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NPTEL ONLINE COURSE
Transform Techniques for Engineers
Properties of Fourier transform (continued)
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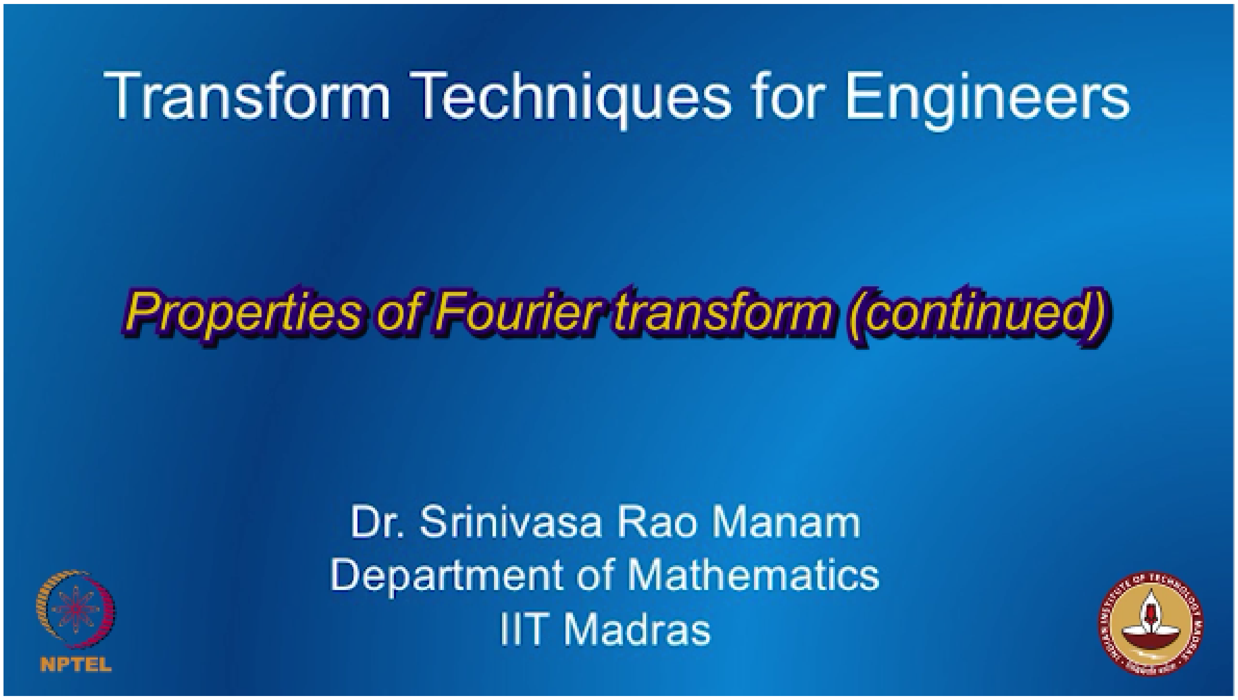
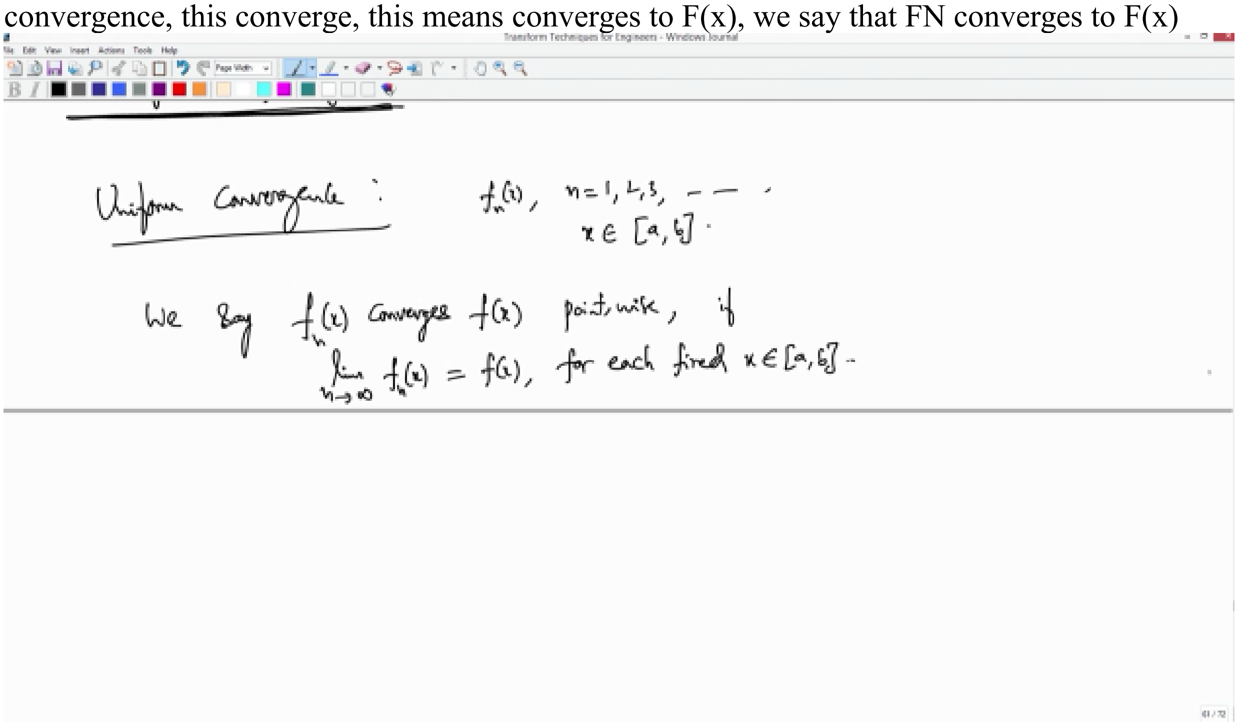
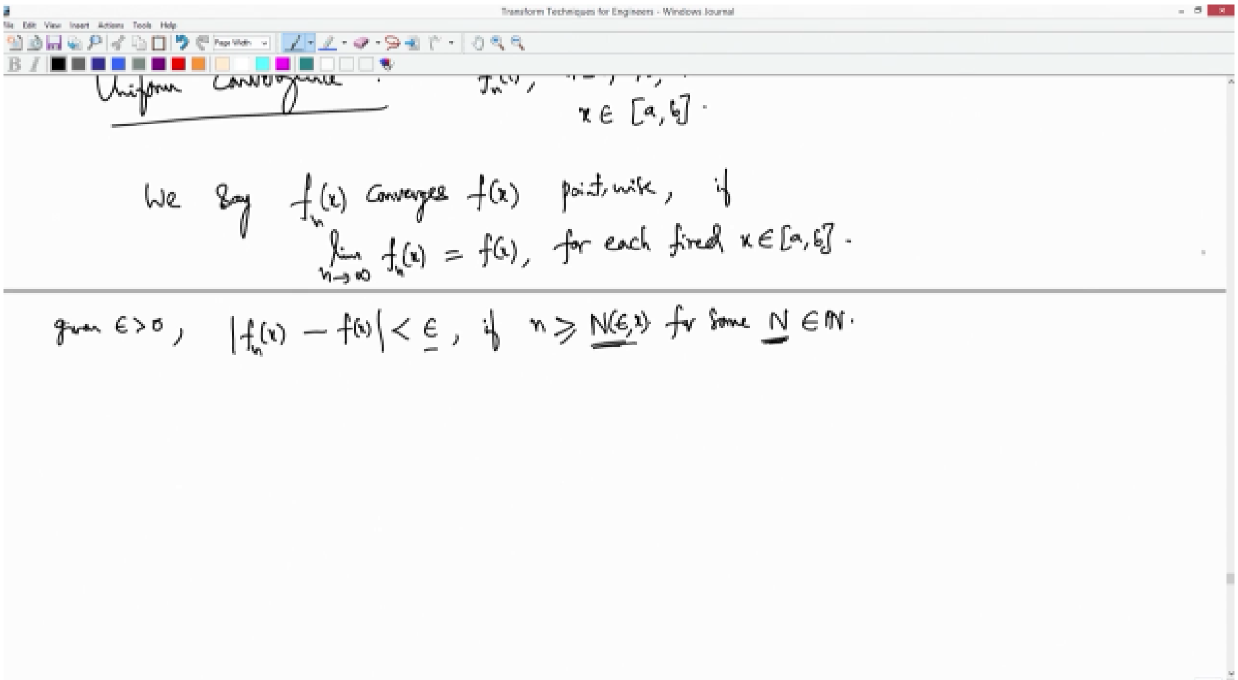


The last video we were looking at some of the properties of a Fourier transform, now one of the property that we had yesterday is, in the last video is this when F and G both piecewise smooth functions, periodic functions with period L. And if you consider the product, product of these two functions F into G, Fourier transform of product of these functions which is given as a convolution sum, that sum from F cap(k) into G cap(n-k), so in the proof of this property we have used, what we have done is the integral we just, we passed the integral inside the infinite sum, so that is not legitimate in general unless that series sigma, K is from - infinity to infinity whatever that series function of K, that series should be uniformly converging, so we have defined what is, I'll just now, in this video we will just define what is a uniform convergence again, we just look into the uniform convergence and under what conditions the Fourier series is actually uniformly convergent.
So after proving this so, it so happened that when F is F1 G are piecewise smooth periodic functions, this product is also piecewise smooth periodic function that implies its Fourier series actually uniformly convergent, so we will prove this result using , for that you need just a small result, one is the Weierstrass M-test and uniform convergence anyway we’ll just prove, we'll just define what is uniform convergence.
I'll explain what is uniform convergence, and also I'll just state what is M-test, you just need these results always, these are actually coming from calculus and then once you have this you need Cauchy-Schwartz inequality for the sequences, that is basically series, I'll just give you some inequality using that one can prove that if F is a piecewise, piecewise differentiable,

actually piecewise smooth that means what you need is F dash is piecewise differentiable function, if such is the case for F the derivative is piecewise smooth, piecewise differentiable function, that means two derivatives second derivative so D square F/DX square this should exist as a piecewise continuous function, then you have the Fourier series converges you can prove that the Fourier series of this function F(x) is uniformly convergent, uniformly and absolutely convergent, so this is what we will see, for that I'll just state, I'll start with what is the uniform convergence, so I'll degress here with uniform convergence.
So let's see what is the uniform convergence, so before you know about this uniform convergence, what is the convergence of a sequence of functions, so if you consider a sequence of functions FN(x), this is a sequence those is defined for every N, 1, 2, 3 and so on, for example you have sequence of functions, and X belongs to let us say some A, B, that's define over a closed interval A, B, and you say that the FN(x) converges to F(x) point-wise convergence, this converge, this means converges to F(x), we say that FN converges to F(x)



