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# Transform Techniques for Engineers

## More properties of Fourier transforms

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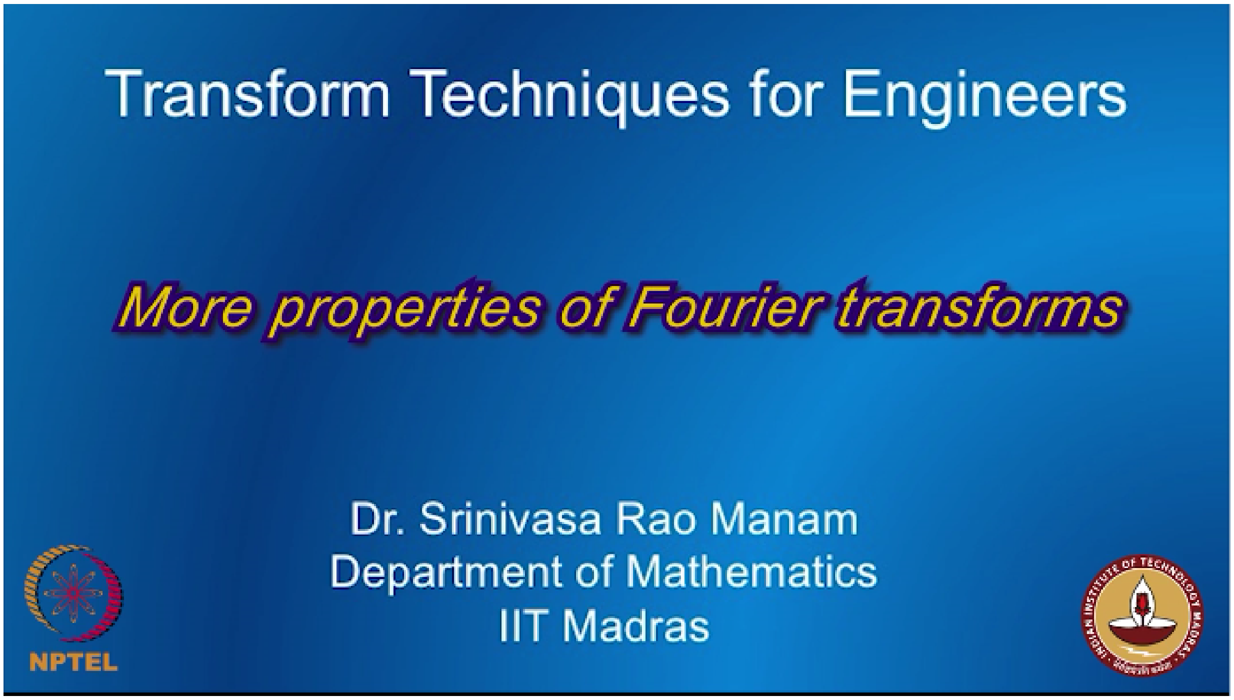
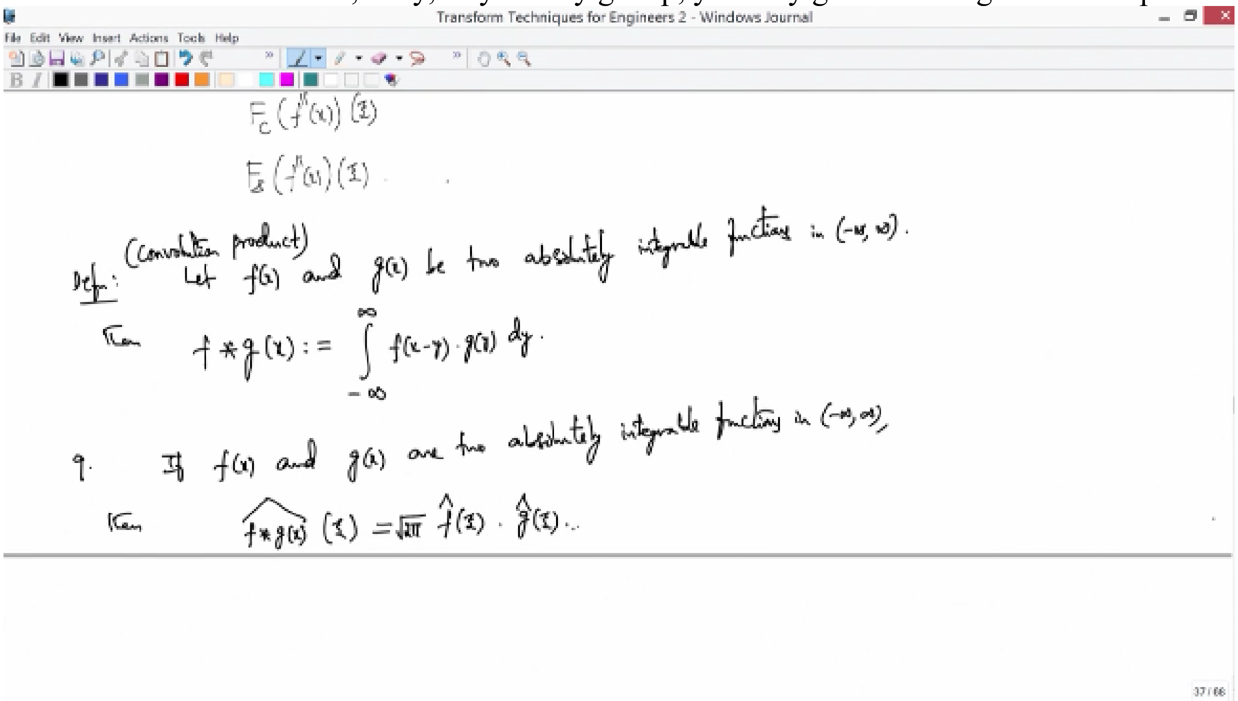


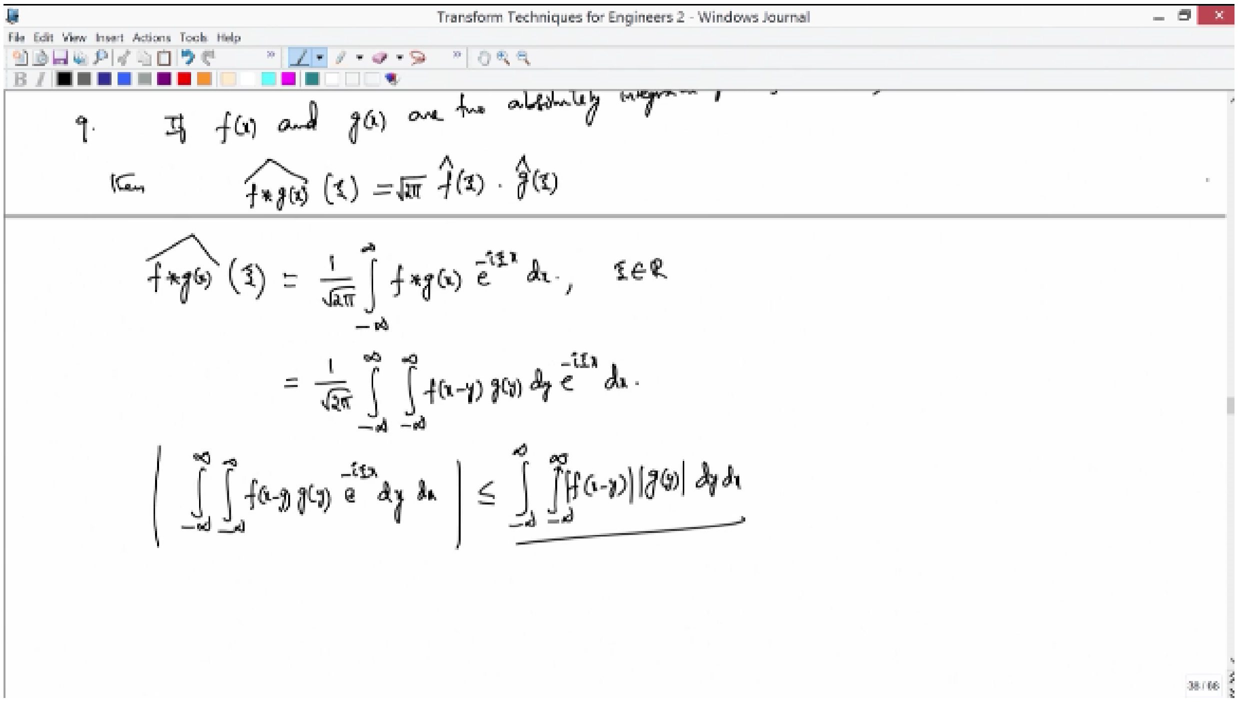

Welcome back, we were discussing about the properties of the Fourier transform, so last video we have done certain properties, today we will do some more properties, some very important properties by on convolution product, we define what is convolution product and Fourier transform of, it is a product of the Fourier transforms, and we see some important equality, it's called Parseval equality, and these things along with Riemann Lebesgue Lemma and then eventually we will try to solve, we will try to prove Fourier integral theorem which is the backbone of this Fourier transform, so through which we have defined this Fourier transform, okay.

So let's do some more properties, so another important thing is $9^{th}$ property let’s say, if it's 8, it’s 9, property 9 is if I define, so let me define what is a definition of convolution, convolution of two functions, let F and G, F(x) and G(x) be two absolutely integrable functions, integrable functions, absolutely integrable functions in -infinity to infinity. Then F star of G this is the convolution product of this, which is a function of X as you define it as the integration over a product of these functions in such a way that you can define F(x), F(x-y) into G(y) DY, which is function of X.

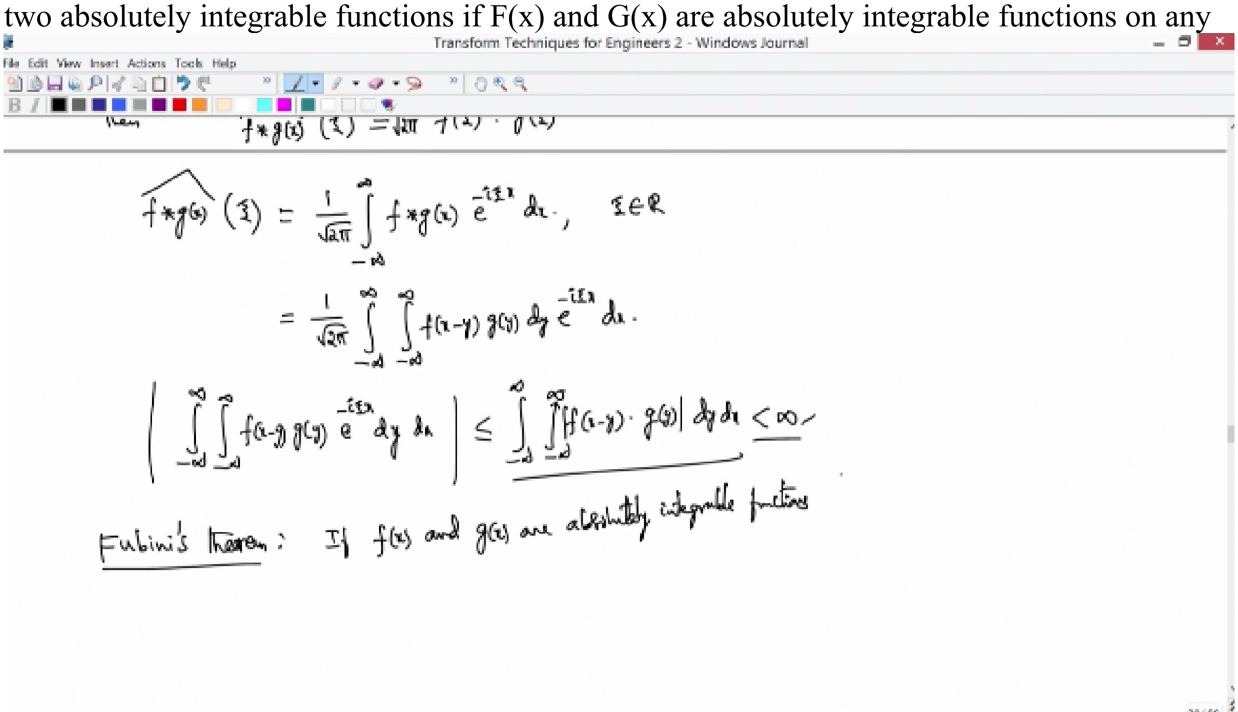
So this is how if you define you can see exactly, so we will try to define in such a way that so we may have to add some constant if required so that we will see, this is the definition of this convolution, convolution product is this, so now the property tells you that if F and G are two absolutely integrable functions, if F and G, F(x) and G(x) are two absolutely integrable functions and –infinity, infinity, then what you have is you take the Fourier transform of this function convolution product which is a function of xi, is will become Fourier transform of that and Fourier transform of G, okay, so you may get up, you may get something like root 2 pi so




