Numerical Analysis
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Lecture 23
Numerical Solution of Ordinary Differential Equations-6
Predictor- Corrector Methods
(Adam - Moulton)

In the last class we studied about the Predictor- Corrector Method by Euler's Method which solves the initial value problem dy by dx is equal to f(x ,y) and an initial condition is given at x 0 as y(x 0) equal to y 0. If you want to obtain the solution say at a point x 1 which is x 0 plus h then Euler's Method says y 1 is y 0 plus h into f (x 0 y 0). So typically if the information is given at x n as y (x n) and that is equal to y n and you are asked to compute what y n plus 1 is?

Then Euler's Method says y n plus 1 is y n plus h into f(x n y n). Then we also saw a modified form of Euler's Method in which the curve solution curve in an interval x n to x n plus 1 is approximated by a straight line passing through x n y n and having its slope to be the average of the slopes at the point x n , y n and x n plus 1 , y n plus 1.

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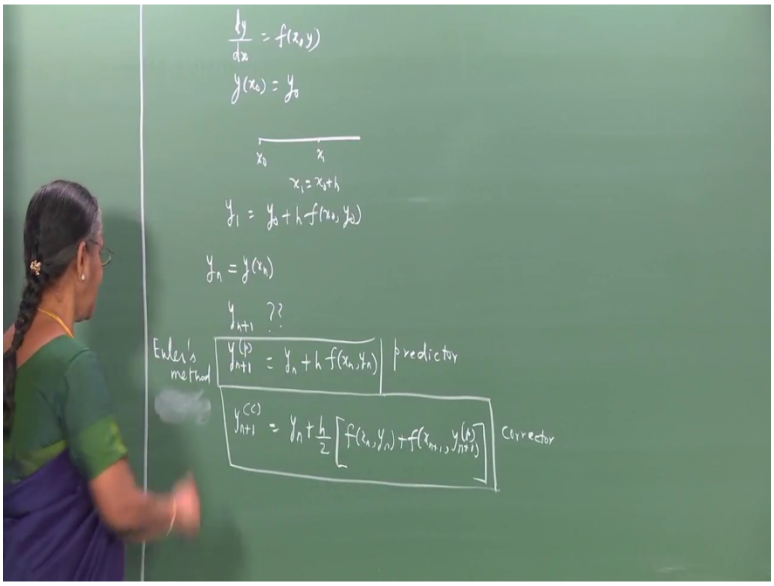

And so we obtained modified Euler's Method given by y n plus 1 equal to y n plus h by 2 into f(x n y n) plus f (x n plus 1), (y n plus 1). And we observed when we derived this formula that the unknown y n plus 1 appears not only on the left side but it also appears here and we need the value of the function at x n plus 1 y n plus 1. But our goal in deriving the method was to obtain what y n plus 1 is.

So this suggest that we can use the value of y n plus 1 here as that is predicted by the Euler's Method. So I call Euler's Method as a predictor and I use this predicted value of y n plus 1 and find what is f(x n plus 1) (y n plus 1) predicted which is the value of the function at x n plus 1 y n plus 1 because of we know what is x , y.

So once we know what the function here is I can evaluate y n plus 1 that is going to be a value that I get for y n plus 1 which is obtained using the predicted value of y n plus 1. So I have essentially corrected the value of y n plus 1, so I call this as a corrected value. So I now have two methods with the help of which I am able to predict y n plus 1. So I call that as a predictor method and using that predicted value here I am able to correct it and so I call this as a corrector method.

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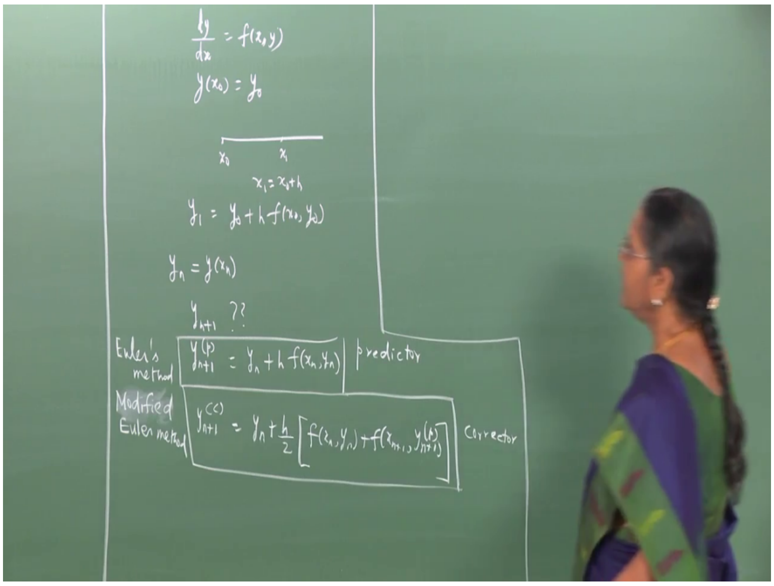

Euler's Method and corrected Euler's Method serve as a Predictor- Corrector pair for computing the solution of the initial value problem of the form dy by dx is equal to f(x,y) and y(x 0) is y 0 and one will be able to obtain the solution at the next point knowing the information at a previous point. So this suggest that more efficient methods can be deviced by employing Predictor- Corrector Methods so that whatever that is predicted can be corrected and successively recorrected till the desired degree of accuracy is attained.

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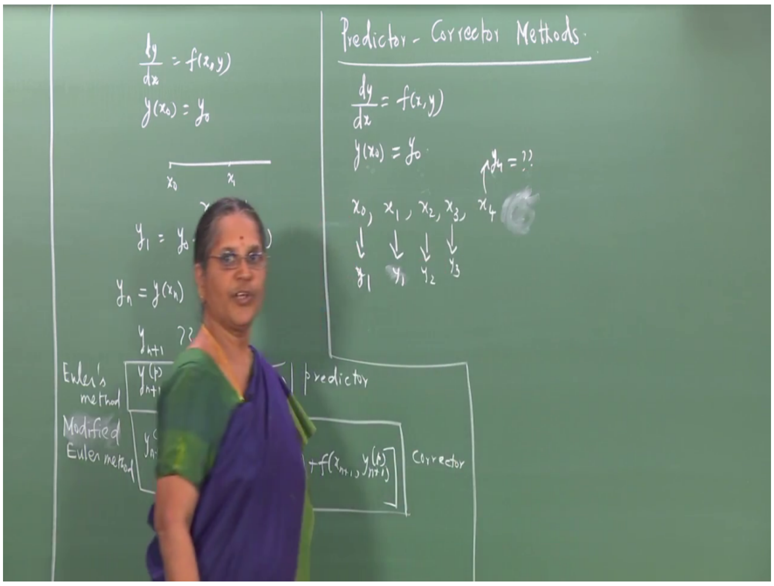

So in this class we shall look for some more Predictor- Corrector Methods. So we shall look into so we will derive some Predictor- Corrector Methods in this class which help us to solve dy by dx is equal to f (x,y) given that y(x 0) is equal to y 0. If suppose we have information about the solution of this problem at points say x 0 which is given by this initial condition at x 1, x 2, x 3 and if we are asked to compute the solution at say x 4 which means the solution at the point x 4 makes use of the information at the previous 4 points. Namely the solution at x 4 is y 4 and that is what is needed by us.

So the solution at x 3 x 2 x 1 and x 0 are given to us, with this information we should find out what y 4 is. So the computation of solution at y 4 involves the knowledge of the solution at the previous points. So the methods that we derive by requiring that information is needed at a set of previous points in order to obtain solution at this point then such methods are called multi step methods.

