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NPTEL ONLINE COURSE

Transform Techniques for Engineers

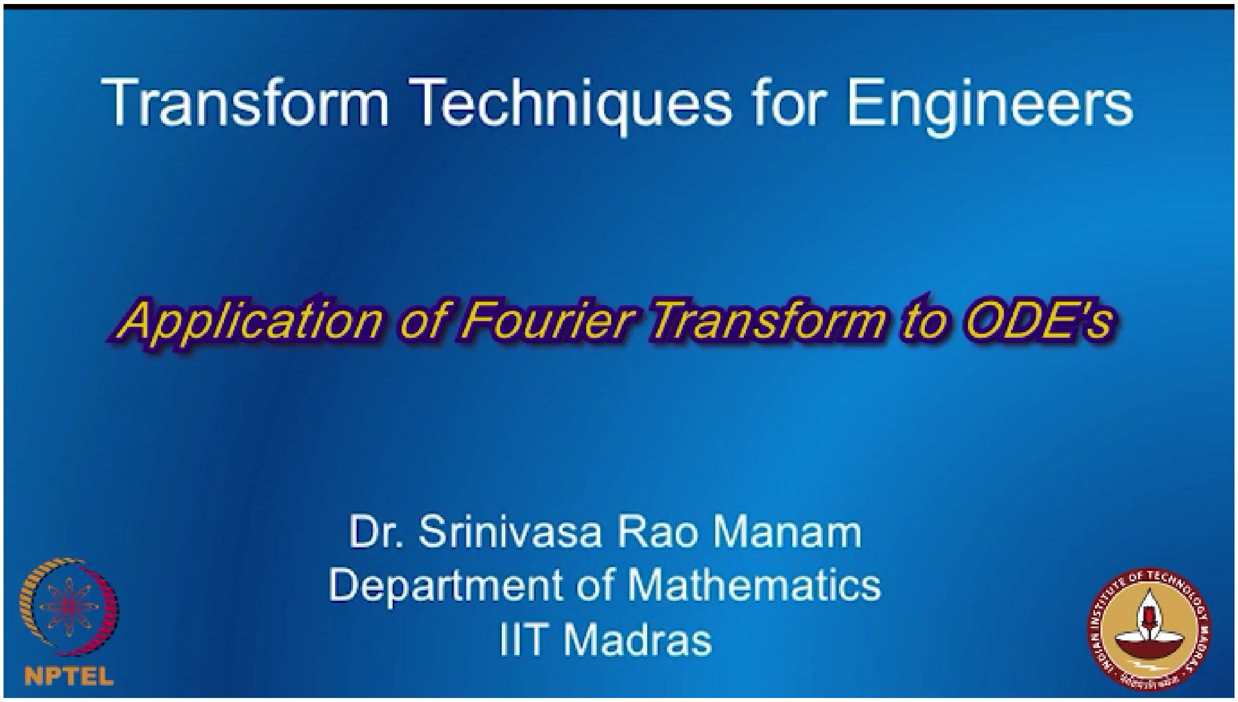
Application of Fourier Transforms to ODE's

With

Dr. Srinivasa Rao Manam

Department of Mathematics

IIT Madras



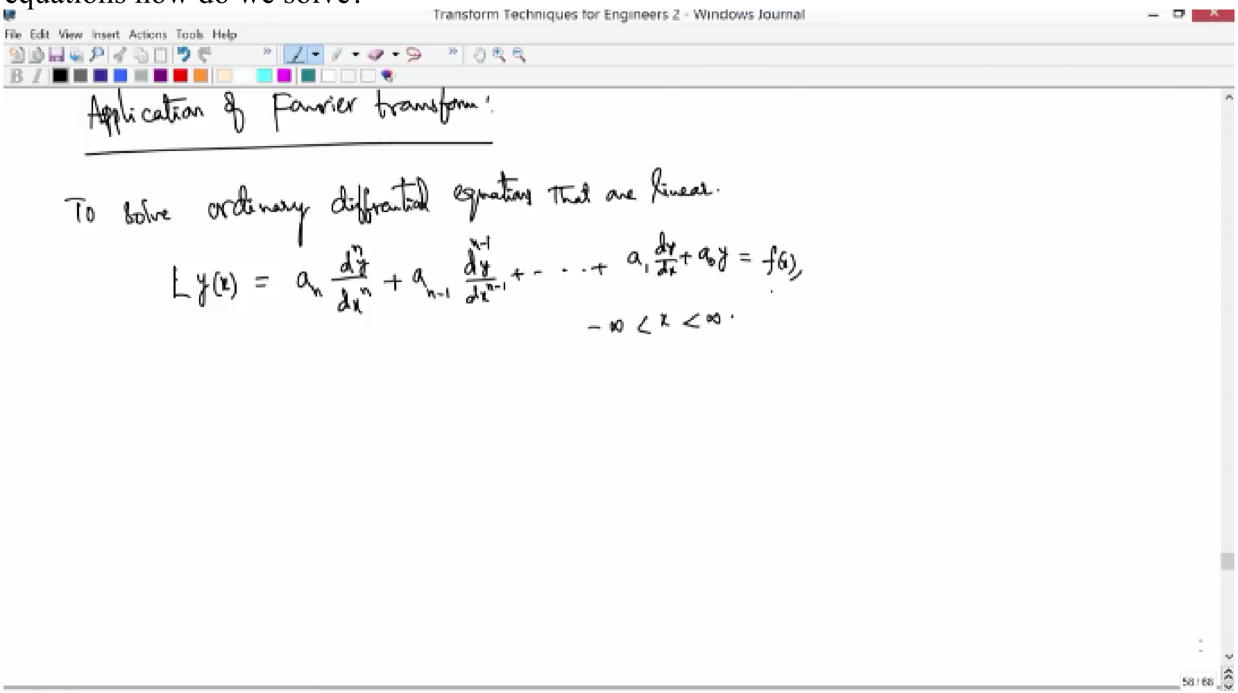
Welcome back,the last video we have seen therigorous proof of Fourier integraltheorem, so like that given a function F(x)if you start with and do the Fourierseries,write its Fourier series and youfix some X values you will see that youend up getting some numberseries, and that number series, the value of the number series, someof the series is known.

The same way inthe Fourier integral formula you cansubstitute some non-periodic signalthat is a piecewise continuous andabsolutely integrable function F(x), left hand side is F(x) and the right handside you have a double integral and if you can evaluate to onepart of the integral and whatremains is the other integral, so we will see that some integration, some integrals values you knowanalytically you know from this formula, okay, that we will do later that is oneapplication of this Fourier integralformula.

So once you have this Fourierintegral formula, so now we can definewhat is a Fourier transformer and its inversetransform, so that is legitimized, whatever we have defined its Fouriertransform and it's inverse transform andyou have certain properties that we haveproved based on these properties youmake use of all these properties and wetry to solve some problems, some firstthing is what do you mean by problem,problem is, you try to applythis Fourier transform to solve certainproblems in physics and engineering, sofor example problems means either thereare ordinary differential equations withinitial conditions or ordinary differential equation over adomain with boundary conditions, sosomething like that if you have someordinary differential equation, linearordinary differential equations you caneasily solve by applying Fourier transform.

And the same way if you have somePDE’s, partial differential equations,so partial differential equations if youhave seen in differential equationscourse any linear partial differentialequations in two variables so that isthat you can classify into three type oftypical equations,so a typical of such equationsfor elliptic type wehave a Laplace equation, and forparabolic type it's a heat equation, andfor hyperbolic equation, hyperbolic typeequations you have a wave equation, soyou have typically three equations which are a wave equation, heat equation, andLaplace equation that is you can alsothink of 2-dimensional version ofparabolic heat equation that is a steadystate of a heated plate, okay, so ifyou think of like that so it's you willget a 2-dimensional Laplace equation,and so these are the 3 typicalequations which you can thinkof as a from, any second,in two variable, second-orderpartial differential equation in 2 variables.Then you can solve suchproblems over certain domains that'swhat we have seen in the differentialequations course if you have seen early,if you have done differential equationsfor engineers you have seen how to solvepartial difference, such partialdifferential equations in certaindomains.
So those similar such problems,these linear equations, these areactually linear equation all three types,these equations with some boundaryconditions are initial conditions, you canactually solve by using the transformtechniques either Fourier transform thelater on you can apply a Laplacetransform at other transforms which youcan is, okay, so you have to suitably, you haveto use these transforms and try to solvethis boundary value problem, so that is one such application so you cansolve ordinary differential equationswith initial conditions, partialdifferential equations with initial andboundary conditions and this is oneapplication and you can also, based onthe properties of the Fourier transformyou can evaluate some integrals othertype of equations that you can solveother application of partial Fouriertransform is solving integral equations,so integral equation is you havesomething like same like it'sanalogous to the differential equation,in the differential equation what wemean is an unknown in the derivative, soin the same way if you think of, if yourunknown is in the integral under some,under integral that is called theintegral equation, so if you cansolve certain type of integral equationI will give you the certain form of suchlinear integral equations you can solveby a Fourier transform, so that isanother application of this Fouriertransform so we will see one by one.
Solet's see the applications, application of Fourier transform,Fourier transform nowwe mean many, so what we mean is Fouriertransform so either Fourier coefficientthat is a Fourier transform as in acomplex form over a finite interval oryou can think of if it is a evenfunction or odd function depending onthat you have either sine transform or cosine transform you call this CN,N evenyou call this cosine transform orrather if you are working with CN, CN’s are a complex Fourier transform of afinite signal, and if you have Fourier series in terms of AN,and BN,if AN’s are 0 and BN’s are nothing but Fourier sine transform,and AN’s are Fourier cosinetransform is just for the namesake, sothose are over for transforms over afinite intervals, okay,so later on what we have is a Fouriertransform over a full real line that isin terms of exponential functionintegral minus infinity infinity F(x) into exponential function.
And also wehave defined in a different place so ifyour function is defined only on from 0to infinity and semi-infinite intervalsif you have, you have a cosine transformand sine transform and it's inversetransform, so depending on what is yourdomain,we will try to apply there one ofthese transforms ordinary differentialequation, first application is to solveordinarydifferential equations,equations that are linear, linear of any order so let uschoose LY, so letme call this LY is anunknown, this is depending on only onevariable, one independent variable X sothat is what is called ordinarydifferential equation, this is you havesome constant AN so and you have thenth derivative of Y and you have AN-1,N-1 derivative of Y andso on you end up getting A1

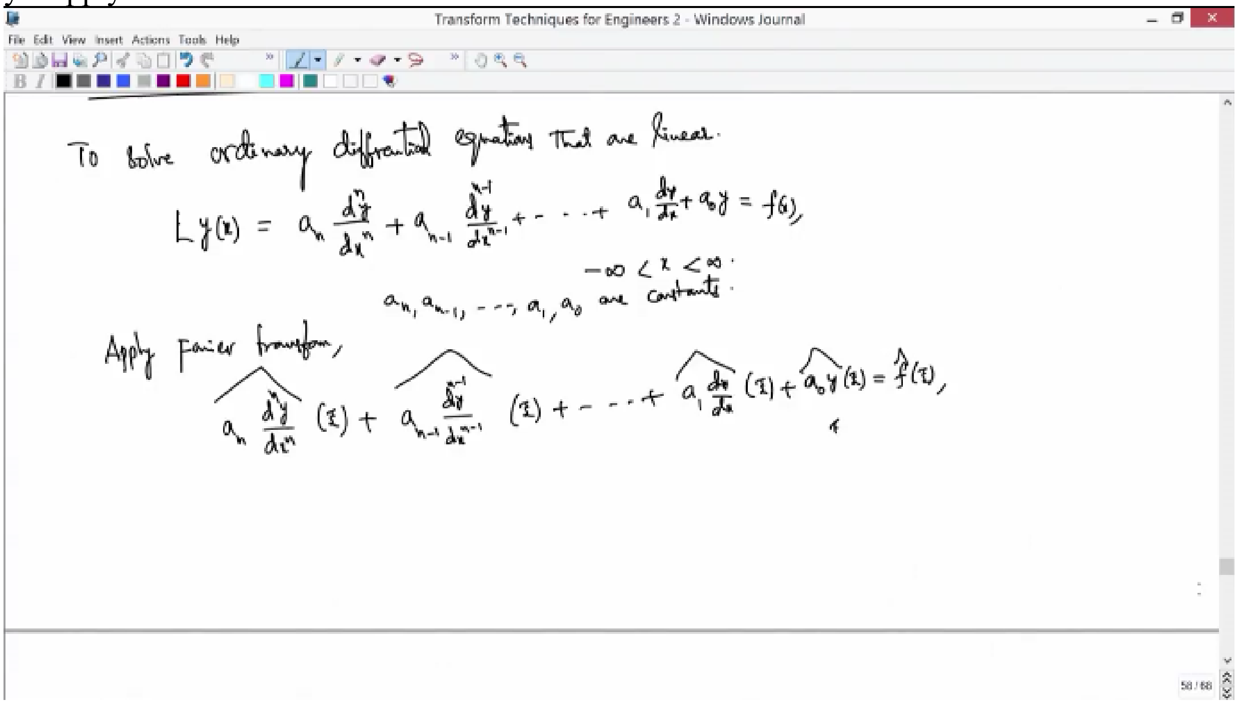
DY/DX+ A naught,A naught of Y equal to, ifyou have some forcing, so in the systemso normally you get the right-hand sideF(x) this is called a forcing term,that means an applications basically ifyou have some force outside force thisis the governing equation will have on the right hand side that is anon-homogeneous term, so you have thissuch equation is if you get aequation like this, this is a linearordinary differential equation of orderN,okay, so such equations how do we solve?



