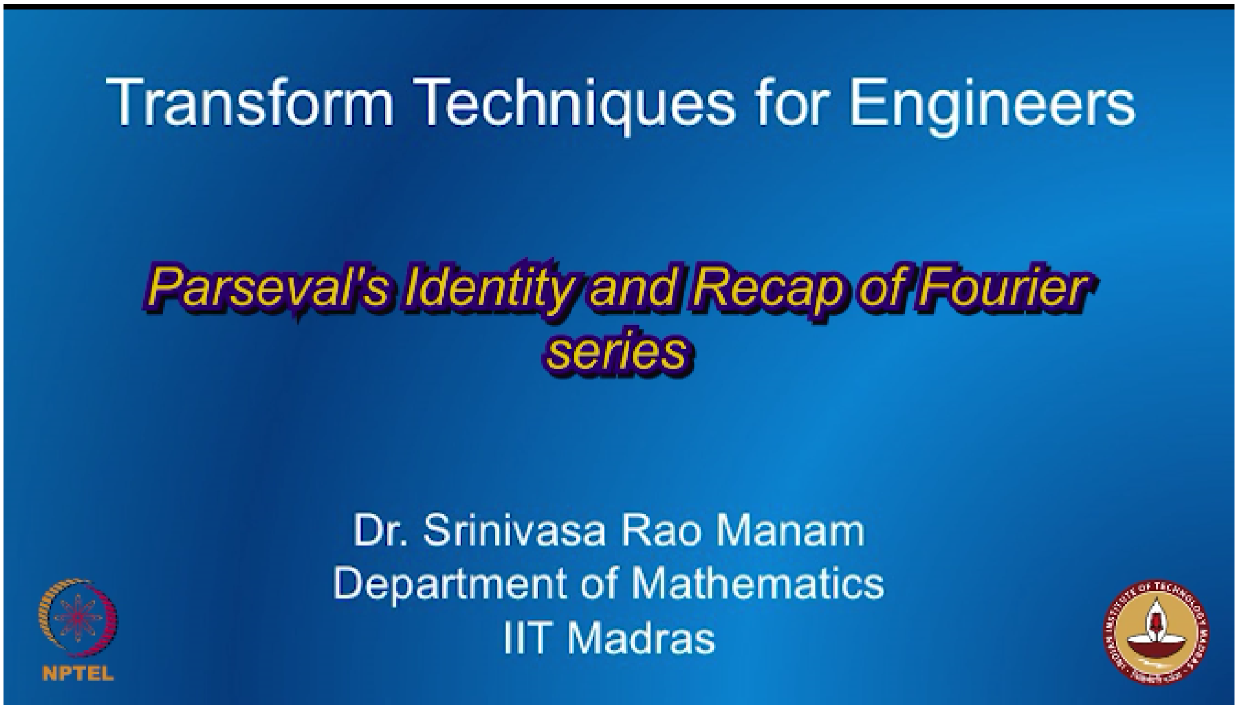
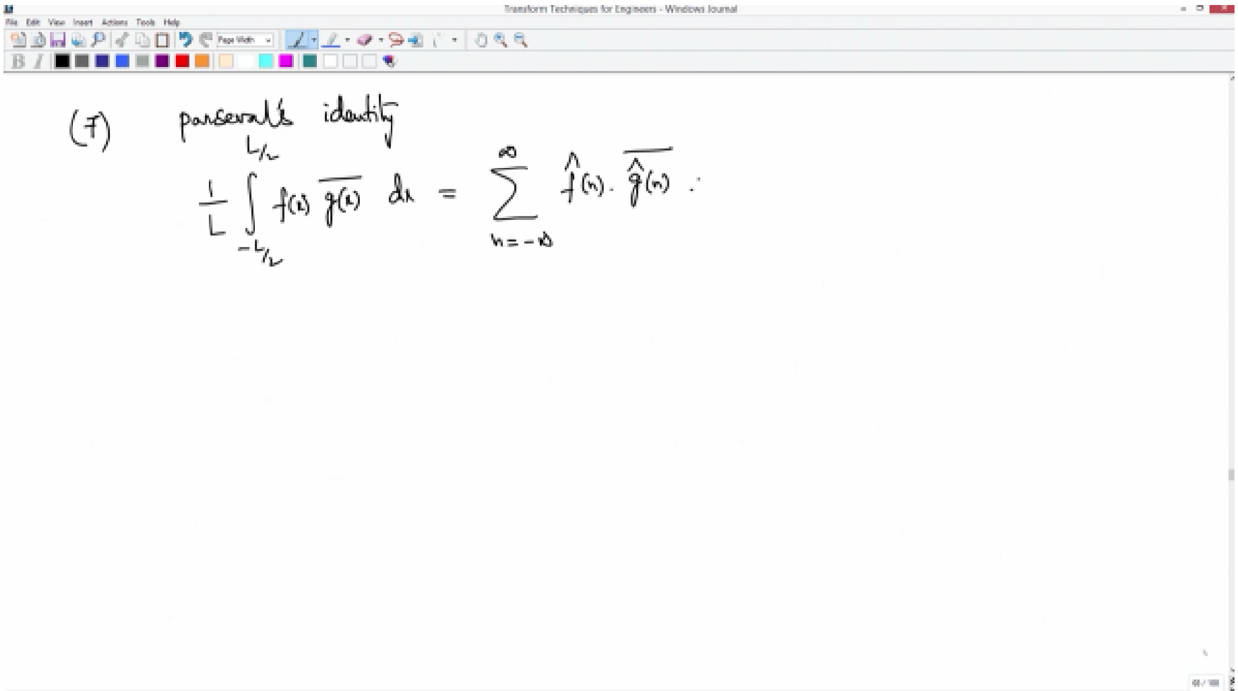
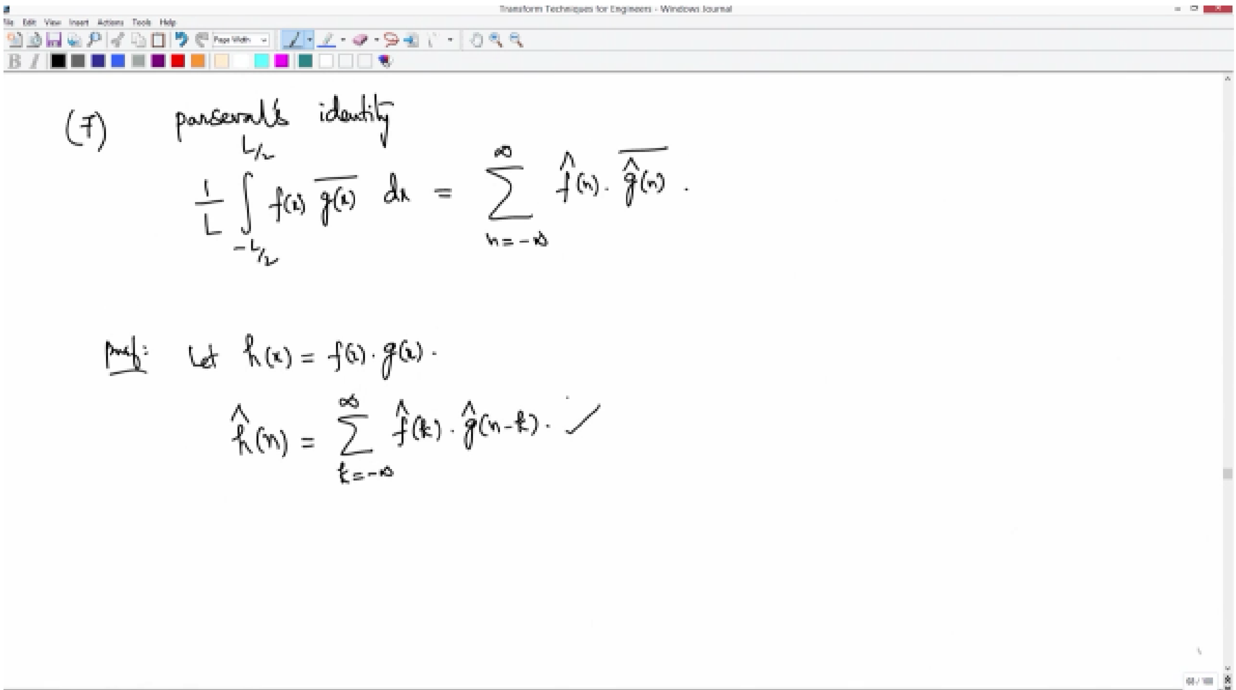
NPTEL
NPTEL ONLINE COURSE
Transform Techniques for Engineers
Parseval's Identity and Recap of Fourier series
Dr. Srinivasa Rao Manam
Department of Mathematics
IIT Madras



Welcome back, last video we have seen certain properties of Fourier transform of a periodic signal and we will have one more property, that is just the equality we can derive if you are given a two such functions, two time signals if you are given we will just write that is called Parseval's inequality.



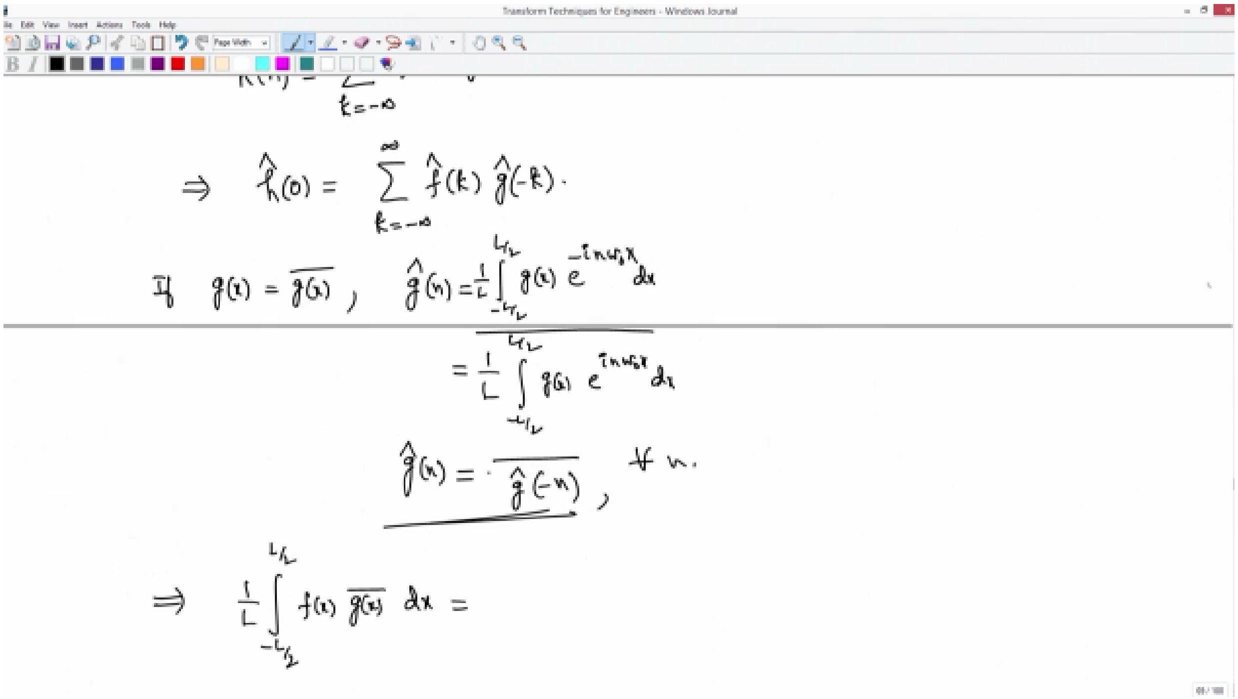
So property number 7, which is Parseval's inequality, Parseval's identity, so that is L/2 to L/2 F(x) G(x) bar DX is equal to N is from - infinity to infinity this you can get F cap(n) into G cap (n) bar, so this is a Parseval's identity, so you will know its importance when we do the Fourier transform on non-periodic function, that is a function defined a signal over a full real line, so here we do, so what is a cap denotes the Fourier transform of a periodic signal both F and G are piecewise smooth functions, so to prove this we consider H(x) be F(x) into G(x), so we already know if we are given a product of functions, what is its Fourier transform? So H cap(n) is actually equal to you get this as a summation from – infinity to infinity, F cap(k) into G cap(n-k), we have seen as a property number earlier, property number 4, I think 4, right, so property number 5 that you have seen this equality.