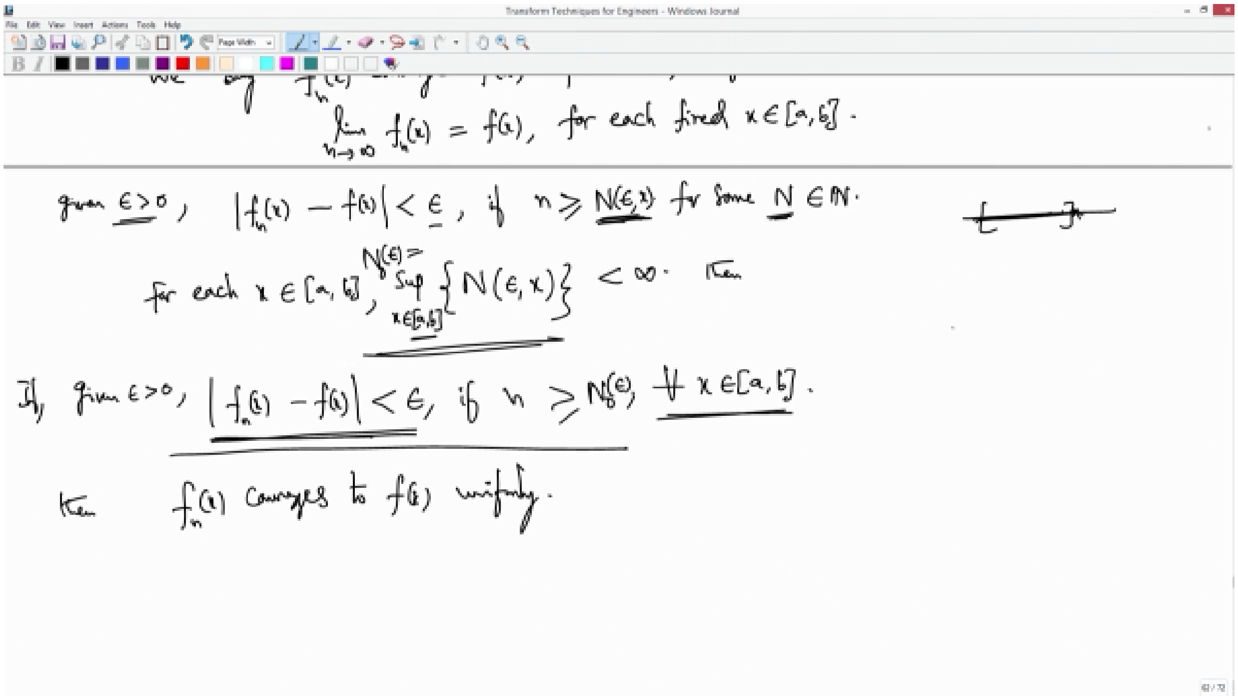
point-wise if FN(x), limit of FN(x) as N goes to infinity, so you just write here converges, converges is equal to F(x) for each fixed X in A, B. What is the meaning of this? This means a FN(x) – you fix X and this difference, this is a given epsilon positive, so you give any epsilon positive and what you get is, this will be less than, so this quantity is less than epsilon, so whenever you have N, N is bigger than capital N, for some N so give epsilon there is a N for some I'll write for some N belongs to natural numbers, okay.
So this N, give epsilon there is a N, so such that for every N bigger than or equal to N or bigger than also N this is also, okay, so let's use N greater or equal to N, N depending on epsilon here because you fixed X, okay, so for every X you have and once you fix X so that is also depending on, the moment you fixed your X and you fixed your epsilon there is a N, so basically it depends both on, both this number N that belongs to the natural numbers that depends both on epsilon and X, okay, when you say that this converges point-wise.



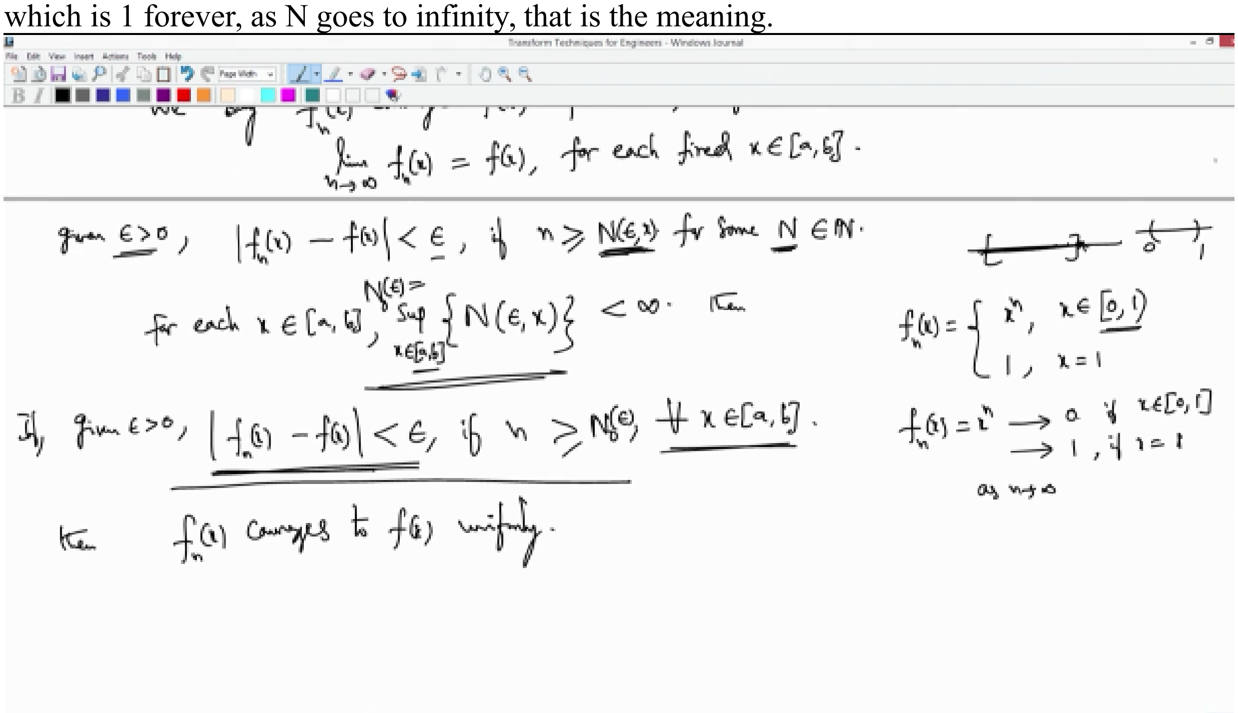
And suppose you have for each N, for each X, X in A, B okay suppose you have this N, for each N and fixed epsilon, let us say epsilon is fixed given epsilon, you fix epsilon the moment you give epsilon fixed, now for each X you have this epsilon X then you consider the supremum of all this, supremum of all this X belongs to A, B, so that means basically maximum of them.

Something this supremum means, some prove something to say this is the set for example this is the set, this is the supremum , supremum means which is always bigger than, so all kind of maximum, okay, so is the maximum so because it is a maximum, supremum or maximum because it is a closed unbounded set, so you can see that this is a maximum, for a continuous function if it is continuous function on a closed unbounded is the maximum always exists, but let us otherwise you'll say supremum, so supremum is always so the some number, this means some number that is always bigger than the set, okay, so that means N epsilon X, X belongs to, when X is running over this closed interval, and you consider that set, that set there is a number which is called the supremum of all these which is a bigger than or equal to all these numbers, okay.

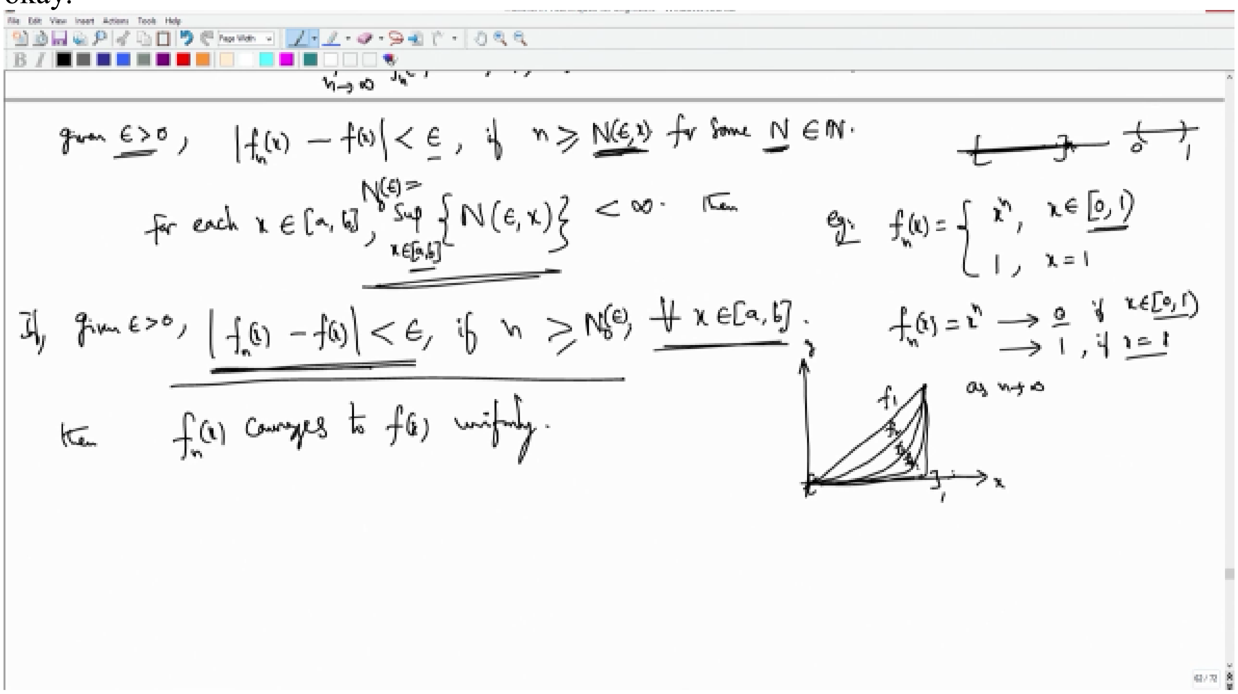
Supremum, if it's a supremum is finite, okay, then you call this N naught, N naught is this supremum, then what happens? FN(x) - F(x) is less than epsilon if N is bigger than this N naught, okay, and this is true for every X in A, B, right if such a supremum exists then you say that then this N, N is no more depending on X, so this N naught is actually depending only on epsilon now, so let's write this N naught only depends on epsilon, so one, so the moment given epsilon, so given epsilon positive if you have this difference, this difference is less than epsilon, for every N bigger than or equal to some number that depends only on epsilon but works for every X whatever you choose then you say that FN(x) converges to F(x) uniformly, okay.



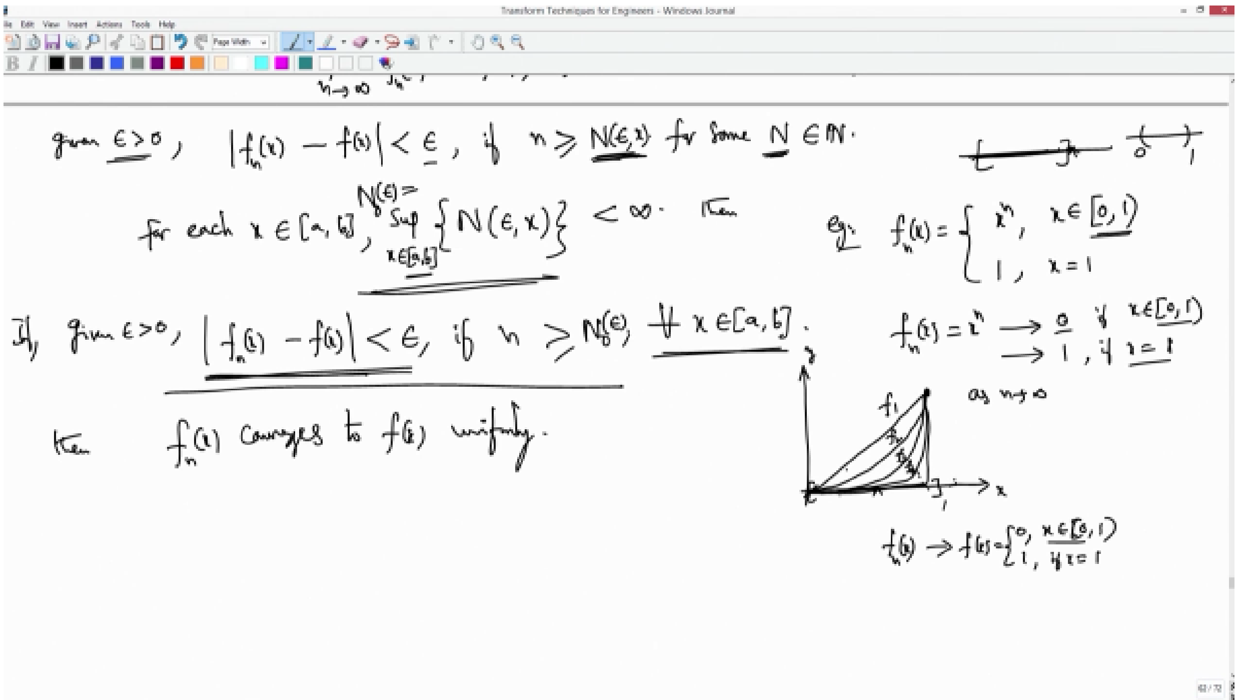
So for example I'll just give you some example FN(x), if I define X power N, X belongs to open 0, 1 okay. And then otherwise okay closed 0 and open so, otherwise its 1, X = 1 let us say so if you define like this and what happens to this function FN(x)? FN(x), you fix X, X within here FN(x) is X power N because X is less than or equal to, if X = 0 this is anyway 0 so, this goes to 0 if X = 0, if X is between 0 to 1, if it is 0 to 1 this is always, because this is 1 by, so this is less than 1, X power N is actually, as N goes to infinity because this is X is less than 1 this is going to 0, okay, if X belongs to the open 0, 1, and if this converges to 1, if X =1 because when you once you put X = 1, FN(x) is, FN(1) is always 1, so that converges to a constant function which is 1 forever, as N goes to infinity, that is the meaning.



