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Transform Techniques for Engineers
Application of Fourier Transforms to Differential
And Integral Equations
With
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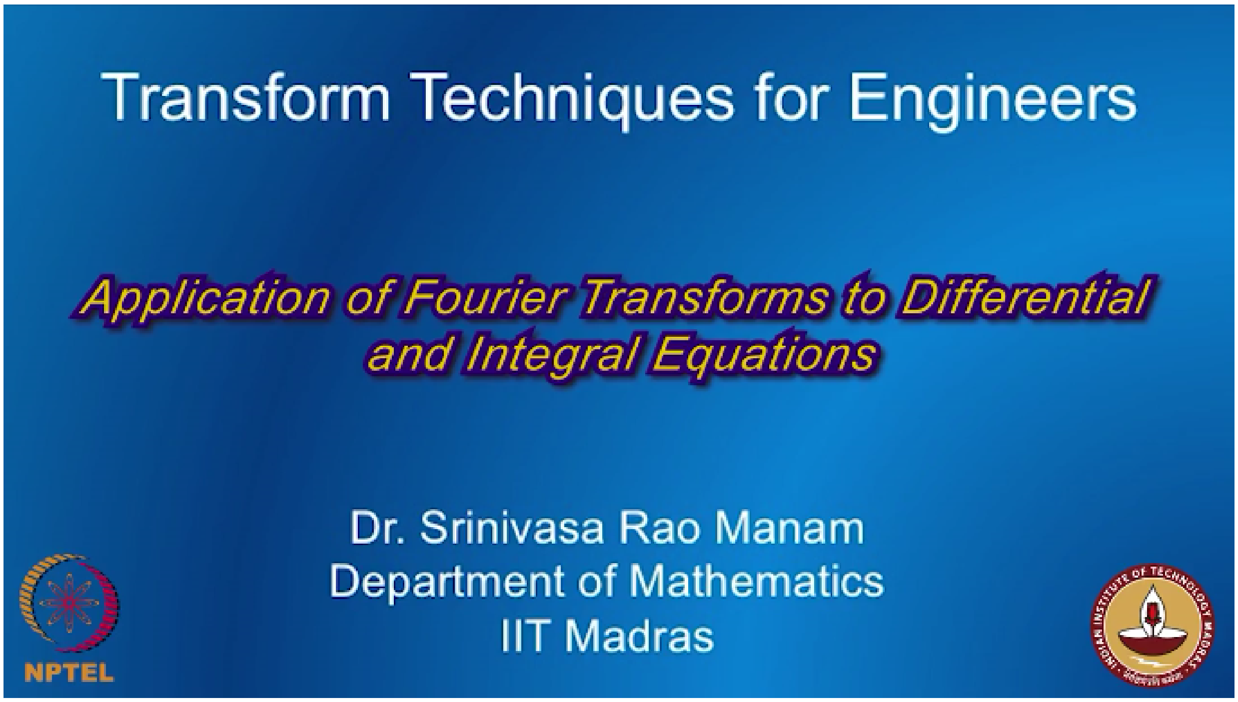
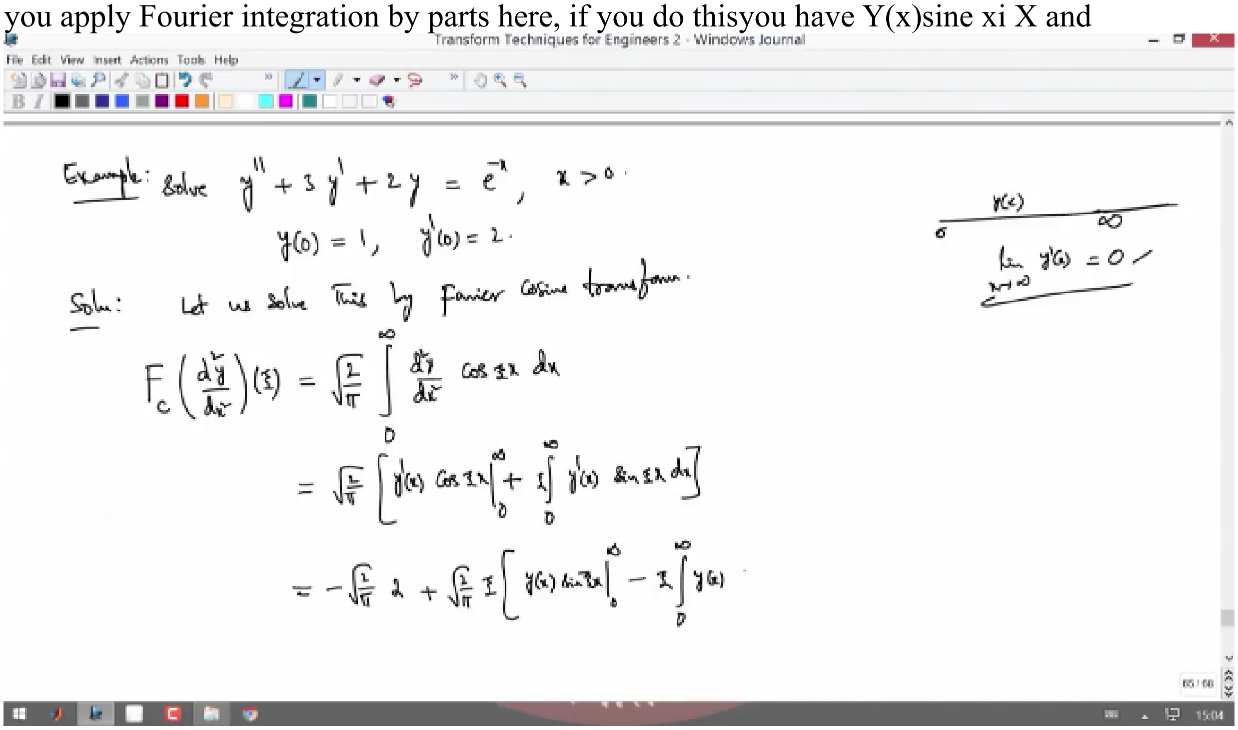
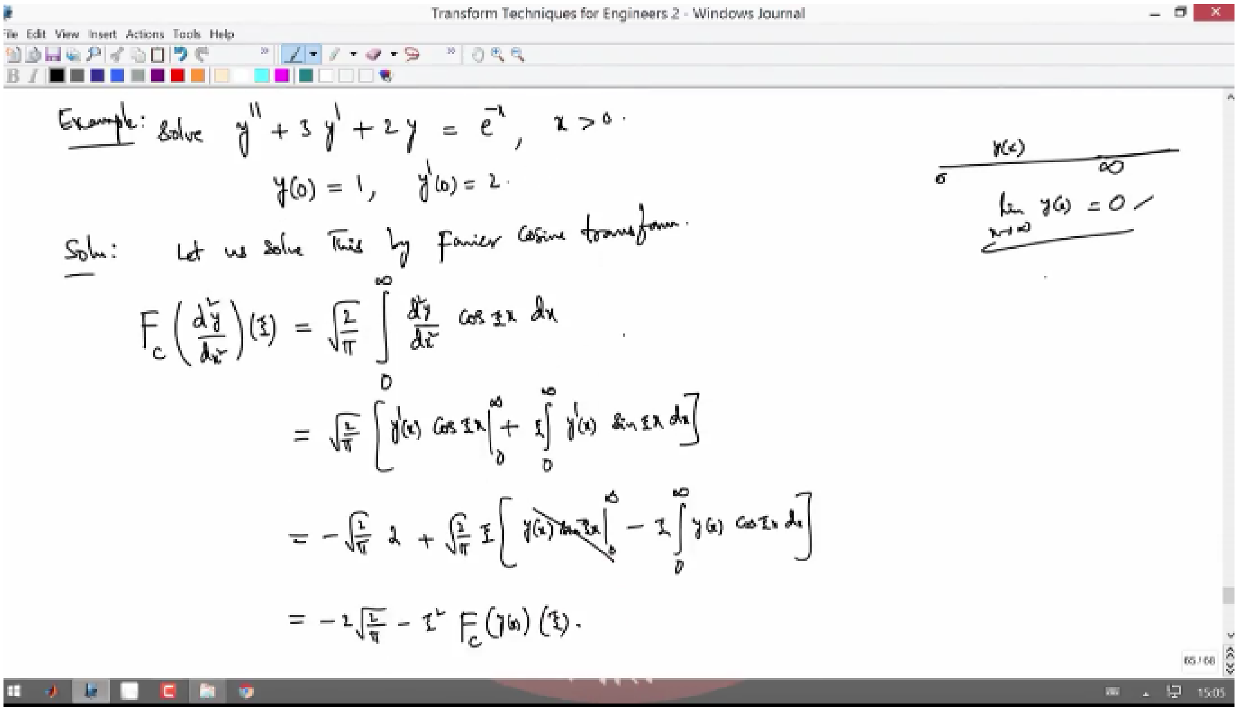



Welcome back, the last video we haveseen how to solve ordinary differentialequations with initial conditions usingfully Fourier transform, we will try tosolve this, so when you have so thoseproblems when we have 0 boundary kind,0 initial conditions that means Y at0 and Y dash at 0 is 0, if theyare given and you can extend that whenit is given the ordinary differential equation is givenin the positive side that means 0 toinfinity you can extend this to theminus infinity to infinitywith right hand side by taking X positive anyway is given, and the negative sideyou make it 0 so that it will be interms of Heaviside function for whichyou take that, you can take the fullFourier transform and try to invert after that you invert and get thesolution.
So this is possible if you have0 initial conditions, if the conditionsare not 0 at 0 that means initialconditions are not 0 conditions then youmay have to, you cannot use this, theseconditions are not automaticallysatisfied if you extend it as aHeaviside function in the right handside.
So in that case you may have to useFourier sine transform or Fourier cosinetransform, we will try to solve I'll justgive you just for the sake of doingsolving how to use this cosineand sine transform, any one of them youcan use if your domain is 0 to infinity,so we'll just try to solve one or twoproblems on that in this video.
So we’ll consider this problem example, so let'stake the same problems that we havesolved but with nonzero conditions,let's do this one, so let's take Ydouble dash,so how do we solve this? Ydouble dash + 3Y dash + 2Y= E power –X, and X ispositive, initial data is given as Y(0) is 1,Y dash(0) = 2,so clearly this data is not 0, it'snot 0 so you cannot extend this togiven domain