So essentially we will be deriving what are referred to as Multi step methods. Once we derive them we can conveniently use these methods Predictor- Corrector Methods and obtain the solution of the given problem correct to the desired degree of accuracy.

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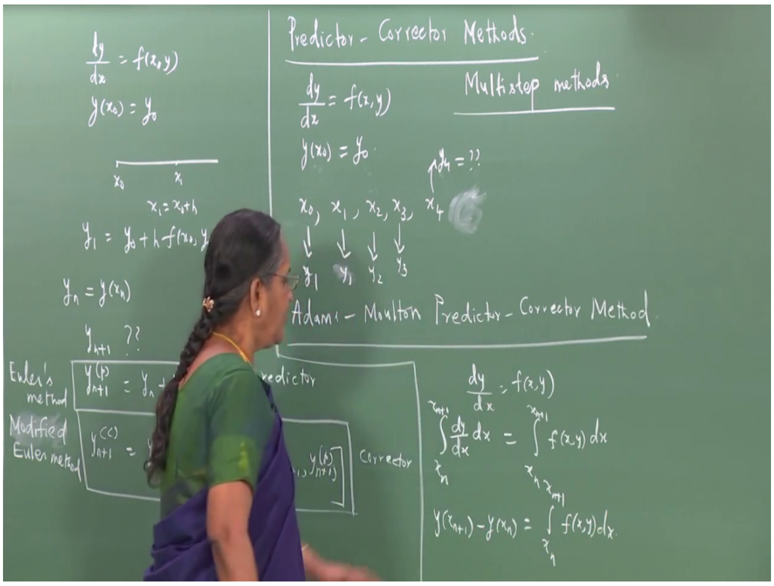

So let us first derive what is known as Adam's - Moulton Predictor- Corrector Method. So it solves a differential equation dy by dx is equal to f(x, y). Let me integrate both sides with respect to x from x n to x n plus 1. So dy by dx into dx will be integral x n to x n plus 1 f(xy) integration with respect to dx. So this tells you y(x n plus 1) minus y(x n) is integral x n to x n plus 1 f(x y) dx.

So at this stage I shall approximate the function f(x,y) using Newton's backward interpolation polynomial.

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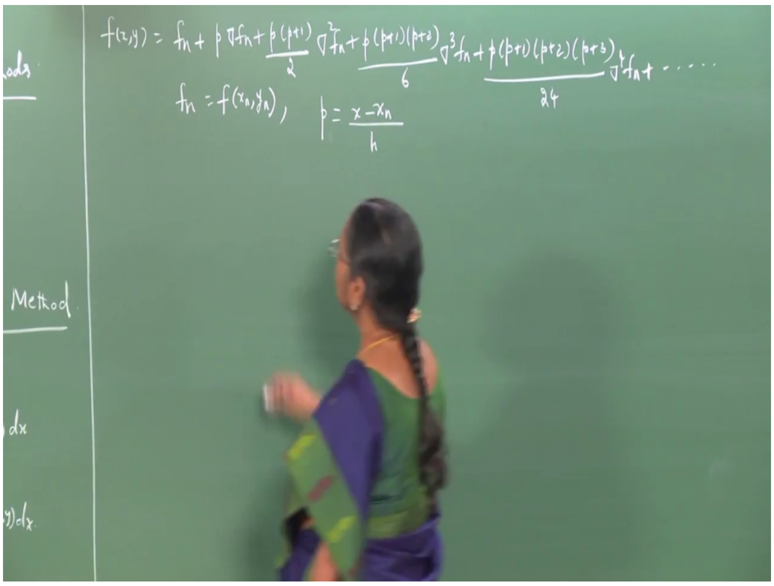

So f(x,y) is f n plus p into delta f n plus p into (p plus 1) by factorial 2 into del square f n plus p (p plus 1) (p plus 2) by factorial 3 into del cube f n plus p (p plus 1) (p plus 2) (p plus 3) by factorial 4 into del power 4 f n plus etc. So by f n I mean the value of the function at (x n y n) and by p I mean x minus x n divided by h.

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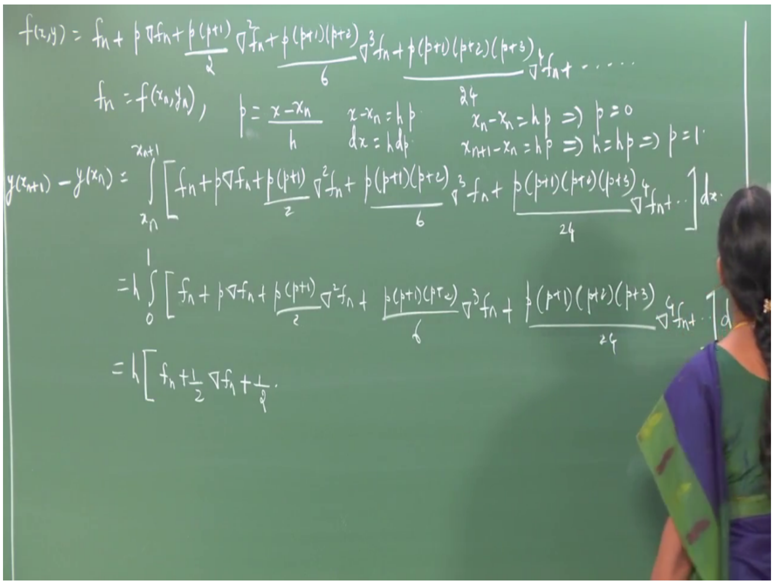

So I substitute for f(x,y) here So I get y (x n plus 1) minus y (x n) is integral x n to x n plus 1 f(x , y) so f n plus p delta f n plus p (p plus 1) by 2 into del square f n plus p (p plus 1) (p plus 2) by 6 del cube f n p (plus 1) (p plus 2) into (p plus 3) by 24 into del power 4 f n plus etc integration with respect to x.

Now I observe that the variable here is p and the integration is with respect to x so I use this transformation so I have x minus x n to be h into p, so dx will be equal to hdp. Further when x is equal to x n then this tells that p is 0 and where x is equal to x n plus 1 then we have h to be equal to hp which gives you p to be equal to 1.

So the integration is h times integral 0 to 1 f n plus p delta f n plus p (p plus 1) by 2 del square f n, So I write down all these terms as they are and then perform the integration. Integration is now with respect to p. So the first term will give you h into f n dp integrated with respect to p from 0 to 1 so it is f n into p between 0 and 1. So it is simply f n, the second term p dp so p square by 2 between 0 and 1 so half of delta f n then plus half of p into (p plus 1).

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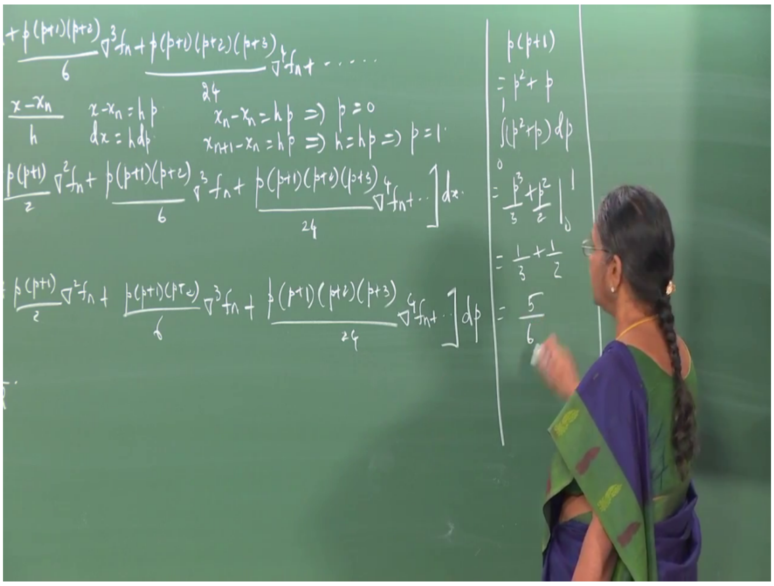

So that will give you p square plus p and that has to be integrated with respect to p, so that will give you p cube by 3 plus p square by 2 between 0 and 1. So this will give you 5 divided by 6.

