Differential Equations for Engineers

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Lecture – 61

Laplace equation over circular domains

Welcome back in the last few videos we have seen how to solve the laplace equation, steady state, of the steady state heat equation so the last video we have seen how to solve the laplace equation in a rectangular domain we can solve the laplace equation in a kind of strip kind of thing it is a rectangular but strip okay so we will just I will give you that remark and I may give you as an exercise that problem and then we will solve laplace equation in circular domains okay.

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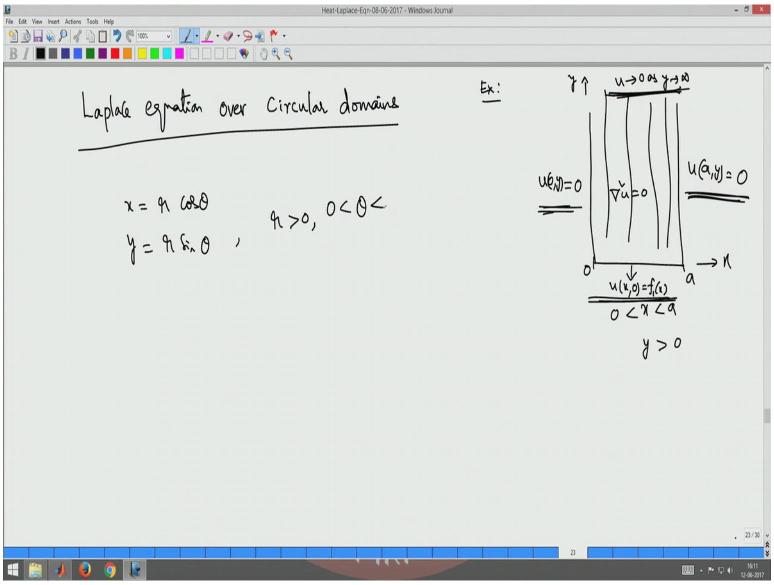

We will just look into this to do that so what I want to say is see you have a finite thing so this is 0 A as a x domain you have a finite and you can close this here so you can consider this as a domain y equal to 0 so x is between 0 to A y is between 0 to infinity, so if this is your y axis and this is your x axis, axis between 0 to A and y is actually greater than 0 so in this domain, this is your domain where you have a laplace equation is satisfied and then on this you provide some boundary condition so that means u so gradient square u equal to 0 and u at x y 0 is given as some f1 on x and here because the steady state heat equation so you want the temperature at infinity will be 0.

So you can give that infinity condition so if you give then only you will have a solution exist u goes to 0 as y goes to infinity okay so that is what you can give so for all x between 0 to a so this is the kind of boundary condition at infinity okay so that means you need you cannot allow non-zero solution at infinity so if you do that and you give the boundary condition here so that is u at x is 0 and y is g1 of y and u at a y equal to g2 of y.

So this problem also you can handle similarly in a similar way that I have sought the laplace equation for a rectangular domain so the idea is you find this you make this 0 condition, you make this 0 conditions so make this 0 so that is what we have seen in the last or just for this you have to give 0 boundary condition because this contributes where finite places X equal to 0 and x equal to A you should have Sturm-Liouville problem so that is why you should have homogeneous boundary conditions.

If you give this boundary condition then you can solve in this semi-infinite statement so how do we do this so you extract the problem for X of x which is for with the boundary condition x of 0 is 0 and x at A is also 0 okay and then and once you solve this and you get the eigenvalues and eigenfunctions corresponding Y of y you can find out the general solution okay and you find the general solution with lambda so and then you apply as u goes to 0 as y infinity.

So for that so that you can remove one arbitrary constant so we will have because of this boundary condition you will only get for each eigenvalue you will have only one arbitrary constant for each n suppose lambda n for each n lambda n you have A n as arbitrary constant so that arbitrary constant you can get it from this boundary condition so with that so in that sense separation of variable that can be applied for this semi-infinite domain for the laplace equation though it is a rectangular so we use only Cartesian co-ordinates okay.

This is what I may give as an exercise so you can also work at if you want, if you are interested then just simply work out okay so today we will just look at the laplace equation in a circular domain so if you want to use a circular domain so you have to convert the laplace equation into polar co-ordinates so we have seen that x equal to r cos theta, r sine theta are polar co-ordinates so that r is positive and theta is between zero to 2 pi okay.

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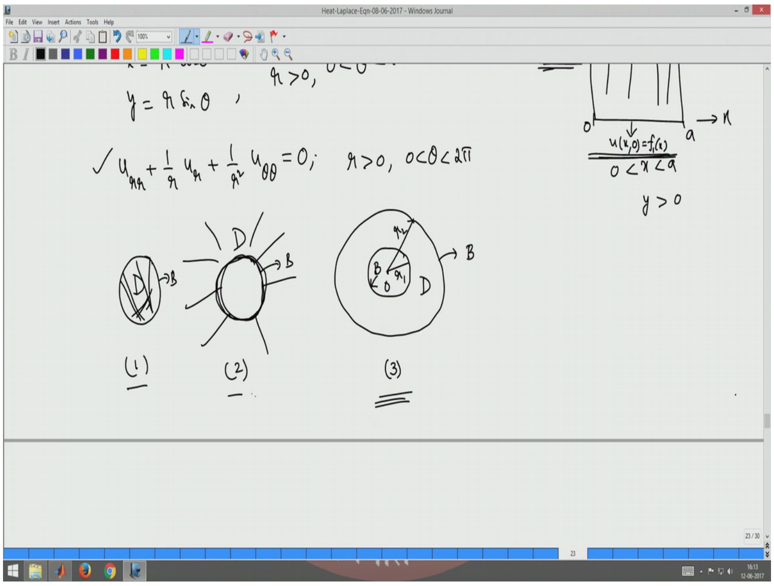

So you have a new variables r and theta so if you convert this del square u so del square u that is u xx plus u yy equal to zero so this becomes laplace equation becomes with this new variables you have u rr plus 1 by r ur plus 1 by r square u theta theta equal to 0 so that is what is the laplace equation r positive and theta is between 0 to 2 pi so this is a laplace equation in the full plane, okay.