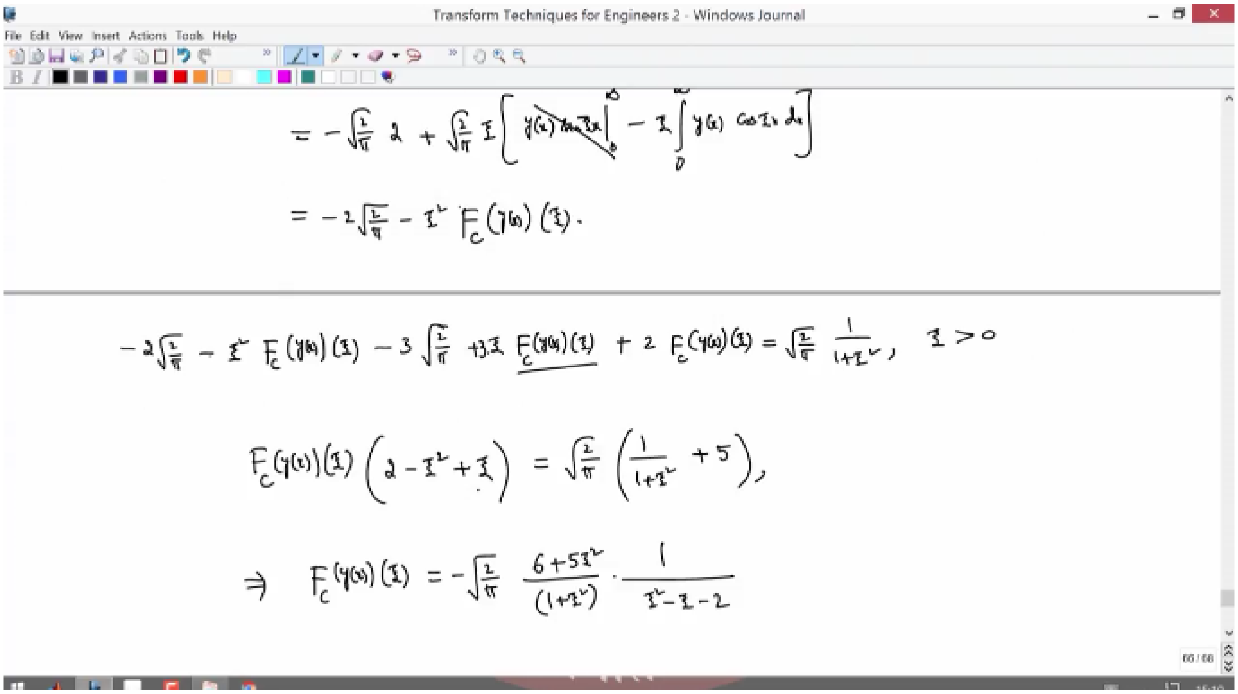
is 0 to infinityyou cannot extend this to minus infinityto infinity with the Heaviside function onthe right and use the full Fourier transform we cannot do this because they're not0 boundary conditions, so what we dois we try to apply a Fourier sine orcosine transform, so let's use any one ofthem, let's solve by Fourier cosinetransform, okay, let's solve this byFourier cosine transform, you can alsouse sine transform doesn't really matter,so if you use the Fourier transform so what you'll have the firstterm is D square Y/DX square,X ispositive,X is 0 to infinity, so if youapply the Fourier cosine transform onthis, so F of FC of this which is,which becomes a function of xi that is aroot 2/pi that is a Fourier transformwe have defined,Fourier cosine transform0 to infinity D square Y/DX squarecos xi X DX,do the integration by parts root 2/piyou get Y dash(x) into cos xi X right, so you bring this derivative over hereso for that you differentiate, so do theintegration by parts so you get 0 toinfinity- integral 0 to infinity Y dash(x) and you have this xi comes out,because of cos, cos derivative is - sine xi X into xi, so that xicomes out and makes it negative sign becomes plus, this is DX.
Clearly so you're looking for Y dash and if you want to apply Fouriercosine transform Y has to be absolutelyintegrable function, okay,Y(x) has tobe absolutely integrable function from0 to infinity that means limit Y(x)has to be 0, as X goes to infinity has tobe 0, it has to go to 0 at biggervalues of X, so and Y and Y’s derivatives, okay, you're applying for Y dashalso, Fourier cosine transform Y doubledash also, so in any case this Y dash hasto be 0 and cos thing when you apply, so - root 2 pi first term becomesthe contribution will be cos 0 is 1and you have this – sign, so you haveY dash(0) that we have as 2, so youhave 2 times that + root 2/pi xi times again you apply Fourier integration by parts here, if you do thisyou have Y(x)sine xi X and



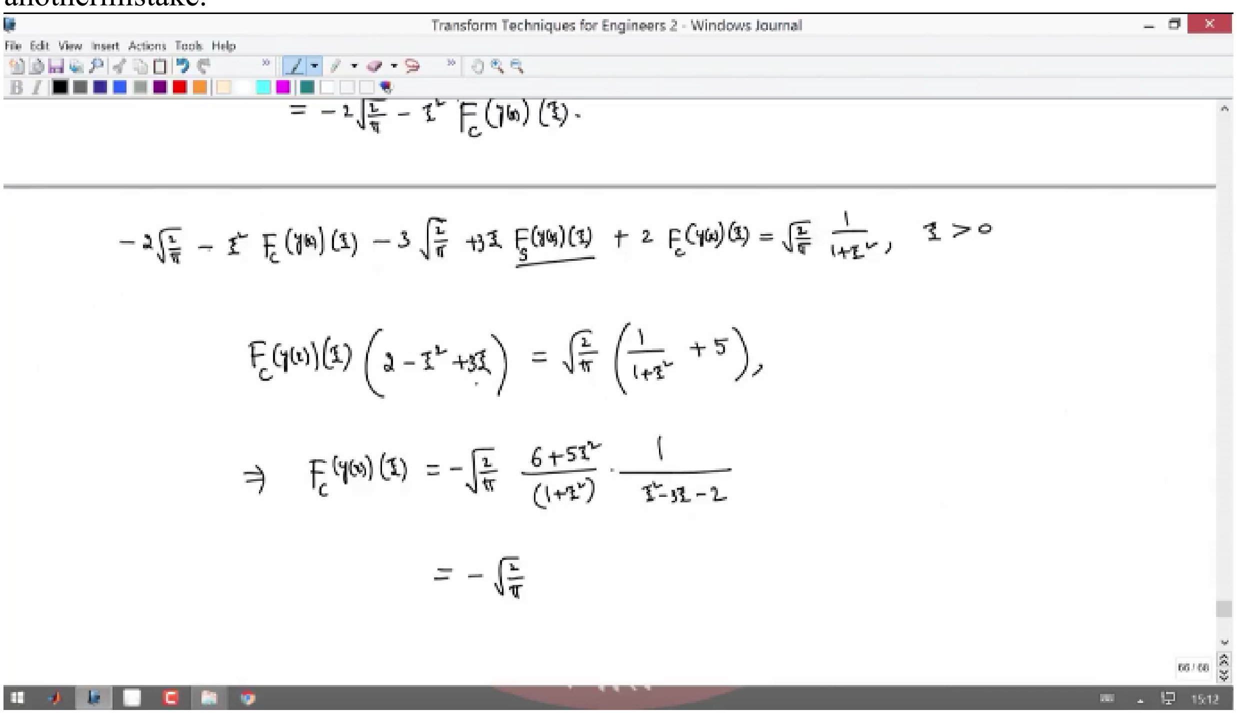

you have 0 toinfinityand – xi, xi comes out, so one more xi comes because of thederivative of sine, and we have xi 0 toinfinity, Y(x) cos xi X DX, so this is aFourier cosine transform of Y,so what you have is -2 root 2/pi+ root 2/pi xi square because thisis 0 contribution sine xi X at xi X = 0 at 0, Y at infinity has to be 0,Y and Y dashboth, okay, so we use these conditions Y and Y dash both are 0 at X goes to infinity, sothis you have a minus and what you getis, what is left is this integral, thisintegral is with this coefficient issimply Fourier cosine transform of Y(x)okay, so this is what you have for thefirst term.




Second term you do the sameway, so in second term if you do, if youapply for second term, so you try to apply,so I am just applying to the equationFourier cosine transform then we get -2 root 2/pi – xi squareFourier cosine transform of Y(x) xi + 3 timesYdash, this is Y dashif you apply again same technique,instead of Y dash you have simply Y, if youhave a Y dash here so oneintegration by parts will give me Y(x) cos xi X, here you have Y(x) here itself,so you can if you apply this Y at 0 sowhat you get is -3 root 2/pi andcos xi X,so cos 0 is 1,Y at 0 is 1, sothat is 1, so this is what you get forthis boundary term and here you get + xi times and that old root 2/piintegral will become Fourier cosine transform of Y(x) and xi, so this wholething is the Fourier cosine transform andthis is equal to this plus and you haveone more term 2 times Fourier cosinetransform of Y(x)xi, which is equalto E power - X if you calculate Fourier cosine transform of E power -Xcos xi X, 0 to infinity root 2/pi thisis the Fourier cosine transform. And onthe right it should have this integralso that value is 1 by, so if you can dothe integration by part and solve it, sothis is going to be root 2/pi 1/1 + xi square,okay, that's what you have, so if you dothis you get root 2/pi 1 over 1 + xi square, so you have xi positive sidethis xi is Fourier cosine transform, so tryto get what is your Fourier cosinetransform of Y(x)xi, thiscoefficient of this is – xi square, soyou have 2 here, 2 – xi square and here you get +xi, +xi and thenthis is equal to root 2/pi 1/1+ xi square and this is going tobe plus, so minus this is going to be +5root 2/pi,so +5 you can put itlike this, so this is root 2/pi,sothis is 5+,so let's solve and do that in the next step, sothis becomes Fourier cosinetransform of Y(x), this is root 2/piwhat you get is divided by 1+xi square you have 5+, so you have 6+ 5 xi square that is the righthand side, and if you divide this sideleft hand side you have xi square, so youhave a – sign, and you have xi square– xi – 2, I think we made amistake, so this 3 so when I applied 3

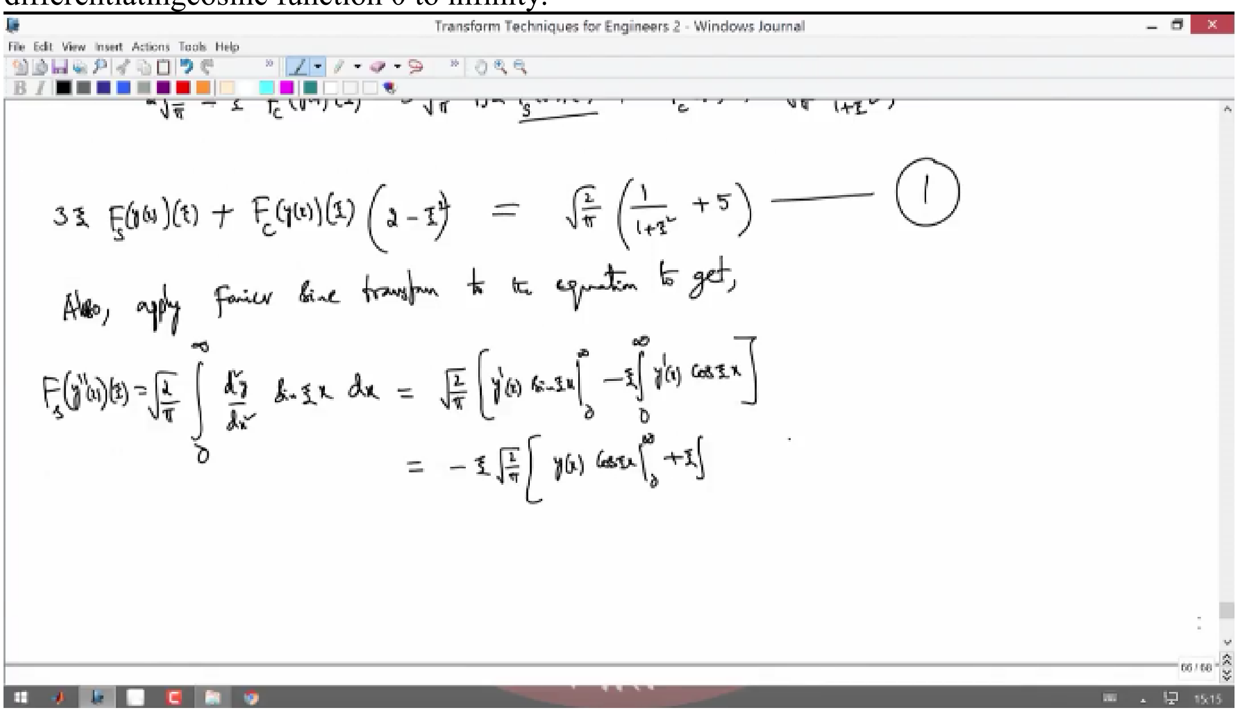



times Ydash, 3 times this and you have 3 xi, so I think that is a mistake I have, so you have 3 here, 3 missing for thissecond term, and you have this, you have 3xi, 3 xi, xi square - 3 xi, -2 yeah, so that's what you have, xi square -3 xi – 2, so ifwe actually do this, do the partialfractions –root 2/pi, so partial fractions you can have for, one is so here what is left here isFourier sine transform, okay, when I applythis one this is going to be Fouriersine transform this is,that is anothermistake.