So is this converging point-wise? Yes, but you fix for every X value is going to some number 0, if you are here this is converging to 0, if X = 1 it converges to 1, so it is point-wise convergence is true, this is a point-wise convergence this is what is true for this example, okay, whether it is uniformly convergent that is what we have to see, so that's how do you see that this actually not true I'll just graphically show you, so once you have this X power N between 0 to 1, this is 0 to 1, F1 looks like, at 1 is 1, this is 1, so this is 0 to 1, so this is X axis, and this is Y axis, and at X = 1 this is always 1, and as X goes to, between so this is at X = 0 this is always 0, okay at X = 0 and as X goes to so far every, so F1 let us consider F1, F1 is a function which is converging to, which looks like this let us say X = 1, X = 1 basically it's a linear function, so it's like this X square, so it is like this, okay, X cube will go on like this and so on. As X power N goes finally you end up going like this, as X so this is your F1, F2, F3, F4 and so on, that's what happens, okay.

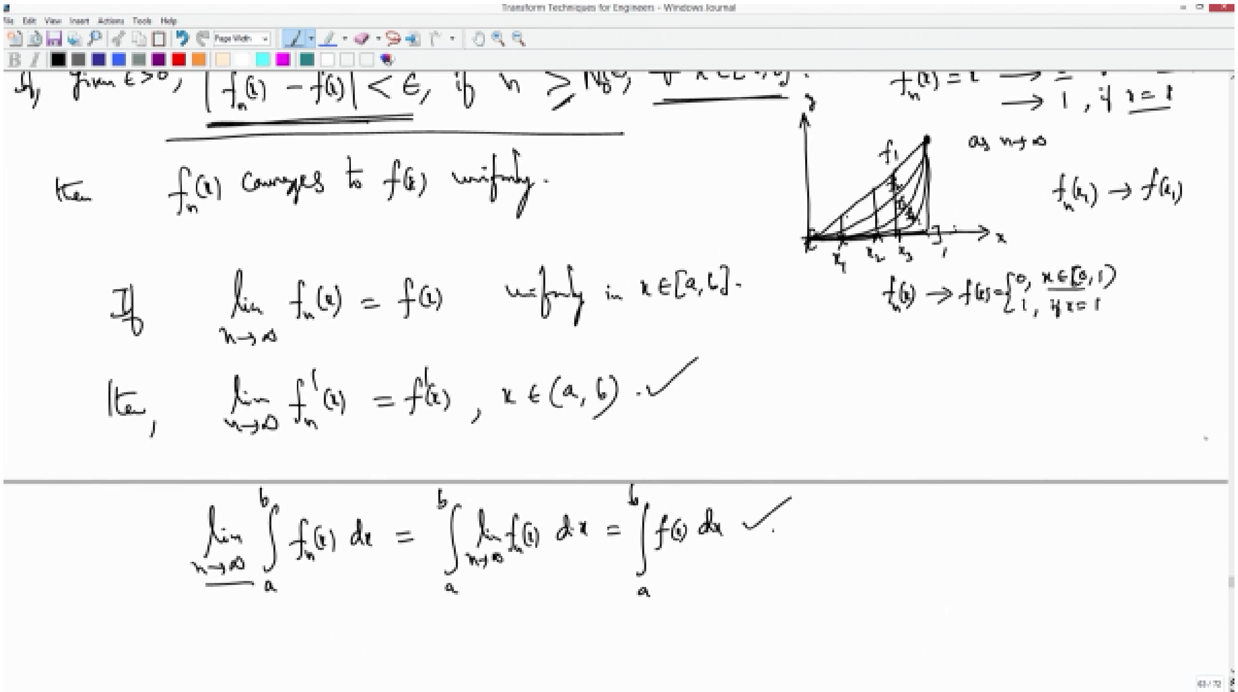


So you see F1, so you fix 1 here, so 1 at every point if you fix, if you fix X value it's finally converging to 0, it's converging to 0, 0, 0, even at including at X = 1 the value is always 1, so it is 1, okay. So at 1 always function value is always 1 for all N, so it converges to 1, so point-wise convergence is there, why is it not converging uniformly? Because you see that there is no N, so if you fix your X here, if you fix your X there is some N beyond which this is always less than this, this quantity FN(x) -F(x) so you can see that FN(x) if you want that for every X and what is FN(x)? It converges to the F(x) which is 0, 0 if X is 0 to 1, and 1 if X =1, so such a function it is converging point-wise, so what happens when you take the difference FN(x) -F(x) this quantity is always less than FN(x) for whenever you are here is always less than, so as this quantity is going to 0, so you see this is how shall I say, you see to tell graphically speed with

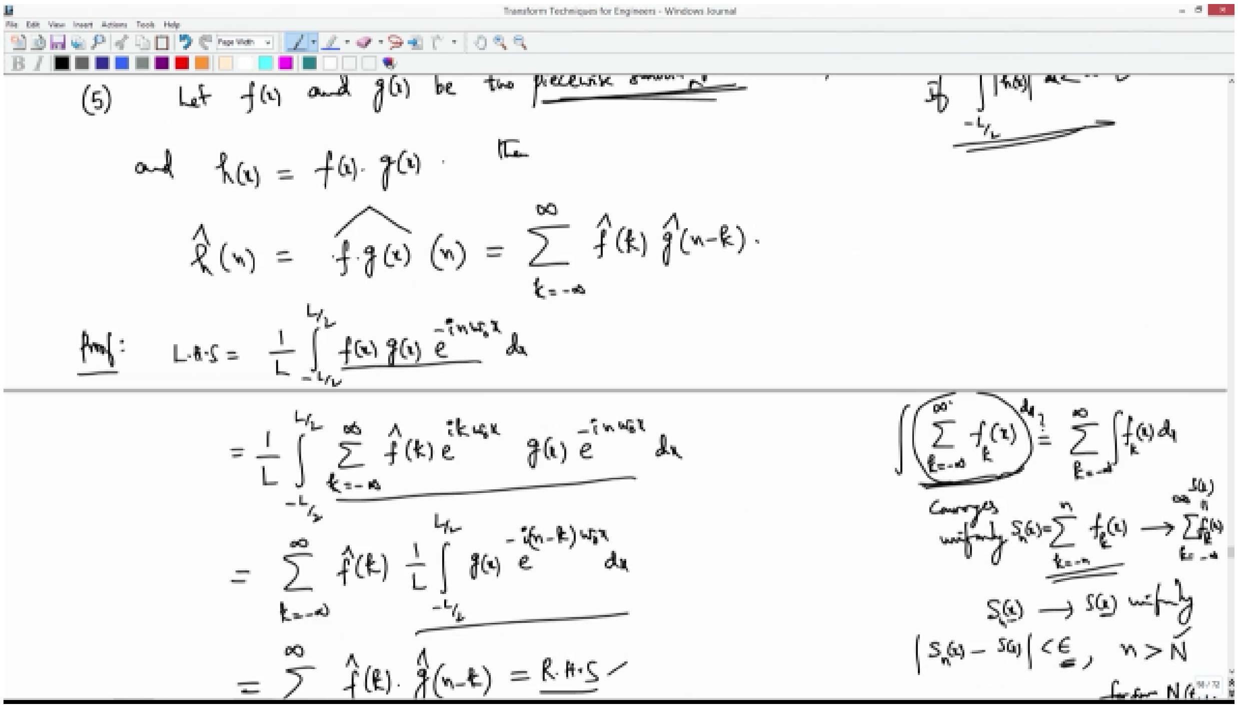


which this is converging to 0, so we fix X value and speed it converges to 0, and you fix any X value speed at which it is converging to 0, so that is among all the speed at which it is converging to 0, and this should have minimum speed among all of them, that should be valid if you have such a minimum speed, least speed that works for all convergence, suppose at you fix X1, X2, and X3, okay, the speed at which this converges to, if you are at X1 FN(x1) converges to F(x1), you will find the speed, speed of convergence, the way the speed it converges, the speed converges here, and the speed converges here, if you look at like that everywhere if you have the least speed of convergence, so with that least speed it works for every, that is the minimum speed so you should have a minimum speed, and for all points if there is a minimum speed that works for at every point, that means there is a N, so then you have a uniform constant, there is a convergence with that speed at every point, so basically that means you have uniform convergence.

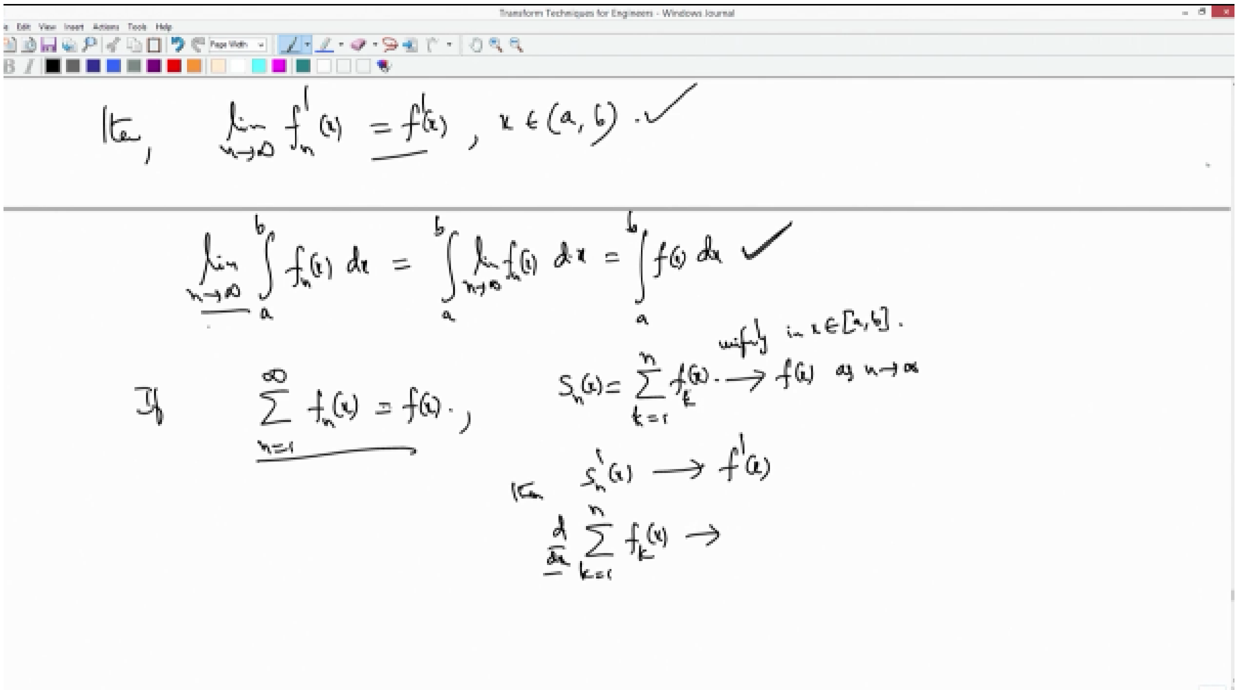
So roughly you can say like that, so mathematically it says that you see there you have some N that works that depends only on epsilon, okay, and but not it doesn't depend on the domain X value, so you can just look into some calculus books what is the uniform convergence you can get this, so if you have a uniform convergence such a uniform convergent sequence of functions and what happens? A limit FN(x) as N goes to infinity = F(x) uniformly, uniformly if you have such thing I can differentiate term by term FN dash(x) = F(x) okay, uniformly in X in A, B, suppose, suppose you have this and you can do term by term integral, so the derivatives limit also goes to that 1, and you can also X belongs to A, B, so derivatives means is defined only in the open interval, so this is this and you can also integrate from A to B, FN(x) DX this limit N goes to infinity, this is same as integral A to B you can take this limit inside FN(x) DX, as N goes to infinity, so this is nothing but A to B F(x) DX.