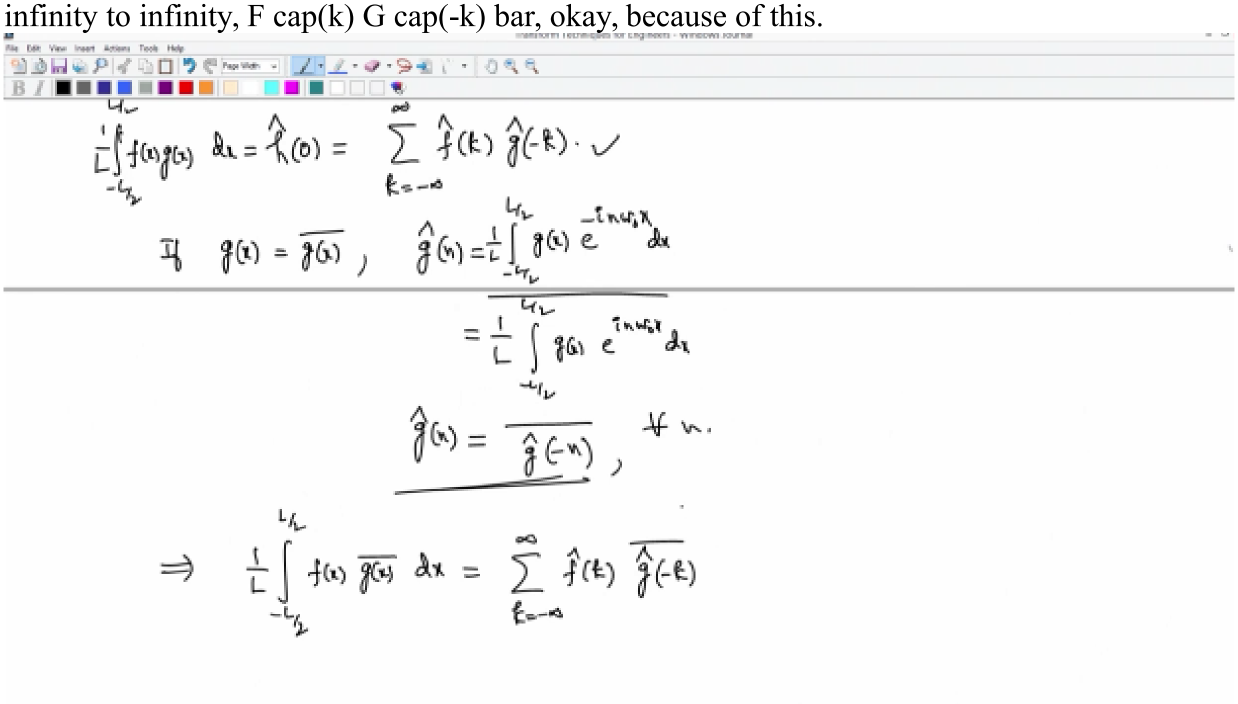
Once you look at that, once you have the product of two functions which Fourier transform is given as convolution product of these sequences F cap and G cap.

Now what I do is here I choose N = 0, so H cap(0) will be equal to, K is from - infinity to infinity F cap(k), so once you choose this N = 0 and you have G cap(-k), so what I choose is, if I choose G(x) if I take it as G(x) bar if I replace okay, if I replace this with the G with G(x) bar what happens to G cap(n), this is integral 1/L –L/2 to L/2 G(x) into E power - IN omega naught X DX, so this is equal to 1/L, G(x) is G(x) bar if you put E power -IN omega naught X DX, -L/2 to L/2, so this is what happens.

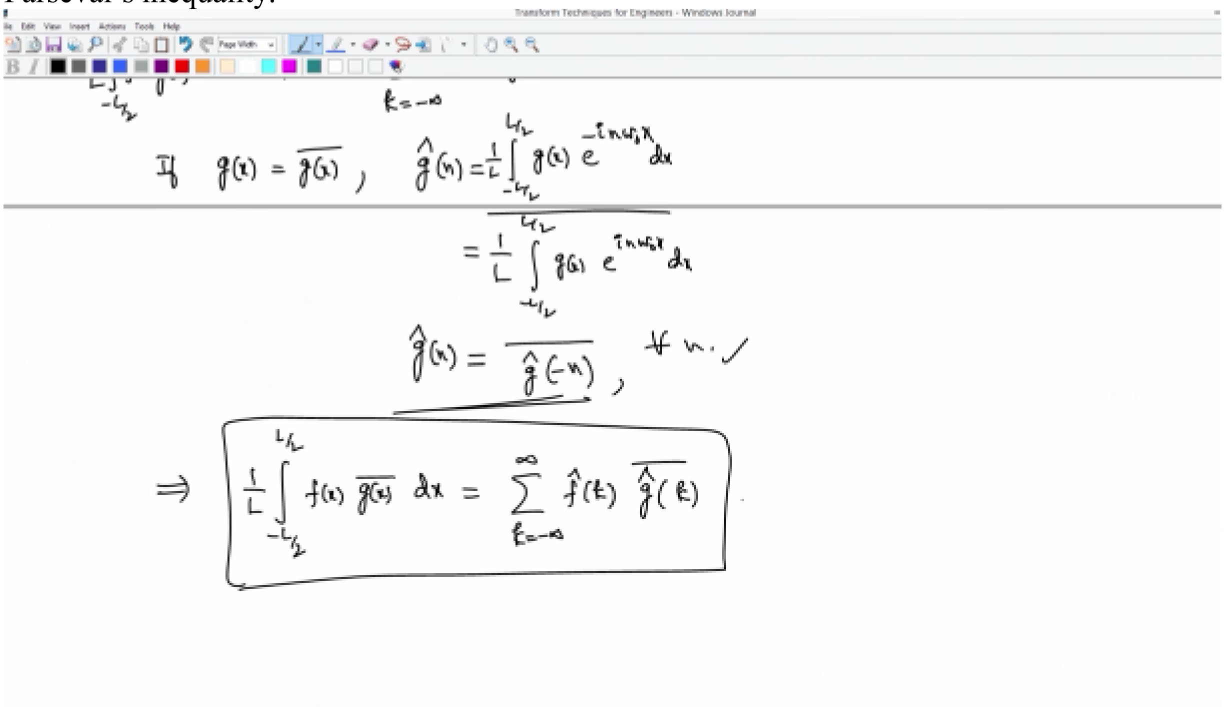
So now if you take the bar 2 bars then one more bar if you take 2 bars so if you apply the lower bar here so this goes and it's going to be plus, okay, this is exactly this is bar of G cap(-n) so what you get is G cap(-n) this is true for every N, if G(x) is G(x) bar, okay, so I choose so you have the, in your Parseval identity what you have is this relation, this integral F(x) into G(x) bar



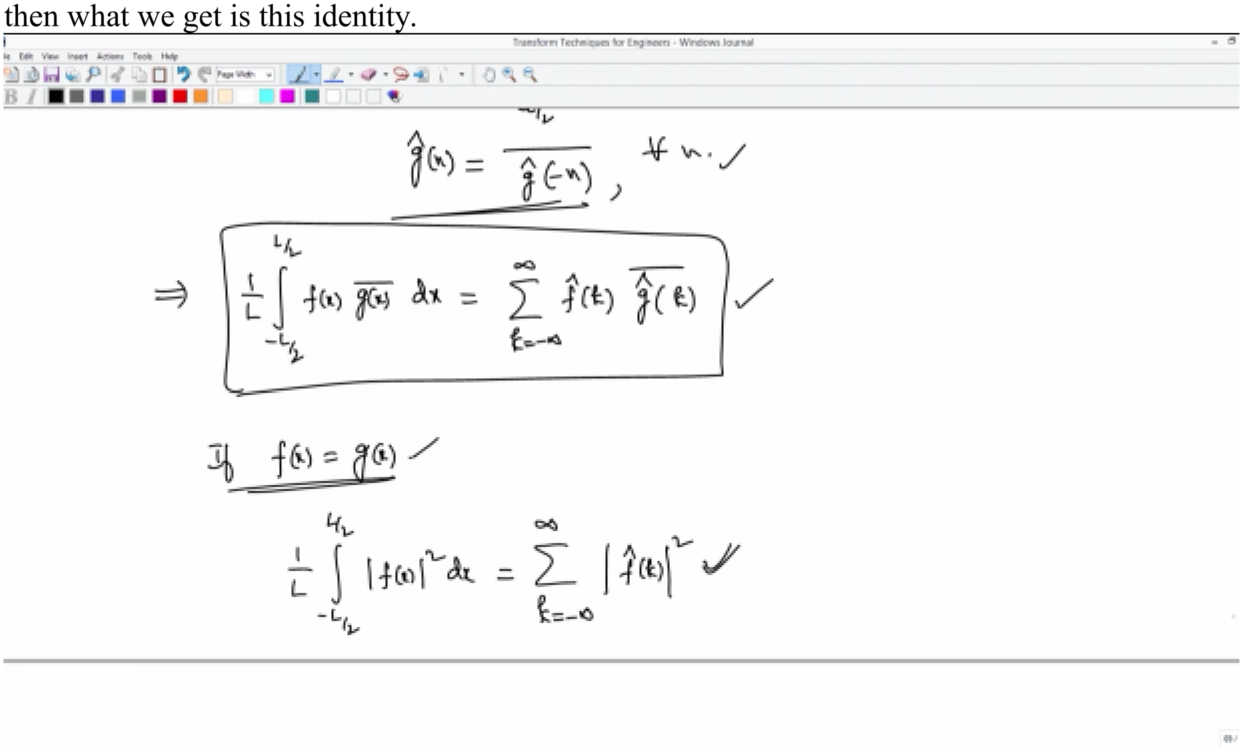
DX, this is equal to, so that is exactly your H dash(0), so what is H dash(0)? H dash of, H cap (0), H cap (0) is exactly 1/L integral –L/2 to L/2 by definition, this cap is H(0) is H(x), H(x) into that is F(x) G(x) E power IN omega naught X is 0, so N is 0, that is going to be 1, so DX so this is exactly your H cap (0), so this one is exactly equal to this from this relation you can say that -infinity to infinity, F cap(k) G cap(-k) bar, okay, because of this.