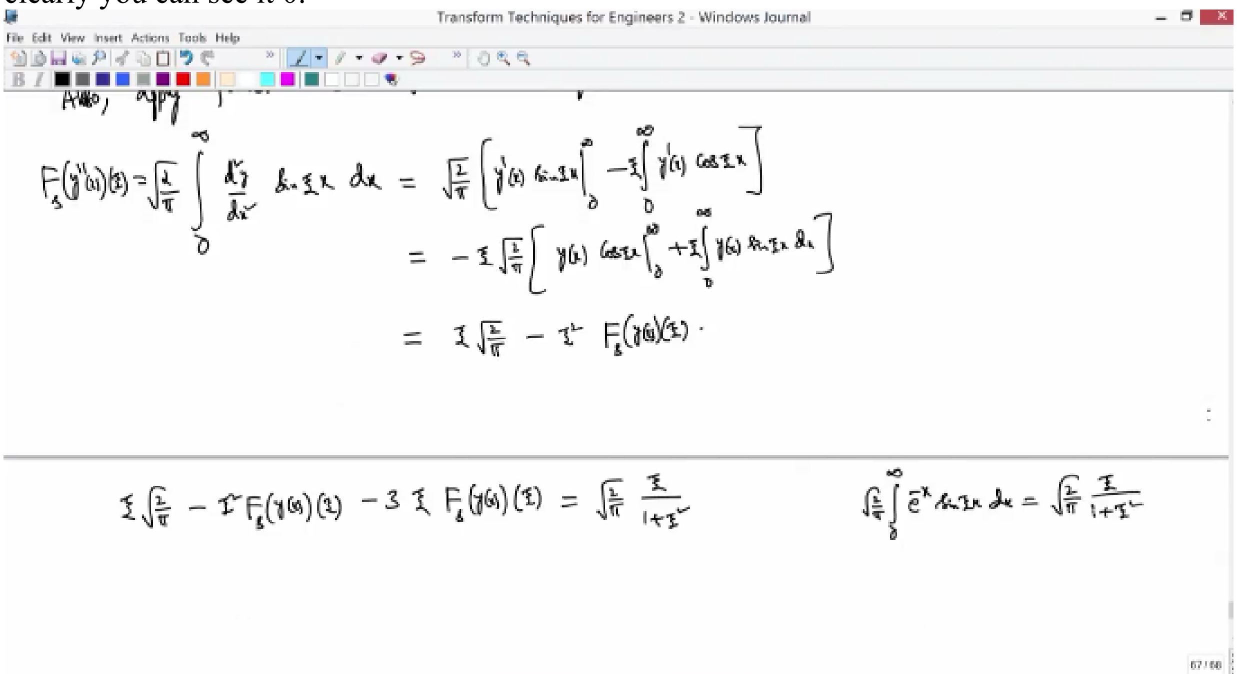

Fourier sinetransform whichyou have, okay, so what you get is thisone, this 2 – xi square times this+ 3 times,3 xi timesFourier sine transform of Y(x)(xi) + this is equal to this one, this isone equation, okay, so when you actuallyapply the Fourier cosine transform sothat means you end

up with both Fouriersine and cosine transform, okay, so youcannot solve this equation directly soyou need another, you can also apply Fourier sine transform for this and get onemore equation like this,okay, so let's apply Fourier sine transformnow, so basically if you want to solvethis ODE with nonzero initialconditions you have to apply both theFourier transform and Fourier sinetransform get the two equations, solvefor this Fourier cosine transform orFourier sine transform and invert it you will get your F(x), that is what exactlywe need to do.
So apply for Fourier sinetransform again, so you apply not againso, also apply Fourier sine transform to the equation to get,what do you get if you apply?Fourier sine transform, so you have Fourier sinetransform, what is this?Let's calculate,if you apply Y double dash DX square andyou have Fouriersine transform isthis, instead of cosine xi X, you have sinexi X DX, this is your Fourier sinetransform of Y double dash(x) which isfunction of xi, so this if you applyyou get root 2/pi Y dash(x) sinexi X for which you apply 0 to infinity,and this is 0 because of sine, and you have- integral 0 to infinity xi comes outbecause sine you differentiate withrespect to X so you have Y dash(x) sine becomes cos, cos xi X, okay,so again if you apply so this is 0, soyou have –xi of course you have root2/pi,root 2/pi xi times, do one moretime if you integration by parts Y(x) cosxi X, 0 to infinity you have –xi comes outthis becomes + after differentiatingcosine function 0 to infinity.

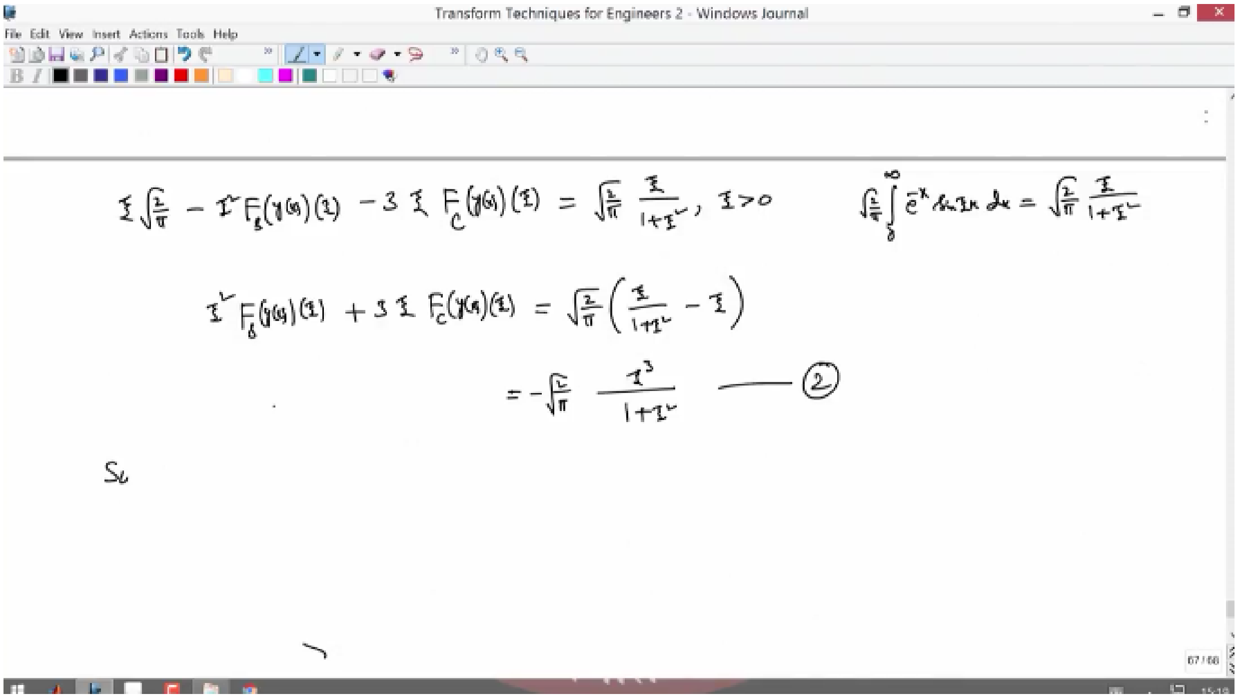



Now youhave Y(x) sine xi X DX, so this is exactlyso what you get is this is a – sign,–Yat 0,-Y at 0 is given as 1, so 1 isminus minusplus xi times root 2/pi O at 0 is 1, okay, so that is what is this boundaryterms, and this becomes xi square root 2/pi this is Fourier sine transformof Y(x), which is function of xi.
Andthen what happens to, now if you apply tothe equation this Fourier sine transformyou will see that if you apply you haveY double dash so that is this, so let meput this so xi root 2/pi – xi square Fourier sine transform of Y(x) of xi + 3 times, now Y dash, in the place of Y dash you have Fouriersine transform for Y dash if wetake, again instead of Y dash you have Y,so this will be you get,that's a minus sign because of thissecond term here, so –xi timesFourier sine transform of Y(x)function of xi, because this coefficient is 0, becauseof this boundary terms even with Y at X, and sine xi X that is 0, and this isequal to right hand side Fourier

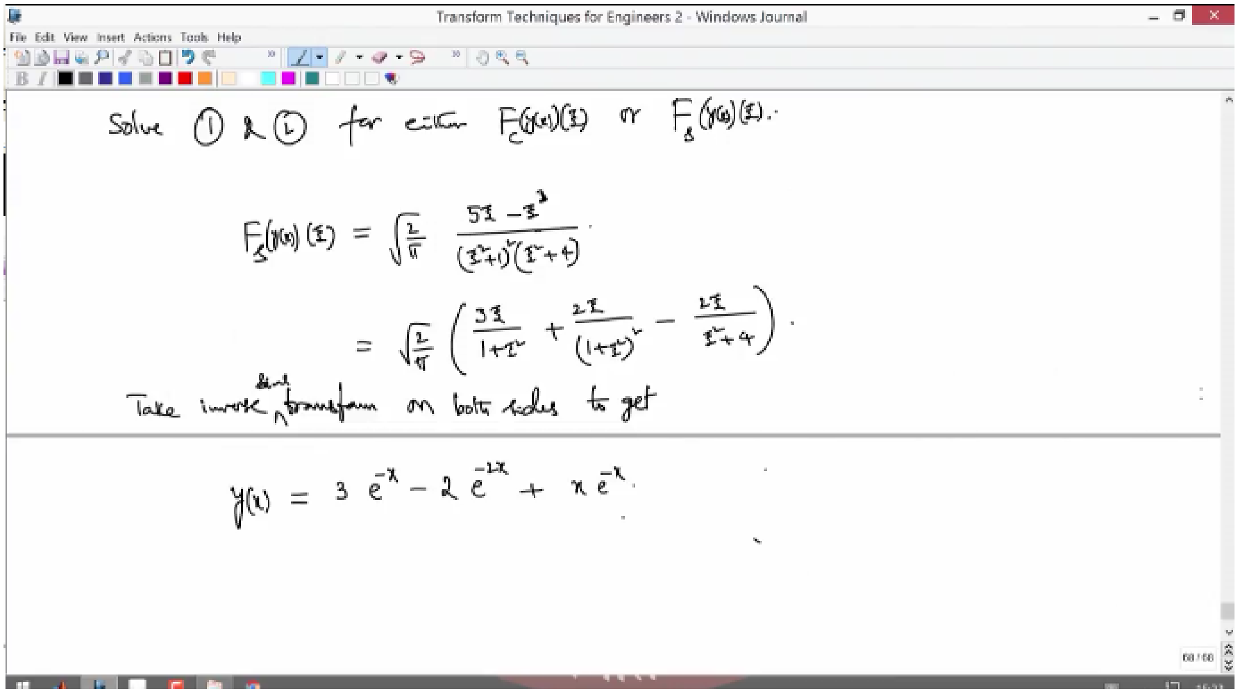
sinetransform of this will be xi divided by 1+ xi square root 2/pi xi dividedby 1 + xi square this is for thisintegral, so if you try to calculate thisintegral 2/pi 0 to infinity E power-X sine xi X DX is actually equalto 2/pi xi divided by 1 + xi square, okay, so that's how it is so 0, xi is0it's clearly you can see it 0.