So now we will just look at the general problem, general boundary, general domain so I will just consider so you cannot solve all domain so only circular domains for example you can solve this equation in this circle inside the domain is D here or you consider exterior circle, exterior of the circle that is D, D is your outside so that still the boundary is same this is the boundary okay this is the boundary here so here also this is the boundary.

So to get this kind of two things, two domains this is one particular problem okay this is the second problem so if you want to solve this laplace equation in the domain D okay and you give the boundary condition on b so considering this as a domain D is here domain is here outer exterior of this circle and here interior of the circle so D is interior of the circle.

So these are the two problems so in the two domains you can solve this laplace equation and to do this we give a general, so these are just particular case as a particular case you can get out of a general problem, general domain so that I consider as a annulus region so let me consider some annulus region that is where my D is, D is here this is 0 and you have this with r1 and this with length r2 two circles okay.

So your domain is D so your boundary is this is your boundary B are also this interior inside this is also your boundary okay so B is basically boundary inside the inner circle and outer circle okay, so we will solve this equation so the problem for three or domain three if you do this one and two will be particular cases as a particular cases you can get it

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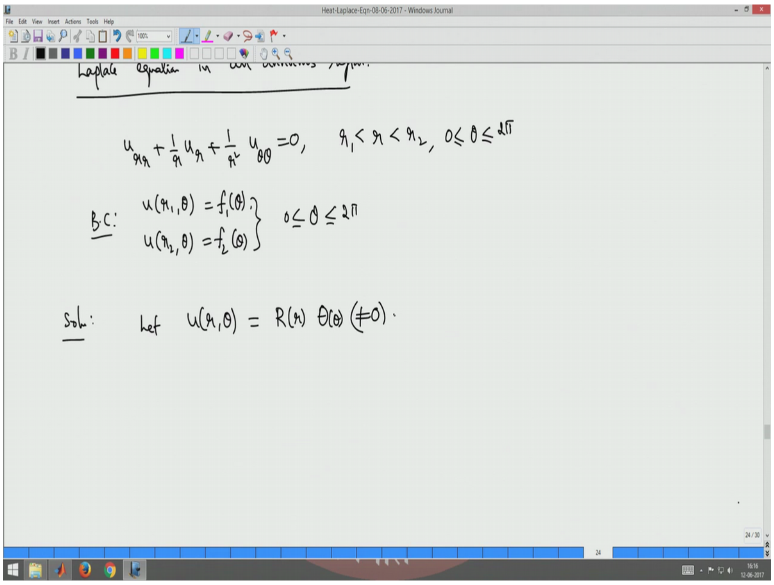

So let me write the problem for this annular region so laplace equation in an annulus region okay so let me write this so to take the same laplace equation so which in polar co-ordinates u rr plus 1 by r ur plus 1 by r square u theta theta equal to 0 now what is your r so the domain is d so r is between r1 and r2.

So that is the region D and theta is always because it is a circular domain so you have 0 to 2 pi so that is your, so you have this includes 0 to 2 pi okay so this is the domain over which you have the laplace equation is given now if you want to give boundary data. So what is the boundary data you want to give so you simply give the dirichlet data, so you provide a temperature on the boundary that is ua, u of x y equal to some so you can see that this is only functon of this r is fixed here okay.

So that is u at r2 theta so it is a function of theta, r2 is fixed okay r is fixed here on this domain because the circular domain so it should be some F1 of theta okay so let us call this f2 of theta so that inside here, inside also you can provide so where u at r1 theta on the inner circle you can provide as f1 theta so these are the two domains this is the boundary data 0 r theta is between 0 to 2 pi so you can provide that so you have u of r1 theta is f1 theta and u at r2 theta is f2 theta so that is what is the theta is between 0 to 2 pi.

So with these are the boundary conditions so this is your laplace equation you want to solve this equation so to do this so we will just work out the solution by the separation of variable methods so again let with solution, let u of r theta so now you have the variables r and theta as capital R of r as a function of r and capital theta of theta let us call this way as a non-zero solution so you assume that this is a non-zero solution.

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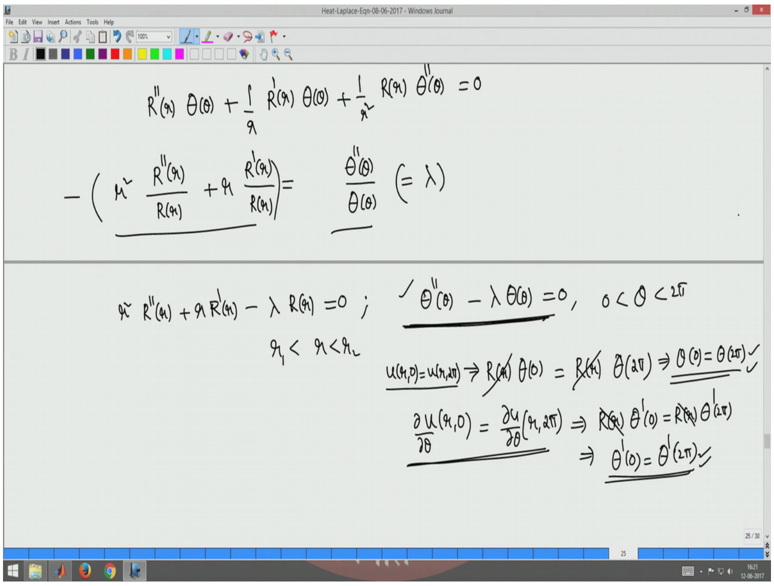

Then if this is the solution of this laplace equation is just simply substitute if you substitute into this equation what you see is R double dash of r, theta of theta plus 1 by r, u r is R dash of r and theta of theta plus 1 by r square R of r into theta, big theta double dash of theta equal to 0 so that is what is the equation become.