that you get good identity, why we add this? So I think this is what we have, okay, so this you can adjust by redefining your convolution product you can remove this depending on, okay.
So let's prove this one, so we use this way so let us define this and then we will get this one, if we don't get this, if we don't get this that means we will have to redefine, okay, so let's see what we get, is this really true we'll see, okay.
So what you have to do is you take this LHS that is F star G(x) for which you take the cap Fourier transform by definition this is 1/root 2 pi – infinity, infinity F star G(x) into E power –I xi X DX, xi belongs to real numbers this is the thing, so what you get is now 2 pi, now you write this exactly what this definition of convolution product and what you get is –infinity, infinity, F(x-y) G(y) DY and E power –I xi X DX, so because F and G are two absolutely integrable functions and you can see that this is –infinity, infinity, -infinity, infinity of this whole thing F(x-y) G(y) E power - I xi X DY and DX this is less than or equal to –infinity, infinity, -infinity, infinity, F(x-y) into modulus of G(y) DY DX, so this because F and G are two absolutely integrable functions, F(x-y) is also absolutely integrable function, and then also G(y) is also, this product as you can see this product is also absolutely integrable function, you

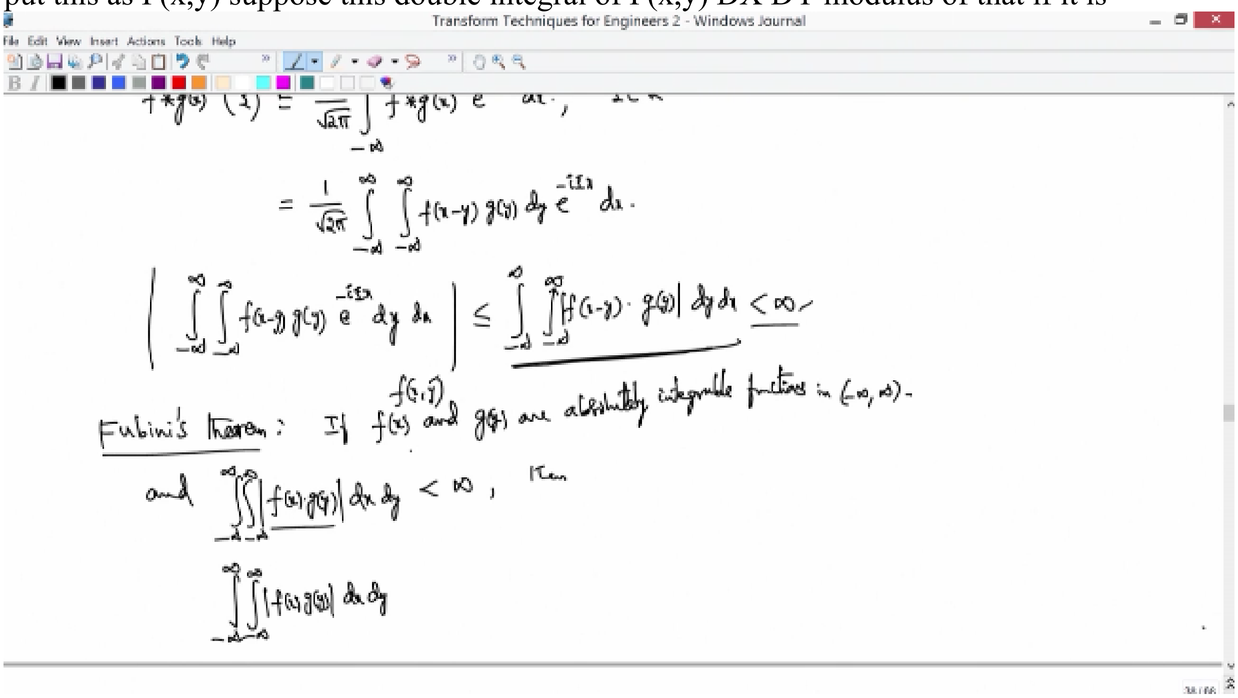



can say this is a product of G, this product is also absolutely integrable function, so because this is, that means this is finite that is the meaning, so because of this there is something called Fubini's theorem you can just have a statement this is very useful and important, if F and G are two absolutely integrable functions if F(x) and G(x) are absolutely integrable functions on any



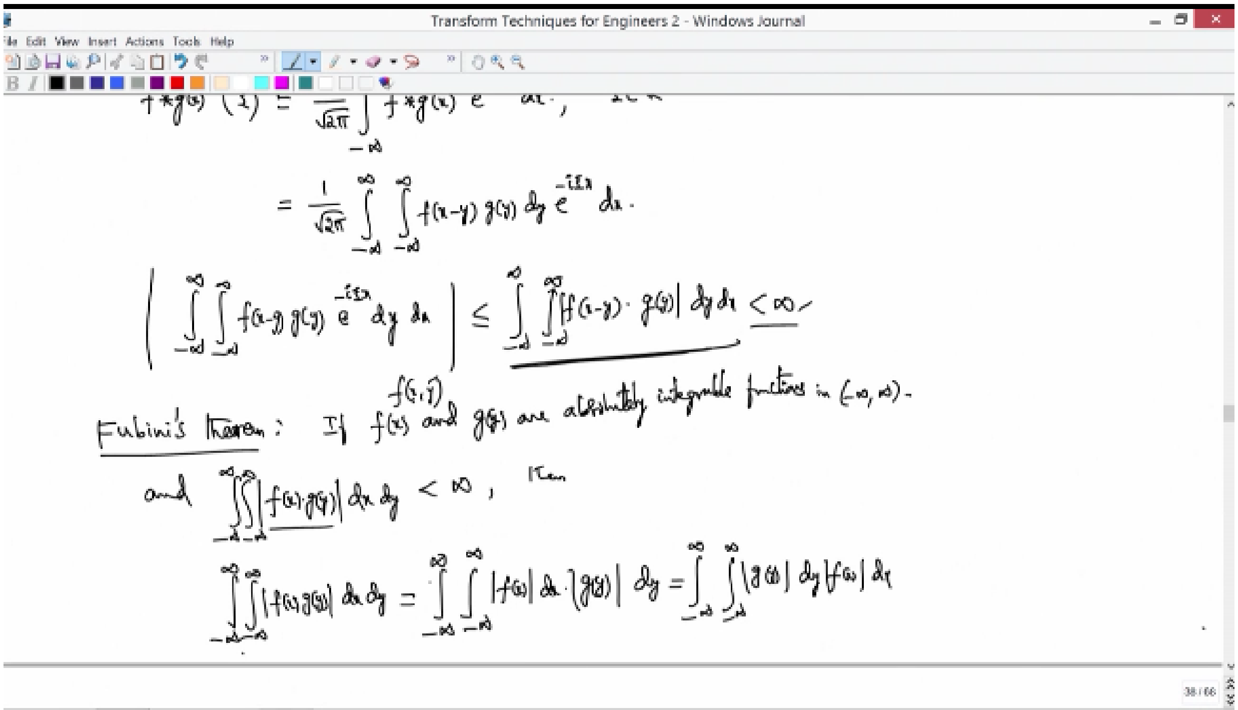

domain, so you can take finite interval or any subset of real numbers, or you can take the full real number set, so let us use the full real numbers that is functions in –infinity, infinity, and what you have is this double integral -infinity to infinity, -infinity, infinity, this is double integral of F(x) into G(x).

So let's say G(y) let us say G(y) okay, F(x) G(y) DX DY if this is finite, okay, this product if this is finite then that is exactly what we have, okay, so then if this is the case F(x-y) G(y) is also absolutely integrable function, so in such a case what happens if this is the case then all these things whatever you wrote as a double integral, as a double integral this F(x) G(y) DX DY rather to be more general you can write F(x,y) as a two variable function you can write, okay, for any fixed X F(x,y) is an absolutely integrable function for -infinity to infinity, if you fix Y F(x,y) as a function of X is absolutely integrable function in –infinity, infinity, so then you can put this as F(x,y) suppose this double integral of F(x,y) DX DY modulus of that if it is



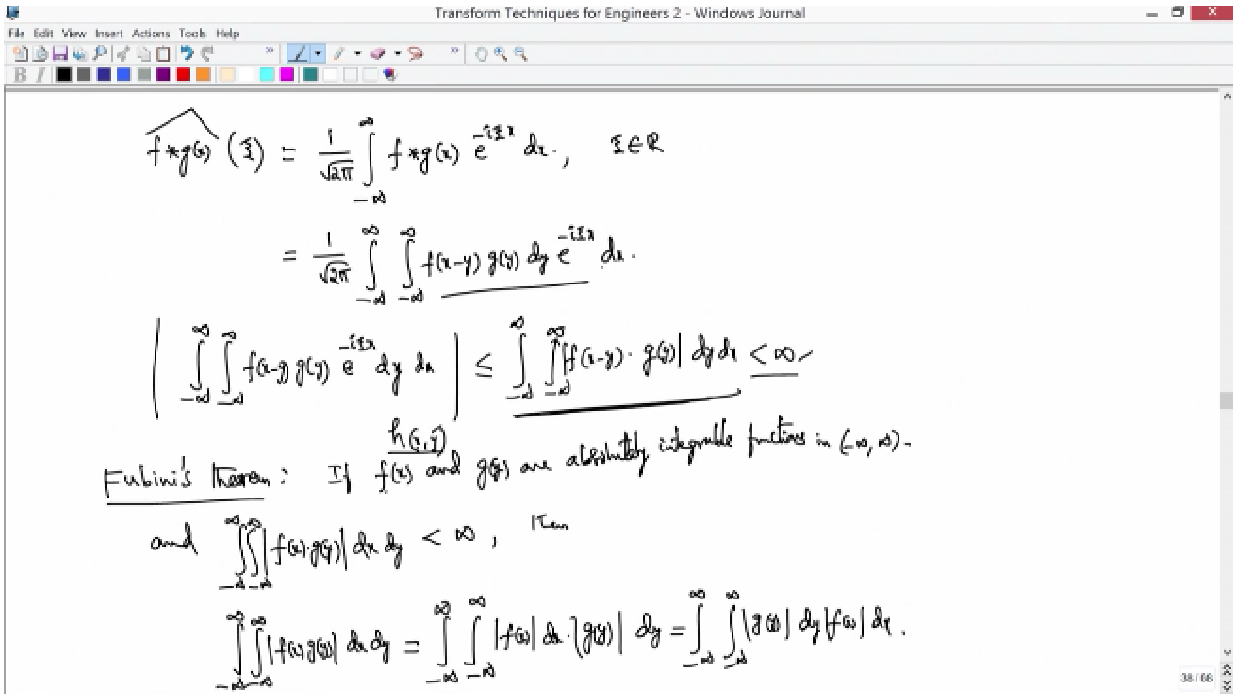

less than finite and then what you get is modulus of F(x,y) so here instead of F(x,y) we have separately you take the product of F(x) and G(y) okay, so as a double integral that means, what is the meaning of integral? Integral means you take a finite interval, so the domain full plane – infinity, the double integral DX DY means you have a full integral, full plane each part you break it as a finite, small, small rectangles or each of that you calculate, you sum it, you take a sum and then finally you take the limit so as that rectangle goes to 0, area of the rectangle goes to 0 that is the meaning of double integral you don't evaluate separately as a single integrals, okay.

