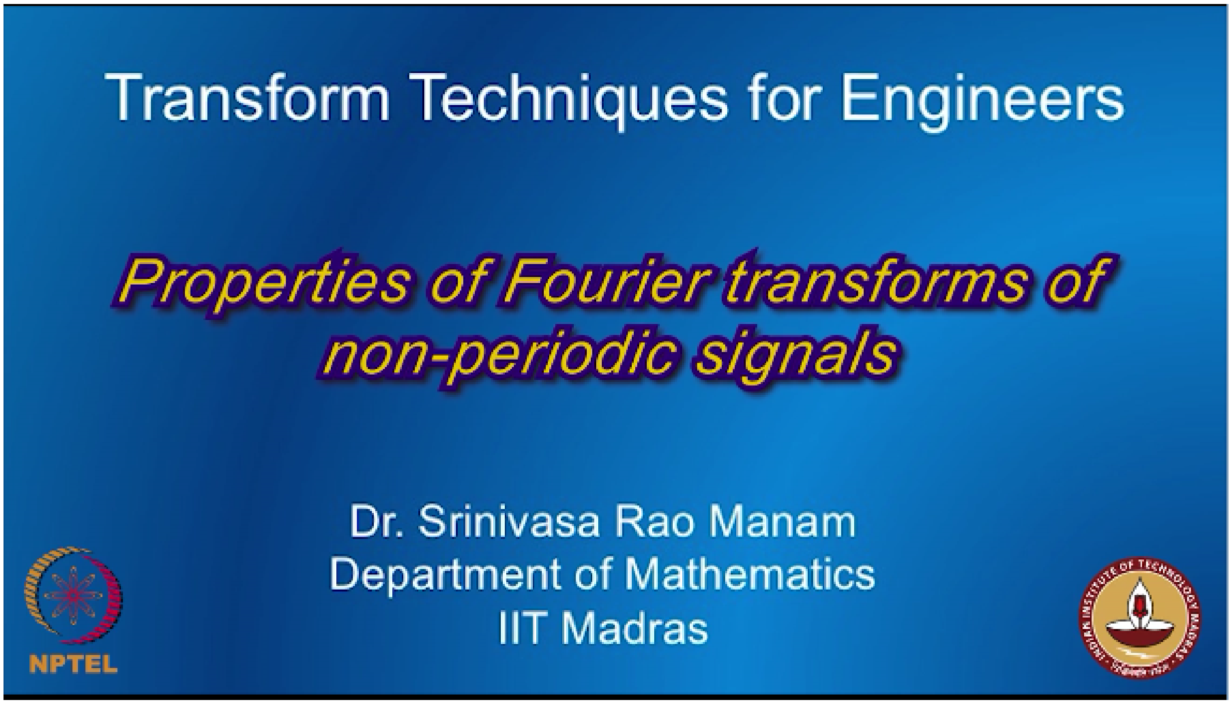
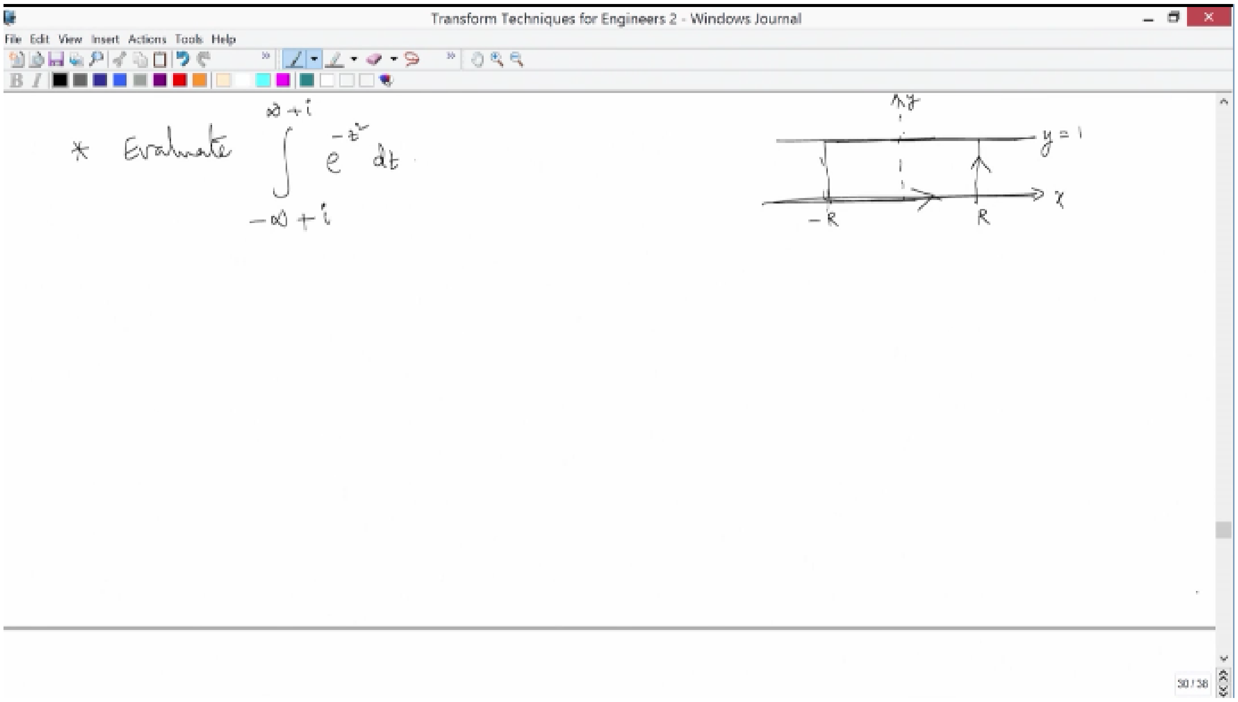
NPTEL
NPTEL ONLINE COURSE
Transform Techniques for Engineers
Properties of Fourier transforms of non-periodic signals
Dr. Srinivasa Rao Manam
Department of Mathematics
IIT Madras

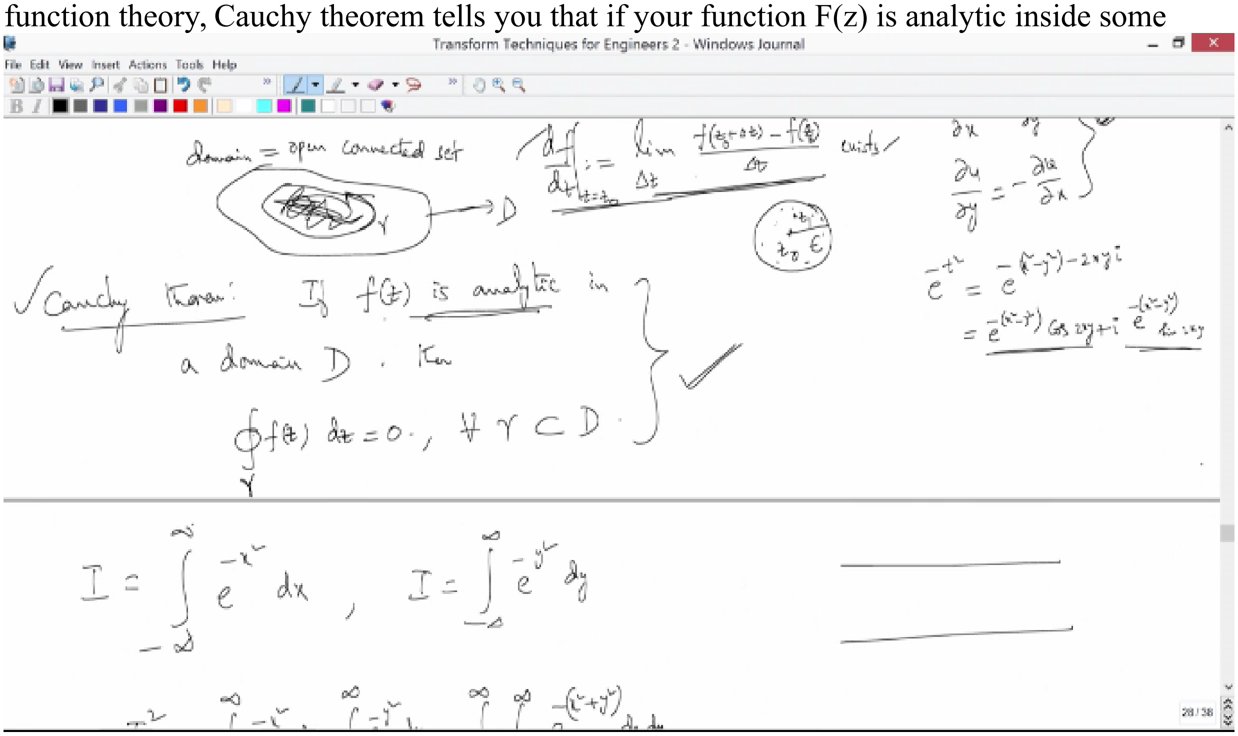


Welcome back, we were discussing about the Fourier transform of rapidly decreasing function yesterday in the last video, and today in the process what you have seen is you encountered an integral over contour that is not actually from – infinity to infinity, but it is a kind of - infinity +I into I to – infinity + infinity +I, so it's basically a line parallel to X axis, at Y equal to some point, okay, that is in the complex plane, it's called because it's a piecewise, it's a continuous curve so it's a straight line, so it's called a smooth curve in the complex plane.
So we explained that for this function F(z) which is E power -Z square a contour in the integral over that contour is actually same as integral over a contour Y = 0 that is X axis, so we'll try to show that again I explained you briefly and used it in the Fourier transform, today you will try to little digression, we will just look at that how to evaluate that integral.
So just let me, how to evaluate the integral let us say if I call - infinity +I, so I try to choose the line like this, this is X axis and I choose at X = 1 line, instead of xi/2A I choose Y = 1, so this is X axis is Y = 0, and this is Y axis, okay, so you want to do this, what you have is this infinity +I, so this is the minus, so this is the contour over which you have this one this integral E power because this is a complex number, so integrand this variable should be Z square DZ, so I want to evaluate this integral, I will show that this is actually equal to the same as it is over here, okay, so how do I do this? I will try to take this, let's choose from some let us say - R to +R, you make a rectangle here and you try to make a close it by this way, you close it so in this direction you are going so, or if you don't want what you do is you try to have a positive orientation as I said, if you move along this domain inside this rectangle should be on your left

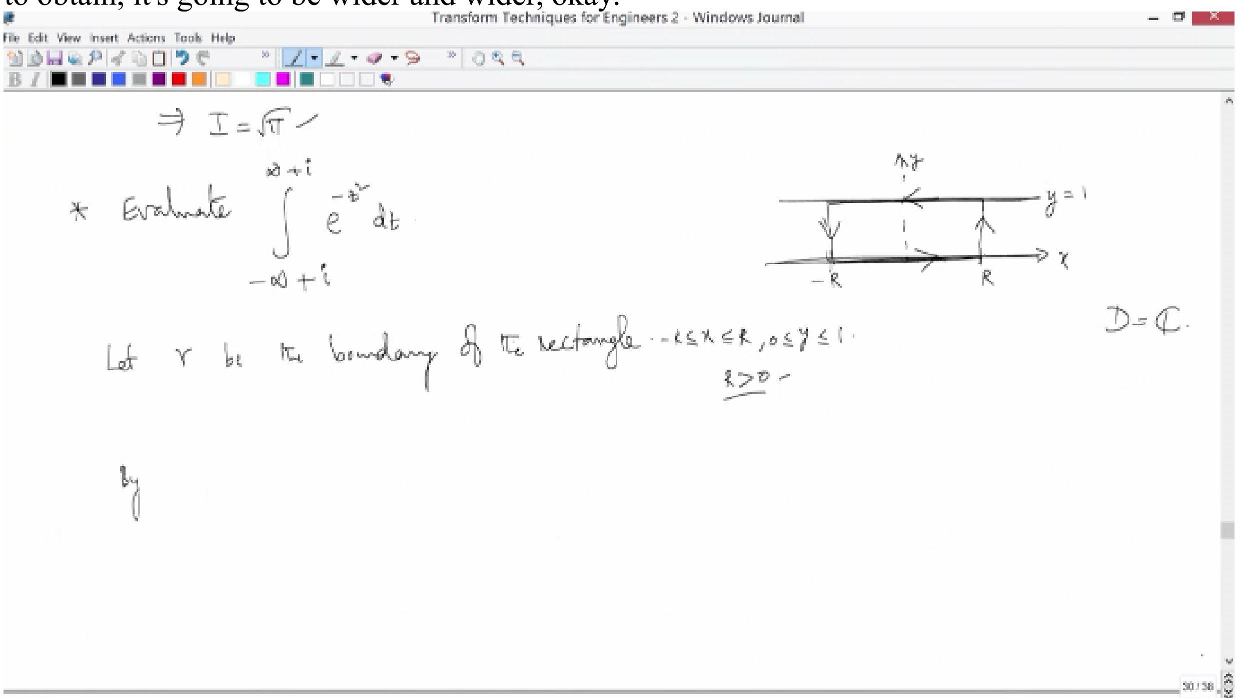


hand side, so your thing will be like this, so I consider this as gamma, okay, so let gamma be the boundary of the rectangle, boundary of the rectangle, this one so that is X is between –R to R, and Y is between 0 to 1, so that's your rectangle.

