Graph Theory
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Lecture- 16
Part 1
Augmenting Path Algorithm

Welcome to the 1st part of lecture 16. In the previous lecture we have learnt what is residual network and Augmenting Path we will continue with this again. In this lecture we learned how to find maximum flow using the Ford Fulkerson algorithm or the concept of augmenting path and residual network.

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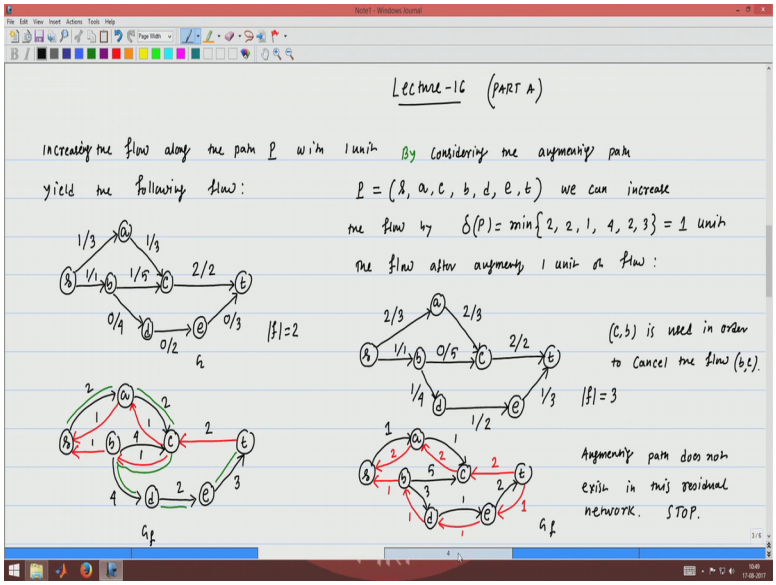

So, this is my example in lecture 15. So, as I said that this is the residual network with respect to this flow.

Now, to increase the flow amount we will look for a augmenting path in the residual network. So, here we can see that there is a augmenting path, you go from s to a a to c and then you can go from c to b using the backward edge. So, this is the first time you can see that the augmenting path is using a backward edge and then from b to d d to e and e to t. Now, by considering the augmenting path P which is s, a, c, b, d, e, t, this is a path from s to t in the residual network, that is why it is augmenting path. We can

increase the flow by delta unit. So, the delta P is the capacity of this path this is minimum of cot minimum of 2, 2, 1, 4, 2, 3.

So, this is minimum of 2, 2, 1, 4, 2 3 which is equal to 1. So, we can increase the flow by 1 unit because the capacity of this path is 1 unit. The network for the flow after augmenting 1 unit of flow is the following. So, here this is very important part because here you can see that the augmenting path consider a backward edge. So, the use of a backward edge means basically you are cancelling this flow. So, look at the new flow now, this is s a a b c t d and e.

Now the augmenting path is this 1 s a so along this augmenting path I am sending 1 unit of flow extra so; that means, along s a the new flow value will be before it was 1. Now it will be 2, 2 and the capacity is 3, now along ac before the flow was 1 now it will be 2. So, 2 and the capacity is 3, ct is not involved in the augmenting path you can see the augmenting path in green color. So, there will be no change in the flow value and similarly s b there is no change because this was not involved in the augmenting path. So, the flow value will remain 1 and the capacity is 1.

Now, the change this is a very important step see here we have used a backward edge, c b which is in red color and the flow is 1 the capacity of this backward is 1. So, use of a backward edge in the augmenting path is basically cancelling this flow you are cancelling this flow. So, the new flow will be 0 and the capacity is 5, this is very important step. So, here I want to also write down that in the residual in the augmenting path c b so c b is a part of the augmenting path c b is used in order to cancel the flow b c.

So, you just remember that use of a augmenting use of a backward is in the augmenting path is basically cancelling that flow, here is the use of residual network I will explain it again. Now the flow along b d it was 0 before now it will be 1 and capacity is 4, the flow along d e it was 0 before now it will be 1 and capacity is 2 and the flow along e t it was before 0 now it is 1, now you can see that the flow value the flow of the previous network the flow value here it was 2.

Now, the flow value here it is has been increased by 1 the new flow value is 3, now is this 3 is the maximum flow possible in this network we do not know what you have to do is that you again compute the residual network with respect to this new flow and if you can find an augmenting path then you can augment that amount of flow along that path if

you do not find an augmenting path you stop there and you declare that the current flow value is the maximum flow value.