Suppose this is finite in general and if the, then iterated integrals when they exist they have to be finite, they have to be same, they have to be same as double integrals so here also that is the case this will be equal to the –infinity, infinity, if it is this itself is finite the iterated integrals will exist and they are finite this is like F(x) separately DX and G(y) DY so these are separately, I first evaluate and then I evaluate this one so this is same as –infinity, infinity, -infinity, infinity and so now what I have is G(y) DY into mod F(x) DX, so these are the two iterated integrals they are also same. If double integral has double integral from the calculus if it

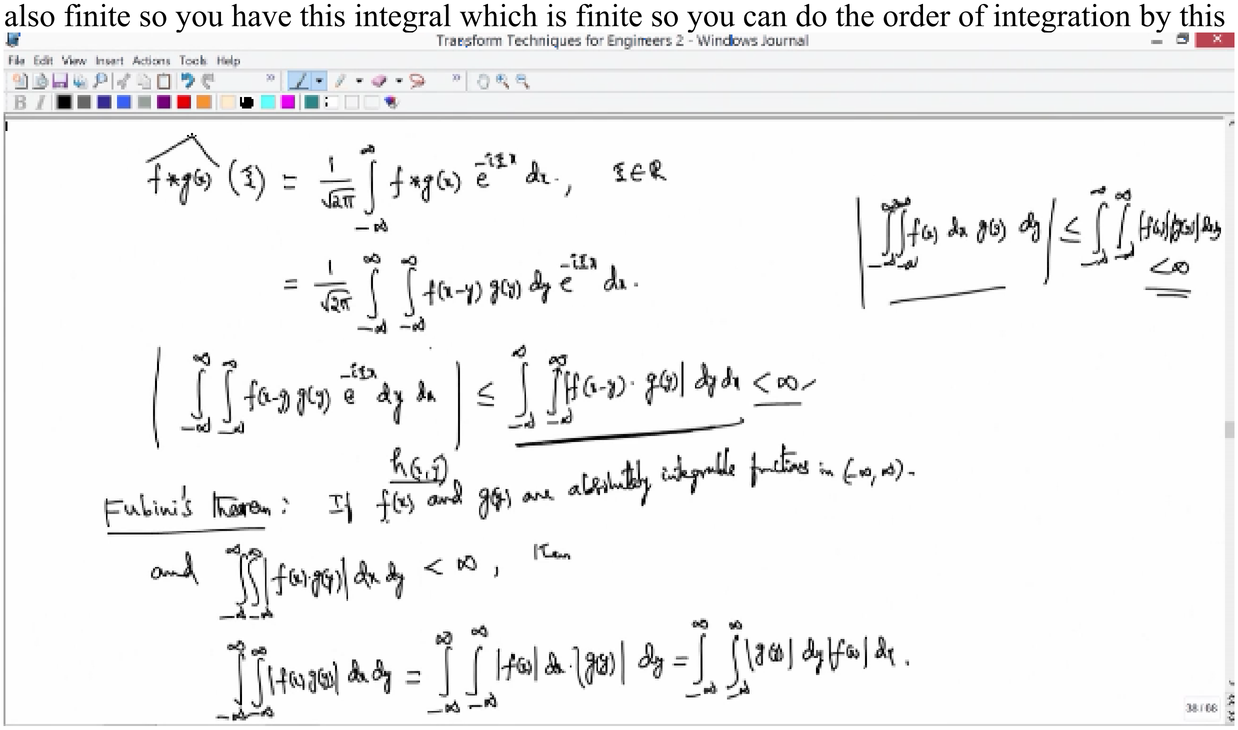


is a two variable function a double integral is well-defined so you know you should know that what is the meaning of double integrals just like how you define single integral as area under the curve, so here you view this double integral as the volume under surface Z = F(x,y), so F(x,y) is the function defined over some domain in the R2, in the plane, so such a thing if you have so double integral such a thing exists what you have is iterated integrals separately evaluate and those things should be same as this one, so that is the Fubini's theorem, so here this F(x) and G(y) is our variables are separated this need not be like this you can choose the F(x) into G(y) as some, let us say H(x,y) which is a function of X, Y, so H(x,y) this statement you have to replace if I choose instead of F and G if we choose H(x,y), you fix X, H(x,y) is absolutely integrable, you fix Y, H(x,y) is absolutely integrable as a function of X in the full real line –infinity, infinity.

And the double integral modulus of H(x,y) is finite then you can write these iterated integrals these two will be same as this double integral that is the meaning of this Fubini's theorem that is exactly what we use here, so because of this, this is same as you can now interchange order of integration that is the meaning, okay, so if you change the order of integration so you see that

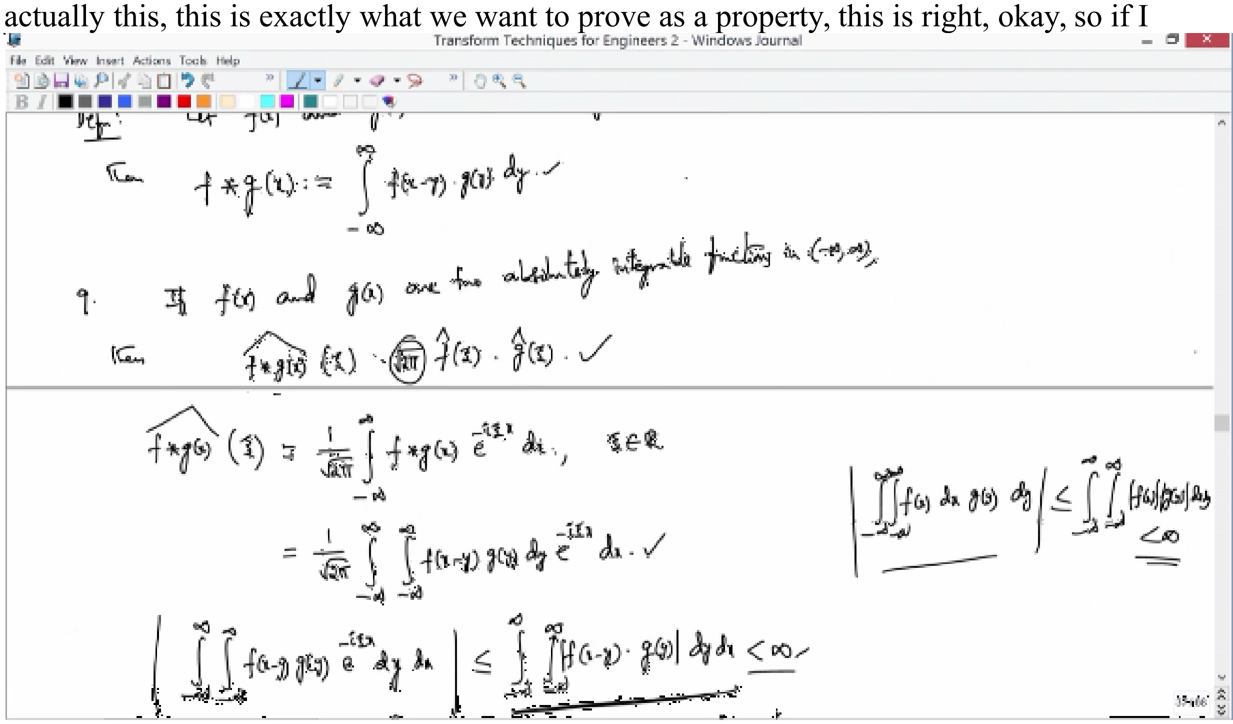



is the meaning of, so because F and G now, now from the statement F and G both are absolutely integrable function, so this is absolutely integrable functions F and G, so that double integrable makes sense, okay, so this modulus of this is always less than or equal to this one, modulus of, so this once you have this without modulus F(x) G(x) D(y) DY, this modulus is less than or equal to this is a double integral of this, F(x) G(y) and DX DY which is finite anyway, so this is also finite so you have this integral which is finite so you can do the order of integration by this



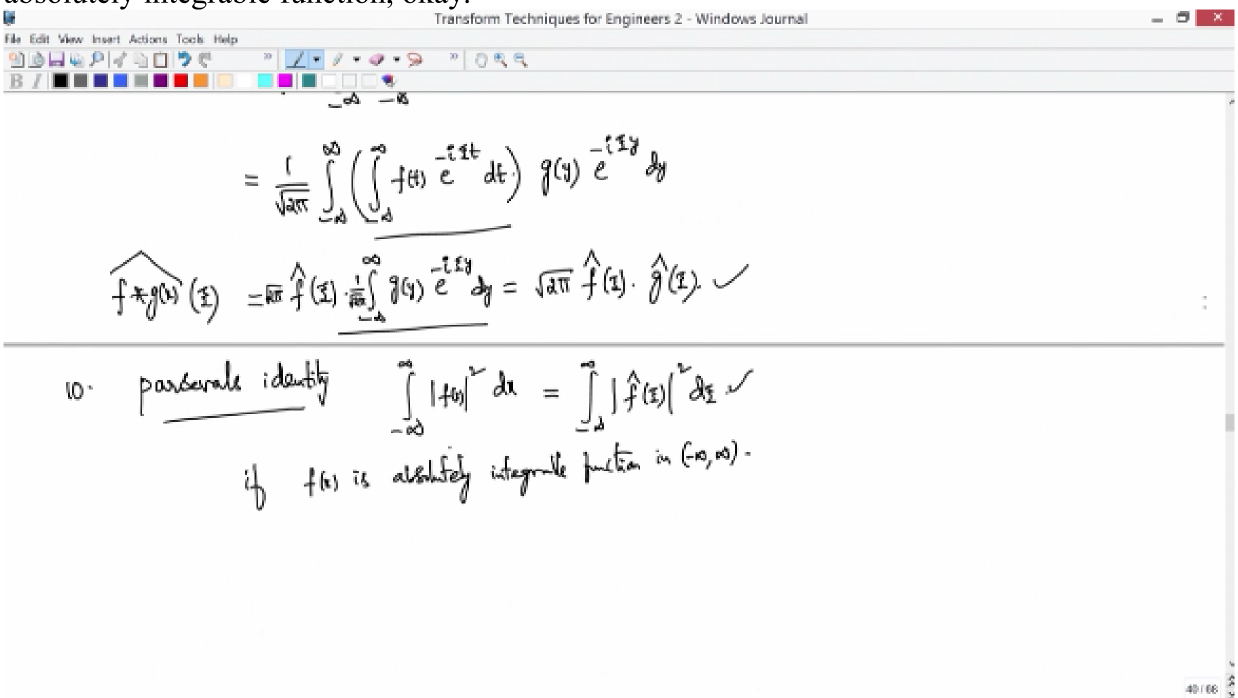

Fubini's theorem we don't know whether we do that or not, so we will see, we want to have separate, we want to separate these X and Y variables, so what we do is let's choose X-Y I try to replace X-Y as a new variable if I do this and X-Y = T, if I use this what happens to that