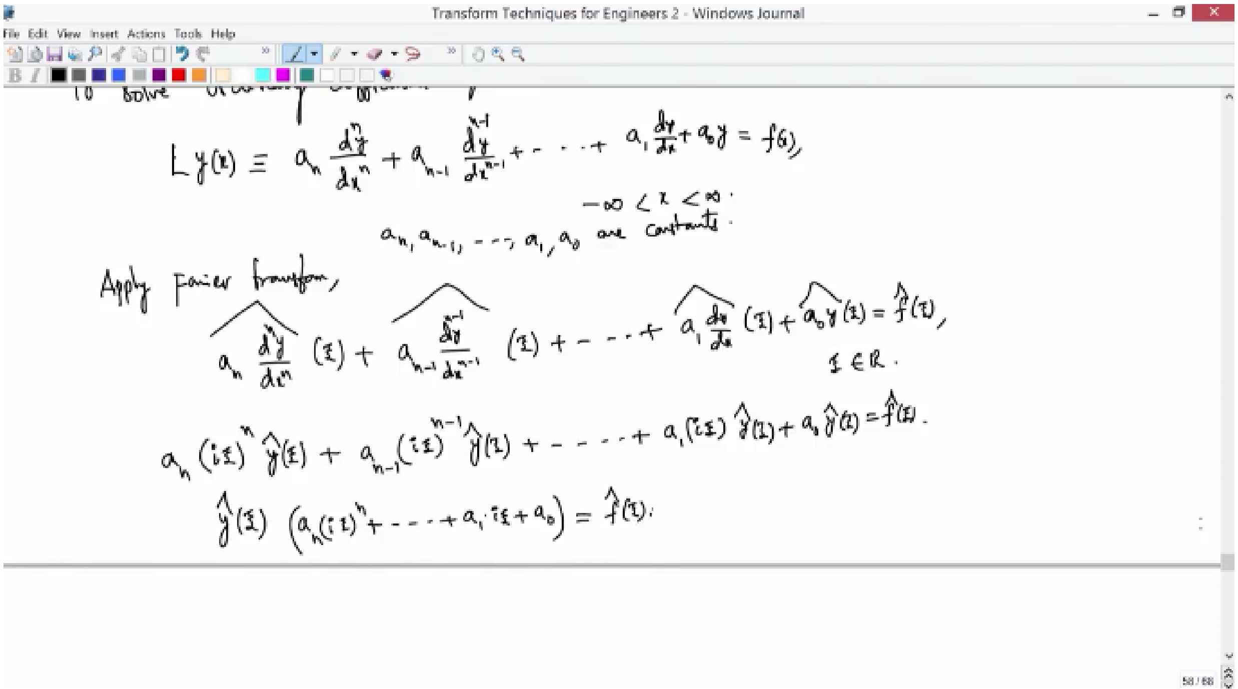
So first of all to see this equationimmediately,what is your domain? Domain is fromminus infinity to infinity, so that iswhat it means if nothing is givenmeans our domain is minus infinity toinfinity, so how do I solve this by usingFourier transform?So you will have aFourier transform over full real line soif you apply Y cap you apply Fouriertransform, forexample if your domain is 0 thenI apply Fourier cosine transform or sinetransform, okay, and once you have thisdomain the boundary is this, X = 0is the boundary on that you should havethe information that is N initial conditions and boundaryconditions you should have, okay,so at X= 0 this is an initial valueproblem so once you have this 0 toinfinity because there is a boundary inyour domain,so at which you have to give theboundary initial conditions, at 0 youshould provide the initial conditions, soY at 0 and Y dash at 0 up to YN- 1th derivatives at 0 you have to provideconditions, so once you give this conditions you apply this cosinetransform or sine transform, so right nowwe have this minus infinity to infinity, sothese things we will see as and when youdo the problems,that depending on thedomain how we apply the Fourier sine transform or cosine transform, we willsee that when we do the applications andwe do the problems.

So right now I have to consider this general problem, generaljust equation so how do we solve thisapply, we apply Fourier transform, so whatyou get is AN DNY/DX power n for which whole function of xi and you havesimilarly AN-1 DY, these areconstants, okay, where AN, AN-1 up to A1, A0 are constants, let us take itas constants, okay if it is function of Xstill have to worry so because it's aproduct of two functions, so you have toconsider such a transform, so if it is aconstants as of now it is easy, so you willconsider like this DN-1 derivative,for this if you take the Fouriertransform and so on, so you end upgetting A1 DY/DX for which you takethe

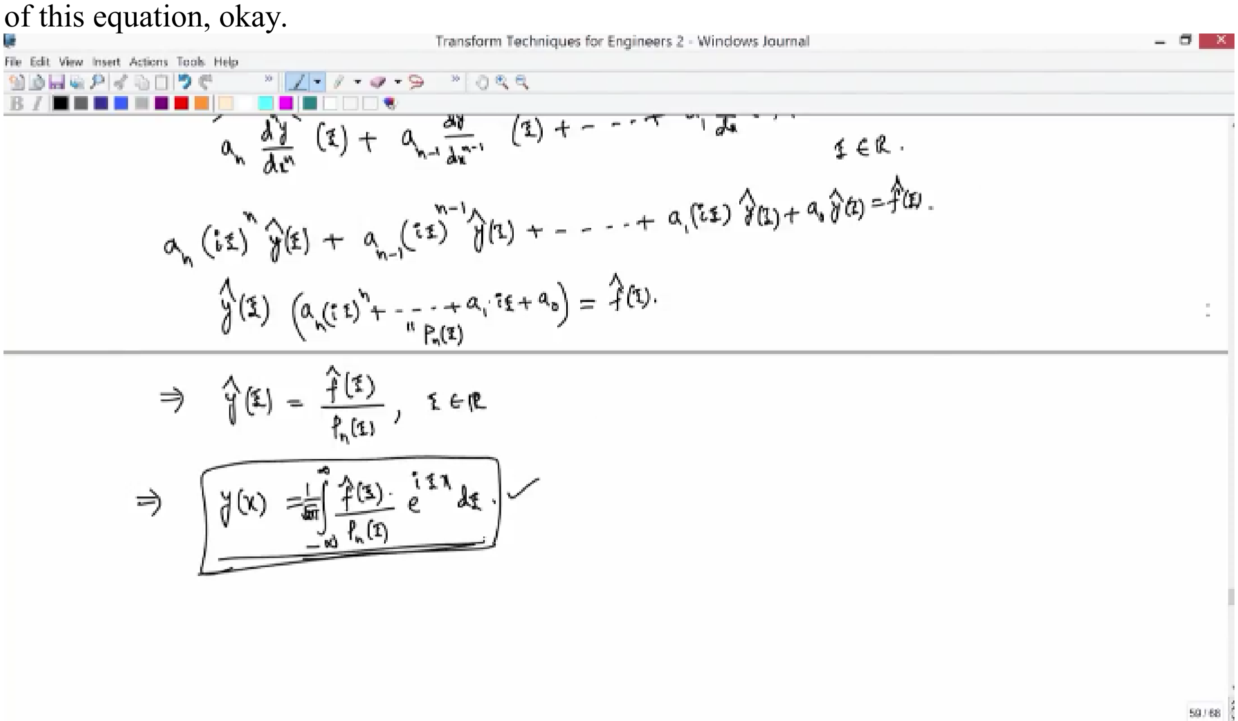
Fourier transform and you have A naught Y transform, Y is a function of X, A naught is constant, so what you get is Fcap(xi), xi belongs to full realline, so this is what you have if youapply.



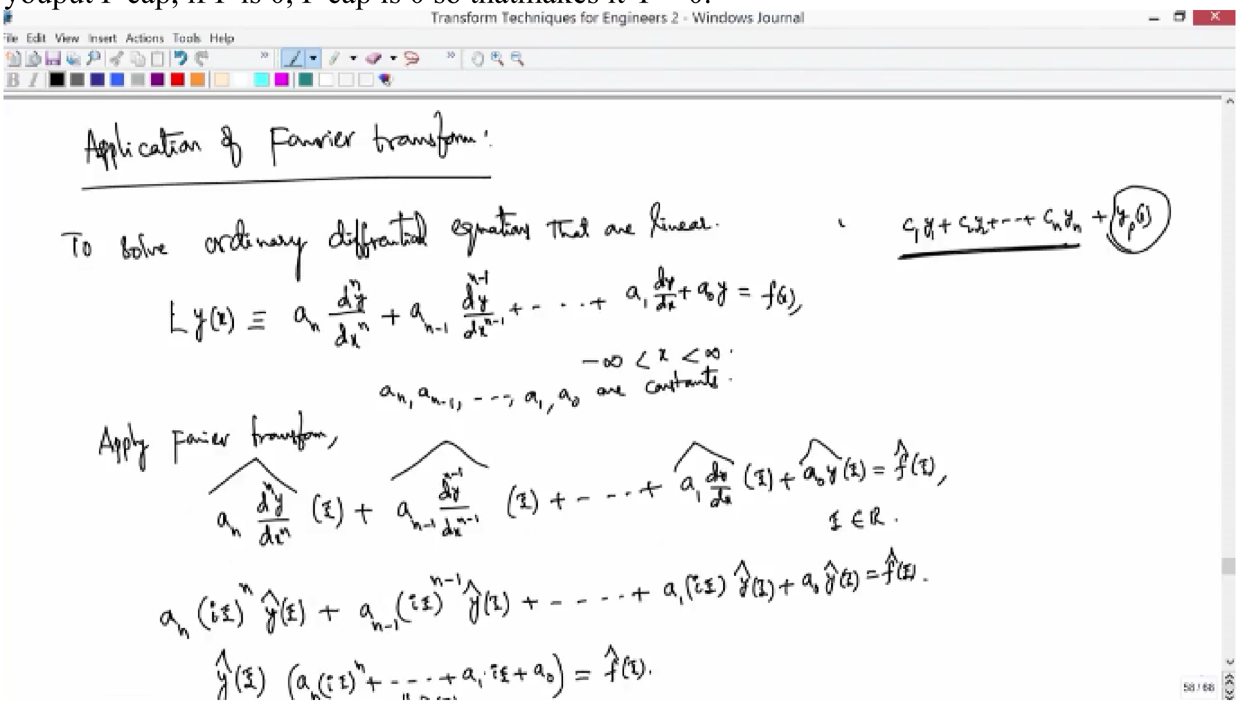
Now because AN is a, it’s linear,transformation is linear so AN is because it's a constant it comes out andthis nth derivative is I xi power Ninto, if you repeatedly do theintegration by parts and in thedefinition of the Fourier transform youbring, you send all these nth derivatives on to exponential functionand what you get is Fourier transform of Y,and this is what is the Fouriertransform of nth derivatives. So likethis you go on soyou have AN-1, constant comesout, I xi power N-1 into Y cap(xi), so you see that every time you areending up only Y cap, Fourier transform ofY so when you get A1 I xi into Y1cap, Y cap (xi) + A naught Ycap(xi) which is equal to F cap(xi) this is what you get, so what weneed is, so Y cap is, Y cap(xi) iscommon,Y cap(xi),and what is this one?This is actually equal to if you becausethe way you have defined LY is this, sowhat you end up is so L of, so this is apolynomial right so you cannot write interms of L though L is some operator, solet me write it as a polynomial, so AN I xi power N up to A1 into I xi + A naught, this is the polynomialin xi, which is equal to F cap