Now from this you can do you can separate this r and the theta variables by just dividing with R of r into theta of theta that is the solution which is a non-zero solution so you can divide both sides so if you do that what you left with is R double dash of r by R of r plus 1 by r times R dash of by R of r because theta of theta gets cancelled so plus 1 by r square R of r goes now you have theta double dash of theta divided by theta of theta equal to 0.

So what I do is I take this side minus I bring this r square here so you have r square here so you multiply r square okay so simply write like this first of all so if you want so you bring this r square here so if I remove this bring this r square here so you have r square so you have r square r r goes so you left with only 1 r so this is what you have okay.

So these are functions of r, these are functions of theta so this should be a constant okay unless they are constant it cannot be same because two functions of two different variables, two infinite variables are not theta so this is what you have so the laplace equation becomes two ordinary differential equation one is r square R double dash of r plus r R dash of r this R of r into lambda so that becomes plus lambda into R of r equal to zero this is one equation one ordinary differential equation other is theta double dash of theta minus so you better you make this minus here so okay bring this minus here so they make it plus so that you have a minus here so that this Sturm-Liouville problem for theta what you get is minus equal to minus lambda theta of theta equal to 0.

So theta is between 0 to 2 pi and here r is between r1 to r2 okay so we extract the Sturm-Liouville problem for theta because if you see that theta is between 0 to 2 pi and what are the conditions you do not have a boundary conditions you have to look at this figure and see that is a circular domain so theta at 0 and theta 2 pi both are same so they are basically and it satisfies a laplace equation so it is a derivatives are also continuous okay.

You differentiate with respect to theta they are actually continuous so you can see that u theta, u theta is actually continue, u theta derivative is twice differentiable so u theta is continuous and u is also continuous that means as a theta as a function of theta it should be continuous variable so if you use that r you simply think of this way so u of, boundary conditions are like u of r theta which is R of r and theta of theta.

So you put theta as 0, this should be same as R of r theta of 2 pi okay now same way u dash of so that is dou u by dou theta is also continuous at r theta, theta is 0 should be same as dou u by dou theta at r 2 pi so both are same so this will give me, so this first of all this will give me theta at 0 this is 0 okay, theta 0 so it should write at u r at 0 should be same as u r at 2 pi because it is a circular domain it is a periodic domain.

So this will give me this so r r goes what you get is 0 theta at 0 is same as theta at 2 pi okay this is one boundary condition for this Sturm-Liouville problem so similarly so these are (()) (13:57) this need not be given okay that you should, you are getting it from the because of the circular domain you can easily see this one.

So if you use this one R of r theta dash at 0 should be same as R of r times theta dash at 2 pi, so this will give me r r goes both sides okay this cannot be 0 so r r because it is a function of r you can it should be non-zero so it should be cancelled so you get theta dash of 0 equal to theta dash at 2 pi so these are periodic boundary conditions that we set for periodic Sturm-Liouville system.

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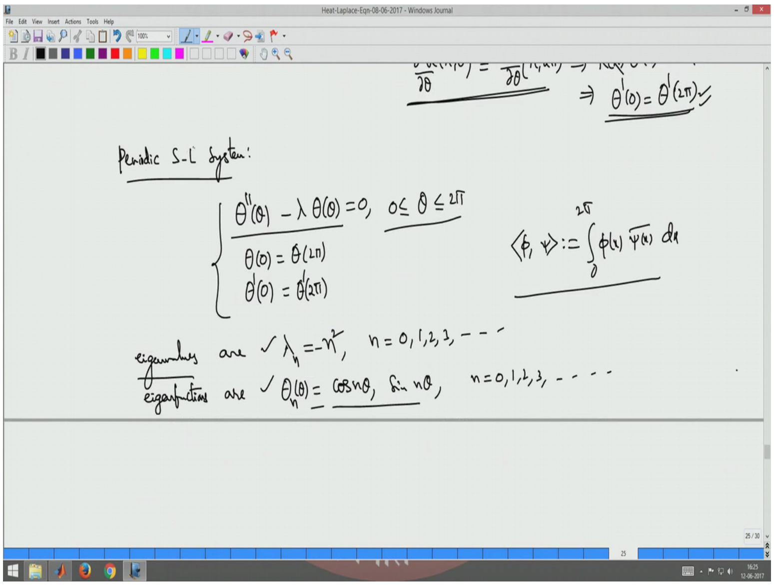

So periodic Sturm-Liouville system is regular type of equation with these boundary conditions if you give this boundary, periodic boundary condition it will become periodic Sturm-Liouville system what we got is periodic Sturm-Liouville system, that is theta at, theta double dash of theta minus lambda theta of theta equal to zero theta is between 0 to 2 pi and theta at 0 equal to theta at 2 pi this is one periodic boundary condition and other one is theta dash at 0 is same as theta at dash at 2 pi.