If I replace G by G dash here so what you get is, and from this you can easily see that this is this, so this is going to be a plus, right, G cap(-k) if I replace G by G bar and what you have to replace is G cap(n) is bar of G cap(-n) so I have a bar G cap of, I have already -K so it's going to be a –(-K) that is going to be a +K, so this is exactly the equality we derive, okay, so this is a Parseval's inequality.

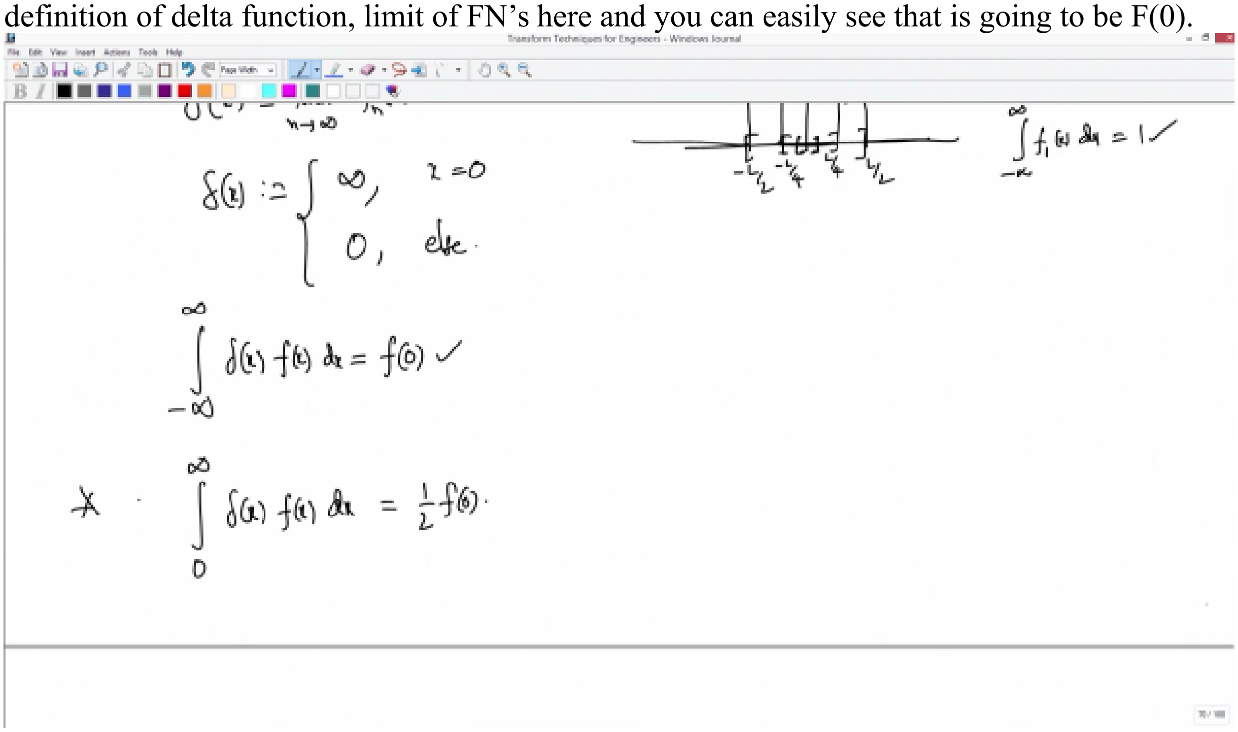


So if F(x) = G(x), if I choose both are same and what you get is 1/L integral –L/2 to L/2 modulus of F(x) square DX, this is equal to K is from - infinity to infinity, this is going to be F cap(k) square, so this is called Parseval's identity, this is also you can, this is a implication if you choose G as, or F as G(x) or G as F, if you simply consider G(x) =F(x), it's above identity, then what we get is this identity.



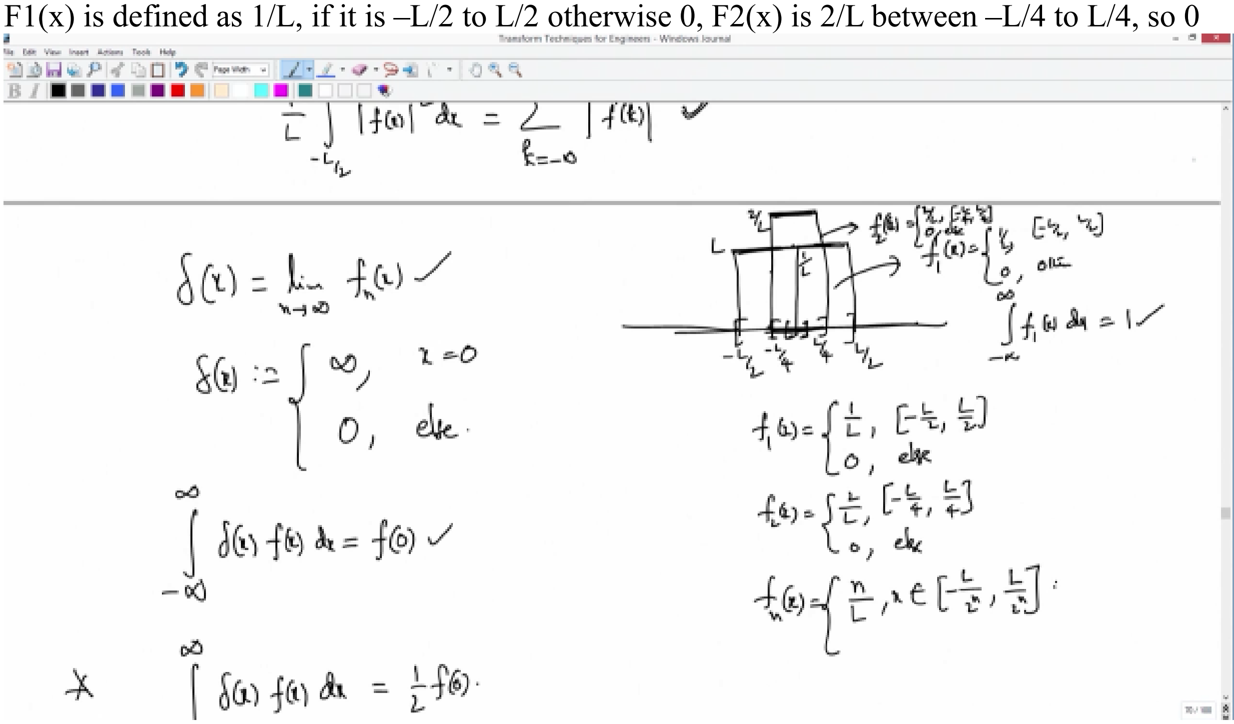
So this is one important property whose importance you will see eventually when you do the Fourier transform, so we'll just have a recap of what we have done before I do this I'll just explain we have defined what is a delta function, delta function is basically you consider all these functions you define –L/2 to L/2 some step function like this, so that whose area is 1, so that is your F1(x) is 1 between –L/2 to L/2 0 otherwise, so you define it like this that is your F1 so that whose area of F(x) DX is or - infinity to infinity is equal to 1, so that makes this height is, that height is simply L and so on, so you go on and doing like this, next time you do it from 1/2 of the distance –L/4 to +L/4, +L/4 if we define the same way so what you do is you have to increase your height that is your F(2) F2(x) that is going to be again 1, not 1 so some quantity is so that so when it is, what is that quantity? If you choose this as L then this is going to be 1 for F1.

Similarly here I have to choose L/2 between –L/4 to L/4, right, otherwise 0, if I choose like this right so this is a integral of this is L/2, so this is 1/L so this is going to be a 1/L, and this is going to be 2/L, 2/L and so on, so whose height is 2/L, whose height is 1/L and so on, okay, this height is 1/L, this height is 2/L, so you can go on doing like this finally you can get some sequence of usual functions FN(x) as N goes to infinity, so as N goes to infinity, as N goes to infinity what you see is this length of this interval is coming smaller and smaller, so going to be what you see is exactly infinity that is not going to infinity as L goes to 0, this is going to be infinity, as N goes to infinity L is going to 0, as L goes to 0 this height of this function every time is depending on 1/L is going to be infinity, so that is at X = 0, 0 everywhere else, so this is your delta function, that is how you construct, you defined it, and if I choose if I need only and then so one of the property of the delta function is - infinity to infinity, delta(x) DX is equal to, this is simply delta(x) acting on, so this is into 1 DX, okay, so delta(x) DX is actually equal to 1, or simply if you multiply delta(x) to F(x) and this is going to be, this is also we have seen in earlier video that you can easily show that this is actually equal to F(0), simply allow this definition of delta function, limit of FN's here and you can easily see that is going to be F(0).

