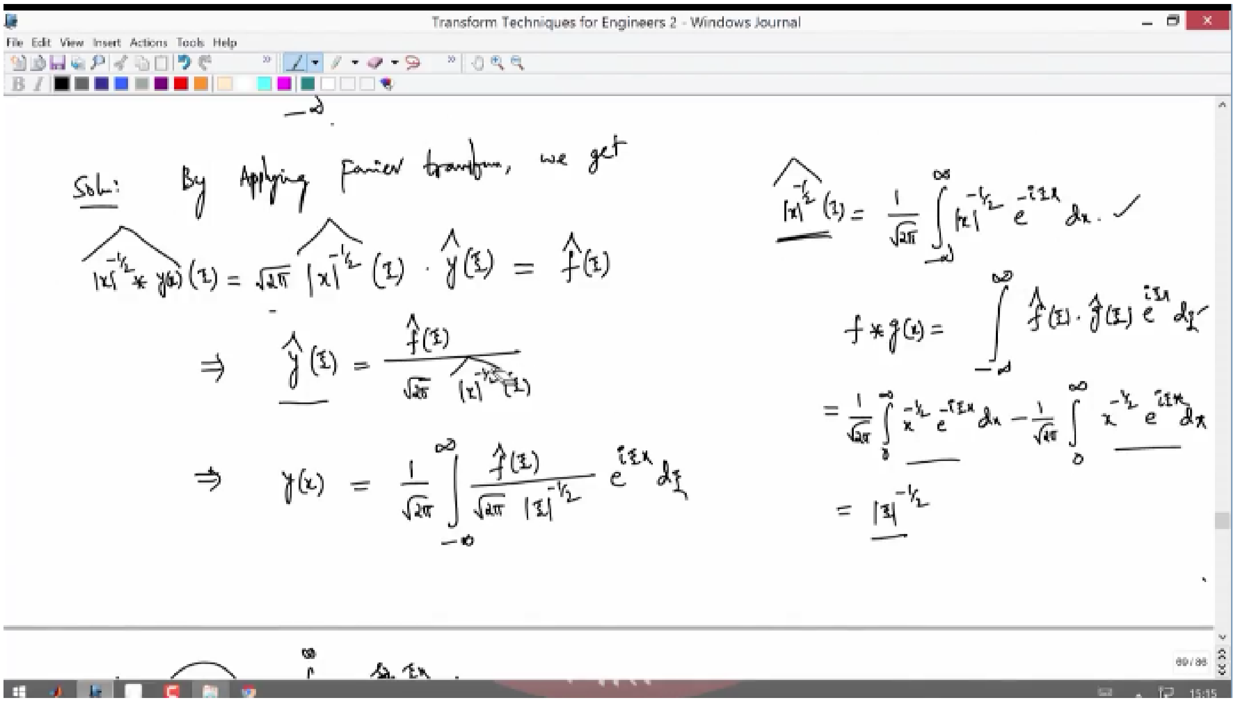


So if you compare thecoefficients real part and imaginarypart if you equate what you get is, youcan see that it's a E power -I xi X, so this is going to be cos xi X DX is actual equal to this cos pi/2sine pi/2 is 1/root 2, 1/root 2, okay,so what you have is root pi/2 times xipower -1/2, okay, xi is positive, soyou have to choose that xi is positive, soyou have to evaluate that xi is positive,so as though you evaluated this way,this integral, so here you have xiispositive, and similarly if you compare theimaginary parts X power -1/2 sinexi X DX is equal to, it's also same xi power -1/2 xi is positive here again, sothis is what you get this integral, sousing this integral, so we'll just goback and try to write this how do youfind this Fourier transform of this, yourewrite this as 1/root 2 pi, 1 is from0 to infinity and when X is positivethis is going to be X power minus, this modX is X, X power -1/2 E power -I xi X DX this is what exactly, thisintegral we evaluated just now, and theother integral is 1/root 2 piintegral - infinity to 0, mod X whenyou have mod X, when X is negative this is-X power -1/2 and you have E power – I xi X DX, somod X -1 power -1/2 that is 1 over -1 under rootthis is exactly 1/I, so you have 1/I times 1/X power 1/2 or X power-1/2, so you have -I times of this isexactly this one, so -Xpower -1/2 is and actually this one,okay, so this if you substitute so yousee that is going to be -I and thisI can replace this with this simply Xpower -1/2 these are the twointegrals which you know now.

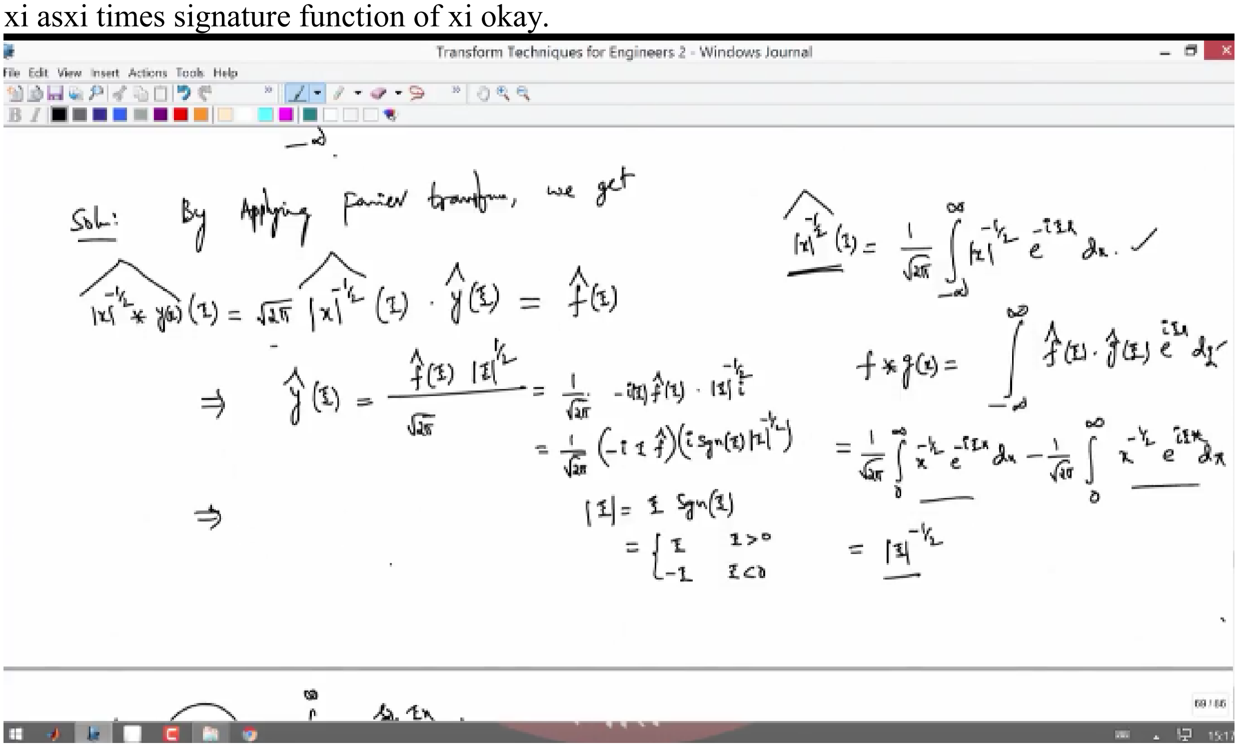


So if younow use again if you try to use X = -T if you put, okay, if you evaluateX = -T DX is minus minusplus and this is going to be infinity to0, and you have -X power –1/2 1 by so you get -T power -1/2 that comes again as -I times root T,T power -1/2,so if you do that - I comes againoutside that is going to be I squarethat is plus okay and this is going tobe, so T power -1/2 and you have Epower +I xi T and this is DT, okay soagain when you replace -0to infinity this is going to be minus, sothis is how if you replace a dummyvariable T with X and this is alsowith X, this is also with X, this isexactly what you have, this Fourier transform this is this now if you usethese two integrals we have just now evaluated and put it herewhat we see is you can get whatexactly the value of this Fouriertransform. Fourier transform of X power modX power -1/2 will be,what you will see is it's going to be1 by,see if you eventually if you calculate put in C, you will see thatit's going to be 1/xi, or xi power -1/2 as your Fourier transform, we can justsubstitute and see that this is what youget.

So anyway, so when you do this if youactually see this substitute and seewhat you end up is you will see that isgoing to be mod xi power -1/2, we'll justcalculate take it as an exercise and youwill see that this if you substitutethese two integrals you evaluate it likeI have done in those two integralsyou try to calculate like here how tocalculate this integral evaluatedthis integral, now we evaluated +I xi you see that what exactly you get andyou substitute into these two integralshere and you end up getting this, so Fourier transform of X power -1/2 xi will be

