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Transform Techniques for Engineers
Evaluation of an integral- Recall of complex function theory
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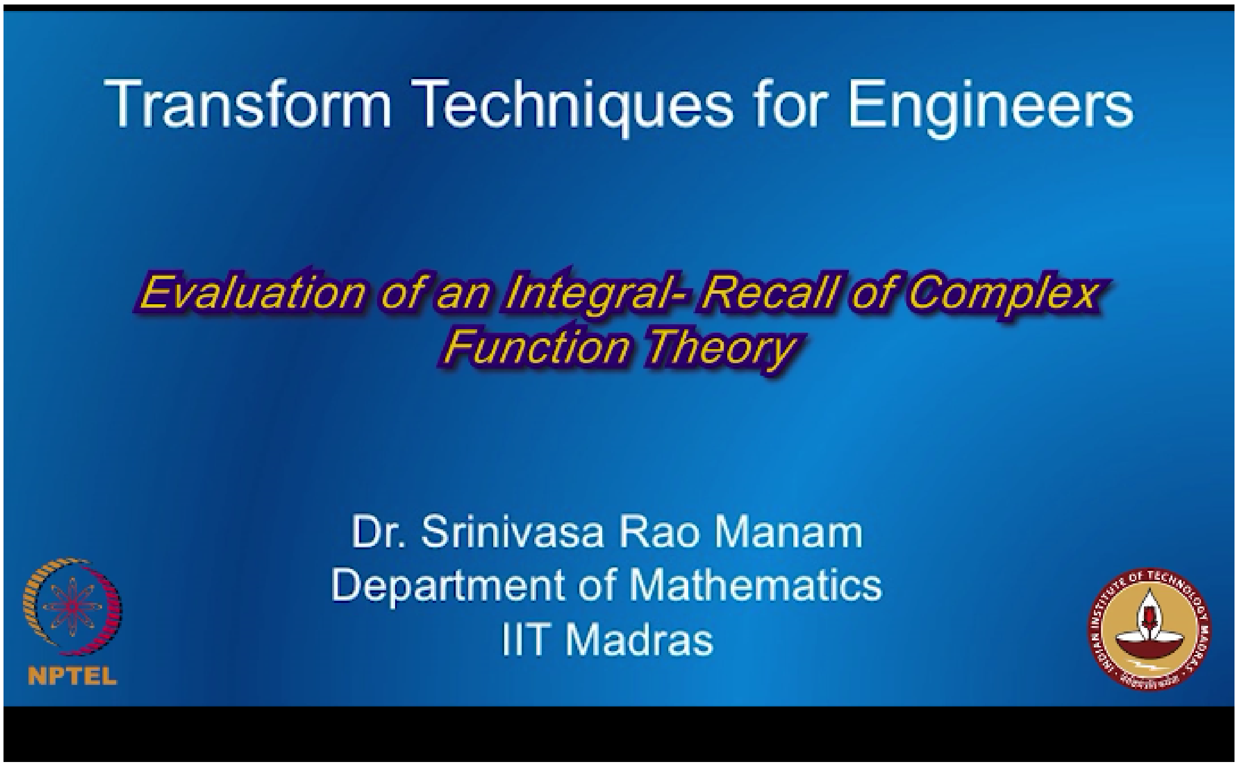
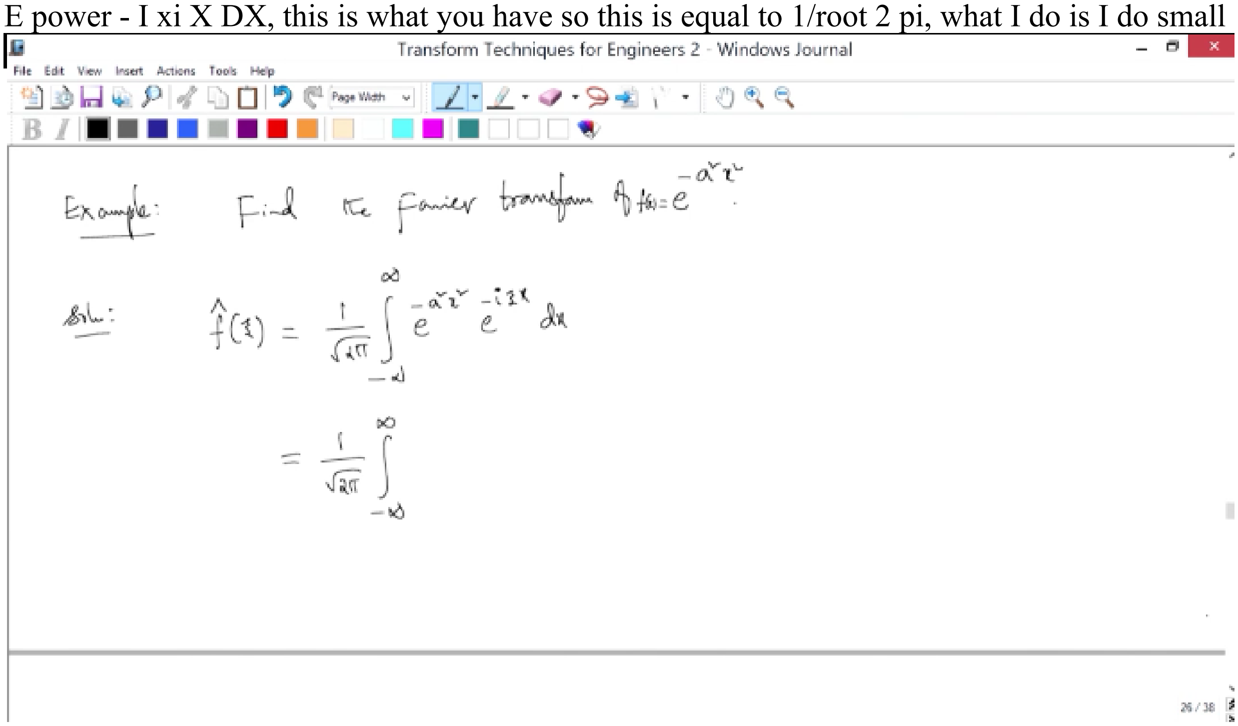
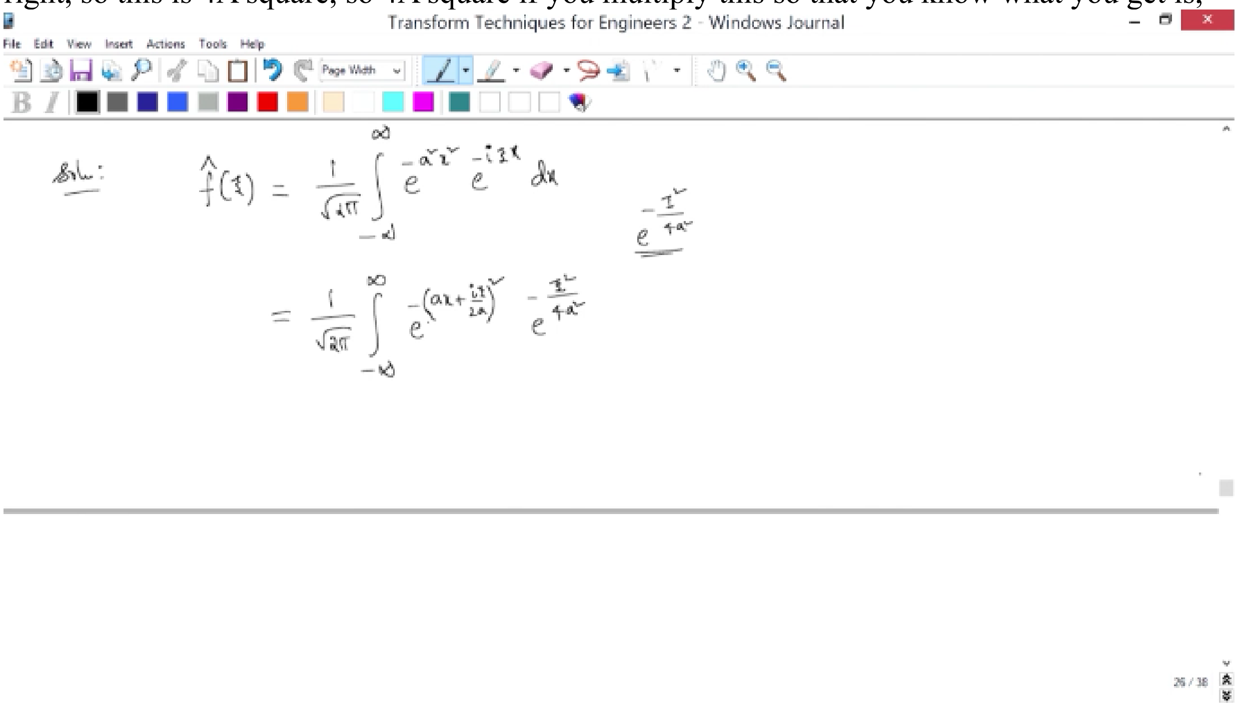


Welcome back, so far we have seen, we have only given intuitive proof for the Fourier integral theorem, and defined what is a Fourier transform, just to give you a motivation why we have to define Fourier transform like that, and Fourier integral theorem will give you inverse Fourier transform of it, okay of the Fourier transform.
So we have defined both of them, with that we though it's, we are yet to prove rigorously what is the proof for the Fourier integral theorem for in piecewise absolutely integrable function over a full real line, that means it's a non-periodic signal, for a non-periodic signal we want to have a Fourier integral representation so that gives you the definition of Fourier transform and inverse Fourier transform.
And we have seen many examples of finding Fourier transform and it's inverse transform, right from the usual functions and also on generalized functions such as delta function, and when you apply for usual functions when you see that Fourier transform definition is actually it's not converging definition, if we take this it is actually you have seen that it is actually in terms of generalized functions, for example constant function if you find the Fourier transform it involves a delta function or piece-wise, what is that, a step function that is heavy side function you have seen that is in terms of, it's in terms of the generalized function delta function, okay, generalized function that is delta function.