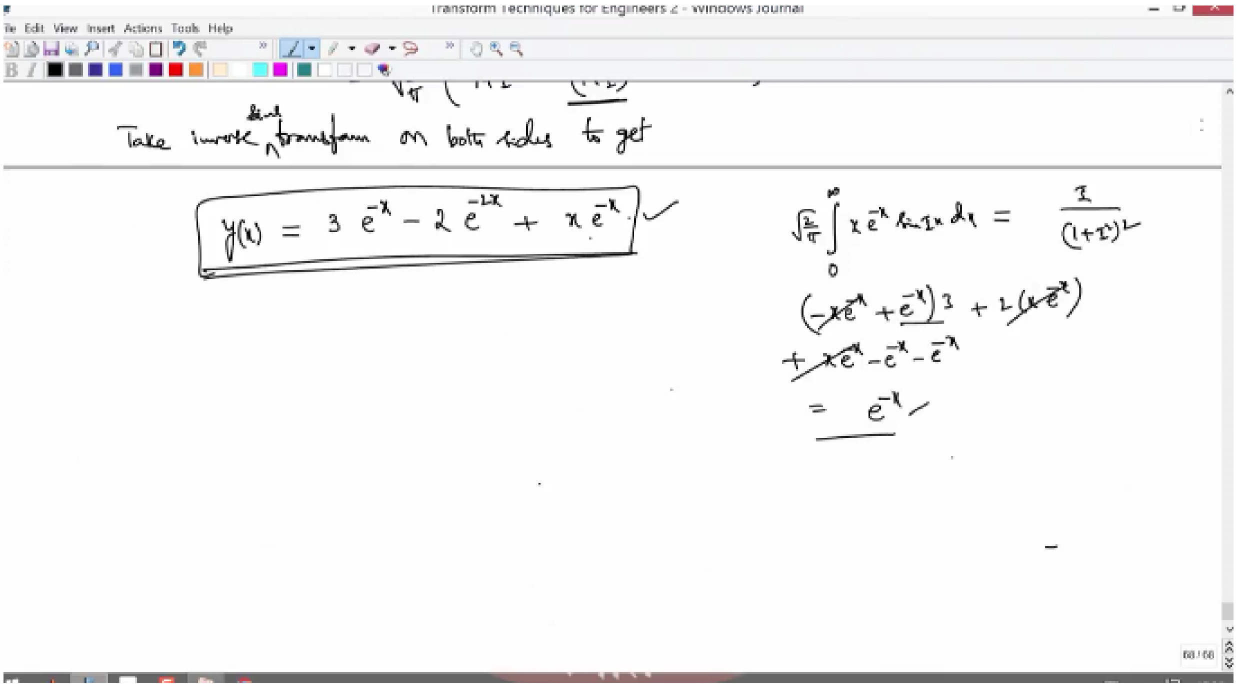

So if you, so this is what you get fromxi positive, now what happens to your, okay again I think I made a mistake what doyou have here Fourier, if you apply thefirst one so for the third term thisis not Fourier sine transform if you applya Fourier sine transform for onederivative, one integration by parts willgive me what you have is here this iscosine transform, Fourier cosinetransform, so you have xi squareFourier sine transform of Y(x) xi + 3 xi, Fourier cosine transformof Y(x) which is function of xi, which isequal to root 2/pi, what you have is xi/xi square 1 + xi square here – xi, so which is 1 + xi square comes out, root 2/pi you havexi – xi – xi cube, so you have –xi cube. Right, yeah, so this is exactly what youhave as a second equation, so you try tosolve 1 and 2, so these are the linearequations for Fourier sine transform andcosine transform, okay.



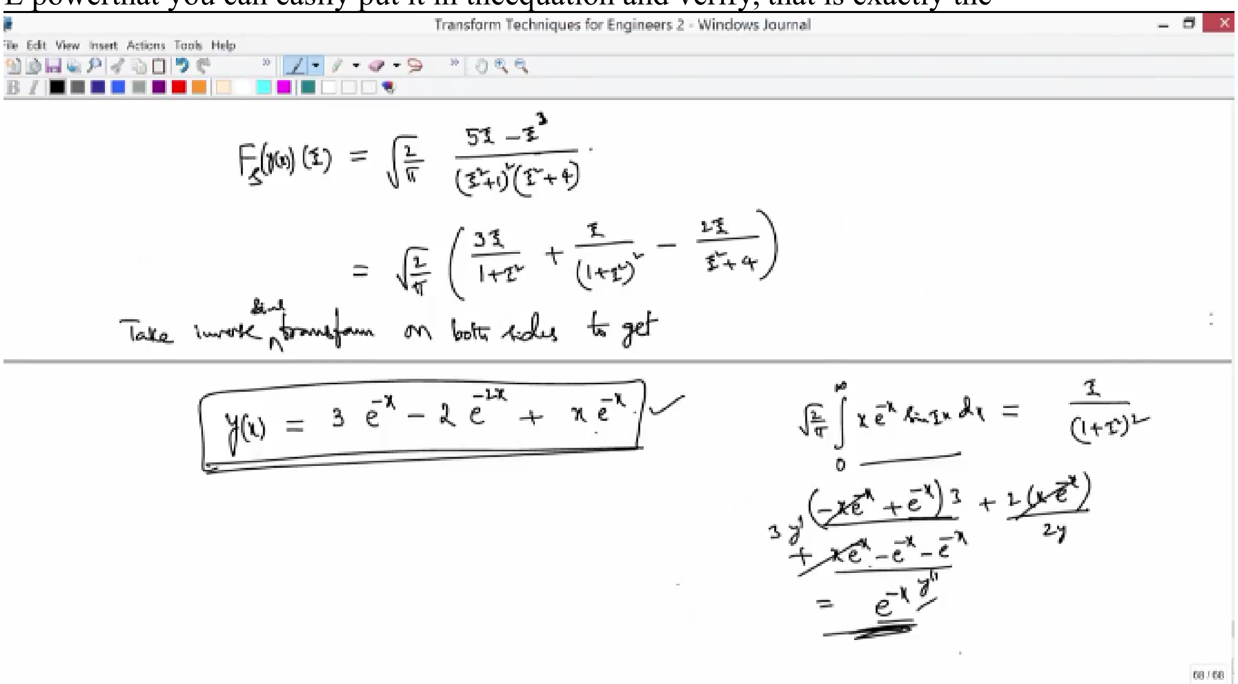

Let's eliminate Fourier cosine both theequations 1 and 2, solve 1 and 2 for eithersine FC Y(x)of xi or FS(YF) this is a transform for thefunction, unknown function Y(x), okay, so ifyou do this you eliminate the FCyouwill get FS, FS will be let me write itdirectly, so your FS of Y(x) as a function of xi, this is equal to I'lldirectly write what you get, so you canjust verify, you'll see that this isgoing to be if you solve it these twolinear equations, you eliminate Fcosine and if you do that I'm directlywriting, so if you do this you'll seethat it's going to be root 2/pi, root 2/pi 5 xi –xi cubedivided by xi square + 1 whole square,xi square + 4, this is what you willget if you actually solve for Fourier sine transform,1 and 2 you solve justget this Fourier FS, FS if you calculatethis is what you get, this you try towrite it as a partial fractions, if youwrite the partial fractions for thisterm this will be 3 xi/1+ xi square, please verify this 1 + xi square whole square – 2 xi divided by xi square + 4, so this iswhat you'll get, so these things, now Iknow what exactly the inverse transformof this, because I know that I see thatthis is, this quantity is Fourier sinetransform of E power –X, so if hereif you do that, if you dothis one now so Fourier transform if youtake the inverse transform, take inversetransform of on both sides,inverse sine transform rather okay, sine on both sides to get, what you aregetting?So left-hand side you willget the unknown function Y(x), if you takethe inverse transform you have 3 timesroot 2/pi xi/1+xi square ifthere's a Fourier inverse transform thatis going to be E power -X +, theother one the second term so you havehere again, okay so last term ifyou see, if you look at the last term thatis going to be -2 again similarthings xi square + 2 square, so insteadof 1 so if you actually see thisintegral E power AX if you use what Iget is here I get A square + xi square,so this is what you have, so instead of AI have 2 now here,2 square.



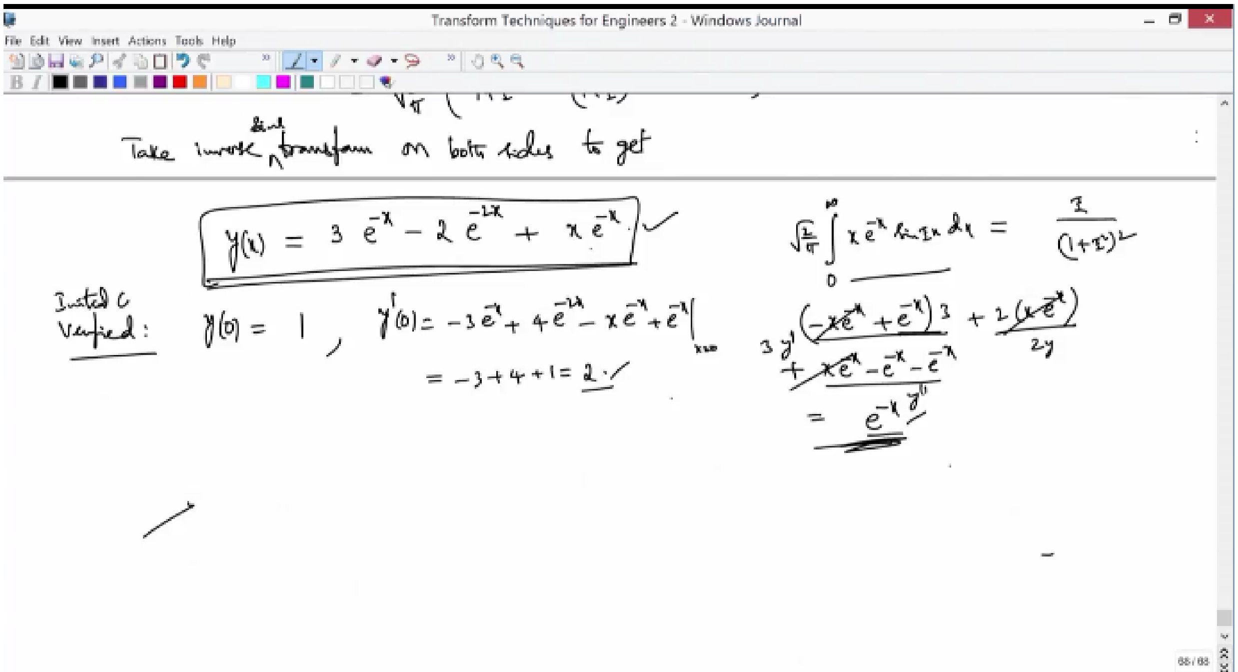

And thethird term, so you get Fourier inversion of this, I inverse Fourier, sine transform will give me -2times root 2/pi xi/xi square + 2 square, that is E power -2X,andhere because 1 + xi square wholesquare you have so this will give me X into E power –X, okay, so that means it should be, I don't knowreally let me see X into E power X, whatis the Fourier sine transform of that? Integral 0 to infinity root 2/pi Xinto E power -X sine xi X DX, sowhat is this?This is the inversetransform, right, so if you take theFourier transform I should get back this,so this will give me root 2/pi sothere's no way you will get this number2, okay, but X E power -X if youactually calculate as let's solveit you take the derivative of X E power-X that is going to be X -Epower -X that's one derivative,other one is E power -X one moredifferentiate, if you differentiate thisis going to be X E power -X and youhave -E power –X,-E power–X, and this is, this one + 3 times of this, this is a secondderivative, and + 2 times of X intoE power –X, what is this value, isthis 0? 3, here you have 2, that's going to be 1, 1, 1, it's going to be, is it E power –X, 3Epower -X and here you have, soit's E power –X, fine, 2, 3, 2 here 3and -3 that is going to be 0,cancelled.So X into E power -X it'sfine, so what you should be getting here is xi divided by 1 + xi squarewhole square, so the partial fractions, this2 is not there, okay, so thisis corresponds to this one, so you havethis, this is your general, this is thesolution that satisfies the initialconditions Y at 0 is 1and 2 these areused when you apply the Fourier cosinetransform and Fourier sine transform.