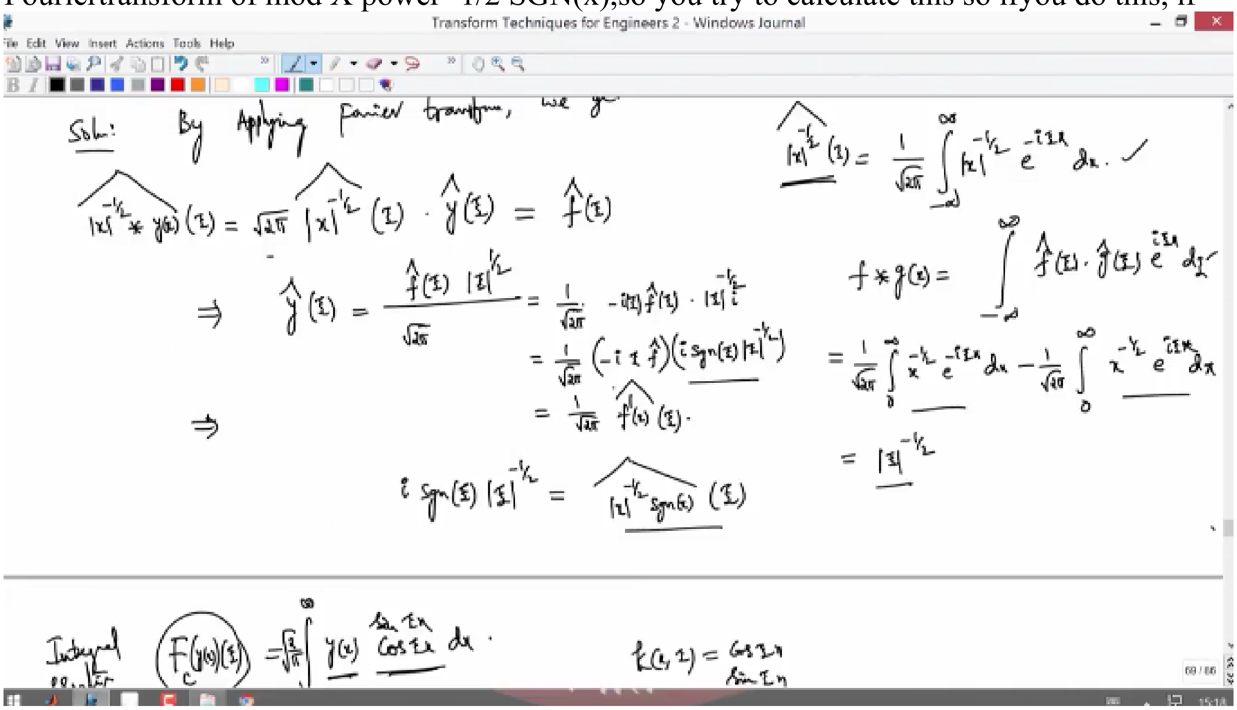


mod xi power -1/2, so what Ido is this one you simply this is, soinstead of writing here I write here so that you have mod xi power 1/2 oneside you take it up, okay,so here I will try to invert here sothis is going to be, this is actuallyequal to 1/root 2 pi times, what Ido is -I square I use –I xi Fcap(xi) multiplied with I already have xi here so I need to havexi, okay, here you have a mod xi, ifyou multiply mod xi here I havealready, I need to have mod xi power 1/2 so you have mod xi power -1/2 times,-I to compensate with thisyou need to multiply I so that this isexactly same as that, so this is equal to1/root 2 pi -I mod xi times Fcap of, okay, so here I split this mod xi asxi times signature function of xi okay.

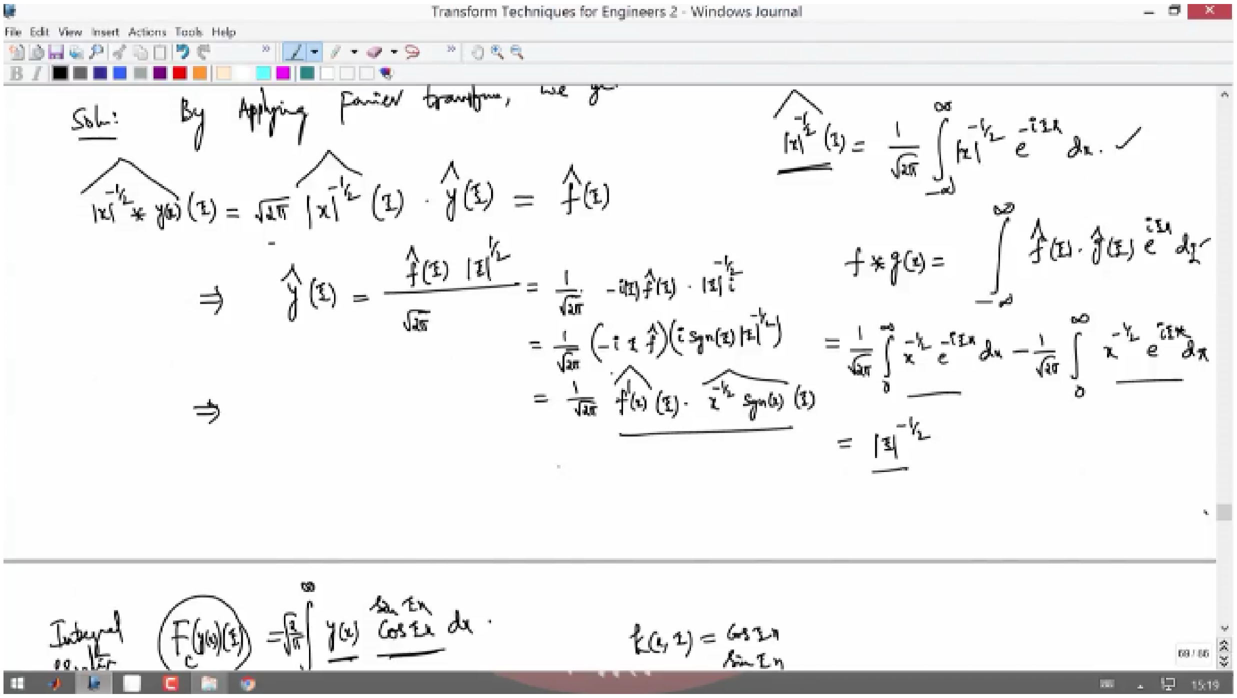


What is signature function of xi? When xi is positive this is going to be 1 soyou have xi, and xi is negative this isgoing to be -1 so you have –xi, this is exactly mod xi function,so mod xi is so you also multiply with theSGN signature of xi that I include here, SGN(xi) I times of this mod xi times-1/2, so these are the two functionsI use, so if I now use this,if I now use this one so I will removethis, so how do I integrate, so if I thisis a product of two Fourier transform of2 such functions this I know, thisis the Fourier transform of what you getroot 2 pi times, what is the Fouriertransform of, Fourier transform of F cap(x) let us say, what is this one?This isactually -I xi F cap(xi), soin the place of this, in the placeof this I put this one so Fouriertransform of 1/root 2 pi, and this I can write F dash(x)Fourier transform, okay, which is a function ofxi times.

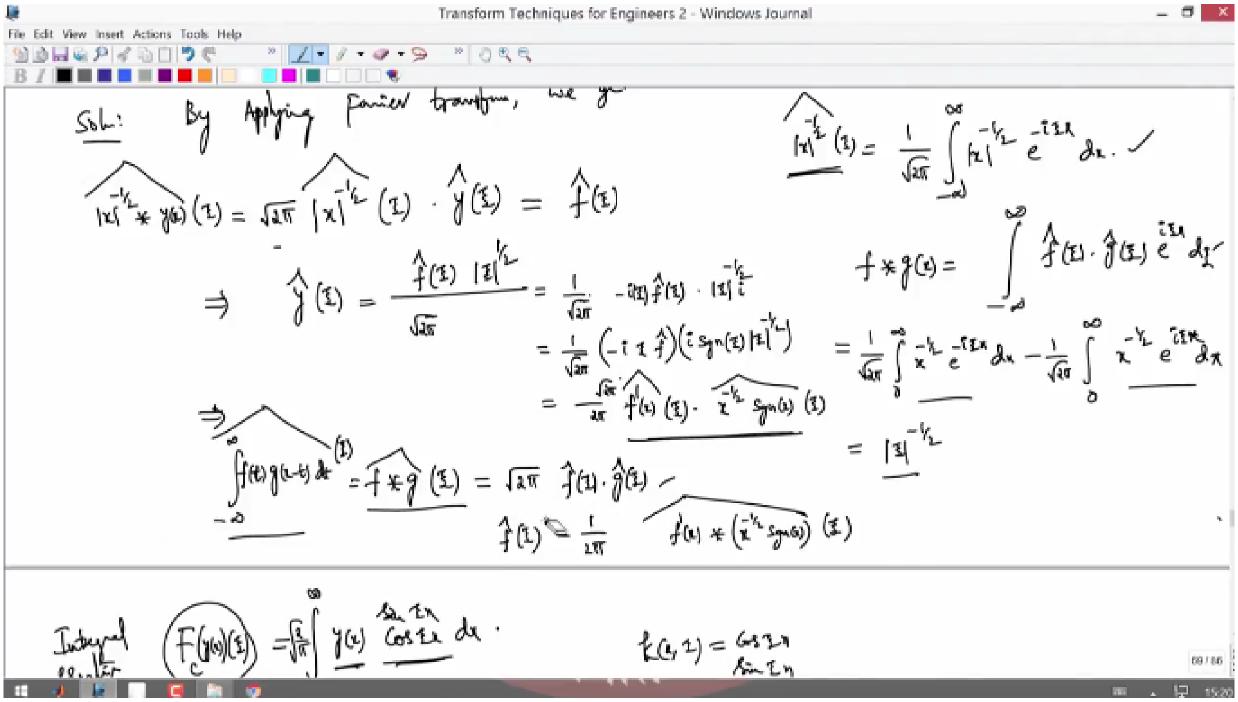
Again one more what about this? Isine xi, so if you use, make use of thissimilar thing or the make use of thisintegral that I have evaluated so if youuse, I give as a, give it as anexercise I'll directly write here andthis I sign, I signature of xi times mod xi power -1/2 is actually equal to Fouriertransform of mod X power -1/2 SGN(x),so you try to calculate this so ifyou do this, if