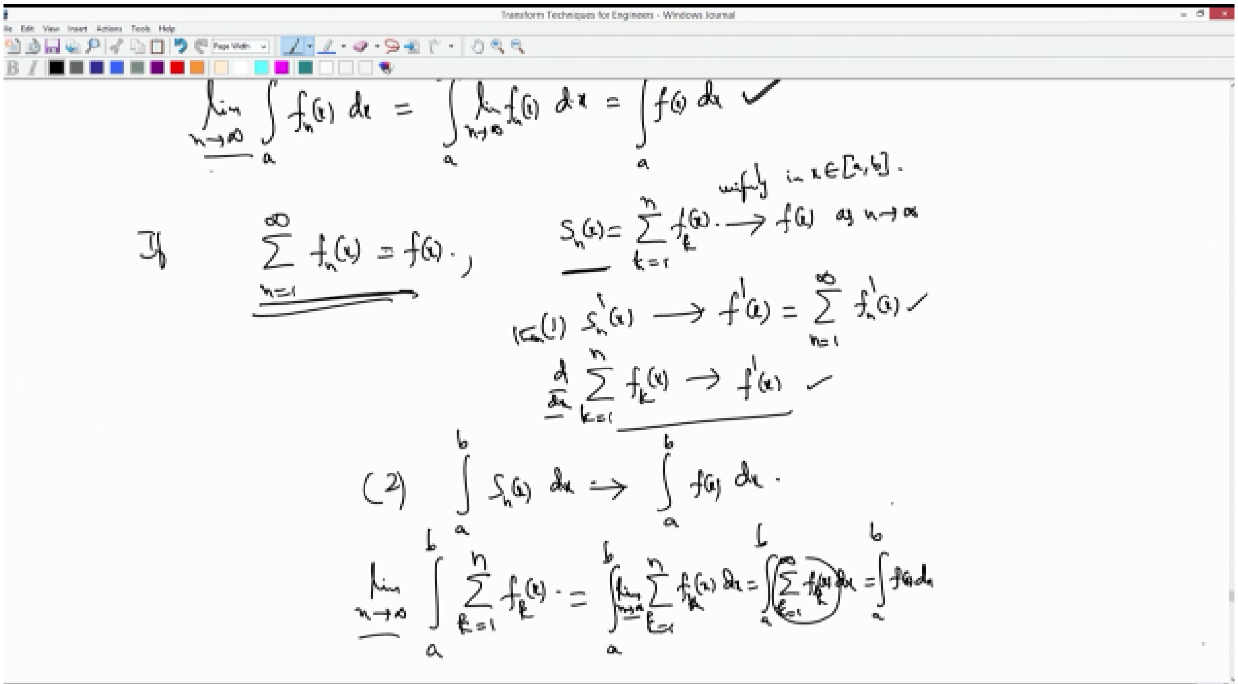
So if this is the case, if it converges uniformly then we have roughly these results, this is one and this is two, you can do term by term, you can do differentiation and the derivatives you can pass the limit inside, okay, this is basically what happens I can pass this limit inside the derivative, this is DDX I can pass this limit inside the derivative so that means this is DDX of this, this we already know that this is F(x) so DDX F(x) is F dash, so when I have DDX here this will be simply F dash once I pass this limit inside that becomes DDX of limit of FN(x), that is limit of FN(x) is F(x), that is exactly this one, so whatever I am doing here I am passing this limit inside this integral, here I am passing this limit inside DDX derivative, so these two results are true whenever you have uniform convergence, so there is the sufficient conditions under which you can do this operations.



And what is its relevance here? In the property 5, if you see a property 5, and when you are given two piecewise periodic functions with periodic L you want to show this result, so when you do this somewhere here, in this I allowed this integral I passed this integral inside this summation, so far we have done so I'll just do the analog early what is this so, if this is uniform convergence analog is the equivalent things also you can put it, so if you have uniform converging so you can also see this series of functions, for example if you look at the series of functions N is from 1 to infinity you can view this as this, this series, okay, this if you call it F(x), this you can view it as SN(x) which is a partial sums, N is from 1 to or rather K is from 1 to N, FK(x) so you consider this as SN, this converges to, as N goes to infinity this converges to F(x), right, it's clearly as N goes to infinity, as N goes to infinity this certainly its converges to F(x), because this sum is this, if this converges to uniformly, uniformly in X belongs to A, B then you apply those results, so you can write SN dash(x) is F dash(x), what is that SN dash? SN dash converges to F dash(x), what is SN dash? That is simply K = 1 to N FK, DDX of this of FK(x) this converges to, what happens to this one? So basically you can pass this limit inside okay, so basically you can pass this SN, DDX of SN that is FK dash, so that is finally becomes F dash(x), okay.



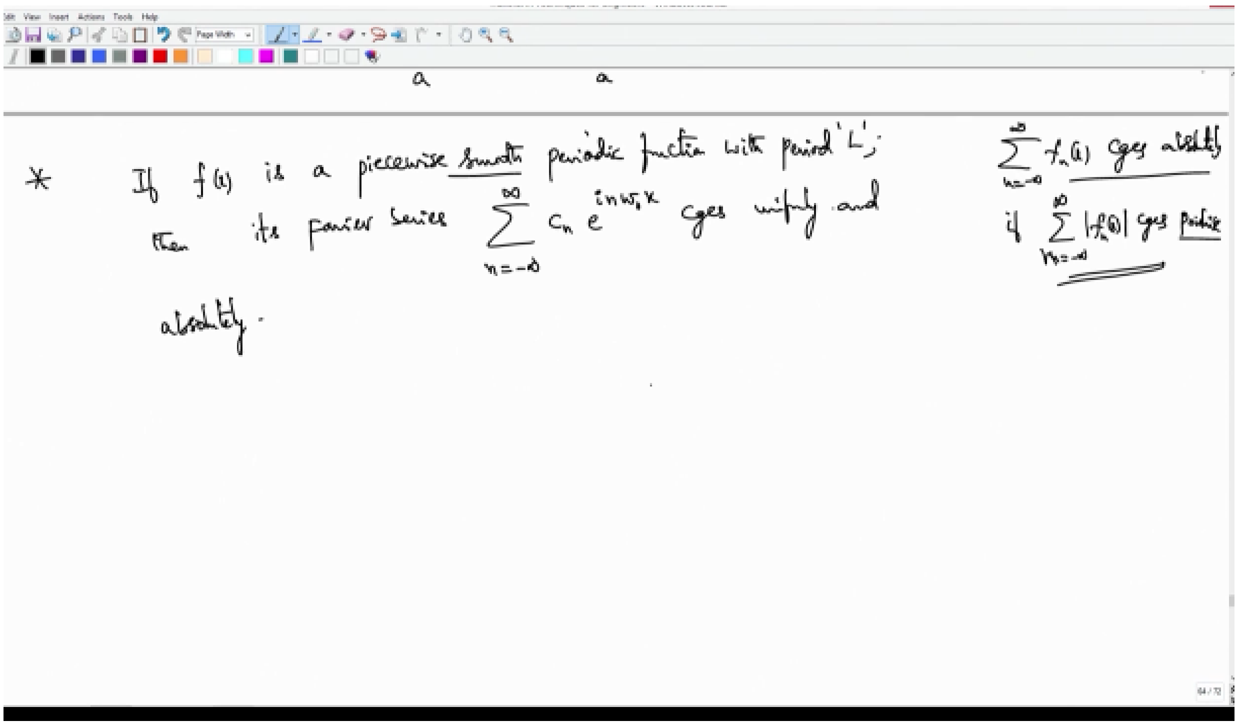
So if you say F dash(x) is simply FN dash(x), K is from or N is from 1 to infinity, if this is your F dash(x) and this converges to F dash(x), so you can view this as a sequence of partial sums then apply these results, now you have a sequence that converges to sum, and similarly and this is one result, and the second result is you can integrate term by term, so integral SN(x) it will be DX is same as this converges to integral, so here I have a limit outside I pass it on that is going to be SX, so limit SN(x) as N goes to infinity that is F(x), that to be DX, so the same thing so you get, so what does it mean? So this means this sigma N is from 1 to infinity, sorry, K is from 1 to N FK(x) this is my SN(x), now I have this for this integration it will be, I have this limit N goes to infinity, this is you know I can pass this limit inside this integral, because it is uniform convergence, A to be sigma, K is from 1 to N, FN(x) as a limit inside now, DX this is same as A to B, this going to be K is from 1 to infinity, and once I pass this limit this is FK(x) DX.



So what is this one inside? It’s nothing but A to B F(x) DX that is the sum, okay, so you could pass this inside limit and you end up, what exactly is this? Is this one, so you pass this one, so both are possible, so either you look at series of functions or a sequence of functions, series of function is equivalently you can put it as a sequence of partial sums, so once you have the uniform convergence, if you assume the uniform convergence and you can do these two operations, so that is what we do. So once F is, if F is a piecewise smooth periodic function that means F dash is a piecewise differentiable and also F double dash is also piecewise differentiable or any nth derivative is piecewise differentiable function, and so in that case you call this piecewise smooth function, F is piecewise smooth function if you any derivative is also piecewise a differentiable function.

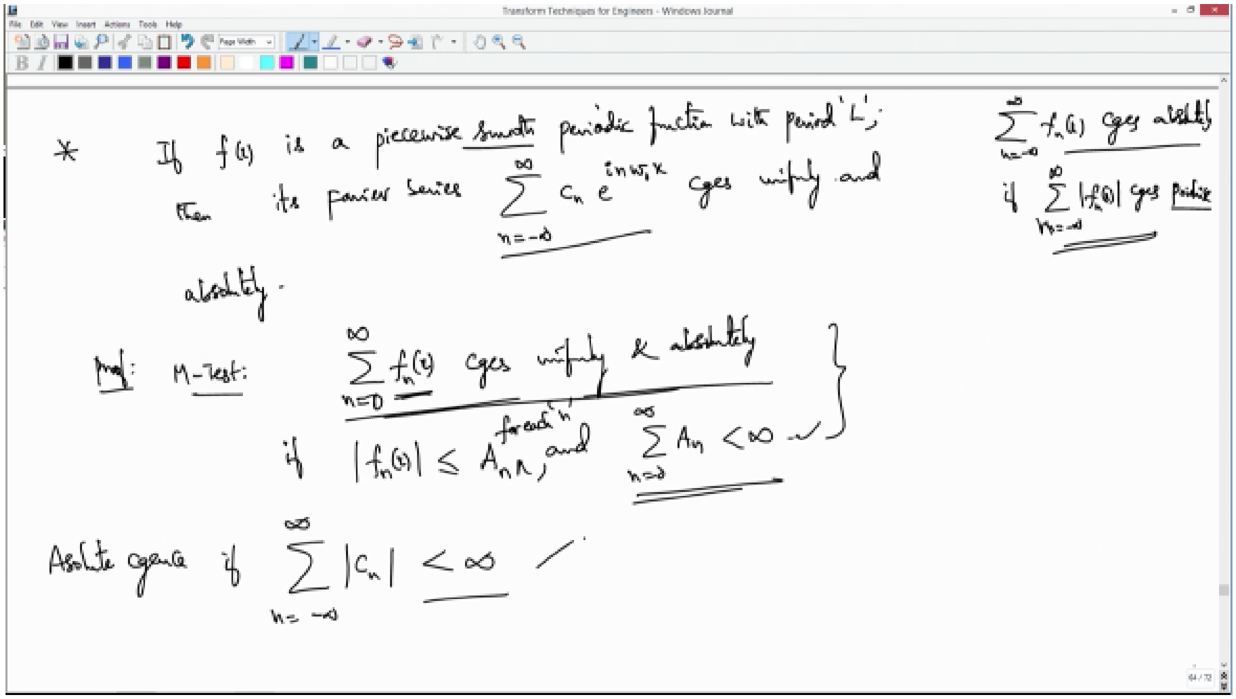
In that case we can pass this limit, one can show that the partial the series the Fourier series is actually converging uniformly and absolutely, so basically we use this uniform convergence, so to show this uniform convergence so we'll just see this one, so if F(x) is, I’ll just write it as a result so if F(x) is a piecewise smooth function, smooth periodic function with period L, then what you have is it's Fourier series, obviously once it is piecewise differentiable it has a Fourier series that is CN E power +I so, IN omega naught X, N is from - infinity to infinity, so this series converges uniformly and absolutely as well.