This is what the Ford Fulkerson algorithm is let me just complete this example what I will do is that to check whether this flow value 3 is the maximum flow or not, again I have to compute the residual network with respect to this new flow which has value 3 and I look for a augmenting path if I do not find an augmenting path I will stop. Let me just draw the residual network with respect to this new flow s a b c t d e. So, here I think by now you understood what is how to compute the residual network. So, here I will just draw the first all the forward edges forward edge with capacity 1 let me do 1 by 1.

So, and the backward edge there is a backward edge of capacity 2, for ac there is a forward edge of capacity 1 and there is a backward edge of capacity 2, for c t there is no forward edge there is a backward edge of capacity 2, for s b there is no forward edge there is a backward edge of capacity 1, for b c there is a forward edge only of capacity 5, b d there is forward edge of capacity 3 and a backward edge of capacity 1, b e there is a forward edge of capacity 1 and a backward edge of capacity 1, e t there is a forward edge of capacity 2 and the backward edge of capacity 1. So, this is the residual network with respect this new flow G f.

Now, can you find and augmenting path can you find a path from s to t, from s you can go to a you can go to c maximum and from s you can go to a and then c and then from c you cannot move to t or you cannot move to a you have to again go back to a sorry you cannot from c you cannot go to b. So, it is clear that there is no path from s to t in this residual network. So, you can say that Augmenting path does not exist in this residual network and you stop here this is the stopping criteria for the algorithm. So, there is no augmenting path with respect to this residual network means you have to stop here and then declare the current flow is the maximum flow.

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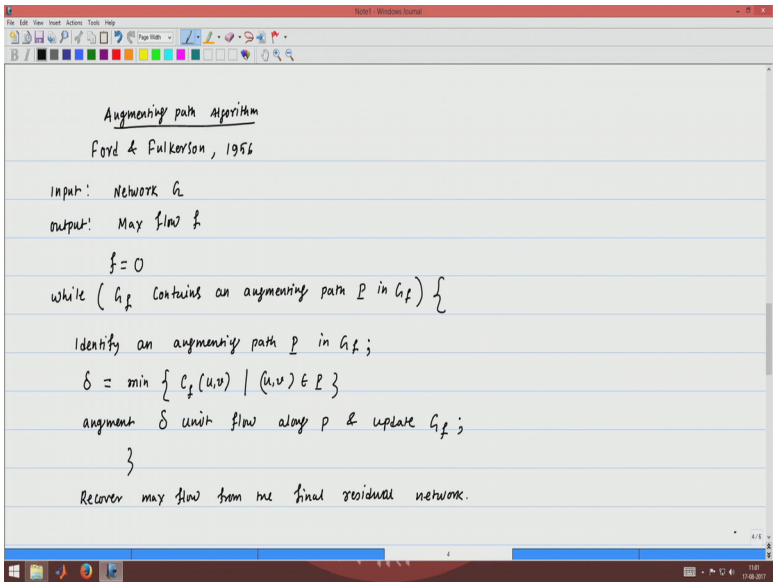

So, this is what the Ford Fulkerson Algorithm is and let me just write down the Ford Fulkerson Algorithm formally 1956 this is also called augmenting path algorithm. So, input to this algorithm is a network G and output to this algorithm of this algorithm is maximum flow f. If you remember that we started with arbitrary flow which has capacity 1. In fact, you can start with a 0 flow because, that arbitrary flow could be a 0 flow. So, initially your flow value is equal to 0 and you compute; that means, everywhere sorry here everywhere you put 0 here the flow value is 0 here.

So, we started with this flow and we computed the augment residual network and then augmenting path, but the algorithm says that you can start with a 0 flow; that means, everything all the flow value all the flow values are equal to 0. So, you start with 0 flow and while G f contains an augmenting path P in G f you do the following you identify an augmenting path P in G f. So, identify an augmenting path P in G f and you compute the capacity of that augmenting path that is delta equal to minimum of c f u v where u v belongs to P, you understand what is this delta I have explained everything in the previous example and then what you do is that you augment you augment delta unit of unit flow along P and update G f; that means, you compute the new residual network.