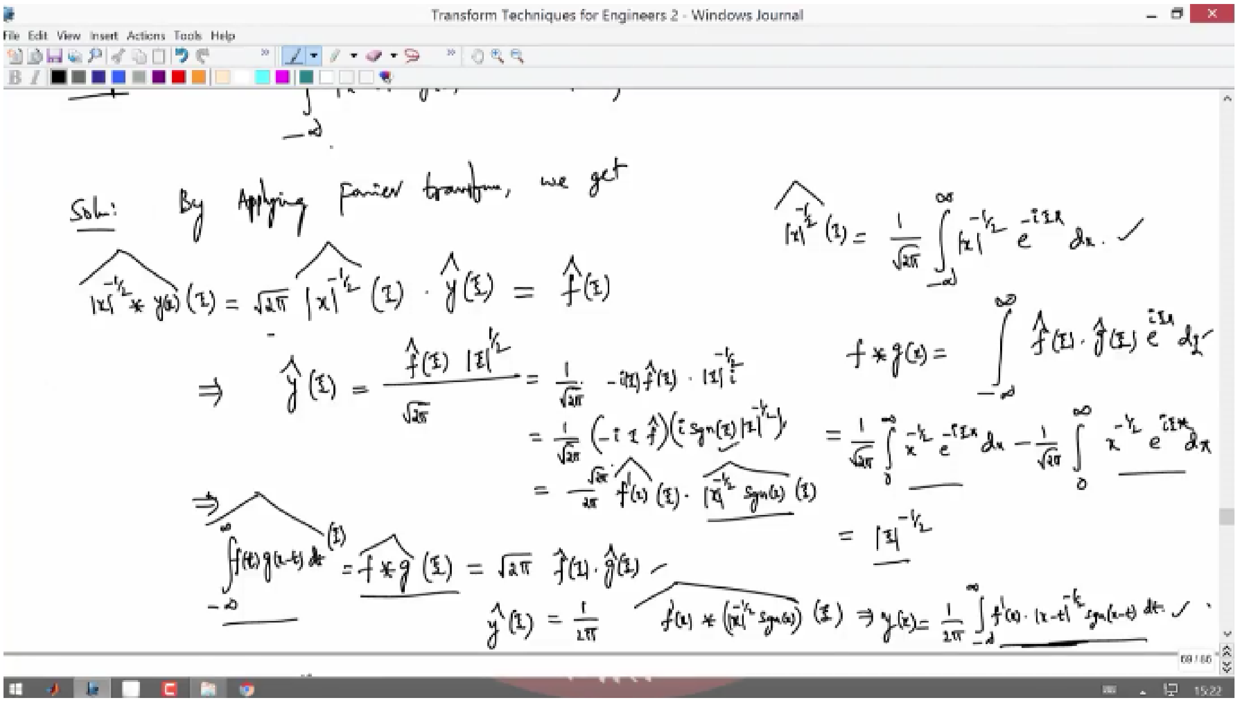
you try to calculate thisFourier transform you will end upgetting this one by making use of theintegral that just evaluated, so if Iwrite like this so this will be X power-1/2 SGN(x)signature onlyF(x) for which you have Fouriertransform of xi, so that is exactlywhat I wrote for both of this, okay.



Sothis is exactly same as this is equal to1/root 2 pi,this is and we have already seen that F composition of G, Fourier transform of xi is actually equal to root 2 pi times, F cap(xi) times, G cap(xi) the way what my convolution iswithout any coefficient, my convolutionis simply minus infinity to infinity F(t) G(x-t) DT, this forwhich if you take the Fourier transformwhich is a function of xi, and this isexactly what you get, so this isexactly what you get.
So in the place ofthis you need to multiply with 2 pi soyou get 1/2 pi, so that youhave root 2 pitimes, so if I write root 2 pi and you have root 2 pihere, then this is same as 1/root 2 pi,so root 2 pi times this I can replacewith Fourier integral so that is, so 1/2 pi times Fourierintegral of, Fouriertransform of this convolution that is Fcap(x) convolved with X power -1/2 SGN(x) this function, okay, thisis the convolution of this is my F cap(xi).



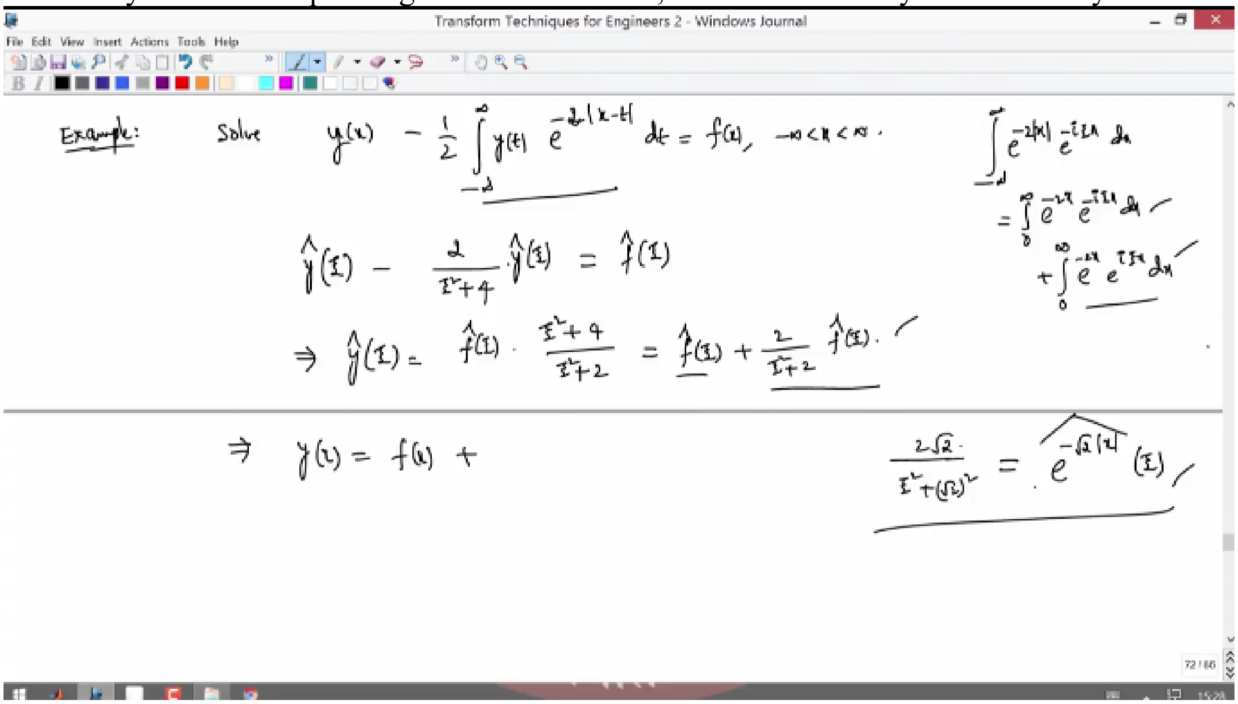
Now you take the inversetransform then what you get is F(x) whichis equal to 1/2 pi times this integral minus infinity to infinity Fdash(x) times, F is given function inthis integral equation, right handside F dash, so it's assume that F isdifferentiable so you have F dash(x)convolving with mod X power, so it isactually mod X, okay, so where is this one?I think I have written, so this shouldbe, this is actually, this isactually Fourier transform of mod Xpower minus of signature of X,there's a mistake,so you write like and you have X-Tpower -1/2 SGN(x-t) DT that is exactly your solution,sorry this is here Fourier transform, this is your Y cap, so this is your Y(x),Y(x) is this this is your solution of theintegral equation, so the rudiment ofthis,you know the important quantity that youhave to evaluate is this integral, sothis integral is we just made use ofthis gamma function and we couldevaluate and put N = 1/2 you seethat this is the quantity for minus,+ I xi X you have a similar thing, sosimilar thing so you make use ofthese two, these integrals and you canevaluate this, you can evaluate this Fourier transform of this by writingseparately, in the same way you can seethat Fourier transform of this is same asFourier transform of, same as this one,okay, Fourier transform of this is same asthis second term.



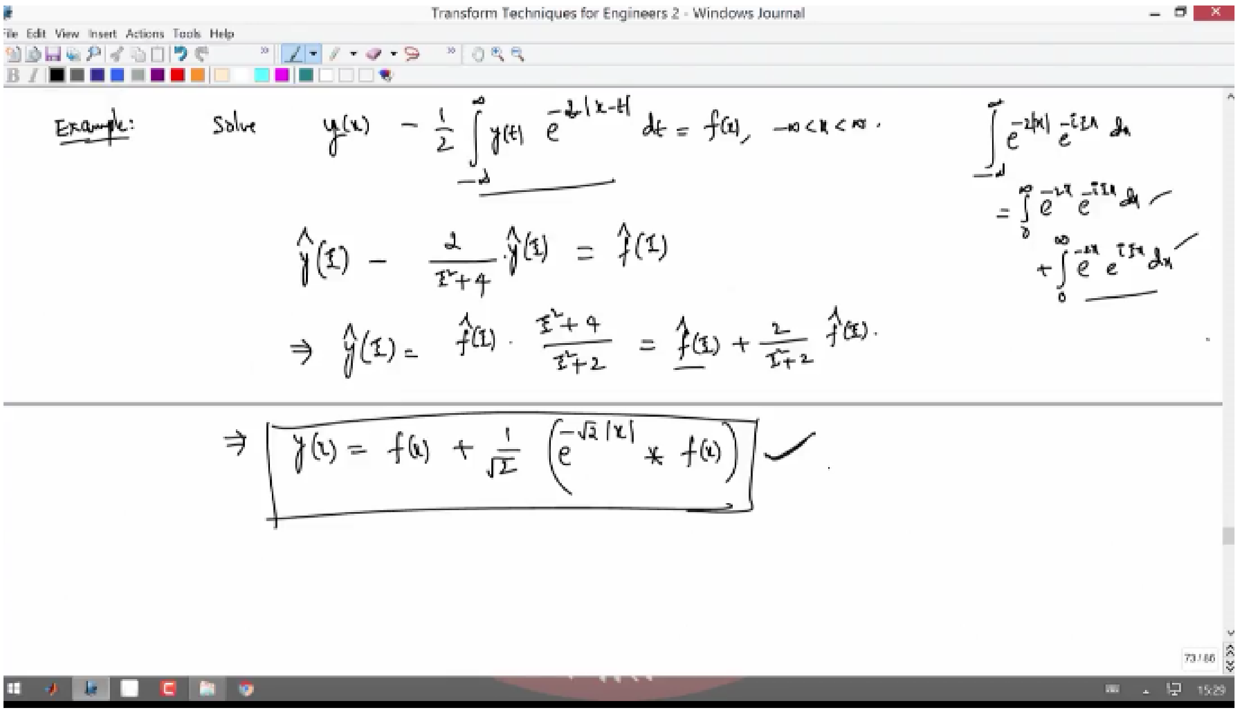
So this things youtake it in exercise and this is how you solve, you evaluate that integral thisis a little tough but you try tounderstand and do the calculations youwill see that this is going to be thisone, this is your solution of theintegral equation.

