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NPTEL ONLINE COURSE
Transform Techniques for Engineers
Definition of Fourier transforms
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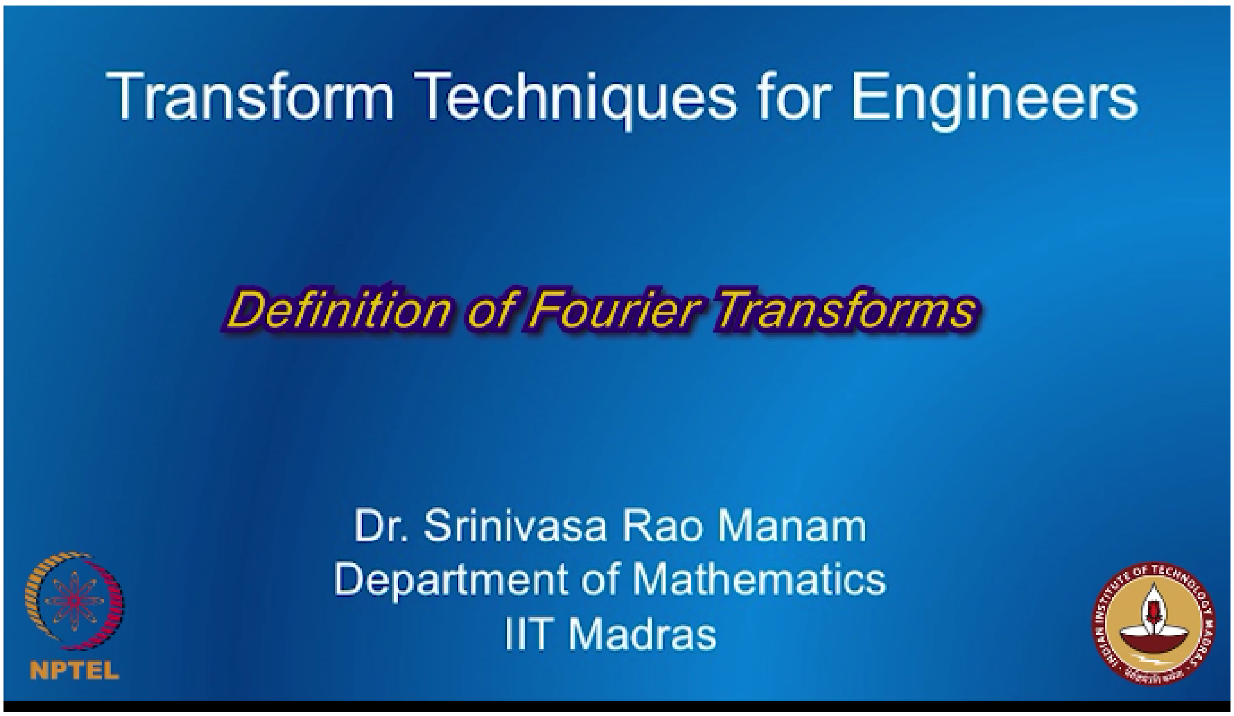
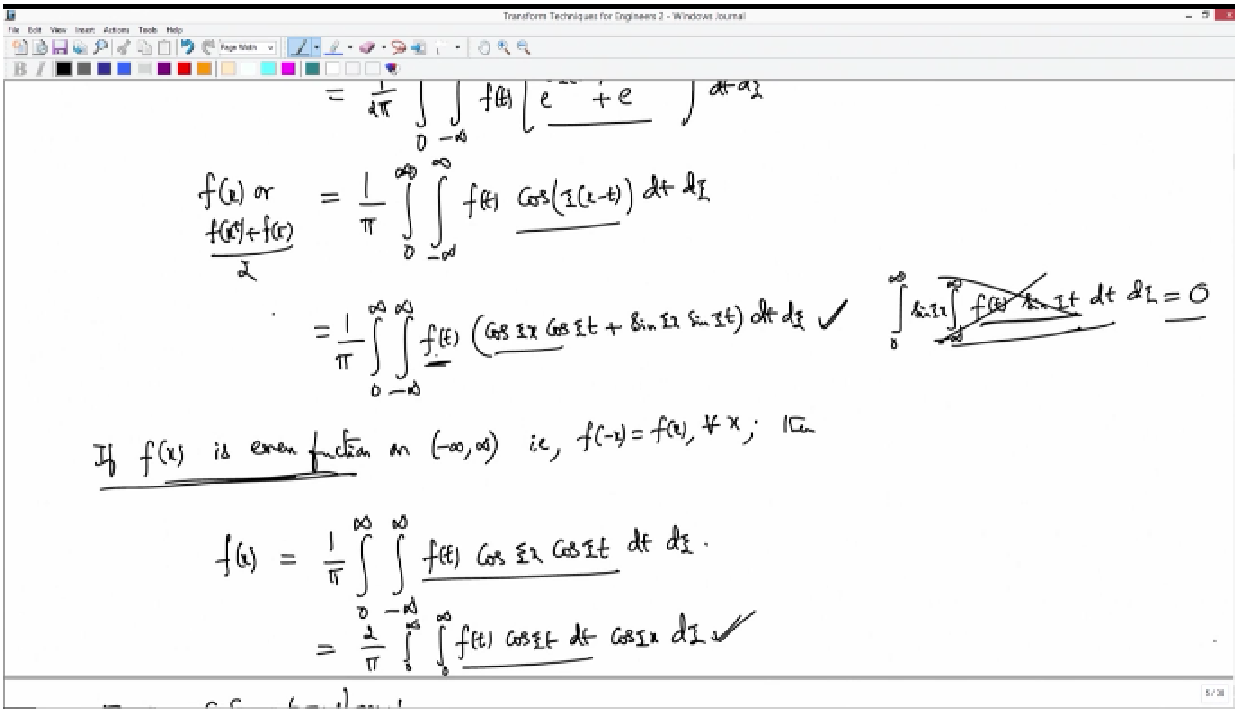
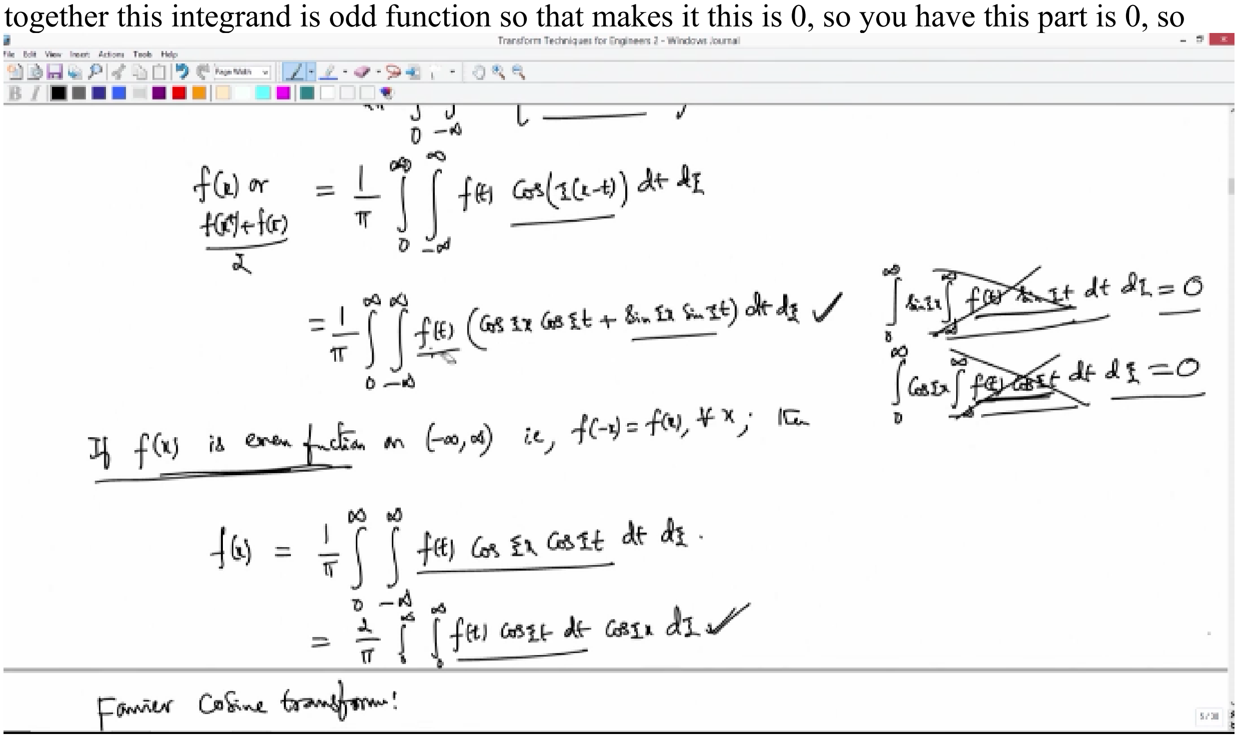


Welcome back, last video we have seen Fourier integral theorem and which you have given not so rigorous proof, that means some kind of intuitiveness we have involved in the proof of that theorem, and we defined what is Fourier transform and it's inverse transform, and also we rewrote the Fourier integral theorem when the signal, the non-periodic signal or the function F(x) on the full real line is even function or when F(x) is given only over 0, even your signal is given only from 0 to infinity, if you extend that as even function from minus infinity to infinity then that case you can define Fourier, so your integral theorem becomes in terms of cosines you can write it as a cosine functions and though which you have define what is Fourier cosine transform and it's inverse transform of it.