This is what you do inside the while loop while there is a augmenting path with respect to the G f you identify 1 augmenting path and you compute the capacity of that path and you augment delta unit of flow along that path and you update your G f and again you

see whether the new residual network has an augmenting path in G f, whether the new residual network G f has an augmenting path, if it is true again you identify all 1 augmenting path and compute capacity of that path and augment delta unit of flow along that path.

So, this continues till there is no augmenting path in G f and you are out of this while loop and there is no augmenting path that is the stopping criteria for Ford Fulkerson Algorithm and once you are outside this while loop you recover maximum flow from the final residual network. So, I hope that you understood this augmenting path algorithm or the Ford Fulkerson Algorithm to find the maximum flow in a network, but somehow I feel that you might be wondering why we need residual network, why we do not just find the path in the original network and augmenting that amount of path that amount of flow in the network so why we need this residual network concept at all.

For that let me just go back to the example here suppose you, this is the network given at the beginning and you do not want to use the concept of residual network fine, you just find a path in the original network from s to t. Suppose you found this path s b c and t and you have computed the capacity of this path, the capacity of this path is 1 because the minimum capacity is 1. So, you augment 1 unit of flow along this path.

Now you do not go for the residual network concept you now is there any another path along which you can send flow yes probably you can you can choose s a c t and you cannot send 2 unit of flow the minimum capacity of this flow of this path is 2, there will be problem see you cannot send 2 unit of flow along this path, you can send 1 unit of flow along this path. Let me just draw this graph once more, let me draw this flow network once more, this is say s a b c t d e.

So, this is the network I have and the capacity is 3 here, the capacity along this is 3 capacity here is 2 here it is 5 s to b it is 1, s to d it is 4, d to e it is 2, e to t it is 3.

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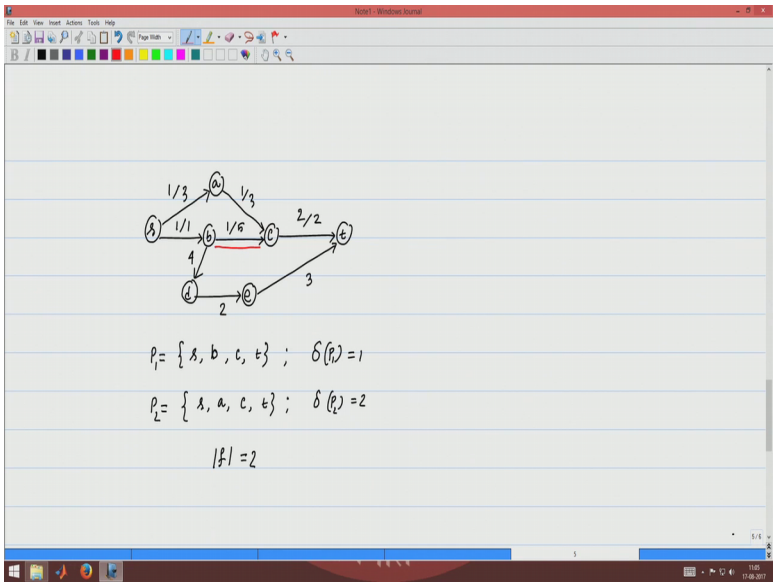

Now, what I wanted to say is that what is the use of this residual network I can directly find the path from s to t suppose I got the path P to d say s s a sorry s b c t and I know that capacity of this path is 1. So, I can send 1 unit of flow along this path sorry I drew it wrongly this is not here this is here this is 4. Now if I will try to find another path from s to t, say I get the path P to b, s, a, c, t and the capacity of this path at this moment see I you have already sent 1 unit of flow.

So, maximum we can sent another unit of flow along this path along this edge. So, that though the capacity of this of this path is path P 2 is 2, but we cannot send 2 unit of flow along this because there will be a problem here. So, I can send maximum 1 unit of flow. So, this 1 will be replaced by 2 and that is all so after this there is no path from s to t because you can see that you cannot use this edge for a path from s to t and there is no other path from s to t.

So, you have to stop here and the flow the maximum flow that you can send is equal to 2, if you do not use the concept of residual network, but a concept of residual network what it does is that if there is a wrong choice of path, basically this is the wrong choice of path you do not know which path is wrong. If there is a wrong choice of edge or path in the at the beginning the residual network will take care of that residual network at some point of time it will cancel that path and it will find the correct path which will keep the

maximum flow. So, that is what the use of residual network in the Ford Fulkerson algorithm, I hope that you understood this one.

Thank you very much.