So these are the periodic boundary conditions okay so this is the periodic systems so which we know already so already in self adjoint form so we have seen just look back the video on periodic Sturm-Liouville system, so in the Sturm-Liouville theory we have seen that this is exactly periodic system we solved okay so the eigenvalues are basically, eigenvalues so eigenvalues I will write directly eigenvalues are lambda equal to n okay so you can say that lambda is n and eigenfunctions are let us call this theta n, so lambda n I call it n, n is running from 0, 1, 2, 3 onwards okay and then theta n of theta is actually equal to, so eigenfunctions are cos and theta and sine and theta okay.

So n is running from so 0, 1, 2, 3 onwards so when you put n equal to 0, cos n theta is 1 and sine n theta is 0 so what you affectively it is for n equal to 0 only one eigenfunction remains for n equal to 1, 2, 3 onwards you will have two eigenunctions so these are all already because 0 to 2 pi these are already orthogonal so they are self adjoint they are actually complete orthogonal set of eigenfunctions.

So this is what we have seen in the Sturm-Liouville theory, so the dot product of these functions theta or the solutions, solutions of this Sturm-Liouville system phi psi as usual this is 0 to 2 pi and phi x and psi bar of x dx these are all real valid functions so bar does not matter so this is the definition of the dot product because this is a periodic Sturm-Liouville system 0 to 2 pi okay.

So with this, so you know that these are the eigenvalues lambda, lambdas are n so you, once you know the lambda you put here as a lambda, lambda equal to n and try to see what happens so when you put lambda equal to n so actually what you get is I think (())(17:30) miss is should be n square so when lambda equals to n square you will get actually minus n square, minus n square you will get okay for lambda equal to minus n square you will get cos n theta sine theta as a solution okay.

So these are the eigenvalues you can look back into these periodic Sturm-Liouville system okay in the earlier videos you see that these are your eigenvalues and these are your eigenfunction so lambda n you get as a minus n square so if you go back and put it here okay so you can see that minus of this equal to lambda so you what you get is, this will be plus so this is the equation, this equation, this ordinary differential equation this is actually kind of Euler Cauchy type equation so we will write it here.

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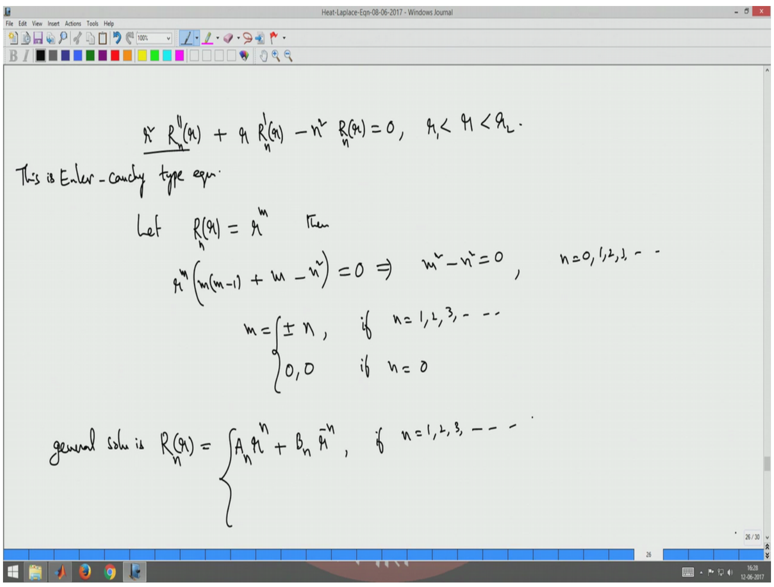

So r square capital R double dash of r plus r times capital dash of R plus what you have is plus lambda R of r so lambda is minus n square so I can put minus n square R of r equal to 0, r is between r1 to r2 so what is its solution this is Euler Cauchy type equation okay this is Euler Cauchy type equation the reason is you have Y double dash of x as the product, so co-efficient is x square so you have a Y dash of x this is co-efficient is x so r into R dash of r and here you have a y with a constant so this is Euler Cauchy type equation.

So the solution you should look for is y of x equals to x power m okay so in the same way here R of r so let R of r be r power m then if you look for solution in this fashion so if you substitute into that equation so what you get is r power m times m into m minus 1 if you replace here if you put r dash so the m into r power m minus 1 this r r goes so you still again you get r power m into m minus again r power n is to substitute that is a common so minus n square equal to 0.

So what you get is m m goes what you get is m square minus n square equal to 0 so n is, n we know that n is actually 0, 1, 2, 3 onwards so m is equal to plus or minus n so two roots if n is from 1, 2, 3 onwards otherwise 0 0, two solutions okay there are repeated roots if n equal to 0 that is what you get so if these are repeated roots first of all we will write so the general solution is general solution of that ODE, general solution is R of r is, so depends if n equal to 0 if n equal to 0 you can write, if n equal to 1, 2, 3 onwards you have plus or minus n so you have for each n so you can write you can label them as Rn capital Rn okay so you have a capital R n for each n is running from 0, 1, 2, 3 onwards.

So you have a capital for each n you have so you are writing so n is from 1, 2, 3 onwards what you get is r power m is n so I call this some A n arbitrary constant is also solution plus B n r power minus n as a solution if n is from 1, 2, 3 onwards because they are repeated roots they are actually distinct roots plus or minus n, n is non-zero.