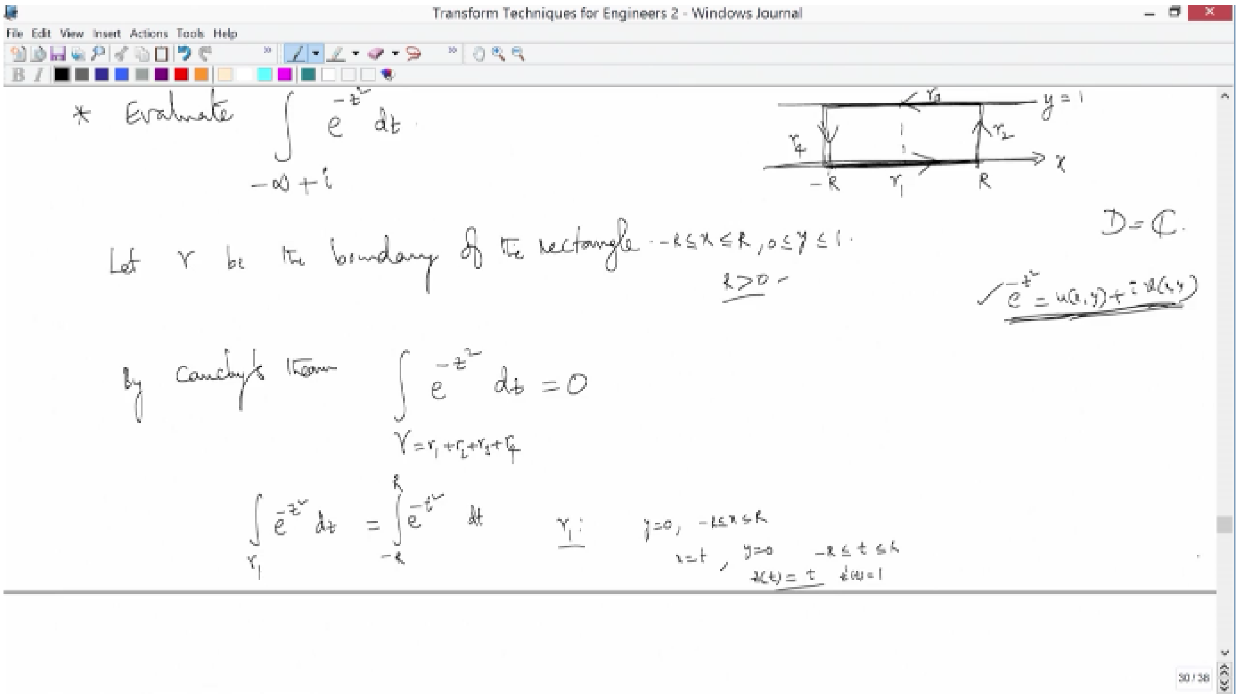
So R is any positive number so that's how we have chosen, if you choose like that then by Cauchy theorem that's what I explained without proof, I'm not going to give you a complex function theory, Cauchy theorem tells you that if your function F(z) is analytic inside some



domain you take any closed curve inside the domain the function has to be 0, that integral value of the function over any closed contour has to be 0, so this is and what you have is here and so your domain D is actually any domain containing that whole complex plane you can think of as your domain, D is full complex plane, D is actually full complex plane, in that you have this rectangle, as R goes to infinity every time this expands but this is fixed, these two lengths are fixed, these two sides are fixed and the other side this is as R goes to infinity this is keep going to obtain, it's going to be wider and wider, okay.

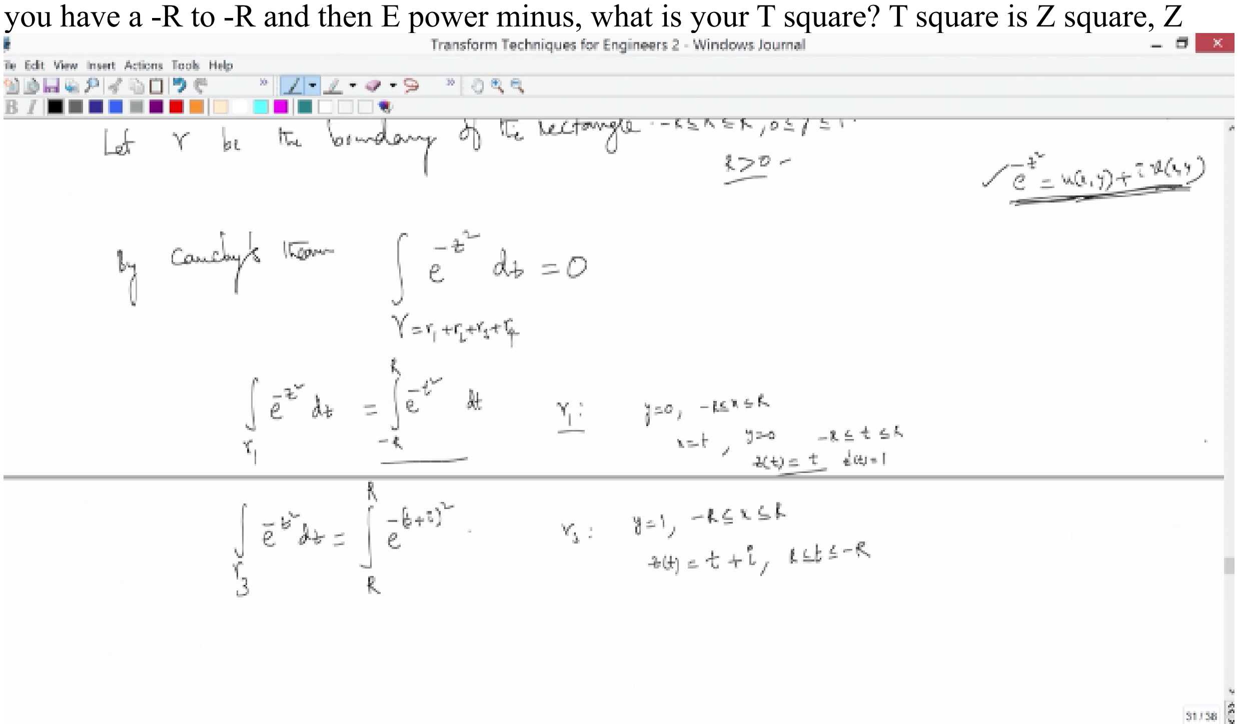


So by Cauchy theorem what we know is over this contour if gamma is this, gamma is this plus this plus this plus this, okay, this over this F(z), now my F(z) is E power -Z square because E power –Z square is a differentiable function, because you can just calculate E power –Z square as a U(x,y) you write it in this fashion I times V(x,y) and you calculate is U and V will have a partial derivatives and they are continuous functions of two variables and then you have a partial derivatives and they satisfy those two equations that I have shown, I have given yesterday, so these are the Cauchy-Riemann conditions that are satisfied, okay, so once you have this, so okay, you can look into books complex function theory books so to see that E power -Z square is analytic function, so this is DZ= 0, what does it mean? Over this contour, but this contour is not, you cannot represent this parameterize as a single function, so this means this is over -R to R, right, so this is let us say gamma 1, gamma 2, gamma 3, gamma 4, so gamma is a gamma 1 + gamma 2 + gamma 3 + gamma 4, so first of all I write this as a gamma 1, gamma 1 is over E power – Z square DZ, so what is the gamma 1? Gamma 1 is if you parameterize this is Y = 0, X is between -R to R, so you take X = T, Y = 0 so what you get is T is between -R to R, and you have Z(t) is, X(t) is T + I times, Y(t) is 0, okay, so this is exactly what you have, so Z(t) is T, so because of that Z dash(t) is 1, so you have a DT, Z dash(t) DT is your DZ, so that you write it as DT and then this is between -R to R, and you have E power – Z becomes, Z is simply Z(t) that is T square, so you have only T square, okay.



So this is your integral first one, so I1 is this, okay, so just let me write only this one, I’ll not use many notations.

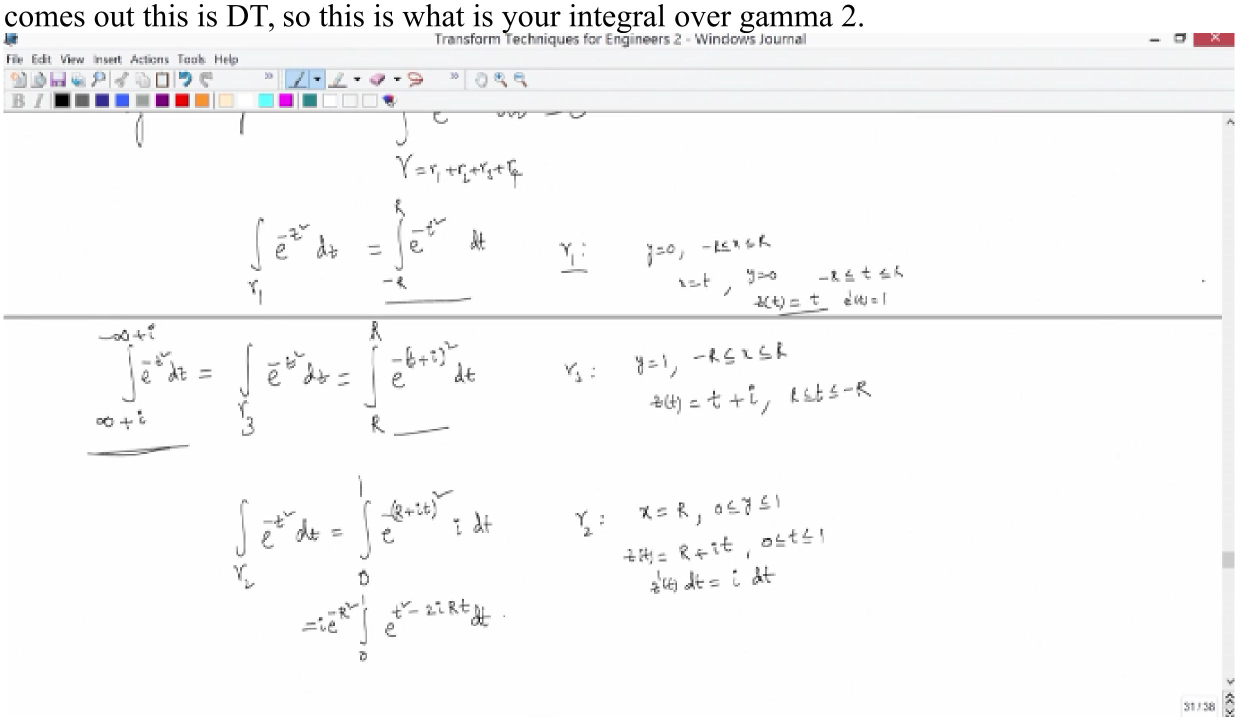
So similarly you can calculate what is your gamma 3? Gamma 3 E power -Z square DZ is, in the same way if you do the parameterization this is actually Y = 1 -R between X is less than or equal to R, so X(t) is T, Y is 1, so you have I1 into I, T is between –R to R, so this is your parameterization for gamma 3, so that means the parallel piece. Now because it is going from this direction to this direction, so what you have is, this is actually going from R to –R, okay, so you have a -R to -R and then E power minus, what is your T square? T square is Z square, Z



square is T + I whole square, okay, T + I whole square and then DT, Z dash(t) is 1 so that is DT, so you have DT.