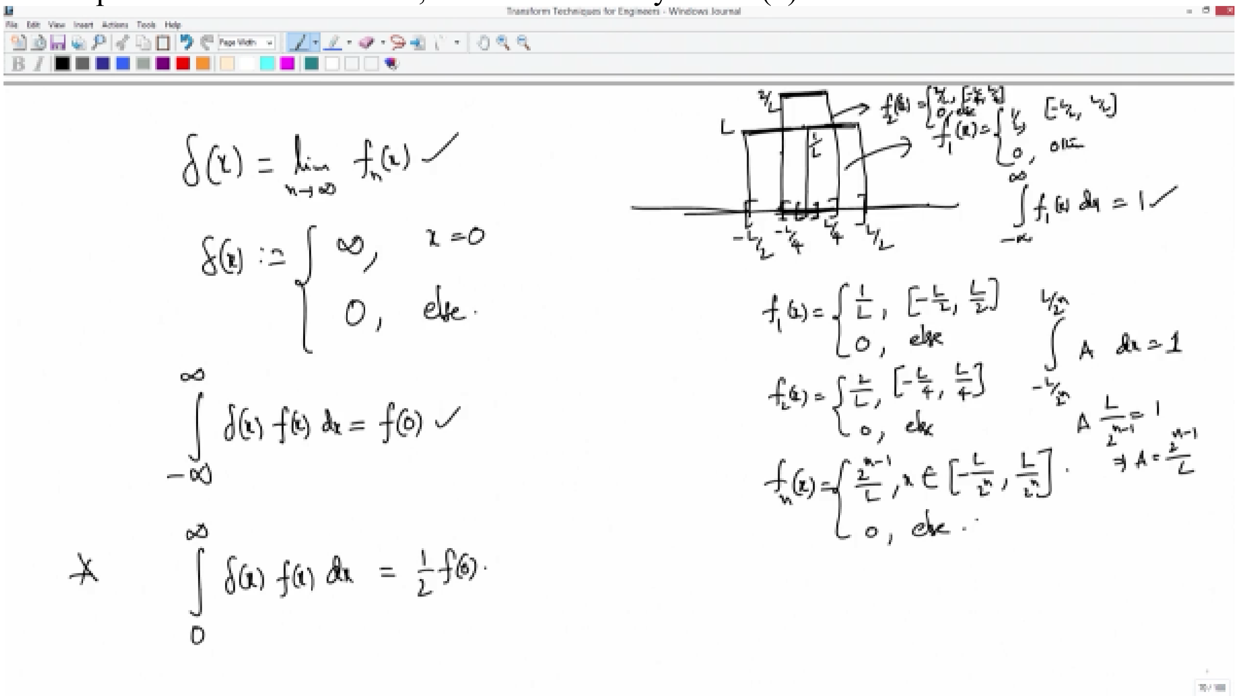


Now what we need is if I have 0 to infinity instead of - infinity to infinity, if I have a 0 to infinity I have delta(x) F(x) DX, what is the value of this? This is actually equal to 1/2 of F(0), okay, so that is the result we want to show, so how do I show this? I just apply so the LHS is, you have LHS equal to, you can allow this limit FN(x) F(x) DX, as N goes to infinity, so assume that you can do this limits outside, limit N goes to infinity, 0 to infinity FN(x) F(x) DX, okay, so if F is piecewise continuous functions, smooth functions these are converging things, so you can uniform convergence, we will allow you to take this limit outside, so that is not difficult to justify, so you have this limit outside, now you can evaluate these are the usual functions, this limit N goes to infinity what you have is it's going to be 0 to L/2 FN(x) is, if you have one is F2 is 2/L so this is like N/L, right, so you have N/L between 0 to L/2 otherwise outside it is 0 into F(x) DX, okay.

So this is equal to, so again so you have this limit the same idea that we have used and this is the integral function so N/L times, so integral 0 to L/2 F(x) DX that is L/2, so this B-A if I use the fundamental theorem of integral calculus B-A into F(c) where C is between 0 to L/2, so this L L goes limit N goes to infinity, so 1/2 comes out and this N into F(c), sorry there is a mistake so you see that F2 is 2/L but the length is every time is –L/2 square so you have –L/0 to minus, because this interval is FN of function is over defined over –L/2 power N to L by 2 power N, so that is what you have, so you have, this is from 2 power N, so L/2 power N, L L goes 2 power N, so 2 power N into F(c), so you have N divided by 2 power N F(c), now as L goes to 0, C is going to 0, so what you get is, and N divided by 2 power N is, it's going to be 1, the limit is 1, 2 power N is, N power, do this, N power N, so L/2 power N – 0 into F(c), C is this, L L goes, and what you get is, so we have to define, I think we should have -L/2 to L/2 that is your F1(x), F1(x) is defined as 1/L, if it is –L/2 to L/2 otherwise 0, F2(x) is 2/L between –L/4 to L/4, so 0



otherwise, so you have a FN(x) is the N/L, it is if X belongs to –L/2 power N to L/2 power N, L/2 power N so L/2 power N -1, so what you get is, this is not 2/N, so this is not N/L it is going to be L/2 power N + L/2 power N whose area, area is that it's going to be A into this, so this is L/2 power N-1 is equal to 1, so A = 2 power N-1/L, right, 2/N power, so if you want that area of FN(x) DX = 1 that is between –L/2 power N to L/2 power N, if you want this and what you get is, if you make it as a constant, so we have A it should be equal to 1, so area should be 1, so what is this area is? L/2 power N - 1 into A = 1, so this makes it A as 2 power N-1/L, so you have 2 power N-1/L otherwise 0, so this is what is your FN(x).



$$\delta(x) = \lim_{n\to\infty} f_n(x) \checkmark$$

$$\delta(x) := \begin{cases} \infty, & x = 0 \\ 0, & \text{else.} \end{cases}$$

$$\int_{-\infty}^{\infty} \delta(x) f(x)\, dx = f(0) \checkmark$$

$$\times \quad \int_{0}^{\infty} \delta(x) f(x)\, dx = \frac{1}{2} f(0).$$

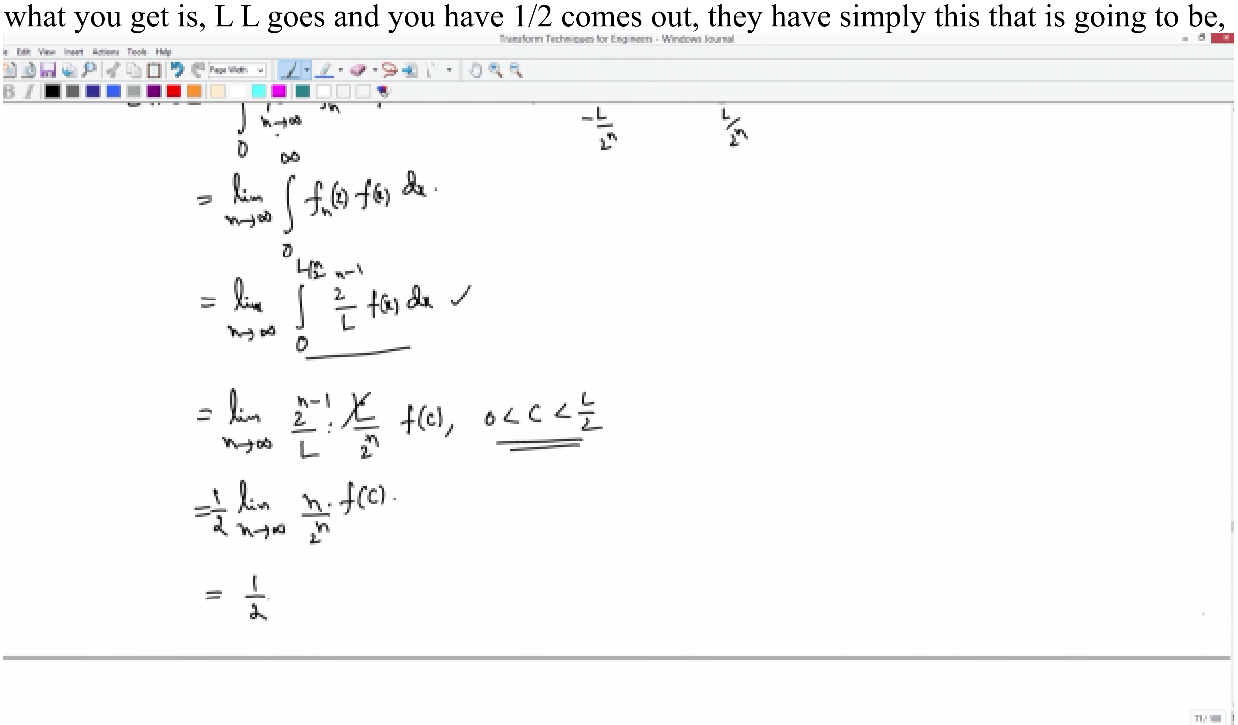
$$f_1(x) = \begin{cases} \frac{1}{L}, & [-\frac{L}{2}, \frac{L}{2}] \\ 0, & \text{else} \end{cases} \qquad \int_{-\infty}^{\infty} f_1(x)\, dx = 1 \checkmark$$

$$f_2(x) = \begin{cases} \frac{2}{L}, & [-\frac{L}{4}, \frac{L}{4}] \\ 0, & \text{else} \end{cases}$$

$$f_n(x) = \begin{cases} \frac{2^{n-1}}{L}, & x \in [-\frac{L}{2^n}, \frac{L}{2^n}] \\ 0, & \text{else.} \end{cases}$$

$$\int_{-L/2^n}^{L/2^n} A\, dx = 1 \qquad A\frac{L}{2^{n-1}} = 1 \Rightarrow A = \frac{2^{n-1}}{L}$$

So not exactly N/L is actually 2 power N-1/L, so here instead of, what you have, now you can get this 2 power N-1 divided by L times, this difference L/2 power N-0 that is this into F(c) as L goes to 0, C goes to 0, so this is going to be, so 2N 2N goes, finally if 2N 2N goes 1/2 is there what you get is, L L goes and you have 1/2 comes out, they have simply this that is going to be,



$$= \lim_{n\to\infty} \int_0^\infty f_n(x) f(x)\, dx.$$

$$= \lim_{n\to\infty} \int_0^{L/2^n} \frac{2^{n-1}}{L} f(x)\, dx \checkmark$$

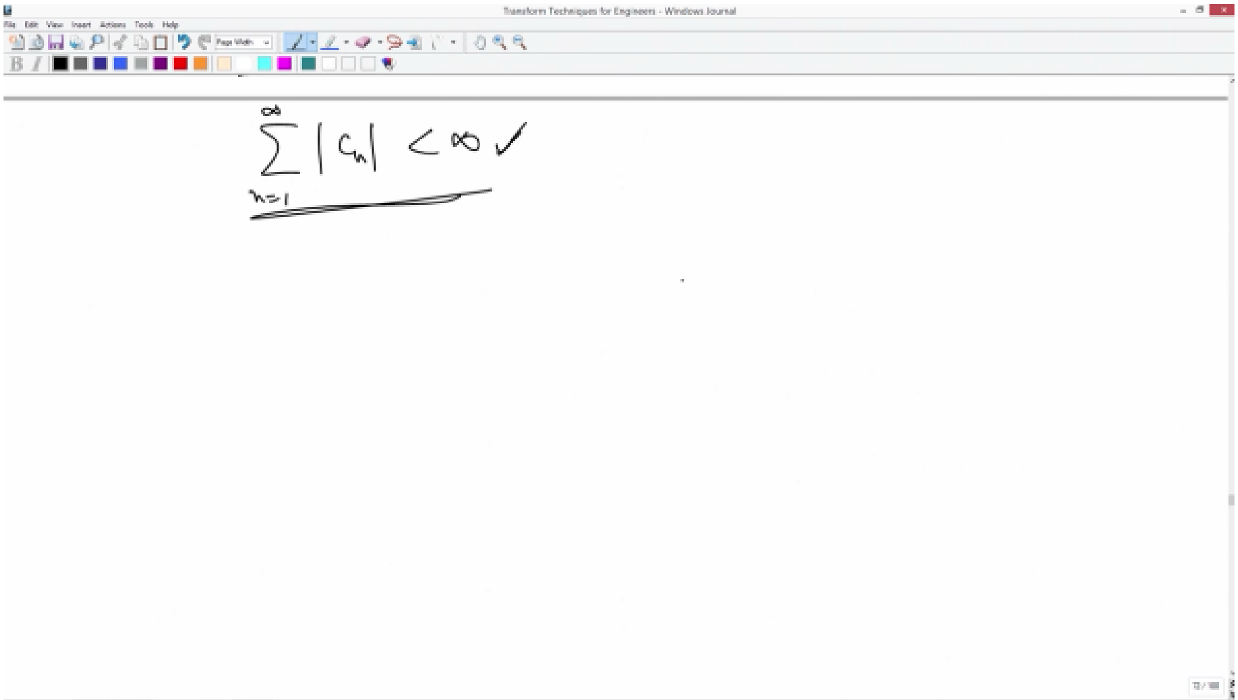
$$= \lim_{n\to\infty} \frac{2^{n-1}}{L} \cdot \frac{L}{2^n} f(c), \quad \underline{0 < c < \frac{L}{2}}$$