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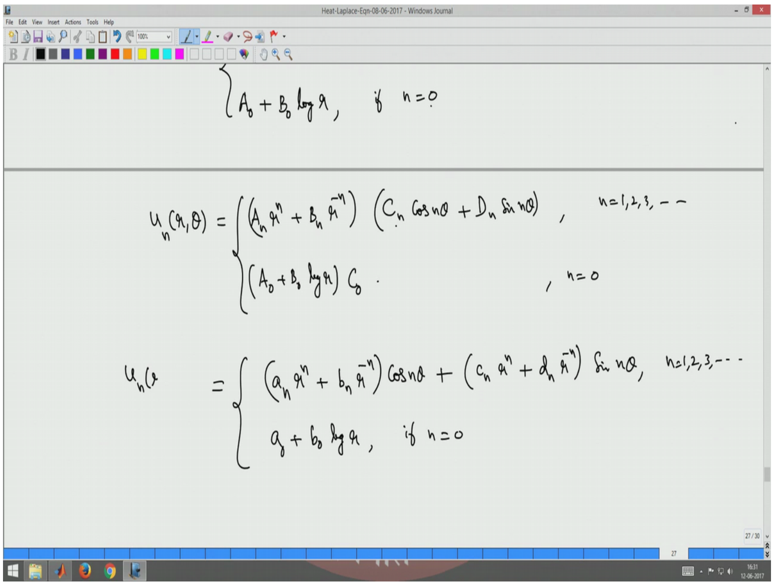

If they are repeated roots we know that it is actually some Cn so actually n equal to 0, if n equal to 0 so you call this A0, A0 r power 0 so r power 0 is 1 plus B0 and you have r power 0 into log x right, so x power same repeated root m is the repeated root if m equal to 1, 1 what you get is r power 1 is one solution r power 1 into log r is another solution okay just like x power 1 and x power 1 into log x so if you do the same thing here so you have m equal to 0, 0 so r power 0 is 1 and you simply get log r.

So log r is, this is your solution these are the solutions you get r power as a general solution if n equal to 0 this is this otherwise this is the solution so now you can see that you can write the product of these solutions what is your actual what you are looking for is u of r theta so for each n you can write as u of r theta, u n of r theta for each n un of r theta is R n of r that is R n of r is you consider A n r power n plus B n r power minus n times this is r and (())(22:43) into theta n of theta.

So theta n of theta is so if theta n of theta is cos n theta and sine n theta okay, so cos n theta you can put cos n theta, cos n theta so now we can see that cos n theta sine n theta are solutions okay they are also linearly independent solutions cos n theta sine n theta so you can have a linear combination okay.

So you can put linear combination so let us call this some Cn cos n theta pus Dn sine n theta this is for n is from 1, 2. 3 onwards so if n equal to 0 you are left with A not plus B not log r times if n equal to 0 only one constant, so you have a Cn 1 so Cn into 1 so C0 into 1 so C0 into A0 if C0 is arbitrary constant A0 is arbitrary constant so I will just define, okay let us call this C not times n equal to 0 simply 1 so that is the only thing left okay.

So you can rewrite this as, you can write C n times this you can take the C n times out so what you get C n into A n you can define a new one so let us call this a n small a n so let us call this big C n and this is a big D n so these are big C0 okay so now I putting as a small cn so capital A n into capital C n I am calling it a n r power n capital B n into C n is small b n into r power minus n into cos n theta plus d n, a n I can call something else small c n r power n D n, B n capital B n, capital D n I am calling some small d n into r power minus n into sine n theta this is from n is from 1, 2. 3 onwards otherwise here A not C not let us call this A not plus let us call this B not into log r, if n equal to 0. So this is what is my un of r theta

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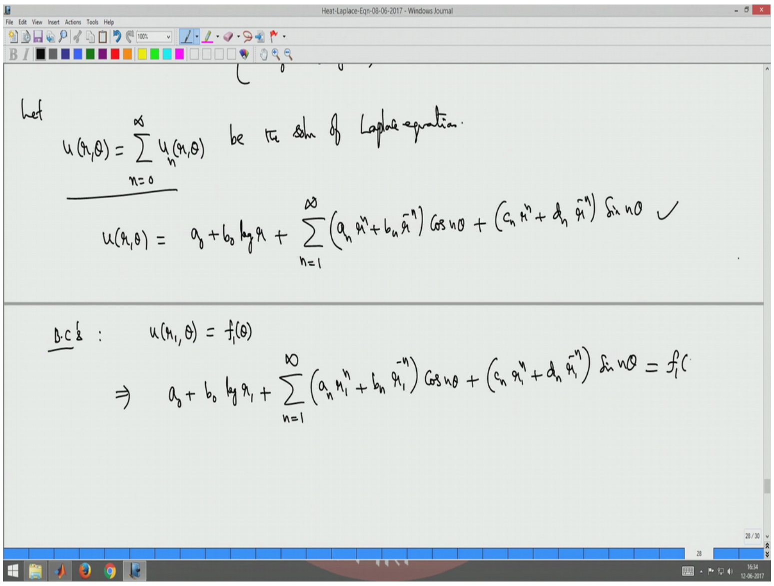

So you can superimpose all these solutions you can write some of un of r theta n is running from 0 to infinity as your solution so let u of b be as the solution of laplace equation okay so far we have not used as a boundary condition okay so we actually intrinsic boundary condition that is periodic boundary conditions we used and we extracted the Sturm-Liouville problem with periodic boundary conditions we derived this solutions and we super imposed them this is what you see that we assumed that this is the solution and what is the this one you can write u of r theta as now n equal to 0 you can write separately A not plus B not log r plus this sum I can put it as 1 to infinity a n r power n plus b n r power minus n into cos n theta plus c n r power n plus d n r power minus n sine n theta okay.

So this is the general solution what you so far okay now you are on the annulus so let us use the boundary conditions so boundary conditions let me use first u at r1 on the inner circle u at r1 theta is equal to f1 of theta so what is this one u at r theta you can put it here so you will get u a not plus u b not log r1 plus this sum n is from 1 to infinity now you can write a n small r1 power n plus b n r1 power minus n into cos n theta plus c n r1 power n plus dn r 1 power minus n sine n theta which is equal to this is what is u of r1 theta which is f1 of theta.