(xi),so if you call this some Q(xi) orwhatever, so let's call this some PN(xi)nth degree so you have Ycap(xi) is actually F cap(xi)divided by PN(xi), so what weneed?We have now this one xi belongsto R, now if you can take theinverse transform here so what you get,if you take the inverse transform for Yyou get your YX, that is exactly thesolution of this equation, so such asolution is actually inverse transformof this that is actually minus infinityto infinity, 1/root 2 pi F cap(xi)/PN(xi),E power I xi X D xi, so this is exactly your solution if youcan evaluate this nicely that is whatyou will get as a solution, so this isthe general procedure to find thesolution of this equation, okay.



So thisis one so if you actually if you knowthe differential equation theory becauseif it is the nth order differentialequation the solution you might haveseen C1 times Y1, and C2 times Y2, andCN times YN,which is homogenoussolution, plus some particular solution YP(x), what you get here by thissolution is called particularsolution, when you put Y equal to, F is0 you're getting Y = 0, okay, so ifyou take F = 0 so your solutionwhat you got by Fourier transform youput F cap, if F is 0, F cap is 0 so thatmakes it Y = 0.

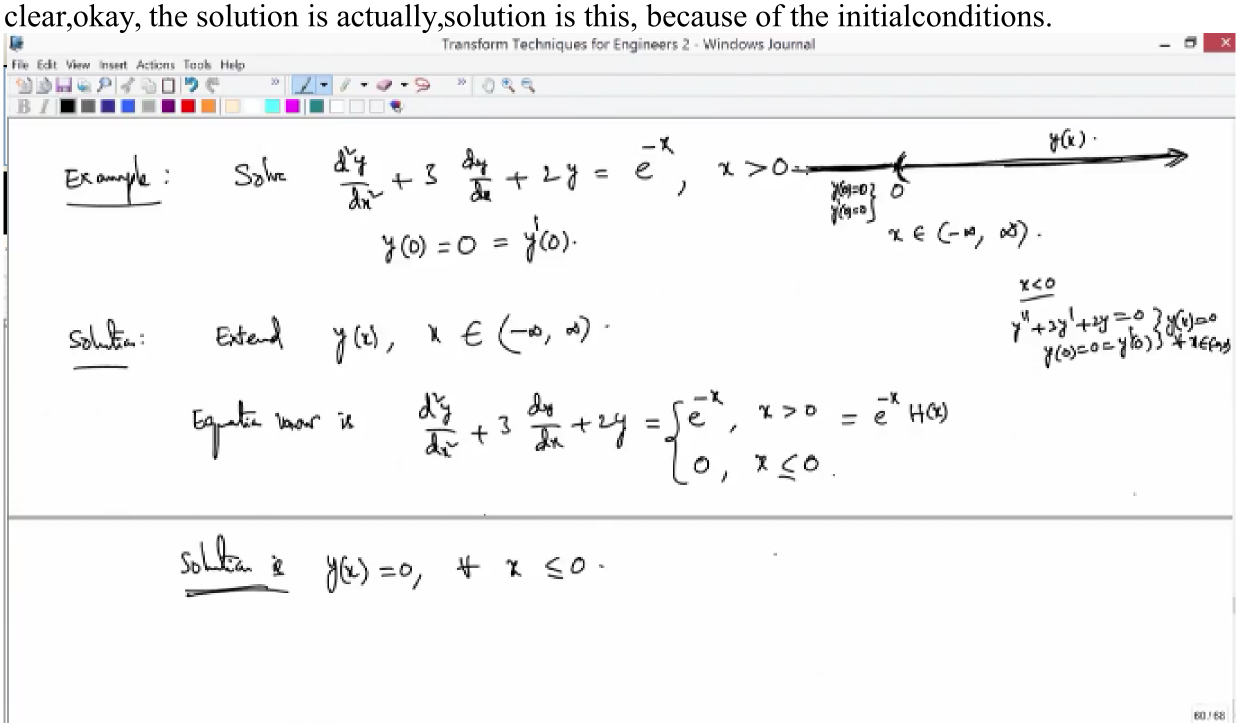


If F = 0you get Y(x) = 0, so from thismethod, okay, from this is what you get, sothat means you're not actuallycalculating homogeneous solution butthis particular solution okay, that isactually the general solution of thisequation okay, so that's how wesolve, we get only a particular solutionhere, so by just if you do like this you geta particular solution because there isno boundary involved here, okay, so if youare given a full solution full domain ifyou apply the Fourier transform what youget is only particular solution, okay.

Solet's do some examples, we start with anexample here where we use full Fouriertransform it's an example what weconsidered here is D square Y by, sosolve this problem D square Y/DX square + 3 DY/DX+ 2Y = E power –X, sowhat is surprising here is if I take Xfull real numbers E power –X, when Xis negative this is unbounded thing sothat you cannot take the Fourier transform so that is not absolutelyintegrable function, from -0 tominus infinity or minus infinity to 0, so what is given the domain is actuallypositives in this you have to solve, andthe boundary initial conditions are, sothis is Y(x), the domain is this 0 toinfinity this is the domain and onwhich you have initially because it's asecond-order equation you have got givetwo initial conditions so those are Y dash(0) and Y(0) they are given as 0, okay, so and what is the other boundary?This is infinity you can assume that Yand Y dash they are going to 0 as X goesto infinity, so this is because you havethe infinity, okay.

Once you have such athing so this is your problem, so if youview this as a initial value problem andyou can consider this as initialvalue problem if you choose onlythis one, and if you choose one of thevalues, one of these conditions here andthen if you use this one that is aboundary value problem, so let's solveonly initial value problem as of now, solet's take this initial value problemlet's solve and then solution goes likethis, so how do we solve this?What isgiven is on

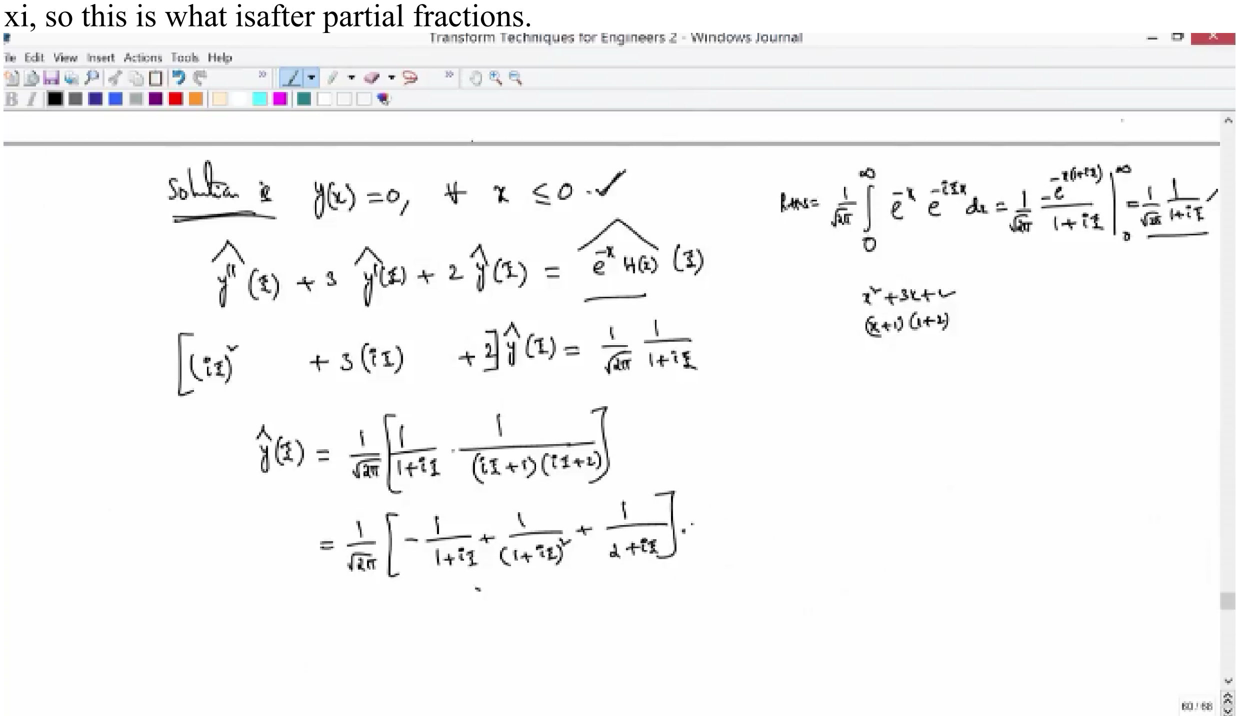
X positive side you havethis Y(x) is defined, and so how are wegoing to solve this so, but we have, if you want to apply the full Fourier transformyou need from X belongs to minusinfinity to infinity, so I extend this Y(x) from X,from 0 to infinity to minus infinity toinfinity, so how do I do this?So asthough Y is actually, Y is ifyou restrict to 0 to infinity it is thissolution, otherwise it is as though it isextended from minus infinity to infinity.
Then what is the equation now?Soequation now is D square Y/DX square and DY/DX + 2Ywhere Y(x) is here, and then this isequal to now I have E power –X, whenX is positive that is X is positive, andwhat I do is I take 0 when X isnegative, those are equal to 0, thisis actually same as saying E power -X times H(x), so this is how if Itake that extension, because thenegative side it's already Y(0), so Y(0)is actually 0, and Y dash(0) isequal to 0, so suppose these two are conditions are given atthis point, okay.
Now if you take X lessthan 0, less than side negative side whathappens, because these two initialconditions your equation is Y doubledash + 3Y dash + 2Y = 0,so have a homogeneous equation with homogenous boundary conditions which arehomogeneous initial conditions that aregiven the solution is actuallycompletely 0 for every X in minusinfinity to 0, so if you choose thisconditions so and your equation, becauseyour equation and the negative sideis 0 you have completely negative sidefor all X, solution YX is alreadythere, the way you choose yourinitial conditions that makes it Y(x) is actually 0,Y(x) is 0 for every Xless than or equal to 0, that's clear,okay, the solution is actually,solution is this, because of the initialconditions.