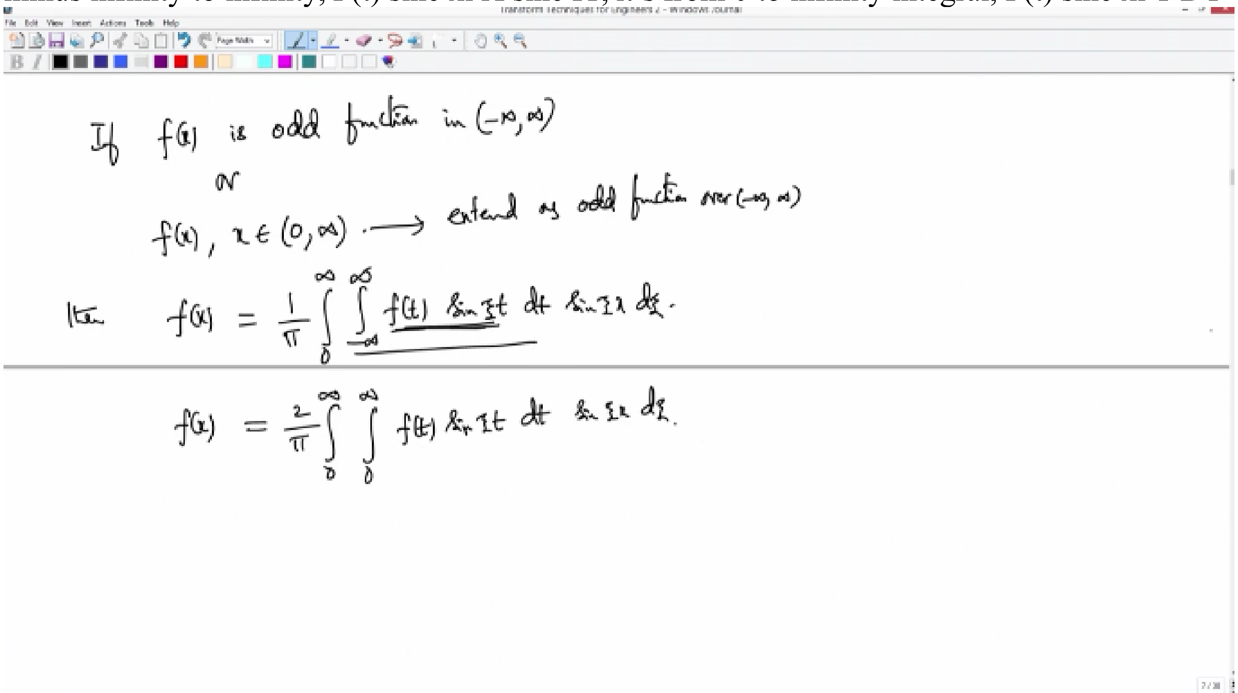
So today we will just consider, in this video we'll look at if F is even, if F(x) the signal is odd function, odd function in minus infinity to infinity, or F(x) is defined over X belongs to 0 to infinity and this we extend as odd function, if your signal is given only in the positive side you extend as an odd function over minus infinity to infinity, in that case what happens?



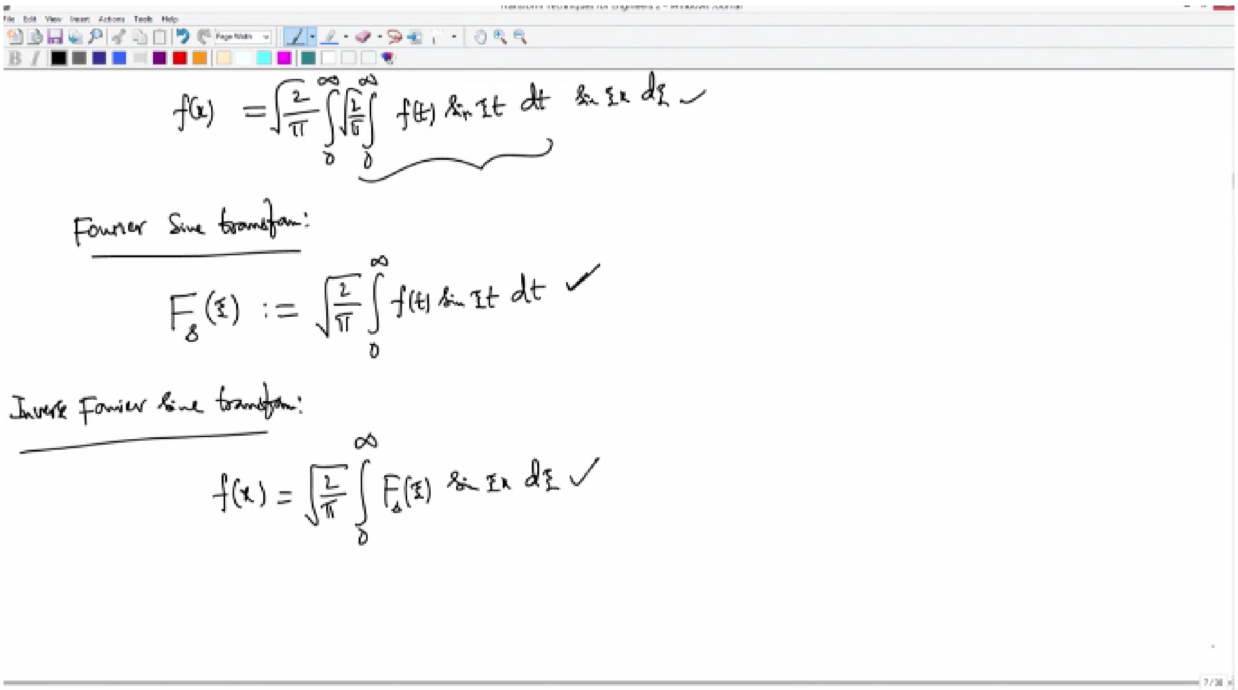
Then you look at the Fourier integral theorem, so if you look at up to here so if you look at this is true, if you look at this part if F is odd here what happens to this part, if F is odd this DT integral F(t) into cos IT, F(t) is odd, cos IT is even function, so F(t) into cos IT, so you can do the same thing so like here, so 0 to infinity you have cos IX comes out you have minus infinity to infinity, F(t) cos IT DT D xi, so this part because F is odd function, cos IT is even function together this integrand is odd function so that makes it this is 0, so you have this part is 0, so



this contribution is only coming from the second term now if F is odd function, so if F is odd function we can write from the Fourier integral theorem we can write F(x), or its average part is actually 2/pi 0 to infinity minus infinity to infinity, what you have 1/pi, so 1/pi it's not 2/pi so minus infinity to infinity, F(t) sine xi X sine IT, it’s from 0 to infinity integral, F(t) sine xi T DT



sine xi X D xi, now from this you can define what is called Fourier sine transform. Definition is, so we define now FS, this is the definition of xi, S transform is over a function signal F(x) defined over 0 to infinity, and this is by definition is if I take it from here this part with root 2 pi, you split root 2 pi here and we put 1/root 2 pi, so 2 pi by, I split it into root 2 pi into root 2 pi, so then you can take this as our definition 2/pi, 0 to infinity F(t) sine xi T DT, so this is your Fourier sine transform and Fourier inverse transform, inverse Fourier transform, inverse Fourier sine transform, this is you can get back your signal that is F(x), that is directly from Fourier integral theorem, so what you have is 2/pi so this whole thing is integral 0 to infinity this whole thing is your FS, FS(xi) and you have sine xi X D xi, so this is your inverse Fourier transform, so if this is your definition this is the fact from Fourier integral theorem, so

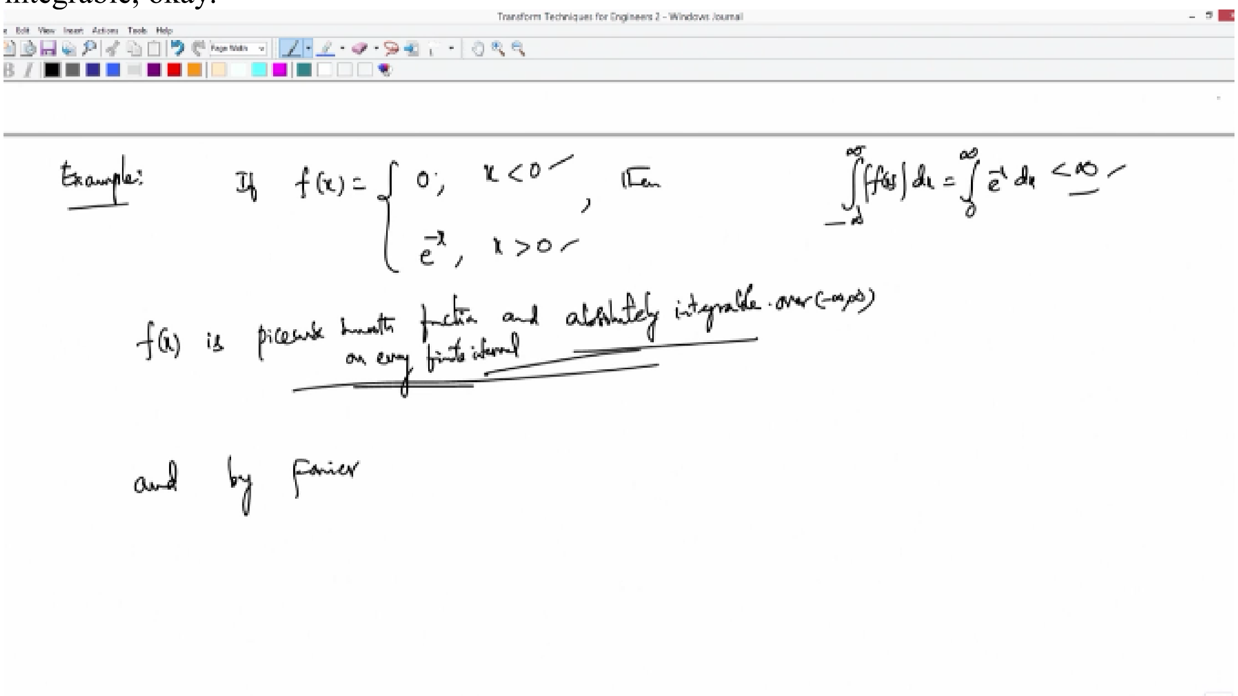


inverse transform is always like earlier, even for Fourier series you have defined only Fourier coefficients and the fact that every signal you can decompose in terms of, you can compose in terms of these coefficients and fundamental signals that is actually inverse transforms.