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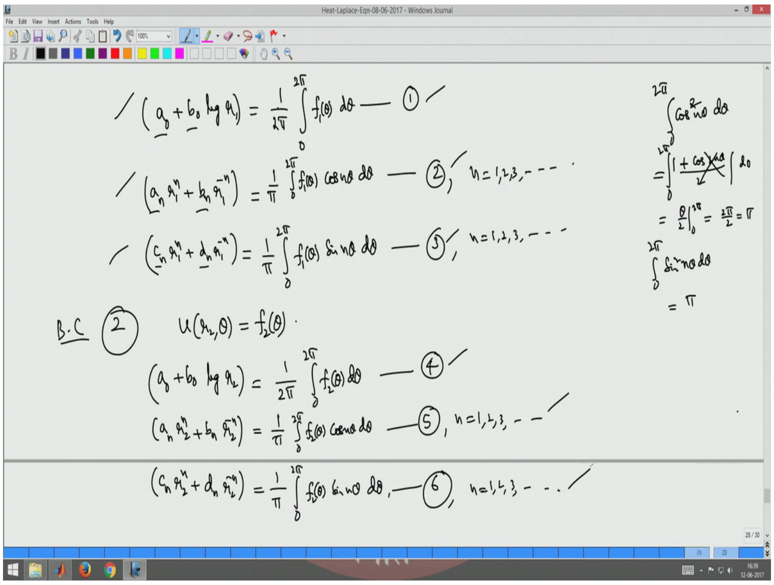

So what are your eigenfunctions cos n theta sine n theta and this is a simply function of as is simply a constant so 1 into this one this constant okay so this is 1 into this one so you have a eigenfunctions are here so this is 1 and this is 1 and this is 1 so you have a function of theta which is written in terms of all this eigenfunctions so that means I can get this constants by just by applying this dot product so I can apply both sides dot product with the eigenfunctions then I can get this constant.

So start with a0 plus b0 log r1 equal to you can simply integrate so dot product if you use this side, this side is simply integrate from 0 to 2 pi 1 square, 1 into 1 b theta so it is actually 2 pi okay, so 2 pi times this equal to or simply so this will come as a denominator so you have 2 pi so the numerator will be so 1 divided by 2 pi into numerator will be, that is the right hand side so 0 to 2 pi f1 of theta into 1 d theta so this is what you have so this is the one constant okay this is 1 and now you can write for each n you can get this constant just by applying cos n theta both sides and integrate from 0 to 2 pi.

So if you do this one you will get a n r1 power n plus b n r 1 power minus n this is a constant equal to again so 0 to 2 pi our right hand side is f1 of theta I multiply cos n theta both sides b theta and integrate and the left hand side remains only will be cos square n theta so what is cos square n theta integral 0 to 2 pi so this you can find out that is 1 minus cos 2 n theta divided by 2 that is simply 1 by 2 theta by 2, 0 to 2 pi okay so what is its value.

1 plus right, 1 plus sine square theta plus cos square theta cos square theta minus sine square theta so 2 cos square theta so this is what it is, so this is, this you are integrating from 0 to 2 pi d theta so if you do this theta by 2 so half theta that is 0 to 2 pi this will be, this will not contribute so what you get is 2 pi by 2 so you will simply have pi so I have again 1 by pi you get this one this is the equation number 2.

This is actually valid you have infinitely many equations here for each n you have 1 equation so you have the system of equations and similarly you can get cn small cn r1 power n this constant dn r1 power minus n this constant I can get it again this time you will see that integral 0 to 2 pi again sine square n theta d theta this is also you will get instead of plus you will get a minus so you again finally contribution is same so you have 1 by pi integral 0 to 2 pi f1 of theta cos instead of cos you are multiplying with sine n theta d theta again this is the system of, again n is from 1, 2, 3 onwards so this are the, from these see you have three equations kind of thing okay three equations you can find a not an cn or three so you have unknowns are 1 2 3 4 5 6 okay.

So now so far you have got only 3 equations I applied only boundary condition number 1 so boundary condition 2 if I apply so that is u at r2 theta equal to f2 of theta okay so if I do this so everything is same instead of r1 you have r2 instead of f1 you have f2 so you can have these three things will be repeated that is if I write this you get a0 plus b0 log r2 equal to 1 by 2 pi integral 0 to 2 pi now instead of f1, f2 of theta d theta so this is the equation number four and similarly a n r2 power n plus b n r2 power minus n equal to 1 by pi 0 to 2 pi f2 of theta cos n theta d theta this is equation number 5, now equation number six is cn r2 power n plus dn r2 power minus n which has 1 by pi 0 to 2 pi f2 of theta sine n theta d theta this is equation number 6.

So these two equations for 5 and 6, you have actually n is from 1, 2, 3 onwards, so this is how you get if you apply this, apply the boundary condition 2 you get this one this is again so you get the same similar type of equation instead of f1 you get f2 instead of r1 wherever in this constant r1 you get putting r2. So that is how I get this 4 5 6 now I have six unknowns a not a n, c n and b n, d n six unknowns and I have a six equations okay.