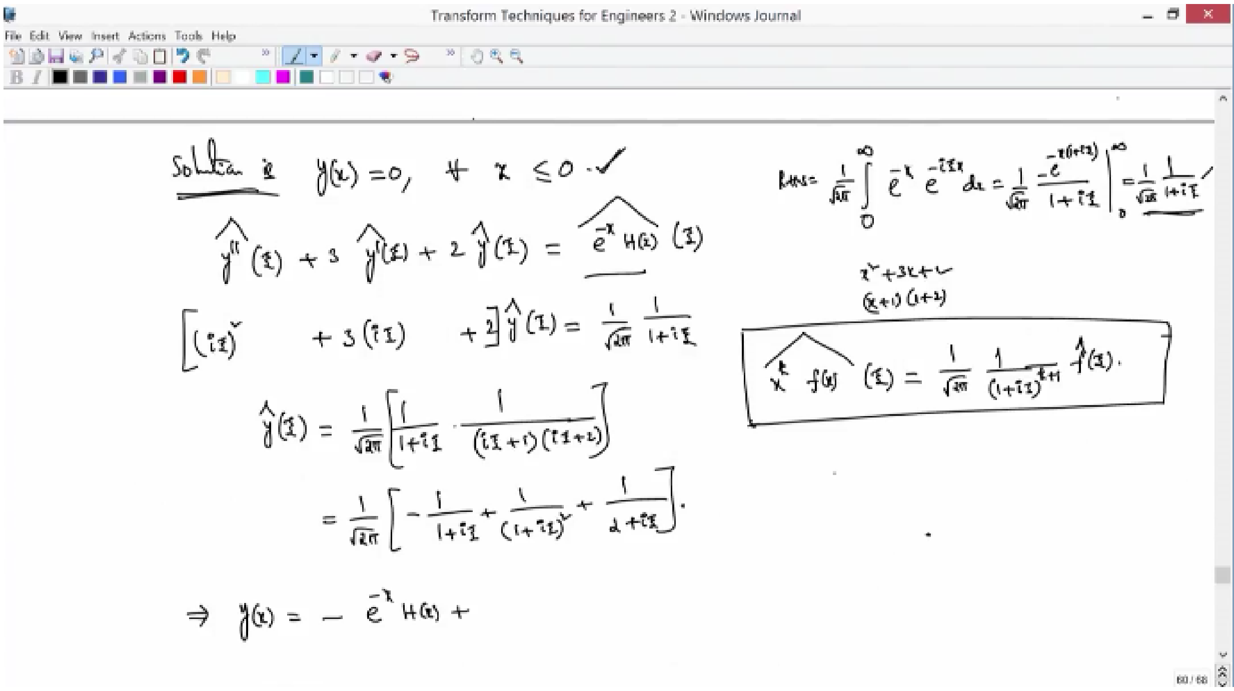
Now only thing youhave to worry about X positive side that is whatwe chose X positive is E power –X, so as such so thisis at how crazy you look at onlynegative side and at X = 0 theseconditions are satisfied, YX is actuallycompletely 0, so these conditions arealready utilized the negative side, sothat way now you consider what happensfor X belongs to minus infinityto infinity if you look at it, this is Epower -X times H(x), so wheneveryou have these 0 boundary conditions youcan extend it this way with Heavisidefunction so that your solution negativeset is always 0, okay.
And then, so theinitial conditions are anyway satisfied okay,these conditions are alreadyutilized that sense, okay.Now if we applyfor this full equation, if you apply Fourier transform and so

what is that apply, ifyou apply?D square Fourier transform of Y dash if I write D square Y/DX square as Ydouble dash Fourier transform of xi + 3times DY/DX that is Y dash(xi), cap ofxi + 2 times Y cap(xi) equal to,you have E power -X H(x) this capof xi, that's what you have to choose,so if you calculate this one so what isthis Fourier transform of 1/root 2 pi,so right hand side what we need is 1/root 2, Fourier transform is 1/root 2 pi E power -X H(x), H(x) is a negative side is 0, so 0 to infinityis 1 and you have E power - I xi X DX so what you get is 1 over 1 + I xi, if you and 1/root 2 pi is aconstant, and so E power -X times 1 + I xi, okay, so of course minus is there, so if youapply 0 to infinity this is actuallyequal to 1/root 2 pi 1 over 1 + Ixi, so the inverse transform of this isactually E power - X H(x), ifyou are given like this you can easilysee the inverse transform as E power-X H(x), so here because it's aderivative if you take the Fouriertransform what it comes out is I xi square for 2 derivatives into Y cap(xi), it's a property of Fouriertransform and here E power 3 times I xi times,Y cap(xi) and here 2Y cap(xi), so Y cap(xi)is common herein all places, so you have this equal tohere you can write 1/root 2 pi, 1/1+ I xi.

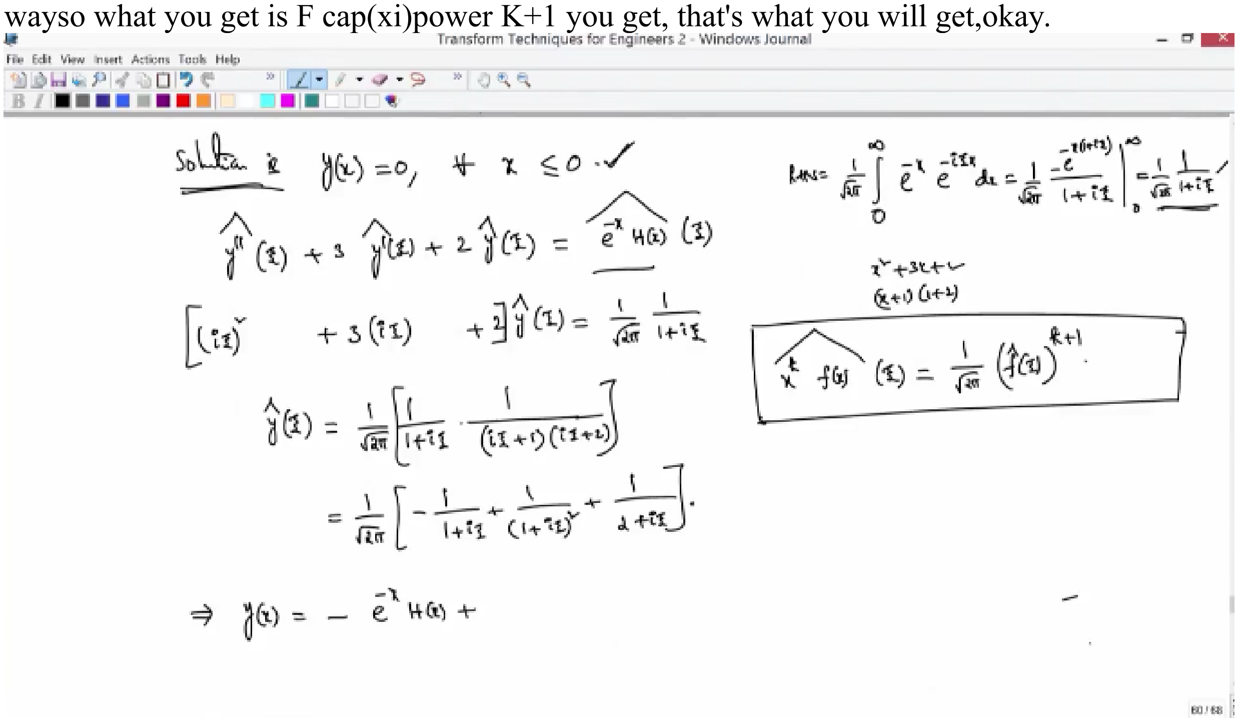
So what is your Y cap(xi)? It is 1/root 2 pi, 1/1 + I xi into 1 divided by, I have I xi squareand it's like if I put I xi as X square+ 3X+ 2, so you have (X+1)(X+2) you can write this, so if you writelike that you have I xi + 1 I xi + 2, so this if you write it as apartial fractions, so let me write thisas a partial fractions if you write-1/1+I xi + 1/1+I xi whole square + 1/1+, rather 2 + I xi, so this is what isafter partial fractions.



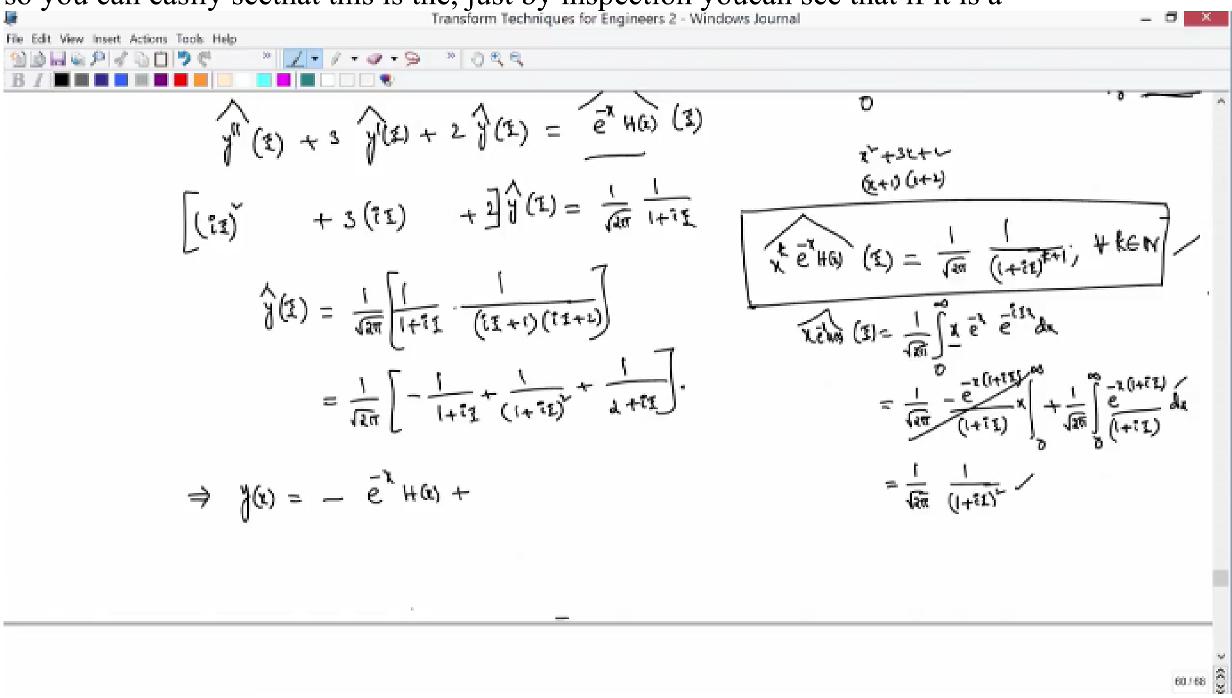
So now you caninvert it Y cap, if you invert your Y cap, take the inverse Fourier transformthat will become Y(x) which is equal toby the integral,Fourier integral theoremand then if you do here so for this ifyou apply Fourier integral theorem whatyou see is you can easily see, for thisinverse is E power -X H(x) so minusof E power -X H(x)and here +, and this will be so if youhave a square, square what you do is youtry to get this Fourier transform of Epower -X times H(x) for which youmultiply X, for this full transform ifyou multiply X any X power K and if Ichoose this as a function, F(x) this is actually equal to, if you take itas a property you can prove



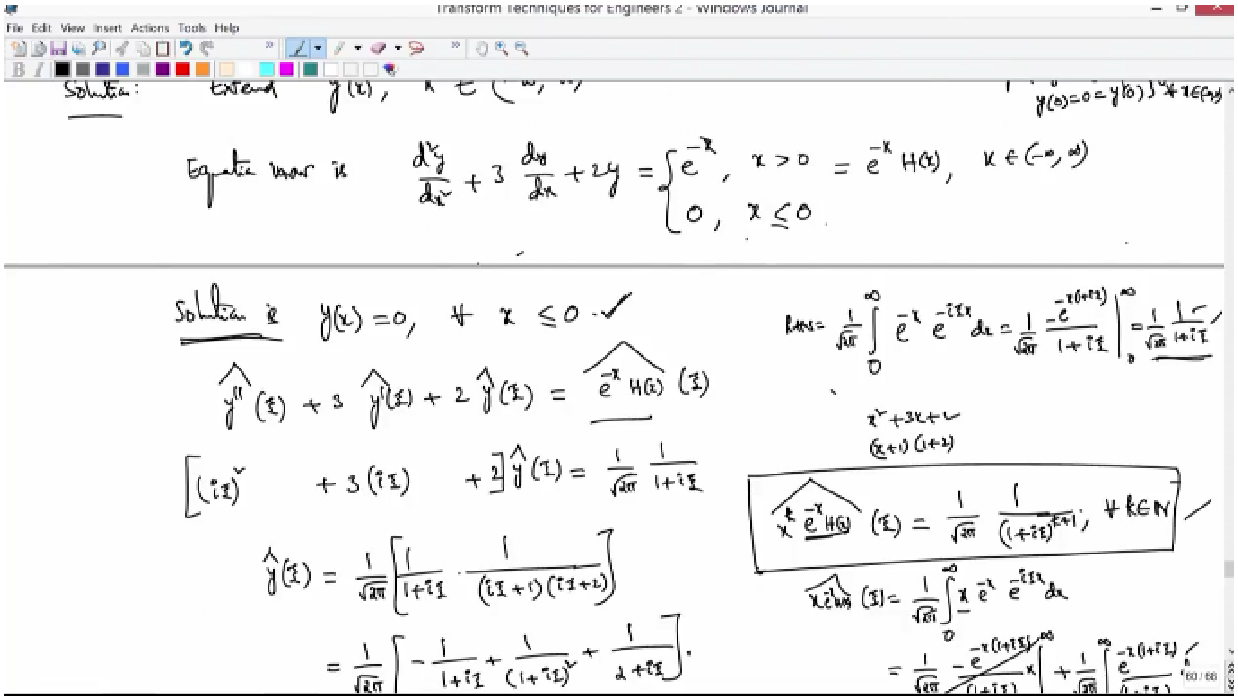
it, this isyou can actually show that this is equalto what you get is 1/root 2 pi 1/1+I xi whole power K+1 times Fcap(xi), no something else you will get soapart you get, no, no, no, not this wayso what you get is F cap(xi)power K+1 you get, that's what you will get,okay.