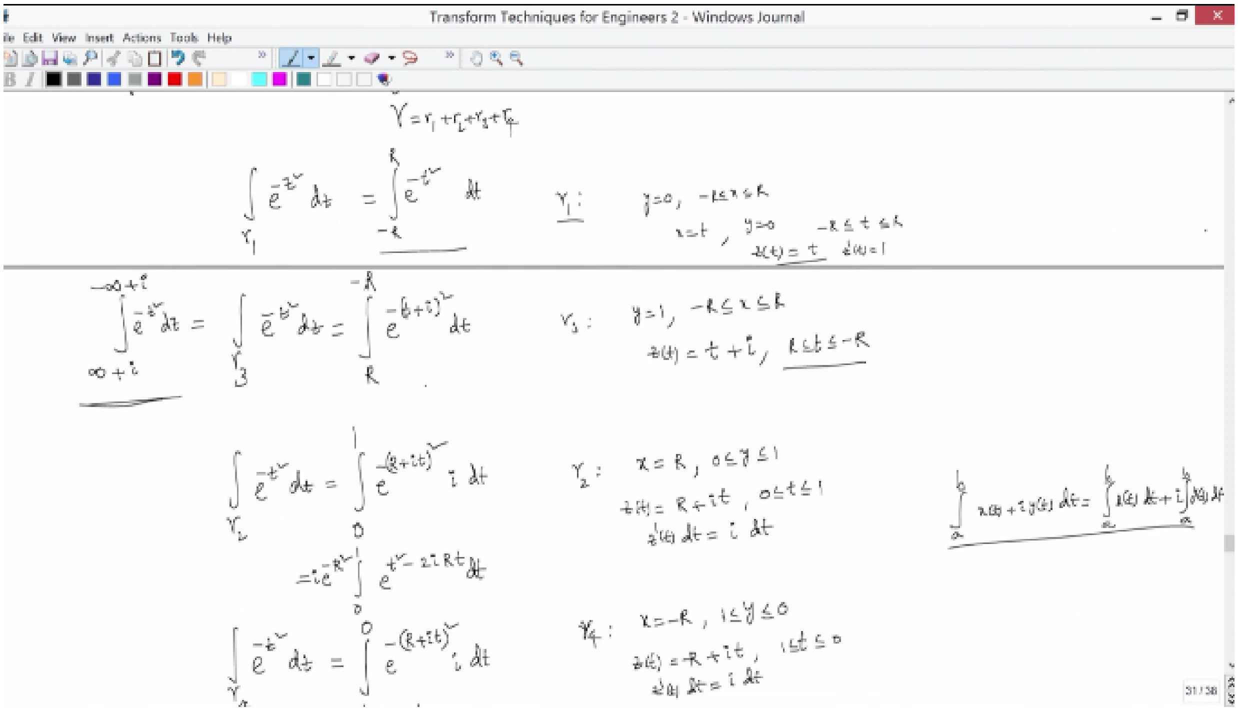
This is actually what you want, actually this is nothing but you can also see that this is also Z3, this is nothing but this is our +infinity +I to -infinity +I E power -Z square DZ both are same, so this is exactly what you want to evaluate, okay, so this is what is that piece over this piece, now if you try to do this gamma 2 and gamma 4 what is this one? Over gamma 2 E power -Z square DZ this is gamma 2, if you look at gamma 2 that is let us say X = R, capital R, Y is between 0 to 1, so how do you represent this? Z(t) which is equal to, R is fixed, X is fixed so R + IT, T is between 0 to 1, so your integral will be Z dash(t) DT is simply I times DT, so you have IDT, T is running from 0 to 1 because you're going from this to this, okay so E power -, what is your Z(t), so that is simply -R +IT whole square.

So what is this one? This is actually 0 to 1 E power -R square, anyway comes out, E power -R square comes out and then what you get is E power T square and then you have -2IRT IDT, I comes out this is DT, so this is what is your integral over gamma 2.



And similarly you can get this gamma 4 E power -Z square DZ that is also, so again if you look at gamma 4, X is now –R, Y is coming from because it's coming from opposite direction and it should be running from 1 to 0, so you have a Z(t) is -R +IT, so T is between -1 to 0, sorry 1 to 0, so you have 1 to 0, E power -R + IT whole square, and then you have when you do this Z dash(t) DT and simply IDT, so this is also again you have a minus you can convert this to 0 to 1 and gamma 3 is R to – R, right, okay, so that is what they have to use, so here this is 1 to 0 becomes -1 to 0, –0 to 1 and then I comes out, and then you have E power -R square and then you have E power IT square -2IRT DT, now this is integrable function because 0 to 1 finite, real part, imaginary part separately, what do I mean by this integral, finite integral A to B, U of I mean Z(t) that is X(t) + I times Y(t) DT means A to B X(t) DT + I times, A to B Y(t) DT so this you can easily see this type of thing, this is how it's define, okay.

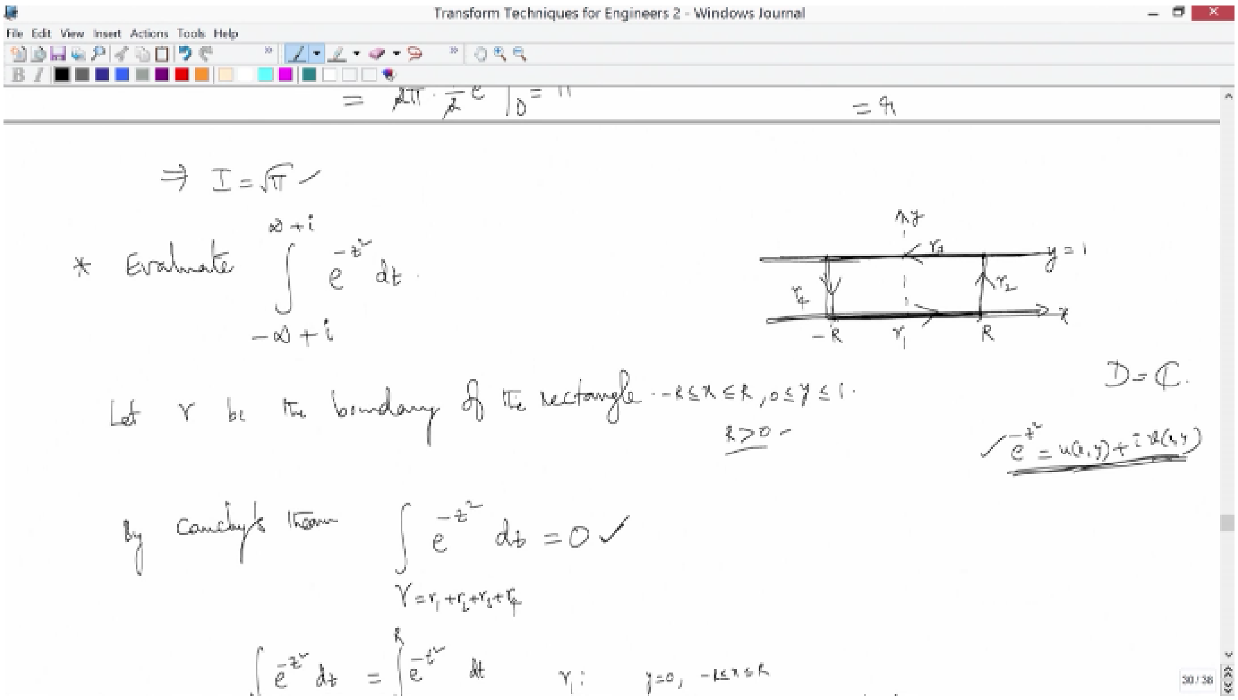
So this is this, so that's how this is integrable functions, these two, these are integrable, this integrand is integrable function, so now you have gamma 1, gamma 2, gamma 3, gamma 4, you



all combining together it is 0, so what is this one the second part? Second part is nothing but this is from minus off –R to R, E power - T + I whole square, okay, so it doesn't matter so we really don't use this one, okay.

As R goes to infinity so this is not really, this is not –infinity, so what you have is -R to +R that's the meaning, okay, so once you combine it because of the Cauchy theorem what you have is as R goes to infinity this goes to 0, as R goes to infinity, this also goes to 0 as R goes to infinity, okay, and what is left here? This goes to, what you want, as R goes to infinity it is going to be alpha infinity +I to -infinity +I E power -Z square DZ this integral, I don't look at here, okay but this one -infinity to infinity E power -T square DT as R goes to infinity, so as R goes to infinity you can write as R goes to infinity so that integral over gamma E power -Z square DZ which is equal to, which goes to, which converges to integral over gamma 1 + gamma 3, E power -Z square DZ, because the other parts are 0 which is 0, so this means integral over gamma 1 E power -Z square DZ converges to integral over gamma 1 is minus, this is what you get is X = 0, gamma 1 becomes as R goes to infinity this converges to let me write it properly, so this is going to be -infinity rather + infinity +I to -infinity +I this one plus, this is going to be -infinity to infinity E power -T square DT = 0, but that is what is the Cauchy theorem.

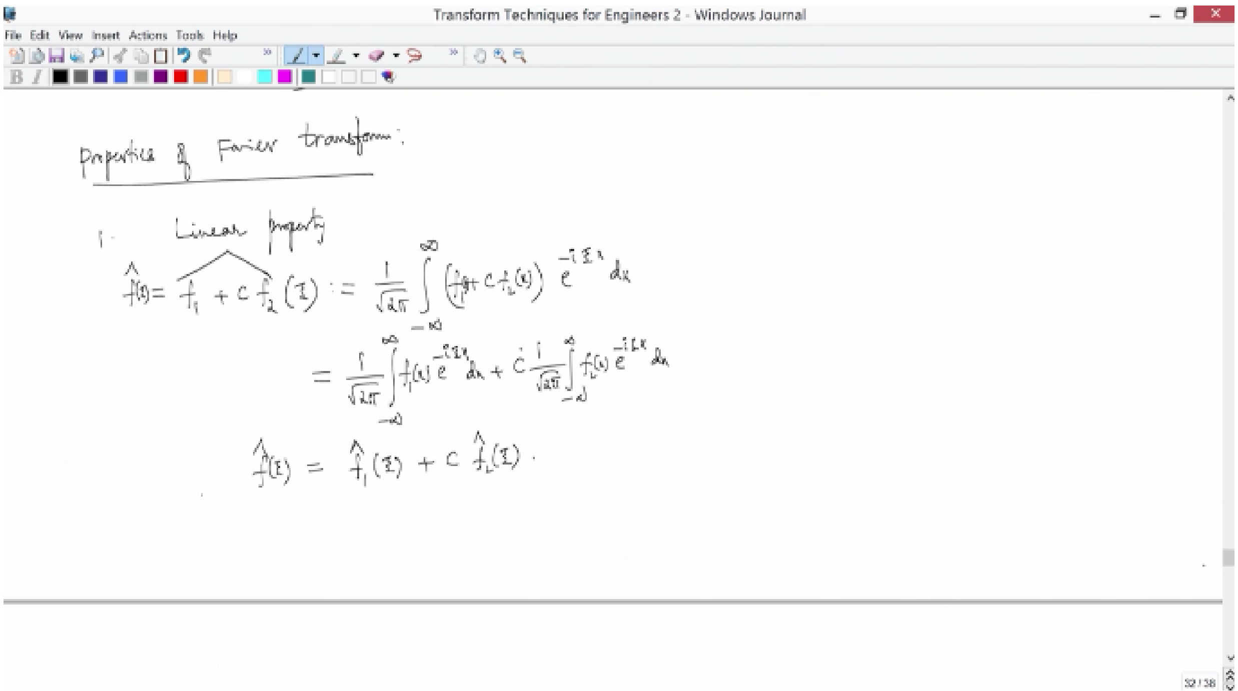
I simply have this, this is all four integrals are 0 together, as R goes to infinity every time this gamma 2, gamma 4 which is going to 0 so that makes it only these two integrals are left, so this is exactly what you want and this is nothing but you can reverse this limits that makes it -infinity to infinity, -infinity +I, + infinity +I, this is actually, because this is a minus of this is plus this or this so that gives you - infinity to infinity E power -T square DT, now T is -infinity which is the real line X-axis, so this means when you have an integral over this line the same as



integral over this line, okay, but in the same direction, so now I have taken from - infinity to +I to infinity +I, this is how you see this by using complex function theory you can see such integrals you can evaluate, so as and when we do the applications of Fourier transform on differential equations, or in other words when you try to solve boundary value problems, initial boundary value problems with this Fourier transform we will see if we encounter any of these integrals if you need to evaluate we will try to use some complex function theory to evaluate, okay, we'll use the counter integration technique and as and when it is required I will try to give you the information about this complex function theory that is required so that we can move on. So now we'll just move on to explain the properties of the Fourier transform, so in this way you can, before I do this properties we can go on doing so many fun, so many, whatever given function you can calculate the Fourier transform in the process you may encounter to evaluate, you may encounter integrals to evaluate that you use complex function theory or otherwise calculus to do that, so there's no end to go on doing these examples so, as and when it is required how we will try to do and we do the applications, so first now try to give you the properties of these Fourier transform that we have defined based on the intuitive proof of the Fourier integral theorem, so after giving these properties I will try to give you the rigorous proof for this Fourier integral theorem, so that whatever we have defined the Fourier transform and it's inverse transforms that are well-defined, and you can believe because it's legitimate, okay.