So I’ll leave it herewe'll try to do some other problem whichis a little easier to do, so let me dofor the sake of solving some simplerproblem, let's solve one more example,solve some U(x) times or let me use Y(x)-1/2 minus infinity to infinity Y(t) this integral equation if you solve,you want to solve this DT = F(x), Fis anything and you have X is betweenminus infinityto infinity, so again the procedure is same you apply the domain is full domain, you canapply the Fourier transforms so that youhave first term is Y cap(xi) and this one, this is actually minus Fouriertransform of Y cap, so this is aconvolution product of E power -2Tconvolving with Y, okay this is exactly your function, sothis is a convolution function,convolution of these two functions sowhen you take the Fourier transform forthis with 1/2, okay,1/2 times if youtake the Fourier transform of this, thisis actually this on which you take theFourier transform what you see is it'sgoing to be minus, this is going to be 4 so A square,this -2 is A, so Asquare divided by, so what you areevaluating is this is going to be 1 over, rather 4 divided by xi square+ 4, xi square + 2 square, okay, sowhat exactly is a Fourier transform of 2power 2X E power -I xi X integralminus infinity to infinity DX, this isgoing to be,and now this is actually mod T, this is mod T, so E power -2 mod T, so if you do this so you have tocalculate for this, you have to calculate this one so you write 0 toinfinity E power -2X,E power –I xi X DX that is 1 + -infinityto 0 E power -2X, when X isnegative this is going to be +2XE power -I xi X DX, so once yousubstitute this with -X with X isgoing to be 0 to infinity and you haveminus and this is going to be plus andthis is exactly, these are the two thingsyou evaluate and try to substitute yousee that, you will see that this is whatis the quantity, so 2 2 goes so you have2 so you will see that this is what youget, so this is into your Y cap(xi), Fourier transform of this will bethis, and Fourier transform(y) is this, okay, so this is equal to F cap(xi).

So this will give me Y cap(xi) as Fcap(xi) times xi square + 4 divided by,xi square + 4 - 2 that is xi square + 2, so this is same as F cap(xi) you write xi square + 4 as xi square + 2 + 2, so then one cancancel the first term and you have F cap(xi)+ 2 divided by xi square + 2 andyou have F cap(xi), now you take theinverse transform for this both sides, you'll get Y(x) as if you take Fourier inverse transform that is F(x) and thisone and you have again, so this is likeyou can write like 1/root 2 dividedby, okay, so you write like xi square + root 2 square divided by 2, 2 root 2, if youactually see you will see that it'sgoing to be 2 root 2, so I'm notevaluating this, so you'll try toevaluate Fourier transform of E power -root 2 mod X,Fourier transform ofthis is actually equal to this one, okay, ifyou directly use the same way ifyou try to calculate it, split into twoparts for mod I mod X - I have 0 to infinity it is X, - infinity to 0 this is -X if you doand calculate you will end up seeingthat this is the case, so if I 2 divided by this is actually



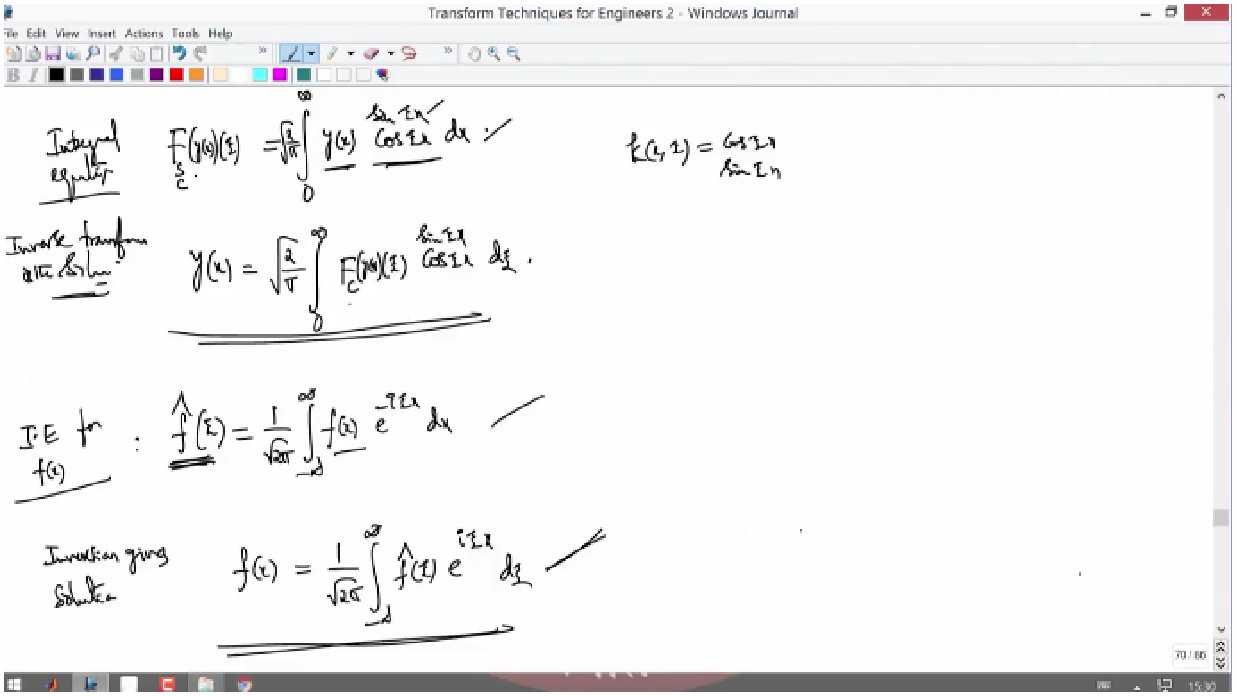
1/½, 1/½, 1/root 2 times Fourier transform ofthis one, what is that?This is actuallyFourier transform of, we have not done this so anyway so you can see you know I'm directly writing, so this is aproduct of two parts, two parts is aproduct you write this as Fouriertransform of, so if you write this is a convolution of, I am writingdirectly, convolution of E power -root 2 mod X convolution with F(x)this function convolution function of Xfor which whose Fourier transform isthis one, so that you can take it as anexercise with 1/root 2 comes out, sothis whole thing if you do that take theFourier transform what you end up isthis product of these two, okay.



So thistake it as an exercise this is how youget a solution of an integral equationjust by applying direct Fourier transformfor each of the terms and then youget this unknown term Fourier transform isgiven with known terms, you take the inversetransform that is what exactly thesolution it'll be, okay, so this is how wecan apply Fourier transformer to find thesolutions of linear, simple linearintegral equations.

We'll also do somemore similar problems on using, you canalso work out some Fourier transformsdirectly, direct Fourier sine transformFourier cosine transform or Fouriertransforms, so I will give as anassignment certain problems so that youjust need to find its inverse, to findthe solution of this transforms you justidentify the linear equation as Fouriersine transform with right hand side isgiven, so if you try to invert it that isthe solution of this integral equation which is Fourier sine transformor cosine transform.

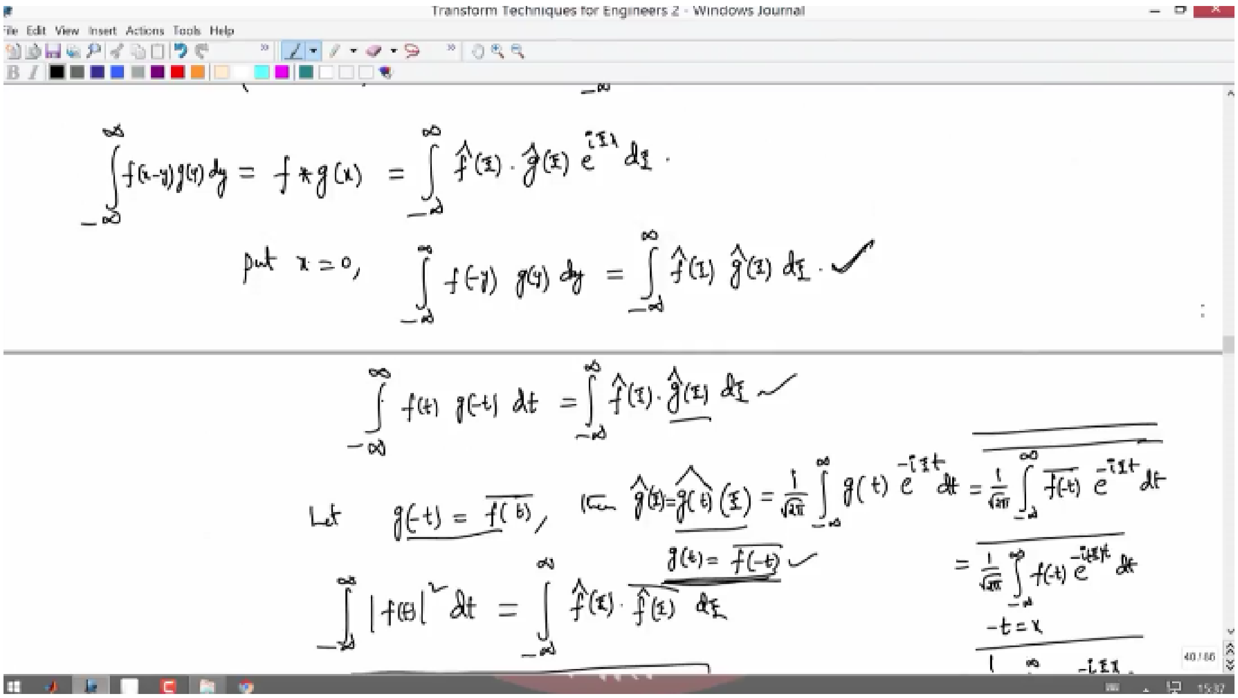
So just for thesake of repeating and let me, which Iexplained in the last video this isexactly, seeif this is your sinetransform of Fourier cosinetransform if it with sine this isgoing to be F(S), okay, either C or S okay, Cor S cosine or sine, or sine forthis, cosine is for the below, so