We will look at one more example before I proceed to find the properties of the Fourier transform, this is some rapidly decreasing function such as exponential function E power -X square kind of function, so this is actually kind of, if you actually plot this E power -X square it looks like a bell curve, so if you actually plot this thing it looks like this bell curve so, asymptotically it goes to 0, and then so it has a kind of bell.
So it's E power –X square kind of function, so if I choose such a thing E power -AX square, A is positive what happens that you will see how to find the Fourier transform, so this comes actually because this is a Gaussian Kernel which comes in the heat equation, so when you try to solve the partial differential equation heat equation with the initial condition, initial boundary conditions, so in the applications of the Fourier transform we will try to encounter this function as a Kernel, so the Kernel is a Gaussian kernel that is also called Gaussian Kernel which is rapidly decreasing.
So as you see it's a heat equation is the diffusion equation, diffusion of a heat so which is rapidly so diffuses, it goes to anything heat is a dissipates very fast, so that models like this, so that's the reason you will encounter this exponential function rapidly decreasing exponential function, so you need to find out, if you want to apply and try to solve that boundary value problem you may need to find a Fourier transform of such function, so we'll just try to show what is that Fourier transform of such function.
So let us start with this example, so find the Fourier transform of E power -A square X square let me take like this, A is positive, doesn't matter so A is negative or positive, A square is always positive so it's fine, so if you take like this so the solution now what you see is a Fourier transform of this one of the function F(x), let us say I call this F(x) so that you can write F cap(xi) that is what you want 1/root 2 pi - infinity to infinity E power - A square X square F(x) E power - I xi X DX, this is what you have so this is equal to 1/root 2 pi, what I do is I do small

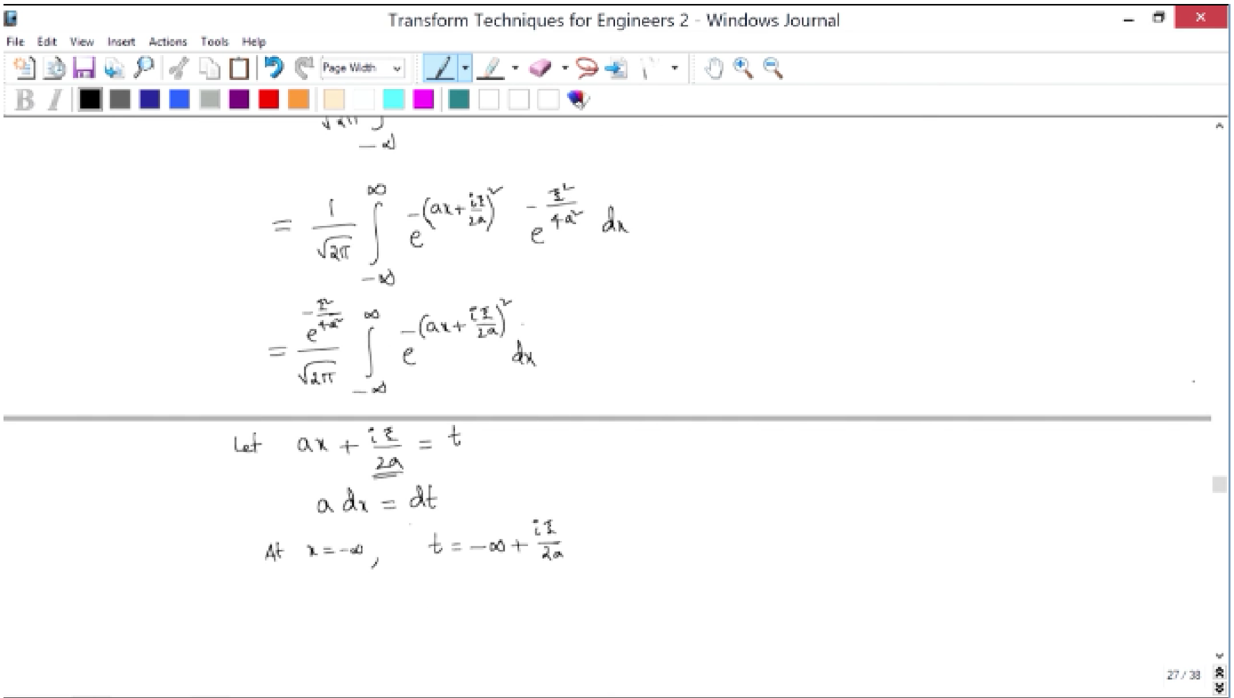


trick here, what you see is you try to put it together so you get the –E, so you try to write something minus square that kind of thing, so AX square, so I have AX and I have finally to get I xi X, I xi X if you want so what you need is I xi, so if you add something, some quantity A square, so what you get is A square X square + A square + 2A AX, so this quantity is actually what you want is equal to - I xi X, right, so not –I, so it's + I xi, because minus I take it out, so this together so this sum should be equal to this one, so that makes it what is A, A equal to, X X goes, I xi/2A, okay, so if you want this quantity to be this, so your A should be that so if I choose S, I xi/2A whole square, this whole square if I take, okay, and then what happens to, so this square is now A square X square is there, when you say take 2AB that to this one and what is left is E power -I square xi square/2A, so this one minus minus plus, so you have E power minus minus plus you have xi square/2A, so that is you already have so you have to multiply with this minus of that, so if you multiply with this xi square/2A, xi square by the 4A square, right, so this is 4A square, so 4A square if you multiply this so that you know what you get is,



this is + and this is -, so that gets cancel so you left with only this one, so this is the same, okay, and you have DX.

So this is equal to 1/root 2 pi - infinity to infinity, so this is nothing to do with this X variable so I can take it out, so you get E power – xi square/4A square - infinity to infinity E power -AX + I xi/2A whole square DX, so I'm doing integration with respect to DX. So what I do is I try to put this AX + I xi, this is a complex number, xi is a real number because it's a Fourier thing, xi is positive, okay, xi is full real line, so xi is belongs to the real number, and you have this, you call this sum T, then what you get is DX you want, so ADX this is a constant, and this is equal to, so I is a complex number so you have DT, so if you do this what happens to these limits? Limits becomes T limits at X = - infinity, what happens to T? T is A into X, AX, AX is – infinity, so let us assume that A is positive, okay, so A is positive it's doesn't matter because here if A is negative at X = - infinity it's going to be + infinity, so if I choose A is positive, so A

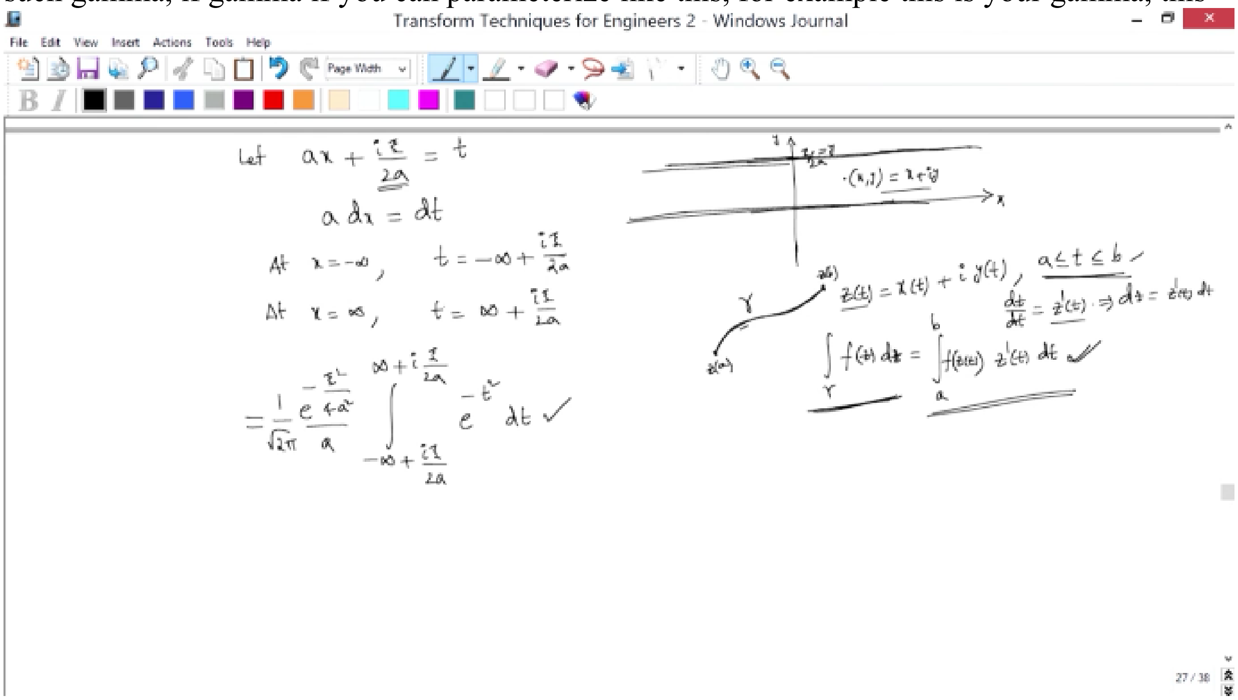


into X will be - infinity + I xi/2 here, okay, I xi/2A, so this is this, whole thing is constant, so you have this one at X = + infinity, T becomes + infinity plus this constant, okay, so try to use that what you get is E power – xi square/4A square 1/root 2 pi times -, now what you get is -infinity + some constant, complex number though infinity plus rather you write the constant first, constant plus infinity so it doesn't matter so you have a plus infinity, plus I times xi divided by 2A, so this is what you have and this becomes E power - T Square DT, DT/A is my DX so you have 1/A comes out.

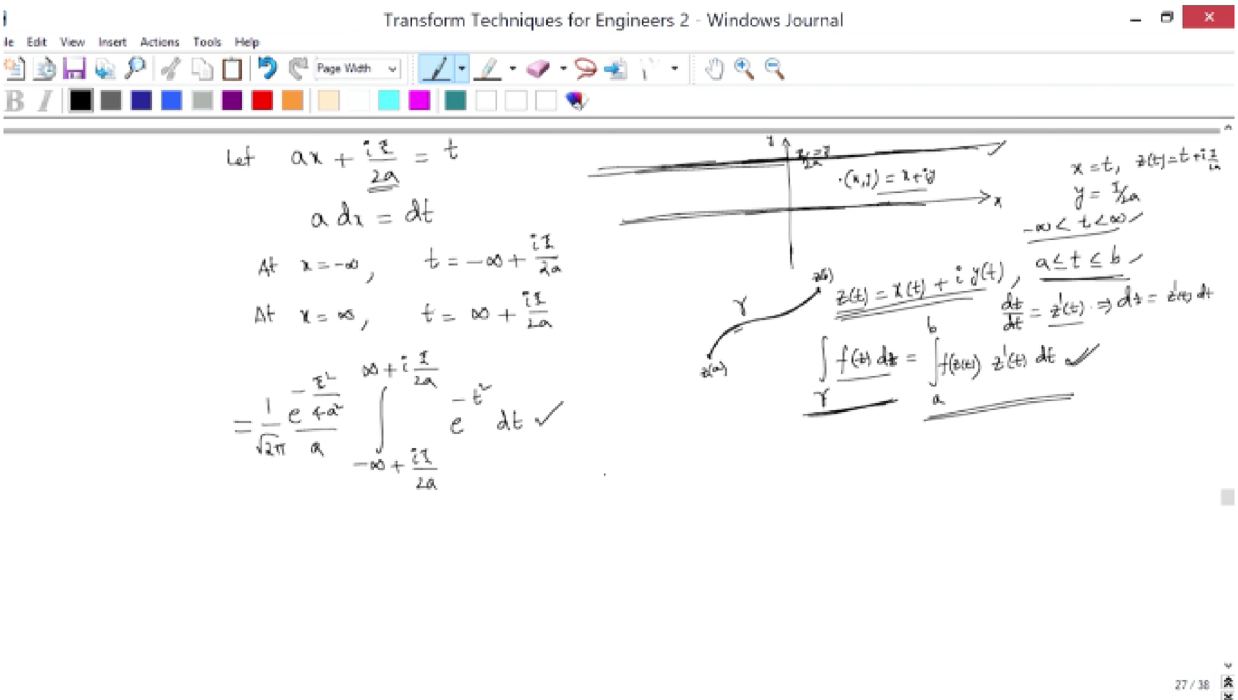
This is what is integral now, this integral if you want to find out because the integration is over, so this is basically if you look at the complex plane, complex plane is actually it's like a plane R square you can identify complex plane with R square, okay, R2 so it's like a usual plane, so what is a main difference in complex plane and a real plane? Real plane so you have - infinity to infinity as your thing, and in the complex plane this you can identify with the C at every point you can identify with this C, so if you think of a complex plane what you need is you need an integral from - infinity to, so let us say if xi divided by 2A is this one, if Y equal to this and this is the line over which you want the integral okay, to get this integral this is your X axis, and this is your Y axis, okay, and to get this integral so every point is like X, Y which is actually X + IY, you can identify like that in the plane.