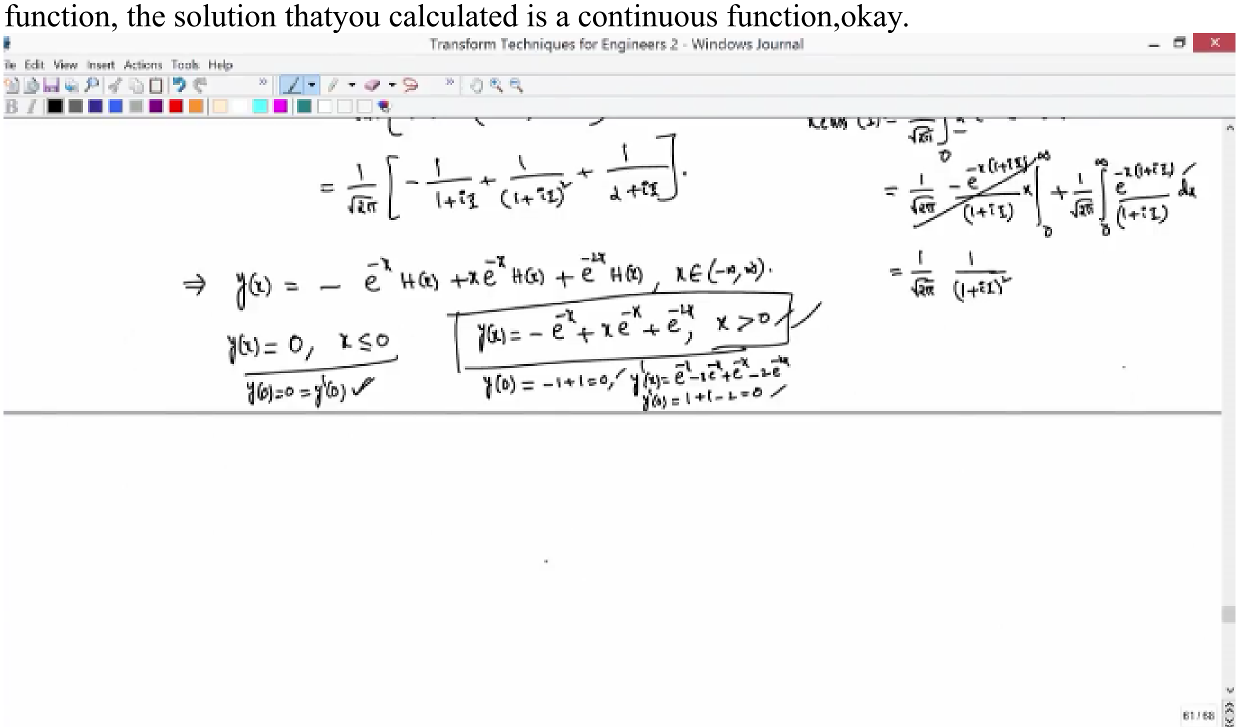
So can we see this one?Ifyou do simply for K = 1 F(x) capof (xi) if you do this 1/root 2 pi- infinity to infinity X F(x)E power-I xi X DX, so I try to take thisas, so integral value of this may be soyou can just if I substitute thisis easier, so if you take E power-X H(x) if you take this isactually equal to 1 over 1+I xi power K+1, so this is at leastthis you can take off so for every Kbelongs to the natural numbers, okay, sowe choose this, so if you do F as this Epower - X into H(x) then what youget if you replace, this is going to be Epower –X, and this is going to be 0, so once you have this, this is equal to 1over root 2 pi and you have E power–X, so what you get is -E power–X1+ I xi divided by 1 + Ixi that is the integration, and youmultiply into X you substitute thelimits, because of X this is 0 and youhave a minus, minus minus plus now 1/root 2 pi minus, so 0 to infinity youhave E power -X times 1+I xi/1+I xi into derivative of X is 1, so we haveDX, so this is actually equal to 1/root 2 pi and you see again so 1/1+ I xi comes out and this integral asyou can see here so that is going to be1, one more 1/1 + I xi, sothere's a square, so you can easily seethat this is the, just by inspection youcan see that if it is a



something,something if you know already the squareof this kind of you, instead of replacingthis with some function F if you knowhow to evaluate so that it's Fouriertransform and then what you end up is power of Fourier transform of suchfunctions power K+1, so this is whatyou see. So you can easily see here thatthis is going to be E power -X H(x) times X so you have X times this andhere this one E power -2X H(x)this is X belongs to - infinityto infinity, okay.
So what happens when Y(x)= 0 clearly so the solution isactually, you can easily see here X isless than or equal to 0, so once you havethis one Y(0) is automaticallysatisfied,Y dash(0) so that is thereason if you are given these zero initial conditions youalways extend your right hand sidefunction with Heaviside function forwhichyou have theFourier transform you know how toevaluate and what happens at Y(x) positive, Y(x)= E power -X+ X E power - X + E power-2X, 4X positive, so this is yoursolution and this is a negative side, howthat?And now even if you use this onebecause let us see whether if it iscontinuous or not, Y(0) is actuallyequal to, so as you see is that this isthe differential equation so what you'relooking for is thesolution Y that has derivatives,okay, right hand side is kind of forcingboth the X positive and X negative side,one side is anyway theseconditions are satisfied, the other sidealso you can



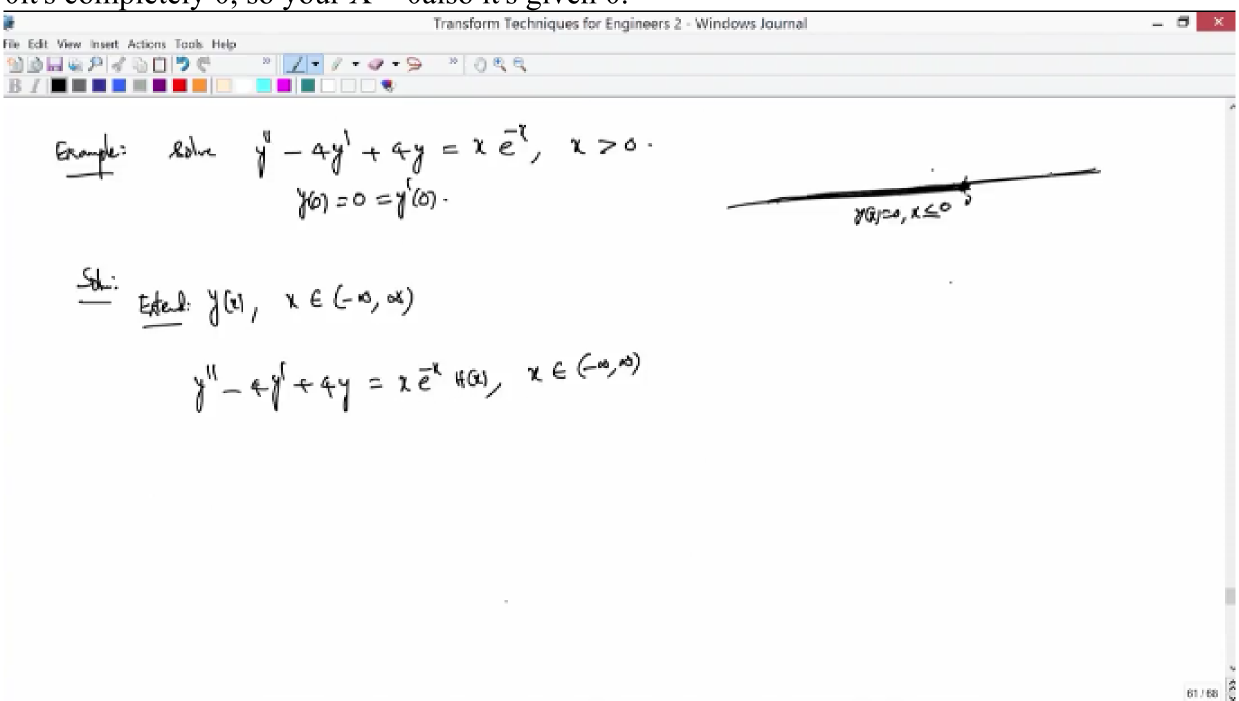
easily see the conditionsthat if you put Y0 this is going to be-1 +1 which is 0, and Y dash(x) is E power -X Epower -X + E power -X -2 E power -2X, so if you put X= 0,Y dash(0) is 1 + 1 – 2, this is also 0, so these conditions areautomatically satisfied because Y is acontinuous function, the solution thatyou calculated is a continuous function,okay.



So this is what you're looking for isthe solution that automaticallysatisfythese boundary conditions.Soand this is not the only way to prove, tosolve this equation, you can also solvethis equation because the domain is theonly positive side you can apply cosinetransform or sine transform directly andyou can, there you don't have to worrydirectly these coefficients will comeinto the pictureso those things whatever may be thevalue when it is nonzero you can usecosine and sine transforms only whenthese both the conditions are 0 you canuse the direct Fourier transform byextending with Heaviside function, okay.

Wewill see some more example in thisdirection, maybe later on we willtry to solve these problems with cosineand sine transform, so let’s do somemore problems for example let me use onemore example,one more example is Y, solve Y doubledash - 4Y dash + 4Y= Xinto E power – X, X positiveso andY(0) is 0which is Y dash(0), if you are givenlike thisso solution as usual you can proceedbecause it is zero boundary conditionyou can extend this to full real line okay,so with Heaviside function, so youextend Y(x), X belongsto minus infinity to infinity, okay, soyou have a solution equation is now4Y dash + 4Y = X E power- X into H(x), now H belongs to minusinfinity to infinity.