So here on analogous to that what you have is, you have a definition of Fourier sine transform and Fourier integral theorem is actually giving you this inverse transform, okay, so like this you can define your Fourier transform, and Fourier cosine, and Fourier sine transforms, okay, so this what, again I stress here that Fourier integral theorem is a cosine part or sine part, Fourier integral theorem is only it's not rigorous proof that I have given, so we'll eventually see the rigorous proof of that when F is the signal is non-periodic function or the signal is piecewise smooth function, in that case we'll actually show the Fourier integral theorem in a rigorous way and then we can mathematically, that gives the completeness that we can believe Fourier integral theorem completely, you can understand that proof in the later videos.

So we'll do some examples now, so if F(x) is, if you take it as 0, if X is negative such a function is you have to choose this E power –X, X is positive so if you are using this non-periodic function which is defined everywhere because you are dealing with the integrals in the Fourier transforms you are dealing with the integral so at one point you need not define, it doesn't matter what you define, okay, but if you define here some value at X = 0 so that should be, that we will see exactly what it is here, okay, so we will see at X = 0 whatever you can define, okay, but then it should be on par with the Fourier integral theorem, so if at X = 0 it has to, what is the limit of F(0+) + F(0-) divided by 2, average value as limit X goes to 0 side this is E power -0 is 1, and this 1 + E power negative side is anyway 0 divided by 2 so it's 1/2, so it's value should be on an average it converges to F(x) is Fourier integral value is actually converges to 1/2, okay, so if you think that it is, if even the average value at X = 0 it has to converge, so it has to be 1/2 here, so it doesn't matter, so let us not give any value here so we will see what exactly, if it is given here is 1/2 at X = 0 we will see, okay, still it is discontinuous point, you still give the value so the Fourier series integral theorem converges to, is actually equal to F(0) at X = 0 that is the average value of this side and this side that is also, that is 1/2, okay, now you need not even define.

So if such a thing what happens to this function, then F(x) is piecewise, it is piecewise smooth function clear right, so these are in terms of elementary functions and absolutely integrable, it is absolutely integrable, so integrable over minus infinity to infinity, piecewise smooth function on every finite interval, because it is smooth anyway less than 0 and positive side, only discontinuity is 0 and that is where you have E power 0 is 1, so it is going to be a jump of as 1, that jump is 1 so it is actually piecewise smooth function every finite interval and is also absolutely integrable over full real line because integral minus infinity to infinity F(x) DX is actually equal to 0 to infinity E power -X DX which is finite, that is why this is absolutely integrable, okay.



And what you have? And you have by and by Fourier integral theorem, so these are the conditions of Fourier integral theorem, intuitive proof we have given in the last video by Fourier integral theorem what we have is F(x) or at discontinuous point F(x+) we have only 0 point here, okay, X+ or F(x-) divided by 2 which is equal to, what do you have? So you have 1/2 pi minus infinity to infinity, minus infinity to infinity F(t) into E power - I xi T DT this is your Fourier transform 1/root 2 pi, 1/root 2 pi comes out and that is E power I xi X, D xi, so this is exactly from your Fourier integral theorem or Fourier transforms, okay.

Example: If $f(x)=\begin{cases}0, & x<0,\\ e^{-x}, & x>0\end{cases}$ then $\int_{-\infty}^{\infty}|f(x)|\,dx=\int_{0}^{\infty}e^{-x}\,dx<\infty$

$f(x)$ is piecewise smooth function on every finite interval and absolutely integrable over $(-\infty,\infty)$

and by Fourier integral Theorem,

$$f(x) \text{ or } \frac{f(x^+)+f(x^-)}{2} = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\left(\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(t)\, e^{-i\xi t}\,dt\right)e^{i\xi x}\,d\xi$$

So you try to put this F(t) value that is given here E power - T as X positive, so what you get is 1/root 2 pi minus infinity to infinity, and you have 1/root 2 pi this is from 0 to infinity E power -T into you have, if you combine with exponential of E power -I xi T, you have 1 + I xi DT into

and by Fourier integral Theorem,

$$f(x) \text{ or } \frac{f(x^+)+f(x^-)}{2} = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\left(\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(t)\, e^{-i\xi t}\,dt\right)e^{i\xi x}\,d\xi$$

$$= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\left(\frac{1}{\sqrt{2\pi}}\int_{0}^{\infty} e^{-t(1+i\xi)}\,dt\right)e^{i\xi x}\,d\xi$$

E power -I xi X D xi, okay, so this is equal to 1/root 2 pi minus infinity to infinity, 1/root 2 pi and this value is 1/1 + I xi, - of E power – T1 + I xi, and this if you do from 0 to infinity to limits, this because of E power -T this is going to be 0 at infinity, so this minus or minus plus E power 0 that is 1, 1 divided by 1/1+ I xi, so you get, when you do this you get this part into you have E power I xi X D xi, this is equal to, you can rewrite, so can write this as 1/2 pi and you have a minus infinity to infinity.

Now we can write exponential function, E power I xi X as cos IX + I sine xi X divided by 1+ I xi D xi, so how do I make this real part? Take this as a real part and imaginary part, if you do this you can write 1/2 pi minus infinity to infinity and you try to rewrite this as, you multiply and divide with 1 – I xi, so that the denominator will become 1 - I square xi square that is 1 + xi square, I is a complex number, which is I is root -1, so you have 1 - I xi, I multiplied and denominator I multiplied, numerator and denominator, so you get this one and cos IX + I sine xi X DX D xi, and this is a real part if you look at it cos xi X, imaginary part is + xi sine xi X, divided by 1 + I square D xi + I/2 pi minus infinity to infinity, I pot you have -I cos xi X -, this

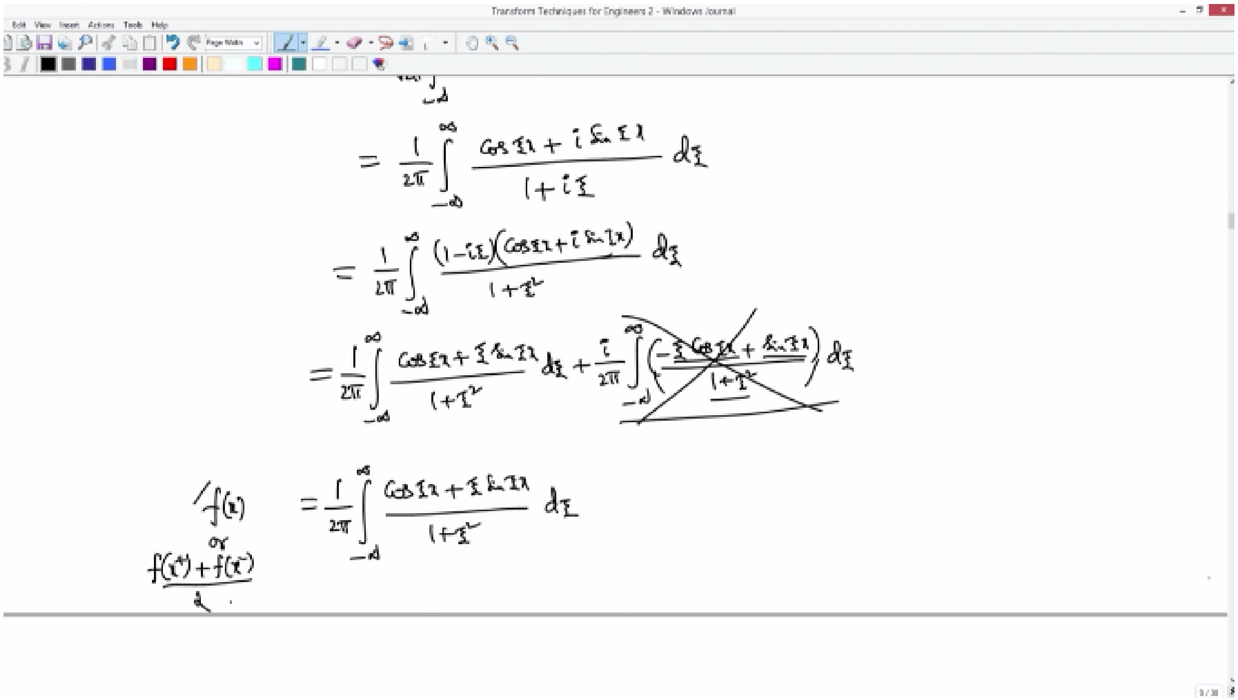
$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} \frac{e^{i\xi x}}{1+i\xi}\, d\xi$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{\cos \xi x + i \sin \xi x}{1+i\xi}\, d\xi$$