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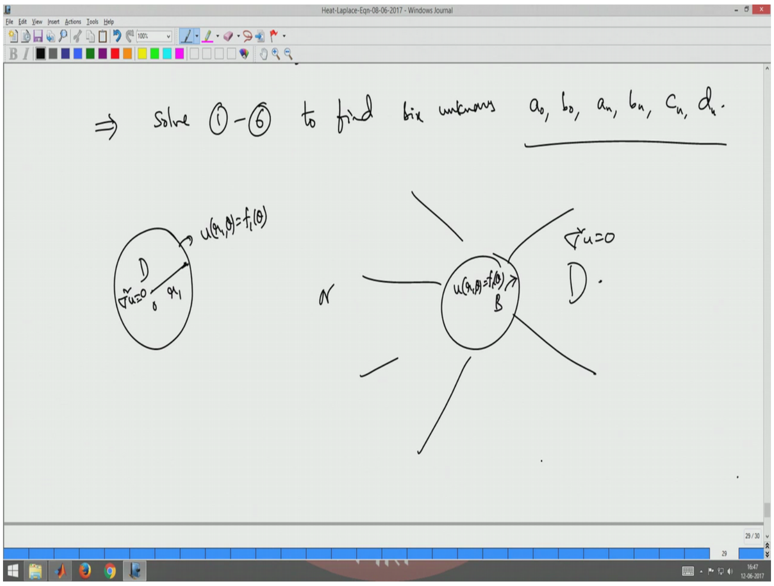


So I can find all of them from this equations, linear equations okay so implies I can get so implies I can get just by solving solve 1 to 6 to find six unknowns what are the unknowns a0, b0, an, bn, cn and dn that is it, okay.

So if you know an bn cn dn so now I have the solutions so solution now everything is known this is the equation this is the required solution so this is the required solution with now unknowns are given by solving this 1 to 6 okay so this is how you solve this is the required solution of this laplace equation the now I use this boundary condition, boundary condition is the data on the boundary on the circle okay both the circles in a annulus region this is how you solve it so far r power minus n is not a problem because it is a annulus region so annulus region you have r1 r2 which are non-zero so you do not have issue okay.

Now let us from this problem let us extract the two remaining problems okay so we solve this problem for the annulus region if you want this circular so let us say you want to solve this problem here so you have a 0 you have r1 okay so if you want if you want this solution for this interior this is D and you provide u at r1 theta equal to f1 of theta so this is your laplace equation in the inside del square u equal to 0 and this for this problem for the interior and the D is interior of the circle solution is so only thing is you have to go here so you take here so this one what happens for the interior you simply take r2 so this is the general solution which we have okay.

So the procedure is same so only thing is when you multiply when you take this solutions for example here so when you get the general solution of this R of r now what is r, r is between now because it is a circle this is circle interior circle r is between 0 to r1 so you have 0 to r1 and because 0 is involved so r is actually 0 to r1 so as r goes to 0 this is going to become unbounded but we know that it 0 this is a steady state temperature, temperature cannot be unbounded it interior of the domain so suppose if you take a circular plate initially you have, you reach, once you reach the steady state of the temperature, steady state when the temperature reach the steady state it cannot be infinite at middle of the point okay so for that Bn's will be 0, because of that now if you take this one Bn's will be 0, so Bn's will be 0 only left with this one.

Similarly this part will be 0, B0 will be 0 okay because log r going to infinity as r goes to 0, so that means this R of r cannot have, will have unbounded solution if you (())(36:04) log r solution so that means the co-efficient should take it as a 0, so here you remove this 2 and A n r power n A not that is your R of rn so that you put it so once you know B n and B0 or 0, so this part will be 0 and what you are left with A n, C n you call this a n, r n and here small b n wherever b n so the you get with r power minus n would not be contributing so similarly here okay this would not be contributing what you are left with only 1 2 3 so you have bn's would not be contributing dn's would not be so this will be 0 okay for the circular domain, for the circular domain where, this is 0 to r1.

So you can apply the initial data so that is boundary condition that is u at r1 theta so if you apply this I do not have bn's I do not have dn's I do not have b0 so what you are left with is a0 equal to this I found directly here not similarly a n r1 power n is this b n, because b n is 0, okay so that means I found a n directly similarly c n I can find directly so I can just solve it so without bothering about so if only thing is you just go back look at the solutions when you before you super impose these solutions you look at the circle so you allow now allow because r is in between 0 to r1 you have r equal to 0 so r power minus n would not be contributing.

Now similarly if you consider exterior of the circle now you take the domain is exterior of the circle so domain is D, so r is between r1 to infinity so at infinity so you want when so you if you have a infinite plate with a hole and it reach the steady state okay initially at some so it is a steady state temperature okay, it is a plate infinite plate with a hole that is the desired exterior domain exterior of the circle okay.

So at infinity it should be temperature cannot be unbounded so that means you should remove this one, a n r power n as r goes to infinity u of r theta should be bounded okay at least you cannot say it goes to 0 because it is infinite plate you can expect it actually it should go to 0 so that it needs to be 0 only bn's will be the solutions r power minus n will be the solution not r power n okay r power minus n will be the solution n is running from 1 to r so if you have in this domain because r goes to infinite u cannot be, u have to be 0 so that means you have to remove instead of bn's you have to remove an's similarly as r goes to infinity log r is also becoming infinity so that means b0 here b0 you have to allow it to be 0 okay so everything is same only an's instead of bn's now you allow an's to be 0.

So you have finally instead of bn's you have an's you allowed to be 0 and b is 0 0 when you take the combination, when you take the linear superposition, when you take the sum of them so un of r theta will be without an's and without cn's you will have you will get that and without b0 so these are the without this you get a solution now instead of bn's you will have a v bn's without this term you will have this okay for external exterior of the circle that means its unbounded domain say it is like a infinite plate, plane plate so plane is like a take a plate and you have a hole in it that is the circle okay and the boundary of the circle you have to provide the data.