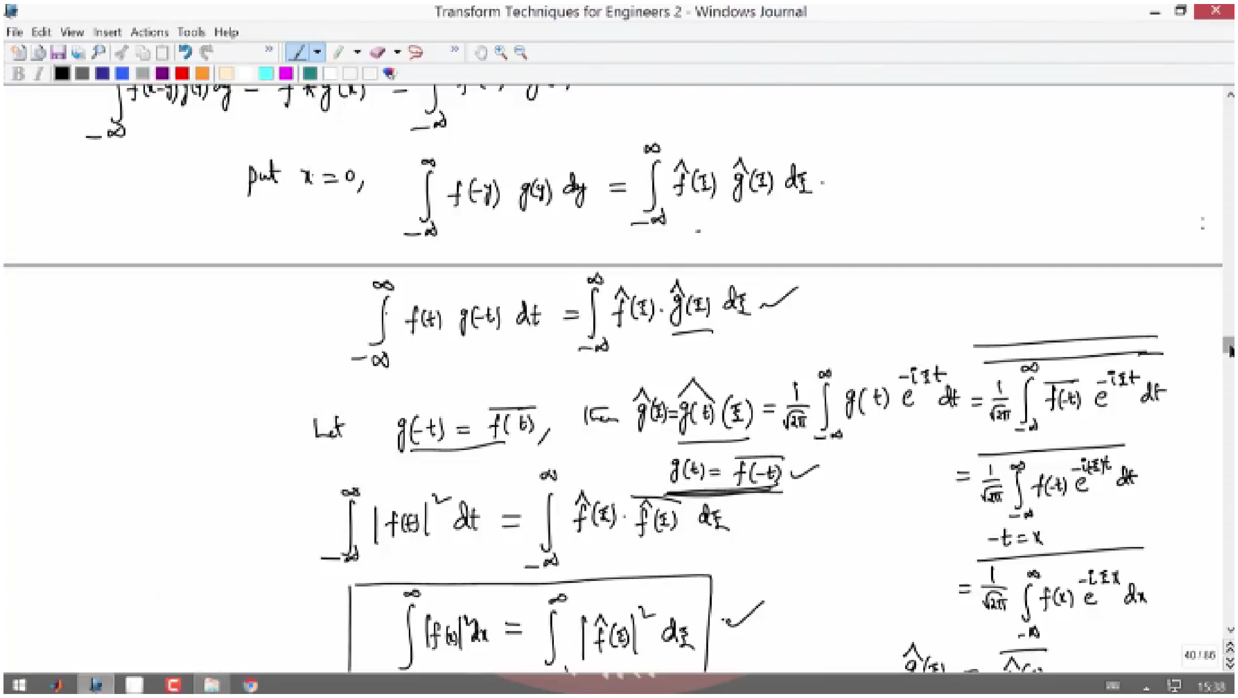
Fourier sine or cosine is this, if thisfunction is given you want to, if unknown is Y(x) inverse transform will give youPhi C or SC okay, this is whatyou get Fourier sine,Sis correspondsto sine, C is corresponds to cos, so anyoneof them, C if you put it so you put itin the bracket, so that if you choose Cyou have to choose cosine. So directlythis is just we know that it's inversetransform of this, this one, given thisone you can calculate this, okay, you can justby inverse transform.

Similarly you canuse the full Fourier transform and it'sinverse transform is, will give you thesolution of this integral equation, ifthis right-hand side means this is given, F cap is given and this isan integral equation, simple linearintegral equation of course first kindbecause this unknown is not outside, okay,so this is exactly the linear equationwhich you can solve just by inversion, sothese things I give as the directproblems in assignment you can do later.

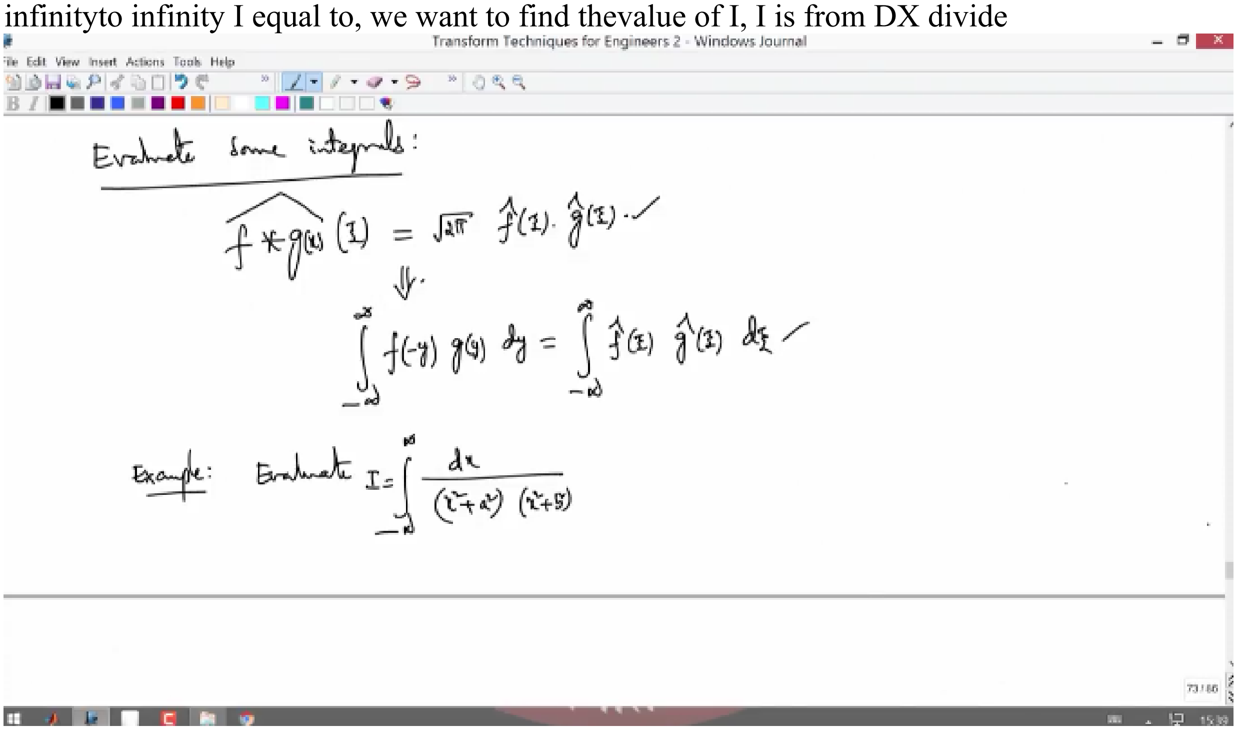
So what are the other applications youcan do?You can also evaluate certainintegrals, certain integrals you canevaluate if I'll just do one example inthis video, I realize that this is not the Fourier transformof this derivative, derivative isactually with our definition this isactually I xi, so that to account for minus you have to write it here, so we put ithere, this minus you put it here so that youcan calculate that minus you can always takeit out and you can write this as Y capis -1/pi and this one, this isexactly what you have, so this issmall correction here so earlier I put -I xi F cap whose Fourier, that is Fourier transform of F dash, now it is a Fouriertransform of F dash is I xi times Fcap, earlier I write – I xi so that isthe only thing, so you will see similarthings such thing okay so you have a minus something like with minus, and you have thissignature thing so you just have toverify whether this is Fourier transformof this, is actually this or not, okay,without this negative sign, sonegative sign you can put it here sowhether this is the Fourier transform ofthis I have to see, okay, so that's onlythen start, okay, so that's the only smallamendment we can make, so we can alsoevaluate, we use a convolution product, wecan use some another application isevolution can evaluate certainintegrals, some integrals we canevaluate just by using the property ofthe Fourier transform that is Fourier transform of this convolution product,this is actually equal to which is aFourier transform of convolution productis a function of X which is finallyafter taking the Fourier transform thisis function of xi is actually equal toroot 2 pi times F cap(xi) times G cap(xi), okay, so we can evaluate by makinguse of this, and this one and wealso used Parseval’s inequality, okay, sowhat we have is, so we just look backwhat is the property that wehave used in the Parseval’s identity youhave this one,Fourier inversion willgive me this one, so we'll go so what you end up is this one, so Parseval’s identity is this,F cap so without this, soyou have, we try to use this one, we try to use this.