So you want integral over this in the complex plane, and so how do I do this? So first of all for any curve in the complex plane, so let's call this Z(t), so this is a curve, curve is any smooth curve piecewise continuous curve that means, so I will not just get into the details what you do is you make any curve which is a parameterization, so Z(t) is X(t) + I times Y(t), and T belongs to some A to B, so if you take, if you parameterize curve so you fix T = A, this is this point, this is X(t), X(a), and Y(a), as your T moves along, move from A to B you actually move over this line, your this curve it's going to be, this is X, this is Z(a), Z(b), this is Z(b) and this is Z(a), so this is XA, YA, and here XB, YB, so this is how you parameterize, though integral over this curve, let us call this some gamma, so you want to have a gamma F(z), so gamma is a curve in the plane, in the complex plane, so you have Z belongs to gamma F(z) DZ, so what is this becomes?

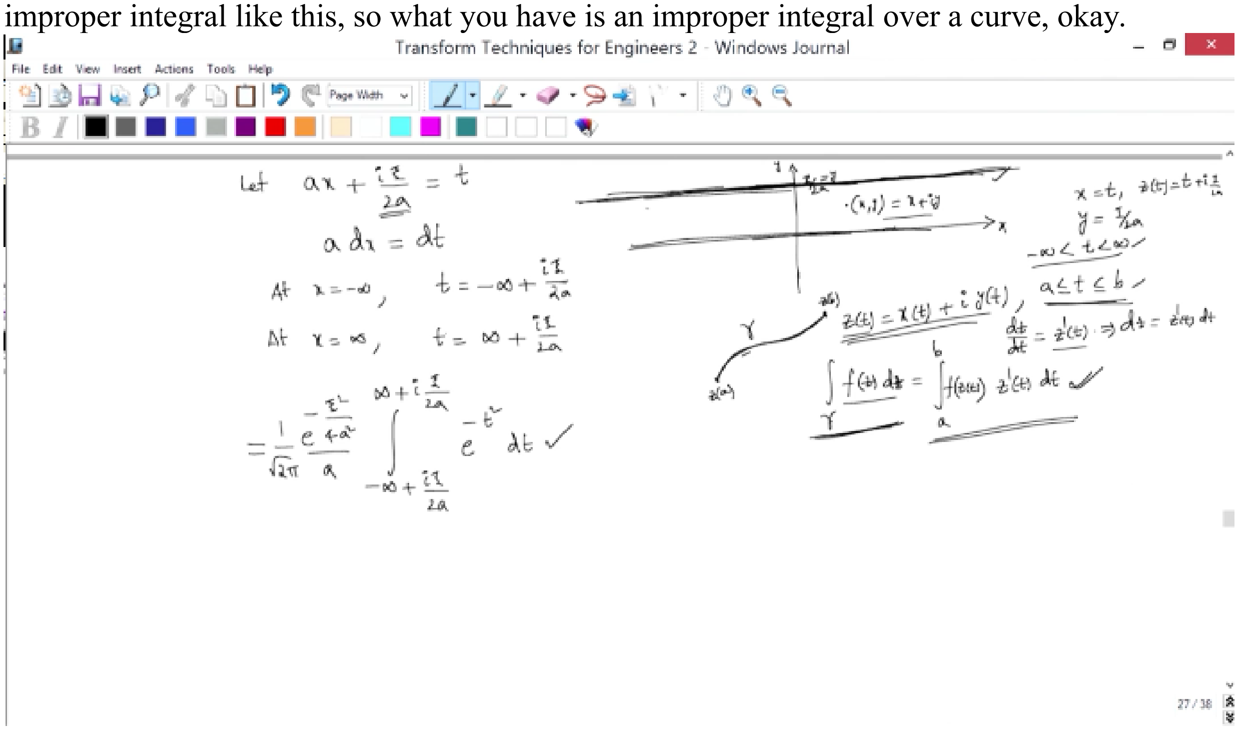
So if you do this one, so if you try to do this one DZ is, what is DZ? DZ/DT if you do that is actually Z dash(t) into right that is a Z dash(t) you can do, because Z is a function of T you can differentiate, right, so that implies what you can write DZ as Z dash(t) into DT, so I write this DZ as Z dash(t) DT, and T is now varying between A to B so you can write A to B, and Z(t), because Z is nothing but Z(t), so everything in terms of T you want, so this is the integral, this is the real integral which you know, usual integral, usual Riemann integral or whatever usual integration which you do normal integration over a real variable, so if you have a complex variable like this, complex integration, integration over a curve which is in a complex plane, and Z is a complex number you parameterize such a curve, such a gamma, if you parameterize such gamma, if gamma if you can parameterize like this, for example this is your gamma, this



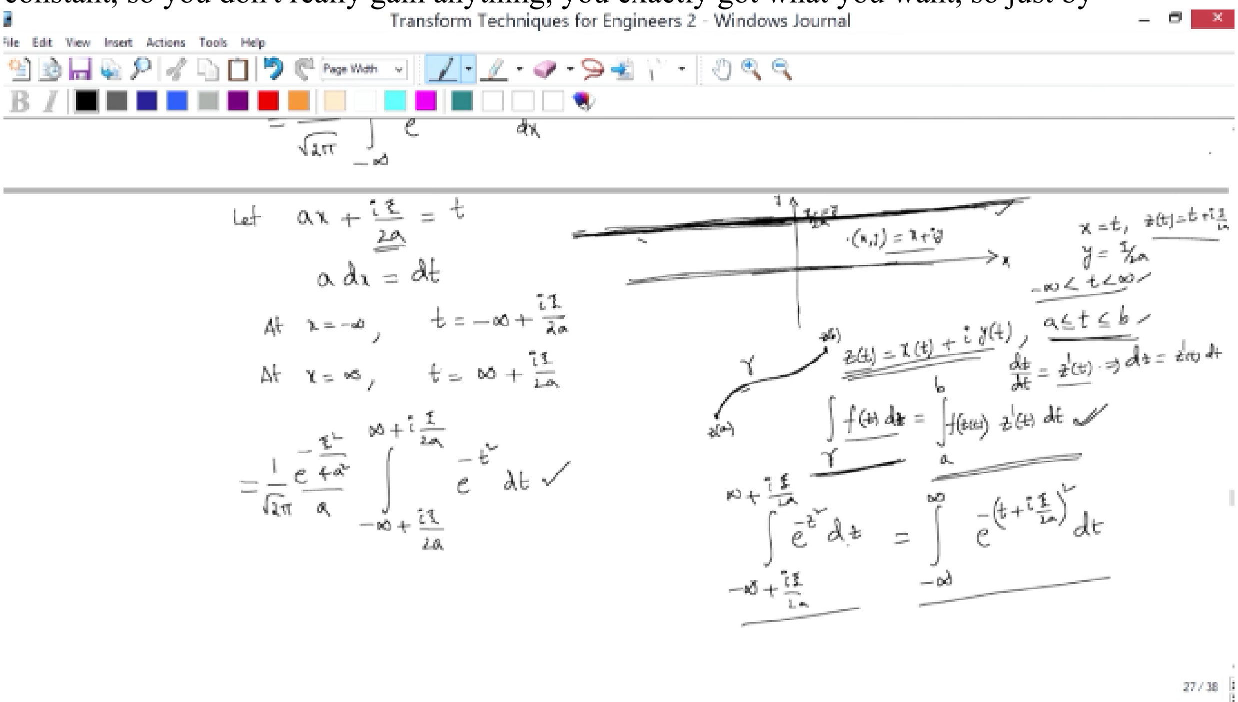
what is its parameterization? X = T, Y equal to simply constants Z/2A, so T is between - infinity to infinity, if I choose like this X + Z(t) is X(t), that is T+I times xi/2A, so Z(t) is T+ I xi/2A, T is between – infinity to infinity, that is actually parameterization for this full line infinite line, okay, but you know only finite line, so what we do, to calculate over a finite line so to get this infinite line, okay.



So because they see, even for real integral what is the meaning of this real integral F(t) DT? This is actually limit some K goes to infinity, you fix it as -K to K, F(t) DT, okay, this is your usual integral, when you finally after calculating if you allow K goes to infinity, that is improper integral like this, so what you have is an improper integral over a curve, okay.

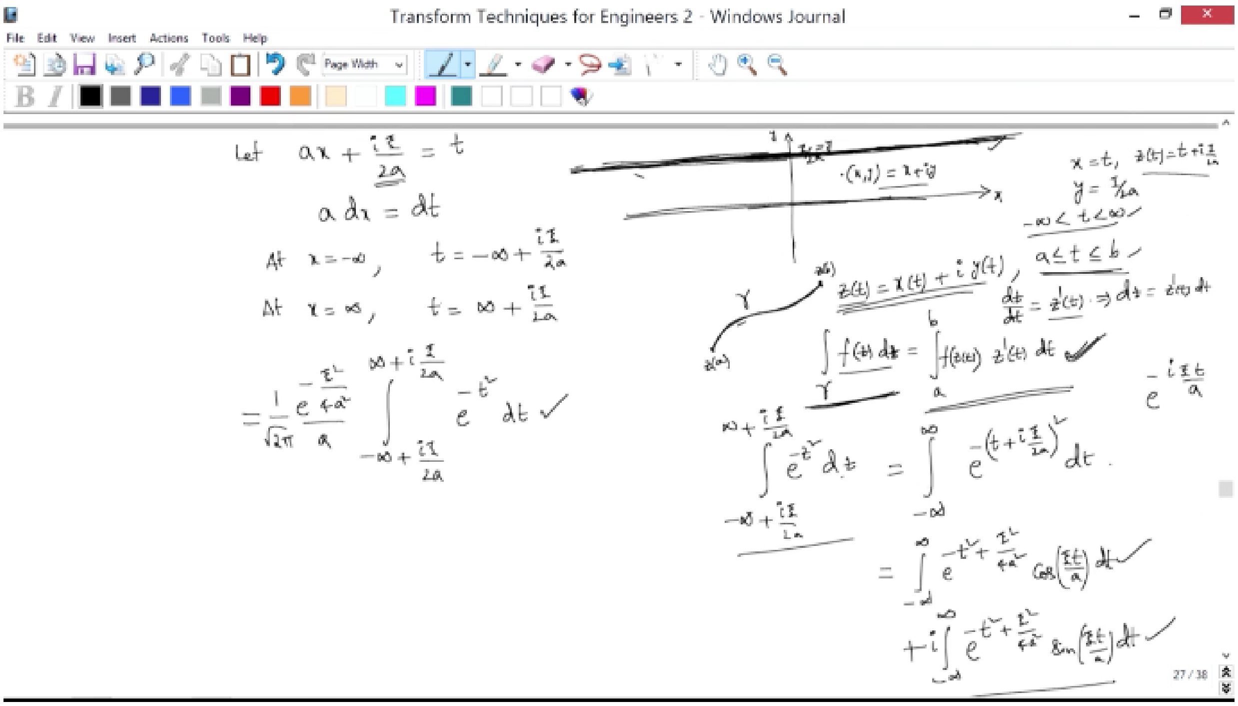


So how do I calculate? So how do I put this? So to calculate over this directly if we put it you will not be able to get it, so you will see even if you put directly so integral over that we can do that, so let us, you see that, so integral over, let us say - infinity to+ I xi/2A, and + infinity + I xi/2A E power - T square DT, okay, so T is now actually Z, because T belongs to this set, - infinity, so this line you are in the complex plane, so this is kind of –Z square, DZ is this one, Z is this, so if you rewrite this on T - minus infinity to infinity and E power minus, now what happens to Z(t)? That is T + I xi/2A whole square, okay, DZ is simply DT, because that is a constant, so you don't really gain anything, you exactly got what you want, so just by