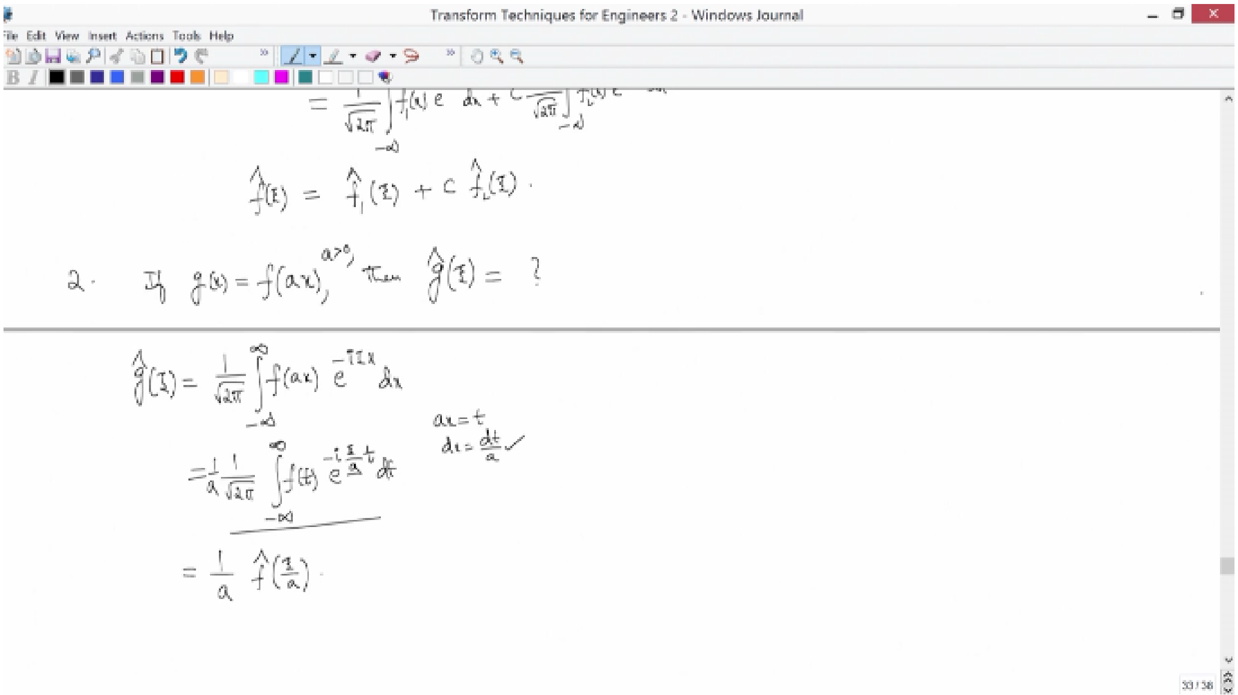
So let me give you the properties of Fourier transform, what are the properties which you have? I'll try to start with this linear property which we have done earlier, so the one is the linear property, what does it mean? If you take F1 + constant times F2, if this is your function F, Fourier transform of this function that means Fourier transform of this function that becomes makes it xi variable which is equal to 1/root 2 pi by definition, -infinity to infinity F1 + F1(x), F1 + CF2(x) okay this old function, so F1(x) + C into F2(x) this into E power -I xi X DX, this is what is the definition, by definition of this transform, so this is equal to 1/root 2 pi you just write separately because you know this, if you sum of the two functions integration is a linear property so you can write this separate 2 integrals F1(x) E power –I xi X DX + C comes out

1/root 2 pi, and F2(x) E power –I xi X DX, so what is by definition this is? F1 cap(xi) C times, by definition this is F2 cap(xi), so Fourier transformation that shows that Fourier transformation is a linear on, take any two function, some of the functions if you apply its some of its Fourier transforms.

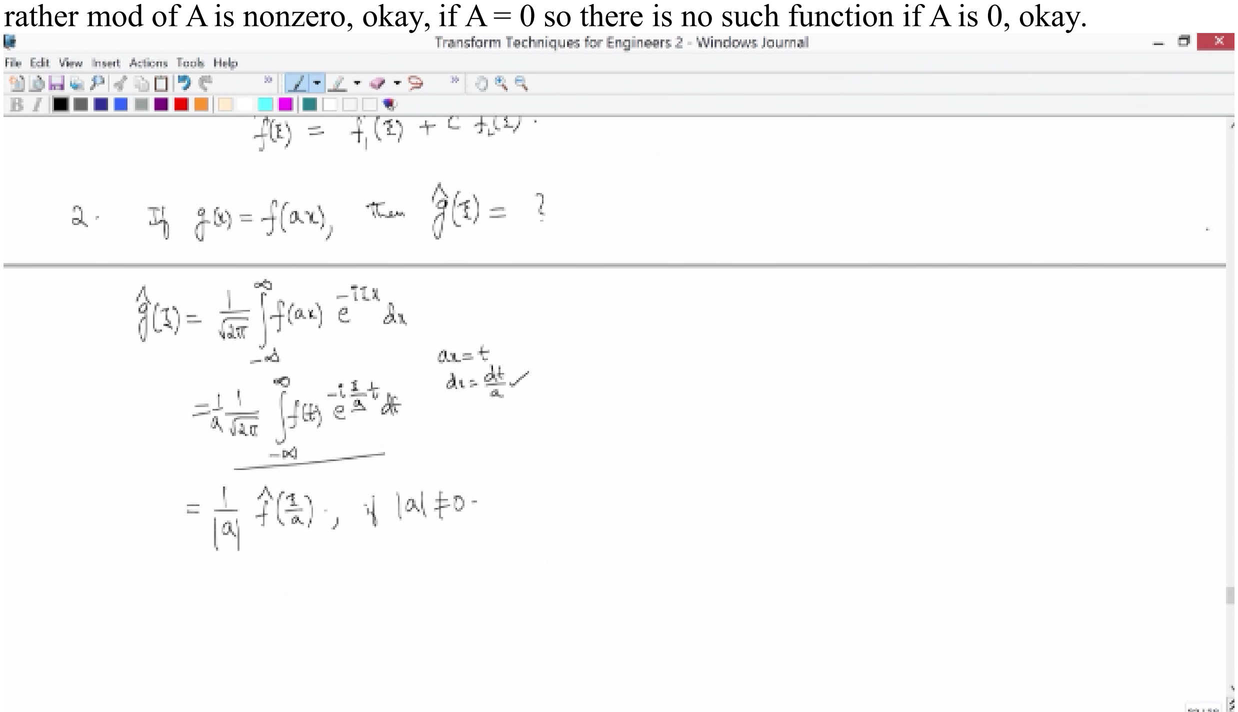


Similarly Fourier transform of a constant multiple of a function is constant, but the same constant multiple of its Fourier transform of the function, so that's what we see here. The second property that we see is what happens if you try to take a function, if your function is like this, Fourier transform of some scaling, you try to scale the variable with some A, okay, so what happens to this? This is a function of X so it becomes xi, okay, so in other words if you write like this if F(x) is or G(x) = F(ax) then G cap(xi) which is equal to, so we will see what exactly and then we write it.

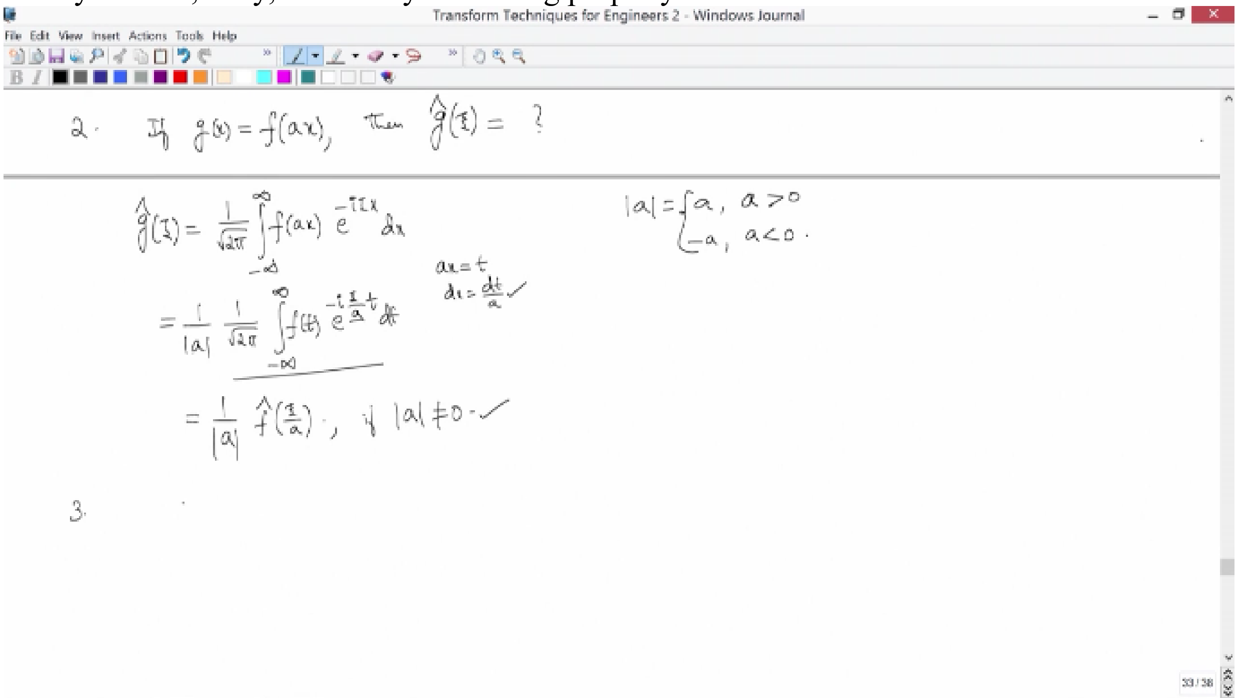
So by definition G cap(xi) is, by definition root 2 pi integral - infinity to infinity, G(x) is F(ax) E power -I xi X DX, and now you simply do this integration, change the variable AX as T, so if you change this AX as T, and ADX, so our DX is equal to DT/A, so if we do that 1/root 2 pi, 1/A comes out because of DX is I am replacing with DT/A, and at X = -infinity is also –infinity, so let's say so A is any positive number, okay, so let's choose A positive. If A is negative what happens? Is going to be infinity to – infinity, so it doesn't matter, so we will see exactly, okay, so if A is positive this limits will not change, so -infinity to infinity F(t) E power - I xi T/A, xi/A into T, okay, you can write this into T, so what exactly how we know is 1/A times, this is the Fourier transform of F cap of, instead of xi you have xi/A, so xi/A, so what happens if A is negative, if A is negative it's going to be, when you substitute those limits X



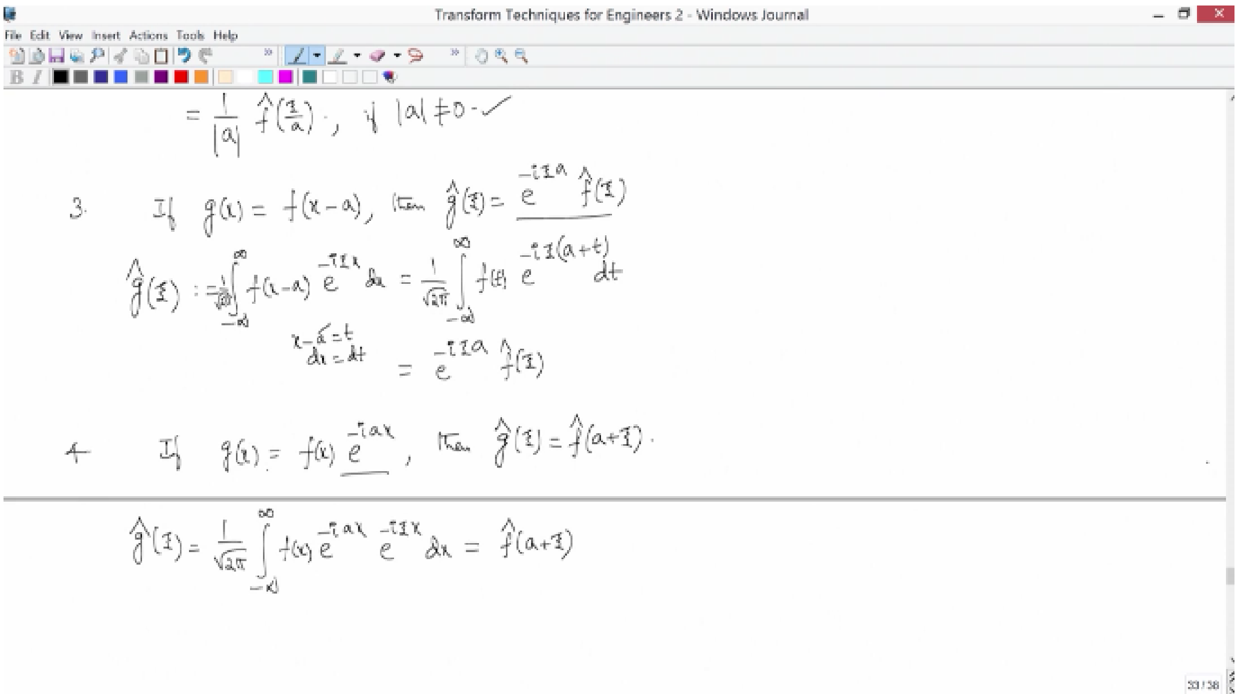
equal to -infinity if A is negative, and T is actually infinity to –infinity, so infinity method that will give you minus, okay, so what you get is minus A, if A is negative what you get is -1/A F cap(xi/A), so that means what you get is the modulus of A, if A belongs to, A is positive or rather mod of A is nonzero, okay, if A = 0 so there is no such function if A is 0, okay.