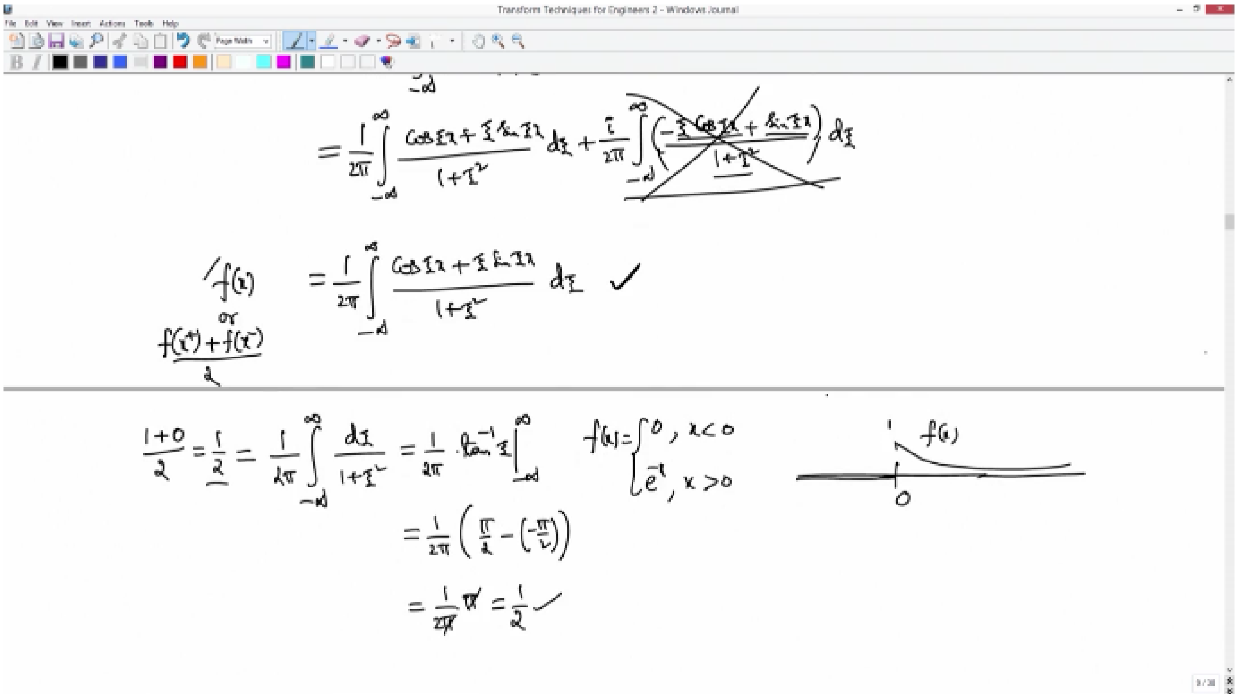
$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{(1-i\xi)(\cos \xi x + i \sin \xi x)}{1+\xi^2}\, d\xi$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{\cos \xi x + \xi \sin \xi x}{1+\xi^2}\, d\xi + \frac{i}{2\pi} \int_{-\infty}^{\infty} \left(\frac{-\xi \cos \xi x + \sin \xi x}{1+\xi^2} \right) d\xi$$

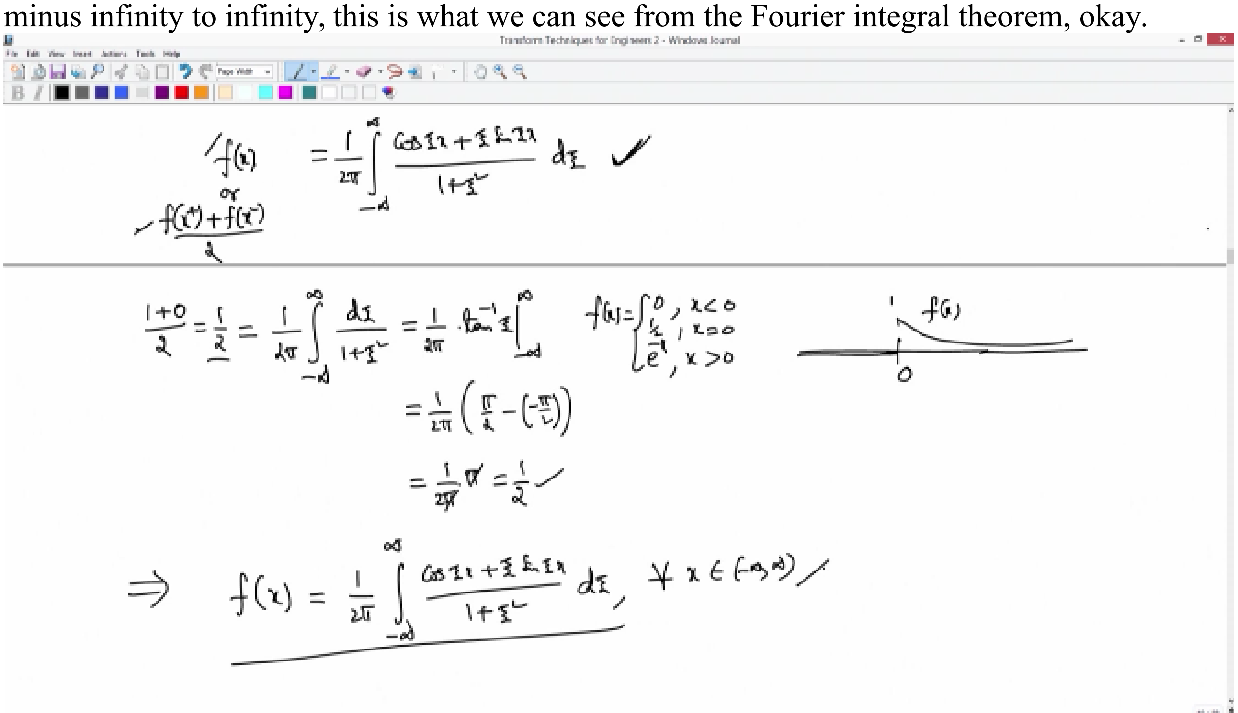
is going to be + sine xi X divided by 1 + xi square into D xi, but if you look at this second integral, this integral is xi cos xi X is odd function, sine xi X is odd function, so the numerator is an odd function, denominator is even function, so together it is an odd function, so this part is 0, so you have to have is 2 pi minus infinity to infinity cos xi X + xi sine xi X divided by 1 + xi square D xi is your F(x) or if it is a continuous this is F(x), otherwise this average value at this continuous point, right.



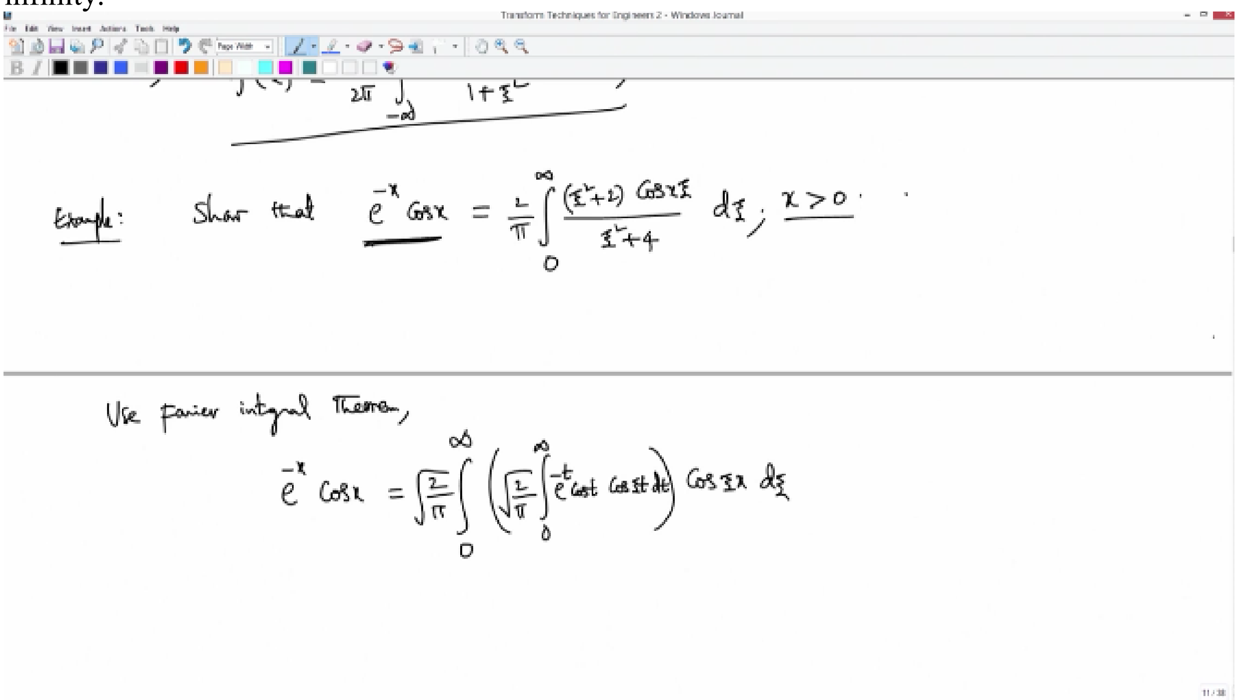
So if you choose xi = 0, xi = 0 that is a singular point for this function, right, what is your function F(x) is 0 if F negative and E power –X, and X positive so you have a 0 is a singular point, this is 0 and what you have is E power –X, at X = 0 it is 1, this is the jump here so this part, okay, actually going to 0 so it's coming to 0 like this, this is a function we have asymptotically it goes to 0, this is your F(x), so 0 is the only discontinuous point with a jump so you because of that you consider a left hand side F(0+) that is 1 + F(0-) that is 0, and you have by divided by 2 that is actually 1/2,1 + 0/2 which is 1/2, which is equal to 1/2 pi and you have minus infinity to infinity, I put X = 0 cos xi X is 1, and if you put sine xi X, X = 0, this is 0, so you have D xi/1 + xi square, okay, so you expect this to be 1/2, is this really true? That means this is, if you actually calculate this integral you have 2 pi pi, and this is tan inverse xi from minus infinity to infinity, so what you get is 1/2 pi, this is pi/2, so this is tan inverse infinity is sine pi/2, so this is pi/2-, tan inverse of minus infinity is – pi/2, so it is actually 1/2 pi times pi, so it is 1/2, so this is actually matching you can easily see that, okay. So this is actually true, so Fourier integral theorem for this example you have seen that is true.