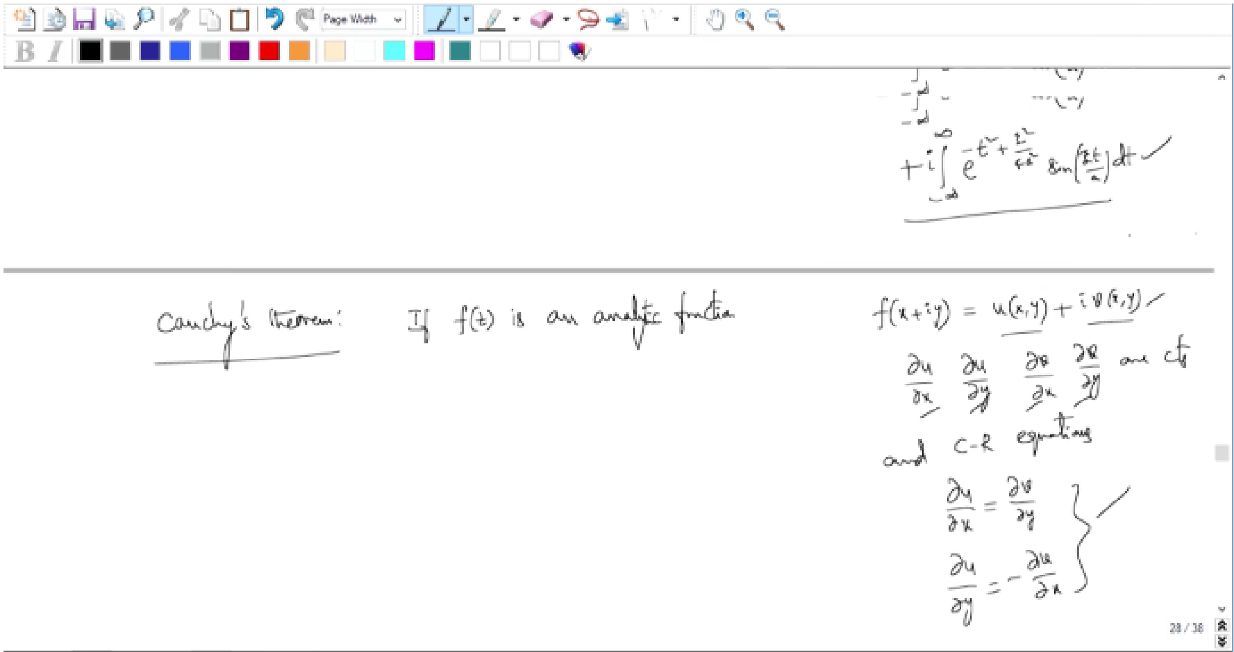
parameterization and evaluating over you will not be able to get it, okay, so this you can get it actually, this is a real function this is F of Z(t) is some function of T + I times another function of T, so it is a complex valued function, complex valued function over a real variable T, so such a thing we have, but here you try to write like that this is actually - infinity to infinity E power -T square, now T is real minus minus plus xi square/4A square into DT that is your real part, and imaginary part is - infinity to infinity, what you get is, no, this is not the real part, what is real part? This is a real part and you have E power -I xi/2A that will give you cos xi/2A, okay, that's what you get, so cos xi square, cos xi square because what you get is E power - I xi X, right, cos xi X, right, cos xi T, what exactly you get? T square - T square minus minus plus, so xi square/A square so 2 times, 2 2 goes, T xi/A, so T xi/A this is what you get, okay.

So what is that one? So your real part if you take cos xi T/A into DT + integral – infinity, this is the imaginary part I times, same E power -T square + xi square/4A square. Now we get sine, sine xi T/A into DT, so these integrals you don't know how to evaluate them, so these are not known, known integrals which you can evaluate easily. But there is a complex variable

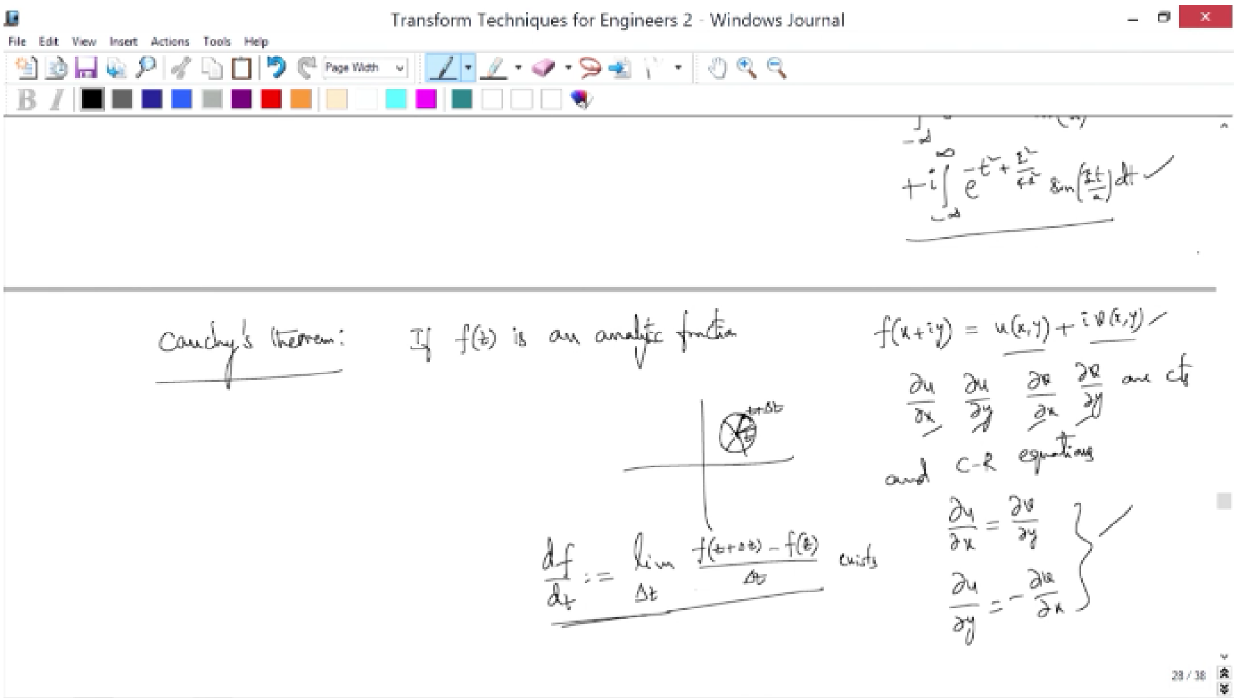


technique in complex variables, in complex variables what there is something called, there is a technique called contour integration technique that is based on a Cauchy theorem, so I’ll digress here, so our aim is to find this integral, so what you get is we digress little bit here, so we write what is called Cauchy’s theorem, Cauchy’s theorem in complex variable.

If F is analytic, what is the mean by analytic? If F is analytic F(z) is an analytic function, analytic function that means F(x) is, F dash(x) all the partial derivatives, okay, so F(z) means, F(x+iy) if you substitute this is actually U(x,y)+ I times V (x,y) this is a complex real part and imaginary part you take and all these partial derivatives dou U/dou X, dou U/dou Y, dou V/dou X, dou V/dou Y all these exists and they are continuous and they satisfy our continuous and you have a Cauchy Riemann equations, CR equations, Cauchy Riemann equations, those are dou U/dou X = dou V/dou Y, and dou U/dou Y = - dou V/dou X, if they satisfy some equations like this, and all these partial derivatives are continuous then you say that this is analytic

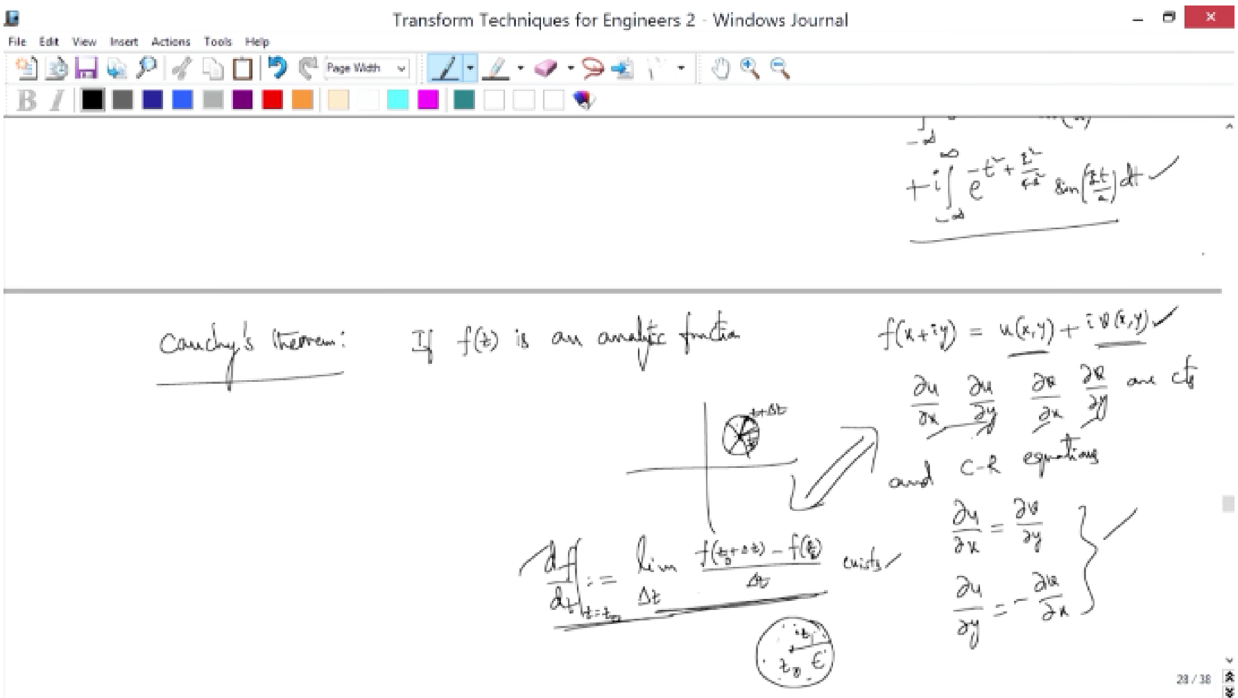


function, okay, so if you just get into some book on complex numbers, complex variable, complex calculus, complex number calculus, complex functions theory if you read you simply calculate what is its derivative of a function, so that you take in the plain, any function if you want to have for F(z), derivative of DF/DZ complex derivative is usual way definition is you take any delta Z that may be any Z function here, F(z-) F(z+), so something like that, so Z is this, Z is this, and you take any all directions in any direction you take Z + delta Z then you calculate F(z+) delta Z that is on here, on the circle, this can be any direction –F(z) that is at Z, that is the center point divided by Z, if this limit exist in all directions this need not be simply you need not come there's a delta Z, okay, if all directions whichever way, all direction if you can come, if we take this, this limit is same then you say that if this exists then you say it's derivative exists, and if its derivative exists it is also, so then we say that it's differentiable at a point, at every point inside and it's derivative exists, you take every point and you take a small



any neighborhood, you take any small neighborhood of Z, you take every point inside here, now Z1, at Z1 this is true DF/DZ exists in the neighborhood of that, okay.