So ifyou directly solve by other methods youwill get this solution, okay, it's just aconfirmation, so I've calculated insteadof calculating this as they so, so instead of, so if you calculate this,there's no way you will get this 2, sothat means I've written wrong thispartial fractions it should not be any 2here, so you just verify, pleaseverify that, so try to get the partialfractions you will see that they willget, you should get this form and so thatyou end up getting X power E powerthat you can easily put it in theequation and verify, that is exactly the

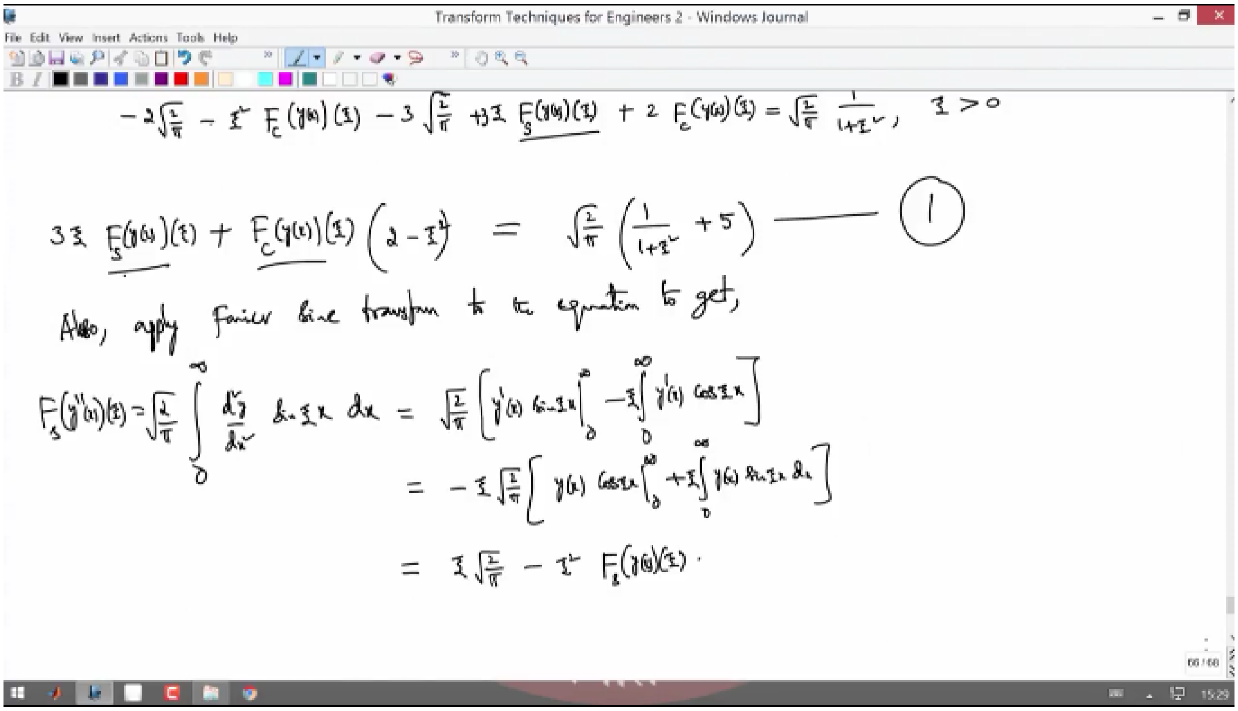



equation, this is your Y dash, Y double dash, this is your 2Y dash, and this is your 3Y,right, this is your 2Y and this is your 3Ydash, which is actually equal to E power–X, so that means X into E power-X satisfies, they give an equation.

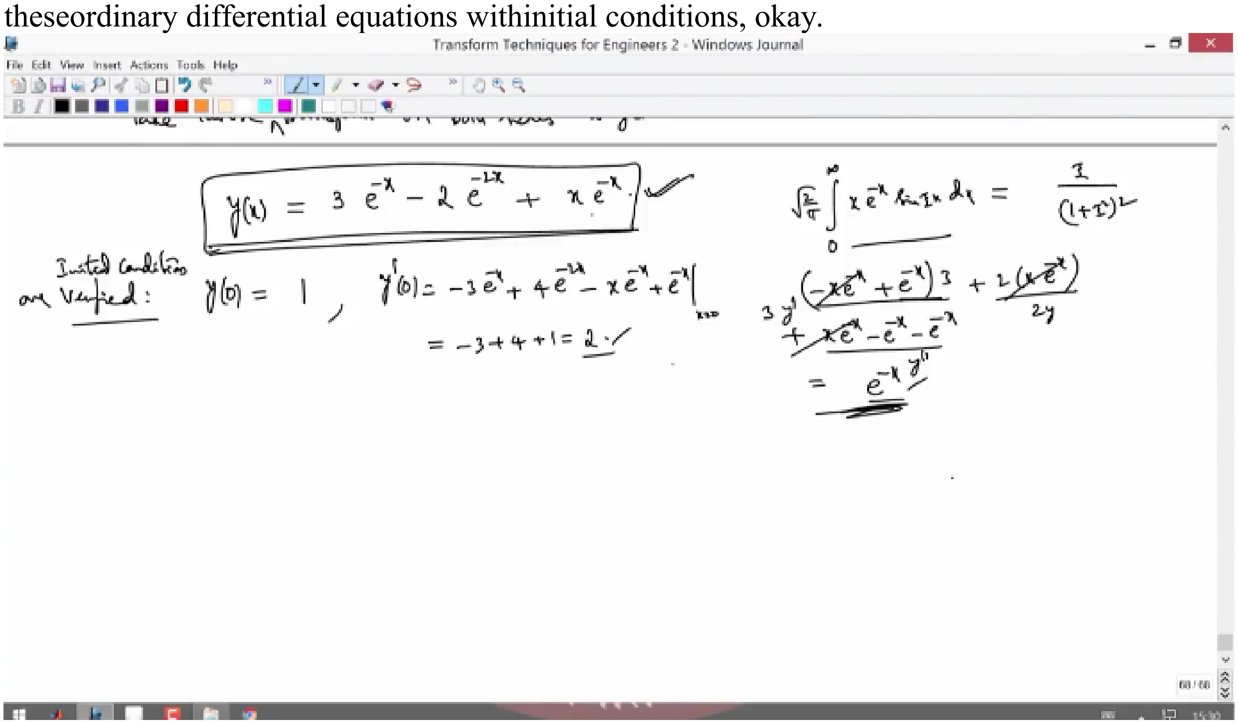



And now we can verify so Y at 0 isactually equal to 3-2 1, and when youput X = 0 that is 0, so Y dash at 0also -3 power -X + 4 Epower -2X + X E power –X, minus of this + E power -X if youput X = 0 here you'll see that-3 + 4 that is 0 + 1 so youhave, that is also 0, that is 5 - 3that's 2, that is exactly what we havethe condition, so given conditions thatit is Y at 0 is 1, and Y dash at 0 is 2that is exactly what we got,they are verified okay, so these areverified, conditions are verified, initialconditions are verified, so this is how yousolve, so given any ordinarydifferential equation you try to applyboth Fourier cosine transform and sinetransform, so if you apply, if you canapply if you have a 0 initialconditions you can extend this functionto the domain to -infinity to infinity,and you can use only one transform thatis full Fourier transform, otherwise ifyour domain is the only positive, and ifyour initial conditions are nonzeroboundary condition, nonzero conditionsthat is Y dash and Y0,Y at 0 and Y dashat 0 if they are nonzero you cannotextend as a Heaviside function and makeuse of the full Fourier transform, so youdon't have choice but to use Fouriersine transform or cosine transform thatare defined over for functions,absolutely integrable functions over,however the domain 0 to infinity.

So ifyou apply one transform you will not beable to solve the equation, we will haveto apply both sine and cosine transformand use the boundary conditions whateverinitial conditions involvedin the problem, you use inside thisboundary terms when you do theintegration by parts and you get twoequations you solve them, either for Fourier sine transform or Fourier cosinetransform and then you invert it you getback the same thing, so from theseequations 1 and 2either you solve for Fourier cosinetransform or Fourier sine transform youwill get, and then you invert it you willget your Y(x), so I have done forFourier sine transform, I solved 1




and 2 for FS,you can also try for solving eliminate YFS and try to get FCand you invert with Fourier cosinetransform you may end up the samesolution, so this is how we solve theseordinary differential equations withinitial conditions, okay.

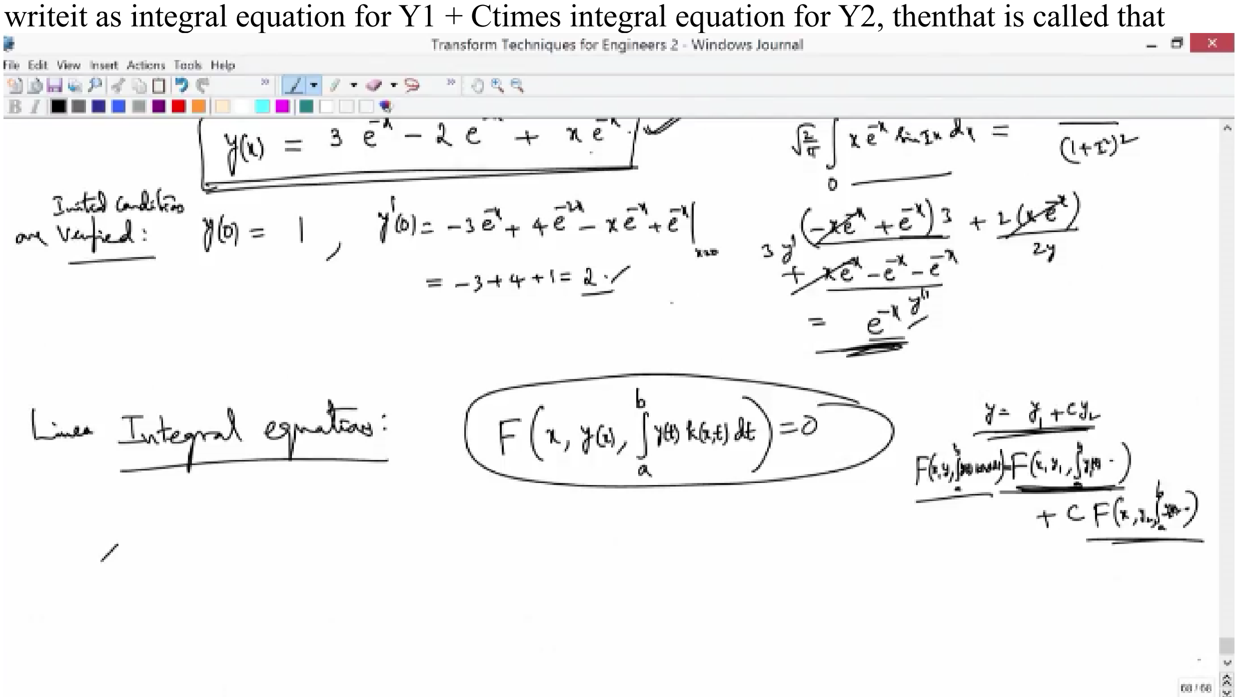



So this is one of the applications whichwe have, this is one of the applicationsof the Fourier transform that involvesboth full Fourier transform or Fouriercosine transform or Fourier sinetransform.

So let's move on to some otherapplications you can also solve tocertain type of integral equations,as I said an integral equation is so letme define now formally what is theintegral equations, integral equationslike differential equations, in thedifferential equations your unknownfunction Y,so any function of X, Y it'sderivative Y dash = 0, this iscalled the differential equation becauseunknown function Y is under thederivative Y(x), right this is themeaning of Y dash.