integral, we get root 2 pi integral –infinity, infinity, so inside and you have X-T so that is I'm integrating DY variable, so DY, X-Y is T so you have -DY is DT so you have a -DT okay, and when you put Y = - infinity it's going to be X -infinity is it X-Y, Y is –infinity, that's +infinity to –infinity, so if I remove this minus and make this interchange these limits this will be this.
And what you get is F(t) into modulus of this and not modulus, without modulus, right, so then we are looking at this integral, so this one G(y) is, so G(y) what should I do G(y) is, I forgot, wait, okay, so I think so this is up to here little bit so, so because of this Fubini's theorem we can do that order of integration I think that is required so that is equal to 1/root 2 pi I'll just do the order of integration because of the Fubini's theorem, so –infinity, so this iterated integral instead of doing this way I get X-Y G(y) E power –I xi X, earlier I have DY/DX DY into DX now I do first DX and then DY, okay.
So now you look at this 1/root 2 pi –infinity, infinity inside DX part that is F(x-y) E power -I xi X DX into G(y) DY, now you look at this one here you try to use X-Y =T, if you do that this is DX will be DT and you have then the double integral becomes 2 pi, inner integral becomes this is limits as any –infinity –Y, Y is fixed so that is –infinity, +infinity - fixed Y is actually infinity, so limits won't be changing you have F(t) and E power -I xi, X becomes Y+T and you have what DX is DT, and G(y) DY, now they are separated, variables are separated now, 2 pi -infinity to infinity, -infinity to infinity now you write F(t) E power -I xi T DT separate and then into G(y) this you write E power -I xi Y DY, this is nothing to do with Y so that is a constant that comes out and with 1/root 2 pi that is exactly F cap(xi) into -infinity to infinity, G(y) E power –I xi Y DY, this is root 2 pi, I need the 1/root 2 pi into I have to multiply it 1/root 2 pi, okay, so this is actually G cap(xi), so F cap(xi) into G cap(xi), this is exactly my F star G that is a convolution product of a, convolution product Fourier transform of that as a function of xi is actually this, this is exactly what we want to prove as a property, this is right, okay, so if I

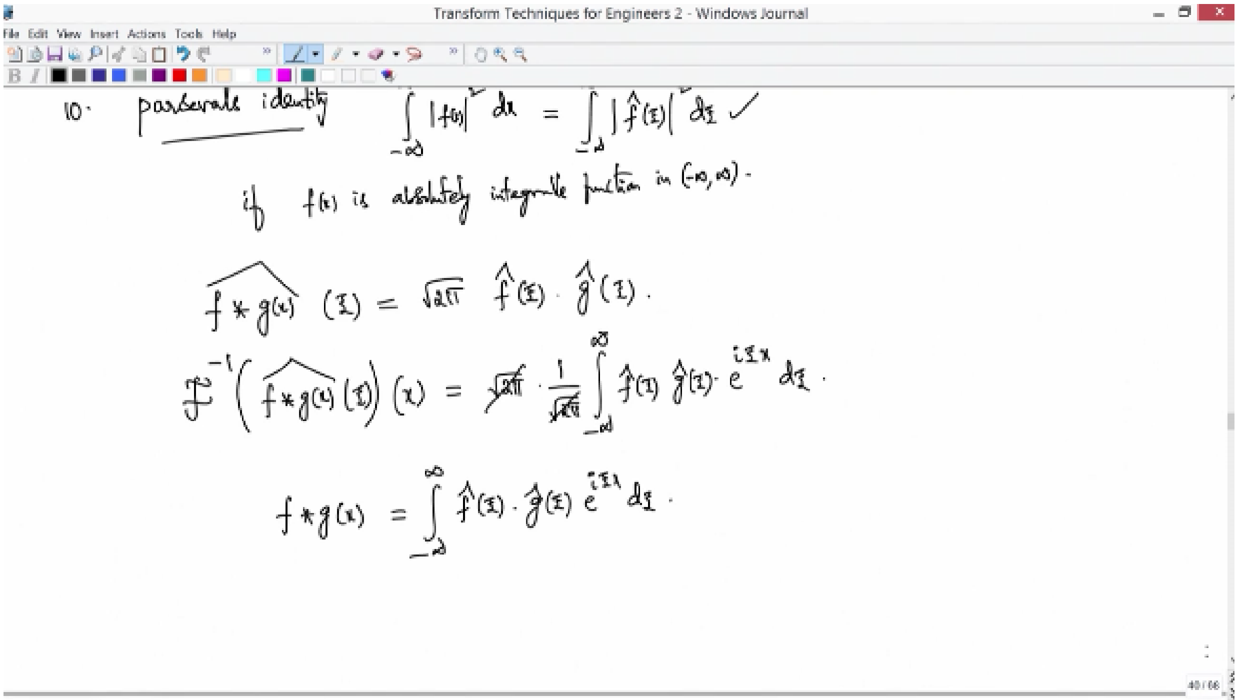



define, if we don't want this part if you don't want this thing you can redefine your convolution product as 1/root 2 pi if you do this then there won't be any, there won't be finally any root 2 pi in the, on the final property, okay, you will not get this one.

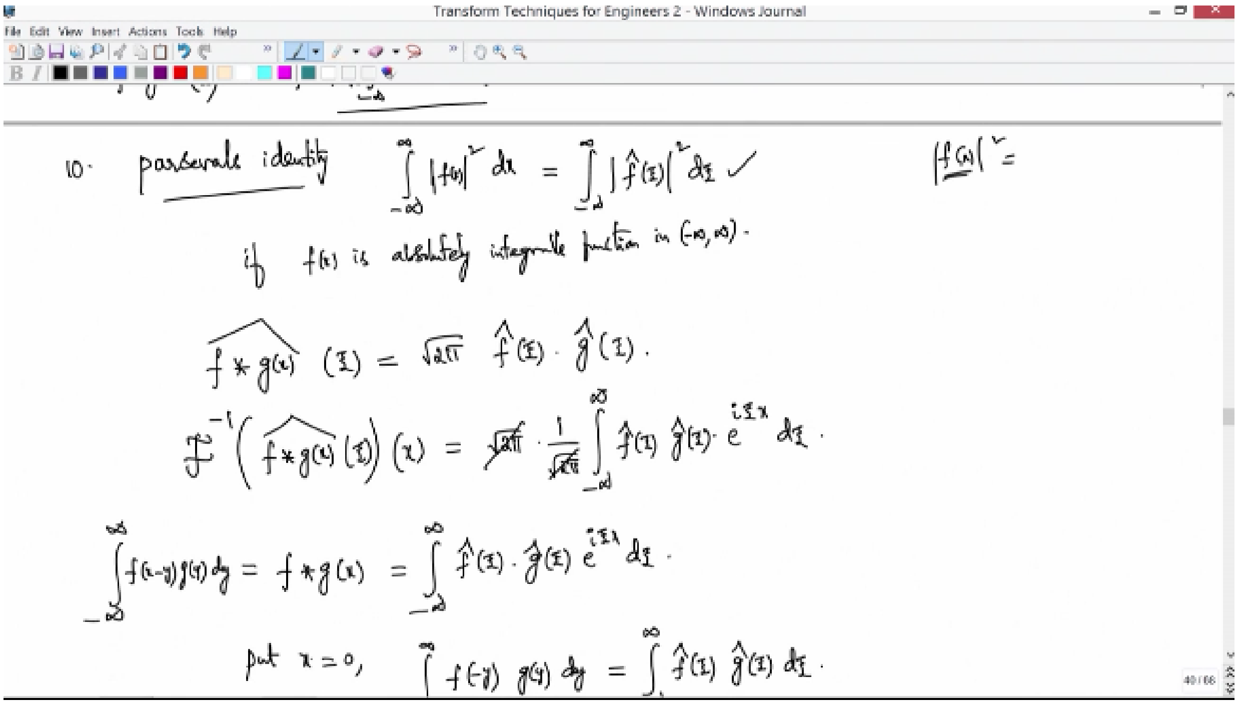
So let's not define, so let me stick to what I am in, well you don't multiply anything simply take the product as usual, you simply take the product of the two functions and one with X-Y, other one is G(y) you integrate it that is a convolution product then what you get is this result so Fourier transform of the convolution is actually with some constant root 2 pi times, Fourier product of Fourier transforms of F and G, one more property that is basically corollary of this byproduct of this property 9 is Parseval's inequality, Parseval's identity you can get nice identity that is, that tells you that double integral of absolute square of this, let's say F(x) modulus square DX is same as this one, so F cap(xi) square D xi, this is what is true, if F(x) is absolutely integrable function, okay.




So if such a thing you have this nice identity, so this physical problems is more generally, energy is represented as square integrable, as a square integrable function so that is why is this identity tells you some kind of energies conservation, okay, so you have this Parseval's identity let's prove mathematically what to do, so is the byproduct of this earlier property and from the earlier property you can easily see that you just write the earlier property you get what you want, and this is exactly root 2 pi F cap(xi) into G cap(xi), now what you do is you take the inverse transform of this, if I take the inverse transform of this let me write inverse transform like this of F star G cap(x), if you do this as a function of xi for this then what you get is finally function of X, okay, so this is root 2 pi inverse transform, inverse transform is I will just write 1/root 2 pi –infinity, infinity for this F cap(xi), G cap(xi) into E power I xi X D xi, this is also function of X, so it gets cancelled. And here this is exactly, this is nothing but F star G(x) which is equal to -infinity to infinity, F cap(xi) into G cap(xi) E power I xi X D xi, okay.