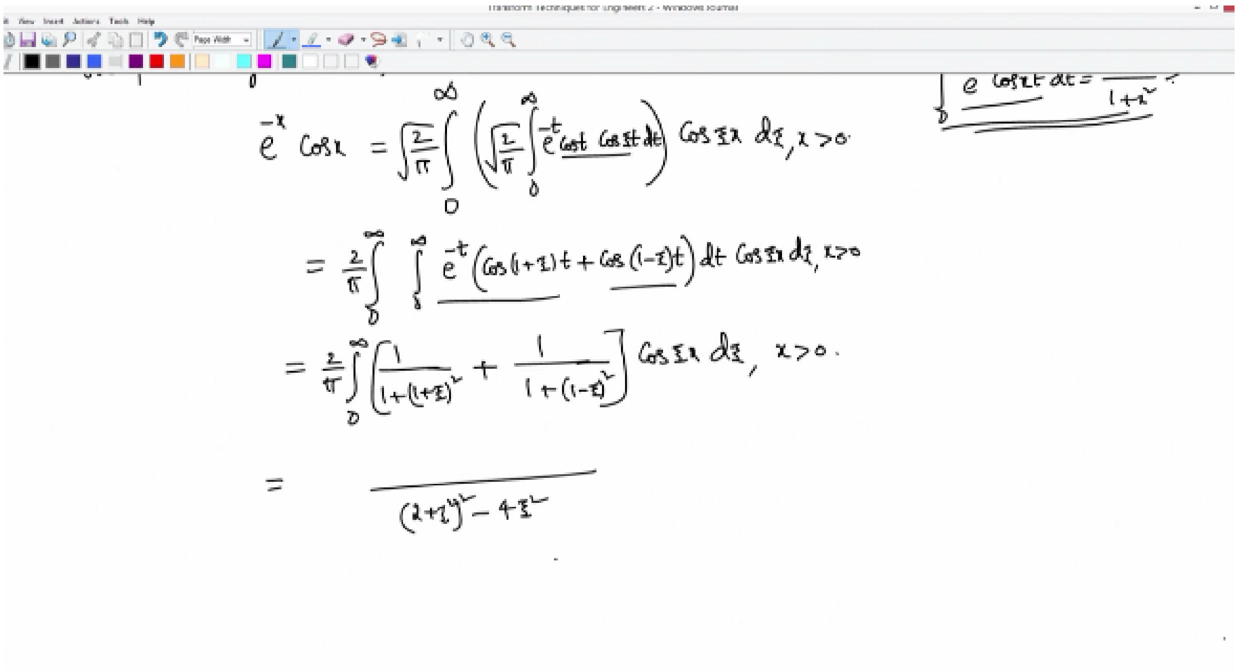
So now you can define your function like this at X = 0, it is actually 1/2,if you define like this then you don't have to consider this part, so it actually converges to F(x), so then if I define like that F(0) is 1/2 at X = 0 then you can simply safely write if any other point it is anyway converges to, that value is converges to F(x) so you can say that F(x) is actually equal to 1/2 pi minus infinity to infinity cos xi X + xi sine xi X/1 + xi square D xi, this is true for every X from minus infinity to infinity, this is what we can see from the Fourier integral theorem, okay.



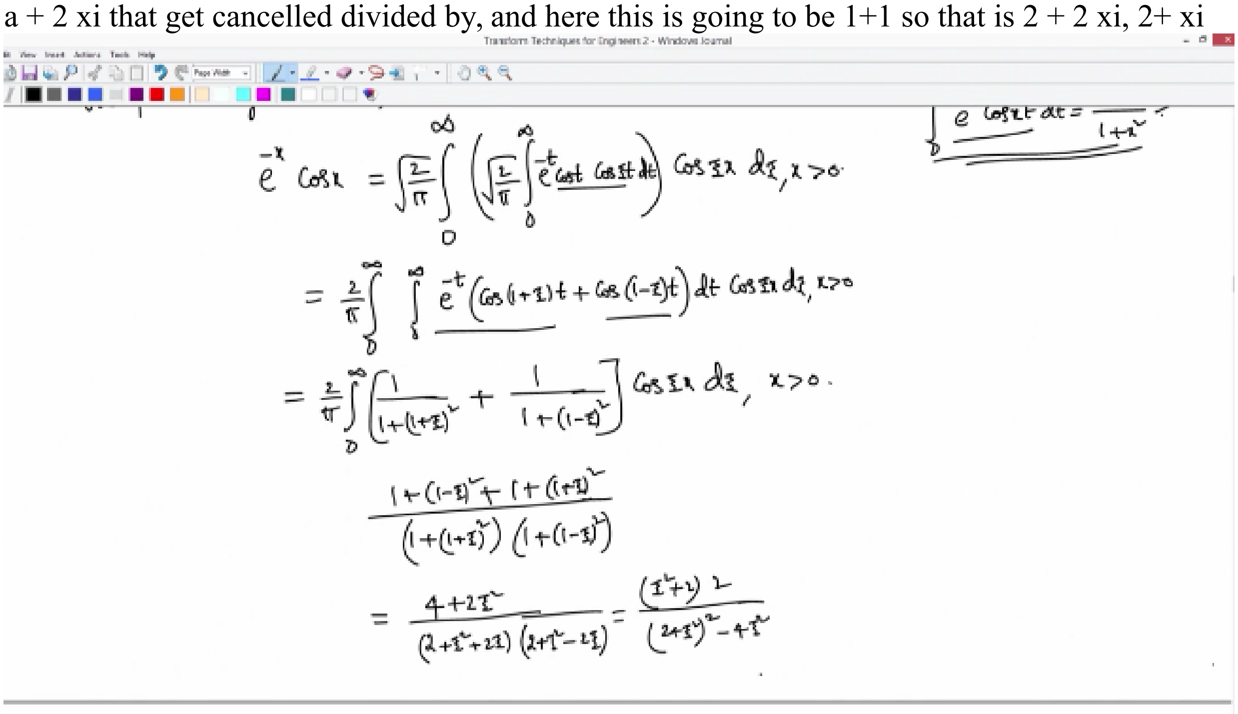
In the same way we can also, we'll do some more example, in this case we will try to show that exponential E power -X cos X, so that this is actually equal to 2/pi, 0 to infinity, xi square + 2 cos xi X xi divided by xi square + 4 D xi, can you show that this integral value is equal to E power -X cos X, so we'll use again, we'll use, so you can clearly see that this is a continuous function so this is true for every X in minus infinity to infinity, again we will try to use Fourier integral theorem for this function, okay, if you do that use Fourier integral theorem for this function or full real line, or because this function is, yeah if you try to do that so if you use that, you want, if it is only for positive, positive side only one side if you want to use you can use Fourier cosine because you want only from 0 to infinity, so you expect, you want to use a Fourier integral theorem in terms of cosines that means it is valid only for X positive, so you cannot give X negative side, okay, for all X you cannot use, so if I give X positive I can safely write the Fourier integral theorem in terms of, is only positive side, so clearly E power - X cos X is only one side is defined, -X cos X is equal to you have 2/pi, is actually you have like this, you would remember like this, you have 2/pi, this is inverse transform, now they have a transform cos xi X D xi, that's what it is, so you here inside 0 to infinity F(t) that is E power -T cos T cos xi T DT this is by Fourier integral theorem for every X positive, so this is 2/pi 0 to infinity.



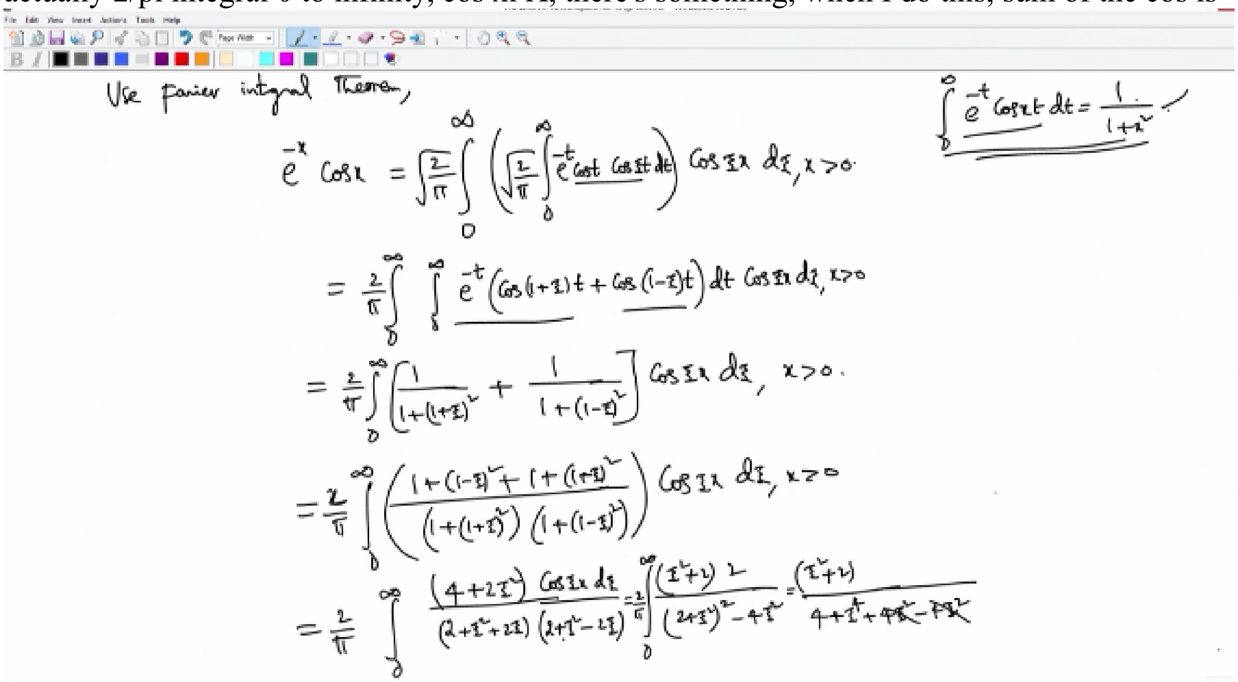
Now if you look at this integral 0 to infinity E power -T and this cos together, cos T into cos xi T you can write it as cos sum + cos difference, so you have a cos 1 + xi T + cos 1 – xi T, xi – 1 cosine doesn't matter both are same, DT cos xi X D xi, this for X positive, so you have 2/pi 0 to infinity, now we’ll try to expand this, this evaluate this integral 0 to infinity E power -T cos XT DT this is actually equal to 1/1+ X square, so you can do the integration by parts and try to bring back the same integral and try to, so if you use the integration by parts you can easily see that this value is this, so I'm directly writing so you have 1/1+1+ I square, X is 1+I + the other part, this part we get 1 divided by 1+1 – I whole square, this DT integral is over, so you have a cos xi X D xi for X positive.