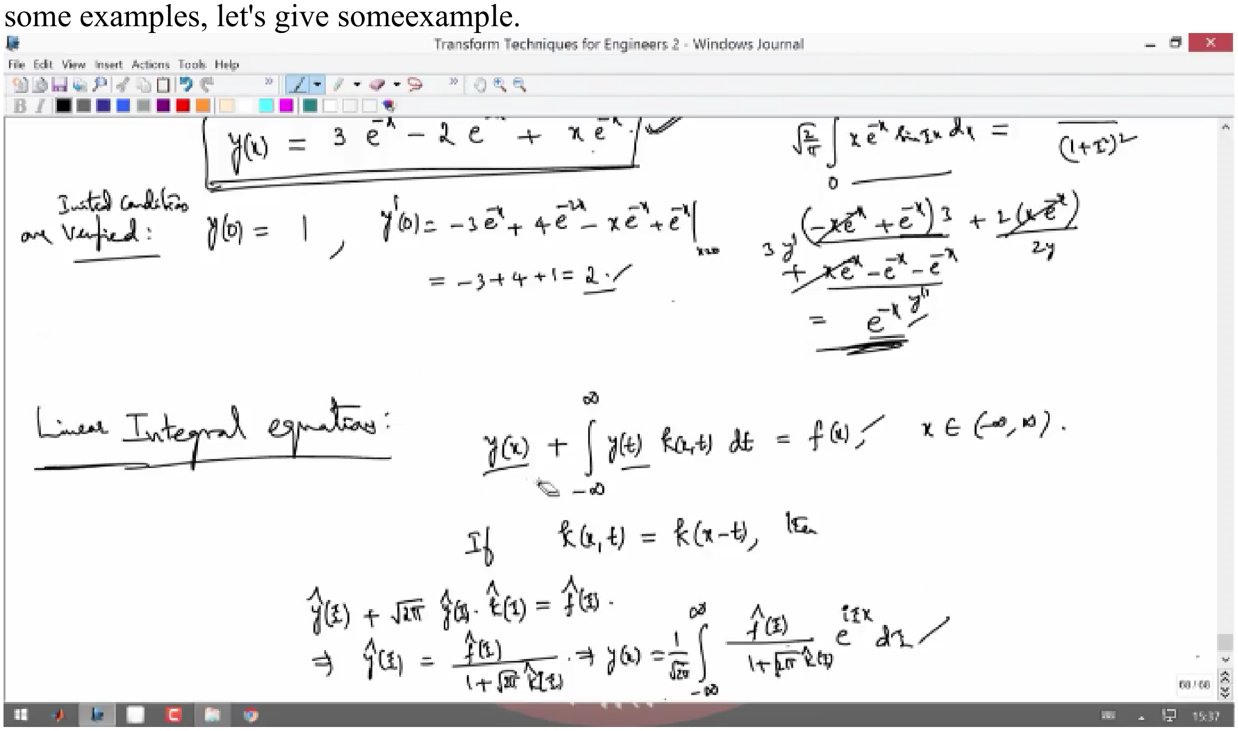
So in your equation youhave an unknown that is under thisdifferential sign, derivative of thisunknown function, integral equation isalso you can think of function of X Y(x)and instead of derivative you have thisintegral, and your domain should be, canbe anything, so some domain let us say Iuse Ato B of Y(x), and if you want thisto be function of X, so let's write Y(t) is an unknown, and some kernel, somefunction K(x,t) and youintegrate under integral sign you havethis unknown function, okay, this youshould have so such equations areintegral equations.Any function of thisform is called an integral equation, sowe can solve certain type ofintegral equations, linear integralequations that means the term, this termwithout Y, linear means you put Y, so Yshould be youonly Y terms not Y square or any higherY’s, let me put it, so this is a linearequation, so as usually the definition soit involves only Y and integral, underthe integral only Y,Y(t) not Y square kind of thing totally. If youformally mathematically if you can writeinstead of Y you put Y1 + CY2, in the place of Y you should getback F(x,y1) under this integralsign Y1(t) whatever okay, so thisplus C times, C comes out again sameinstead of Y you have Y2 integral A toB, Y2(t) K(XT, DT) okay, so that hasto be 0.So in integral equation + for Y1, + C times integral equation for CY2 that is what you should get for F(x,y) integral A to B, Y(t), K(xt,dt).

Integral equation for Y,youshould be able to split, if your Y isin this form you should be able to writeit as integral equation for Y1 + Ctimes integral equation for Y2, thenthat is called that



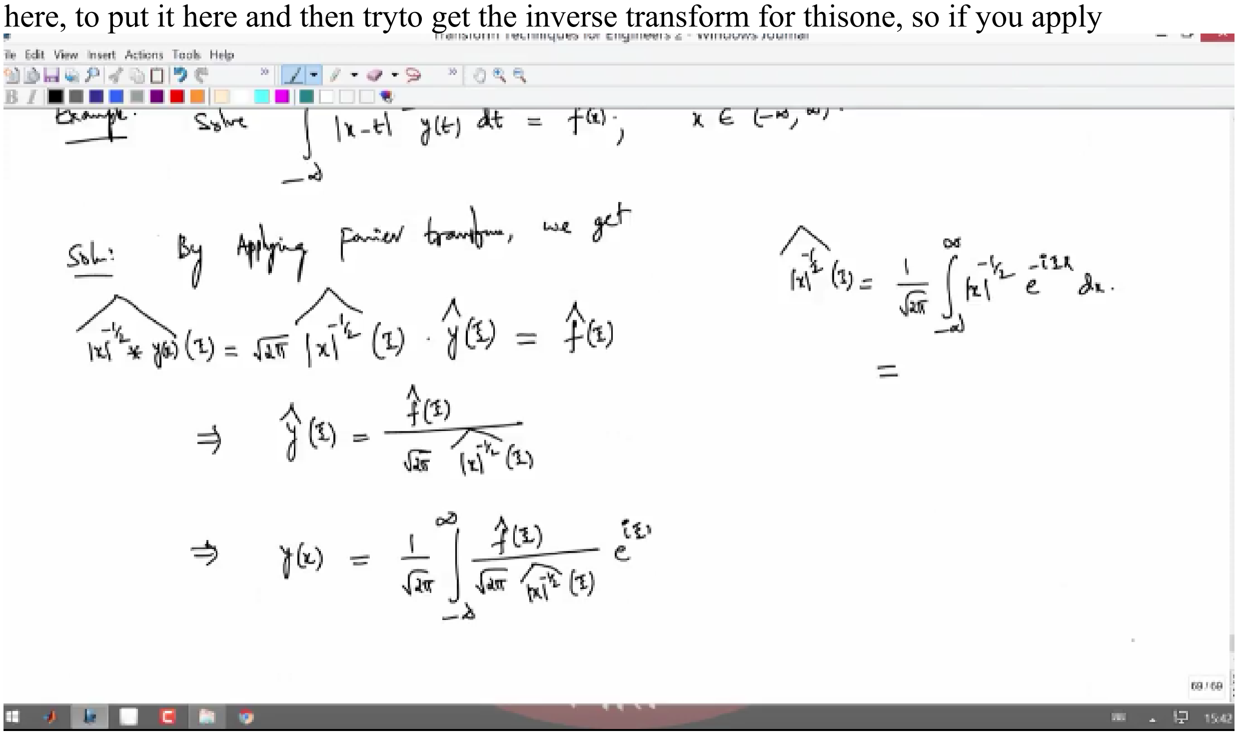

such equations arelinear integral equations, so you cansolve certain type of linearintegral equations, let me write exactlywhat kind of form of linear equationsyou can solve, so I will remove thisgeneral integral equation, so let mebriefly give you what kind of equations you can solve, so you have Y(x), let's say, so let's choose some A(x) it's a given function plus under thisintegral sign let's choose from A to B, Ato B means you can also extend becausewe know let's say let's take from -infinity to infinity, so that we can use thefull Fourier transform, Y(t) here some K(x,t), okay and DT this is equal to F(x), so Xbelongs to - infinity to infinity, ifit is

like this you can apply your Fourier transform okay, so if you apply you candivide both sides if F(x) is nonzeroyou can apply, so you can write likethis, so you can apply both sides say A(x)will, you can write it here, you can absorbit here by A(x), you call whatever hereas F mu F(x), so here K(x,t) divided by XSmu K(x,t), so yourtypical equation can be in this form oryou can remove this and keep it likethis, that is also you can, that is alsocalled linear equation, okay. So different type of linear equations, sothis A I'll put constant let us saysome constant L where L is either 0 or 1,if it is 0 it is a, it's a first kind ofequation, if it is 1 is the second kindequation because outside this integralyou have an unknown, if it is there youcall the second kind of equation of onetype, and if L = 0 this isfirst kind equation, so these are type ofequations which you have, so whatkind of equations we can solve is whenthe kernel,this K is called kernel,this K(x,t) if you want to apply Fouriertransform here, this should be this kindof convolution function, convolutionintegral, so the convolution meansK(x,t) should be K(x-t), soit should be in this form, then I canapply both sides Fourier transform, forexample if I apply Fourier transform, if Ihave this then what if you apply aFourier transform Y cap(xi) for thefirst term + here, if you applyFourier transform, Fourier transform ifyou apply we know that this is aconvolution form Y(t) into K(x,t) isFourier transform of this convolution, okay.
This is convolutionintegral, so Fourier transform will be Ycap(xi) into K cap(xi), alright, sothis is what you get in the lefthand side, the right hand side you haveF xi(xi), so you can easily write Y cap(xi)as F cap(xi) divided by 1 + root 2 pi into K cap(xi), so from thisif you take both sides inverse Fouriertransform you can get your function F(x),that is satisfying this integralequation, Y(x) as Fourier integral Fouriertransform, Fourier integration of, inverseFourier transform of F cap(xi) this isan integral which you have to evaluate,if you really want form of solutionwant, so this is actually you knowalready that 1/root 2 pi integral-infinity to infinity, F xi E power Ixi X D xi, so this is function of Xbecause you're integrating with respectto xi, xi will go, so this is the exactlythe way that you want, that is the unknownfunction that satisfies this integralequation, this is how you solve thislinear integral equations, we will try togive you some examples, let's give someexample.