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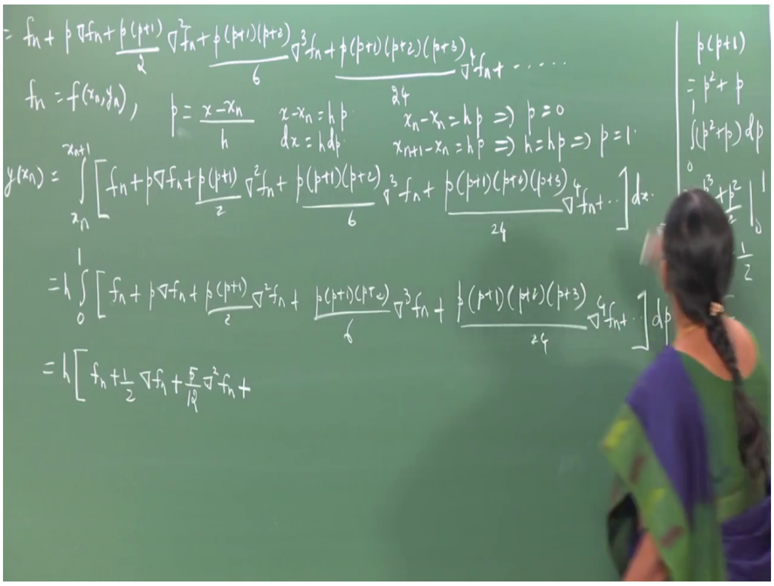

So this term will give you 5 by 12 into del square f n.

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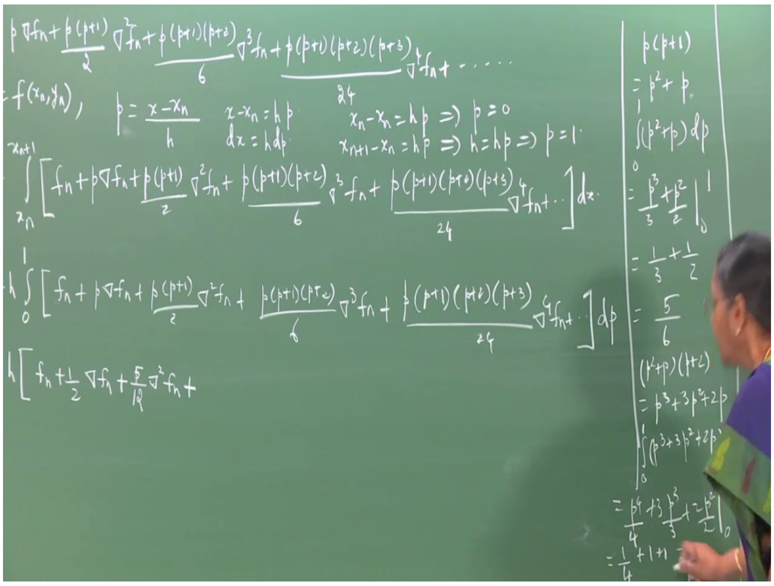

Then the next term p into p plus 1) is p square plus p and that has to be multiplied by p plus 2 so that will give you p cube plus p square plus 2p square so this will become 3psquare plus 2p. So I have to integrate this between 0 and 1 with respect to p. So that will give you p power 4 by 4 plus 3 p cube by 3 plus p square by 2 between 0 and 1. So this gives you 1 by 4 plus 1 plus 1, so we have 9 by 4.

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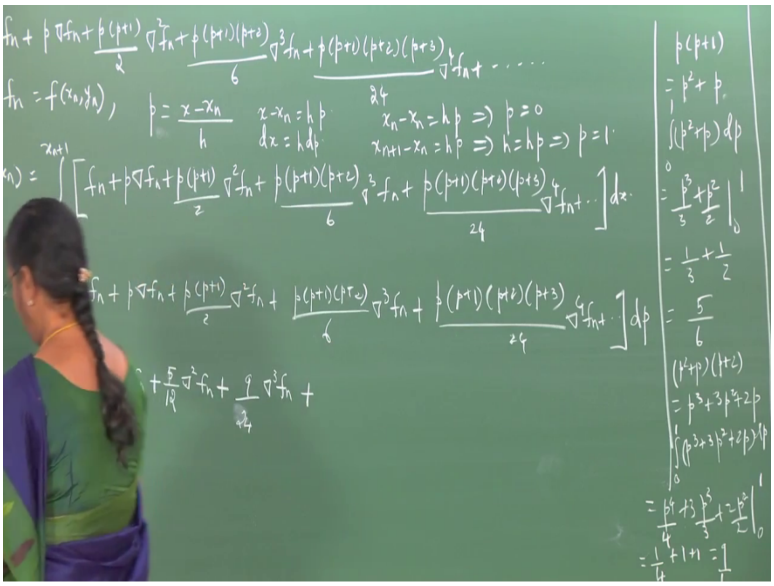

So I have already 6 here so 9 by 24 into del cube f n and then I have to perform integration of this term. So let us work out the details. So I have p plus 1 p plus 2 given by p cube plus 3p , p square plus 2p.

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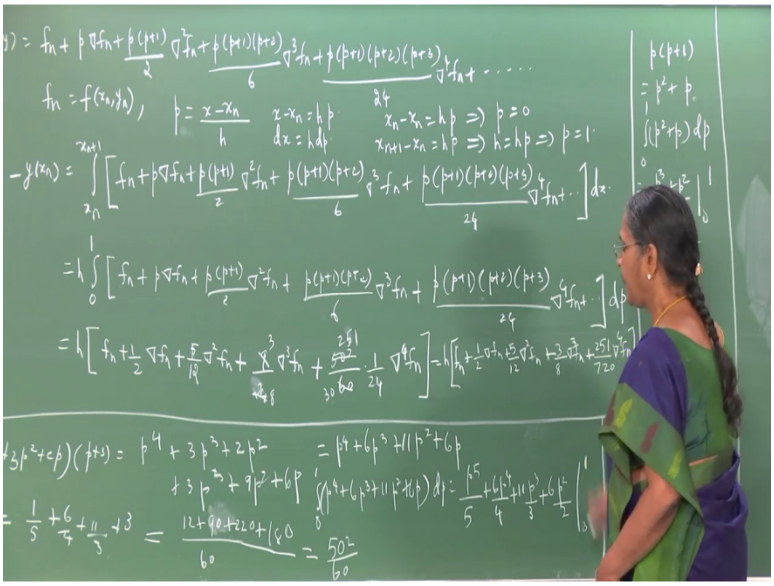

So we have p cube plus p square plus 2p now that has to be multiplied by p plus 3 so and that will give you p to the power of 4 plus 3 p cube 2 p square and then plus 3 p cube plus 9 p square plus 6 p which will give you p power 4 plus 6 p cube plus 11 p square plus 6 p. Now I have to integrate this with respect to p. And that gives you p power 5 by 5 plus 6 p power 4 by 4 plus 11 p cube by 3 plus 6 p square by 2 between 0 and 1.