So absolutely means some series sigma FN(x) X converges, N is from - infinity to infinity converges absolutely, if when do we say this is absolutely? If mode FN(x) X converges, N is from - infinity to infinity, if this is the case, if this is converges to some number okay if this converges point-wise, if you say only point-wise then you say that is absolutely converges, you fix X and you take the modulus value if it converges you say that is absolutely converges, so

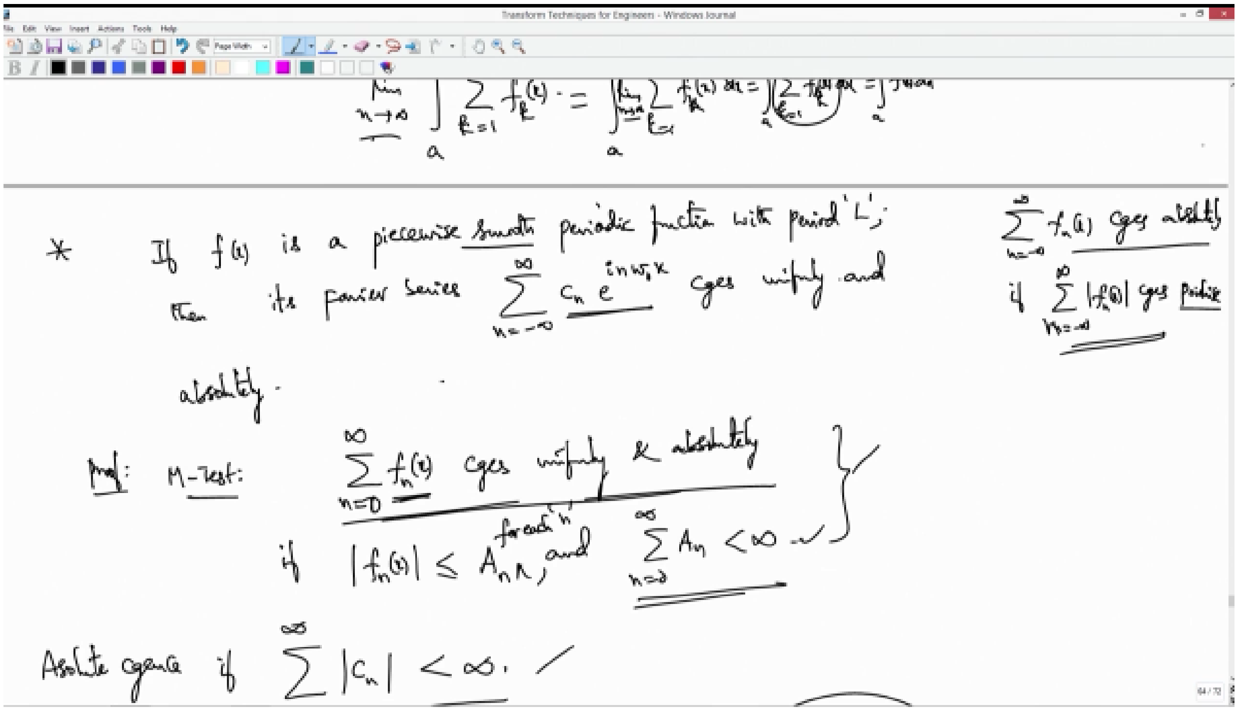


this is extra so this is absolutely convergence is also true, so to prove this one we need so this is basically we use M-test, okay, so what we use is to show this, to show that this is true what we need is we use M-test, if you have M-test is, Weierstrass M-test that tells you that you have two series FN(x), you want to know whether this series is a uniformly convergent or not, you want to see this uniform convergence, converges uniformly and absolutely if mode FN(x), so if you have, if you can take the modulus of this is bounded with some numbers let's call this some AN's they are bounded, they are bounded and sigma AN, N is from 0 to infinity or - infinity to infinity also you can take, okay, and this is bounded if once this is true so you have a bound for every FN, so this is true for every N, for each N okay, if this is true for each N, each N you have some number AN which is bound for that function.

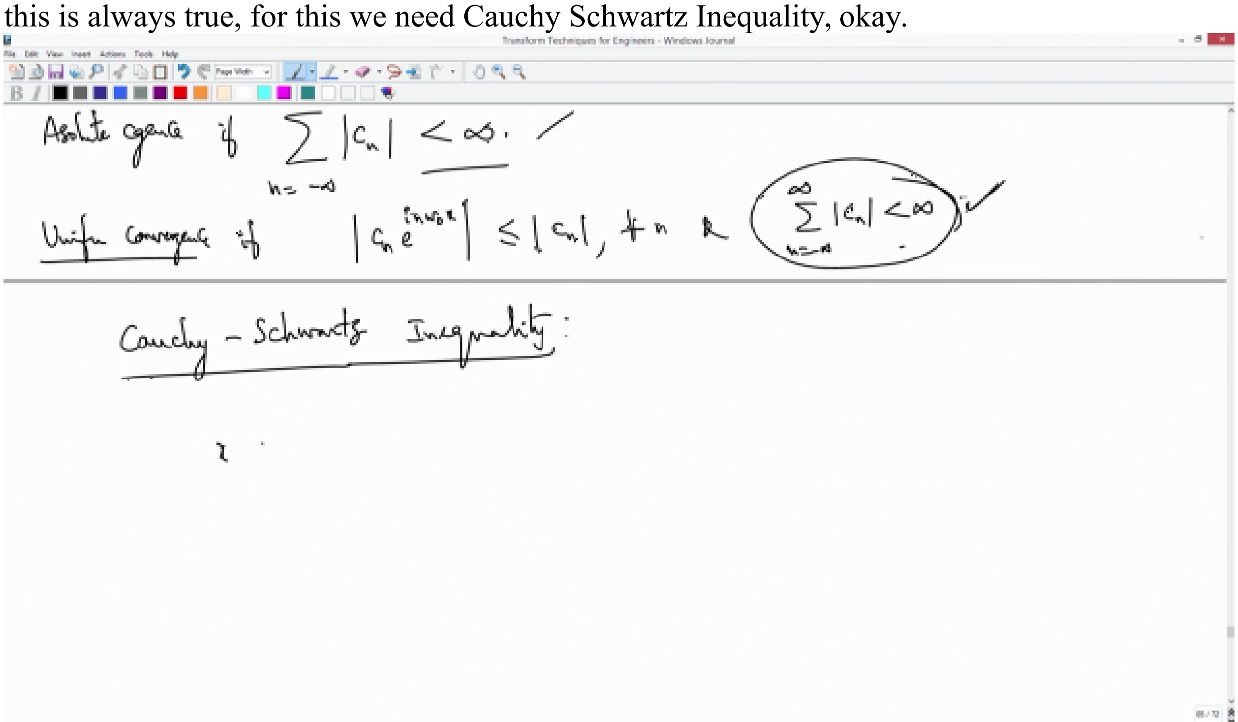
And then, and you have those numbers, is this number series is finite, then this converges uniformly and absolutely, that is what is the M-test, so we use directly this, this is a very important result in the calculus, so we will try to apply this to this Fourier series, what happens? When do you, so you have this, you want to show that this series is, Fourier series is a uniformly convergent or absolutely convergent, so absolute convergence if you see, what is that



absolute convergence? You take the modulus of this that is only simply mode CN, okay, absolute convergence if this is finite period okay, so if this is true, this is absolute convergence. Uniform convergence by this M-test, if at least this is a sufficient condition, if you see this term is bounded with, this is the modulus of that is CN, mode CN, so you have this CN E power IN omega naught X, this modulus is obviously less than or equal to modulus of CN for each N, and if sigma CN, N is from - infinity to infinity, if this is bounded, okay, so uniform convergences of this Fourier series is guaranteed, where is the Fourier series? Here, so this series is guaranteed, uniform convergence of this Fourier series is guaranteed once you have, each of the terms are bounded with this CN's that is known, and that's not right, this modulus of this exponential function E power IN omega naught is 1, so this is this, and if I know that this quantity, if this is true then from the M-test I can say that this is uniformly in absolute convergence, okay, then I can simply pass the limit, so I can integration here is same as this is equivalent to summation, I can take this integral inside, okay, that's what I will do then you can justify the property that I have proved in the last video, fifth property that I have proved in the last video.

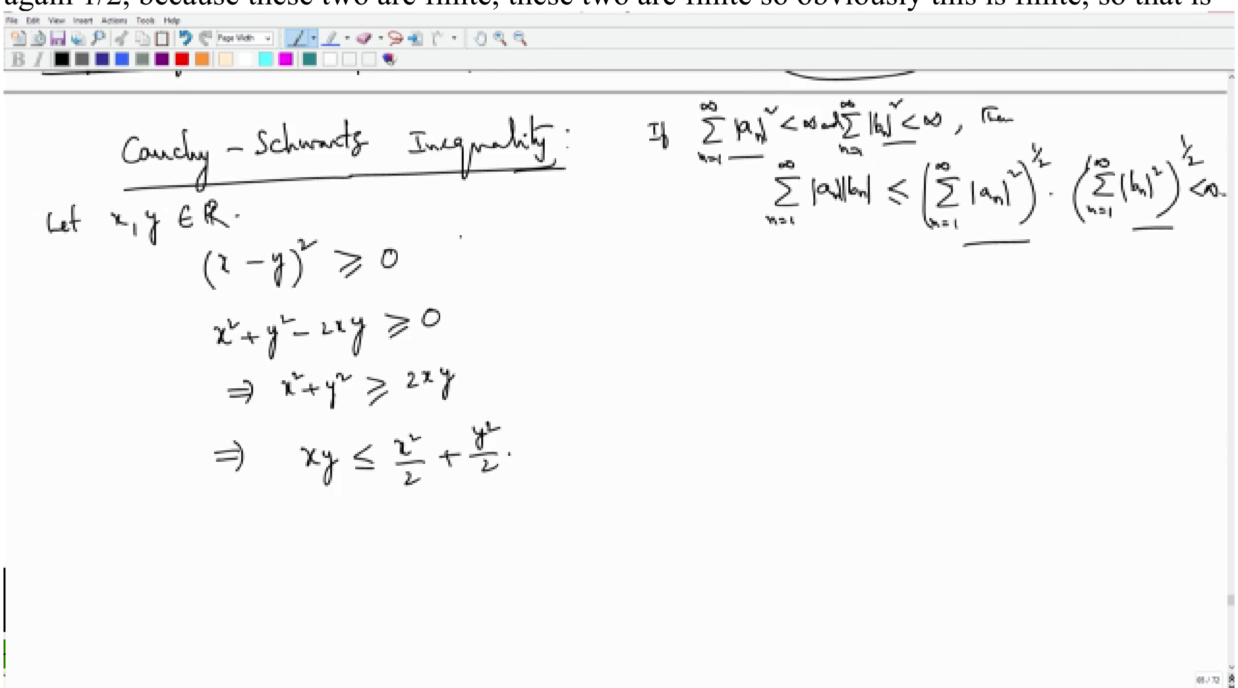


So to show this one, to show this one so given a Fourier series sigma CN E power IN omega naught X, if that series is uniform convergence you need sigma mode CN should be finite, so this is actually true, if F is a piecewise smooth function, okay, if F is piecewise smooth function this is always true, for this we need Cauchy Schwartz Inequality, okay.