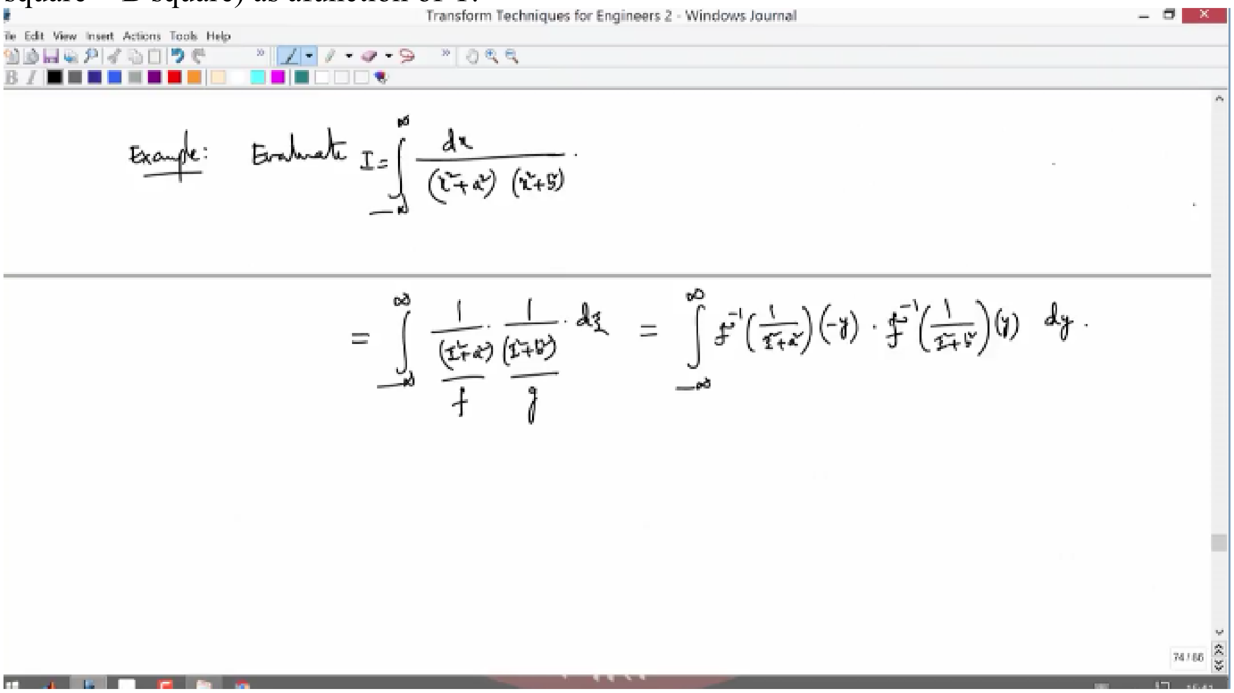
So we will basicallyuse this one, this equality this isfrom Parseval’s identity that property10 if we use, if you want to evaluate, if you knowthat if your integral, integral isinvolving some product of 2 products suppose you have D xi, andeach of this you know what is its Fourier inverse transform, so that you can put ithere and you can evaluate this, supposeafter putting this, after gettingthis inversion if you can evaluate thisintegral this is you can easily evaluatethis integral, and so that finally so this integral will be known,okay, so with this idea we'll try toevaluate.



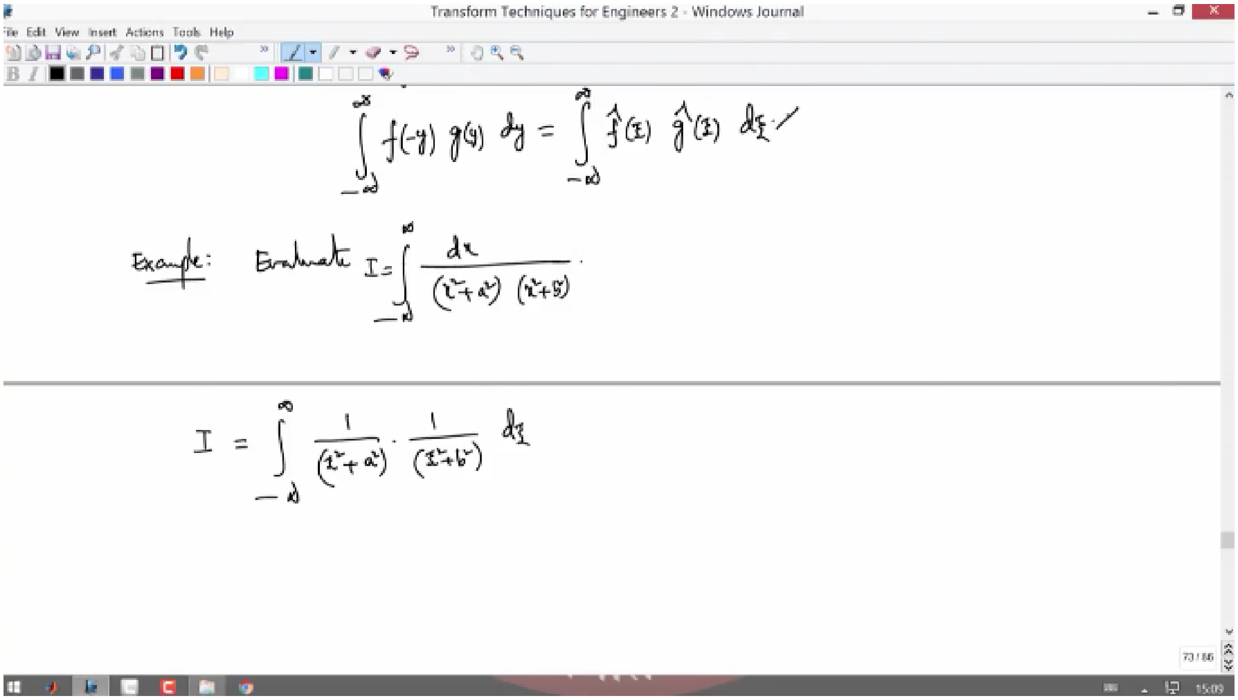
So we'll use F(-y) G(y) minus infinity to infinity, so let me usethis one, so F(-y) G(y) DY,F(-y) G(y) DY is same as minusinfinity to infinity,F cap(xi) into G cap(xi) D xi, so from thiseither this or this both are same so fromthis you got this one, okay as a property,this is one property and making use ofthis we got this property, this is a partof this Parseval’s identity so we make use of this to evaluate someintegrals, for example one example is cando evaluate integral minus infinityto infinity I equal to, we want to find thevalue of I, I is from DX divide



by Xsquare + A square times X square + B square,okay, sothis is equal to you can rewrite likeD xi/xi square + A square, xi square + B square, so as the 2 products you can write like 2 productstypes, one of this D xi, I just changethe dummy variable and then this we knowthat this is inverse transform of thisone,so this is by theearlier this property we can rewritethis is equal to minus infinity to infinity,this is suppose your F, this is your Gcap, this is your F cap and this is yourG cap, then what you have is F capand you have this, this is going to besuppose this is your F and this is yourG, okay, so if this is your F cap what youhave to write is F here so if this isyour F the inverse transform, so we F inverse of F1/xi square + A squaretimes F inverse of which is a functionof, for example here I have to writefunction of -Y times F inverse(1/xi square + B square) as afunction of Y.

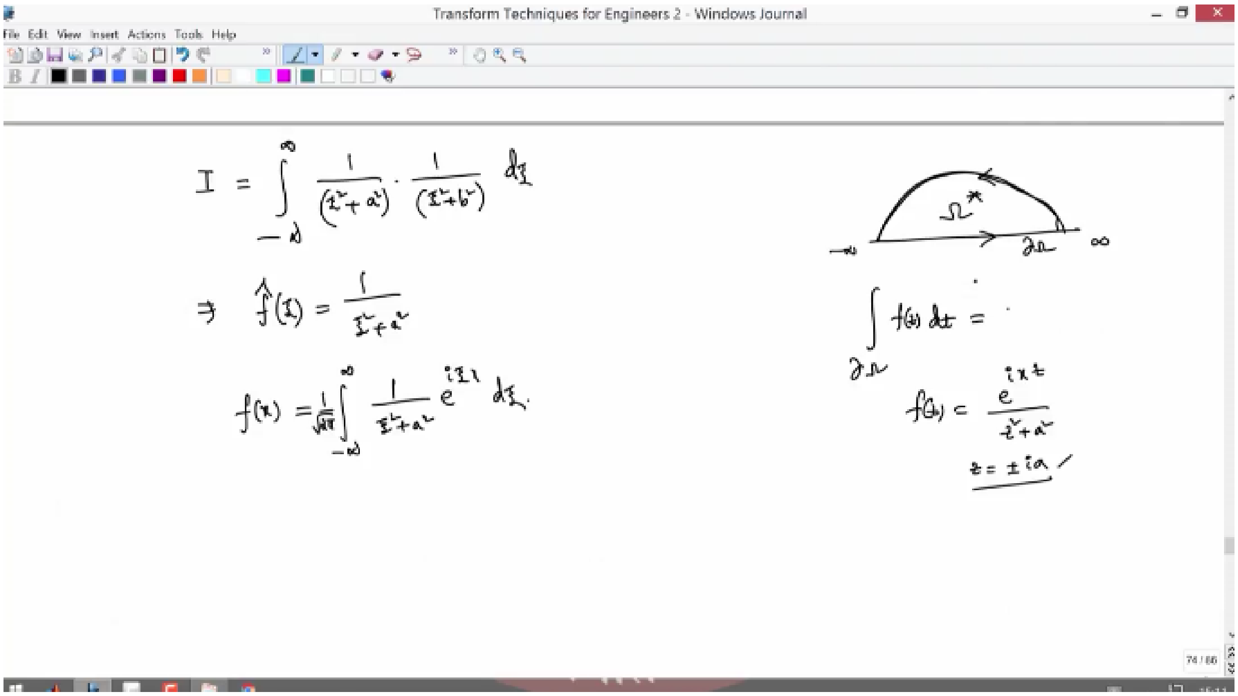


See if this is a G capwhere we are writing G(y) so we areusing this one, so left hand side I’m writing here, so when you get thisinverse transform of F and that as youget a function of Y finally replace Y by -Y and here you do this, so if youuse this you can easily see that this isgoing to be, so you can you know you tryto calculate this Fourier inversionsof both,Fourier inversion of for thiswe already seen if you have seen earlierFourier transform of E power -A mod Xis actually Fourier transform of this ifyou calculate, you try to see that it's going to be onesuch integrals, so you will get 1/xisquare + something into 1/xisquare + X square, so if you have seen earlier I don’t know whether I have done, I’ll just let me see, so we have this integral Iwhich is minus infinity to infinity, so youcan split this as sum xi square + A square into 1/xi square+ B square into D xi, so I justchanged the dummy variable.