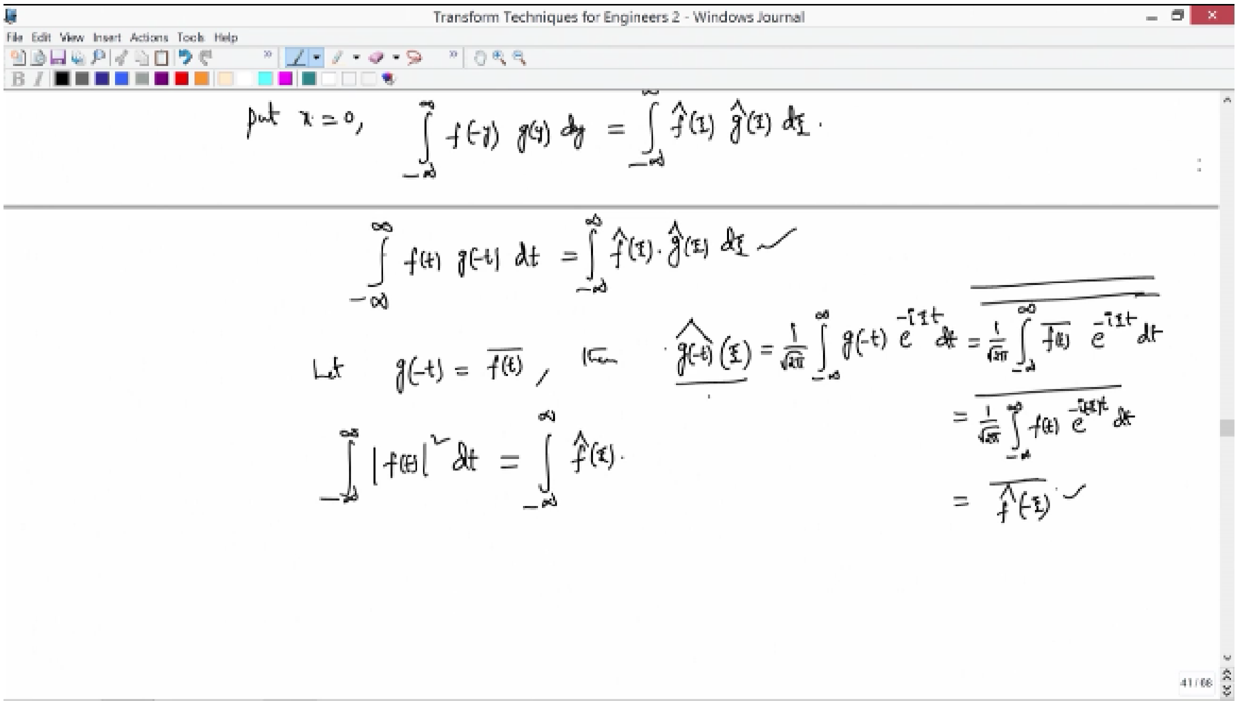



Now what you have to do? Simply put X = 0 here, so what exactly this means? This means in my definition of convolution product I don't have any multiplication constant, so you simply defined as this way F(x-y) G(y) D1, so if I choose like this put X = 0 and choose G(y) as remember in the earlier property, F and G both are absolutely integrable functions, so G(y) you can choose as a function of, how do I choose this function of, once you choose or this is X = 0, once you choose as X = 0, let us see what happens, so X = 0 what you get is –infinity, infinity, F(-y) G(y) DY which is equal to –infinity, infinity, F cap(xi) G cap(xi) D xi, and what you do is, you choose, so this is same as -infinity with, if –Y you replace it as Y that means you take -Y = T for example, so then you have F(t) and then G(-t) DT, DT is -DT and you have, this becomes infinity to -infinity that makes it –infinity, infinity, so both are same and you get this one F cap(xi) G cap(xi) D xi, so what I choose is, I choose G(-t) as F(t) bar, if I can choose this, if F is a complex valued function, F is a real valued function or which is also you can think of complex valued function, so right, so this modulus square means this square means F(x) into F(x) modulus square in complex if you can think of even complex, real number is also complex

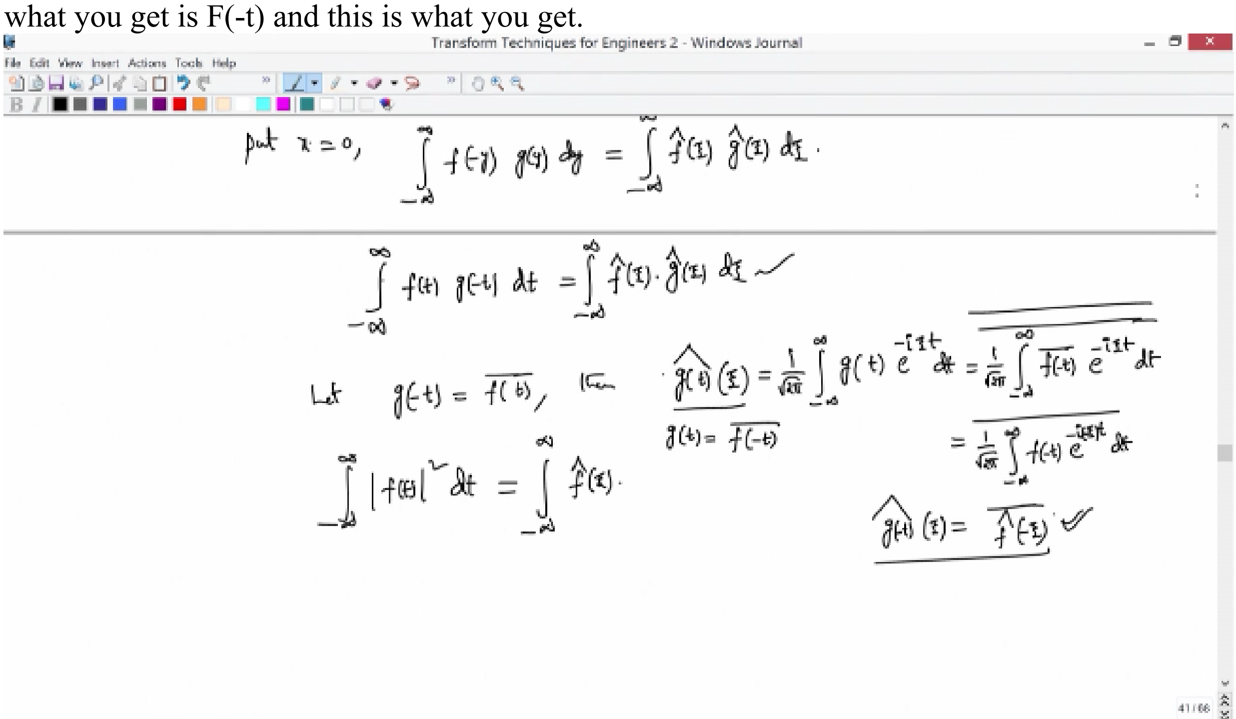



number, right, so F(x) even if it is a real valued function you can think of, you have seen that signals can be even complex valued function, so F(x) is actually can be a complex valued function, so F(x) bars modulus of F(x) square is F(x) into conjugate of F(x), so in that way if F is the absolutely integrable function, F dash of, f bar, conjugate of F is also absolutely integrable function, so you can choose such a, sum G in such a way that which is G(-t) as a modulus of this, if you chose then G(-t) is also absolutely integrable function, okay, then G(t) is absolutely integrable function then what you see is this identity becomes -infinity to infinity modulus of F(t) bar square DT that is exactly your left hand side, and the right hand side so once you have this what is your G cap(xi)? G cap(xi), so what is this G cap(xi)? So this we try to use, so we will try to see this one separately, F(xi) and here G(xi) is, so what you have to replace, what happens to this G cap? If I calculate G cap(-t) what is this one? This is equal to G cap(-t) if I multiply if I take the Fourier transform which is a function of xi, which is 1/root 2 pi -infinity to infinity, G(-t) E power -I xi T DT, okay.

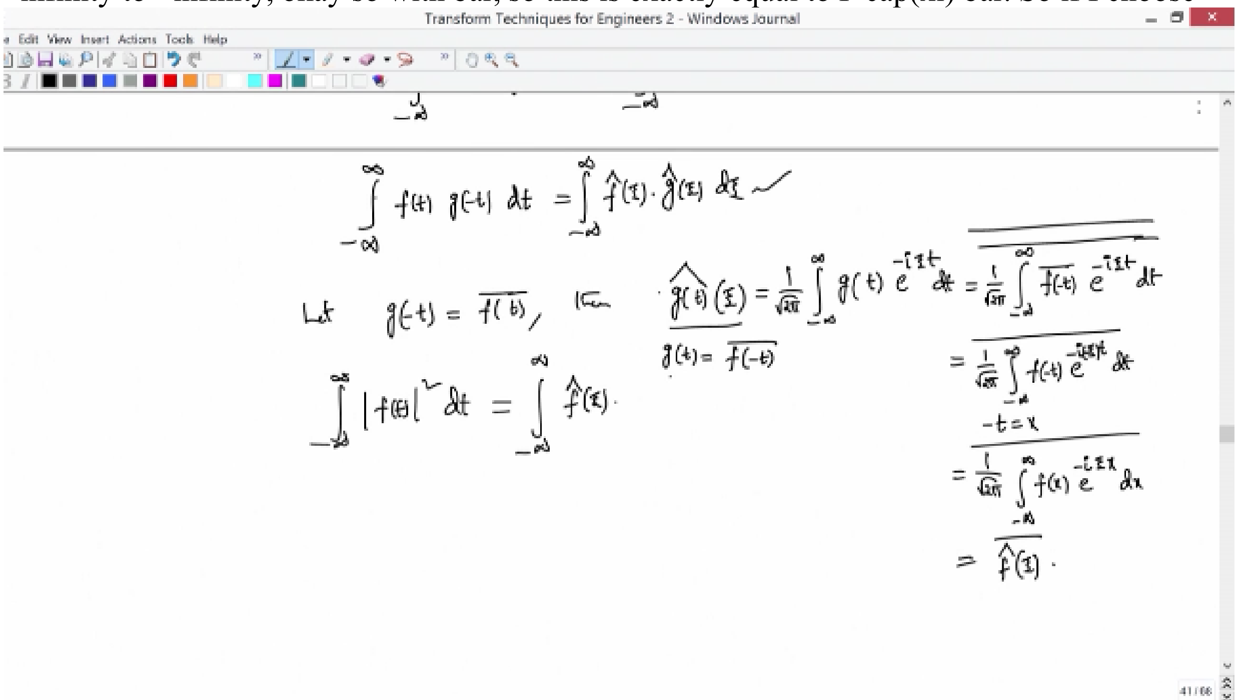
And then, so what is G(t)? 1/root 2 pi, now you can put just G(-t) as F(t) bar E power -I xi T DT, so you can do 2 times conjugate and you allow that one inside so you have a bar 1/root 2 pi –infinity, infinity, if you take this bar for this, this is a complex number so as it is, limits are complex numbers so real number, so bar conjugation doesn't matter, and this becomes F(t) and this becomes E power I xi T, so this is become +T, right, DT is a real number so this is what you get, so this is actually equal to bar off, and this you can write minus of –xi, okay, so that