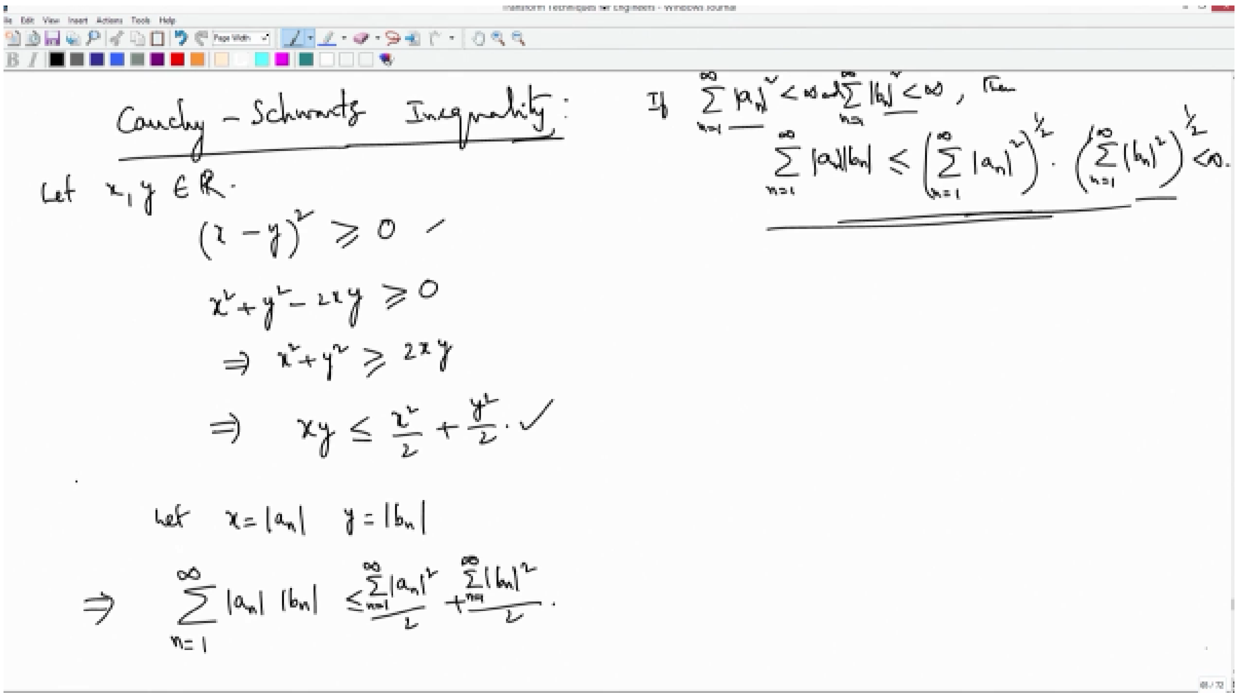
So what is this one? So you start with simple, you take two real numbers, you call this X and Y, if you take the difference between X and Y, let X and Y belongs to real numbers, 2 real numbers you take, you take the difference and square it, what is its value? Is always positive or 0, so if you expand this X square + Y square - 2XY is greater than or equal to 0, so this means X square + Y square is greater than or equal to 2XY, so what does it mean? So this is always will give me

XY is less than or equal to X square/2 + Y square/2, so this is a starting point to show this Schwartz inequality that will show for the sequences, so Schwartz Inequality tells you, I'll just write directly, so if you have a two series, if sigma AN square, N is from 1 to infinity, if this is finite okay, let us say mode so they're all positive numbers so that I don't put the modulus. And sigma BN square, N is from 1 to infinity, let us say and this BN square these are number, this is also finite then sigma AN, BN okay, so if you have we can also keep the modulus square if you want, so there can be even complex numbers, so this modulus of AN, and modulus of BN both together N is from 1 to infinity, question is whether this is finite, okay, it's in fact if these are finite this should be finite because of this inequality sigma mode AN square, N is from 1 to infinity you have this with root and sigma BN modulus square N is from 1 to infinity, for this again 1/2, because these two are finite, these two are finite so obviously this is finite, so that is



what is actually this Cauchy Schwartz inequality tells you, so we will prove that so that this using this we will try to show that this is finite okay, the sigma mode CN is finite, so to show this Schwartz inequality we will try to start with, we started with two real numbers and the difference is always, a square of the difference is always greater than or equal to 0, and that gives you this inequality.

Now next step is I consider simply I take X = mode AN, and Y = mode BN, okay, let's take this, then what happens? Mode AN, mode BN is less than or equal to mode AN square/2 + mode BN square/2 this is true for every N right, so you can just add them up, do you have any quality? That is true for every N, you can sum it up also, N is from 1 to infinity then you have, sum is from N is from 1 to infinity, sum is from N to 1, 1 to infinity, okay, that's what you have, okay.

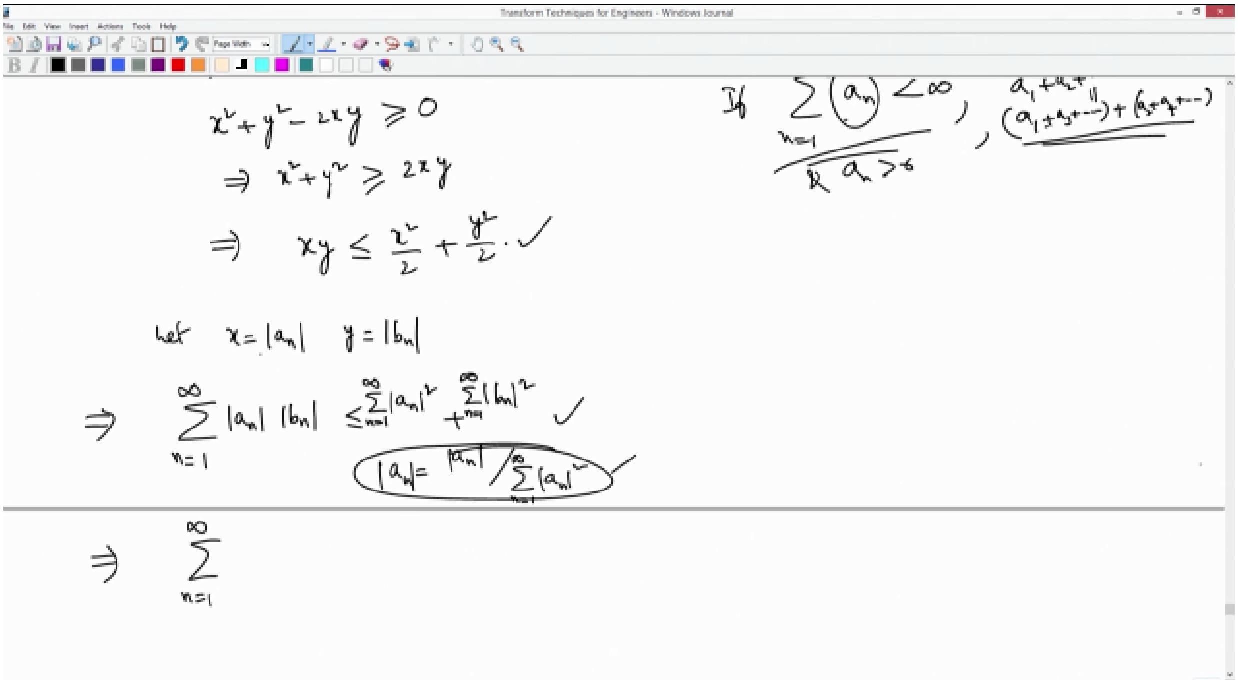


So here we have, we assume that so these are the numbers they are all finite, they are all real numbers and there are real numbers they are all finite, so because they are finite already we know that that sum is finite so given that their sum is finite you can sum them up AN + BN this series, this sum once you know that this is a finite I can see that this I can write like AN + sigma BN, if they are finite this whole thing I can sum the way I like once or in other words once you know that a converging sum, a finite sum AN, N is from 1 to infinity, if this infinite sum is finite, if this is the case I can sum it, this is actually A1 + A2 and so on, I can also add, I can rearrange these terms and then add it, okay, I can do A1 + A3 and so on first, and then plus A2 + A4 and so on, so this two sum together will be same as this one, if this is finite okay and they are positive numbers, this should be, AN's are positive, if AN's are positive and this sum is finite this is what I can do always, so if I use that, that is what I can do, so once I sum it up what you have is, summation is actually A1+B1 + A2+ B2 and so on like that, so now I rearranged such that A's I put it together and B's I can put it together on the right hand side, this is what is the result, okay.

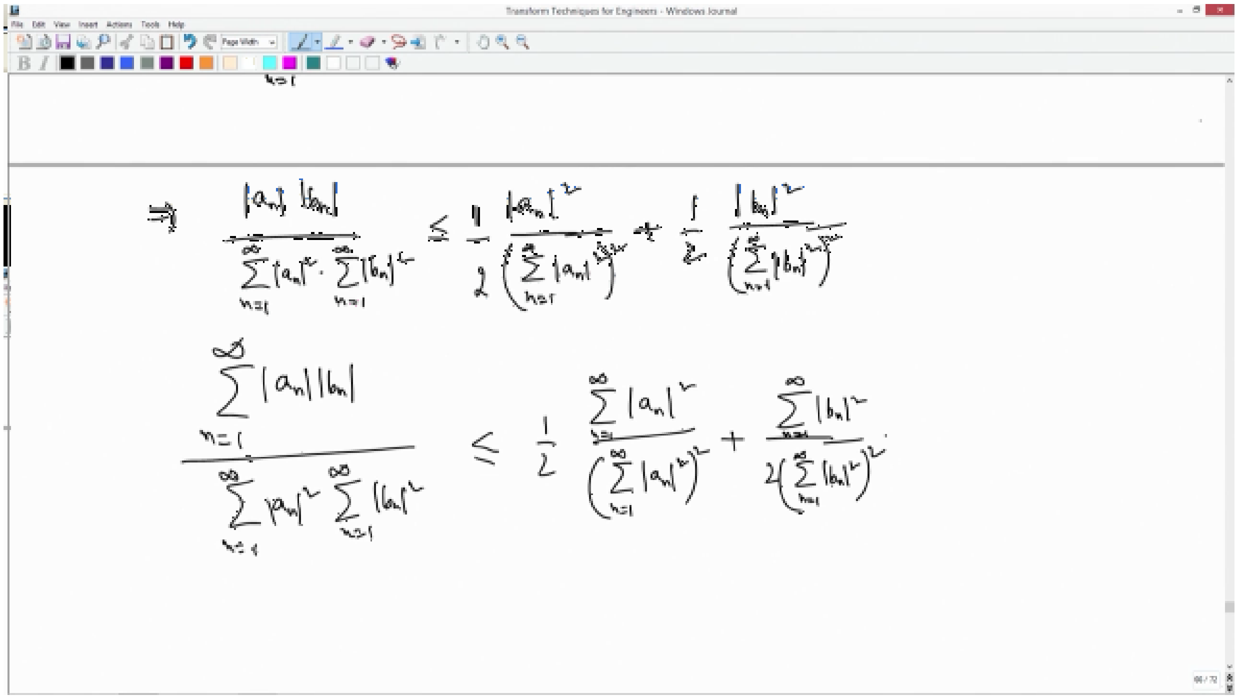
Now next step is I consider I simply, instead of choosing this I choose because this sum is finite sigma AN mode square, N is from 1 to infinity, here also I divided with N is from 1 to infinity mode AN square assume that is positives okay, because it's positive and it should be nonzero okay if I choose this one again like earlier if you substitute so instead of simply taking this if you substitute, so instead of X, instead of AN I will replace this term, right, I can simply I have chosen YN, BN like this and then I summed it up.