So let us substitute for p the upper and the lower limit that gives you 1 by 5 plus 6 by 4 plus 11 by 3 plus 6 by 2.So fi you evaluate this then you get 502 by 60 so I substitute here that will give you 502 by 60 into 1 by 24 into del power 4 f n. So I shall truncate this terms here at this stage so y (x n plus 1) minus y (x n) which is this when the integration is performed it gives you this result which can be simplified and it is given by h into f n plus half delta f n plus 5 by 12 del square f n then the next term 3 by 8 into del cube f n plus this gives you 251 by 30 into 24 so that is 251 by 720 into del power 4 f n.

So let us now substitute for the backward differences delta f n del square f n etc and then express this in terms of the function values.

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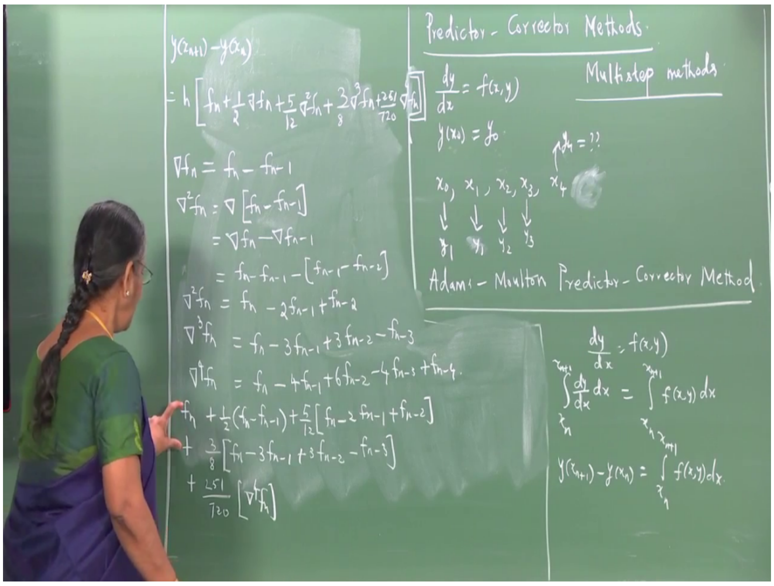

So we obtain y(x n plus 1) minus y(x n) is given by h, let me rewrite that expression once again that will be easier for me to make the computations So [f n plus half delta f n plus 5 by 12 into del square f n plus 3 by 8 into del cube f n plus 251 by 720 into delta power 4 fn]]. So we use the definition of the backward difference operator on f n so that gives you f n minus f n minus 1. So what is del square f n it is the backward difference operator operating on f n minus f n minus 1 so it is delta f n minus delta f n minus 1.

And that is f n minus f n minus 1 minus f n minus 1 minus f n minus 2. So that will give you f n minus 2 f n minus 1 plus f n minus 2 that is what is del square f n. We require del cube f n so that will be f n minus 3 fn minus 1 plus 3 f n minus 2 minus f n minus 3 and finally we have del power 4 f n and that will be f n minus 4 f n minus 1 plus 6 f n minus 2 minus 4 f n minus 3 and plus f n minus 4. So we have obtained the backward differences on f n which appear here and express them in terms of the function values of different points.

So now we have to substitute these here and collect the like terms and then write down the result finally so we have the first term to be f n the second term is half of delta f n which is f n minus f n minus 1 then the next term is 5 by 12 into del square f n that is f n minus 2 f n minus 1 plus f n minus 2. Then I have 3 by 8 into del cube f n so that gives you f n minus 3 f n minus 3 f n minus 1 plus 3 fn minus 2 minus f n minus 3. And finally this term 251 by 720.

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So we shall retain that term as it is as del power 4 into f n I do not want to substitute that in terms of the function values. Let us simplify these terms. So I shall collect the terms involving f n so the first term has coefficient 1 then we have half 5 by 12 then 3 by 8 that is the coefficient of f n. And then the next term is f n minus 1[minus half which comes from this then minus 10 by 12 from this term then minus 9 by 8]. Then I will collect the terms involving f n minus 2 that is 5 by 12 from there and then 9 by 8 from the next term and we have f n minus 3 whose coefficient is minus 3 by 8.

And I have the next term which is 258 by 720 into del power 4 f n. So we have to simplify these so I shall take say 24, 24 plus 12 plus 10 plus 9. So 36 , 46 so 55 by 24. So coefficient of f n is 55 by 24 then I have minus f n minus 1 into so I shall now take 24 as common denominator so I have 12 plus 20 plus 27 47, 49 so 59 by 24 into f n minus 1.

Then this term again 24 so 10 27 so plus 37 by 24 f n minus 2. And then if I take 24 here then it will become 9 by 24 f n minus 3 and then the last term 251 by 720 into del power 4 f n. So we have simplified the right hand side so we write down the result therefore y (x n plus 1) is equal to y (x n) plus I have h as a coefficient here so h by 24 into [55 f n minus 59 f n minus 1 plus 37 f n minus 2 minus 9 f n minus 3 plus 251 by 720 into h into del power 4 f n.

Thus we have y(x n plus 1) to be given by y(x n) plus h by 24 into a linear combination of the function values f(x,y) at x n y n this is at x n minus 1 y n minus 1 x n minus 2 y n minus 2 and this is at x n minus 3 y n minus 3. And this is obtained by approximating the function f(x,y) by means of backward interpolation polynomial for equally spaced points.

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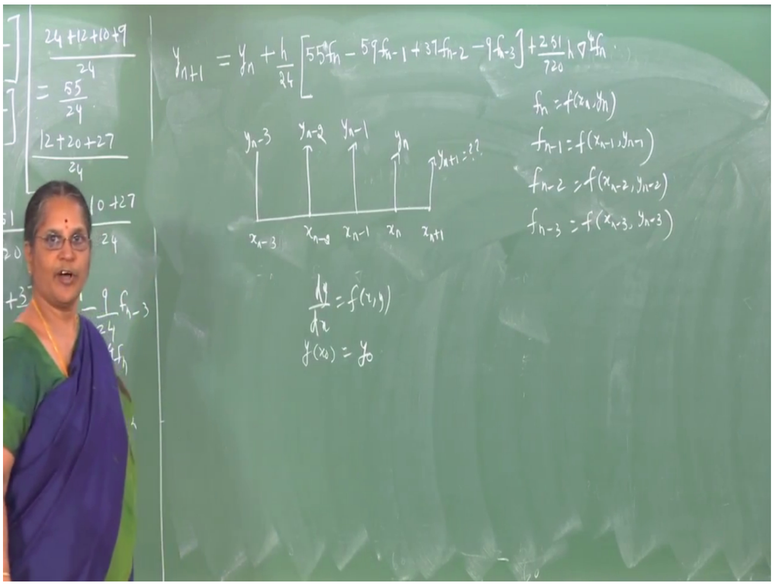

The result that we get is an approximation value to y (x n plus 1) so we denote it by y n plus 1 and that is y n plus h by 24 into 55 f n minus f n minus 1 plus 37 f n minus 2 minus 9 f n minus 3 plus 251 by 720 into h into del power 4 f n. So what does this method indicate? Suppose you require a solution at this point x n plus 1 namely y n plus 1 is what you have to find. Then you require the knowledge of the value at x n which is y n and what about this f n is f(x n y n).

So when you know what is y n at x n then you can compute what is f (x n y n) so this term is known what is f n minus 1 it is f(x n minus 1) (y n minus 1). So the information at the previous point namely x n minus 1 should also be available to you and you call that as y n minus 1. And once you have this you find what f n minus 1 is.