$$= \frac{1}{2} \lim_{n\to\infty} \frac{n}{2^n} \cdot f(c).$$

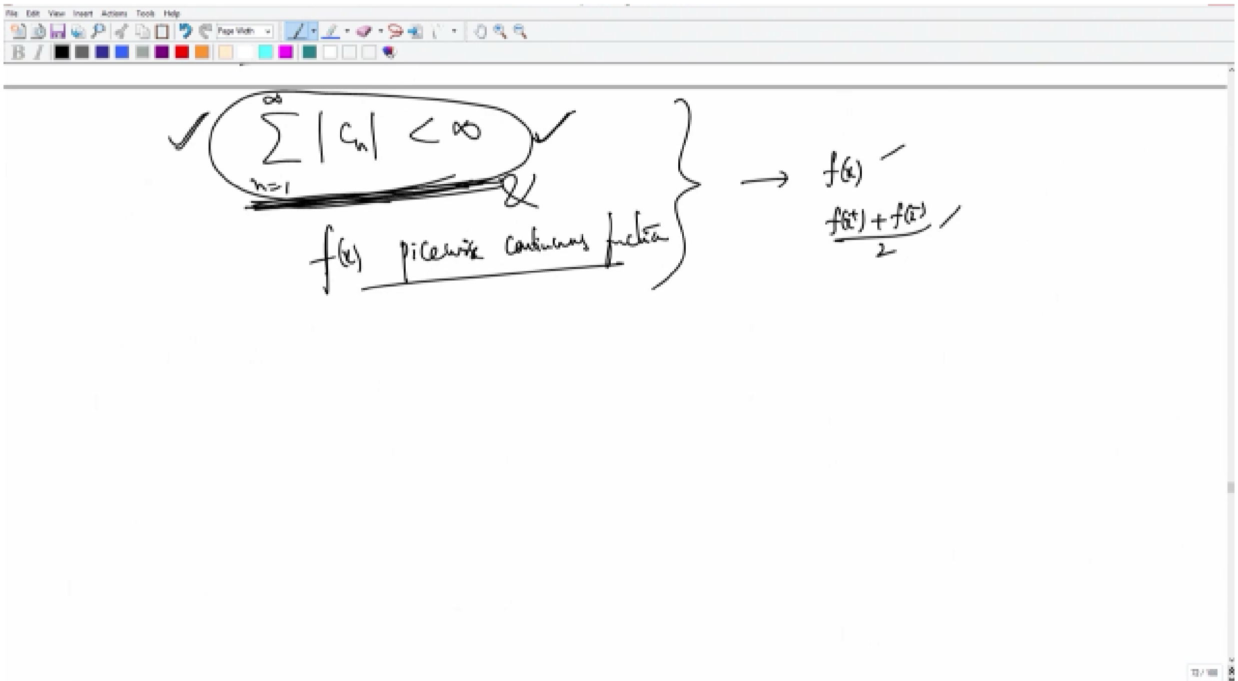
$$= \frac{1}{2}$$

as N goes to infinity, L goes to 0, as L goes to 0, C goes to 0, so this is going to be F(0) because F is smooth function, so such a thing, so F is piecewise continuous if you take this is what is the

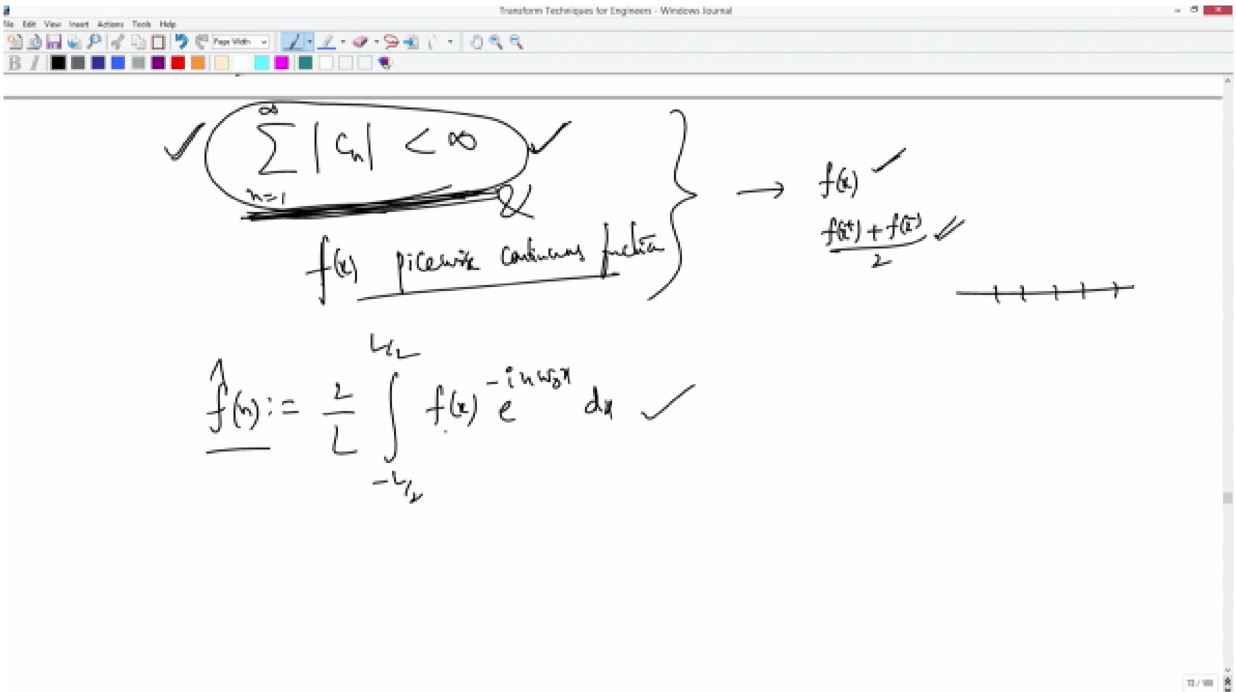
result, so you can see that if you multiply delta function to a function to a suitable function F(x), what you end up is, piecewise continuous function if you take, what you see is this, this result 1/2 of actual value of F(0).
So if you take the full thing, if you take - infinity to infinity delta function you have F(0) otherwise only 1/2 contribution will be there at the pole, at 0, at X = 0, okay, so that is what is, it will be useful when you do the Fourier transform later on, with this I will have a recap of what we have done for Fourier series.
Fourier series so far what we have done is if you consider a signal that most probably you can have a piecewise that means you have finitely many discontinuities, and those discontinuities are jump discontinuities and it is continuous function, it's a continuous function or piecewise differentiable function, piecewise smooth function means it's many times its piecewise differentiable function, N derivatives if you consider, N derivatives if FN is, any N derivatives of the function FN any nth derivative of the function of the signal that is FN, nth derivative of function is piecewise smooth, piecewise continuous function, piecewise differentiable function, when we say it's something is piecewise differentiable function you simply consider its derivative which is piecewise differentiable piecewise continuous function, if it is piecewise continuous function of first derivative you say that is piecewise differentiable function, any nth you consider which is piecewise for every N you say that it is piecewise smooth function okay, that smooth means you have all derivatives, many derivatives exist, every derivative you take it's a piecewise continuous function then we say that it's a piecewise smooth function, so when you have a piecewise smooth function what you get is you have a Fourier series you can define, you can define the Fourier coefficients and you composed all of them as Fourier series, and this Fourier series is you have shown that it is uniformly convergent, a Fourier series itself is uniformly convergent and absolutely convergent series, so that you can do some manipulation with the series, you can differentiate term by term, you can integrate term by term, because of the uniform convergence of the Fourier series, okay, for that we have used many results that, we have used only few results that are required here.
To start with, to show the convergence of the Fourier series when you have a piecewise differentiable function what you had is Bessel's inequality and Riemann Lebesgue lemma. what happens to the Fourier coefficient as M goes to infinity, that is a Riemann Lebesgue lemma, using them if you have a piecewise differentiable function and you have seen that Fourier series actually converges to the function value peace, point-wise convergence you have seen, the Fourier series actually converges, if you fix X value a Fourier series of the function F(x) converges to the function value of F(x), if F is continuous at X, otherwise both sides limits at X, suppose it is not a continuous it should be discontinuous at the point X, but they are jump discontinuities are only allowed, because that is how we restricted our signals are, and both sides limit exist if it is jump means both sides F(x+) and F(x-), both sides limits exist, and we take the average of them, so the average jump, half of the jump is exactly where this Fourier series converges to the value.
If it is piecewise smooth function we have actually seen that this Fourier series is converging uniformly implies by the corollary of Bessel's inequality, you can see, because the signal by the Bessel's inequality sigma CN square, we have seen that sigma, absolute series of CN's this is a finite, you have seen, this is the result when it is piecewise smooth, once you have this that means once you see that this is true implies Fourier series is uniformly convergence that means Fourier series is actually converging even point wise, once it is uniform convergence it is also