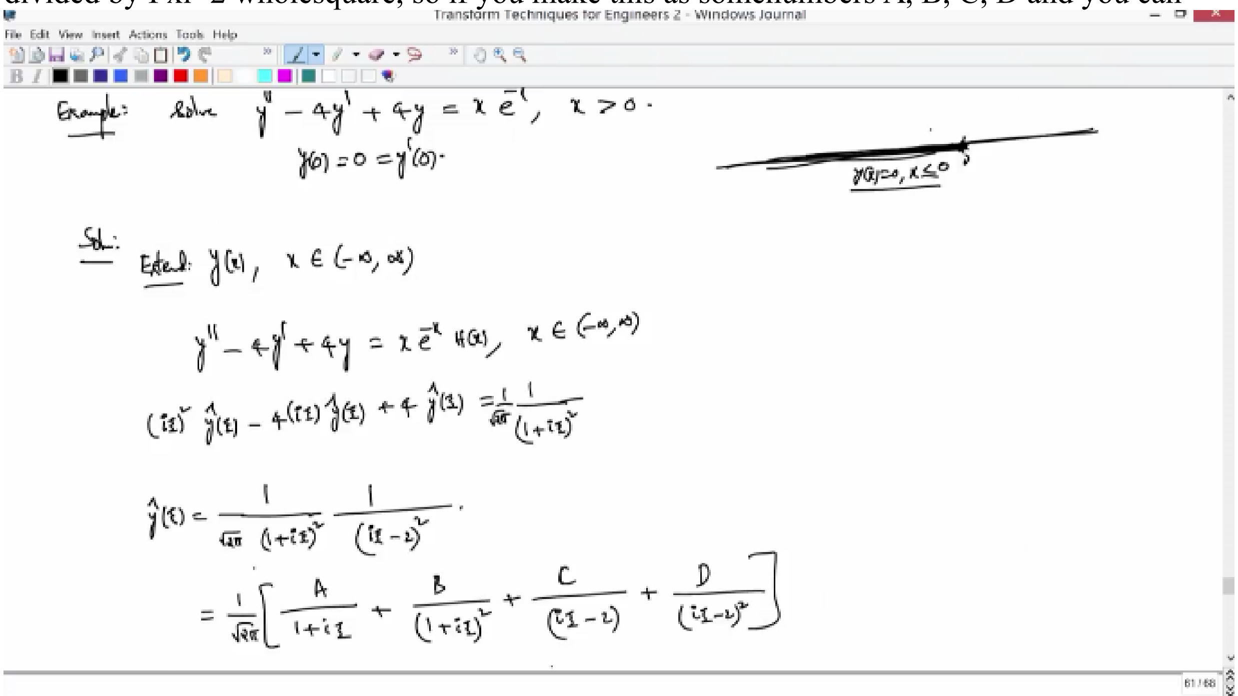
Now your conditionsare automatically satisfied as you seenegative side is always at that point is0, it's a homogenous equation thenegative side the solution is actuallyyou know that homogenous equation with 0initial conditions is always 0,completely identically 0,Y(x)= 0 for X negative, so these conditionsare automatically satisfied for thisequation because at X equal to 0it's completely 0, so your X = 0also it's given 0.



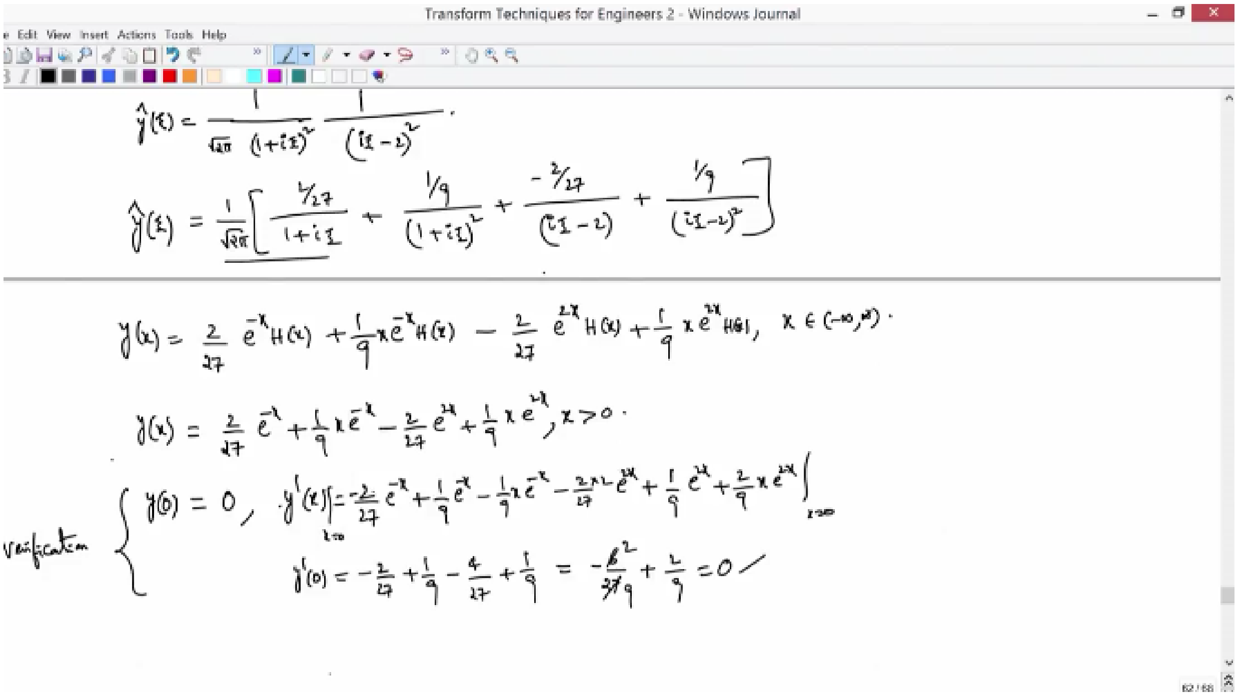
So let us see whathappens, so let us solve we are notsolving by splitting it like that,and we will try to solve this togetheras though this is one function okay, wedon't split the domains anddo this, even if you split you can seethat this one you clearly see that these conditions are automaticallysatisfied, and negative side at X = 0.

So now look at as usual, apply thefull Fourier transform you see that I xi whole square Y cap (xi) -4 I xi times Y cap(xi)+ 4 times Y cap(xi)and here is a Fourier transformof this you have seen earlier that thisis 1 over 1 + I xi whole square,okay, so if you apply this one X squareso if you look at this one, this is Y cap(xi)= 1 over, this is when you do, when you get this you see that 1/root 2 pi, so you have 1/root 2 pi missing, and what you getis 1/root 2 pi, 1 + I xi squareand here 1 over, if you divide so I xi square -4 I xi + 4 that is likeX square -4X + 4 that is

goingto be I xi – 2 whole square, right,I xi -2 whole square that is Ixi square - 4 I xi + 4, so that iswhat is the left hand side, so this iswhat you have.
Now if you use the partialfractions 1/root 2 pi, if you use thepartial fractions I write directly here,so what I do is first 1/1 + I xi you have something divided by 1/1+ I xi is something some numberinto 1/1 + I xi whole square + something this is here,2 –R I xi– 2,I xi –2 + what is divided by I xi -2 wholesquare, so if you make this as somenumbers A, B, C, D and you can

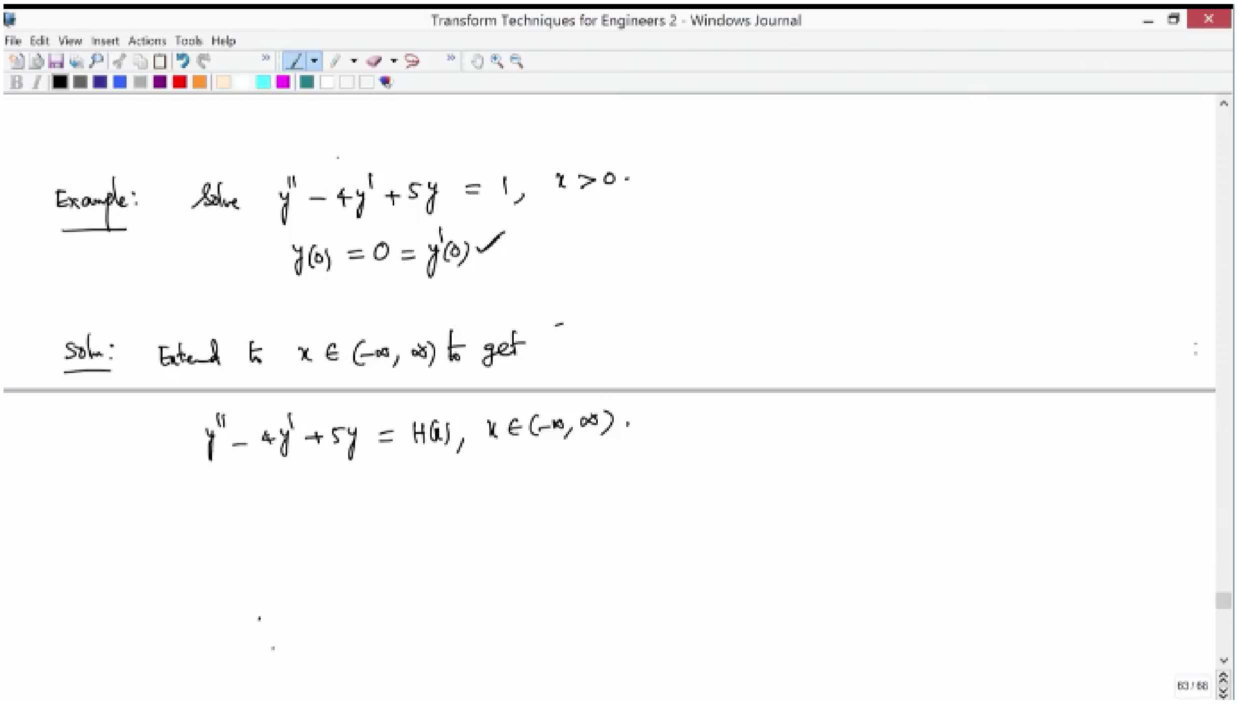


multiply andthen we can see we can equate thecoefficients of, by partial fractionmethod you can find I'll write directlyit is going to be 2/27 you can verifythis and it's going to be 1/9 andthis is going to be -2/27 andthis is 1/9,okay, so this is exactly our Y cap(xi), if you invert this a first term willgive me 2/27 E power -X H(x) and + 1/9 E power -X into X H(x)and here it's going to be -2/27 and this is going to be E powerminus, this is E power 2Xinto H(x) becauseof -2, okay, if you calculate thisFourier transform you will see that it'sgoing to be this one, and this one willbe +1/9XE power 2X H(x), andwhere is this valid?From - infinityto infinity, so if you look at negative side it's completely 0 thatis what we get, and for positive sidethat is what we want which is equal to 2/27 E power -X +1/9 XE power–X,-2/27 E power 2X+ 1/9 XE power 2X, this is for Xpositive side that is what we want.
Nowif you verify this Y(0) 2/7 and-2/7 that is 0, clearly Y dash(x)if you calculate 2/27 E power-X with – I and here +1/9 E power -X -1/9 X Epower – X, - 2/27 times 2 E power 2X+ 1/9 E power 2X + 2/9 XE power 2X. Now this if you put X= 0, okay,so what you get is Y dash(0) -2/27and you have +1/9 and here -4/27and you have 1/9, and this is actuallysame as 0 again, so -6/27 that is 2/9 already you have 1/9, 1/9that’s 2/9, this is 0, so both theconditions are satisfied this is what weverified, okay, so the actual solution whatwe are looking for even