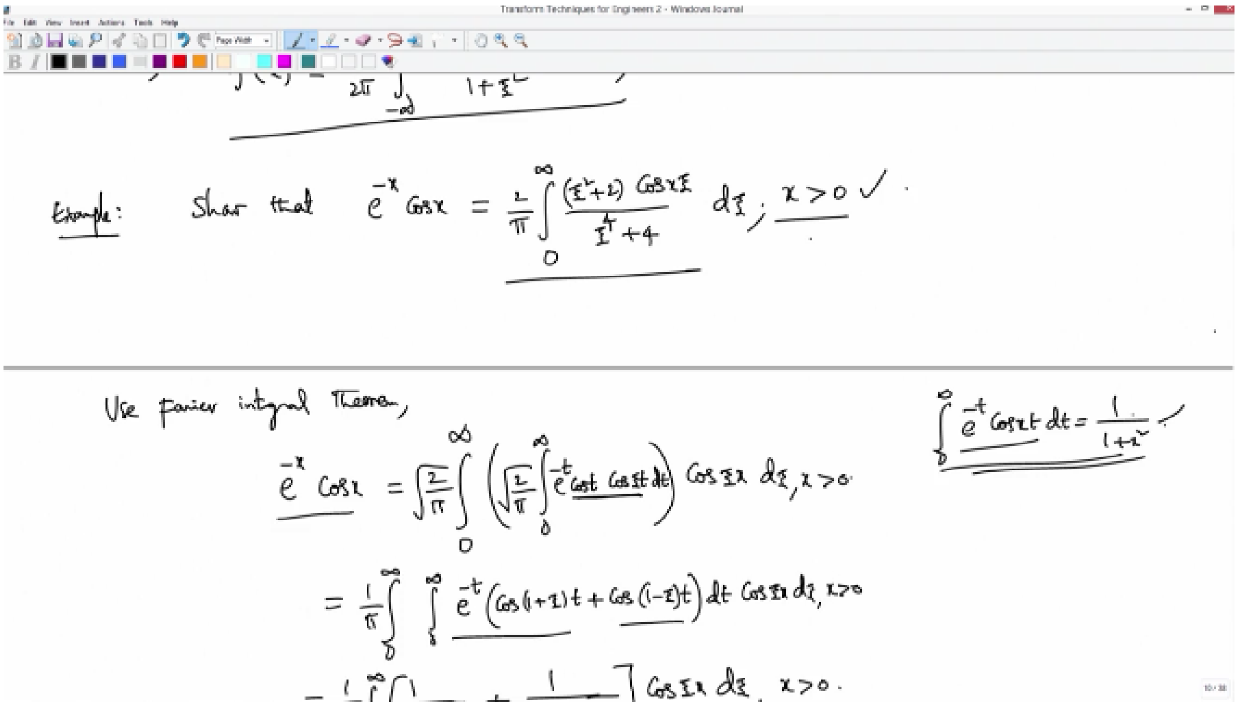
So you just do this what you get is, this is going to be 2 + xi square + 2 xi into this one 2 + xi square - 2 xi, so that is 2 + xi square whole square - 4 xi square, right, or simply take this sum this one, what you get is 1/1+1+ xi whole square into 1+1 – xi whole square, and here you get 1+ 1 – xi whole square + 1+1+I whole square, this is equal to, this is what is integral 0 to infinity, so let us work out what this is, this is equal to, this is as it is so you have a numerator will be 1 + 2 + 2 that is 4, 4 + 2 xi square, 4 + 2 xi square and you have a -2 xi here and there's a + 2 xi that get cancelled divided by, and here this is going to be 1+1 so that is 2 + 2 xi, 2+ xi



square + 2 xi and here 2 + xi square – 2 xi, so you get xi square + 2 into 2, this 2 here, we get 2 + xi square whole square - 4 xi square, so what is this one? You get 2+ xi, so A square, A+B –A square – B square, so what you end up is xi square + 2, so you're not getting here, what is this? Divided by this is going to be 4 + xi power 4 this is going to be 2 2, 4, 4 xi square - 4 xi square, yeah this is fine, so this is going to be 0.
So if you see this one so 2/pi integral 0 to infinity this is what it becomes cos xi X D xi X positive and here as well this is equal to 2/pi integral 0 to infinity cos xi X D xi, and this is actually 2/pi integral 0 to infinity, cos xi X, there's something, when I do this, sum of the cos is



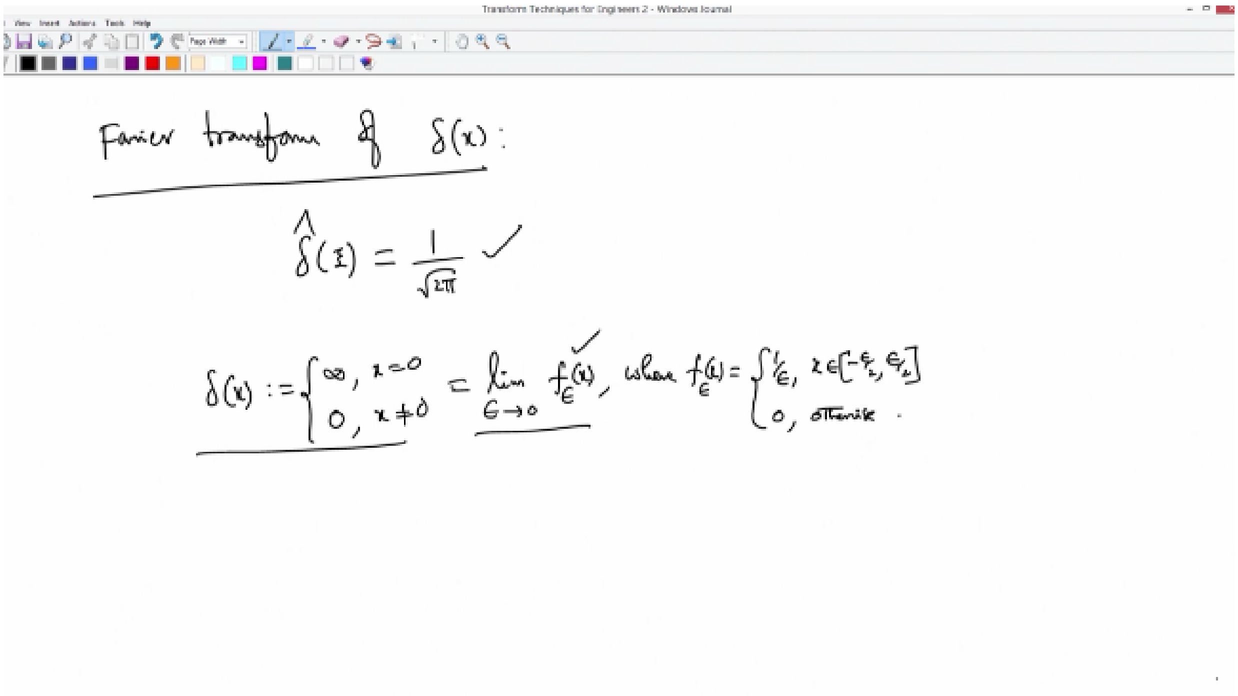
if you replace this you should have 2 times, sum of this is 2 times, so this is going to be 1 here, so this is actually 1 that is a mistake, so this is 1, and what you have is simply 1, so you finally get 2/pi, 2/pi integral 0 to infinity, xi square + 2 here anyway cos xi X D xi, so this is but 2 comes out cos xi X D xi divided by, and this is going to be, if you expand this and you can see that xi power 4 + 4, okay, so this is for X positive.
So actually, so this is something wrong in this typo, should be X power 4, xi power 4 this is a correct problem, should have xi power 4, and if it is xi square this is actually, it's not integrable, if it is xi square, if you see this xi square, xi square is like order of 1 cos XS it should be X power 4 xi, and it's 1/xi square order, so it is integrable and its value is finite here so for every fixed X, so it is X power 4, so this is a correct thing, so you have shown now that this integral E power -X cos X value is this integral by cos E, by Fourier integral theorems in terms of cosines, this you can use if your function is even function or here it is certainly it's not even function or not neither even function nor odd function, but if or in other case when it is defined only one side you can extend as an even function or odd function, so I extend as, because it is given for



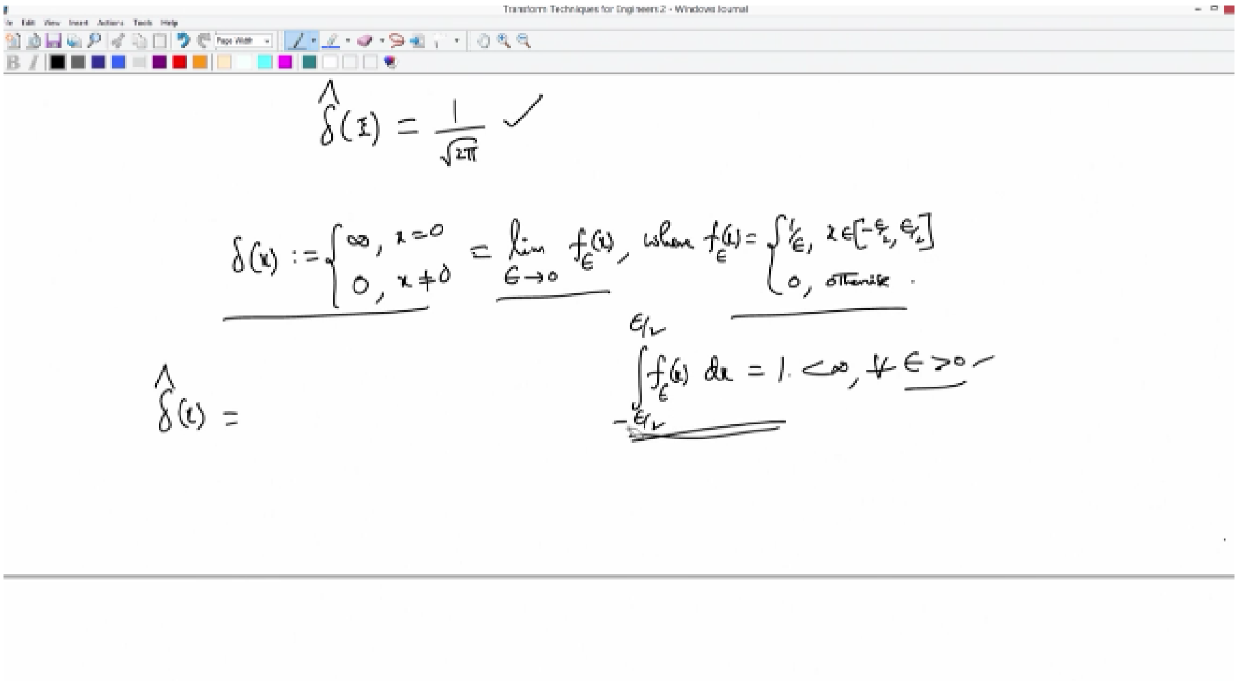
X positive you can extend this as an even function and then use a Fourier cosine integral theorem, in terms of cosines you have Fourier integral theorem, so if you use that you can show this one, that's how we have done this.