Similarly you compute the values of f n minus 2 which is f (x minus 2) (y n minus 2) and f n minus 3 which is f (x n minus 3) (y n minus 3). So that is going to be attained by using the information at x n minus 3 where the solution is y n minus 3. So in order to get the solution at x n plus 1 you require information about y n at a previous four points.

So essentially what is it that you have done? You have approximated that function f(x,y) by a polynomial passing through these 5 points x i y i such that this will be a fourth degree polynomial and hence the fourth order backward difference is constant and the fifth and the higher order backward differences are all 0. And that is why you truncated your Newton Backward interpolation polynomial at the term which contained del power 4 f n.

The Question now is , how do you get these values? because your problem is dy by d x is equal to f (x ,y). And you will be given information at some initial point x 0 y 0.

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You are asked to compute the solution at x n plus 1. So you require the knowledge of the solution at the previous four points x n, x n minus 1, x n minus 2, x n minus 3. Divide the interval x 0 to x n plus 1 into four equal parts and name those points as x i and then try to obtain the solution at other three points using single step method which you have already learnt . Namely you can use Euler's method or Taylor series method or Runga Kutta method of order 2 or order 4 and compute the solution at the points at which you require the information.

So step 1 would be to collect information at a set of previous four points once you have that information now use this method to obtain the value of y(x n plus 1) which is denoted by y n plus 1 because you know every value with the help of the information that you have computed and hence you can compute y n plus 1.

So this is a multi step method because the value at x n plus 1 which is y n plus 1 is obtained by having information or knowledge about the solution at the previous points which can either be given to you a priory or you will have to compute using the single step methods which you have already learned. And using those information at the previous points you can use this method and compute what y n plus 1 which is the value of y at x n plus 1 and it is an approximation to the value of y at x n plus 1. So it is a multistep method.

So now we shall derive another multi step method which the calculations the arguments and the discussions are analogous to this so I will not be doing in detail these calculations I will just indicate the method and present what finally the method is and then tell you how we can use the method that we have derived now and the one that we are going to derive now can be used as a Predictor Corrector pair and solution to a particular problem can be obtained correct to the desired degree of accuracy.

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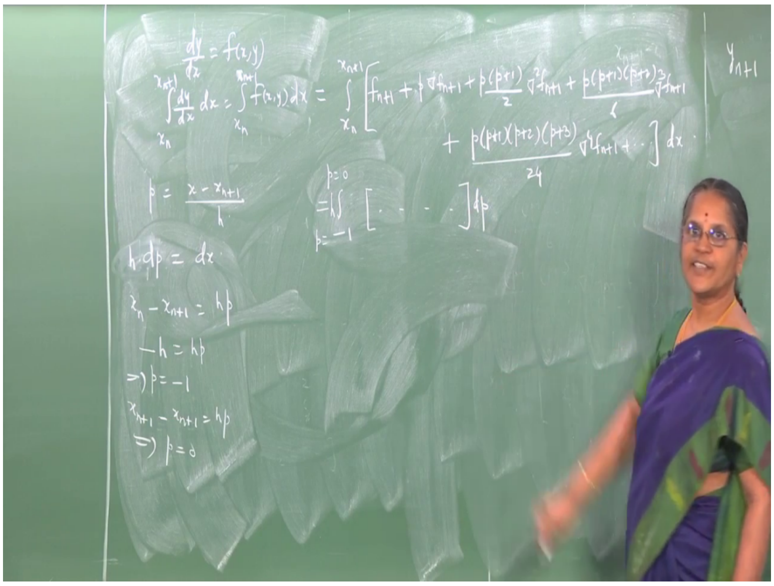

Here p is x minus x n plus 1 by h this derives you dp into h is d x. So when x is x n you have x n minus x n plus 1 to be hp. So minus h is hp so p is minus 1 , then x is x n plus 1 then this gives you p equal to 0. So the limits of integration are p is equal to minus 1 to p is equal to 0 of the terms within this bracket and integration is now with respect to p. So you have to work out the details of this integration we shall give you the result.

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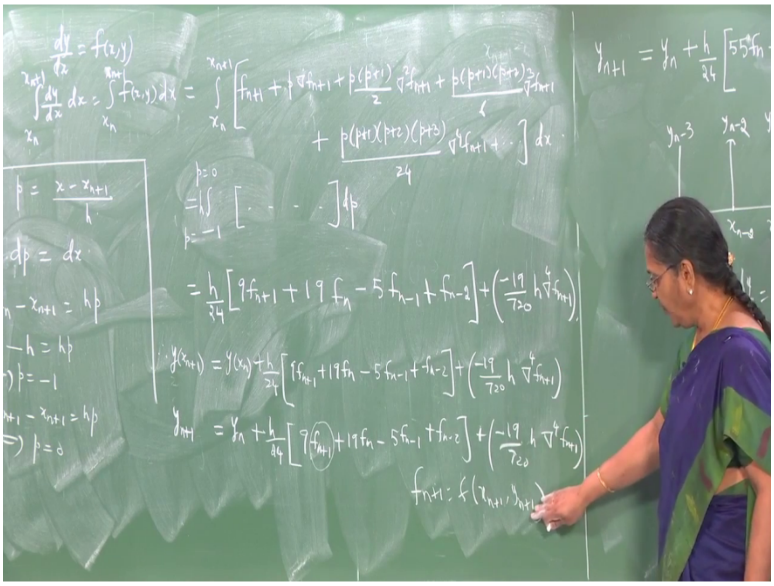

So we have y (x n plus 1) will be equal to y(x n) plus h by 24 into [9 f n plus 1 plus 19 f n minus 5 f n minus 1 plus f n minus 2] plus (minus 19 by 720 times h into del power 4 f n plus 1). And since we have approximated the function f(x,y) by means of backward interpolation polynomial about x n plus 1 we obtain an approximation to the value of y (x n plus 1). So we have y (n plus 1) to be given by y n plus h by 24 into the terms which are written here.

So let us look at this method that we have derived. Again we observe that it is a multi step method it requires the knowledge of the solution at x n which is y n at x n plus 1 which will be y n plus 1 only then we can compute what f n plus 1 is namely f n plus 1 is f (x n plus 1) (y n plus 1) and we already have seen f n is f(x n y n) f n minus 1 is f(x n minus 1) (y n minus 1) and f n minus 2 is f(x n minus 2) (y n minus 2).

Now we see that the left hand side gives you y n plus 1 and on the right hand side we have f n plus 1 which requires the knowledge of what y n plus 1 is. We are only trying to gt what y n plus 1 is but that involves the knowledge of y n plus 1 in getting f n plus 1.

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From where do we get it we make use of the y n plus 1 that we already derived in the earlier method. Recall this is the method that we had derived earlier. Where we are able to write down what y n plus 1 is when we have information at a set of previous points. So we use this y n plus 1 obtained using this method in evaluating at f n plus 1 is, then we know what f n plus 1 is of course the other values can be computed with the available information.

So we obtain the value of y n plus 1 this is the second time that we get y n plus 1 at x n plus 1 for the same problem governed by the differential equation. We used the method which we derived earlier to get y n plus 1 that is used here and in turn f n plus 1 is used here and that gives you a new value of y n plus 1, so we say that we have corrected the value of y n plus 1 by the new method that we have derived and this was done by using the y n plus 1 obtained by the earlier method as a Predicted value.

So I call this as a predictor and the method that we have obtained now as a corrector. So how do I get f n plus 1 ? It is f (x n plus 1) (y n plus 1) is nothing but the predicted value obtained from the predictor. So thus we have a predictor corrector pair for solving an initial value problem dy by dx is equal to f (x,y).