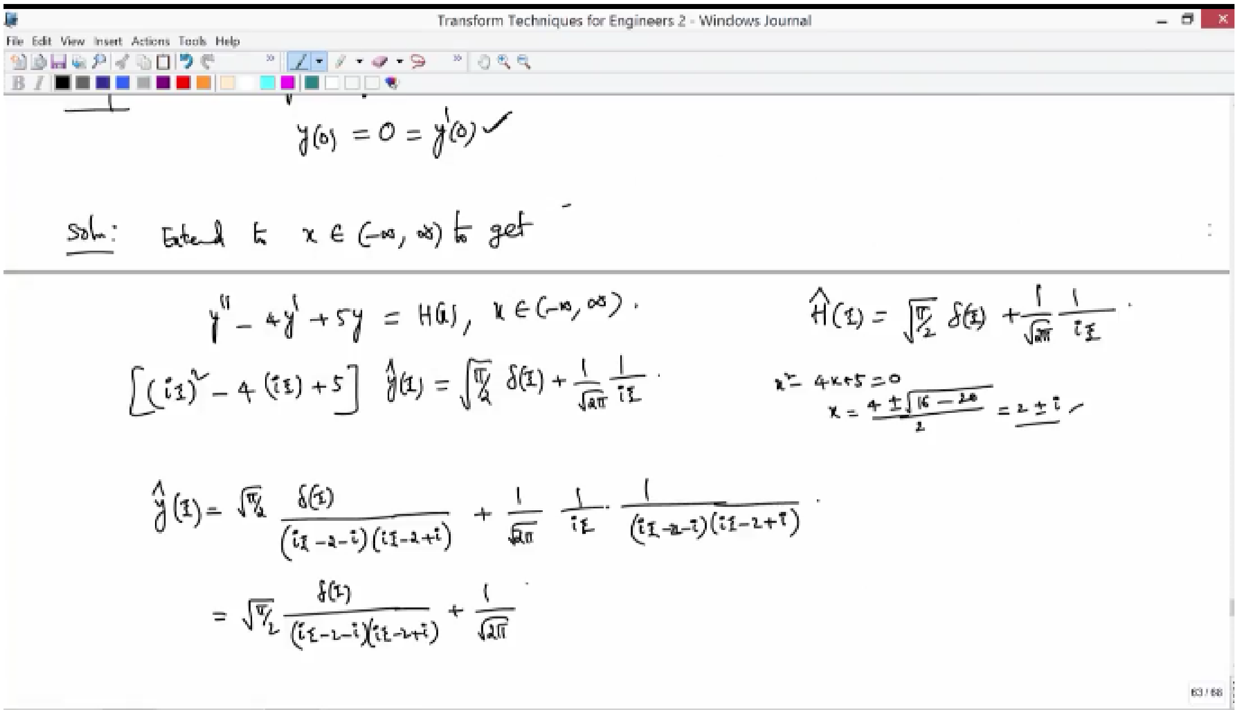


if you solve byother methods you should get you'regetting this, this is your solution ofthat initial value problem that isgiven as here.

If we solve also by acosine transform or sine transformbecause it’s givenin the semi-infinite domain so you cansolve even by those method, that wewill see later, okay.So letus solve one more example but this isnot that straightforwardthis is let's do this solve Y doubledash - 4Y dash + 5Y= 1now, so this is X positive, so a solutionwe know already what is its solution andso the conditions are also same givenas 0, Y and its derivatives at X = 0 okay, so if you do this what is so if youhave, if you follow the same way themoment you see these initial conditionsyou can extend this, extend to Xbelongs to - infinity to infinity toget the equation Y double dash -4Ydash + 5Y= H(x),now this is X belongs to minus infinityto infinity.

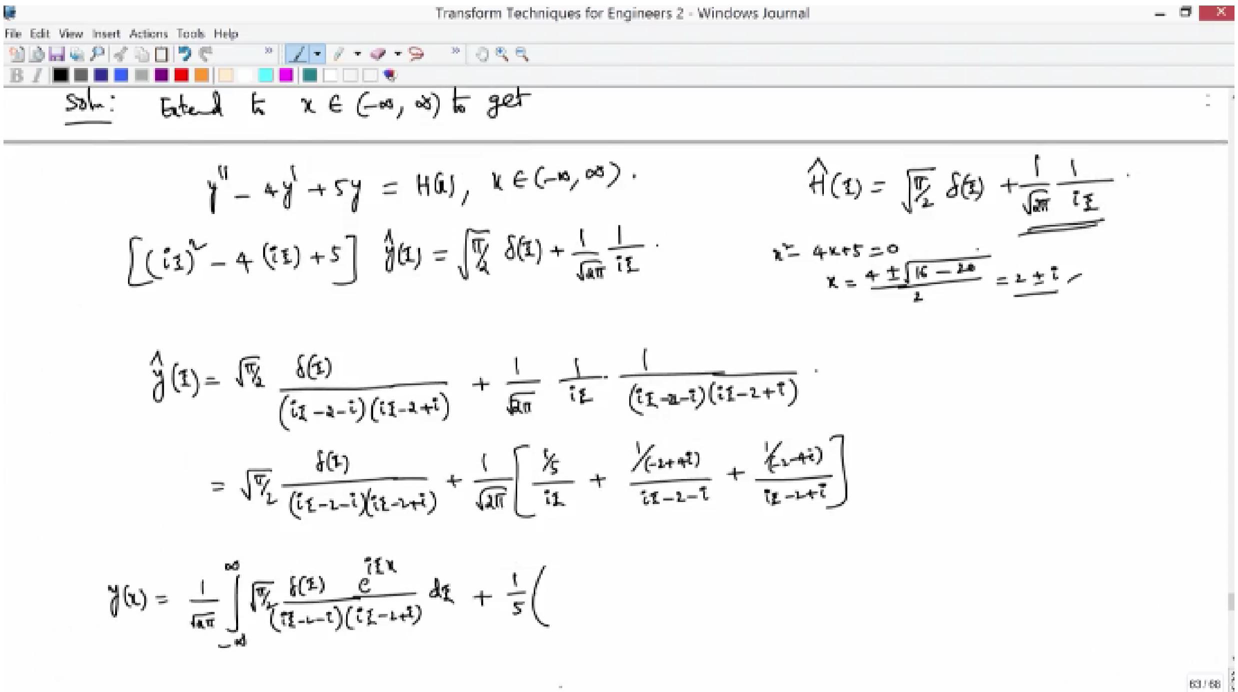


Again these conditions areautomatically satisfied the negativeside, is identically zero solution, so andwhat happens now?So what happens if youapply the Fourier transform like earlier I xi square - 4Ixi + 5 timesY cap(xi) it's a left-hand side, whatis a Fourier transform of H(x) that is Hcap(xi) that is H cap(xi)we have seen earlier that this is, thisis not, this is actually what you get iswith our definition root pi/2 deltafunction of xi + 1/root 2 pi 1over I omega I xi,so this is yourFourier transform so if I write this hereroot pi/2 delta function of xi andthen + 1 over root 2 pi, 1over I xi, so what I do?What I get is Y xi is, so Y xi is now root pi/2 timesdelta xi divided by, so what is this one? X square -4X +5 this I canwrite like X-2, so what is this one?So if you get the roots of thisequation you will see that roots of theequations are X = 4 + or -square root of 16 - 20 divided by 2,that is going to be 2 + or - I2,so alright 2 + or -, 2 divided by 2so that is going to be this, these arethe roots, so because of those are the roots so youcan rewrite this as I xi, -2 –I for 2+Iand other one is I xi -2 + I this is one term, other term if we write like this 1/I xi 1 over the same, this division the left handside I xi –2 – I, and I xi – 2 + I, so this is what you have, soif you try to take the Fourier transform,so before you take the Fourier transformso what happens here?So this is going tobe root pi/ 2delta xi, this you write

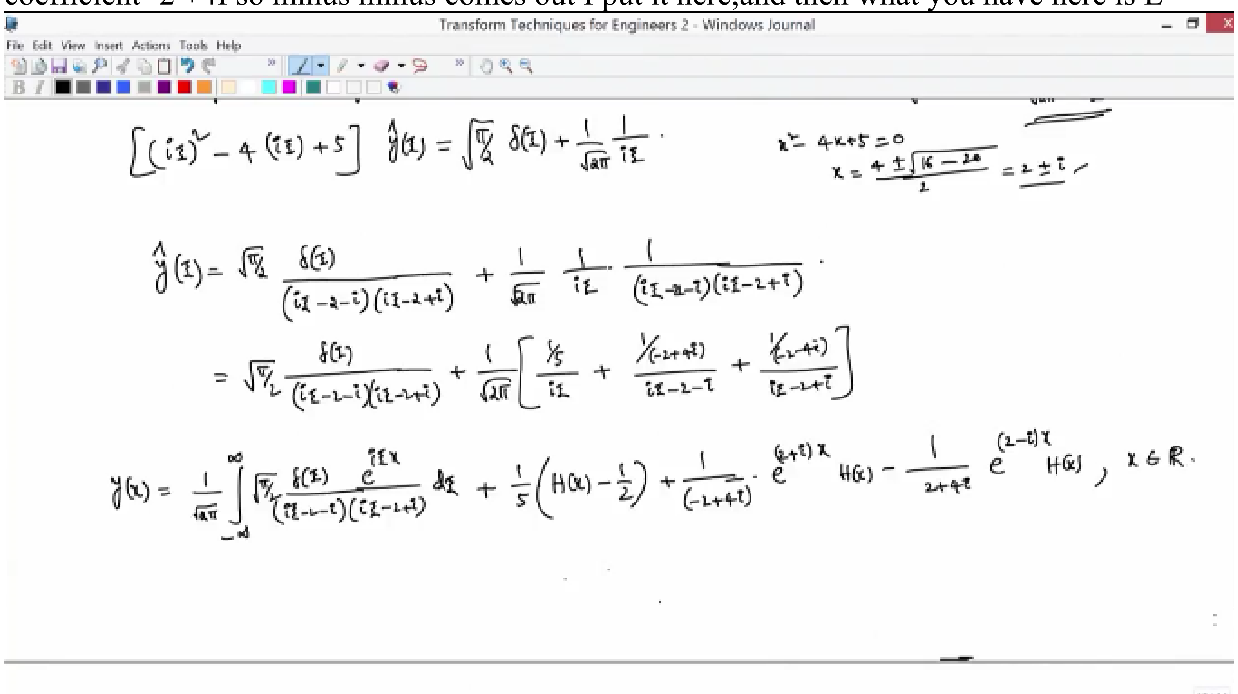


as it is,I xi -2 –I, I xi -2 + I this one again you apply partial fractionsand rewrite, after writing partialfraction or directly write as 1/5 divided by I xi + 1 over, if Iwrite -2 +4 I divided by I xi - 2 –I, and here one morething is 1 over -2 -4I dividedby I xi - 2 + I, you can verifythat this is same as this the secondterm here.

So once you write like thisnow you can take the inverse transform,what is the inverse transform of this?The inverse transform of this one, inverse transform of this, inverse transform ofeach of these 4 terms, so first one isif you don't know exactly what is theinverse of this, because it involves adelta function inverse you will use theinverse transform definition that is 2pi -infinity to infinity, root pi/2delta function divided by I xi – 2 -I and then I xi -2 +I timesE power I xi X D xi, and this isyou can easily, because of the deltafunction simply substitute xi = 0 that will do, that will give the value of this integral.Another one,other integral is you can, 1 over 5comes out and the remaining 1/root 2pi I xi is E power minus, okay this is this if youagain if you use this E power I xi, so this is what, for this if youwant inverse transform, inverse transform of this minus this, what is theinverse transform of this?Is H(x)minus, now thisis, this we have seen already this is inverse transform of this is 1/2, because Fourier transform of 1/2 isactually you get this value, so theinverse transform is 1/2, so H(x)-1/2 okay.