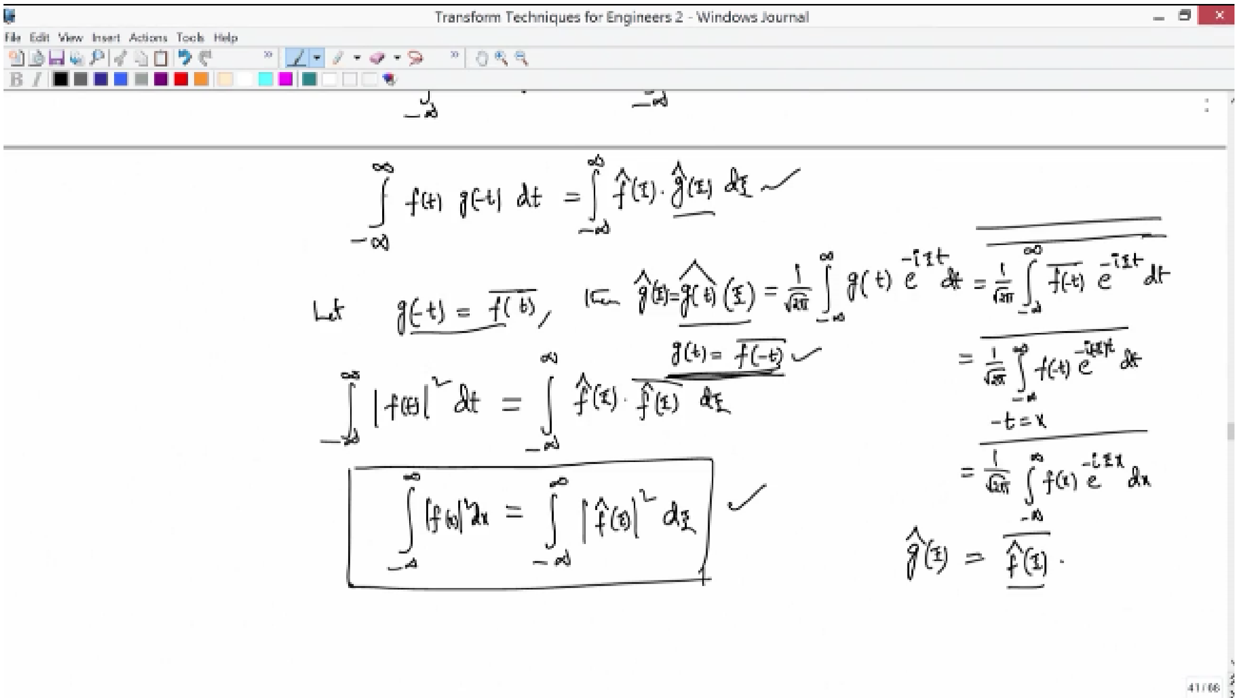

makes it bar of F cap(-xi) okay, so what you got is G cap(-t) if you take this is what you get, so if you take, what do you need? If you take so this implies you get G cap of, so G cap(-t) which is a function of xi, this is what you get, this means G cap(xi) is what? This is G cap of, or you start with G(t) is F(-t) let us say, okay, G(t) is once you choose like this first of all, once you choose like this then what you get is G(t) = F(-t) bar, so G(t) so if I replace T then you try to calculate only G(t) okay, then only G(t) this is what happens, and F(-t) I have to put, right, so what you get is F(-t) and this is what you get.



And then you do the change of variable here, so here if you try to do the change of variable T equal to, -T = X if you do again what you get is bar of 1/root 2 pi -infinity that becomes +infinity –infinity, F(x) E power –I that's anyway I xi T, so -I xi X, DX is –DX, so that makes it –infinity to +infinity, okay so with bar, so this is exactly equal to F cap(xi) bar. So if I choose




this G(t) is this, so F cap of G bar is, so this is exactly my G cap(xi), so this means G cap(xi) which is a function of T for which if we take the Fourier transform which is a function of zeta, it's a transform variable, so you have G hat(xi) is F hat(xi) bar, if your G or F are related by this which is same as this, okay, so in that way if I now become, so what do I have to do, I have to again replace G cap(xi) with F cap(xi) with conjugate D xi, so this is exactly –infinity, infinity, absolute value square, F cap(xi) square D xi, so this is exactly what you want.

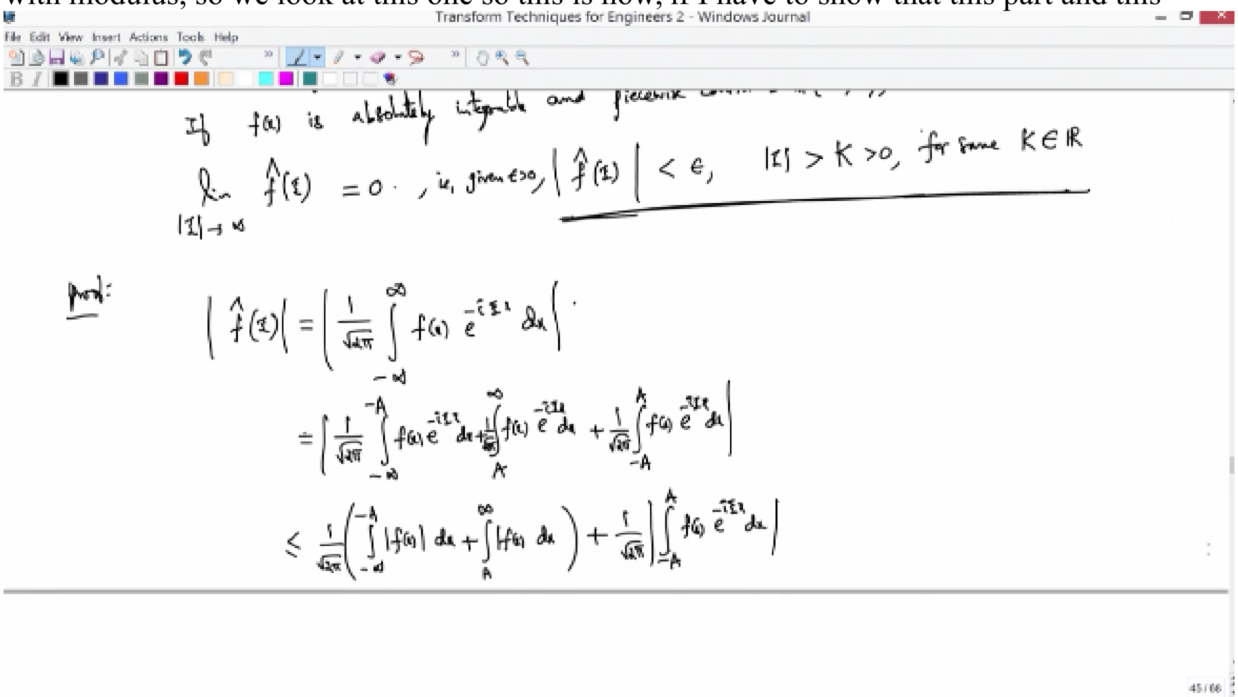



F(x) let us write DX, so this is the Parseval's identity, so which is you're making a conservation of energy kind of thing, because the square integrable function normally energy is represented in like in this integral representation, square integrable square, absolute value of the square integration in general, so most of the physical problems if you see energy is represented this way, so you can think of this as a conservation of energy before transformation, after transformation, okay, so this is one property, important property and one more will do so with that we'll try to prove the Fourier integral theorem.

We look at the property it's called Riemann Lebesgue Lemma, we have already seen this for piecewise, we have seen this Riemann Lebesgue lemma for periodic signals that is if you have a signal of defined for X belongs to –L/2 to L/2, so what you have these Fourier coefficients goes to 0 as mod N goes to infinity, so this is exactly this is the finite that is a Fourier series version of Riemann Lebesgue Lemma, so in this case what is this one? So what we do is now we want to do for non-periodic signal that is F(x) is defined our full real line if such a function F(x) is a signal F(x) is, and it is assumed that is absolutely integrable and piecewise continuous, you have only finitely many discontinuities, and –odd to odd, -infinity to infinity, so such a function if you have then what you have is limit, the Fourier transform of this mod xi goes to infinity is actually equal to 0.

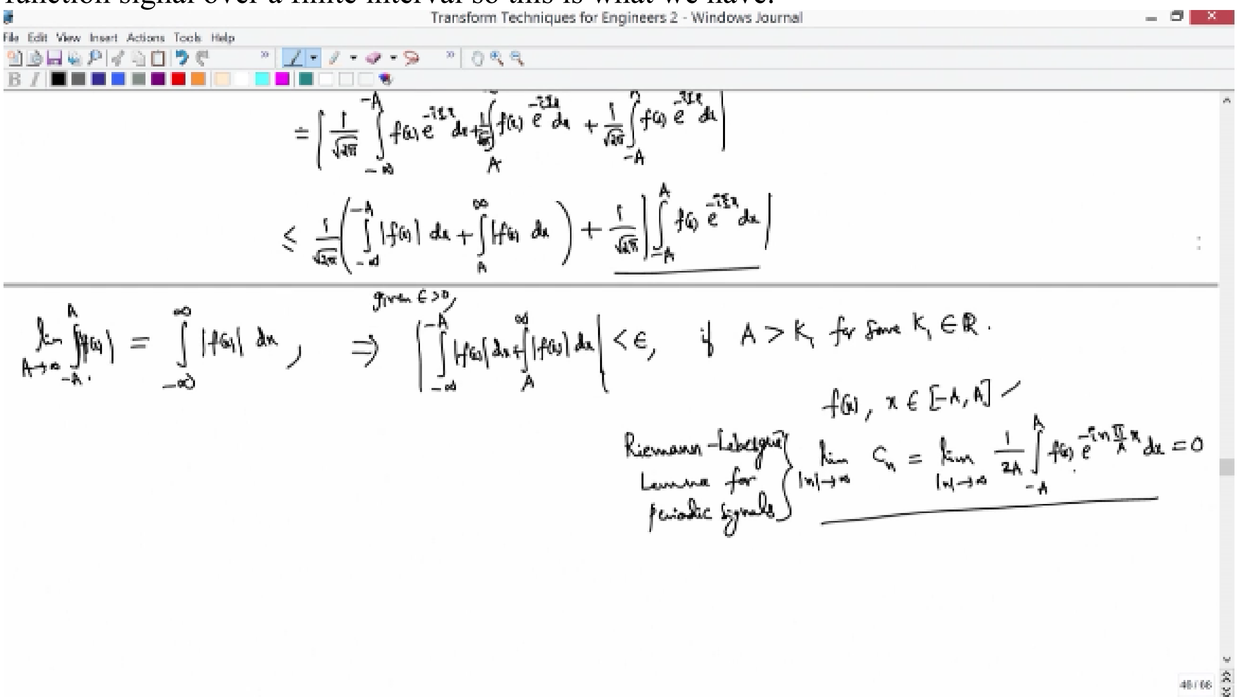
So look at the proof, so what we do is we try to take this F cap(xi, we apply this limit so we will try to apply so what is this meaning? That is as a limit definition if you use this is F cap(xi) this is less than epsilon, whenever mod xi is bigger than K, some big, let us say some K which is positive for some K belongs to a real number, so if such a thing happens so this means, this is always true given epsilon positive that is so you can write like this, for given epsilon positive we have this thing, this if you have this for some always K then you say that this limit is equal to 0, that is what we are going to show, so we are going to show this one so for that you write Fourier transform which is by definition 1/root 2 pi –infinity, infinity F(x) E power -I xi X DX, so for this you take the modulus what you want is this one so you take the modulus, so now we have the modulus and this one is, you split this into two parts this integral you can split this into

two parts, so what I do is 1/root 2 pi this is from -infinity to some number, some big number A, -A F(x) E power -I xi X DX+ A to infinity, same F(x) E power -I xi X DX and then plus 1 by, of course 1/root 2 pi you have, 1/root 2 pi, now what is left is -A to A F(x) E power -I xi X so I split this integral into 3 parts, so this is less than or equal to 1/root 2 pi and integral -infinity to infinity, mod F(x), I take this modulus inside so this whole integral, say this whole integral modulus which is again less than or equal to, I can take this modulus inside this integrand that is mod F(x) into E power –I xi X, that is actually mod F(x) DX, this is –A, +A to infinity mod F(x) DX, so this is one part.
And the other part is +1/root 2 pi –A to A, so let me write here -A to A F(x) E power –I xi X DX with modulus, so we look at this one so this is how, if I have to show that this part and this