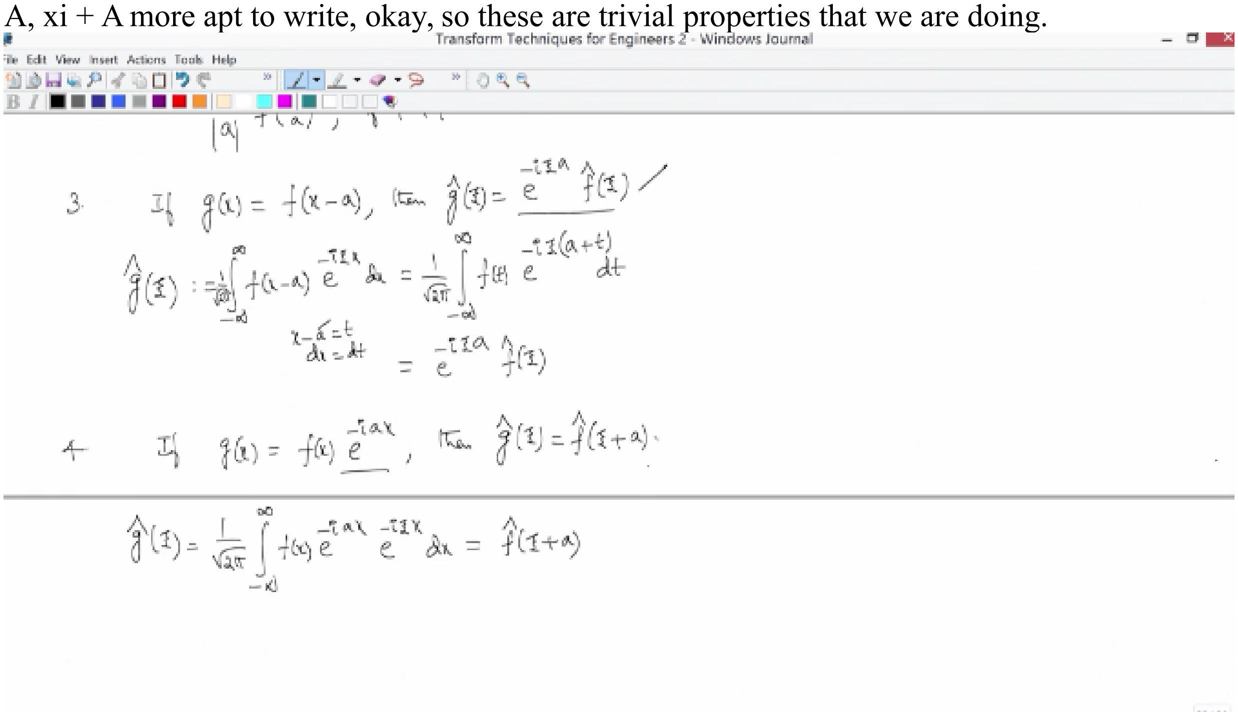
So now I can remove this A positive and what you get is, if you can see if A is depending on, A is positive one way it comes out, if A is negative it will simply come out as -1/A so that is exactly the definition of mod A, so mod A definition is A, if A is positive, okay, and –A if A is negative, so that makes it and once you substitute this is what, once you substitute X = - infinity when A negative is, it’s going to be infinity, infinity to -infinity that is minus off -infinity to infinity integral, so that minus comes here so -1/A that is exactly 1/mod A, so this is exactly what you have, okay, so this is your scaling property.



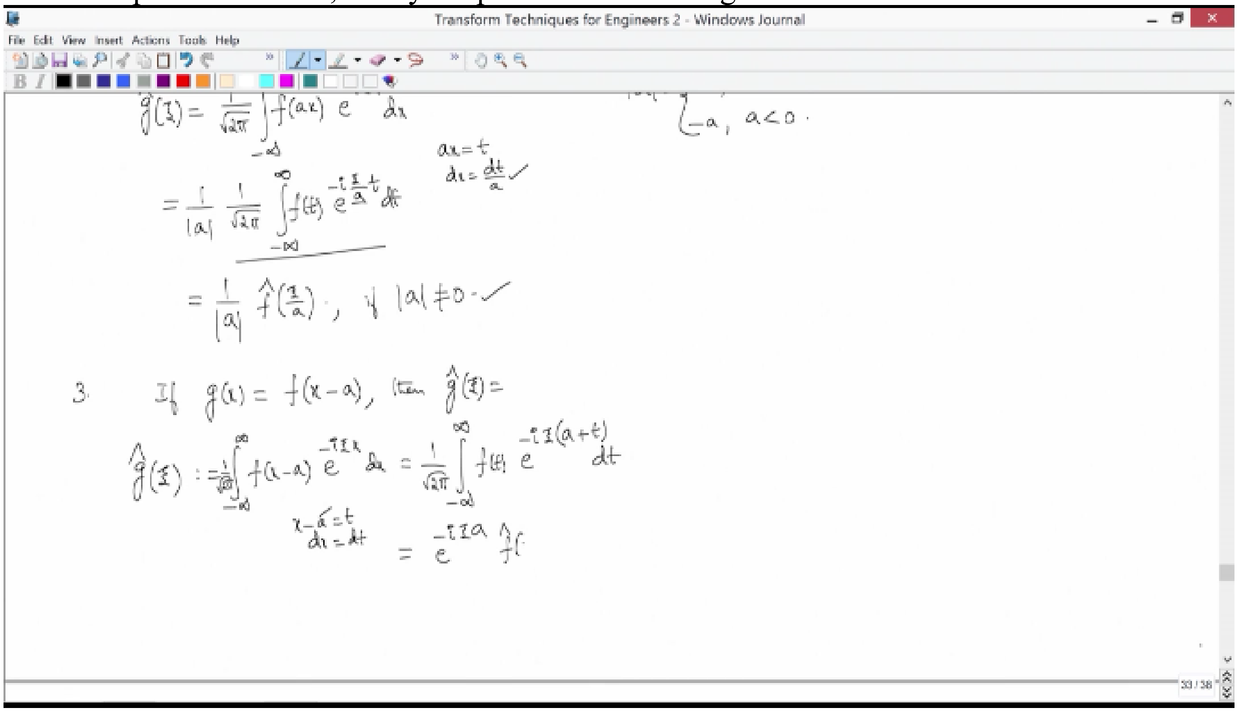
Now you can go on doing like this some more shifting properties for example, what is the Fourier transform if so what you have is 1/mod A times F cap(xi/A).
Now if I have G(x) as F of some kind of shifting in the variable X – A, it can be any real number then what happens to your G cap(xi)? So this we will see, okay, we'll see and try to write, so we'll calculate this G cap(xi), G cap(xi) is by definition 1/root 2 pi - infinity to infinity, F(x-a) E power –I xi DX, again X-A you take it as T that means DX is DT, because A is a real number when you put as a finite number, so X if you put -infinity it's going to be –infinity, so no change with the limits, whatever may be the, A3 is positive or negative the limits are as it is, and you have F(t) and what you get is E power -I xi, X is A+T now, DX is DT so this is exactly now what you get is E power -I xi A comes out and that makes it, the remaining thing is Fourier transform at xi, so what you get is here E power -I xi A F cap(xi), if it's +A you will get E power +I xi A into F cap(xi), so this is a third property, there is a trivial properties that we can easily see, okay.
Next thing is, so this will also tells you the stories that if you have Fourier transform is a, Fourier transform of some function into exponential function of E power – I xi R + IA, that is actually Fourier inverse transform of that is exactly F at the shift X-A, okay, so this will give you that information, shifting with respect to or rather so you know if you want to put it as another one if G(x) is F(x) into E power -I AX, then what is the G cap(xi)? So for this you try to, if you calculate this is by definition Fourier transform of G, G is here F(x) E power -I AX, E power -I xi X DX, so this is exactly equal to, so this is simply, what is this one? So you combine this, this is exactly Fourier transform of F at A + xi, okay, so this is F cap(A + xi) so when you multiply with this thing you just add its Fourier transformation, A + xi.



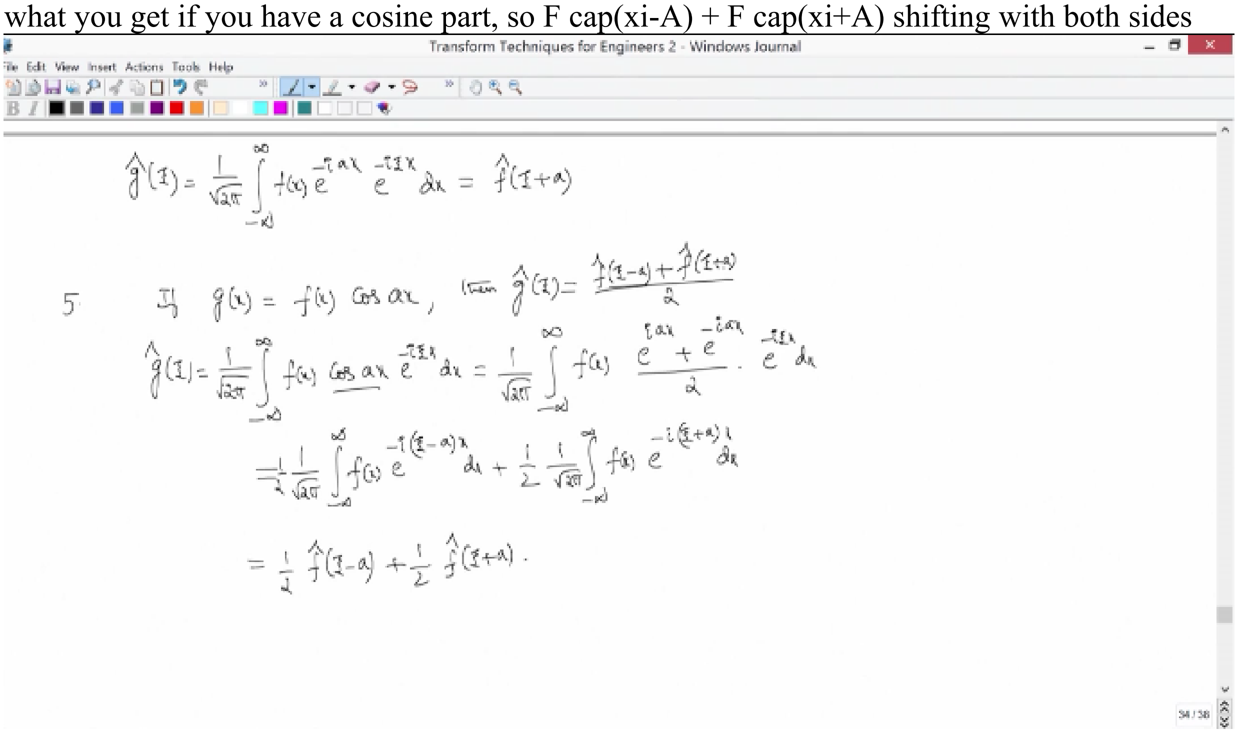
Inverse transformation side if you have multiplication with this, inverse transform of this you simply have scaling, shifting with -A here, here shifting with +A, so here you have to write xi + A, xi + A more apt to write, okay, so these are trivial properties that we are doing.