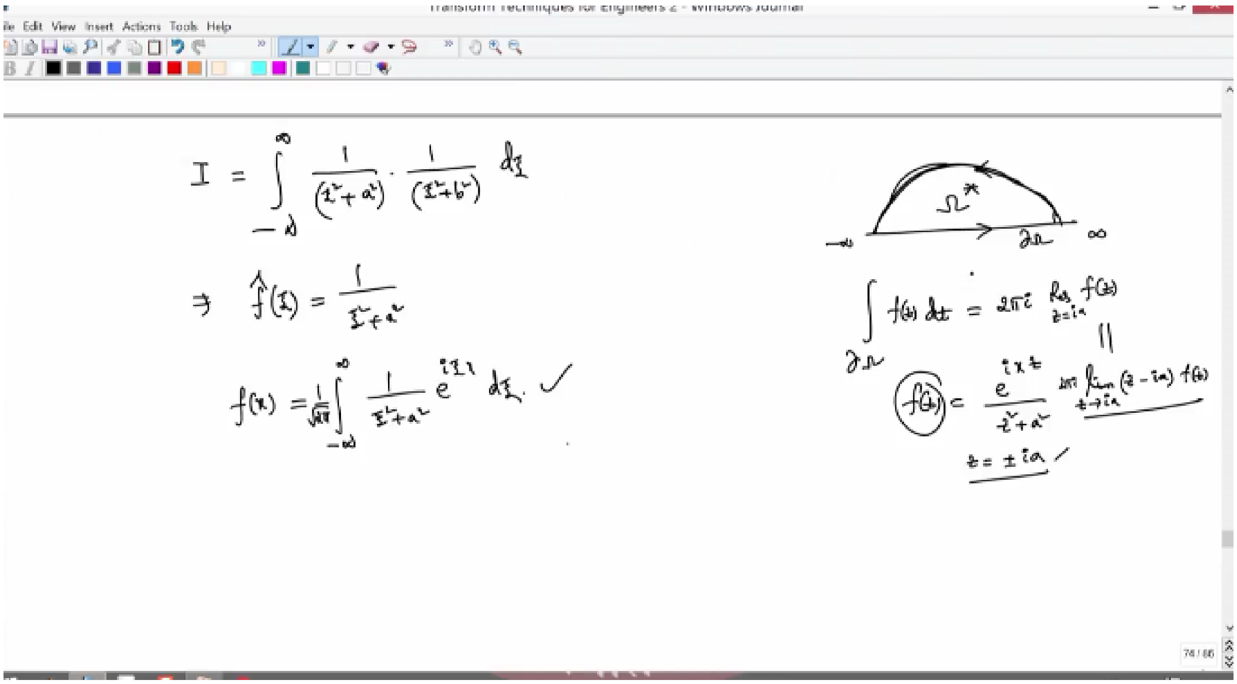


Now we makeuse of this inequality here, we make use of this equality soif you use this so you want to seeidentify this as right hand side so thatimplies what you have is the F cap(xi) you're looking for 1/xi square+ A square, what is it's inversion? So inversion you're looking for, soyou have F(x) will be, so integral 1/root 2 pi minus infinity to infinity D xi, this is xi square + A square into Epower I xi X D xi, so this if youwant to evaluate you can evaluate thisand get your function F(x) that is one way,that is if you want to do this you haveto use contour integration techniquethat is you consider minus infinity to infinity, you use this counterintegration and you have xi square, soyour function consider this function FZis over this, within this domain, integralover this domain omega,so consider, so dou omega this is the contour we haveconsider, and that is your dou omega, ifthis is your omega,so contour is douomega so this is DZ.

So what is yourfunction,F(z)is I consider E powerXZ, X is fixed number,Z dividedbyZ square + A square, so thesingularities of this function if youknow a little bit of complex analysis,complex variables you have, Z is –IA, okay, so those are that is what youget is – Asquare, Z square is–A square and Z is + or –IA, so these are the roots, these are thesingularities, one point will be somewherehereother will be down, so you have asingularity

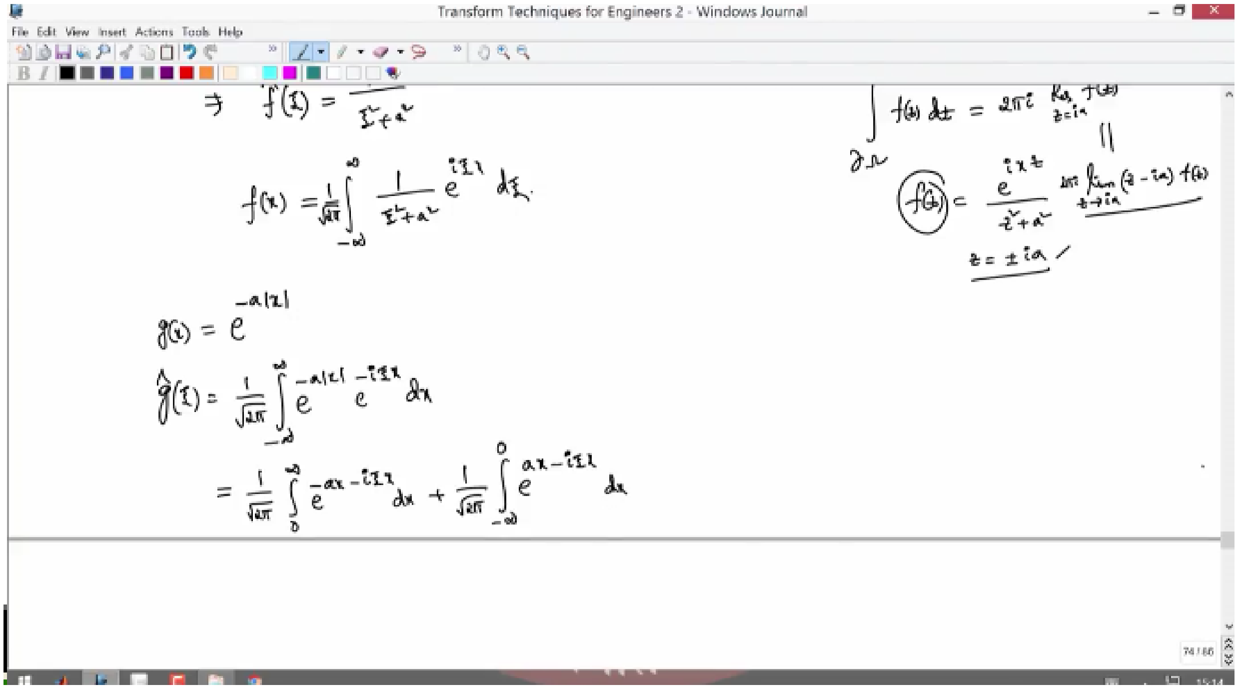


inside there is a theorem thattells you that this is actually equal to2 pi I times, this 2 pi Itimes some residues, residue of this F(z), as Z goesto this IA, okay,at Z = IA, so the meaning of residue is this limit, sobasically if you don't know what isresidue you have to write (Z-IA) times F(z), now you take thelimits Z goes to IA, so that isactually is the residue, so 2 pi I timesis that the whole thing, so this isequal to, the residue means just thislimit and 2 pi I times that, so this isequal to that, so if you calculate thisthat you can easily calculate because weknow what is the F(z) and then youcan calculate this, because you will seethat is, this over which this you canactually prove by some theorem inthe complex variables that over whichthis is 0 because this is exponential,function is you have oscillator functiondivided by some more than order of Z more than Z order, okay, so you caneasily, one can prove that is actually onthis it is 0, so you have this one isactually 2 pi times this, that is oneway of to find evaluate this

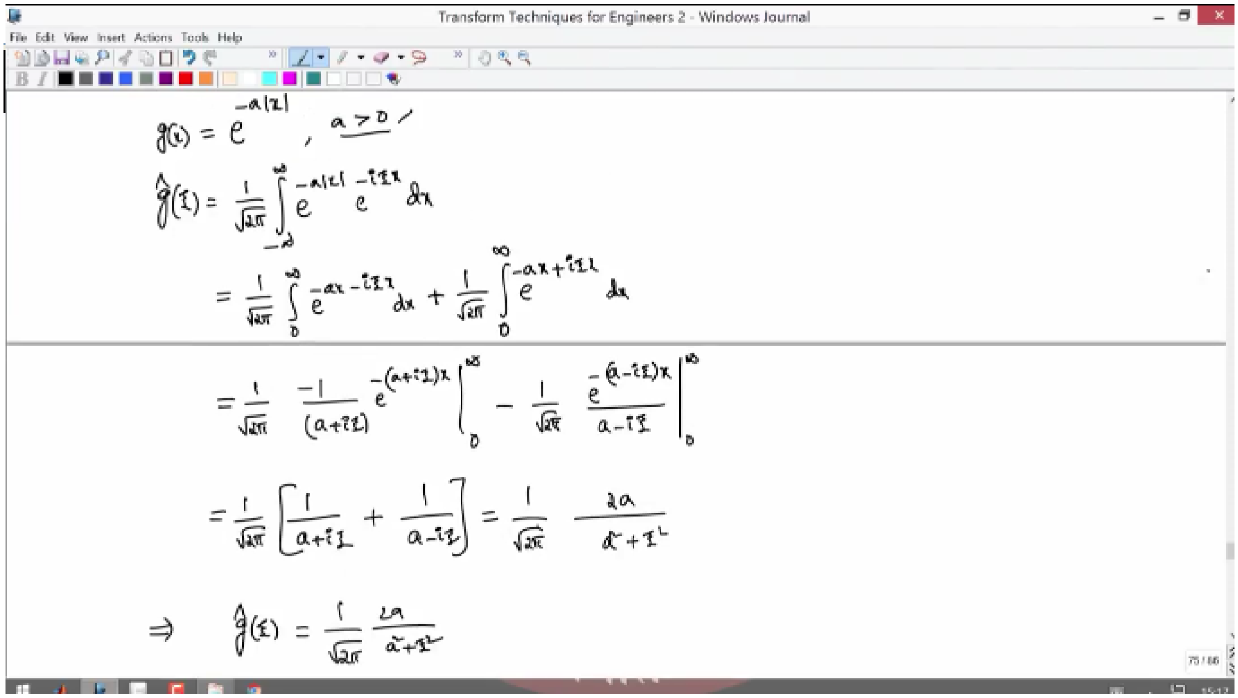


integral, soinstead of doing this way if youknow complexvariables you calculate this and youwill end up finally what is thisfunction, so I do it in a differentway, I'll try to see that 1/xi square + A square, okay, so I consider my I, Apriori I know as though I know this function F(x), if you calculate this complex variable technique you can getit.