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So let us write down what the corrector is. This is given by y n plus 1 corrector is this expression. So my predictor is this and my corrector is going to be this again I observe that the last term gives me the local truncation error in this multi step method. Another observation just look at the local truncation errors in both the methods, right?

So you observe that the local truncation error in the predictor method is several times that in the corrector method and therefore the error that is incurred by using this method in predicting the value of y n plus 1 is corrected using the corrector method. In addition the corrector method can be used successively to recorrect our solution till the dersired degree of accuracy is attained.

How do we do it? We are given a problem dy by dx and we are given enough information that is required to compute y n plus 1 using the predictor. Once the predictor is obtained we use it here and correct it . At this stage we check whether the difference in absolute value between the corrected value and the predicted value is less than Epsilon which is the desired degree of accuracy.

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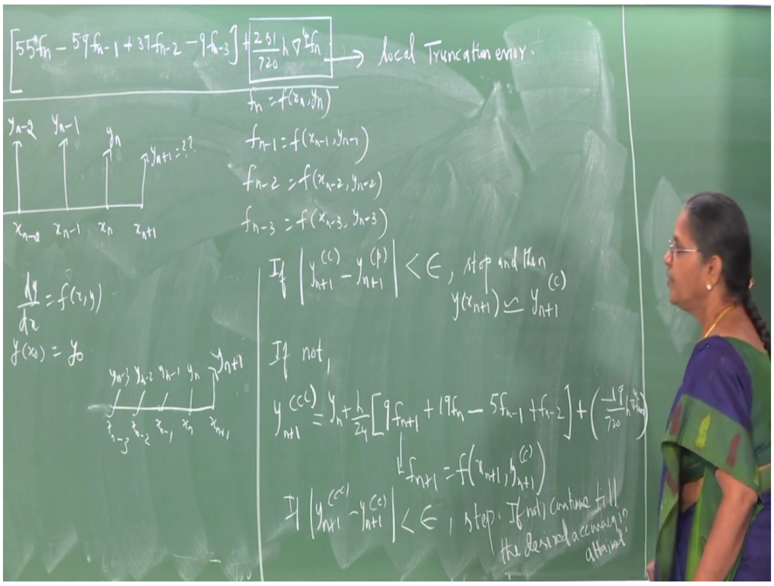

So let us write down the details. So what do we do at this stage? At this stage we check whether y n plus 1 obtained using the corrector minus y n plus 1 obtained using the predictor is less than Epsilon. If so then we stop and then take this y n plus 1 corrected value as approximating y(x n plus 1). Stop and take the solution as approximating y(x n plus 1) which is given by y n plus 1 corrector value.

If suppose this does not happen then what do you do? you use the corrector once again and get the second corrected value how do you use it? The second corrected value is going to be y n plus h by 24 into 9 into f n plus 1 plus 19 f n minus 5 f n minus 1 plus f n minus 2 plus the truncatin error minus 19 by 720 h into del power 4 f n plus 1).

Here the [f n plus 1 is going to be f (x n plus 1) (y n plus 1) but which (y n plus 1) will you use? You have the first corrector value there that is what you are recorrecting so you compute y n plus 1 double corrector value by evaluating f n plus 1 making use of the first corrected value of y n plus 1. The rest of terms will be the same there wont be any change the only change that will occur in this will be at f n plus 1 which is now f (x n plus 1) (y n plus 1) first corrector value.

What you do now? You check wheter y n plus 1 double corrected value minus y n plus 1 first corrector value is less than Epsilon if then stop if not continue this process if not continue recorrecting your solution every time computing what is f n plus 1 by making use of the value of y n plus 1 obtained at that corrected step and then evaluate the new y n plus 1 which is the corrected value till the desired degree of accuracy is attained if not continue till the desired accuracy is attained.

So the Predictor Corrector method helps you to obtain the solution of dy by dx is equal to f(x,y) correct to the desired degree of accuracy by using the Predictor to predict the value and then you can successively recorrect using the corrector till accuracy is attained. So I am sure you have been able to understand the details in the derivation of this Predictor Corrector Method this pair the Predictor and the Corrector together is known by Adam Moulton Predictor Corrector Method or it is also called a Predictor Corrector pair.

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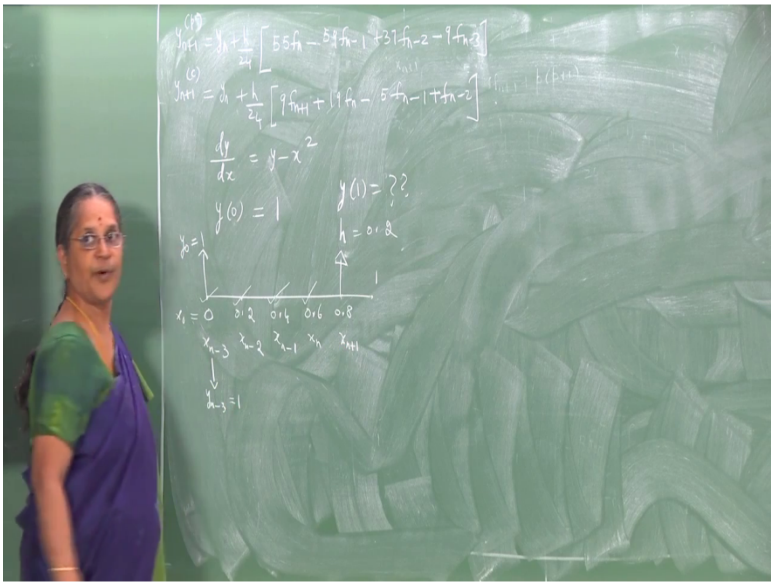

Let us illustrate Adam Moulton Predictor Corrector Method for solving a differential equation. So consider differential equation dy by dx which is equal to y minus x square. And the information given to you is y(0) equal to 1. The question is find the solution of this initial value problem at x is equal to 1 by taking step size as 0.2

So we are given a solution at zero in the form an initial condition. So this is x if I call this as say x 0 y(x 0) is y 0 and that is given to be 1. And I require the solution at 1 and I am asked to go in steps of h which is 0.2. So let us mark these points 0.2 0.4 0.6 0.8 and 1. And I am asked to use Adams Predictor Corrector method.

So I can compute the solution y n plus 1 and predict it if I know information at x n, x n minus 1, x n minus 2, x n minus 3. So I require information at 4 points. So I have information given here, so if I know information at these four points then I can evaluate the value at this point using the Predictor. So instead 1 I shall denote this y x n plus 1 so 0.6 will be x n 0.4 will be x n minus 1 0.2 is x n minus 2 and 0 will be x n minus 3.

And therefore what is the corresponding value of y it is y n minus 3 since x n minus 3 is 0 y n minus 3 is 1 that is already given to me. So I need to now get the solution at these points 0.2 0.4 and 0.6. Only when I have information at all the 4 points i can use the predictor and get the solution at these pnt and then use the corrector to correct it. So my goal now would be to compute the solution at 0.2 0.4 and 0.6 .