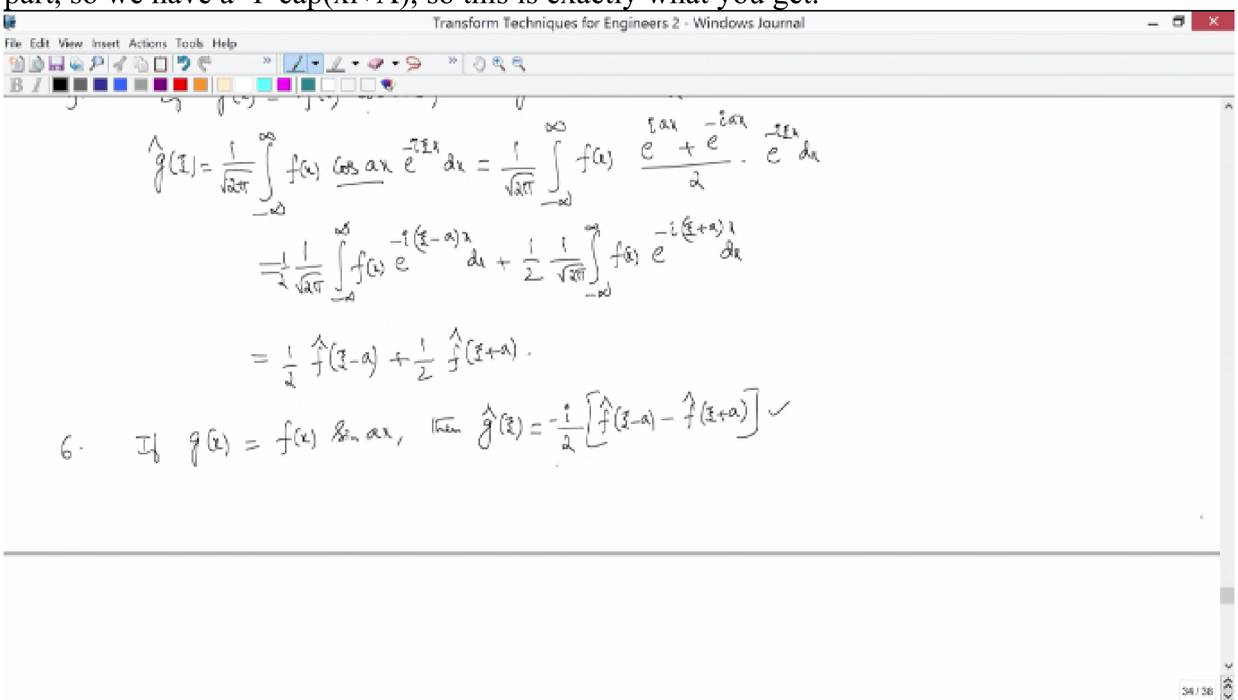
Another important thing is some modulation kind of property that if, what is the, if G(x) is F(x) times, instead of E power –I AX you simply have cos AX, then what is the G cap(xi)? So modulation why I'll call modulation because this function, this signal you multiplied with this cosine function so it's a kind of modulation, so you make it you put it in the oscillatory form, F(x) into cosine part, so what happens to this, if you actually calculate G cap(xi) again, so root 2 pi -infinity to infinity FX cos AX into E power -I xi X DX, so cos AX I can write, what you can write? This is as sum of -infinity to infinity F(x) E power I AX + E power -I AX divided by 2 times E power -I xi DX, now you split this as two integrals because based on the earlier



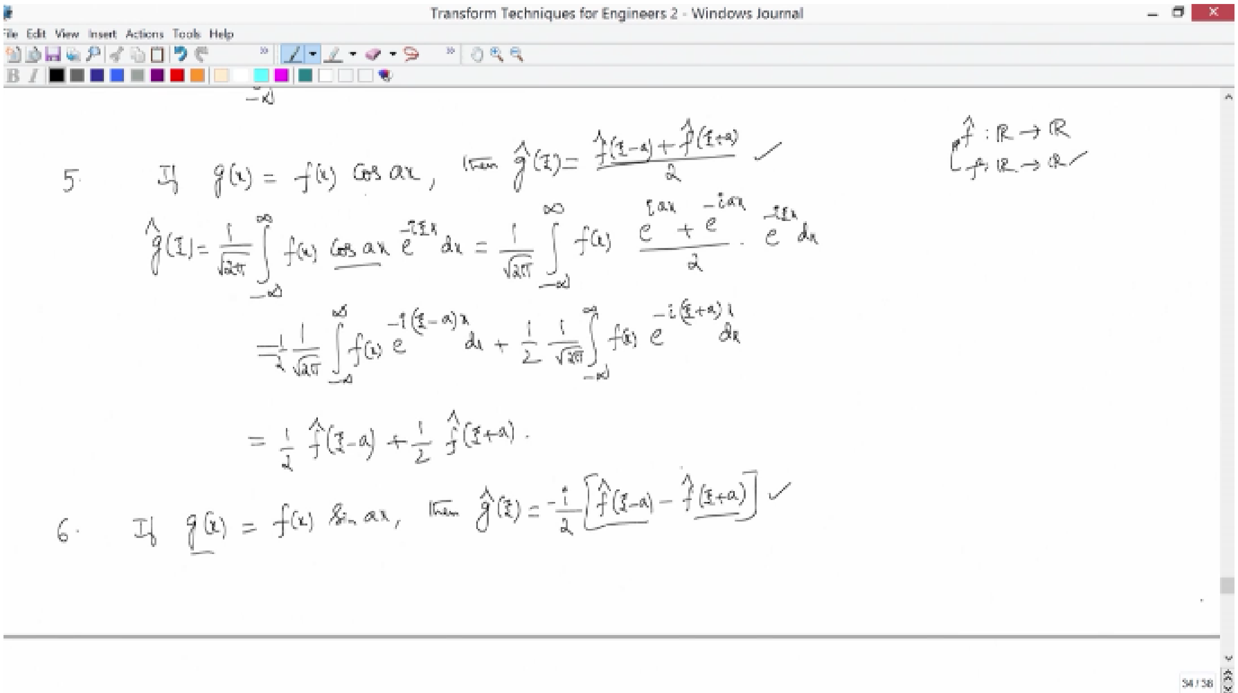
property so what you get is 1/root 2 pi -infinity to infinity F(x) 1/2 times this, F(x) into E power I times A-I times xi-A into X DX + another half 1/root 2 pi other integral part and what you get here is F(x) times E power -I xi + AX DX.
So now this you can write it as 1/2 times F cap (xi-A) +1/2 times F cap(xi+A) so this is exactly what you get if you have a cosine part, so F cap(xi-A) + F cap(xi+A) shifting with both sides



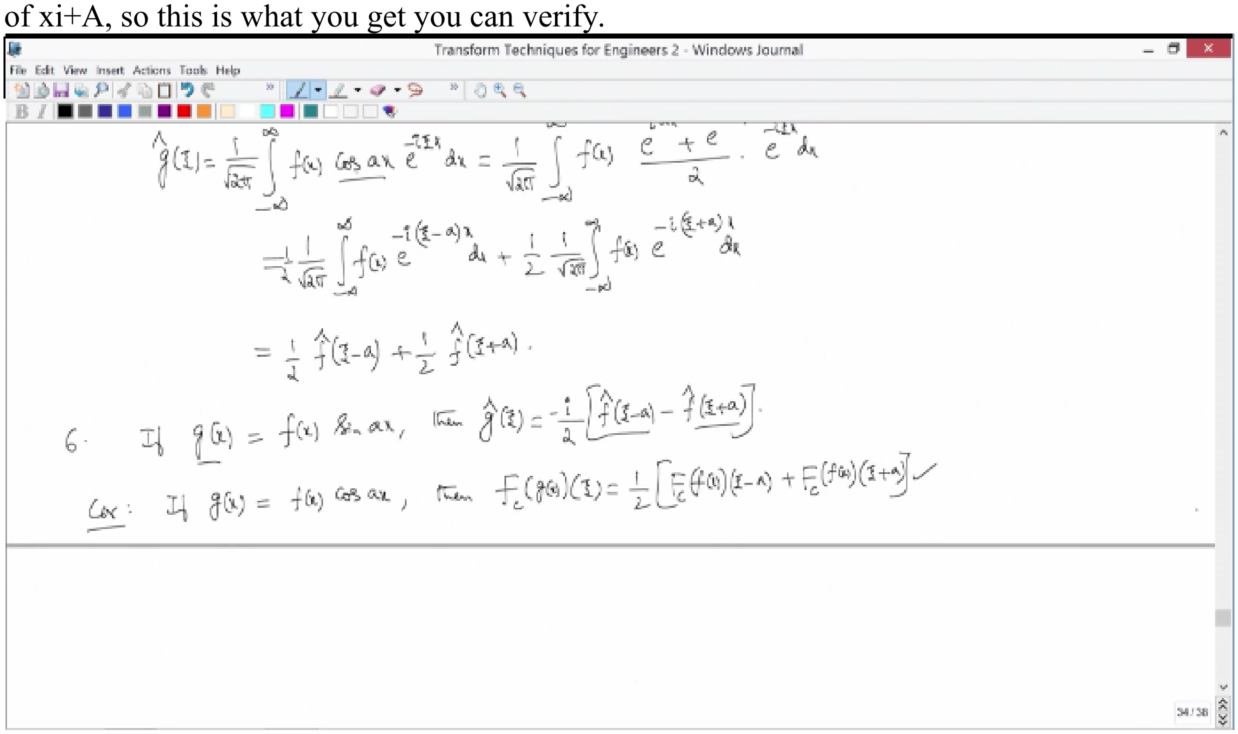
and I would take the average that is what is modulation part, modulation of F(x), F(x) into cosine part then Fourier transform is this, so using this property sometimes if you have a, if function is involving like this you can directly calculate only for F, Fourier transform of F and then use this property you can calculate the Fourier transform, okay, so in some corollaries this part, this property similar property you can work out, I will just try to give you, what if you take sine? Sine function you have a minus here so minus divided by 2I, so you have a 2I, so -I so accordingly you can just write, if it is not cosine you can also write a sine, if it is sine what happens? So let me write it as another property, if G(x) is F(x) times sine AX then G dash(x), you can take it as an exercise or look at this, instead of cosine you have sine, so you have E power IAX -E power -IAX divided by 2I, so instead of 2 you get 1/2, you get 1/2I, 1/2I is - I/2 because 1/I is –I, okay, so because of that this is –I/2, F cap(xi-A), and now you have a minus part, so we have a -F cap(xi+A), so this is exactly what you get.



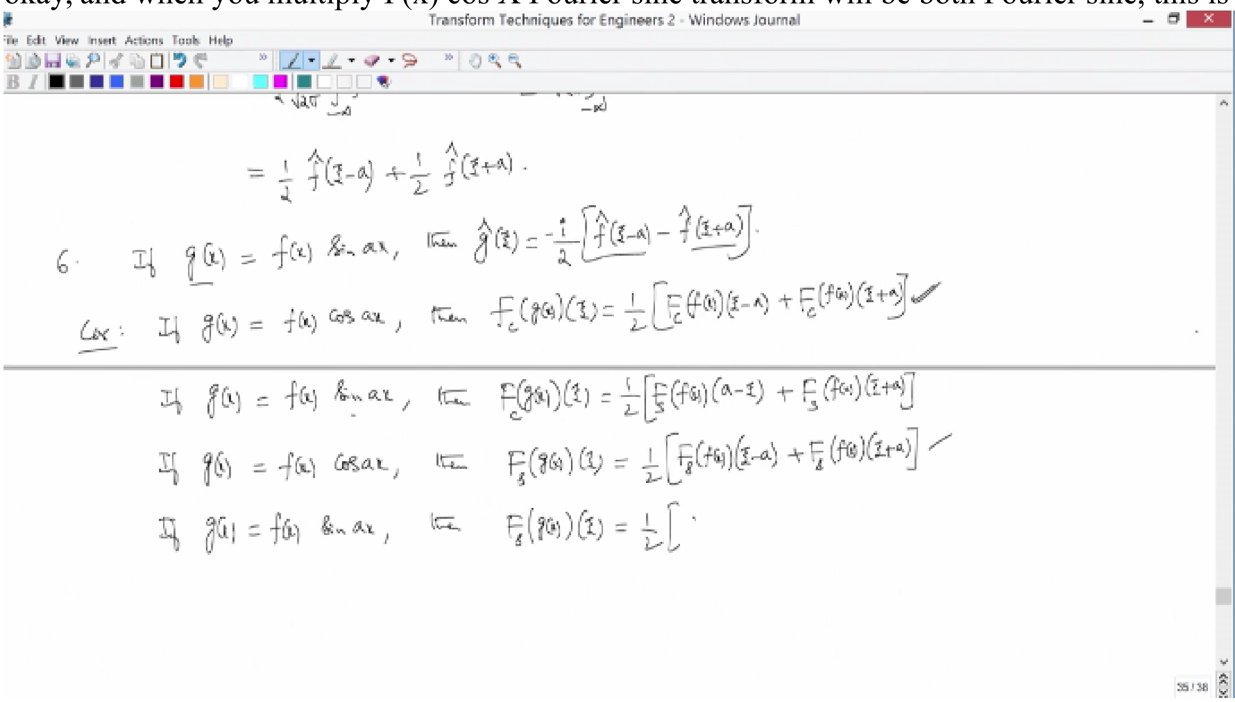
G cap(xi) is, this is what you get, now you see that this is a real part and this is a complex part, so this is a complex number, right, yeah, so that's fine, so you have doesn't matter this is your Fourier transform, you can easily see that this is a Fourier transform, but your Fourier transform, so one question here is if you see that if F is a Fourier transform, if F is a real valued function from R to R, F cap is from R to R, F is from R to R, F cap is also from R to R, so here if these are from real valued functions this is a complex valued, so G cap, G is a real number to real number function, G cap is real numbers to complex numbers, so it's not allowed, so let's not give this property, so we cannot do this, okay.