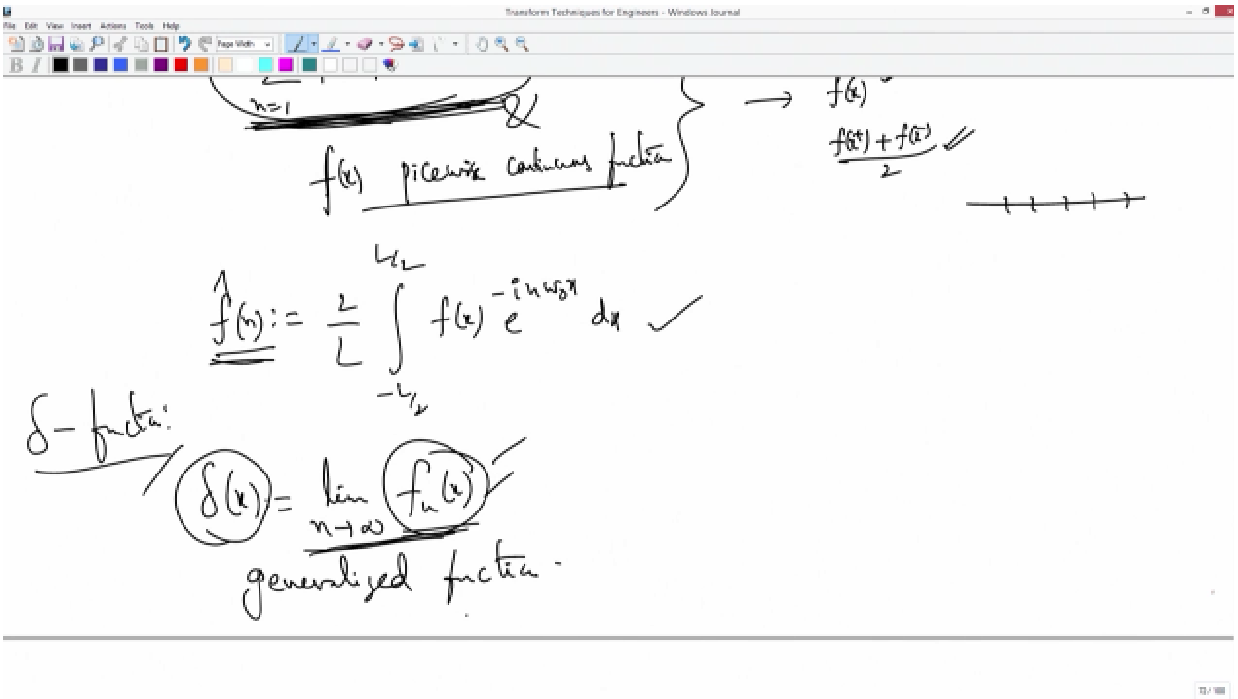
converging point wise that means so you have a Fourier series, composed of a Fourier coefficients and signals, fundamental signals in terms of exponentials E power IN omega naught T, so in terms of them Fourier series that converges point-wise once you know this is true and it has to, where it has to converge that we don't know if F is piecewise, just piecewise continuous function, okay, we know that if it is piecewise smooth this is always true, okay, implies Fourier series actually convergent that is trivial we have seen that if it is a differentiable function piecewise differentiability implies Fourier converges to a function value point wise, but if it is piecewise convergent we know that this series is finite, so that means it is uniform convergence implies, then the Fourier series converges to the function that is also trivial.
So only thing is when, so if you assume this that means if F is piecewise only continuous function, piecewise continuous function only, then you don't know whether this is true or not, okay this result is true you have to include piecewise smooth function okay that means at least two derivatives of the function F(x), F double dash(x) is piecewise continuous function then we have shown that this is actually true that implies uniform convergence, that in place you can, whatever you can actually show that the Fourier series converges to F(x) in the second result, so we use the crucial assumption that this is true, if this is finite, and if it is piecewise continuous function, okay, F is a piecewise continuous function, piecewise only continuous function, these assumptions both are true then the Fourier series converges to F(x) or the average value of it whether it's a discontinuous or continuity depends on that it converges to the function, okay.
But we don't know this result, if it is piecewise continuous function you cannot guarantee that this is true, okay, uniform convergence of the Fourier series it's not guaranteed or in fact convergence, okay, so this shows the convergence if this is actually true implies uniform



convergent that’s what we have seen in the last video, so but in general when you have a signal which is piecewise differentiable, piecewise continuous obviously its derivatives also exist that are also piecewise, basically if you represent your signal with elementary functions they are actually composed of, what you see is if you calculate the derivative they are not differentiable at most at finitely many places wherever they are not differentiable, okay, if it is piecewise continuous at those certain points you have finitely many points at which it is not continuous, at those points it is not even differentiable so within gaps you have a piecewise differentiability, that means a derivative is piecewise differentiability that means, you take any signal you represent by elementary functions, your signal is always piecewise smooth implies Fourier series actually converging uniformly, okay and absolutely implies your convergence theorem, Fourier series actually converges to the function F or its average value depending on its continuity at the point X or not, okay, so that's what we have seen and later on using the Fourier coefficient CN that is 2 /L integral –L/2 to L/2 F(x) this is how we define CN we call this F cap(n) this is the definition of the Fourier transform IN omega naught X DX, okay.



So this you have seen with minus, this is your definition of a Fourier transform once your Fourier coefficients if you represent as a transform, and what your Fourier series is, actually your inverse transform. And you have seen certain properties on these Fourier transforms that include linearity, scaling, time scaling and other things okay, and we have defined product of what is a Fourier transform with the product of the function, and what is a Fourier transform of its convolution product, what is your Fourier transform of its convolution product, and you have also seen today Parseval side identity, okay, these are the important properties through which you can actually calculate, if you are given a complicated function as your time signal you make use of these properties and evaluate the Fourier series, okay, that is the importance of these properties as of now, so this is what we have done in the process we have also introduced what is a delta function which is not a function, which is simply as a limit of usual functions FN(x), usual means what we don't know it can be any some sequence of functions FN, if you take the limit which is not a function, if delta X is not a usual function limit of usual functions, but if you view this as a limit of usual functions then you call this generalized function okay, generalized function, so delta function is one such function, so delta function is a generalized function and whose properties you have seen, how you get this delta function as you have seen how to construct this FN’s so that it converges, it is amounts to what is a limit of that is delta function.

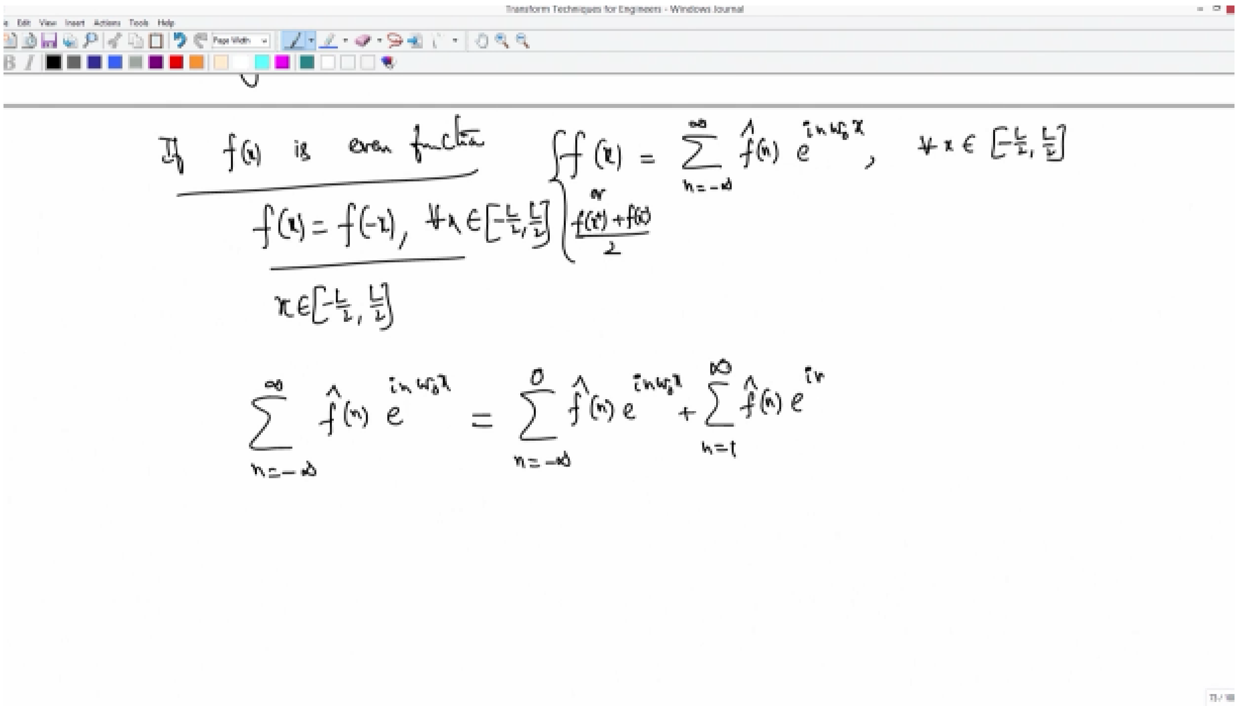


And one of the property of the delta function is, if you multiply delta function with another function and integrate its value, that value is actually function value at 0, okay, and if you take only integration from 1/2 that from this singularity, so from 0 to infinity if you take delta is, basically delta(x) means at 0 you have a singularity, so delta X, delta 0X because at X = 0 this is infinity, okay, so if you integrate only from 0 to infinity 1/2 of that, so if this singularity, for this singular point infinity at which the function is, delta function is infinity at 0 and if you integrate 1/2 of that singular point contribution will be there that is going to be 1/2, if you integrate from 0 to infinity, or - infinity to 0 both sides because of symmetry what you get is this delta(x) DX will be only 1/2, okay this is the result we have seen today.

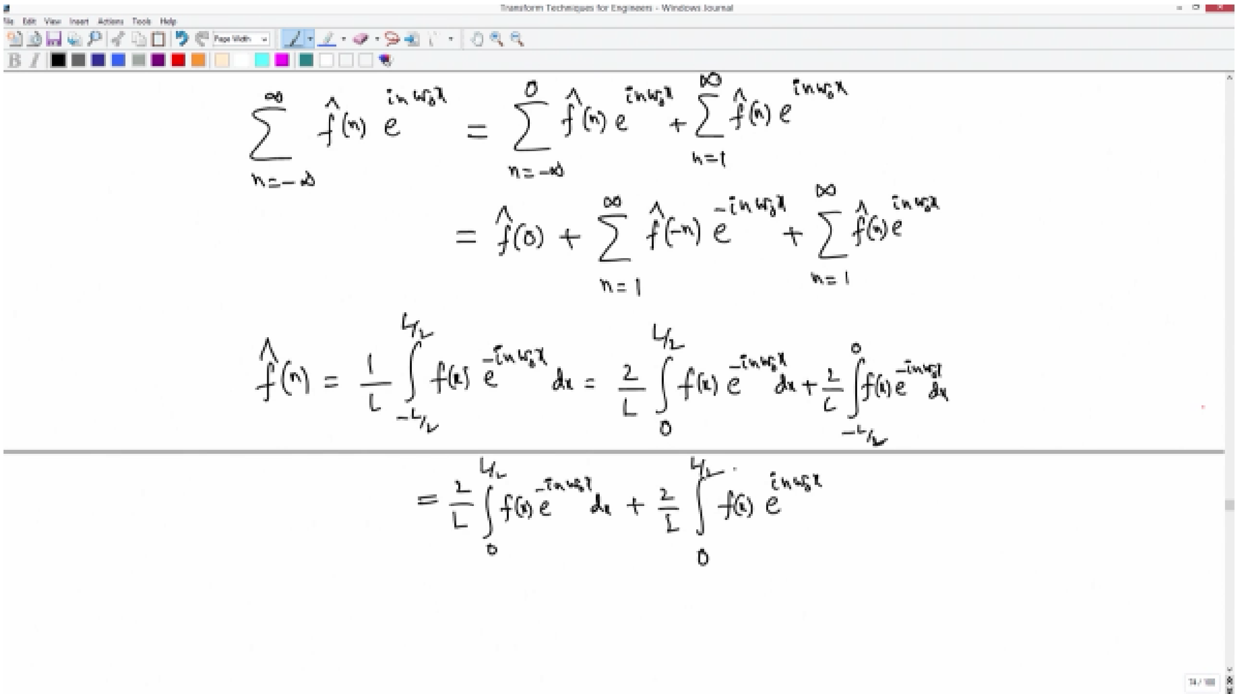
So with this we close the Fourier series part so far you have seen a signal function F(x) as periodic, piecewise continuous or piecewise differentiable function, okay, piecewise smooth function if you represent as elementary functions, but see if your function F(x) if your signal is F(x) is an even function, even function, what we have seen is you can see that the Fourier series is F(x) is assumed that if it is even, so what is the even function? F(x) = F(-x) for every X that is the meaning of even function, so if your F(x) is X is defined between –L/2 to L/2 and you have this is always true, -L/2 to L/2 wherever it is defined this is periodic function which is even function.