Now instead of AN I can choose AN divided by sigma AN, okay, so if you do this N is from 1 to infinity, instead of AN I can also choose AN rather starting, now itself you can do, so this divided by some number S, some number S, so what happens? You have A square and you have here what you get? So you have S square right, S square and S square, it's not the way to say, you know, so we have chosen X equal to this, Y equal to this, and you add it up you get this one, this is what you get, okay.

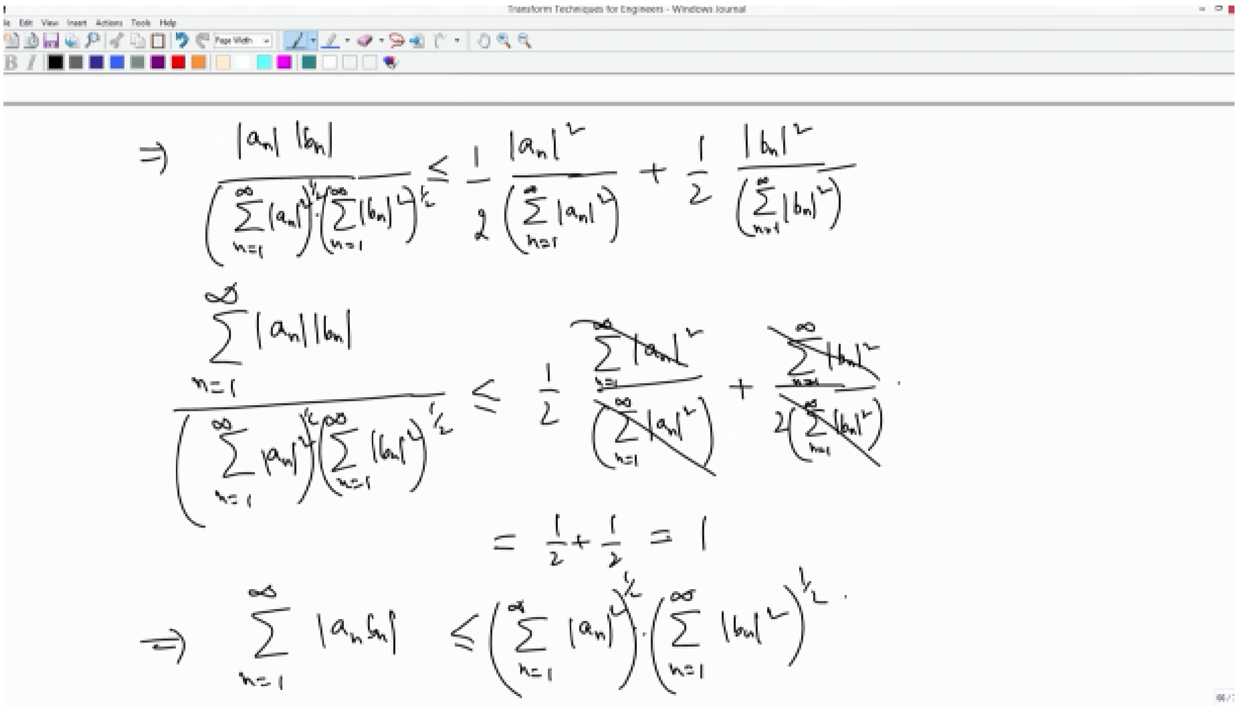
Now instead of choosing this you can also replace mode AN's divided by sigma AN mode square, N is from 1 to infinity, so you choose instead of AN you choose this quantity, so here



itself when X equal to, you can choose this by this quantity, okay, so let me put it that way so let us start with that, so let us choose X equal to this divided by sigma mode AN square, N is from 1 to infinity, and Y = mode BN divided by sigma mode BN square, N is from 1 to infinity, then what happens? AN, BN divided by sigma N is from 1 to infinity, mode AN square into sigma BN mode square, N is from 1 to infinity, this is less than or equal to, X square is AN square/sigma AN square, N is from 1 to infinity, okay, of this square by 2 + by 2 sigma BN square/sigma mode BN square, N is from 1 to infinity this whole square, so this is what is true for every N, now you can sum it up both sides to see that sigma AN, BN, N is from 1 to infinity and you can see that divided by sigma AN square, this is a number fixed number 1 to infinity, sigma mode BN square, N is from 1 to infinity, this one is less than or equal to, now you have this sigma, so sigma mode AN square divided by, N is from 1 to infinity, this is a number fixed number, so this is like with 2, 1/2 this whole thing comes out that is 1/mode AN square whole square, okay + 1/2 this sigma, N is from 1 to infinity mode BN square whole square, now here I get because I summed up over N, this BN square, N is from 1 to infinity, okay.



Now what? So you can cancel this, this gets cancelled, okay, so this quantity is less than, strictly less than 1/2 because this is 1 by some positive quantity, which is less than 1, and this plus another 1/2, this is also 1 divided by this sigma mode BN square is always less than 1, okay, because these are all positive quantity, to avoid this you don't want this inequality here what you can do is instead of choosing X equal to this, here you choose this with root, if you choose with root what happens? So you have this with root, you have this with root, and here there is no squares, okay, because square root once you square it this is going to be without that, so that what you have is then you don't have this one, you don't have this, and now this completely cancels, so what is left with is 1/2 + 1/2 which is 1, so this is with 1/2 with 1/2 so this implies sigma mode AN, BN this product is less than or equal to sigma, N is from 1 to infinity, AN square into sigma BN square, N is from 1 to infinity with square roots, so this is exactly your Cauchy-Schwartz Inequality that is what we have proved.



$$\Rightarrow \frac{|a_n|\,|b_n|}{\left(\sum_{n=1}^{\infty}|a_n|^2\right)^{1/2}\left(\sum_{n=1}^{\infty}|b_n|^2\right)^{1/2}} \leq \frac{1}{2}\frac{|a_n|^2}{\left(\sum_{n=1}^{\infty}|a_n|^2\right)} + \frac{1}{2}\frac{|b_n|^2}{\left(\sum_{n=1}^{\infty}|b_n|^2\right)}$$

$$\frac{\sum_{n=1}^{\infty}|a_n|\,|b_n|}{\left(\sum_{n=1}^{\infty}|a_n|^2\right)^{1/2}\left(\sum_{n=1}^{\infty}|b_n|^2\right)^{1/2}} \leq \frac{1}{2}\cancel{\frac{\sum_{n=1}^{\infty}|a_n|^2}{\left(\sum_{n=1}^{\infty}|a_n|^2\right)}} + \cancel{\frac{\sum_{n=1}^{\infty}|b_n|^2}{2\left(\sum_{n=1}^{\infty}|b_n|^2\right)}}.$$

$$= \frac{1}{2}+\frac{1}{2} = 1$$

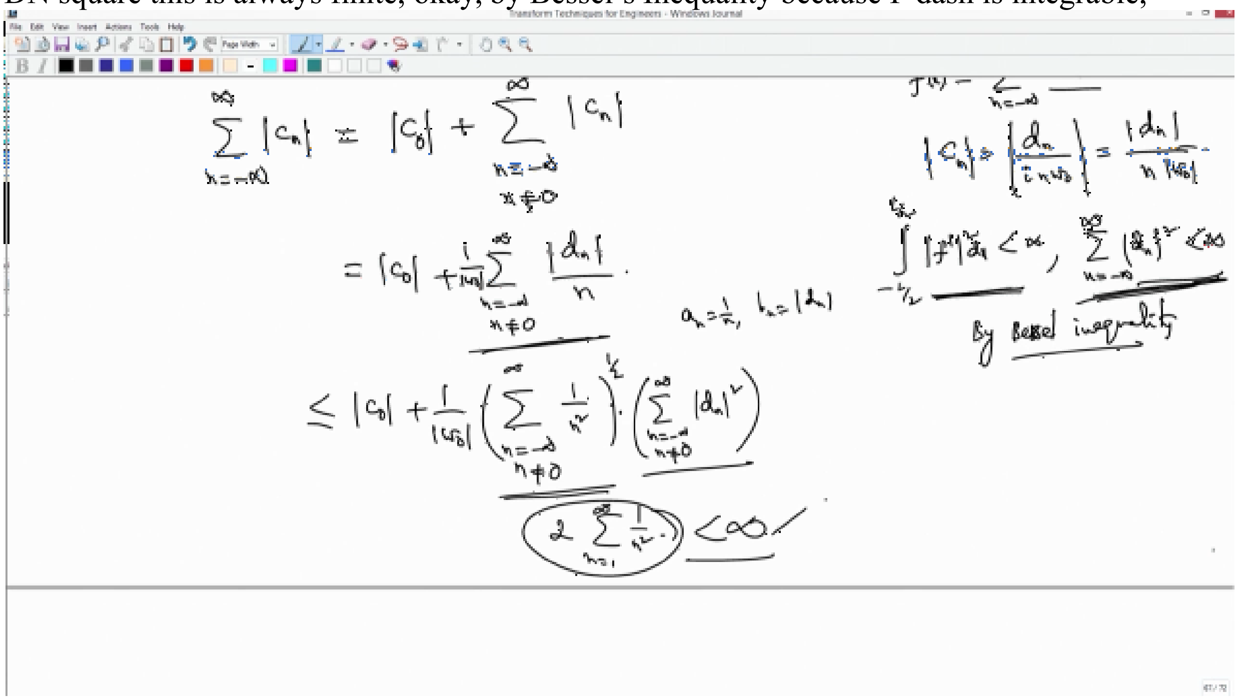
$$\Rightarrow \sum_{n=1}^{\infty}|a_n b_n| \leq \left(\sum_{n=1}^{\infty}|a_n|^2\right)^{1/2}\cdot\left(\sum_{n=1}^{\infty}|b_n|^2\right)^{1/2}.$$

So how do we show that this sigma CN is finite? Sigma CN is finite, now we have to show this one, okay, if CN is a Fourier coefficient of a function F that is piecewise smooth function, if it is piecewise smooth function why is it so? So we'll start with, so we will show that this is this, N is from 1 to infinity or rather you have - infinity to infinity, so this mode CN is actually equal to C0, 0 term I put it outside, mode C0 + N is from - infinity to infinity, N is not equal to 0 and what you have is this CN, right, so because it is piecewise differentiable function and you have seen that F dash, if F is piecewise differentiable function, F dash(x) exists, okay, F dash(x) is a piecewise continuous function, but now we're seeing that if it is a piecewise smooth function F double dash(x) exists, and F double dash(x) is actually piecewise continuous function that means F dash(x) is piecewise differentiable function implies it has a Fourier series, and once it has a Fourier series what we have seen is F dash(x) is sigma, so it has a some Fourier series, N is from - infinity to infinity, and DN into E power IN omega naught X, that is this Fourier series, and we have actually seen that DN's are in terms of F that is IN W naught CN will be DN, CN, okay, DN will be CN, so this is equal to this, E power IN omega naught X, that is my F dash (x).