Next I will try to show what is the Fourier transform of delta function, delta function(x), what is the Fourier transform of delta function? Okay, I will show that this is actually Fourier transform of delta function, delta tilde, delta cap(x) that is delta cap(xi) is actually, once you have a cap is a function of xi, so that is going to be 1/root 2 pi, so this is what is the result.

We will try to prove this, so to show this we will just define what is a delta function again so we have seen that delta function by definition this should anyways infinity at X = 0, 0 at X is not equal to 0, so this I can think of as a limit as epsilon goes to 0 some usual functions F epsilon(x) where epsilon(x) I can think of 1/epsilon if X belongs to – epsilon/2 to epsilon/2 with closed in delta of course, and then 0 otherwise, if you think of them as like this this is your definition, and because why it is delta function is not a function? You can think, this is given as a limit of usual functions F epsilons which are well-defined piecewise smooth functions.



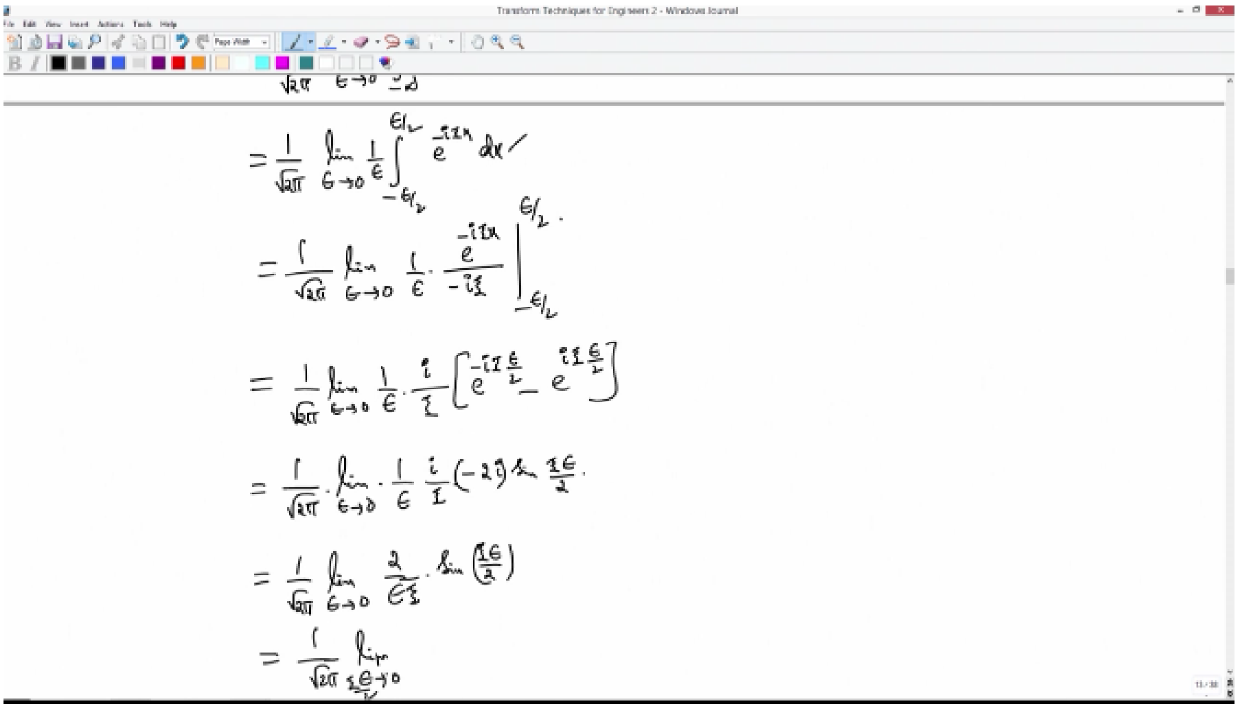
Now I can calculate what is my Fourier transform(x), the limit of this Fourier transform so you have, because this function is uniformly continuous functions or minus, or as epsilon so you fix your epsilon as 1, as epsilon goes to 0, so in the neighborhood of 0, because it's epsilon goes to 0 because this function is, you can think of epsilon = 1 and on which all these sequence is uniformly converge, so uniformly convergence is assured, F epsilon(x) you can look at the limit F(x) converges to, F epsilon converges to delta function, this convergence is not, so you cannot simply and you can easily see, so how do I do this? This integral and you see the way we have defined this integral value DX is always equal to 1 minus infinity to infinity, because of this I can define this integral, integral is finite for all values, for every epsilon, so this is finite for every epsilon positive, so in that sense you have convergence is FN(x), so for all values of so, this is minus infinity to infinity or minus epsilon/2 to epsilon/2 or which this integral is already is always 1, and what is this uniform convergence, so is the question, so FN(x) or just, will just



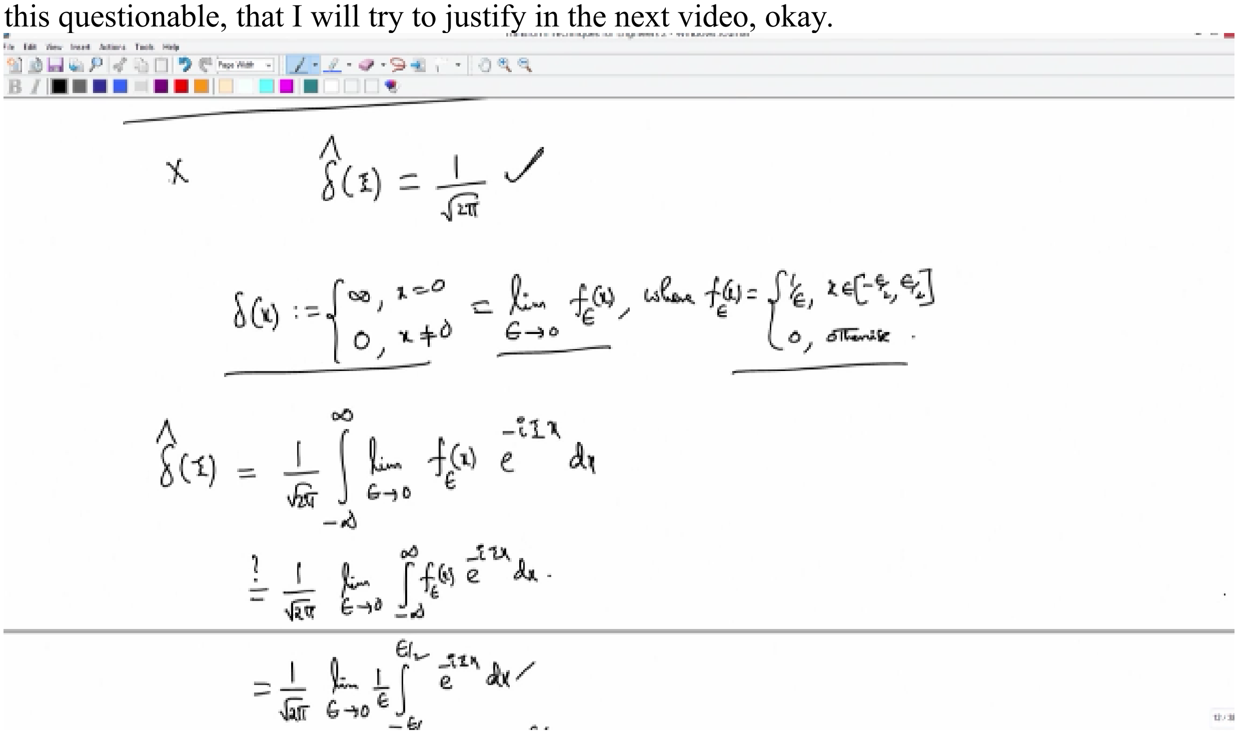
for later anyway, so we still have a question how whether you can do this, so this cap the Fourier transform of this delta function is equal to the limit, so what we do is 1/root 2 pi the definition 1/root 2 pi you have this integral minus infinity to infinity, limit epsilon goes to 0, F epsilon(x) this is my delta X into E power -I xi X DX, that is my definition of Fourier transform.

So this is 1/root 2 pi, you think of, you can bring this limit outside, and now we calculate this F epsilon(x) E power - I xi X D xi DX, if you do this 1/root 2 pi, so this is what is the question, right. So in this case this limit when you can allow this limit to go inside this integral, provided this convergence is uniform, so this is F epsilon(x) is always bounded with 1, right, so because 1/epsilon, no, no, no this is, so we'll see, we will see the justification anyway later, so this is this, this is still question, so assume that you can do this, so 1/root 2 pi and then you have this, this limit epsilon goes to 0, and this you try to expand, so this is actually given as minus epsilon/2 to epsilon/2, F epsilon is 1, right, that is 1/epsilon, so 1/epsilon E power -I xi X by, 1/epsilon comes out DX, so this is 1/root 2 pi, so limit epsilon goes to 0, 1/epsilon times now this integral is by -I xi, and you have E power -I xi X from epsilon/2 to epsilon/2, so this is 1/root 2 pi limit epsilon goes to 0, 1/epsilon I/xi, now I put I/xi, and this is E power, you're integrating with respect to X so you have –I xi epsilon/2 - E power –I xi, this is –epsilon/2, so you have, this is going to be + I xi epsilon/2.