And what is your Fourier series? Representation F(x) is you have a sigma, N is from - infinity to infinity, you have F cap(n) Fourier transforms E power IN Omega naught X, this is your Fourier representation for every X in –L/2 to L/2, okay, and you have F(x) if X is continuous or if it is X here, at X if it is not continuous you simply take F(x+) + F(x-), half of that, average of that, that is what is the representation, this is your Fourier series.

And what happens? So you have, if F is even, if F is even what happens? You can start with the Fourier series N is from - infinity to infinity F cap(n) into E power IN omega naught X which is equal to, I simply split it into two parts, okay, so N is from - infinity to 0, F cap(n) E power - IN omega naught X + N is from 1 to infinity, F cap(n) E power –IN, this is not minus, this is



plus okay, IN omega naught X, so this is equal to F cap(0) + and this you try to replace n/-n then it's going to be 0 to infinity, that is going to be 1 to infinity, F cap(-n) E power - IN omega naught X, okay + sigma N is from 1 to infinity, F cap(n) E power IN omega naught X.
So what is your F cap(n)? This is 1/L –L/2 to L/2 F(x) E power – IN omega naught X DX, because F is even you make use of the identity property that F is even, and this is because it is even you have 2/L times this is going to be 0 to L/2 F(x) is even, right, and this function is, so if you use this F(x) E power –IN omega naught X DX + 2/L –L/2 to 0 F(x) E power -IN omega naught X DX.
So what is this one? First part as it is, F(x) E power -IN omega naught X DX, the second one you replace X by –X, then what you get is 2/L, this is going to be L/2, if you replace X by -X this is going to be L/2 to 0, and F(-x), and which is F(-x) is F(x), because F is even, and E power IN omega naught X, DX is -DX so that minus part you can write 0 to L/2, okay, so this



$$\sum_{n=-\infty}^{\infty} \hat{f}(n)\, e^{in\omega_0 x} = \sum_{n=-\infty}^{0} \hat{f}(n)\, e^{in\omega_0 x} + \sum_{n=1}^{\infty} \hat{f}(n)\, e^{in\omega_0 x}$$

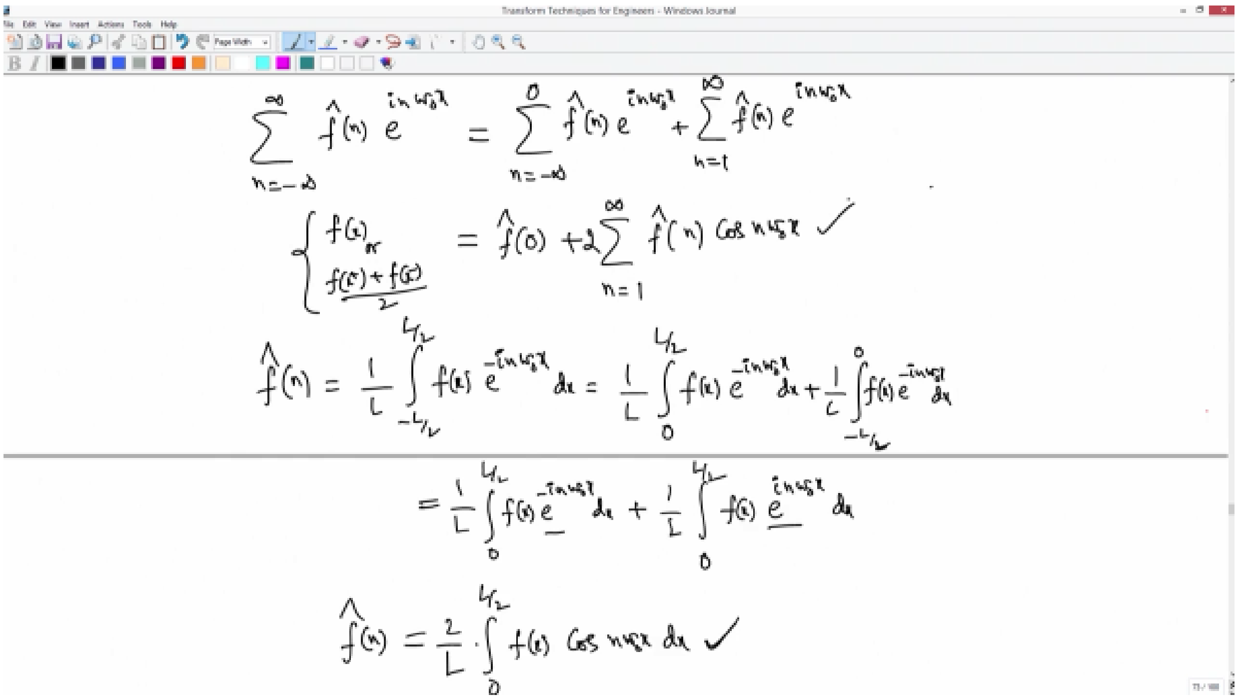
$$= \hat{f}(0) + \sum_{n=1}^{\infty} \hat{f}(-n)\, e^{-in\omega_0 x} + \sum_{n=1}^{\infty} \hat{f}(n)\, e^{in\omega_0 x}$$

$$\hat{f}(n) = \frac{1}{L}\int_{-L/2}^{L/2} f(x)\, e^{-in\omega_0 x}\, dx = \frac{2}{L}\int_{0}^{L/2} f(x)\, e^{-in\omega_0 x}\, dx + \frac{2}{L}\int_{-L/2}^{0} f(x)\, e^{-in\omega_0 x}\, dx$$

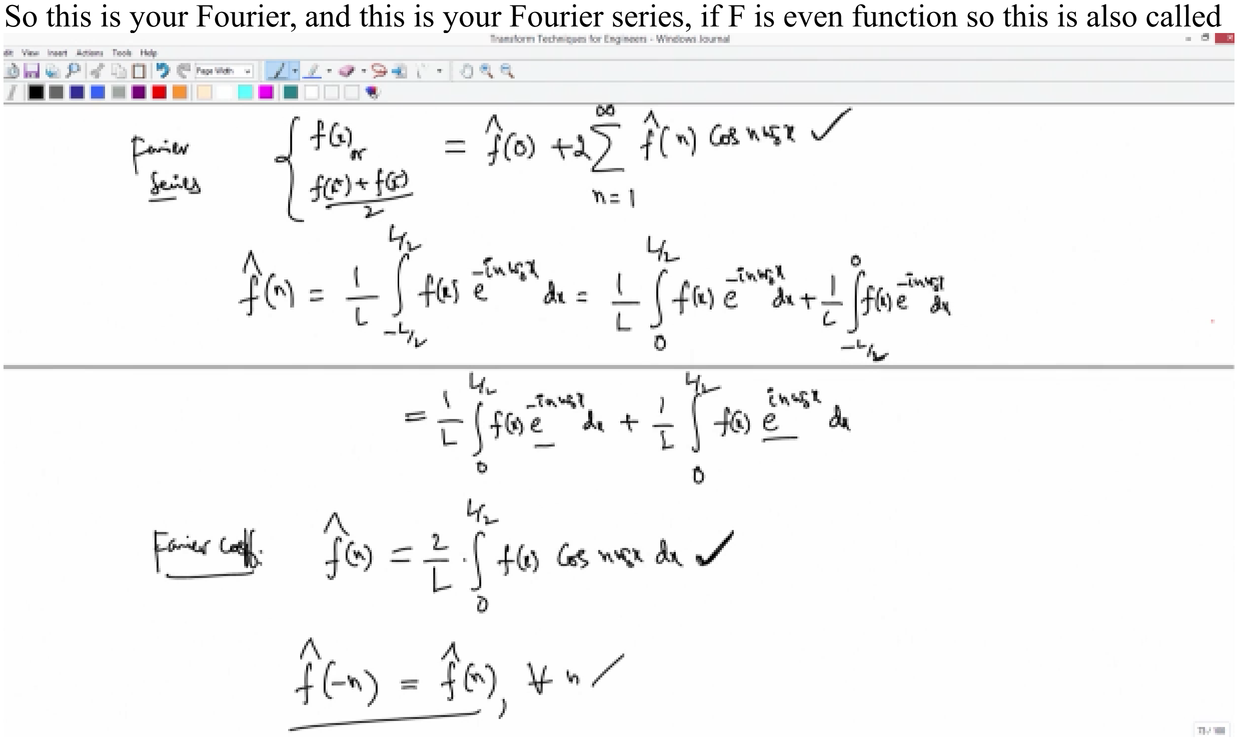
$$= \frac{2}{L}\int_{0}^{L/2} f(x)\, e^{-in\omega_0 x}\, dx + \frac{2}{L}\int_{0}^{L/2} f(x)\, e^{in\omega_0 x}$$

is what you have so, this is nothing but 2/L, why do you get 2/L? It's not 2/L, it’s 1/L, so 1/L is only continuing, I split it into two parts, so what you get is now this is going to be, if you add up together, so you get 2/L, 0 to L/2 F(x) and this is, this you add up so you get cos N omega naught X DX, so your Fourier coefficients are only this, okay, only this.

And now you can easily see that F cap(-n) from this, F cap (-n) is also F cap(n) this is true for every N. Now if you use this here, so these are F cap(-n) I can replace with F cap(n), okay and that makes it I can remove this part and this is what you get, so E power IN omega naught X, okay, the summation, so this summation is nothing but you have 2 times cos N omega naught X is my F(x) or F(x+) + F(x-) divided by 2, so this function, this is your Fourier series, and this is your Fourier coefficient, okay.