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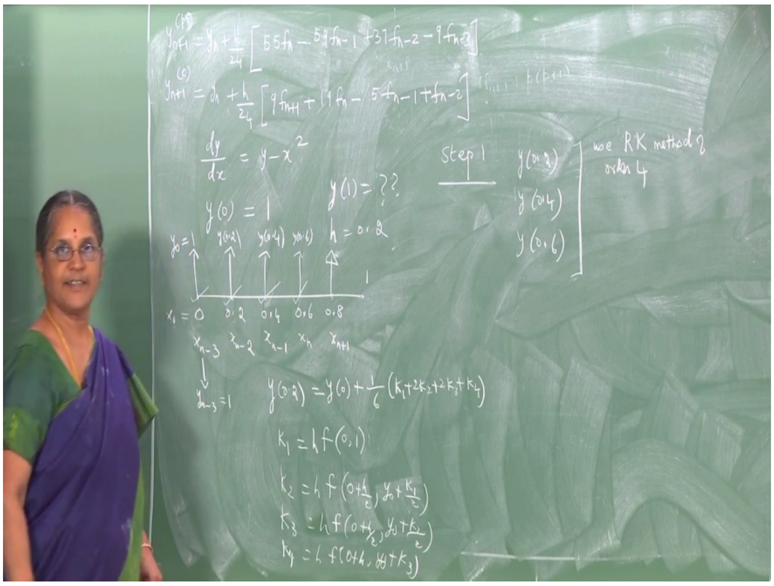

So step 1so given a problem write down what are all the information that you gather from the given problem and what is it that we require? That is your first step. And then continue in your first step the computation of whatever is required so we require solution at 0.2 solution at 0.4 and at 0.6. So how do you get this? These solutions can be obtained using say Runga Kutta Method of order 4. How do you do it ? You know the solution at this point so I march one step ahead and compute the solution by Runga Kutta method at this point isn't it?

What is it/ So I require y at 0.2 What does Runga Kutta method of order 4 says? It says it is y (0) plus 1 by 6 into (k 1 plus 2 k 2 plus 2 k 3 plus k 4). What is k 1 ? K 1 is h into f (x 0 y 0). So 0 what y 0? It is 1 and what is k 2? K 2 is h into x 0 plus h by 2 y 0 plus k 1 by 2 and k 3 is h f(x 0 plus h) by 2 y 0 plus k 2 by 2 and finally k 4 is h f(x 0 plus h) y 0 plus k 3.

So I know what the function is given to me, what are x 0 y 0 which are 0 and 1 so I can compute k 1 k 2 k 3 k 4 substitute here and arrive at what y (0.2) is? So once y(0.2) is obtained then again I use Runga Kutta Method of order 4 I march one step ahead and find the solution at 0.4 and that is y(0.4), I do the same thing and obtain the solution at 0.6. So please carry out the details of these computations I will give you the results namely the values of these points and then we move on with using the Predictor corrector method to obtain the solution at 0.8.

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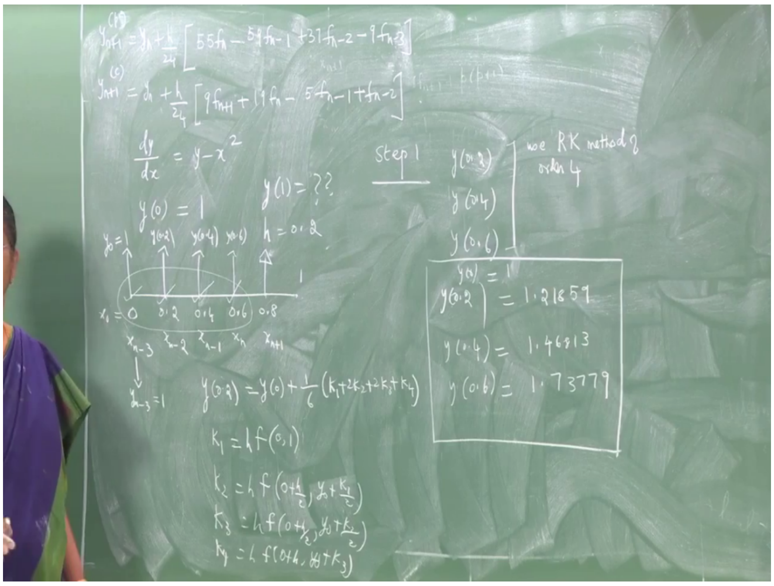

When we do it by Runga Kutta method we get y(0.2) to be given by 1.21859 y (0.4) is given by 1.46813 y (0.6) is 1.73779 and we already know that y (0) is 1. So we have the solution obtained using Runga Kutta method of order 4 at these three points and this is an initial condition that is given in the problem.

So we have information at all these four points which is needed to obtain the solution at the point 0.8 using Predictor Method. What does the Predictor method say this is a Predictor. That is y n plus 1 so I have called this as my x n plus 1 so the value of y at this point is what is denoted by y n plus 1.

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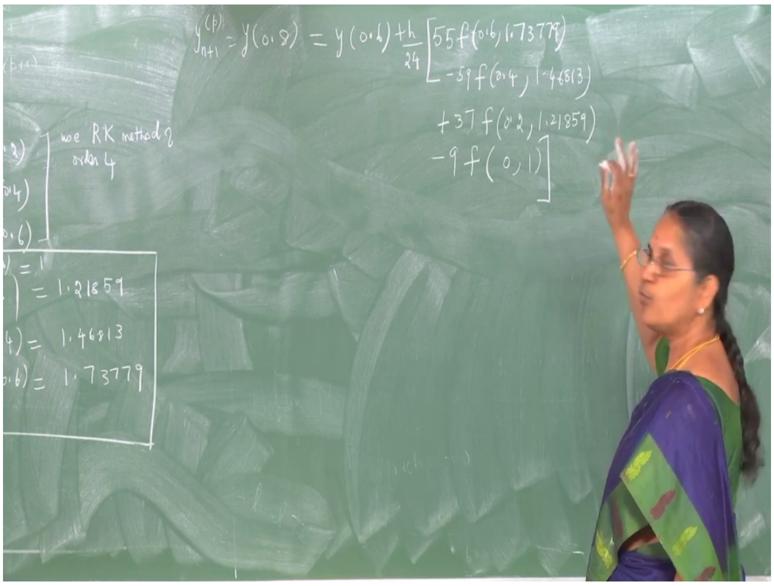

So y n plus 1 Predicted will give me y(0.8) according to minutation and that is equal to y(n) what is y n it is going to be y(0.6) and then plus h by 24 into 55 times f of what is f n it is f(x n , y n) x n is 0.6 what is y n? We have just now obtained its value that is 1.73779.

Then the next term minus 59 into f of what is f n minus 1 it is x n minus 1 y n minus 1 so it is f(0.4) what is y n minus 1 we have computed it to be 1 .46813 then the next term is 37 f n minus 2 and f n minus 2 is f o(x n minus 2)(y n minus 2) so it is f(0.2) , What is the value at 0.2? 1.21859 and the last term minus 9 times f n minus 3. Our n minus 3 is 0 and value there is 1 so it is f(0, 1) s o it is always better to write down using the formula what is it that you want and then evaluate these functions at these points and then simplify to get the answer.