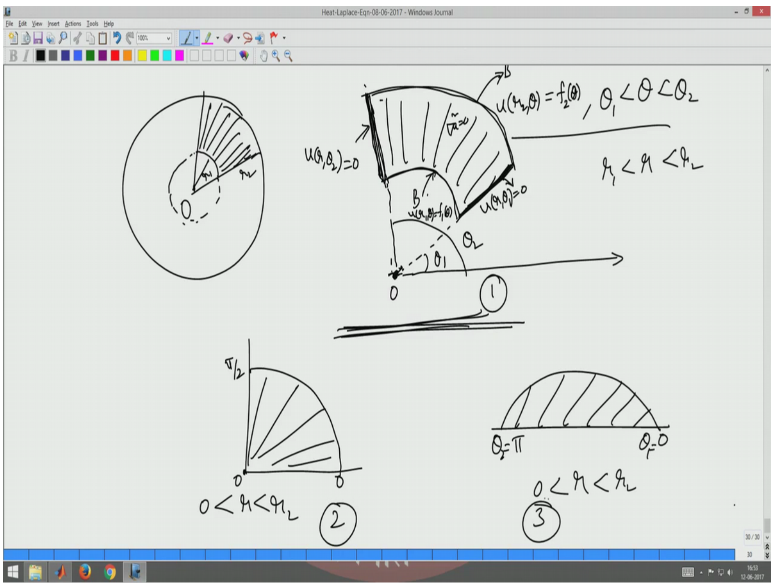
So this is what is the solution without cn's and an's without bn's b0 okay b0, an and cn would not be contributing as a final solution now you apply the initial data what you left with is b0 is 0 so you get anyway A not equal to so you get A not from equation number 1 now from equation number 2 because an's are 0 so you get bn directly from this equation number 2 from cn's are not there equation number 3 will give you dn so that is it.

So you can solve both the problems either this or you consider the exterior domain so as your D and you provide the data here so this is the boundary B, u at r1 theta equal to f1 theta okay so here del square u equal to 0 so these are the 2 problems you can simply extract from this method actually not from the exactly from this annular region, annular region we have not so this is the solution you used if you want the exterior thing you just have to allow r2 goes to infinity okay if you want this circular problem you have to allow r1 goes to 0 so that sense you can here also you can get it so directly you can once you get these solutions you can allow that r1 goes to 0 that is also one way okay.

So allow r1 goes to 0 so you cannot have unbounded things so the bn's would not be contributing okay so you will have something like that so you do not really just do not take the limits instead you part of the solution procedure which you use for the annulus region you use where Rn of r and theta n of theta when these functions when before you superimpose you consider the domain r is the, if you consider the interior of the domain between 0 to r so you consider instead of r is between r1 r2 you consider what is the domain for the interior of the circle that is now 0 to r.

Now you remove whatever is not so at 0 it should be bounded so your solution R of r will be little different similarly but theta n's are same so theta n's are simply cos n theta sine theta this would not be changed okay only R and (())(42:09) will be changed this is how you can solve interior of the circle and exterior of the circle just by the procedure of this annular region okay. So now you know that how to solve this three problems just by looking at the procedure of this annulus region okay.

(Refer Slide Time: 42:25)



So in the next video we will see how we will look at the problem in the sector like this, the sector means it is a circle this is the sector okay this is the 0, let me draw properly so if this is your circle 0 is the center and you take this one part of the circle that is called sector so in this I consider this is at r1 this is a interior circle so you have a one more circle here okay and this is your r2 so I consider this domain that means if I draw again, so next video I will do this one so this is what I do okay.

So this is the domain I consider so that so you have theta is here and this is your so r so this because of this, this is 0 okay so you have this line with theta 1 and because of this line this line let us say so this line with theta 2 okay so your domain is actually this one so here we try to solve this laplace equation is satisfied here, the boundary is now this is one boundary and this is another boundary so B is this, B is this, so on this B we provide a data okay so dirichlet data that means you provide the temperature u at r2 of theta, theta is between theta 1 to theta 2 equal to okay so some function f1 f2 of theta because at r2 okay.

So similarly here you can provide u at r1 theta as f1 of theta, theta is between theta 1 to theta 2 earlier for the circular domain between 0 to 2 pi now here is only because it is a sector so you have to take theta is only between theta 1 to theta 2, theta 1 is smaller so theta 2 is bigger so this theta is actually theta 1 to theta 2 so what you expect is you have to, so what is the boundary now so this is one boundary and you have a boundary here so this is the boundary, this is the boundary for theta so at this theta you have to provide zero boundary conditions because you want to use the Sturm-Liouville problem you want to extract the Sturm-Liouville problem and solve it so for that you provide the data here so this is from r theta 1 equal to 0 and here u at r theta 2 equal to 0 okay.

So with this kind of problem so this will give you this will allow you to get the extract the Sturm-Liouville problem and then finally use this boundary condition to find the unknowns involved in the super position of all the solutions okay so you will look at this general problem in the next video and then now you allow to take theta equal to 0 theta 1 is 0 and theta 2 equal to 2 pi then your domain will be this one circular domain semi-circular domain okay and if you want in the quarter plane you can also get this, if this is you domain you take theta 1 equal to 0 and theta 2 equal to pi by 2 here this is 0 theta 1 equal to 0 theta 2 equal to pi okay this is pi here 0 to 2 pi.

So different domains so if I do this and again you have to allow r1 goes to 0 okay so earlier so here r is between r1 and r2 so here r is between 0 to r2 that means r1 is how to take 0 here also because its (())(46:23) 0 is involved here so r is between 0 to r2 so this is how by looking at this domains and so you can extract the Sturm-Liouville problem and then make use so once you solve this we will solve this in the next video for this general sector part of the sector okay, circular sector so if we once we solve this we can actually solve this two problems again, okay.

So we can solve once you solve this one 1 and you can solve 2, 3 as similar to 1 from the, you can actually see the special cases of 1. 2 and 3 are special cases of 1 problem okay we will see this in the next video. Thank you very much.