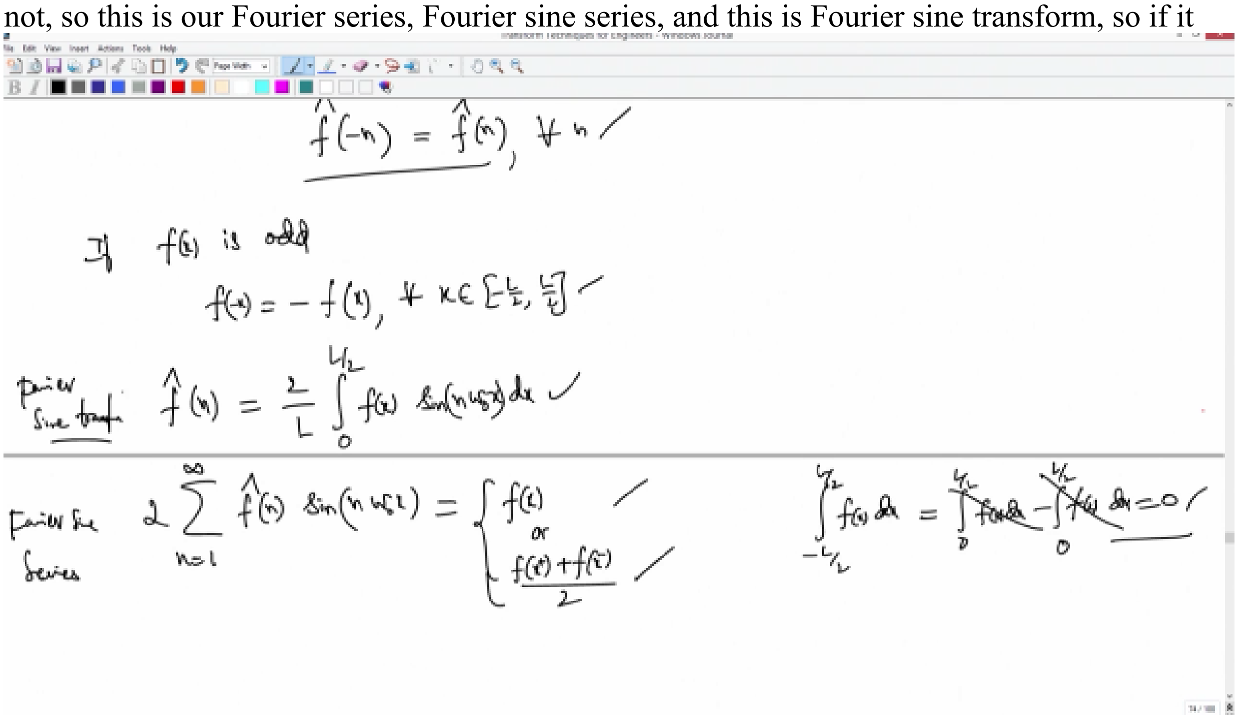


So this is your Fourier, and this is your Fourier series, if F is even function so this is also called



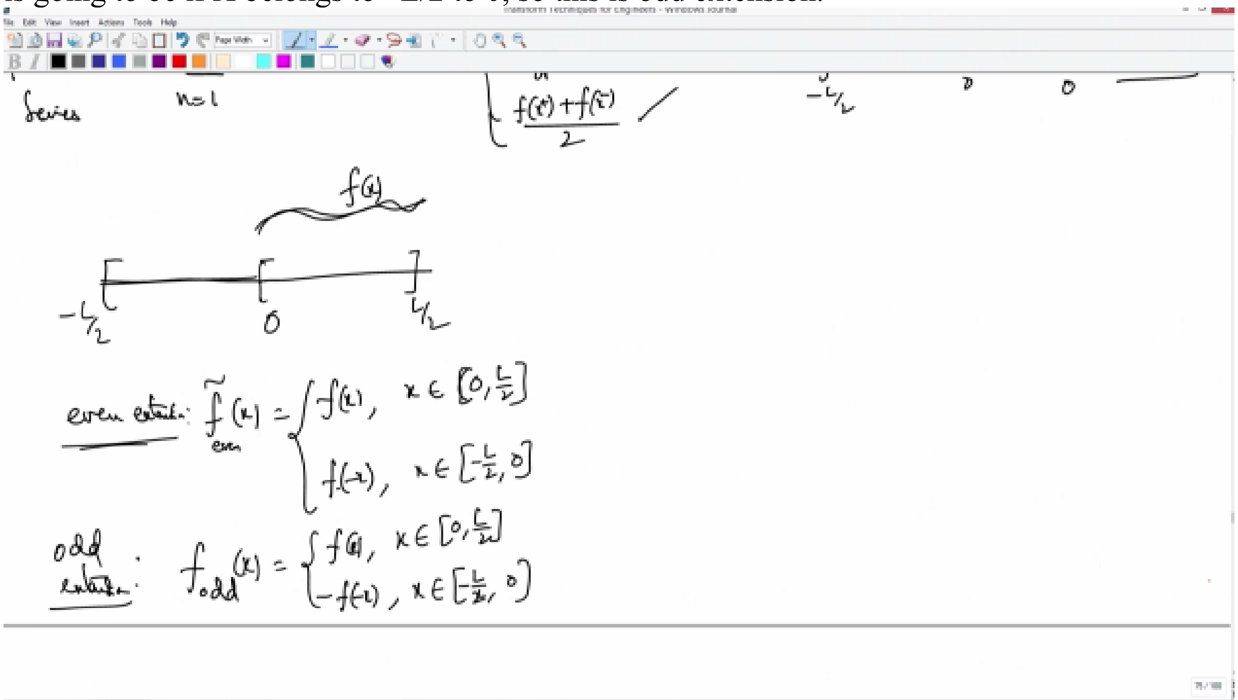
Fourier cosine transform, and this is your Fourier cosine series. What is your F cap(0)? F cap(0) is simply 2/L integral 0 to L/2 F(x) DX that is your single constant, okay, F cap(0) and with this F cap(n) that is only running from 1 to infinity, you have instead of exponential functions now what you have is cosine functions, so this is what happens if it is a even function, if you a priori if you know that your signal is symmetric with X, at X = 0 so –L/2 to 0 is symmetric to 0 to L/2, so if it is symmetric both sides something like a signal will be, so if we take the hat function this is symmetric both sides, okay, symmetric at about X = 0 it is an even function, if your function is like this, if it is like this which is here is the positive, here is negative, so this is

odd function, okay for example you can have a sine X function, sine X how does this look sine X function, between - pi to + pi 0 this is how it looks, so you see that –pi/2 is a negative of that, so this is the odd function.
If you take cos X, but if you take cos X cos pi is -1, cos -pi is -1, cos pi is 0, cos pi/2 is 0, so this is how it looks, so this is even function. So depending on even function or odd function you can work with this Fourier series and in the odd case if F is odd function, if F(x) is odd that means F(x) is -F(-x) or you can write F(-x) is -F(x) for every X in –L/2 to L/2, such a periodic function if you have what you get is F cap(n) the Fourier transform will be in this case 2/N between 0 to L/2 F(x), instead of cosines now you get sine, sine N Omega naught X DX, you can easily see and your Fourier transform will be N is from 1 to infinity because it is even function F cap odd function, F cap of 0 that is integral from –L/2 to L/2 F(x) DX that is going to be 0 because F is odd, right, so you have a –L/2 to L/2 F(x) DX, if F is odd function you can easily see that this is L/2 F(x) DX + -L/2 to 0 F(x) DX.
Now if you put X by –X, you have -FX that is going to be L/2 to 0, and DX is -DX so you can revert these limits, so this is what you get so finally 0, so because of that this cap in the Fourier series this cap, this F cap(0) will be 0 otherwise the Fourier series will be 2 times F cap(n), N is from 1 to infinity instead of cosines you have now sine, sine N Omega naught X, okay, so this is exactly equal to F(x) hard function F(x), if X is continuous or if it is not continuous it have to take the average value of it, that average value of the jump depending on F is continuous at X or not, so this is our Fourier series, Fourier sine series, and this is Fourier sine transform, so if it



is odd and even function this is the case and sometimes you may have a signal only on the positive side 0 to, only half of that if you are given 0 to L/2 only you have a signal, so left side you don't know really but you want to have a Fourier series of period minus period L if you want to have a Fourier series with period L you can extend your signal here as either, as a even function or odd function.
Suppose this is F(x), how do you extend this function over, X belongs to only between 0 to L/2 you can extend this function as itself if X belongs to 0 to L/2 and F(-x) if X belongs to –L/2 to 0, so if you put X = -L/2 to 0 is going to be values here only, so this is the even function, even

extension, this is useful in even impurities this technique we use, you can also, this is like even we can write F even, and you can also extend it as F odd function, and then how do you do this? F(x) X belongs to 0 to L/2, how do we get the extension for the left hand side? Now –F(-x) that is going to be if X belongs to –L/2 to 0, so this is odd extension.



So once you have this, if you have a signal only between 0 to L/2 you can extend it to a length of a period of up to L, with the period L, you can extend as even or odd function and you can give the Fourier series in terms of cosines or sines depending on how you extend the function over the full period L, so that you can get the Fourier series with period L, okay, so using these Fourier sine transform and Fourier sine series or Fourier cosines transform and or Fourier cosine series, okay, so this way you can have, so we'll try to give you some problems on these whatever we have done on the Fourier series, and problems will be given as an exercise you can work out, so in some of the problems you may have to use properties that I have explained in the last videos.

With this we end this Fourier series part, and the next video we'll start Fourier transform, that is you can consider a signal that is a non-periodic that means something is non-periodic means you have a signal that is defined over a full real line, so there is not like a finite part or which it is repeated outside, so it is extended fully if you have a non-periodic function how can you get a Fourier transform and it's inverse transform that is what we will see, so we will see these things as we will define what is Fourier transform and it's inverse transform in the next video. Thank you very much.

[Music]

Online Editing and Post Production

Karthik

Ravichandran

Mohanarangan

Sribalaji

Komathi

Vignesh

Mahesh Kumar

Web Studio Team

Anitha

Bharathi

Catherine

Clifford

Deepthi

Dhivya

Divya

Gayathri

Gokulsekar

Halid

Hemavathy

Jagadeeshwaran

Jayanthi

Kamala

Lakshmipriya

Libin

Madhu

Maria Neeta

Mohana

Muralikrishnan

Nivetha

Parkavi

Poonkuzhale

Poornika

Premkumar

Ragavi

Raja

Renuka

Saravanan

Sathya

Shirley

Sorna

Subash

Suriyaprakash

Vinothini

Executive Producer

Kannan Krishnamurthy

NPTEL CO-ordinators

Prof. Andrew Thangaraj

Prof. Prathap Haridoss

IIT Madras Production

Funded by

Department of Higher Education

Ministry of Human Resources Development

Government of India