So what you get is 1/root 2 pi limit epsilon goes to 0 into 1/epsilon, I/xi this is going to be -2I sine xi epsilon/2, so this is 1/root 2 pi, limit epsilon goes to 0, 1/epsilon so this you have, this is what you have, so it's going to be 2/epsilon xi times sine xi epsilon/2, this is 1/root 2 pi this

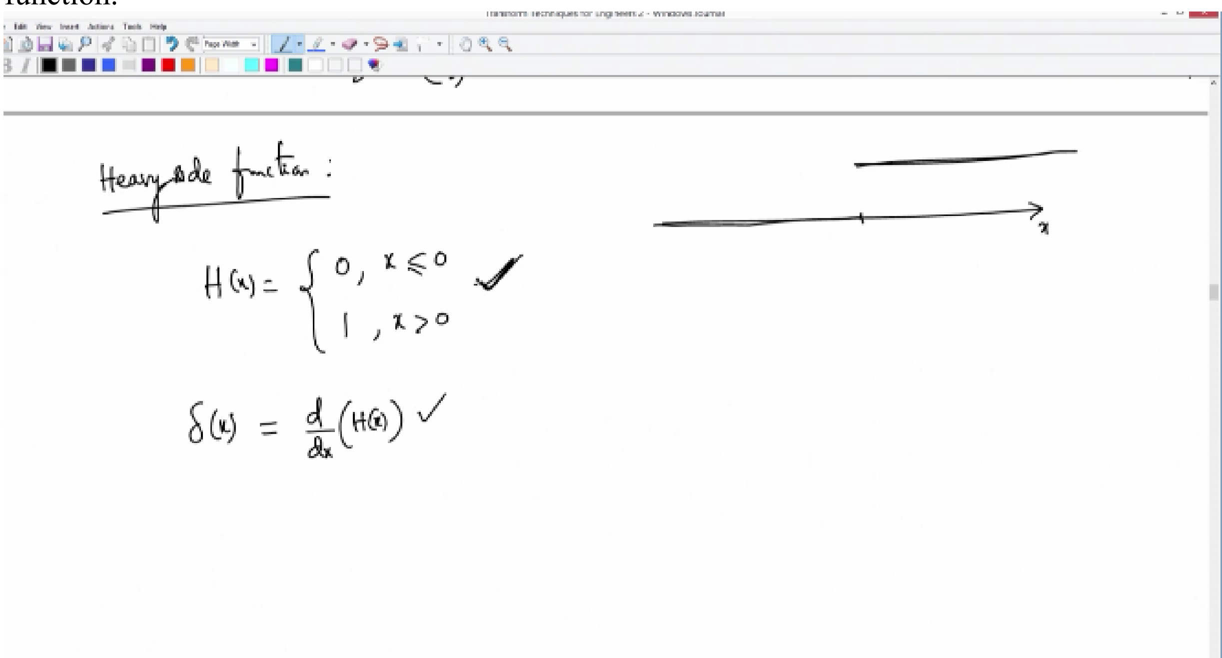


limit epsilon goes to 0 or xi epsilon/2 also goes to 0, what you get is sine xi epsilon/2 divided by xi epsilon/2, and this is sine X/X, as X goes to 0 that is 1, so you have 1/root 2 pi, so only justification that you have to do is here, how you can take this limit from inside to outside, so this questionable, that I will try to justify in the next video, okay.



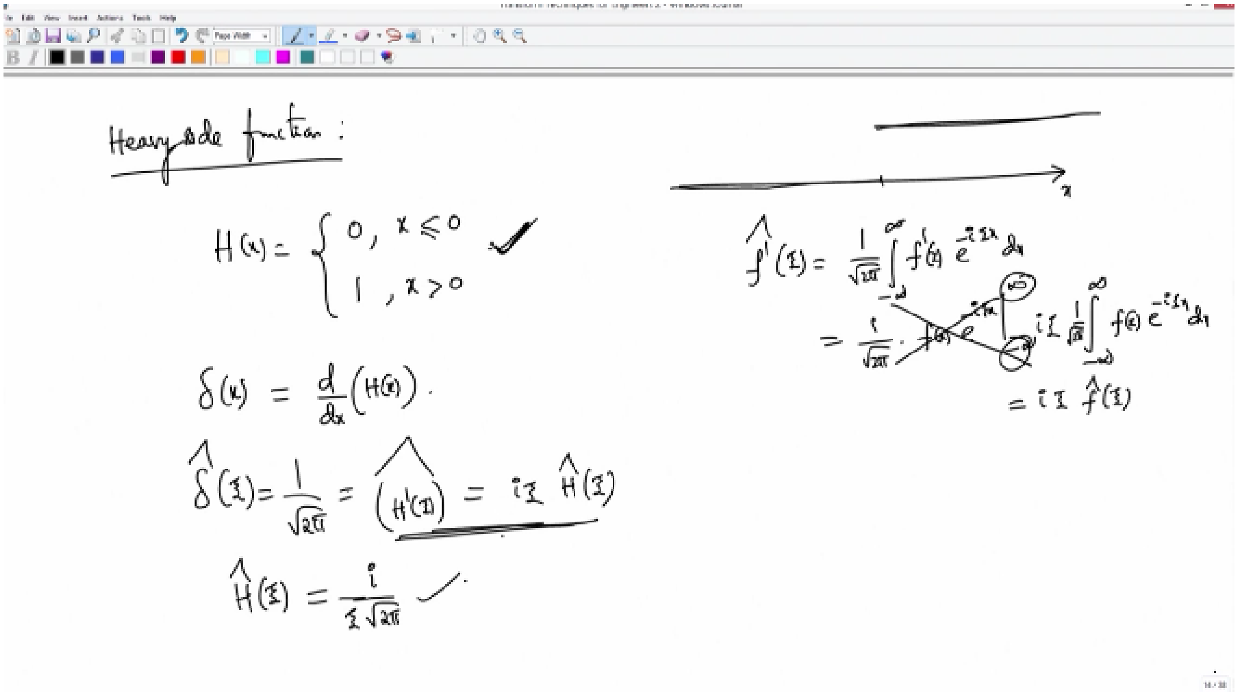
So what you have shown is if you take this delta function you have shown that this Fourier transform of a delta function is 1/root 2 pi, okay, so this is what is the result.

And also in the next video what we see is if you define your heavy side function, heavy side function I'll define, that is H(x) as it's like heavy side function is I take 0 for X negative, at the X positive side, X positive side, up to here it is 0, and when it comes is jump is 1, that is going to be at every positive side is 1, so that is heavy side functions, so 0 if X is negative, and it is 1 if X is positive, okay, such a thing if you take that is called a heavy side function. What we show is delta function that we have defined is actually equal to the derivative of D/DX of heavy side functions, see this derivative so you can see that this is not differentiable at X = 0, so you can think of, so derivative is this side, in a generalized sense we have to define, there's the derivative of a heavy side function, we will try to give you a glimpse of what this is, okay, in a generalized sense, in a generalized functions that is a delta function is one such example you can differentiate this delta any number of times, okay, so like that you can differentiate some sense, you can make some sense mathematically so in that sense you have a non-differentiable function which is heavy side function which you differentiate, what you end up is a delta function.



First of all this is only function, this is the only regular function, this is the heavy side function is a regular function you differentiate this itself it's not legitimate because it's not a differentiable function, so something which is not differentiable function you are differentiating, so that means it should be a generalized function it's not a usual function, so what is the derivative, so that you comes out to be delta function which is a generalized function.

I'll try to give you, I'll try to give a brief account on this how this delta function is derivative of the heavy side function, and we know that this Fourier transform this is 1/root 2 pi, so the Fourier transform of this derivative that is Fourier cap of heavy side function dash(x) okay, dash(xi), this is 1/root 2 pi but I know that this is actually equal to - I xi, if you calculate it is going to be – I xi, H cap(xi), so in that if you do this what you see is heavy side function Fourier transform is, Fourier transform of heavy side function you will see that 1/root 2 pi divided by, so this is going to be I/xi, I/xi so something like this you will try to see exactly, we will see how it is so, this also we will see. If F cap, F derivative if you want to have what is this one? This is actually 1/root 2 pi minus infinity to infinity, F dash(x) E power -I xi X DX, you do the integration by parts 1/root 2 pi F(x) E power -I xi X minus infinity, assume that is integrable function F(x), absolutely integrable function so it has to, at infinity it has to go to 0, so you end up -1/root 2 pi minus infinity to infinity F(x) and you get minus minus plus this is going to be I xi, F(x) E power –I xi X DX, so this is I xi F cap(xi) that is, so you have a plus sign here, so that makes it I xi 1/I xi, so it's going to be –I, this is what you may end up as heavy side function whose of Fourier transform.



We will see, I will try to give you what this all about in the next video, okay, we'll see in the next video with many examples and how to calculate the Fourier transform of this, these functions, heavy side function at least delta function you will have to know, because these are very important for generalized functions, so we will use only these generalized functions either derivative of H(x), heavy side function or delta function whose Fourier transform hold so that we can use them in when you apply the Fourier transform to the, in the applications, when you try to apply the Fourier transform to boundary value problems in partial differential equations or some differentially, somewhat really problem when you're trying to solve you may end up using this Fourier transform on the heavy side functions or delta functions, that's where it is useful. So we will see all that in the next video. Thank you very much.
[Music]

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Funded by

Department of Higher Education

Ministry of Human Resources Development

Government of India

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