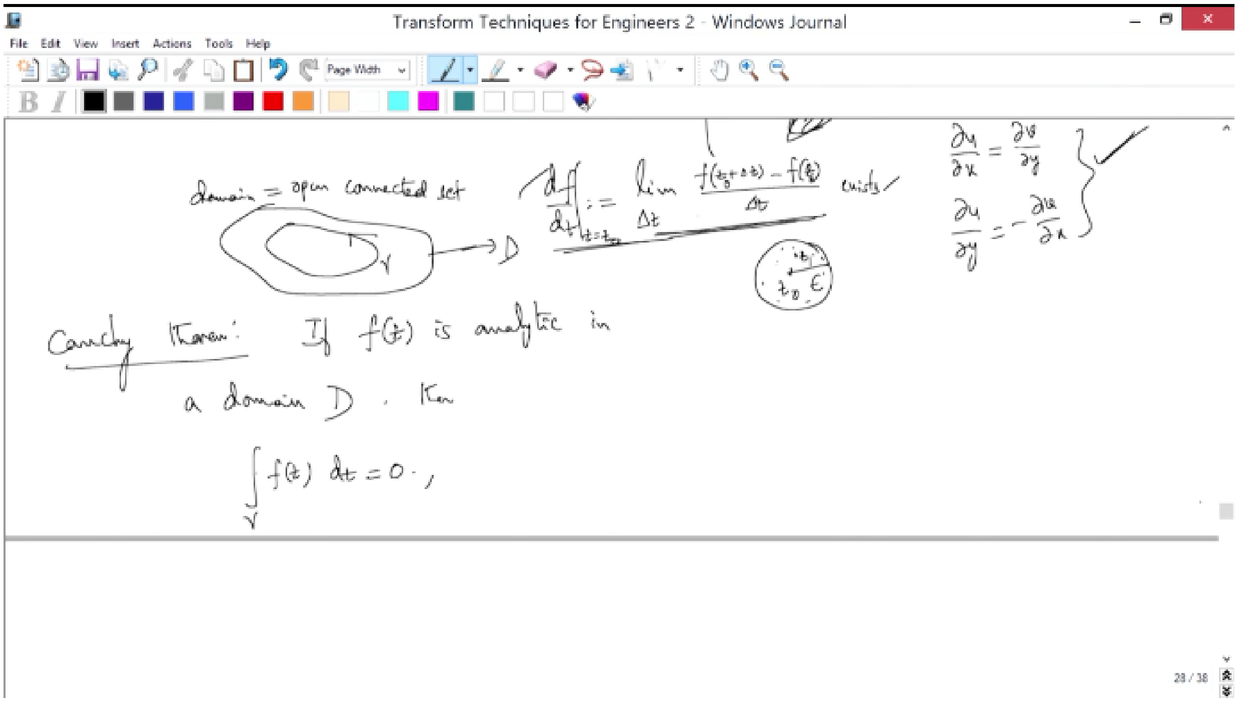
At every point if you have such a thing, at every point inside if it is still differentiable that means this limit exists, so this means a limit exists at Z, that means this A exists at Z, at Z = Z, okay, so let's call this Z0, then it is Z0. So at Z0 this limit exists then you say that F is differentiable at Z naught, and once you have this Z naught and you take any neighborhood every point Z1, if it is differentiable at Z1 also, F is differentiable at every point inside this neighborhood, neighborhood of Z inside this disk that means you take any point Z1 it still this it exists DF/DZ at Z = Z1 if it exists that limit exists then if you say that it is analytic function that is equivalent to say that instead of calculating these limits this is equivalent to say that you write this function as a real part and imaginary part and then you look at its partial derivatives real part, imaginary part, because these are function of two variables, if you remember the calculus, multivariable calculus this is a function of two variables, so you have, you calculate its partial derivatives and these are the, and if they are all continuous functions and if they satisfy these CR equations they are actually equivalent to their analytic function.



So that's why I said that's, that's how we characterize these analytic functions, so if such an analytic function if you take so let me write the Cauchy theorem, Cauchy theorem will write it here so if F is Cauchy theorem, if F is analytic function, F(z) is analytic, analytic in a open, it's in a region so this means, region means it's connected every point to every point you can always connected by a line inside but that is inside that region, if it is open that means open means you take every point I can always construct a circle that is disk entirely in it, that it is open connected means you take any two points I can always connected by a line which is entirely in the domain, so that is such a call is domain, domain means open settle, just like open interval you take every point I can always construct a small interval which is entirely into that, so like there's open connected set, connected set is called domain, so such a thing, connected means you take any points in that domain I can always connect with line entirely into it, okay, so such a thing is called analytic in a domain, in a domain omega, so let me call this omega, omega is the domain inside, okay.

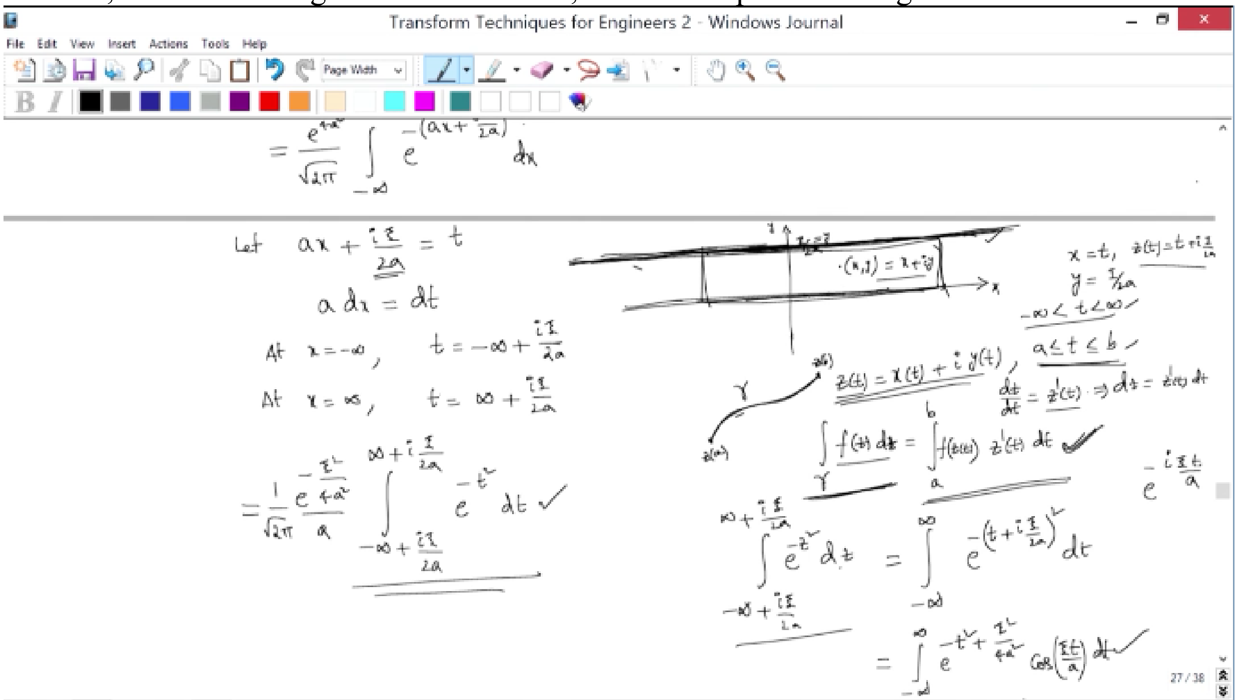
Then and if I call this boundary, boundary of that is dou omega then integral over F(z) over D omega DZ is always 0, this is true for every omega inside the domain D, so let’s call this domain D and then you take this, this omega is into some bigger domain so this is your D, this is actually your D, and in that you consider this gamma is your, omega is your curve, omega is curve, for every omega a closed curve omega, let me call this closed curve omega which is in D, okay, let me say for every domain, for every domain you consider in D, for every domain in this, once you have every domain in D close domain, so once you have say domain, so instead of putting in this way you make, you confuse, so what you do is, so you have initially one domain D which is a connected open set, okay, this is your D.

Now you consider any closed curve, closed smooth curve, okay, so if you call this gamma, so let me call this gamma equal to 0, or gamma for example, gamma closed because it's a closed I give the orientation it's going from anti clockwise direction, so if anti clockwise is a positive orientation if you go with the clockwise direction that is negative orientation, so whichever is you can think of, so why I choose this if you go counterclockwise why you make it positive

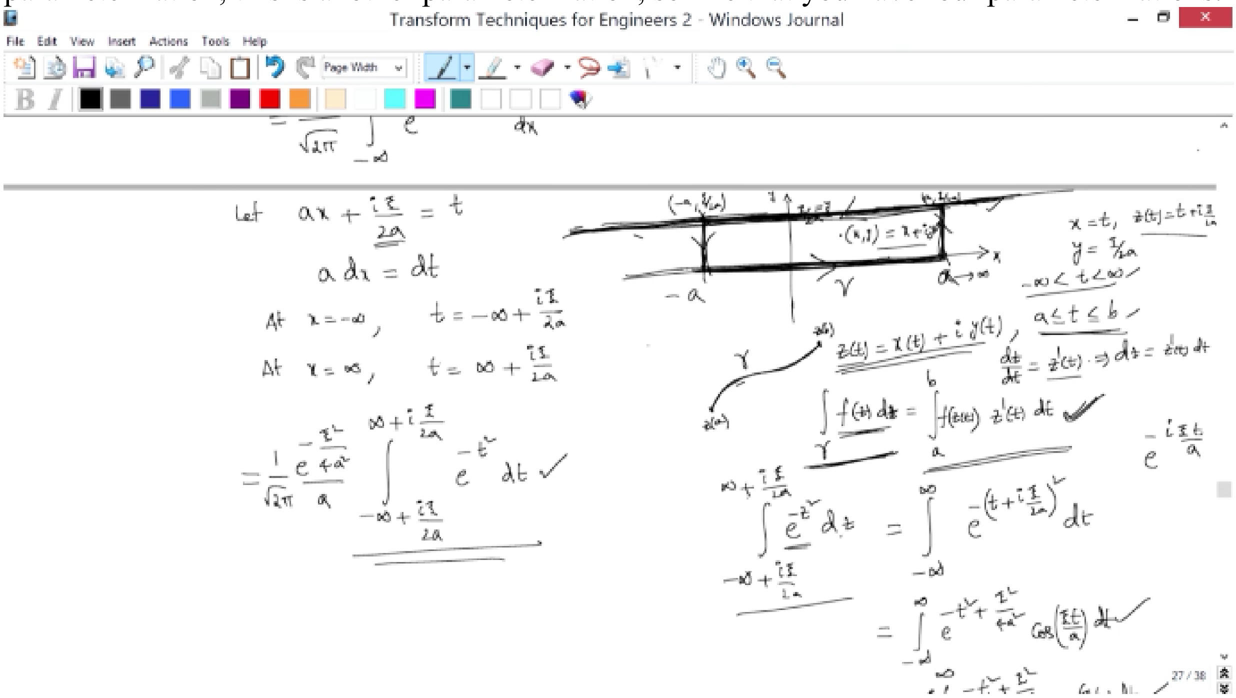


orientation because when you move on the curve we are on the boundary you know as you move along the clockwise direction your domain is on your left-hand side that I fix has positive orientation. If you move along the line so that when you see right side your domain, right side or see this is your domain and if you walk along this inside your domain, this domain is inside if you go along this side right side that is negative orientation, because you are moving along like this you see that always left side is your domain, so that is why it is positive orientation.