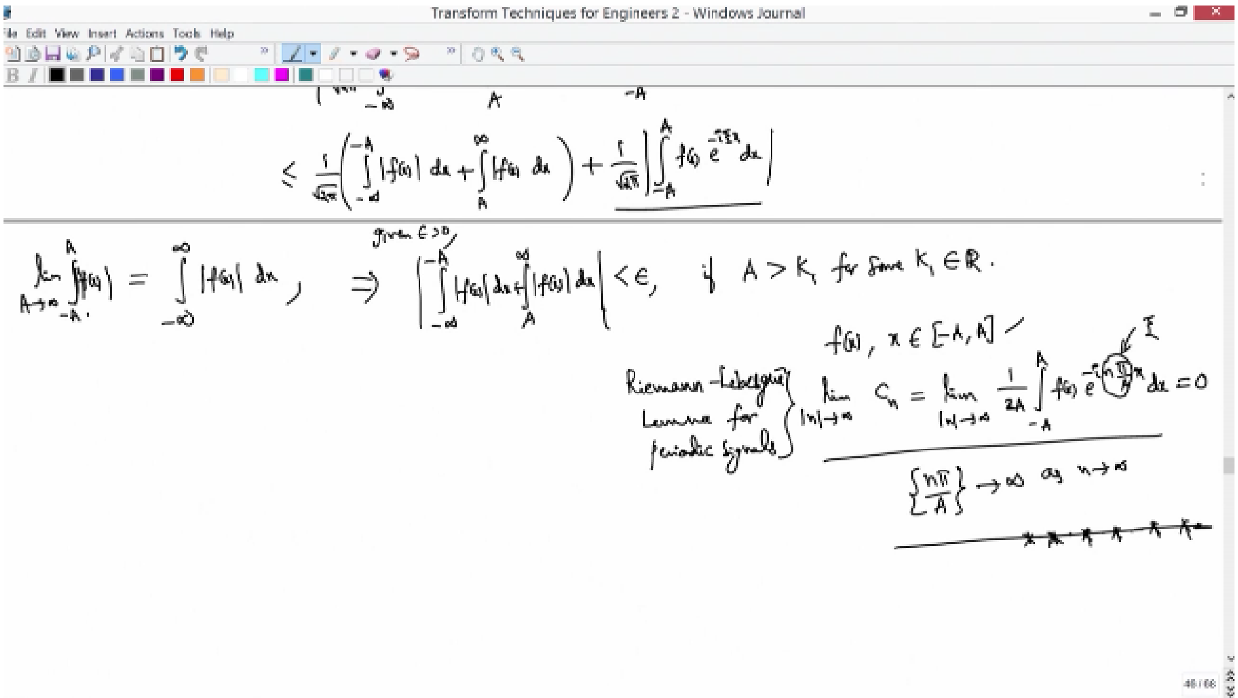



part together they're like, they can be made less than epsilon, so look at epsilon, so because F is absolutely integrable function and what you have is the integral this means -infinity to infinity because it is –infinity, infinity, mod F(x) DX is finite that is the meaning, okay, so what is this meaning of this one? This is actually equal to so limit of A goes to infinity for some A, -A to A F(x), mod F(x) rather okay, this is the meaning of this improper integral, so this is some number, this is the limit, so what does it mean? So this means this minus this so that is that is with modulus, modulus of that that is actually this one, so -infinity to infinity, sorry -infinity to -A mod F(x) DX + A to infinity, mod(x) DX this quantity can be made less than epsilon whenever, so given epsilon positive so you give epsilon any however small it is you give such an epsilon I can make this quantity less than epsilon whenever for some, some A for some A big enough, for some big A belongs to A, so A is, so let us say A bigger than some K1, okay, for A bigger than K so you say like this for rather this can be made less than, if A is bigger than K1 for some K1 belongs to the real numbers, so technically this is the meaning of this limit is the definition.
Now you want to show this one, so what happens to this? For this now this is this means F is defined over interval -A to A, so this is exactly what we do, we try to use, we try to take this function as a periodic signal defined over –A to A, okay, so once you have this you have a Fourier transform so what we have from Riemann Lebesgue lemma for this periodic signal is F

cap over, you have Fourier transform of this over a finite interval they are nothing but Fourier coefficients, CN mod N goes to infinity this is 0, so what is CN? CN is limit, 10 goes to, mod N goes to infinity, the CN is nothing but 1/2A integral -A to A, F(x) E power –IN omega naught is 2 pi by, this difference is 2A, so pi/A X DX, so these are your CN which is actually 0, as this is the meaning of Riemann Lebesgue Lemma, Lemma for periodic signals, that means any function signal over a finite interval so this is what we have.

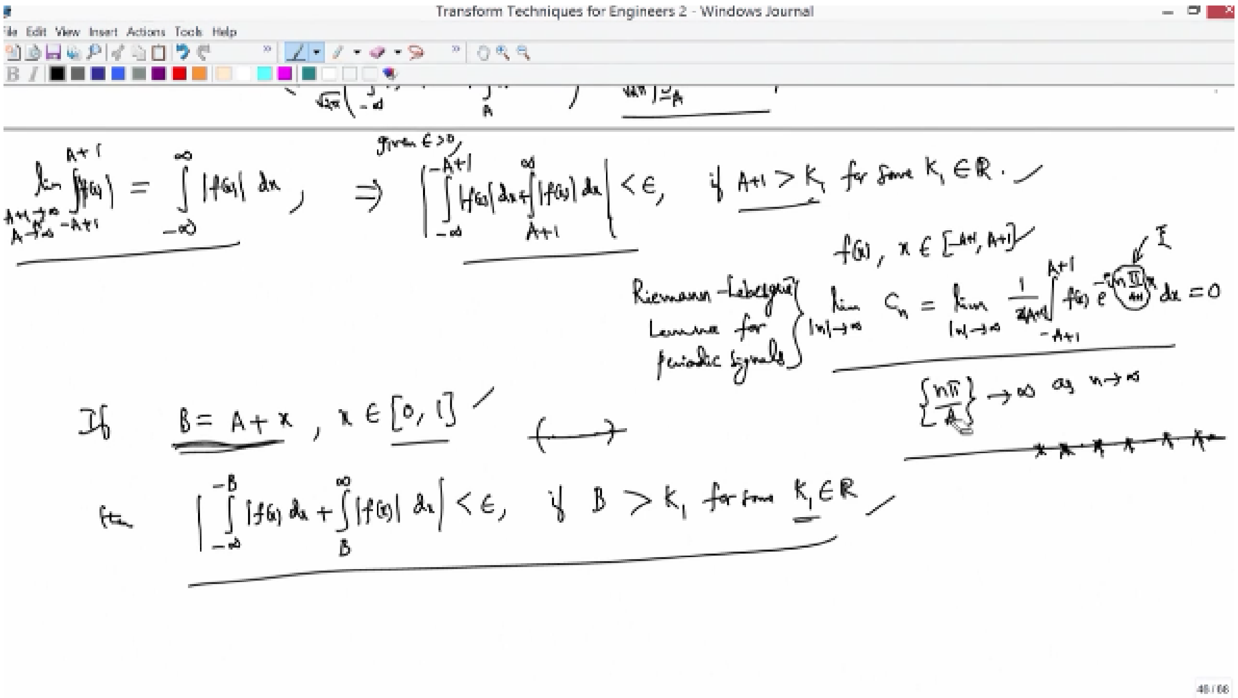



So what does it mean? So but then this tells you that what you need is, what you need is integral -A to A F(x) into E power –I xi X, so I xi is this one here, so this is your xi, xi is a continuous variable, so if you actually see xi is a continuous variable so this tells you that N pi A, as N goes to infinity pi/A, once you have fixed A, N pi/a is N = 1 pi/A so that's a positive number some number and then N = 2 it will be having like this, it will go to infinity, that is how the sequence of, sequence N pi/A, N pi/A sequence goes to infinity as N goes to infinity, as like this, right, this is the meaning of going to infinity, but what you need is xi goes to infinity, so what you require what you look for is xi goes to infinity, so that means if my sequence is here

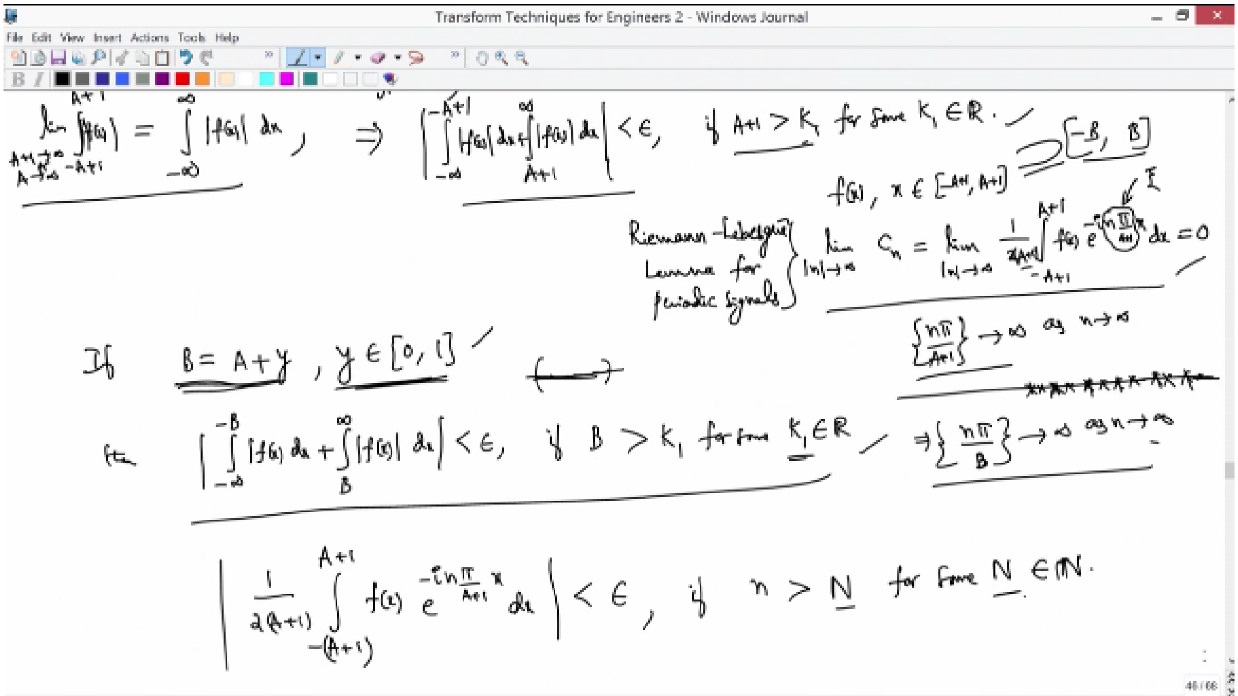



going to infinity like this that also should be accounted for, so what I do is I consider A+ some small interval, so let us choose some small interval let us say 0 to 1 interval, so if you add so what you get is A+1 –A+1 and you just add, you take bigger than this, A you take, for this A once you fix A so such a big A you try to consider A+1 okay instead of A you consider A+1 okay, so if I do that still even if I saw this means if A is this if I choose if I take A+1 here, so if I take A+1, A+1 here still this is valid because A+1 is bigger than still K1.