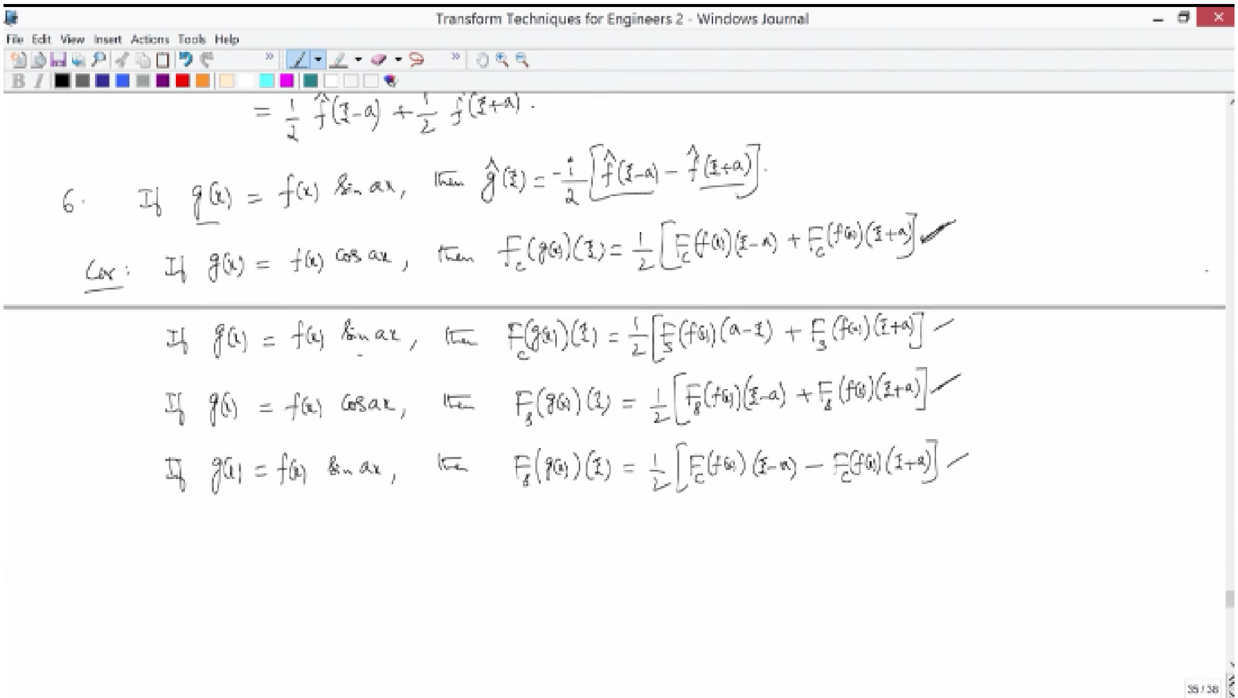
So basically, finally eventually this together it's a real valued function, okay, so it doesn't matter you can still give, so together this is a real valued function, because once you replace this by definition F(x) into sine AX into E power -I xi X, that all together it's a real valued function, so G cap(xi) is actually, xi is a real, xi, okay, so let's do formally like this so you have this, this is one property, some more properties are some corollaries some properties which I simply give you, if G(x) = F(x) times cos or sine, cos AX then Fourier cosine transform of G(x) is, we'll see that 1/2 times Fourier cosine transform of F(x) at you know this xi will be changing, so you have xi+A or xi+A + xi-A, so this is xi-A + Fourier cosine transform of F(x) which is a function of xi+A, so this is what you get you can verify.



Similarly for sine, Fourier sine transform also you can calculate and you replace cosine by sine and calculate Fourier sine transform of G, okay, and Fourier cosine transform of G, so all combinations you can get it, and if you write this, let me write all these corollaries if G(x) is F(x) sine AX, then Fourier cosine transform of G(x) of xi is actually equal to 1/2 times, so what you get is a Fourier cosine transform of F(x) of A-xi + a Fourier cosine transform of F(x) xi+A, this is what if you get, if you have a sine multiplication function.
Similarly you can get G(x) is F(x) sine A or cos AX then Fourier sine transform FS of G(x) now of xi is actually, it's basically same, same as what you get here, so Fourier, so you write instead of cosine transform you write sine transform the FS of F(x) of (xi-A) + FS of F(x) (xi+A) similar to cosine thing.
Last one another combination is F(x) sine AX then Fourier sine transform of G(x) of xi, now this will be you have a minus here so there is a small change, these things you can just work out and see, I think I made a mistake F(x), if G(x) is F(x) into sine AX I have not done it, so let me so you just do this, you take up as an exercise and do it this is not Fourier cosine, so Fourier cosine transform of F(x) into sine AX is what you get is I should get as Fourier sine transforms, okay, and when you multiply F(x) cos X Fourier sine transform will be both Fourier sine, this is

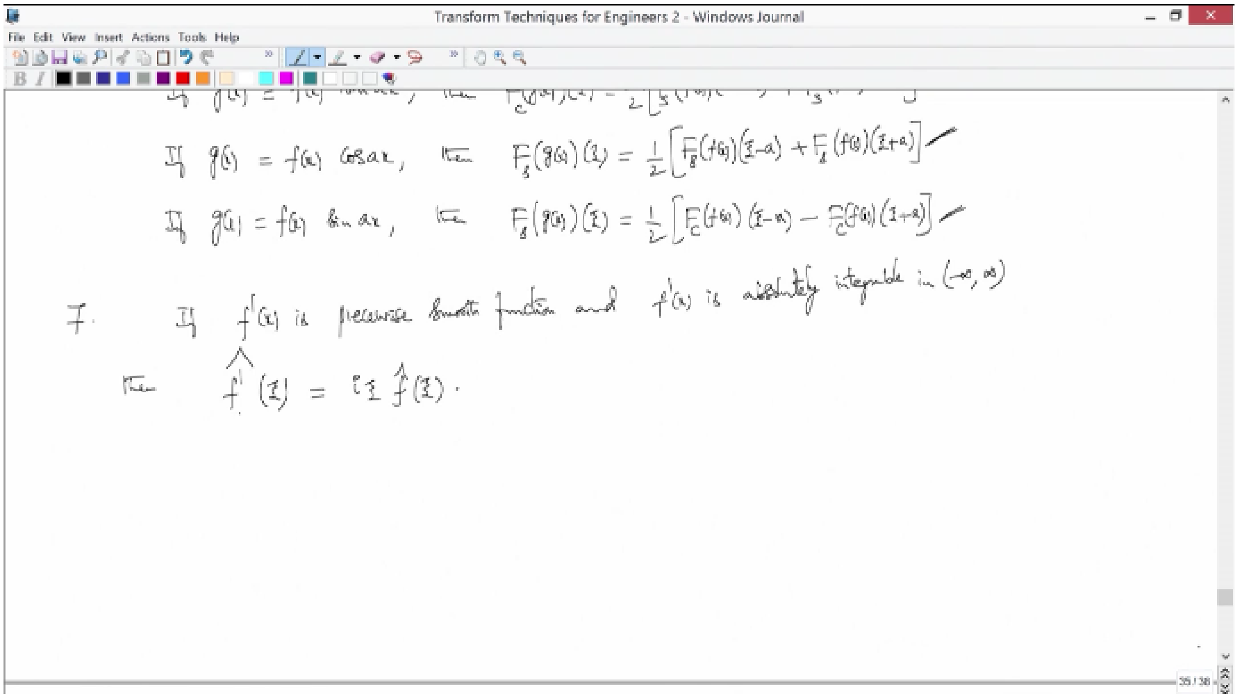


okay like this, but when you multiply sine X Fourier sine transform of G(x) will be 1/2 times Fourier cosine transforms here, Fourier cosine transform of F(x) at xi-A -Fourier cosine transform of F(x) xi+A, this is what I have to verify, okay, so these are the things you can

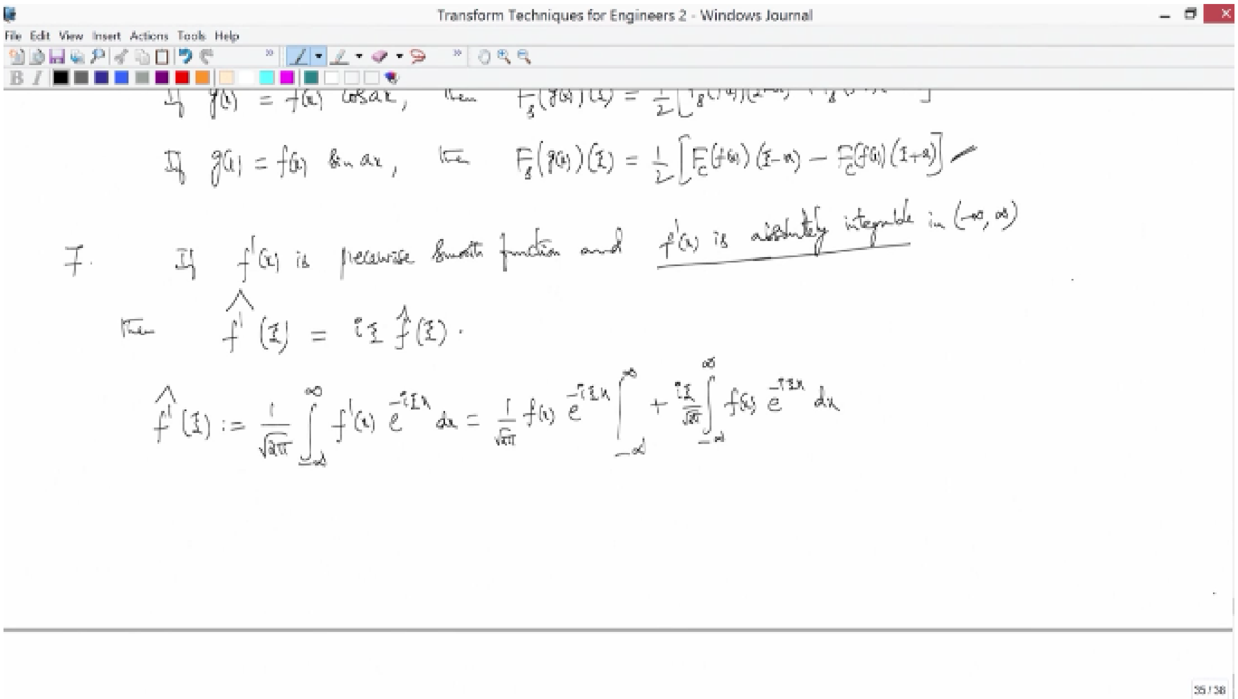


verify, so if you multiplication, if you modulation with sine function what you get is you can see that the cosine transform of such modulus with modulation of sine function, product of sine function with your function whose cosine transform will be in terms of sine transforms, sine transform of function multiplication with the sine function is actually in terms of cosine function, if it's multiplied with cosines, cosines will be in terms of cosines, sines will be in terms of sines, that's what this properties tells you, okay.

So we will define some more properties, so one of the important properties we have already used is F of, if F is our piecewise continuously differentiable function, and F Dash is also absolutely integrable function, if F dash(x) is a piecewise smooth, smooth means continuously differentiable function, okay, that means it's a differentiable that is also continuous, smooth function, and F dash(x) is absolutely integrable, integrable in -infinity to infinity that means you take any finite integrable that is into absolutely integrable, and it will have only and you take any finite piece, now always have one finitely many jump discontinuities for it's derivative, now this is also continuous.