So this is a closed curve for every closed curve gamma, F(z) DZ is actually 0 for every gamma inside D, closed curve gamma, for every, so moment you write gamma which is, if I put right like this is a closed curve, okay, for every gamma inside D, that is the meaning of this closed, so Cauchy theorem, if you apply this Cauchy theorem, if you apply this Cauchy theorem here now come back to this, if I apply to Cauchy theorem to this piece for example, you make a rectangle like this, now the rectangle is a closed curve, now let me put this as a gamma this is

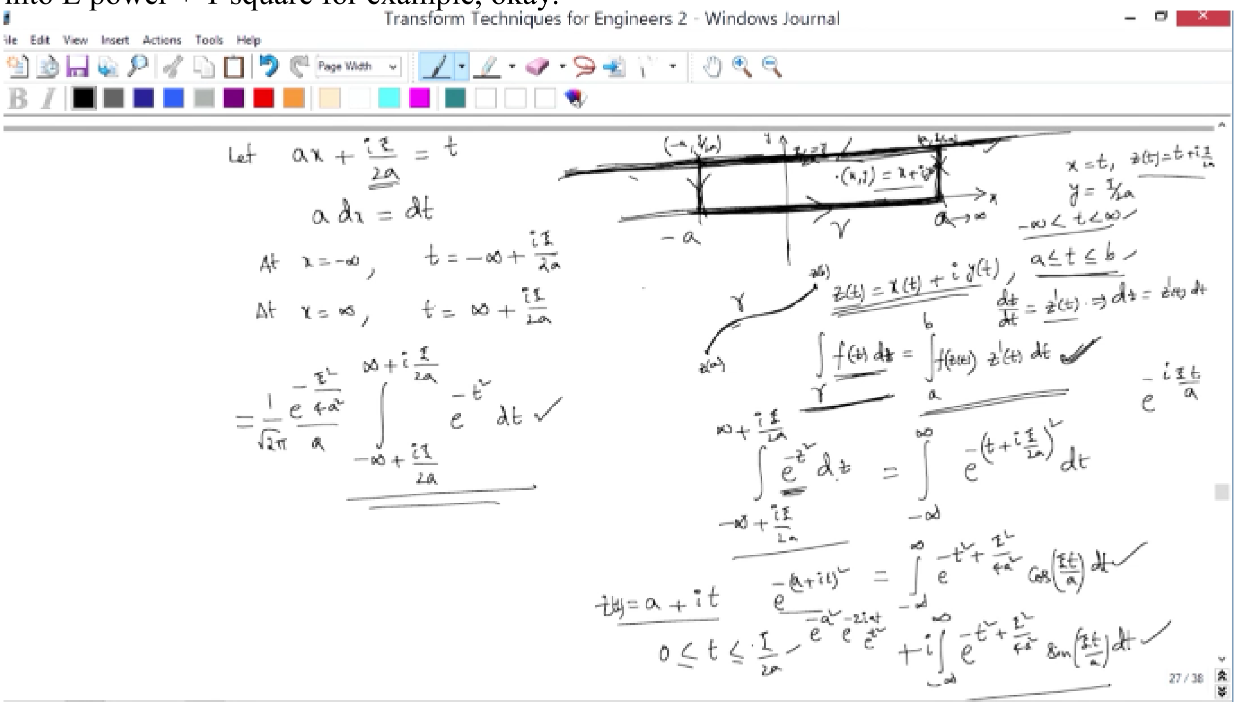


your gamma, this is how you reporting as a, represent your gamma over which you try to apply your integral over gamma, F(z) you take it as E power -Z square, okay, so if we choose like this and what you see is on this if you parameterize this piece and this piece, see each piece you cannot parameterize, see everything you cannot parameterize because this is a rectangle, these only piecewise continuous, if it is now continuous curve but it is not smooth, at this point it is not differentiable so you cannot represent this as a single parameterize curve instead you can represent this piece one representation, one parameterization this piece another parameterization, this is another parameterization, so like that you have four parameterizations.

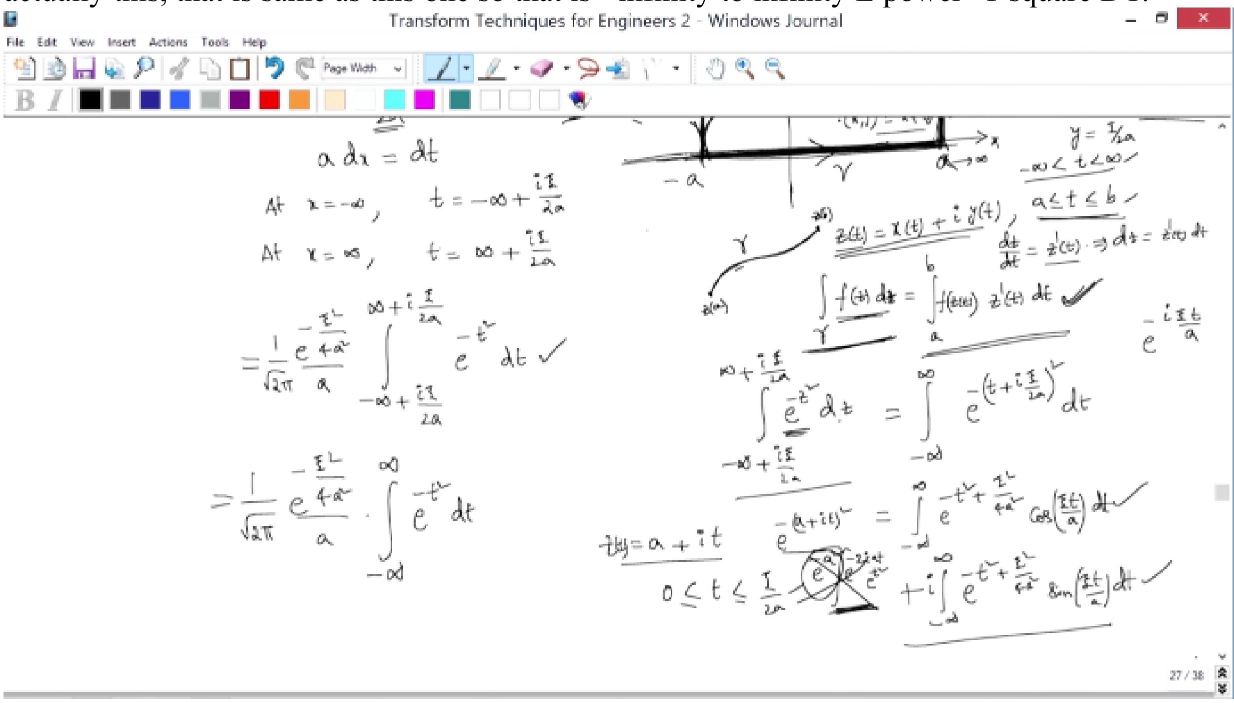


Once you have a 4 parameterizations and over this one, plus this one, plus this one, plus this one is equal to 0, so that means but you see that over this if a parameterize these two pieces it will be 0, as if this is your X, as X goes to infinity eventually you want it over full line, right, so that way you have to allow this X goes to infinity or A, okay, so –A to A and here A+ this is going to be A, xi/2A, and this is, this point is –A, xi/2A.

So eventually when you allow A goes to infinity so you see that this piece will become minus what you want, this - infinity +I xi/2A to + infinity I xi/2A, if this piece becomes as A goes to infinity, this is going to be - infinity to infinity, okay, and this piece what happens? What is the parameterization? Parameterization for that piece is actually X =A, Y = 0 to, Y is T, T is between, so that means A+ IT, T is between 0 to xi/2A, okay, this is your curve, so like this if you do and finally when you allow A goes to infinity because of this E power -Z square, so E power when you substitute this X, Z is this, Z(t) is this, E power -Z square, so A square, A square comes out whatever may be A square Z square, so you can write A + E power –A + IT whole square, so what is this one? So this is E power -A square into whatever E power -2IAT into E power + T square for example, okay.



So if you choose like this, so what happens? So as A goes to infinity this is going to be 0, so whatever maybe the integral, this integral or the small piece that is this piece, this is a finite piece so doesn't matter so the integral value is finite and you have E power -A square, as A goes to infinity this is going to be 0, so like that you can show that this or this is 0, if your parameterize the same way this way opposite direction that is also 0, so maybe we will try to do this in the next video, we will properly calculate and see this one okay, so when once you do this one plus this is 0, and this is plus this one, plus this equal to 0, that means this is equal to minus of this one, because you go in the opposite direction that is the minus of that, so you have from this to this, that is - infinity to this is same as - infinity to infinity, you simply replace that with E power -T square, so if I apply this we will try to do, redo this thing, I will try to calculate this as a complex this, I will try to apply this Cauchy theorem, and try to get this integral in the next video, so what you see is and just right now I'll just orally explain you, and I'll just directly put it, so E power -I square/4A square by A into this becomes over this integral this integral is actually this, that is same as this one so that is - infinity to infinity E power -T square DT.