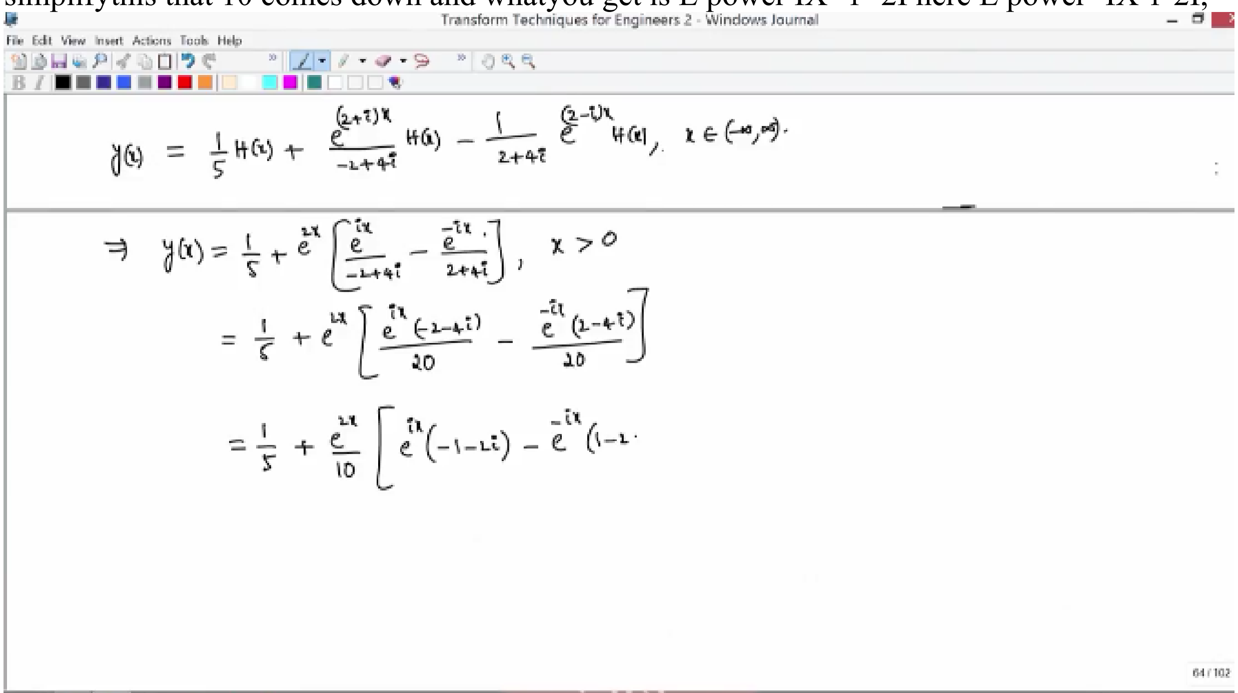
And then here 1 over -2 +4I times and this is going to be, sowhat do you get again like earlier youhave E power because of this minus, and youwill get a plus sign so you have 2 + I times, X times H(x)okay.And then one more, onemore term is you have this partialfraction that coefficient -2 +4I so minus minus comes out I put it here,and then what you have here is E



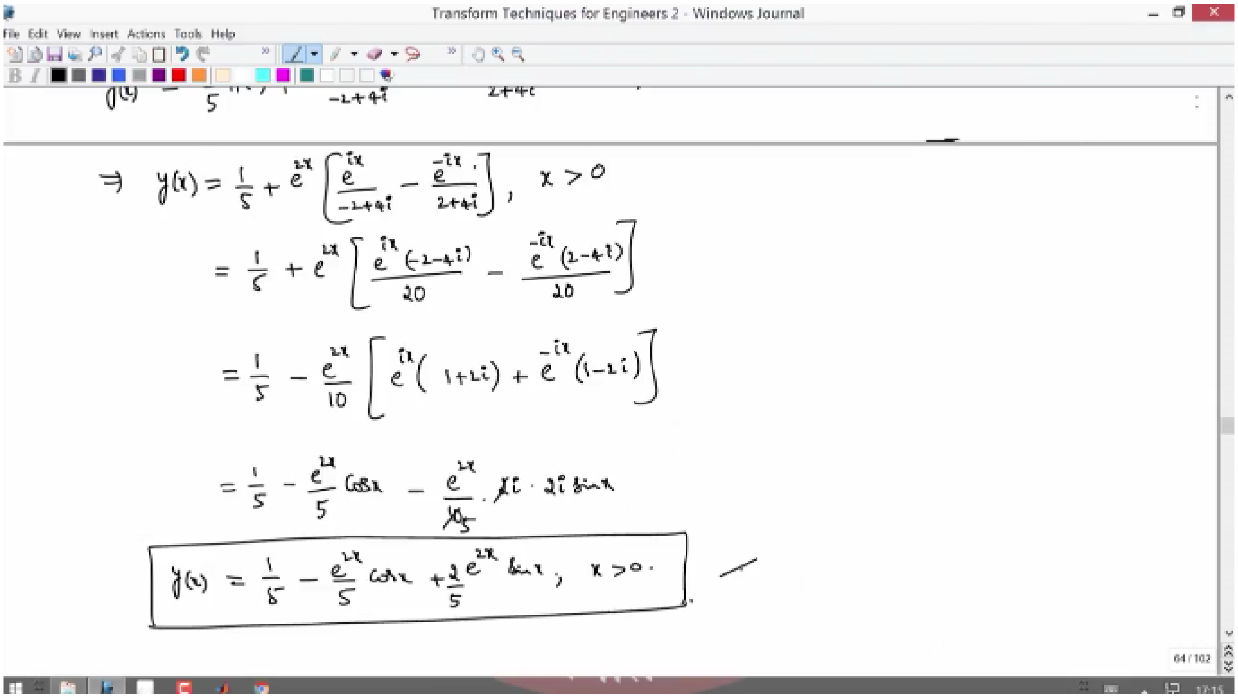
power 2-IX times H(x) this is for Hbelongs to, X belongs to full real linethat is minus infinity to infinity, so thisis exactly your solution, so what happensto the first integral?Because of thisdelta function root pi root pi goes andyou have root 2, both the place in the denominators you have 1/2 and when you put xi = 0 that issimply 1/2-I -2 -Itimes -2 +I so it's going to be4,-2 whole square 4 -I square,so it’s going to be 5, so together 1/5 that is into 10 that is going to be 1/5 into 1/2 is 1/10, so the firstintegral is this.

And the second integraland this 1/4 and -1/1/2 thatis going to be 0 so that is gone, andwhat you have is 1/5 H(x) + andthis is what you have, so you have Epower 2 + IX/-2 + 4I H(x) and 1 divided by 2 +4IE power 2-IX H(x), this is X belongs tominus infinity to infinity, so this gives mea solution as 1/5 for X negative,Xpositive side, X negative side is anyway0 so because I have a 0 initial conditions withhomogeneous equation you have YX is 0completely, identically 0 solution in thenegative side a positive side1/5 + E power 2X is anywaycommon and divided by -2 +, so wemaybe you haveto see this one C power IXdivided by -2 +4I, then - Epower -IX divided by 2 + 4I,rightokay so this is for X positive side.

Sowe'll simplify this 1/5 + Epower 2X, so I multiply -2 -4I multiply and divide so that you have Epower IX times -2 -4I andthe denominator you have a 4 + 16that is 20, and here again I multiply 2- 4I and the denominator, if youdivide the same and you multiply with 2+ 4I that's going to be again 20, 4 + 16, 20, so as you see 1/5 + Epower 2X is common and you can simplifythis that 10 comes down and whatyou get is E power IX -1 -2I here E power -IX 1-2I,

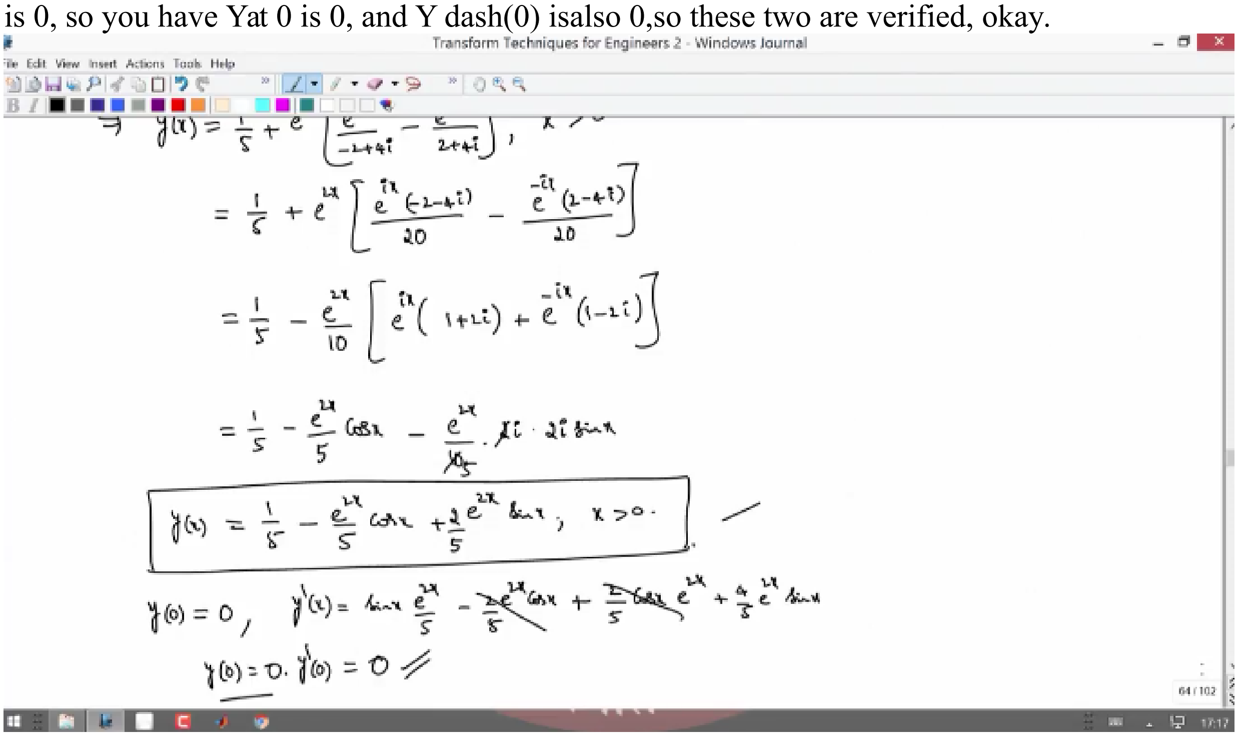


so iftake it out minus comes out this is going tobe plus, and this is going to be plus, so this is going to be 1/5 times Epower 2X/10, and this together is 2cos X so you have 5 cos X,and then hereminus E power 2X/10 and you have E power IX - E power IX so you have 2Itimes E power IX - E power IX is, soyou have again 2I sine X, so thatmakes itE power 2X/5 cos X and here sothis 2 is 5 so you have this is going tobe +I square is -1 so thatmakes it plus,you have E power 2X sine X2E power 2X divided by 5, so this iswhat you have as your solution.



So your YX is this solution,okay, so as you can see when yousubstitute Y at 0, cos 0 is 1 so you have1/5 -1/5 that is 0, this term is 0 so it's clearly Y(0) is 0and if you calculate Y dash (x)whichis, this is if we differentiate this thisis 0 and here you have sine X E power 2X/5 and here are -2 E power 2X cos X/5, again if you do differentiate thisterm so 2/5 cos X E power 2X andthis gets cancelled anyway, so and thenif we differentiate this part we have 4/5 E power 2X sine X.

Now if yousubstitute Y dash(0) here, this term is0 because of sine X and because ofsine X that is 0, so you have Yat 0 is 0, and Y dash(0) isalso 0,so these two are verified, okay.



So withthis we can easily see that this eventhough 1 is not absolutely integrable function in this example, you canactually extend this as with Heavisidefunction, Heaviside function has aFourier transform in terms of a deltafunction because these we see weare minimum by having we are doing the Fourier transform of veryfew generalized functions, usualfunctions anyway we know how to find Fourier transform, but generalized functionsalso that to only 1 or 2 whichcomes in the application, for exampleheavy side function are the deltafunction itself, only these two you seehow to find the Fourier transform, wehave already seen so make use of them toevaluate, to solve this simple ordinarydifferential equation okay with theinitial data when it is a zero boundary,zero initial conditions you can solve thatagain, you’re able to solve just by using the Fourier transform, fullFourier transform.
So you don't have todo this way, so you can also do all theseproblems using Fourier cosine transformour Fourier sine transform, because thedomain is a same infinite that isbetween 0 to infinity, so these things wewill see maybe in the next video. Thank you very much.

Online Editing and Post Production
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Sribalaji
Komathi
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Bharathi
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NPTEL Coordinator

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