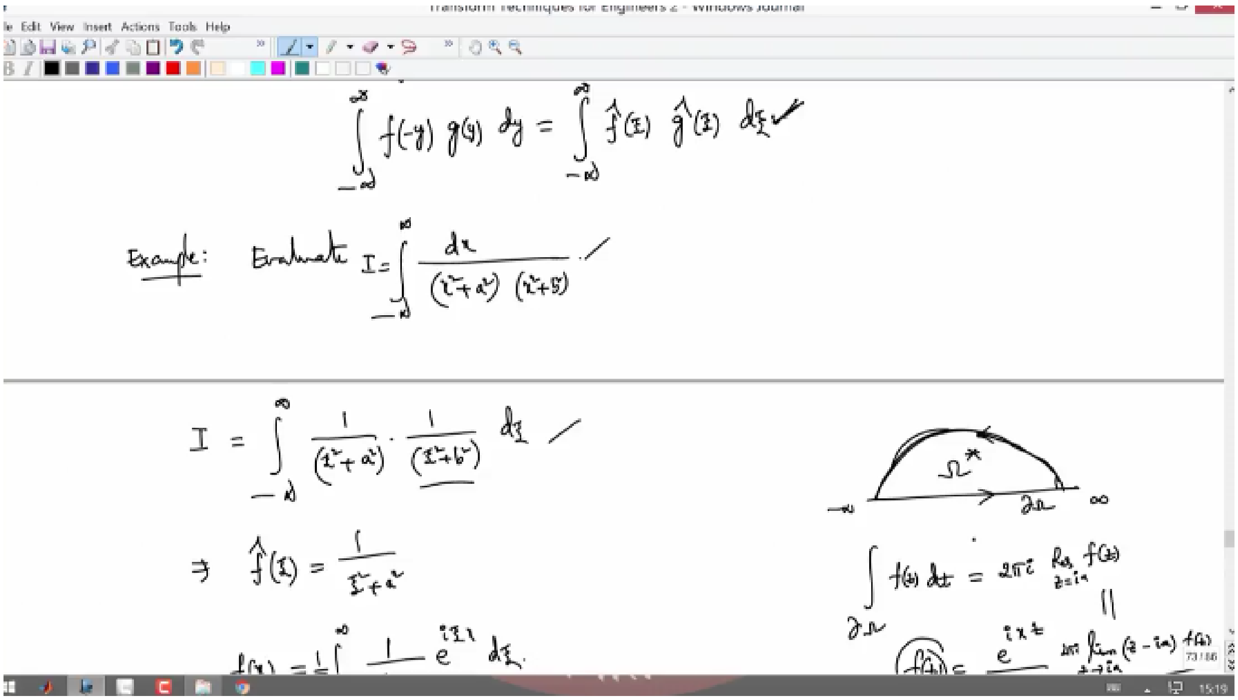
Let me calculate E power -A modX, if this is my function F(x) okay, let ussay some G(x) is this function, so whathappens to its Fourier transform, so Gcap(xi)is actually equal to 1/root 2 pi and integral minusinfinity to infinity, E power -A modX, E power -I xi X DX, this iswhat is Fourier transform, so this yousplit it, split this integrals from 0 toinfinity and you get this will be, ifit's X is positive this is going to be Epower -AX and then you have -Ixi X that is one DX and if it is otherpart will be 1/root 2 pi - infinity to 0, in this case it's going tobe AX when X is negative, mod Xis -X so minus minus plus so you have AX andhere you have I xi X DX.



Now if youroot, if you substitute X/-X soyou get a minus for DX and this is goingto be infinity to 0 and here you have a-AX and you have a +AX, so thisis what you get and because - infinity to 0 so you can make it plushere and then write 0 to infinity, sothese are the two integrals you get, soone is this 1/root 2 pi so this youcan easily evaluate, so this is E power A+ I xi into X, so 1over A + Ixi times E power –A + I xi X forwhich you apply 0 to infinity,
this is one first integral, a secondintegral will be 1/root 2 pi E power –A - I xi times Xdivided by A– I xi, this is alsofrom 0 to infinity, this is equal to 1/root 2 pi and this is going to be, and ofcourse you have a minus sign, minus signyou have in a differentiate you can getthe minus sign, so this minus minus plus,this is minus of this quantity, solet me put this minus here, and so yousee that this is going to be 1/root 2pi, 1/A+ I xi because atinfinity this is A is positive, so A ispositive, let's choose this way so you have E power –AX, it's going tobe at infinity its 0, so this one minus, minus minus plus again so here also youget 1/root 2 pi 1 over A - I xi, so what you see is 1/root 2 pi, nowyou have A square - I square that is+ xi square times, and what you get isthis A-I xi + A + I xi that is 2A.

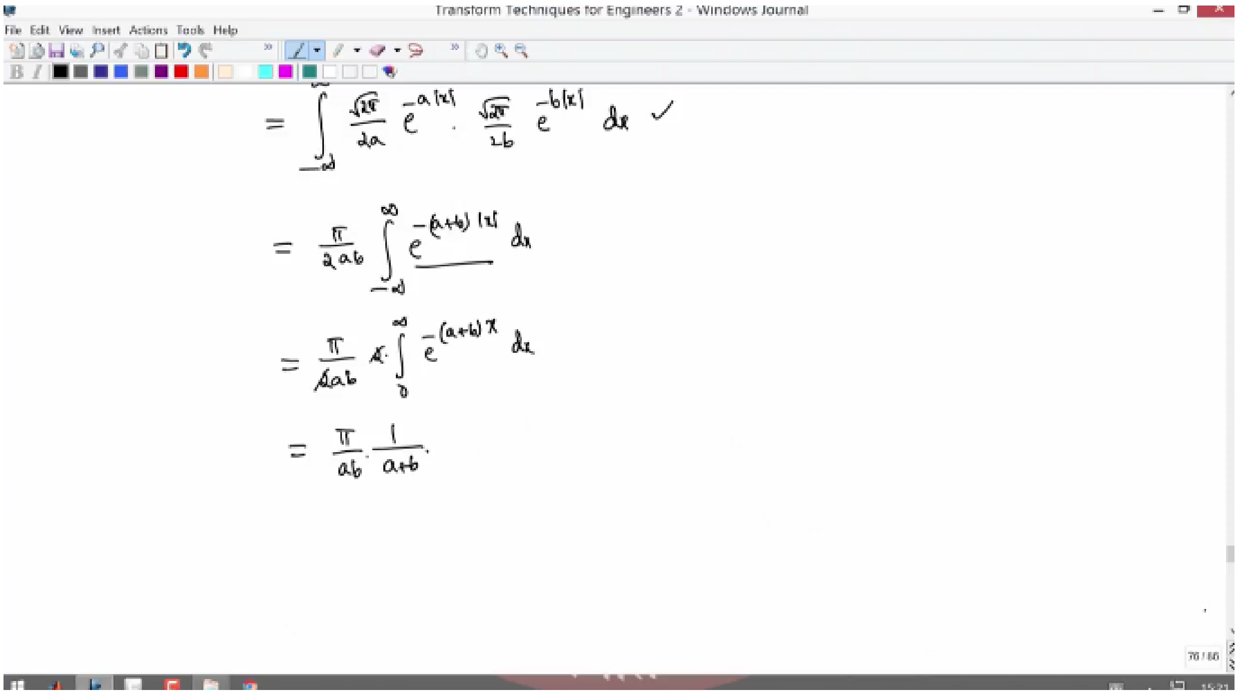


So this implies, so G hat (xi) is actually equal to 1/root 2 pi, 2A/A square + xi square,so if your G is, if I multiply with theroot,if I consider this with 1/2A, so letme choose with 1/2Afor GA I will have 1/2A here, and again I'll have 1/2A 1/2A because that my function Gis that, so 2A2Acomes down youhave 2A, and finally you have 2A comes as a factor, and you have 2A so that gets cancelled, so we end upgetting simply this, this is simply 1, 1over root 2 pi A square + I square, okay,so if you don't want that root 2 pi alsoso you can also multiply here root 2 pi,so you can easily see Fourier transform ofroot 2 pi E power -A mod X dividedby 2A, so let's divided by 2A for which ifyou take the Fourier transform that isactually equal to 1/A square + xi square, so this is exactly you're goingto write here, here in this I, thisintegral I can write this as, so this Iwill be now equal to integral minusinfinity to infinity, so you have root 2pi/2A times E power - A mod Xfor which you have Fourier transform whichis a function of xi, okay, this is onefunction into other function thisinstead of A you have B, so you write likethat Fourier transform of root 2 pi/2AE power, 2B here, so –B mod X which is a function of xi, you haveD xi, so this is exactly what your integral is.



This is the integral youwon't evaluate that is this, but fromthis, this is a Fourier transform of, youput it in this form Fourier transformof this so that function you can writeit here this side so that is a sameassaying, if you make use of this relation isactually equal to minus infinityto infinity, root 2 pi/2A this functionE power -A mod X times root 2 pi/2B times E power -B mod X DX, okay,so because here in this I have to put X/-X and as in the relation here,F(-y) so because it involves the modulus so it is same,modulus of -X is same as mod X so, you have this is the relation so though youractual identity, the integral value is actually this integral, so if you take itout it's going to be 2 pi divided by 4AB so that makes it, divided by 2, okay, 2 pi/4AB and then you havethis integral minus infinity to infinityE power –A+B mod X DX.

Now yousee this integrand is even function, soyou can write this as 2 times integral 0to infinity E power –A+B mod XDX, mod X is when you write X is in 0to infinity,mod X is you can replace asX DX, so you should write like this, nowthis is same as X is positive mod X is



simply X, so this is actually equal to,this goes you finally end up pi/ABtimes and this will give me like 1/A+B, this is how we can evaluatecertain integrals using this Parseval's identity, so we will see some otherexamples and other applications in thenext video.Thank you very much.

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