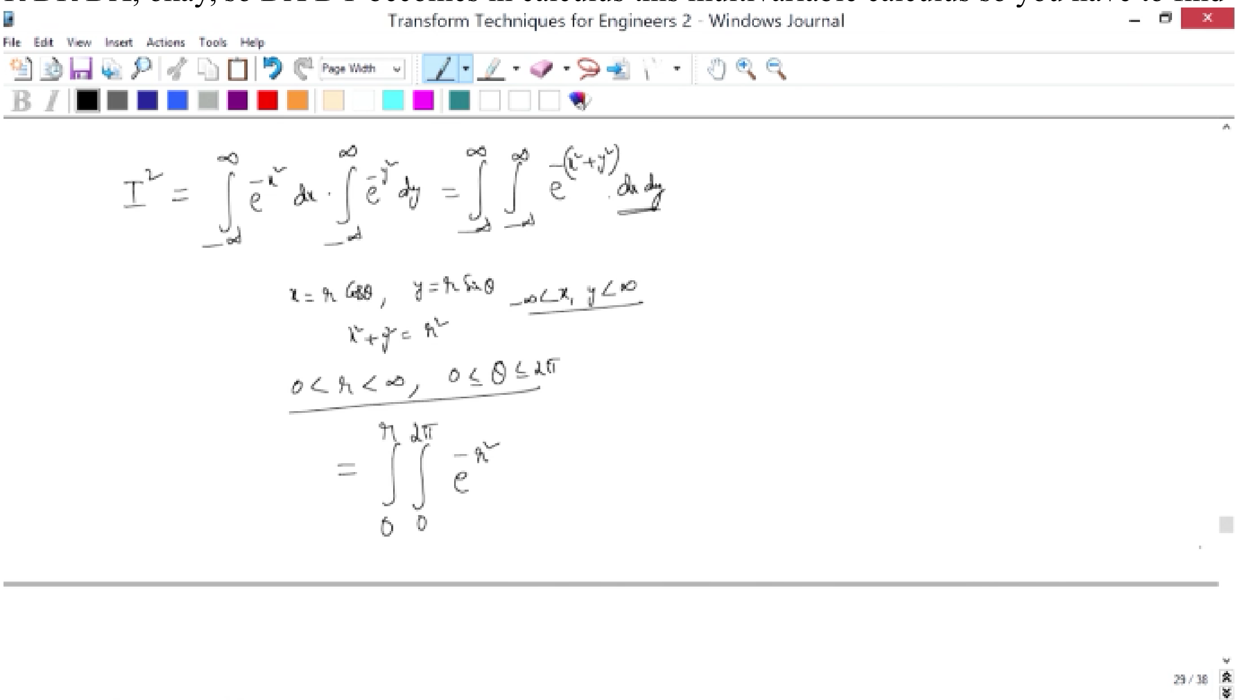


Now T is complex or doesn't matter because you're on the real line, so this is over this, T is always real, here T is complex, so that is why you see that both these integrals are same, so what is this integral? This integral because this is, how do you calculate this one? Now again if you have to use the complex, how to use the calculus, you have to use the calculus, so how do I calculate this integral? Now your question is how do you calculate this integral E power -T square DT - infinity to infinity, so to do this again let me put it so - infinity to infinity E power -T square DT, so what I do is I will try to suppose this is your I, T is a dummy variable, so let me call this X, -X square DX, and I also write I as - infinity to infinity E power - Y square DY, both are same.

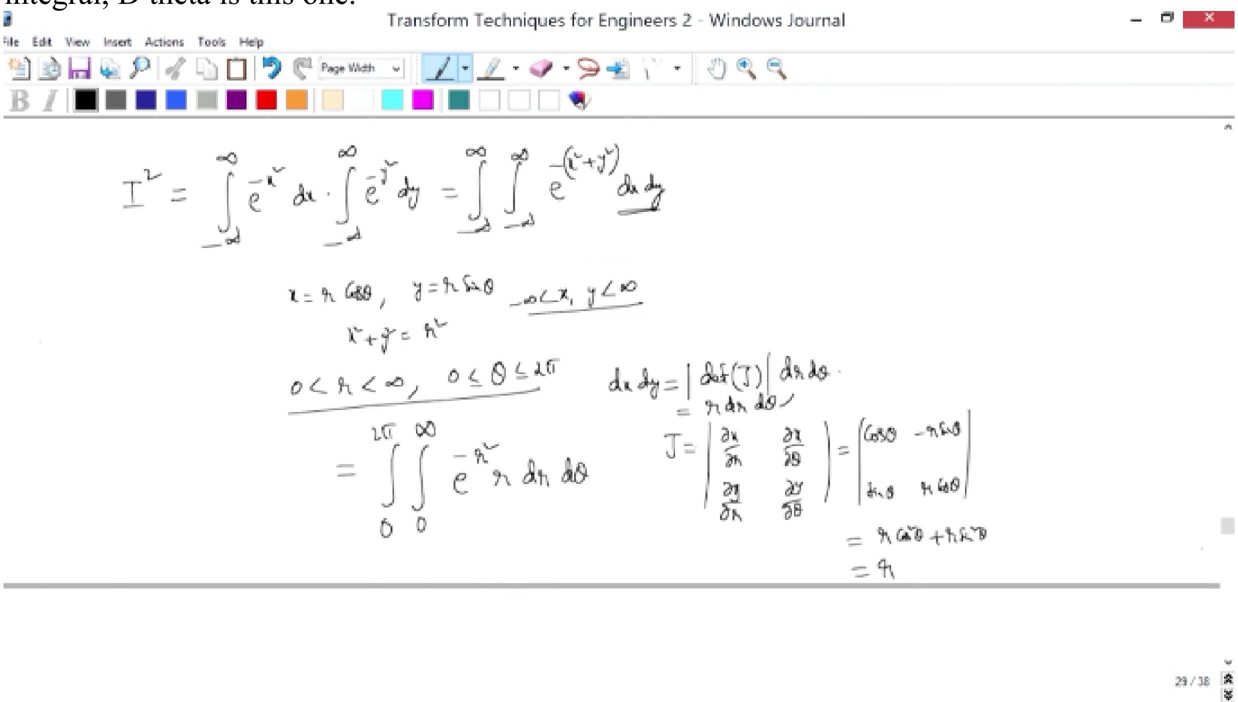
Now if you multiply I square both sides I1 and I2 together I square and what you see is - infinity to infinity E power -X square DX into - infinity to infinity E power -Y square DY, X and Y are separate variables so you can write -infinity as a iterated integral, -infinity to infinity E power -X square + Y square DX DY, this is how you do.

Now if you use this X = R cos theta, Y = R sine theta, then what happens to X square + Y square? This is R square, so what happens, what happens if you do if full plane and you can represent R is between 0 to infinity, and theta is between 0 to 2 pi, okay, so if XY between - infinity to infinity this becomes, the domain becomes this one, so your integral will become now 0 to R, 0 to 2 pi E power -R square DX DY becomes, this is like a change of variable from

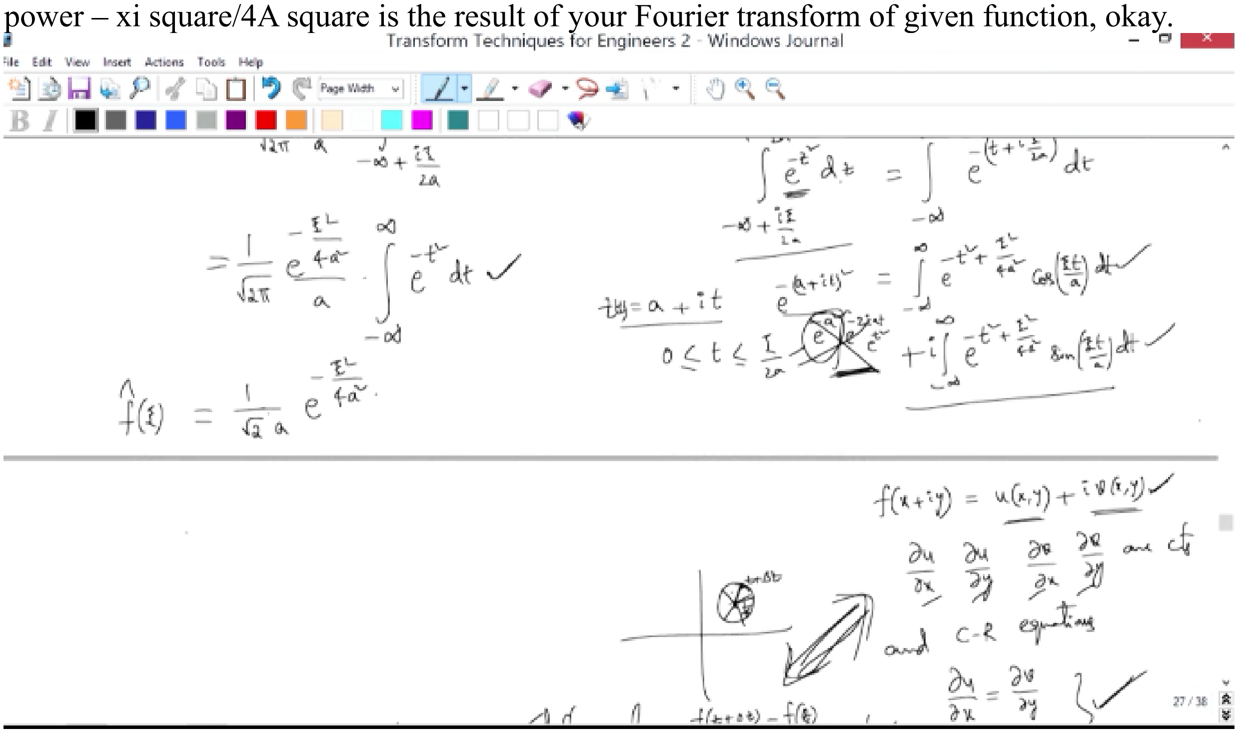
R DR DX, okay, so DX DY becomes in calculus this multivariable calculus so you have to find



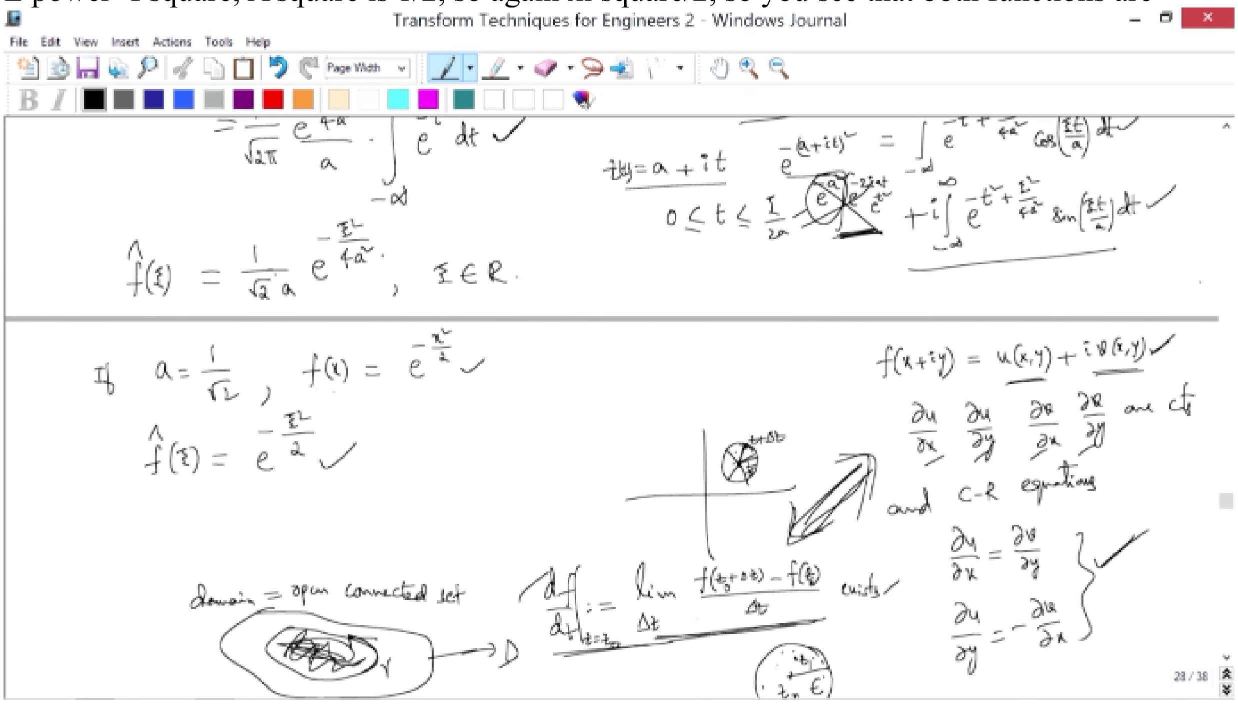
that, how do you find this what is R? DX DY is actually, DX DY is equal to some Jacobean, some modulus, the determinant of, modulus of, the determinant of, modulus of, determinant of some matrix times DR D theta so that is what is this J? J is you have to do DX/DR, so dou X/dou R, dou X/dou R, dou X/dou theta, and similarly dou Y/dou R dou Y/dou theta, if you do this one what is this one dou X/dou R? You have cos theta dou X/dou theta is -R sine theta here, dou Y/dou R that is sine theta, this is R cos theta, so what is this derivative? Is simply R cos square theta minus minus plus R sine square theta, so that is simply R, so that is how you get so DX/DY as R DR D theta, this is polar coordinates, okay. So R DR D theta that's it, so this is from, R is from, so actually you should write 0 to 2 pi, 0 to infinity R DR D theta, DR is inner integral, D theta is this one.