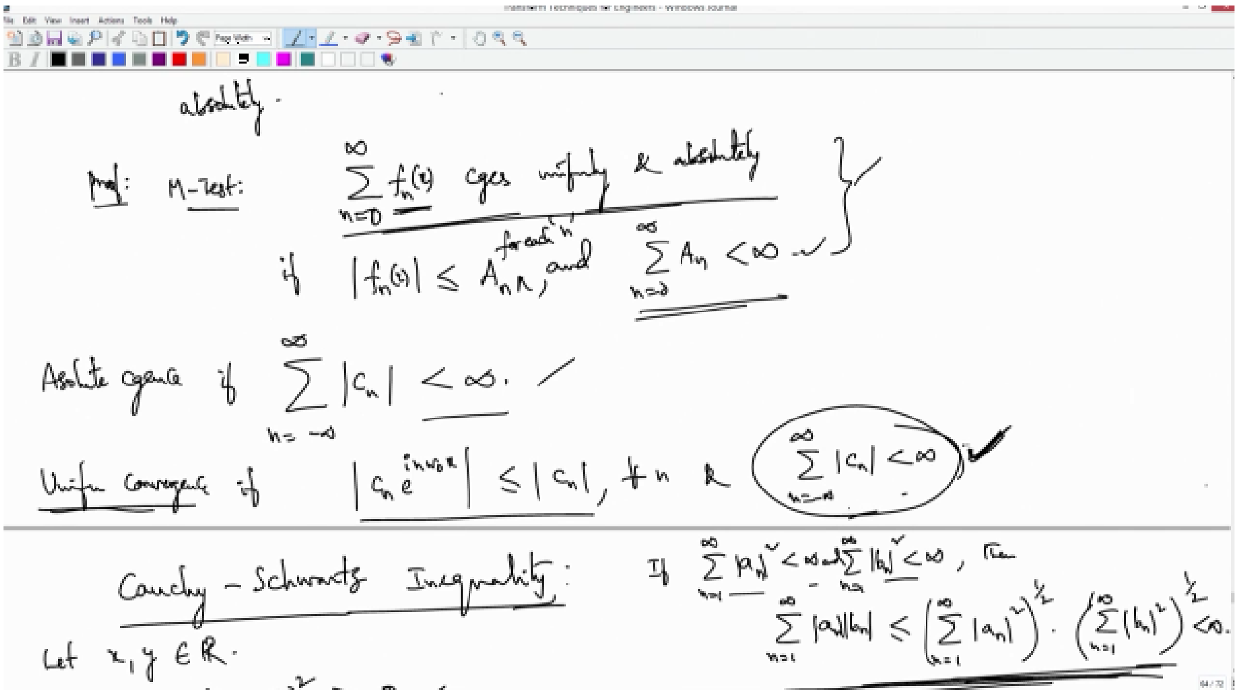
So because DN equal to this, okay, so what is your CN? CN's I can write, CN's I can write, so from this I can write CN's as DN divided by IN omega naught, so mode CN will be mode DN, so this is mode DN divided by -1 so mode N, so this is N mode omega naught, so N into omega naught 2 pi/L this is the constant, okay, so this is 2, so mode omega naught, that is anyway constant 2 pi/N, so you can write like this so this is C0 + sigma, N is from - infinity to infinity, N is not equal to 0, now we have this, this comes out 1 divided by mode W naught that is a constant comes out mode DN divided by N, okay.

And since F is a piecewise, F dash is piecewise differentiable function and it is square integrable function, F dash square –L/2 to L/2 is finite, once it is finite by Bessel's inequality you have this sigma DN say Fourier coefficients DN square, right, so this DN square N is from 1 to infinity, this is also finite by Bessel's inequality, you just see earlier videos, Bessel's Inequality, if we know that F Dash is a piecewise differentiable function it has a Fourier series and because piecewise is a differentiable function it is also square integrable function, it is integrable function and implies absolutely integrable implies it is also because it is over a finite domain, it is also square integrable function, so because of this now because this is finite by Bessel inequality this quantity is finite.

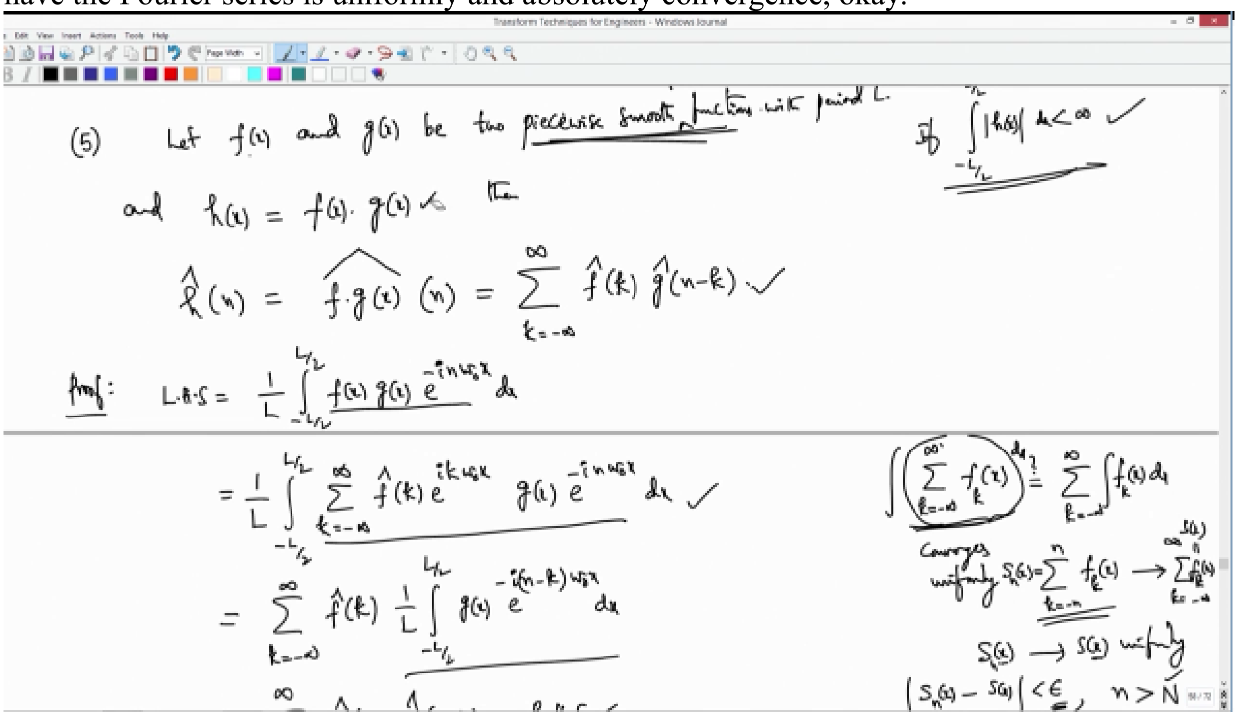
Now you apply Cauchy-Schwartz inequality for this, so you get mode C naught + 1/mode omega naught, now this is less than or equal to, if I apply this Cauchy-Schwartz inequality, N is from - infinity to infinity, N is not equal to 0, and what you have, this is AN is 1/N, BN is mode DN, so what is this one? So AN square, sigma AN square that is 1/N square into this 1/2 into sigma N is from - infinity to infinity, N is not equal to 0, mode DN square, so you see that mode DN square this is always finite, okay, by Bessel's Inequality because F dash is integrable,



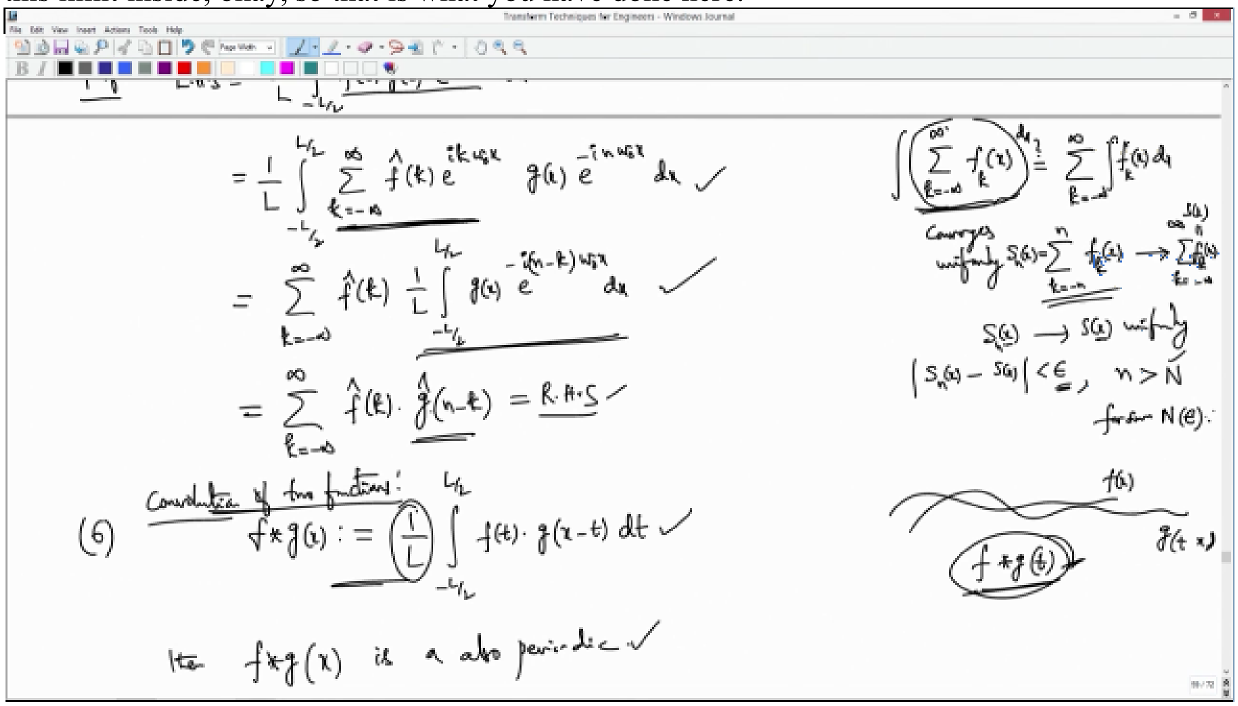
F dash square is integrable so this quantity is finite and we know that this is finite, sigma 1/N square is always finite, this is 2 times, N is from 1 to infinity, this quantity is finite, this is by calculus, so overall this is finite, this C0 is anyway constant that is a finite, W naught is constant so we have shown that this quantity is finite, once this is finite by uniform convergence this is true and this is always true, so this means uniform convergence, because of



this, this, this is true, that is same as this, absolute convergence is also true, so you have your fourier series once F(x) your function, your signal is piecewise smooth periodic function you have the Fourier series is uniformly and absolutely convergence, okay.



Now let me go back here to that property 5, that I have proved yesterday, now what I have is these are the two functions F and G be two piecewise smooth functions with period L, and I consider this product, this product if I consider then you have to show this is the one, and what I choose is the left hand side I started with a definition of this Fourier transform of product of functions F and G, and you introduce because F is a piecewise smooth function you can write F as Fourier series, we put this Fourier series, now I take this integral inside because this Fourier series F cap(k) that is your CN into E power IK W naught, and you can also include this one inside, okay, so only K, so K inside this K integral only you are introducing, you're passing the integral inside, so if you look at this one, since the series is uniformly convergent you can pass this limit inside, okay, so that is what you have done here.



So now this is legitimate and once you pass the limit this is exactly nothing but Fourier transform of G dash at N-K, so this is exactly what we want the right hand side, so this completely justified, a property 5 is proved, so for that you have to use this uniform convergence and Cauchy-Schwartz inequality it's a part of calculus that I have done here, just to recall so I tried to use minimal things to prove this and then prove the property 5, so this is how we can show this property 5, and then maybe next video we will try to do some examples, one or two examples we'll try to, we will have a recap of what we have done so far for the Fourier series, all the results, that limited results that we proved, minimal results we try to prove, okay, when we choose the signal, what kind of signal is, what kind of function it is whether it's always, once you give the signal most probably, mostly it is a piecewise smooth function, once it is a piecewise smooth function what are all the results we have mathematically, and how to calculate this Fourier transform and also some results on delta function that we may need further in later videos, that we'll try to do in the next video and finish the Fourier series part. Thank you very much.
[Music]

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