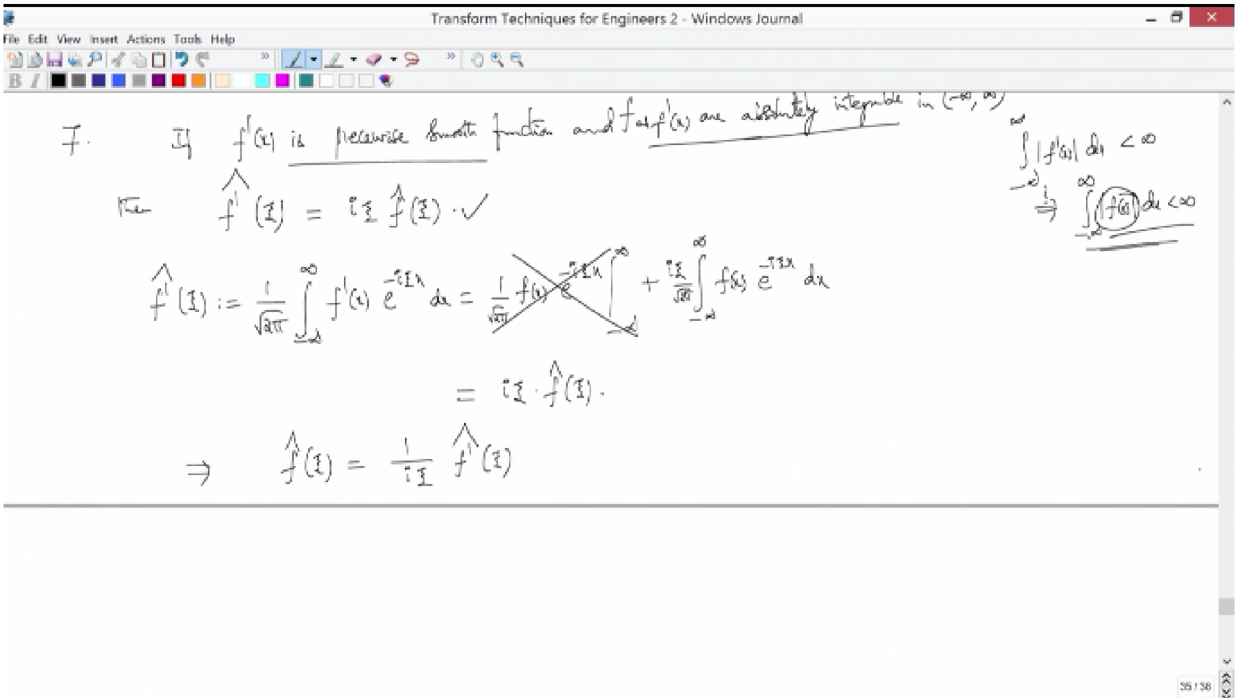


So then what is its Fourier transform? Fourier transform of this cap, Fourier transform of this F dash is actually what you get is I xi F cap(xi) we have already seen this earlier and we were using some, and we are trying to find the Fourier transform of delta function that or generalized function, so that time we looked at this property xi in the sites so we have calculated, so let's do it anyway formally so proof is so you start with the definition left hand side that is by definition 1/root 2 pi - infinity to infinity, F dash(x) E power -I xi X DX. Now you do the integration by parts that makes it F(x) E power -I xi X, 1/root 2 pi for this you apply these limits, minus minus plus, minus and what you get is 1/root 2 pi -infinity to infinity F(x), now you differentiate this exponential –I xi X that makes it -I xi comes out so it makes it plus, I xi will be here, E power -I xi X DX, so because F dash is absolutely integrable function, and F is that implies F is also

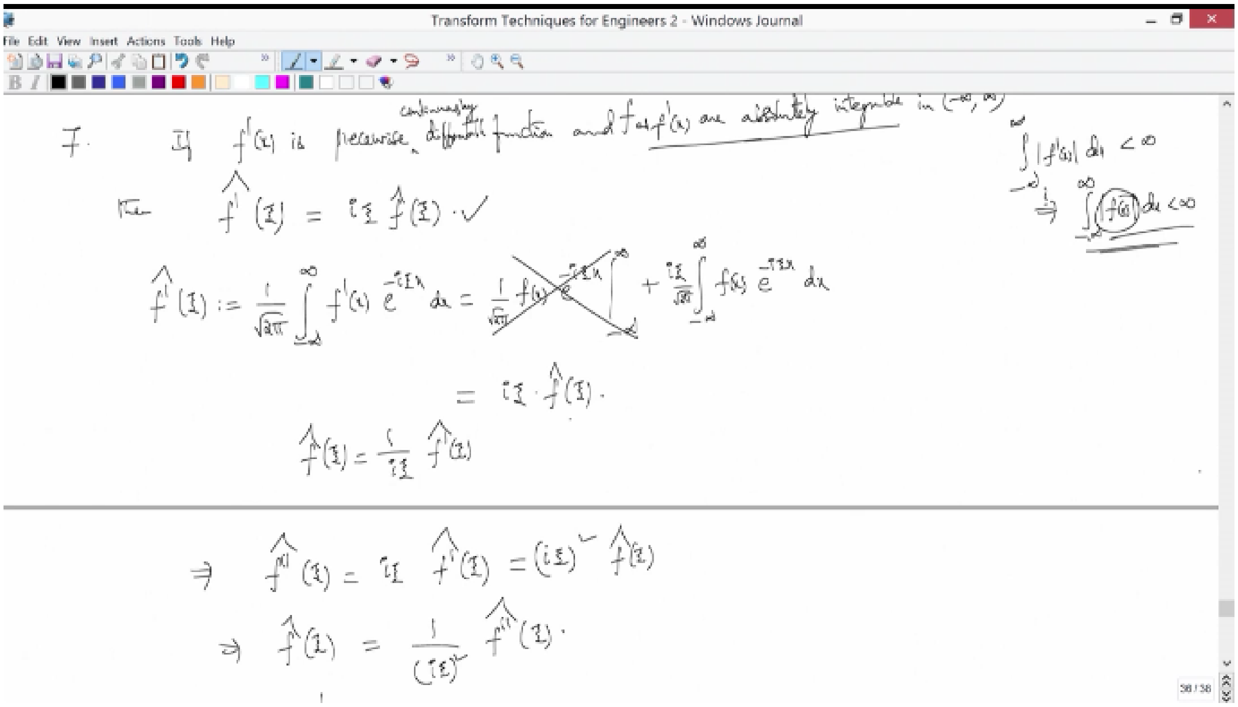


integral function, F is integrable, F dash is integrable, F is also integral function, absolutely integral function, okay, so we assume that, so how do you see that if integral mod F dash(x) DX -infinity to infinity, if this is finite can we say that it implies modulus of F(x) DX is also finite, is this true?

Can you see this one? Is this really true? So otherwise you can include in your hypothesis, and F, F dash both are, F and F dash are absolutely integrable function, if you do then there's nothing so, then if F is integrable, if this means F is at infinity has to be 0, if it is not this cannot be finite number, so that way both these things will be 0, value at the +infinity and -infinity what you are left with is I xi times, remaining thing is simply Fourier transform of xi, at xi, so that is exactly what you want, this is one.



So from this if F is a piecewise smooth function, so from this you can see that F cap(xi) is actually 1 divided by I xi, F dash cap(xi), okay, so this is what you get, so if it's F dash is piecewise smooth, once the moment you say that F is a piecewise smooth function, F dashis piecewise differentiable function, differentiable, continuously differentiable that means it's a differentiable function which is continuous piecewise that we write continuously differentiable function, and these are conditions under which you have this one, if you go on repeating like this suppose F is a smooth function that means you take any derivative FN derivative is piecewise continuously differentiable function, and F, F dash all FN’s up to FN derivatives are absolutely integrable then that means you can repeat this process any number of times to see that F cap(xi) which you can write like 1/I xi times, F double dash of xi, this cap, okay that means so what you get is F cap or rather use this inequality than this what you get is F double dash, F double dash cap is equal to I xi times, F cap of, F dash(xi), again this makes it I xi square, F cap(xi), so that makes it F cap(xi) 1/I xi whole square F double dash cap(xi), here what you get is F cap(xi) is 1/I xi, F dash(xi), F dash cap(xi), Fourier transform of derivative, so you can go on doing like this finally if it is a smooth function what you end up is I xi power N, FN derivatives of N if you calculate for this Fourier transform that is exactly what you get, okay.



So these are one properties, you can also do the same technique for cosine and sine transforms, when the function F is differentiable absolutely integrable over 0 to infinity and you have certain conditions and F is piecewise continuous function, piecewise smooth function on the interval 0 to infinity you can apply Fourier cosine transform for that derivative, and so can calculate at the end, just integration by parts you end up getting some formulas for the derivative of the Fourier transform.

So I'll just write it, write them and we will see the remaining properties in the next video, so the other corollaries, so let me write it as other properties, so if this is, let us say 8 property, if F is a similar property that I’m writing, so if F(x) is piecewise smooth function in 0 to infinity and integral F DX -infinity to infinity is finite, and integral -infinity to infinity F dash(x) this also absolutely integrable, F dash is absolutely integrable over 0 to infinity, and also F is also integrable absolutely over 0 to infinity this is the meaning.

And then what happens to the F cap? So Fourier cosine transform of F(x) of xi is of F dash if you calculate it's derivative one by, what you get is root pi/2 rather root, you get root 2/pi integral 0 to infinity, F cap(x) cos xi X DX, now for this if you do the integration by parts root 2/pi F(x) cos xi X, imply the limits minus when you differentiate cos you get comes out some xi comes out, so xi comes out and that you can replace and you have root 2/pi as it is a constant, and you get F(x) into sine xi X DX, so this is exactly so if you, because F is absolutely integrable function F at infinity has to be 0, and this contribution is 0, and cos 0 is 1 so you have a minus root 2/pi F(0) + xi times Fourier cosine transform of F(x) which is a function of xi, this is exactly the formula what you get for F dash(x) which is function of xi.

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$$\hat{f}(\xi) = \frac{1}{(i\xi)^n}\,\widehat{f^{(n)}}(\xi).$$

8\. If $f(x)$ is piecewise smooth function in $(0, \infty)$ and $\int_0^\infty |f|\,dx < \infty$, $\int_0^\infty |f'(x)|\,dx < \infty$

then

$$F_c(f'(x))(\xi) = \sqrt{\frac{2}{\pi}} \int_0^\infty f'(x) \cos \xi x \, dx$$

$$= \sqrt{\frac{2}{\pi}}\, f(x) \cos \xi x \Big|_0^{\infty} + \xi \sqrt{\frac{2}{\pi}} \int_0^\infty f(x) \sin \xi x \, dx$$

$$= -\sqrt{\frac{2}{\pi}}\, f(0) + \xi F_c(f(x))(\xi).$$

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Like this we can write other formulas try to calculate or I leave it as an exercise, okay, find Fourier cosine transforms, so under these conditions same conditions what are the Fourier sine transform of F dash(x) which is a function of xi, okay, and you can also go on doing for 2, and we can also do FC of F double dash(x) of xi because it's a piecewise smooth and you have that many number of derivatives, okay up to you can write 0 to infinity, FN derivatives(x) that function is also finite, in that case you go up to 2 derivatives so you have, you can calculate this and you can also calculate the sine transform of F, 2 derivatives of X which is function of xi, exactly can you calculate, okay.

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$$= \sqrt{\frac{2}{\pi}}\, f(x) \cos \xi x \Big|_0^{\infty} + \xi \sqrt{\frac{2}{\pi}} \int_0^\infty f(x) \sin \xi x \, dx$$

$$F_c(f'(x))(\xi) = -\sqrt{\frac{2}{\pi}}\, f(0) + \xi F_c(f(x))(\xi).$$

Ex: Find $F_s(f'(x))(\xi)$

$F_c(f''(x))(\xi)$

$F_s(f''(x))(\xi)$.

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So find the values, find this as it’s taken, you can take it as an exercise, but pull up, yeah, yeah I'll give you the conclusion. We have some more important properties for such as, the way we have defined properties for Fourier transform of a periodic signal that is a function defined over a finite interval, we have seen, we have defined what is that if you have 2 side signals you can define what is a convolution, convolution of two such functions here also we can do the convolution of two functions F(x) and G(x) and its transformation as like in there you have a Fourier transform of the convolution product is a product of a Fourier transform that's what you will see.

And also and other conclusion is, and you can derive Parseval inequality and sorry you can also derive a Parseval identity that is the Fourier transform, so square integrable of function, what is integration of square of the function, I mean square integrable function, integration of a square of absolute value of the function, what is its relation with integration of square integration of modulus of the Fourier transform, so that's what we will see as Parseval’s identity.

And then we'll prove this important property that is a Riemann Lebesgue Lemma, what happens to the Fourier transform as xi goes to infinity? At extreme points of xi what happens, like there what you have is Fourier coefficients as N goes to infinity they are going to 0 that is your Riemann Lebesgue Lemma there, here we see that Fourier coefficients are nothing but Fourier transform F cap(xi), as xi goes to infinity, there N goes to infinity, here xi goes to infinity, so here also it is going to be 0 that is what is the Riemann Lebesgue Lemma, all these properties we will see in the next video. Thank you very much.

[Music]

Online Editing and Post Production

Karthik

Ravichandran

Mohanarangan

Sribalaji

Komathi

Vignesh

Mahesh Kumar

Web Studio Team

Anitha

Bharathi

Catherine

Clifford

Deepthi

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Gayathri

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Poornika

Premkumar

Ragavi

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Sathya

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Subash

Suriyaprakash

Vinothini

Executive Producer

Kannan Krishnamurthy

NPTEL CO-ordinators

Prof. Andrew Thangaraj

Prof. Prathap Haridoss

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