Differential Equations for Engineers.
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Lecture-64.
Conclusions.

So we have video course on differential equation for Engineers. In this course you have learnt how to solve differential equations, ordinary differential equations as well as partial differential equations. We started with the definition of differential equations and we considered first-order ordinary differential equations can be linear or non-linear. And we have provided certain form of equations, 1st order equations, all the questions that certain equation that can be solved analytically. So we have given methods to solve those kinds, those types of first-order equations and then we looked at the second-order equations.

Second-order equationsof any order for that matter, any nth order differential equation you can convert into first-order ordinary differential equation but as an equation for the vector valued function. That means you have a system of first-order coupled equations, okay. You have system of coupled equations, that is, that is what it becomes any nth order equation. And so that is general nth order equation but if you consider linear nth order equation or second-order equation, that is actually, a linear equation has certain properties. So one of the important properties is if you are given one solution and it always, one of the important properties is that it should have only 2 linearly independent solutions.

So once you get 2 linearly independent solutions, you can make a linear combination and you can write it as general solution with 2 arbitrary constant. So in the process we devised Abel's formula, we derived Abel's formula and if you are given one solution through which, through this Abel's formula we can find the other solution. And simpler equations, linear equations we solved and when you cannot find any solution, we have actually devised, will be found through power series method, we find both, 2 linearly independent solutions and in that, so that depends on the point of whether you have single point or not.

If you do not have single points, you can actually find 2 linearly independent solutions as power series solutions. If we have a singular point around the point, either this side or that said, you can have, you can have solutions, 2 linearly independent solutions, that is why the Frobenius method, motivated by the Euler Cauchy equation, okay. So we, we had this base, whatever method we have, this power series method and Frobenius series method, based on this we actually used some physical equations such as Legendre's equation, Bessel equation, we apply these methods to find the solutions. And powder solutions only be considered and defined them as Legendre polynomial and Bessel functions, okay. These are, these play important roles in the physical and, physical sciences.

That is where you see, whenever you see, in many physical situations, these kinds of equation play an important role, that you have actually seen in the partial differential equation, when you consider boundary initial value problem for certain partial differential equations, heat equation or wave equation you may come across this kind of ordinary differential equations. Okay. Then so based on something to do the second-order ordinary differential equations, we have given certain properties, so the property is putting into the normal form.

When you put in the normal form, analogous to the symmetric matrices, that is where you have the eigenvalues an Eigen vectors as a real values and eigenvalues are real, completely real and you can find, you can have all the Eigen vectors. So based on, analogous to the matrix arrays, using that analogous to the matrix arrays, we have certain second-order differential equations that is always, you can always put it in the symmetric form so that you can find always eigenvalues an Eigen vectors. That form complete orthogonal Eigen vectors, okay, Eigen functions here because it is, it is not simply finite dimension as in the case of matrices.

Therefore solution of the differential equation is, so the solutions can be from a space of functions, for that is infinite dimension. So you have, so based on this property that you have eigenvalues an Eigen functions, you see that the Sturm Louisville, you develop the Sturm Louisville theory which is eventually we used to find the solutions of the partial differential equations, okay. So we started with the second-order partial differential equations, we classify them, we always, in the classification we translate into new variables.

Instead of independent variables which changed to new variables, the new variables it will be one of the typical forms, either very equation, hate equation or Laplace equation type or the, hyperbolic, parabolic or elliptic. So one of these 3 rights of equations we have, the typical equation that we have considered are wave equation, heat equation and as in the prototype equation for this type, 3 types of equations with equation, heat equation and Laplace equation. And for each of these equations we have considered initial value problems and initial boundary value problems based on the domain, special domain.

And time is always positive, okay. So it happened for heat and wave equations because you series 2 different things. So and for the Laplace equation this is a kind of steady-state heat equation, while steady-state is reached for the heat equation, that is actually a wave equation. When you reach the steady-state for heat equation, that is Laplace equation. So Laplace equation we can only provide the boundary data and you could find, you can find the solutions, unique solutions only in certain domain that we have seen in this course, okay.

And also in the applications you can see that many of the problems that have solved are having applications, physical and inherent sciences, such as drum problem, vibrations of a string, vibrations of drum, two-dimensional wave equation and also that, that is where you solve the, you bring in always Sturm Louisville problems either of the type that we have studies, okay. So you see that most of the problem that I have, we have studied are basically on physical sciences, so that is really useful for you to, if you if you actually involved in, if you are in some project, if you are working with some differential equation, working with some model, so partial differential equation.

So typically if you have, if you encounter a problem, if encounter or if you are working with a partial differential equation, okay. That can be any one of the 3 types. So for this if you, if you can do some transformation and reduced to one of these problems, you can make use of these methods in your in your work. Otherwise what, if you really want to, further to this course, you can always look at certain equations, nonlinear equations, non-linear partial differential equations written study. Main thing is you have to understand the not just methods and solutions, and also look at the characteristics of these methods, of these equations.

They have certain properties each equation is different from the other. So if you start with the wave equation, we can see that speed of, you have a speed of propagation. That means you have a characteristic variable, you have Eta and xi, Eta and xi are 2 new variables, they have, then actually X minus CT and X plus CT. So they have, these are the characteristic equations, so whenever they have, so they propagate, it has a finite speed of propagation as you can see for the wave equation. But it is not the case for the parabolic equation. So where you have UT, UXX you have any see that whenever you have this, you do not have characteristics here but you have infinite speed of propagation.

Whatever happens here it infinitely in no time, if it has a discontinuity in the initial domain and it smoothens out in no time. So as T, after finite time you always, whatever you give initial data as a discontinuous thing, discontinuous initial data will be smooth and out, that means it will be differentiable. But that is not the case for the very equation, if you give the initial data is having some kind of discontinuity, okay, suppose you give U of X0 equal to fx, that fx is a piecewise continuous function for example. So as time goes, still this discontinuity will propagate with the speed of propagation. Okay that is C, that is our hyperbolic equation behaves.

So these parabolic equations behave like the heat equations, it is actually in the heat equation you see that diffusive equation, you call diffusive equation, okay, heat equation. And we have seen the Laplace equation, so that is a typical elliptic equation when you see that boundary data should only be provided as a steady-state heat equation. So when you achieve the steady-state in the heat equation, that is exactly the Laplace equation that we have solved by this method. You may encounter, if you have encountered any of these equations, these methods can be used for, I suggest you to look into other books, advanced books for the 2, and study further on these equations, okay.

And also you can look into the books, partial differential equation book, on nonlinear equations, okay. So thank you very much.