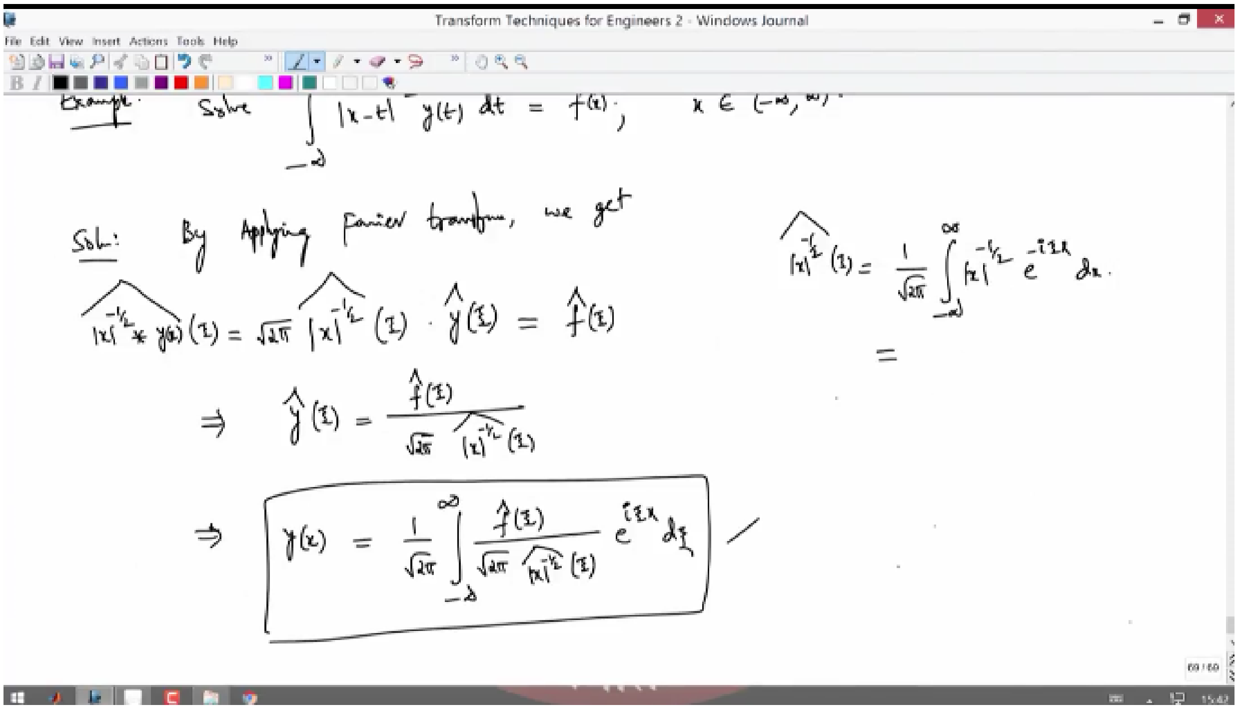


Let's do some example now, solveY integral - infinity to infinity,modulus of X-Tpower - of Y(t,dt) equal to some given function F(x), so forX belongs to minus infinity to infinity,how do we solve this?So I don't have,this is a linear equation because onlyY’s are there,there is no way squarekind of thing and your kernel is inthe form of X-T, so I can solvethis by Fourier transform method, bysolution, by transform method, by applyingFourier transform because the domain isfull space, if your domain is only 0 toinfinity you can apply Fourier sinetransform depends, okay, Fourier transformwe get, what we get, the left hand sideso you have root 2 pi modX for which you take the Fouriertransform whatever you get,okay,that’s function of xi times Y cap(xi) this is because left hand side isbasically convolution of, because why isthis so? Because this is actuallyconvolution of mod X power -1/2 convolving with Y(x) for which ifyou take the Fourier transform this isexactly what you have, Fourier transformof F convolution G is root 2 pi timesconvolution of this, transform offirst function and transform of secondfunction that is exactly you have by theproperty of the Fourier transform.

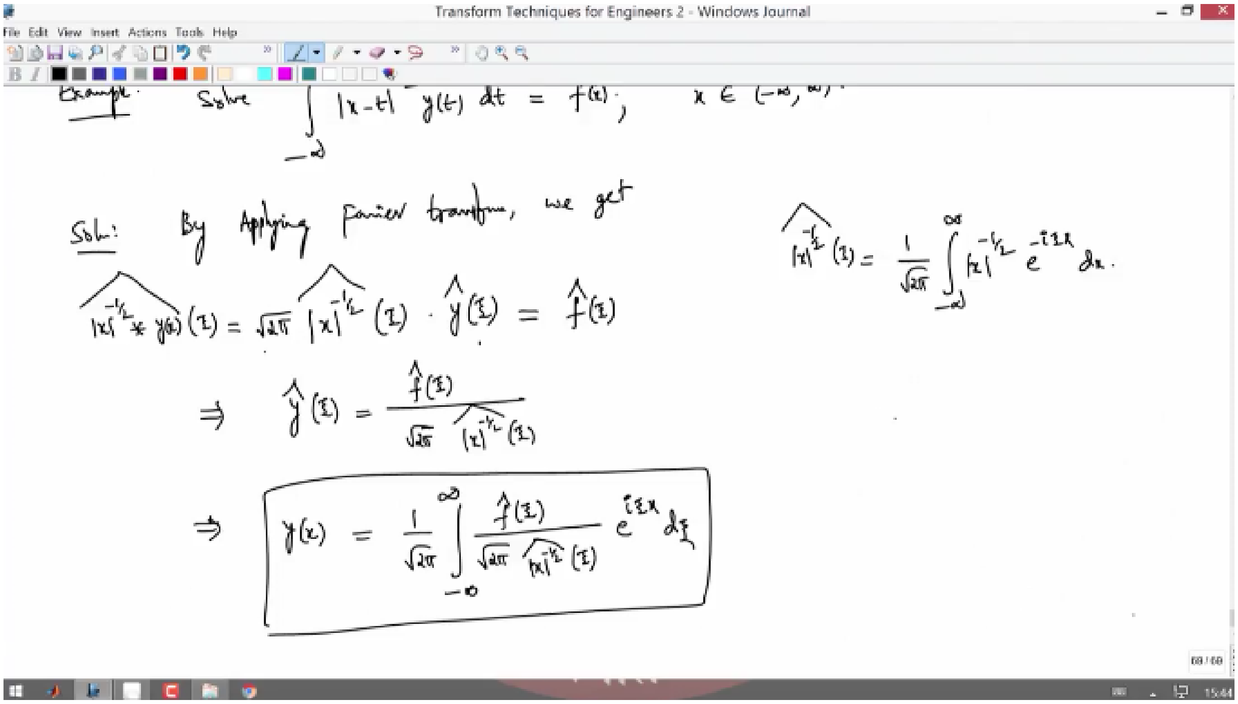
Nowthis is your left hand side, the righthand side is simply Fourier transformof F, so what you get?You need lookingfor Y as a solution, so you try to Y xi as F xi of, F cap(xi) divided byroot 2/pi mod X power -1/2 Fourier transform of this, so if we actuallycalculate that,so you need to calculate what is 1/root 2 pi Fourier transform of, right, sowhat is this one?So you calculate mod Xpower -1/2 Fourier transform of xi, what is this one? By definition 1/root 2pi mod X power -1/2 E power - infinity to infinity, E power -I xi X DX, so this you need to find toproceed here, to put it here and then tryto get the inverse transform for thisone, so if you apply



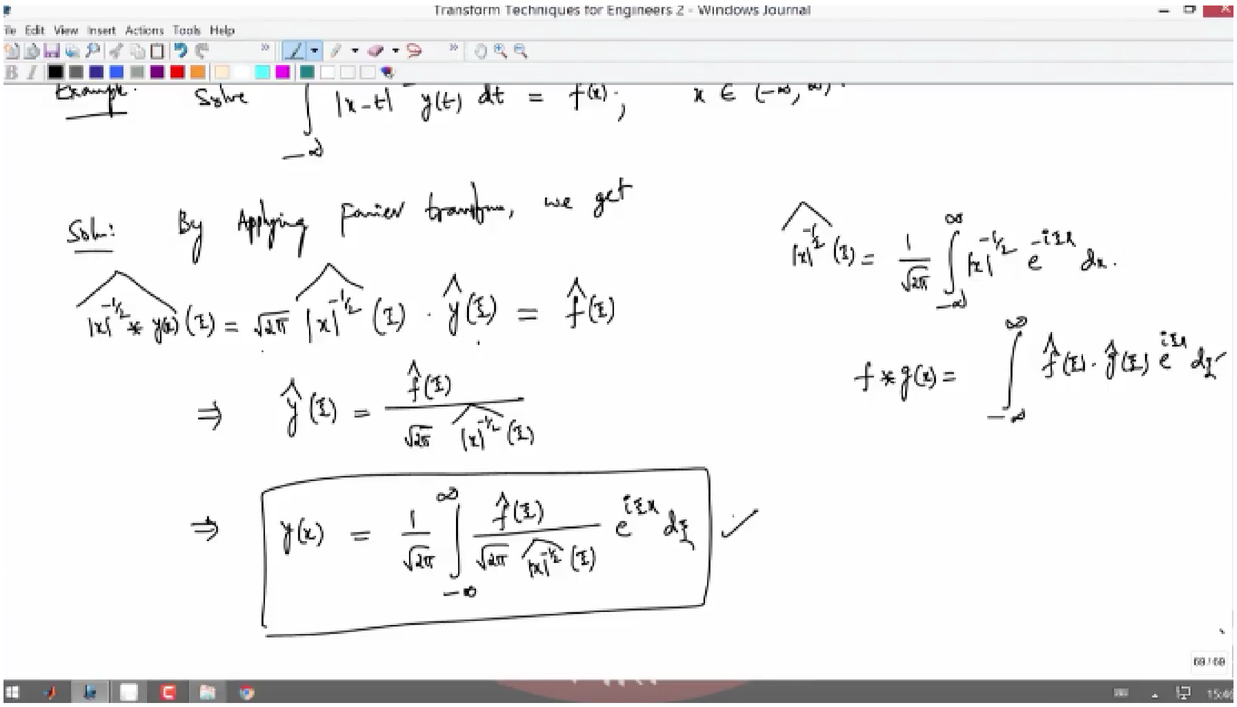

this finally if youtake the inverse transform here 1/root 2 pi - infinity to infinity, F cap(xi)divided by root 2 pi times Fouriertransform of this, after getting this youput it here mod X power -1/2 whichis a function of xi, E power I xi X D xi, this is exactly the solution, okay.



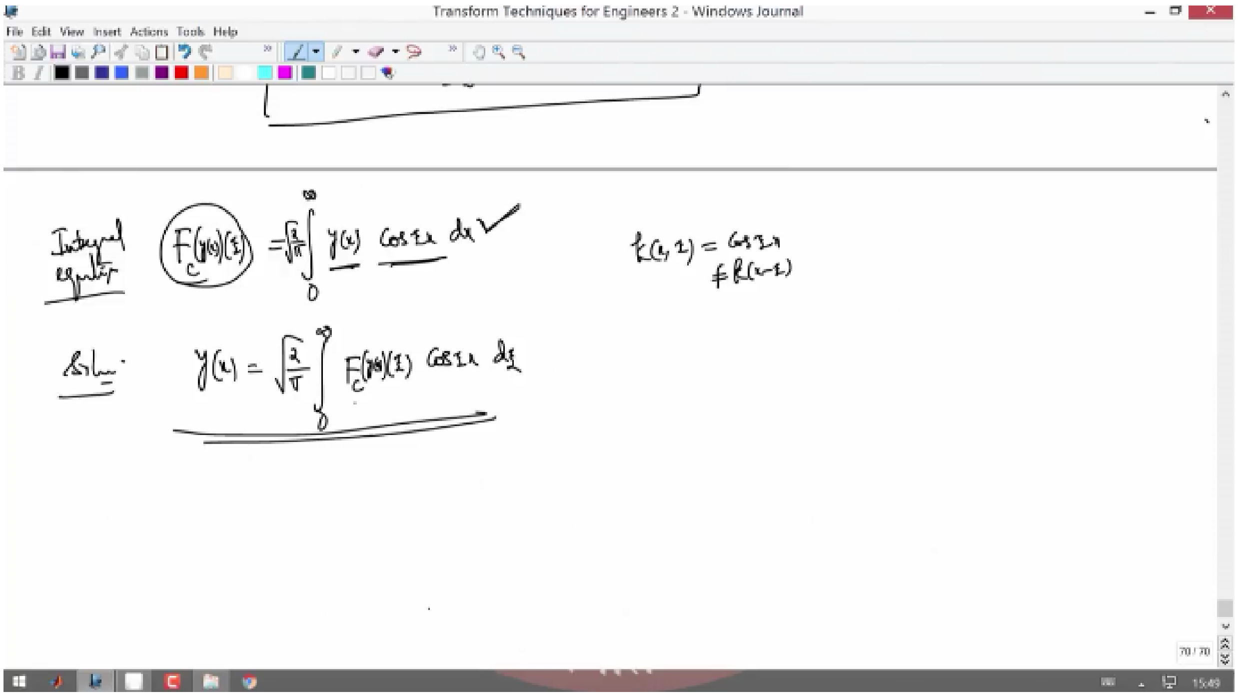

So youtry to get this and put it here, you tryto get this and put it here and you canevaluate what is that you get, if youactually see this so maybe we have notdone this earlier, so maybe thesethings we'll see in the next video howto calculate this part, and then we'llput it here and try to evaluate, how we can evaluate such interiors, andalso so basically so that tells you thatsee if I try to substitute, try to getthis part and put it here and this is, weneed to,we can try to evaluate thistransform this integral, thisintegral we can try to evaluate, soagain we use certain properties of Fourier, certain properties of the Fouriertransform to evaluate such integrals,so that is again so we use the sameproperties, so you have the convolutionif you have a product of 2 suchtransformwhich you know, so root 2 pi times Fouriertransform of something into Fourier transform of something, suppose you have 2such functions you know already, theninverse transform of this is simply,inverse transform of this, this isactually inverse transform of this function here, what you have is here?Thisis the inverse transform of thisfunction, so if this function is what youhave is here, you can write like root 2pi, I mean 2 functions, product of 2 functions into root 2 pi inversetransform of this will be simplyconvolution function of this function,inverse transform of this and inverse transform of this, that's whatthat is the property we use, okay.