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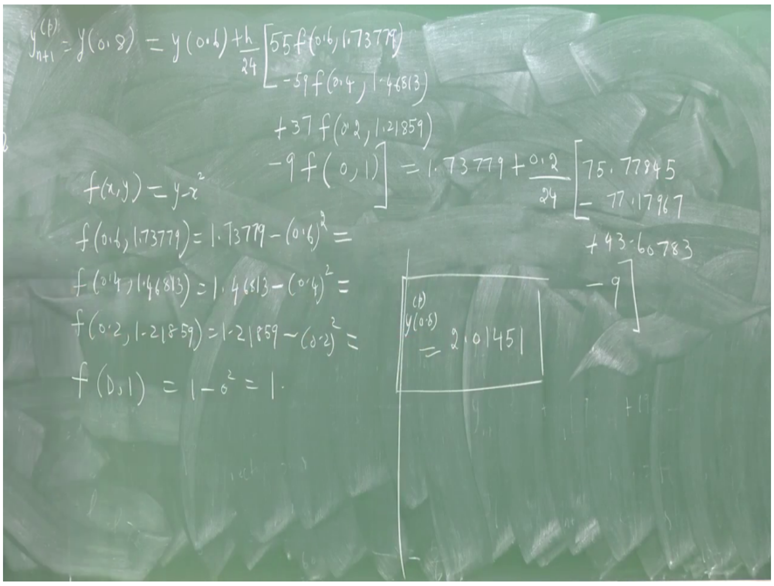

So let us find out what is f(x,y) a problem tells you that f(x,y) is y minus x square so you require f(0.6) 1.73779 so it is 1.73779 minus x square so (0.6) square then f(0.4) 1.46813 so it is 1.46813 minus (0.4) square; f(0.2) 1.21859 so 1.21859 minus (0.2) the whole square and f (0) 1 and that wll be 1 minus 0.

I substitute these values in this formula and that gives me y(0.6) which is 1.73779 h is 0.2 by 24 into 55 times f(0.6) 1.73779 which is this value I have to take 55 times this value and that turns out to be 75.77845 the next value is 77.17967 and 43.60783and finally we have minus 9. And if you simplify this further it turns out to be 2.01451 this is your y(0.8) and that is what is predicted. Having predicted the value at 0.8 we would like to correct it using the corrector.

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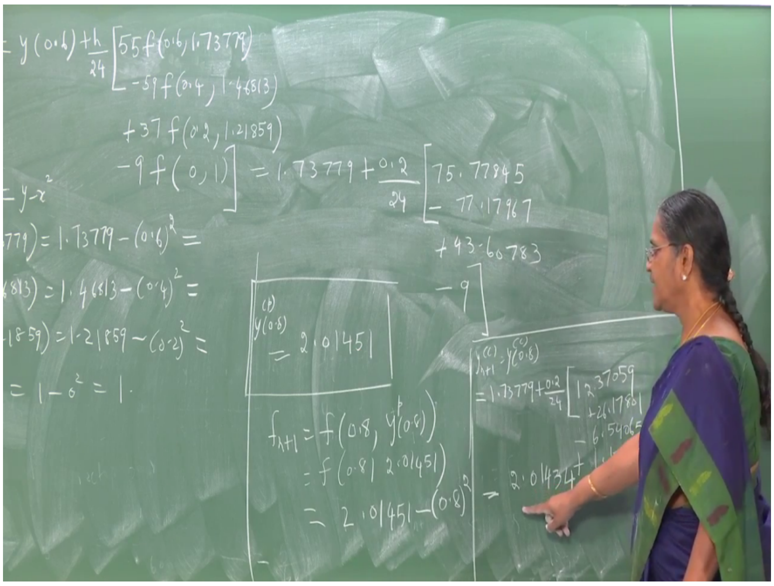

So before using the formula just find out what is this f n plus 1. What is f n plus 1? Here is f n plus 1 is going to be f evaluated at x, y what is x it is 0.8 what is y; y is y(0.8) that is predicted. So it is f(0.8) 2.01451, but f(x,y) is y minus x square so 2.01451 minus (0.8) the whole square. So compute that f n plus 1 first.

Let us now use the corrector and write down the corrected value so y n plus 1 corrected is y(0.8) corrected value and that is equal to y n which is 1.73779 plus 0.2 by 24 into f n plus 1 which is this and that turns out to be 12.37059; then the next term is 19 times f n which is 26.17801; then the third term minus 5 f n that is 6.54065 and finally plus 1.17859 which is f n minus 2 and that turns out to be 2.01434 this is the corrected value of y(0.8) the predicted value is 2.01451 and the corrected value is 2.01434.

If I require that my solution is correct to two decimal places I observed the predicted value is 2.01 and the corrected value is also 2.01 so the desired degree of accuracy is attained at this step that is not the end of the problem. What is it that we want? We require the solution at 1 the question is compute the solution at 1. So what does that mean?

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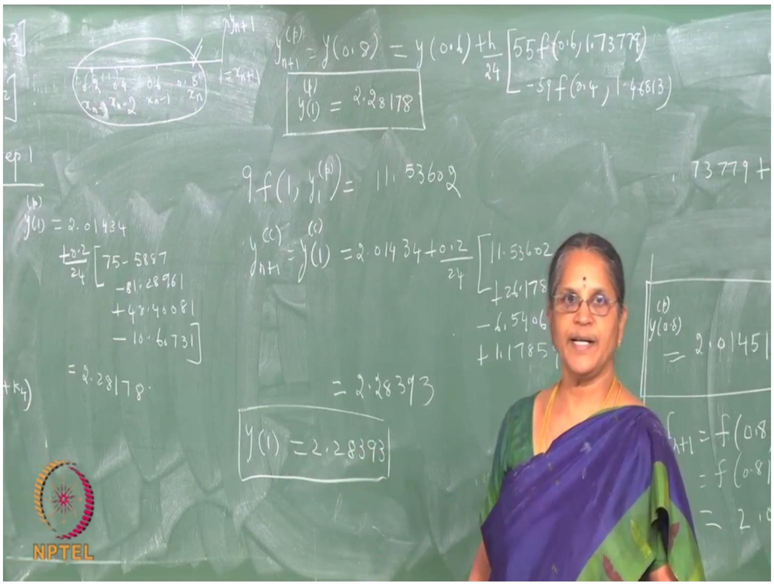

We require the solution at 1 so I should call this as x n plus 1. In order to obtain solution at x n plus 1 which is 1 we require information at the previous 4 points and they are available. The value at this point was just now computed using the Predictor Corrector Method and so we again work out the details as before. Use the predictor and predict this value and then successively use the corrector and obtain the solution correct to the desired degree of accuracy.

So I leave this as an exercise I will give you the answers you can work out the details yourself. The Predicted value turns out to be y(1) predicted turns out to be 2.28178 I can give you the step just before the final answer so that you can check the details your self. y 1 predicted value is 2.01434 plus 0.2 by 24 into [75 plus 0.5887 minus 81.28961 plus 48.40081 minus 10.60731] and that turns out to be 2.28178 that is what I have given.

Now I would like to correct this value using the corrector that requires information of f at this point so f(x n plus 1) (y n plus 1) that is y (1) predicted is required and I also have to multiply it by 9 in the formula. So I take 9 times this and the value turns out to be 11.53602. Once I have the f n plus 1 value I use the corrector so y n plus 1 corrected is y(1) corrected and that turns out to be 2.01434 plus 0.2 by 24 into 11.53602 plus 26.17801 minus 6.54065 plus 1.17859 and the result is 2.28393.

So this is the corrected value at 1 and the Predicted value at 1 is 2.28178 and we observe that if we demand two decimal place accuracy then the difference between the solution of the corrected value and the predicted value is less than the prescribed tolerance and so we have been able to obtain the solution correct to the desired degree of accuracy namely 2.28.

So in the next class we shall derive another predictor corrector method which is known as Milne's Predictor Corrector Method and the computation details are analogous to what we have done here in this case in deriving Adam Moulton Predictor Corrector method we approximated f(x,y) by using backward interpolation polynomial by Newton, whereas in Milne's Predictor Corrector method we shall approximate f(x,y) say by using forward interpolation polynomial by Newton and obtain a Predictor Corrector pair known as Milnes Predictor Corrector pair.

So we shall continue our discussion about this Predictor Corrector method in the next class.