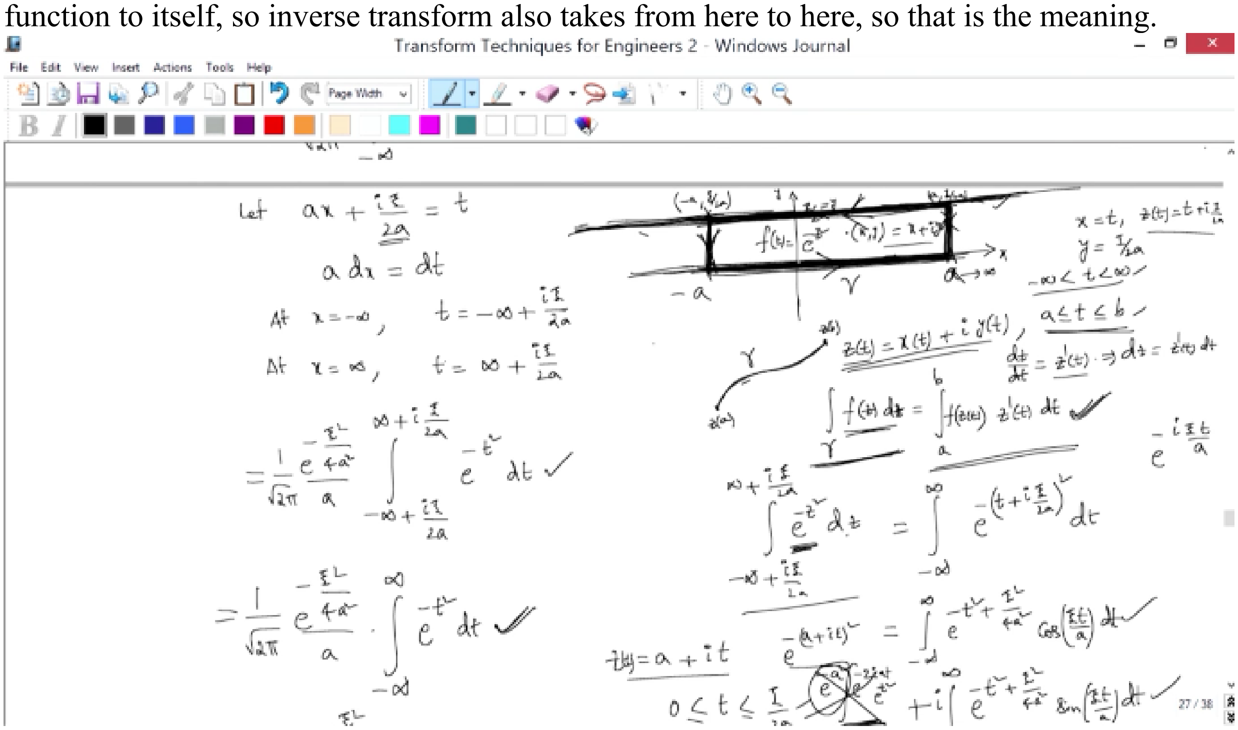
So it's nothing to do with this theta so you have 2 pi times and this E power IR is you can write 1/2 E power -R square minus of that, if you differentiate this minus minus plus, so -2R, so R into E power -R square so for this if you put these limits so 2 2 goes, pi pi, so you get simply pi, okay, simply you have E power -0 is 1, so we have pi, so together you have this pi, so I square is what you got is I square is pi, that implies I is root pi, so what is I? I is actually what you want here, so here this value of this integral is root pi, that root pi root pi goes 1/root 2 into A E power – xi square/4A square is the result of your Fourier transform of given function, okay.



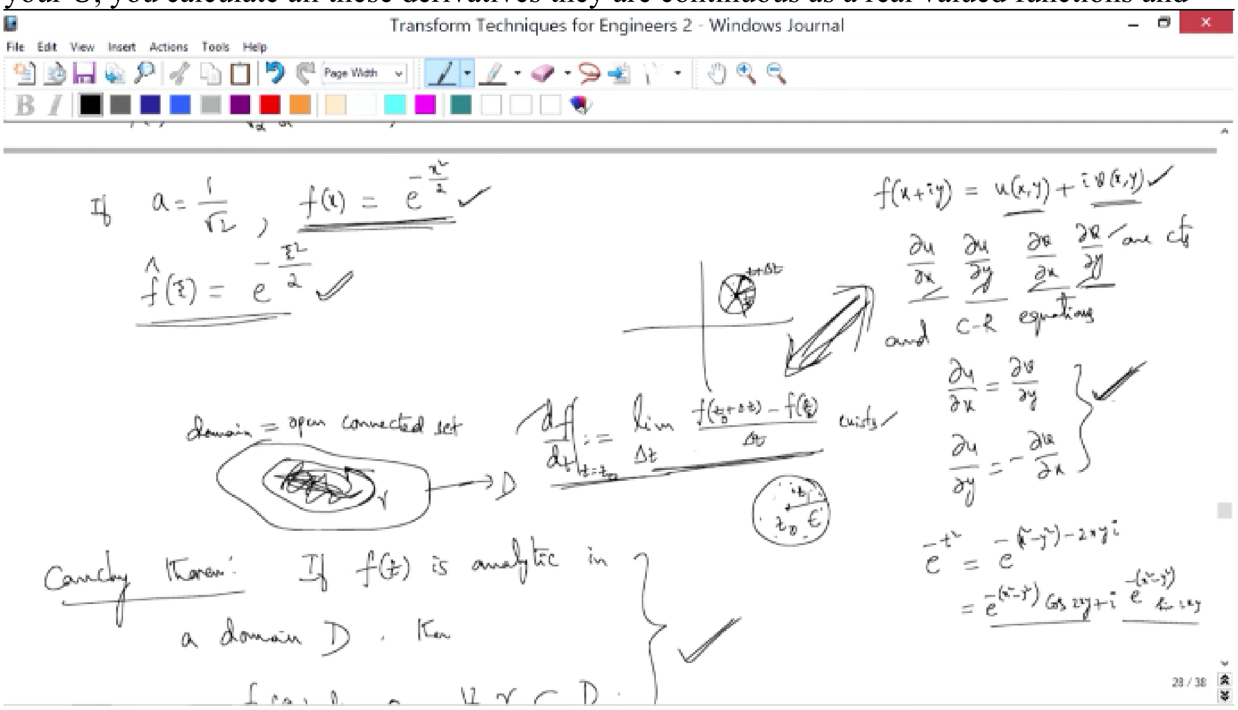
Now xi is actually full real line, so if I choose A = 1/root 2 in your function what happens to your function? F(x) is E power - A square is X square, A square is 1/2, so this is your function. And what happens to your Fourier transform? A is 1/root 2 that goes and what you simply have E power -I square, A square is 1/2, so again xi square/2, so you see that both functions are



same, Fourier transform of this function itself as a function of xi, so it's called self-reciprocal with respect to the Fourier transform this function is self-reciprocal, so basically it's reflexive once you, it takes the function to you know the function itself, okay, Fourier transform takes the function to itself, so inverse transform also takes from here to here, so that is the meaning.



So in the next video we will try to calculate again I will try to rigorously, I'll just do the parameterization for this contour this is called contour because it's closed, piecewise closed curve that is called contour in complex variables you take this contour closed one and your function is E power –Z square that is analytic function, F(z) is E power –Z square analytic function inside, okay, inside and on this domain, on this including the boundary it is an analytic function and to see that, check the analyticity you can verify that all these you can verify as a students, so you can just calculate E power –Z square means E power -X square + Y square, X square - Y square, right, X square, X + IY whole square that is X square - Y square - 2 XY I, right, so such a thing if you take this is your U, so U is E power -X square -Y square, this is going to be cos 2XY+ I times E power -X square -Y square sine 2XY, so this is your V, this is your U, you calculate all these derivatives they are continuous as a real valued functions and



you see that these CR equations are satisfied, Cauchy Riemann equations are satisfied, this is Cauchy Riemann equations they are satisfied by these two and you can verify that is an analytic function, is analytic function in the domain that is that rectangle and that implies by this theorem which you can assume that this over that real, over that boundary that means on the rectangle that equation integral value is 0, the only thing is then this place opposite direction has to be 0 because these parts, I will show that is 0, and finally because on this part is actually minus of this way, so that means this is same as this way, this way that is actually what you want is from this point to this point that should be same as this point to this point, so that's how I have written minus this to this, this is same - minus infinity to infinity this is over real line, this is our that upper piece of the rectangle, so this is this one, okay.

So I'll I'll try to show that one in the next video, so what you have seen today is you calculated E power -A square X square which is rapidly decreasing function which looks like a bell curve and also called Gaussian Kernel in the heat equation, heat kernel is also called heat kernel it comes in the applications of partial differential equations, so there if you want to solve some boundary value problems you want to apply try to watch, if you like to solve by Fourier transform you may have to find the Fourier transform of such rapidly decreasing functions, so we calculate this rapidly decreasing function with that technique we end up getting this Fourier, this complex integral which we see that it's real integral and this is again from the calculus you can show that is actual value is pi, so that pi if you this root pi, root pi, so that root pi root pi goes what you end up is this one, so once you choose A particular value and it is a self-first, this function E power -X square/2 goes to the Fourier transform of that goes to itself that is the meaning of what we have done.

So in the next video we'll try to, I will try to show again this complex, how this complex integral over that upper part and is same as on the real line, okay, we'll see that in the next video along with, and then we will move on to find the transform, find the properties of the Fourier transform start from the linearity and so on whatever we have done for the finite signal, okay, so have a periodic signal, over a periodic signal whatever properties that we have done we will try to see here also, and then we will end up finally eventually we will try to find a Fourier, we will try to prove rigorously Fourier integral theorem, okay, we will see all that in the coming videos. Thank you very much.
[Music]

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