Soconvolution function transform, sosomething like if I write Fconvolution of G(x)Fourier transformyou take root 2 pi because without definition this root 2 pi comes out,Fcap(xi) times G cap(xi) this if youtake the Fourier transform, if you take theinverse transform if i write likeinverse cap okay, and if you apply herethis inversion will cancel this Fourier transform, so you end up gettingthis, so what is the sign means? This isactually 1/2 pi by definition 1/root 2 pi okay, root 2 pi so integral -infinity to infinity, so this root 2 pi thisroot 2 pi goes E power this times Epower I xi X D xi okay, so this isexactly what you have, so you can usethis once you have this convolution youapply Fourier transform and it's inversetransform.so you can see thatconvolution you can write a product ofFourier transform of F and G andwith the exponential function if youintegrate that is exactly what you get,so such property if we use here to evaluatethis in the next video,okay.

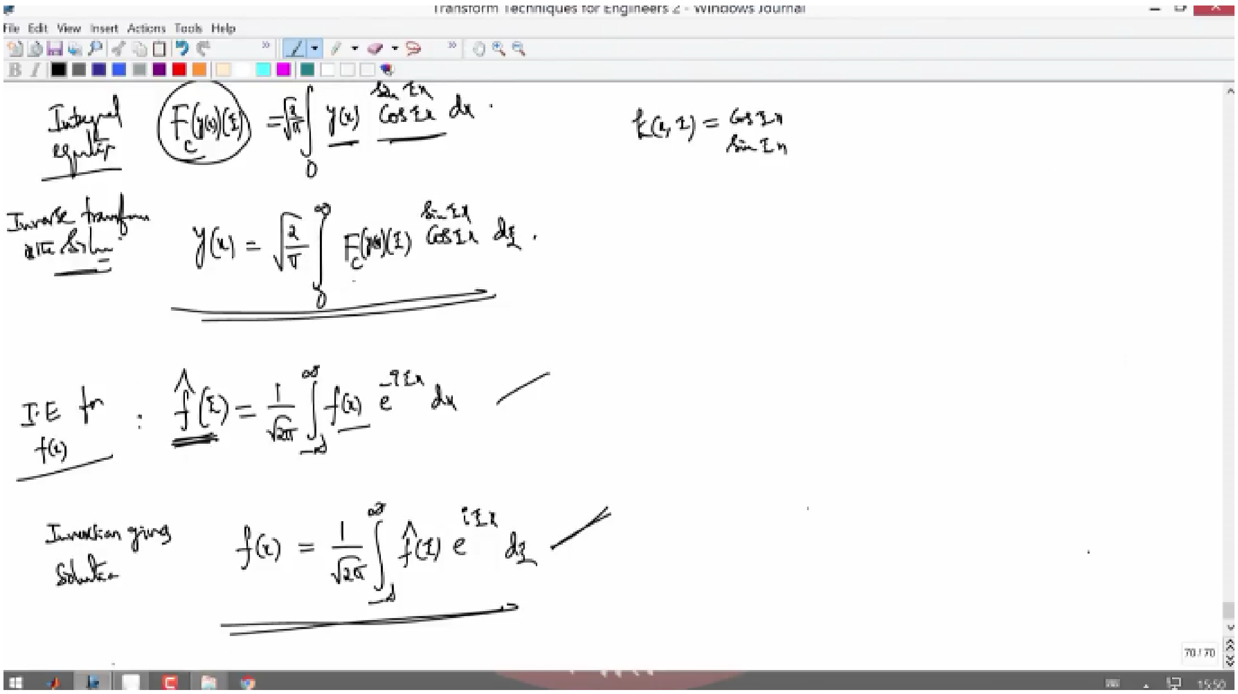



So we'll see that this is one suchapplication of Fourier transform where the domain is -infinityto infinity, and we can use full Fouriertransformand also another, so thisis a restricted, only certain integralequations you can solve when the kernel is of K(x,t) x-t form that isit's already that integral, theintegral equation, the integral term itshould be like convolution type ofintegral, so that we can apply theFourier transform you can apply that, you can apply the property of theFourier transform on the convolution of2 functions, and then you can get yourFourier transform of the unknownfunction, and you invert it you can getback your solution, so this is how you can solve, sowe have already have suchintegral equations, for example Fourier cosine transform Y(x) if you have,what is the Fourier cosine transform?Fourier cosine transform of Y is cosxi X DX, this is from 0 to infinity and wehave root 2/pi as your Y cap of, thisis Fourier cosine transform of Y(x) whichis function of xi, so if this is given ifyou know that Fourier cosine transformof xi this is given and you have Y(x)is this, okay,otherwise let’s write inverse transform, for this if youwrite the inverse transform again root 2/pi into 0 to infinity, Fourier cosinetransform of Y(x) which is function ofxi, cos xi X D xi, so whichever isunknown, so each of these are integralequations for the unknowns Y(x), here the unknown is Fourier cosine transform, ifyou have this if you know this one,and if you know this given that thisis known, you can get this Fourier, ifthis is, if I give you this as some known function, then this is an integralequation with unknown as Y, you invert ityou can get your solution,this is an integral equation, though itis whose kernel is not of the formwhich you choose K(x,t) here X xi iscos xi X which is not of type K(x-xi) though, okay, so but here these are thesimplest those Fourier cosine transformof sine transform simplest, linearintegral equations, okay, so here alsojust by inversion, the solution iswe're not applying so if you apply hereif you apply Fourier inverse transformhereI'm just getting directly a solution,okay.

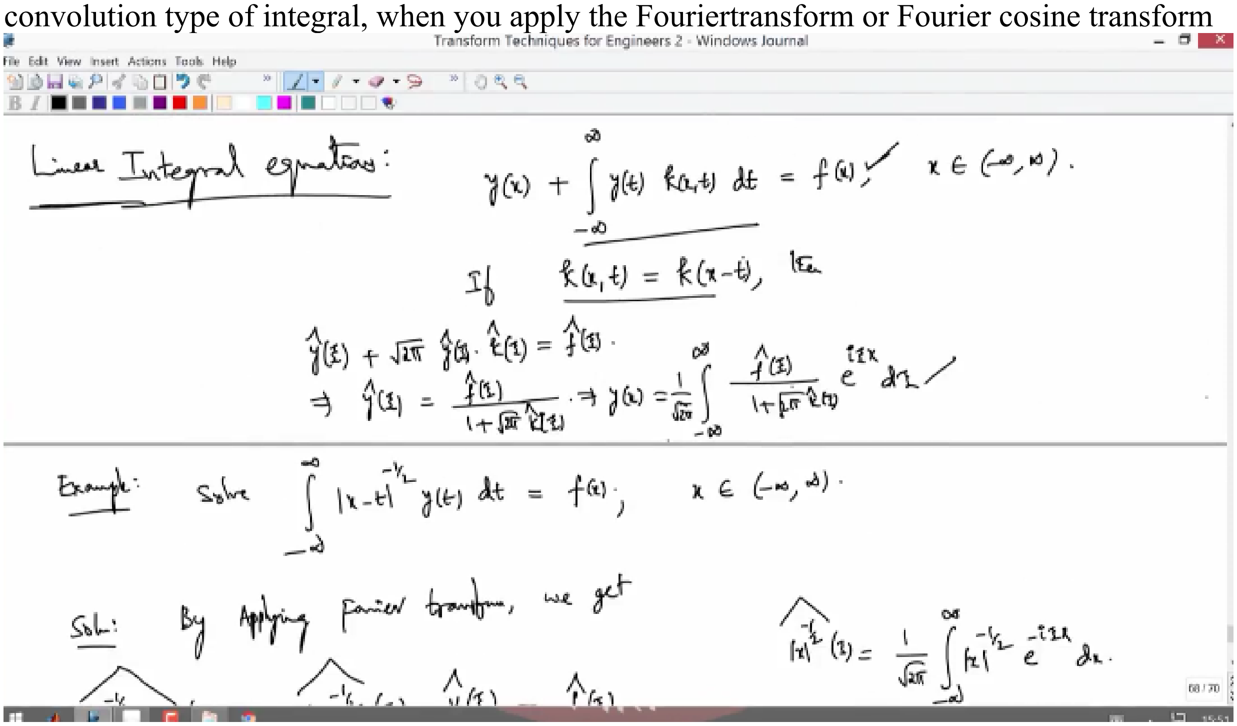



So here if you wantthis as an unknown, if this is an unknownso again same thing, okay, so if thisis your integral equation you have asolution by inverse transform, inversetransform as a solution, is this a solution,so you have the solution for this integralequation though it is not in this form,not only such thing when K is of thisform you can solve by earlier methodotherwise also if it is simple cosinefunction or sine function, if it is asine also same situation, so instead ofcosine you have sine xi X, so you cansimply inverse transformation will giveyou the solution, these are simple linearintegral equations, okay, simplest typeFourier transform and Fourier cosinetransform and Fourier sine transform, theseare simplest kind of integral equations,for example F again F cap(xi), this is aFourier transform 1/root 2 piintegral - infinity to infinity F(x)into E power - I xi X DX, this isan integral equation for unknown F(x), okay.

So what is its inverse?Inversetransform will give you the solution,inversion gives solution, what is the solution? F(x), so inversion actually will give me F(x) = 1/root 2 pi - infinityto infinity, as though this is given, if this is giventhis is an integral equation IE,integral equation for unknown F(x), giventhat F cap is known, so if F cap is knownthis is known thing I'm putting here, andyou know by inverse transform this isthe case, this is your solution, okay, soyou have an integral equation, you have asolution either it is, these are thesimplest integral equations Fouriertransform either cosine, sine, or full Fourier transform, these are all simplest




integral equations for which you havesolutions by the inverse transform, andif your kernel, integral equation, ifkernel is of this form and K(x,t)K(x-t) then the integral termwill be convolution type of integral, when you apply the Fouriertransform or Fourier cosine transform




you can write it as a product of a unknown, Fourier transform of the unknown, and theFourier transform of the kernel.

So that you can get your unknown, Fourier transform of theunknown function you can write it assomething known, and at the endyou take the inverse transform to getthe solution, only thing involved is how to evaluate such integrals of Fouriertransforms, and when you take the inversetransform those integrals you have toevaluate to solve these integralequations, these are the simplestapplication, so you can also solvesome integral equations if it's in thisform, okay, so this is another applicationwhere you can use this Fourier transforms,we will see as I said in this example, we will try to evaluate thisintegral in the next video, you may needsome more Fourier transform required, so for example this one, thisFourier transform you may require andthen to avail it, to know what exactly isthe solution, what exactly this integralyou need to evaluate this Fouriertransform of this, and then you have tosee how to evaluate this using knowntransforms, okay. So we will see that inthe next video.Thank you very much.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