So then that way so this is I can choose some B as A+, now some you can say open intervals or closed interval doesn't matter, okay, so you can choose the interval B is like this, okay, A+ some X, X belongs to this closed interval let us say, so then you can replace this with B, okay, so any B that is still valid, if B is this then still I have that quantity so that -infinity to -A okay, - B mode F(x) DX + B to infinity, mod F(x) DX is still less than epsilon if B is bigger than some K1, for some K1 belongs to R, so that means same K1 will work for every B if K1 is you choose K1 as A+1, so I try to choose A+1 as here, okay, if I choose A+1 this is what is true okay, so both are same right, so A goes to infinity so let's choose like this, okay, this means this minus this as A goes to infinity A+1 is also, I'm just replacing A/A+1 that's all, otherwise everything is same, A+1 goes to infinity is same as A is going to infinity both are same, okay. So that means this is true, so for A+1 when you choose you have such a K1 exists, now if I choose B as A+X, X belongs to 0 to 1, so if we start with A, A+0.5, A+ any number in between these real numbers, okay, this interval anything you choose so this is still valid for the same K1, okay, and then you look at every time if you already have this A+1, right, now you have instead of A you have A+1, A+1 interval and you have everything A+1, A+1 this is what exactly you have right now, okay, A+1, so this is exactly your Z, okay, so if we choose this one A becomes A+1 so this is a Riemann Lebesgue Lemma for this interval, so you have some sequence.

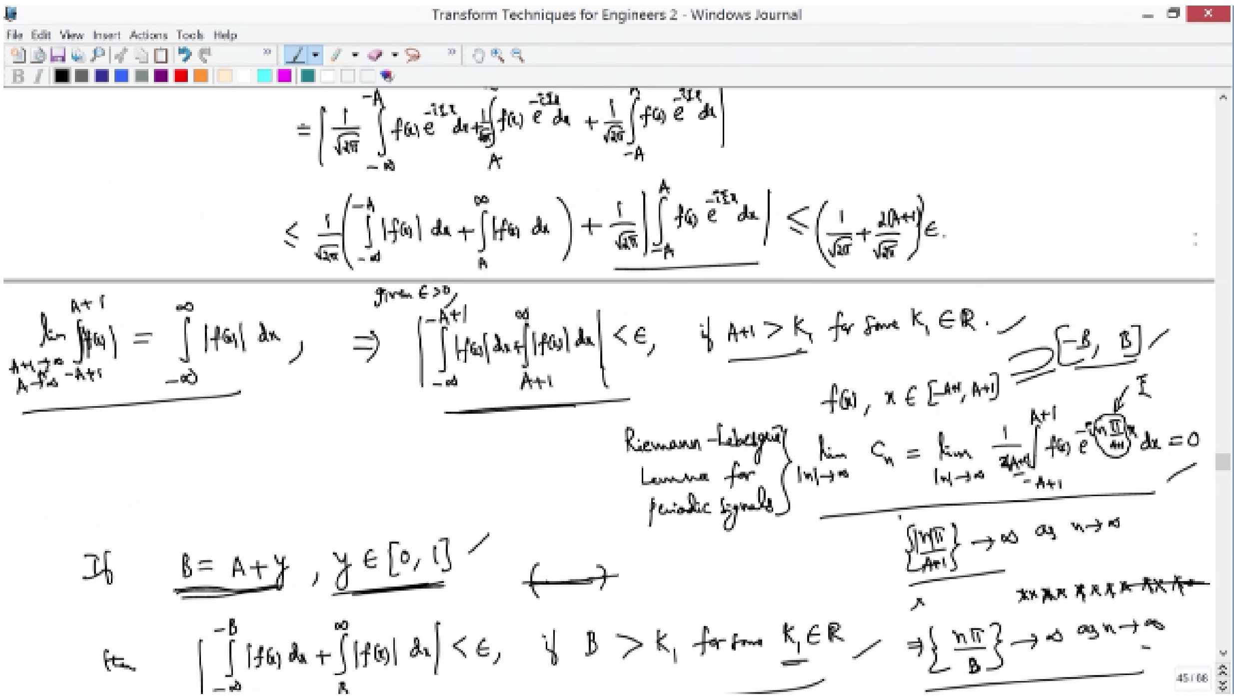



Now if you choose B, that is A +, you try to choose some interval in between this interval, okay, so if I choose this interval as, let us say B, -B to B, okay, this is actually contained in this if I choose B like this, okay, it may be same if I choose X = 1 here, so you don't want to call X, some call this some Y, okay, if my Y = 1 this is exactly –A+1 to A+1, so in this case what happens is that every time you change this one, and your A+1 becomes B, so 1/2 pi B, so you have N pi/B, so as you choose your Y variable here in this interval so you will see that every time the sequence is it covers all sequence, once you choose different thing it will have a sequence that goes to infinity, as N goes to infinity, okay, so this implies N pi/B this sequence going to infinity as N goes to infinity, okay, so this is always true, so what is the definition for this is Riemann Lebesgue lemma definition is 1 by 1/2 times A+1 is 2, -A+1 to A+1 okay, so if I write like this F(x) E power -IN pi/A+1 X DX this quantity can be made less than epsilon whenever if N is bigger than, right, N is bigger than some big number, for some N belongs to the natural numbers, some big number N.

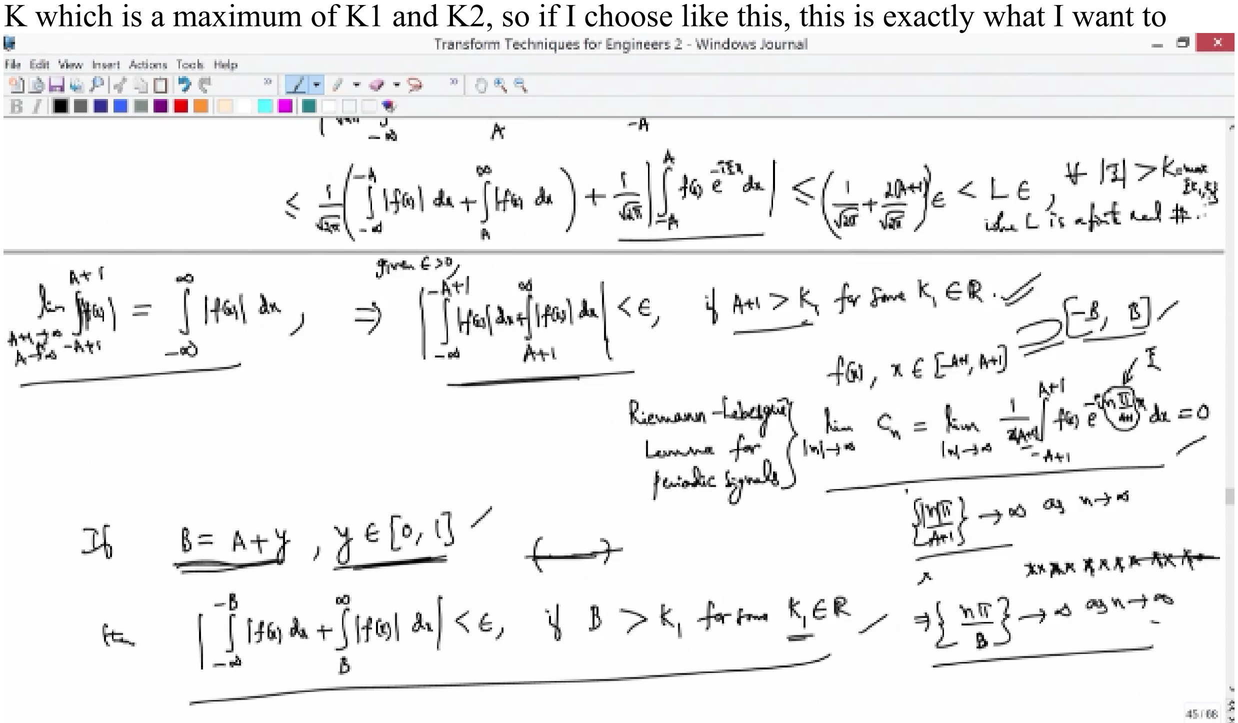



Now if I choose -B to B, this is the same N number will work, but it's a different sequence that converges to infinity, okay, so if I choose so every time you do that what you get is, this quantity is still true so that way if I choose like this, if I choose like this the same N which is valid from –A+1 to A+1 interval this Riemann Lebesgue Lemma when you choose this one it is still valid, so this quantity is always less than epsilon whenever if I replace this by some xi, if I try to put it as xi, and then this you can keep it as the same, okay, then it is still valid so this can be made less than epsilon for every N bigger than this, yeah, N is not, so now instead of N pi/B or N pi/A+1 I replace with xi, so I replace this N, as N pi/A+1 I can replace with some xi, both sides, so mod N goes to infinity this goes to infinity or mod N goes to, mod N into that, okay, so whichever way so even the negative side you have the same sequence, you have a sequence that converges to –infinity, that way also you can look at it and that way I can choose mod xi bigger than N for some N, so you can call this instead of this N you call this some K2 okay, which is positive for some K2, you call this.

Then if you combine these two, this one and this one, if you combine and put it here so this quantity is less than epsilon, so instead of A you have A+1 so it doesn't matter, so you can write this is, A you replace with A+1or A+1you replace finally A, okay, so doesn’t matter this is epsilon already, epsilon/root 2 pi and this one is 1/root 2 pi, this quantity is 2 times what is this one? So this is 2 times A epsilon, okay, 2 times A+1, right, 2 times 1/2A+, yeah that's it, so into epsilon, so this is what exactly so this is a fixed quantity times epsilon, this is a fixed number




finite number into epsilon, so that means what you have proved is some fixed number let us say some L into epsilon, so where L is a finite number, finite real number so that means mod F of, this modulus of this quantity I could make less than epsilon, so where is this valid? When you have this, this quantity is less than epsilon and this quantity is less than epsilon, so that means it is valid for every mod xi, mod xi is bigger than for every mod xi bigger than some K2, okay, and also K1, so K1 is, K1 K2 whichever is bigger you pick up, okay, so K1 K2 so you pick up K which is a maximum of K1 and K2, so if I choose like this, this is exactly what I want to




prove, so K is such a thing so that means this limit is actually 0, so this is how you can prove this Riemann Lebesgue Lemma, we can also prove this Riemann Lebesgue Lemma by some other way, we will try to prove that in the next video, and we will also prove finally eventually later on, once we prove this by other method we will also prove finally Fourier integral theorem, which is what is the requirement for us to believe that the definition of a Fourier transform and it's inverse transform. Thank you very much, we will see in the next video. [Music]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