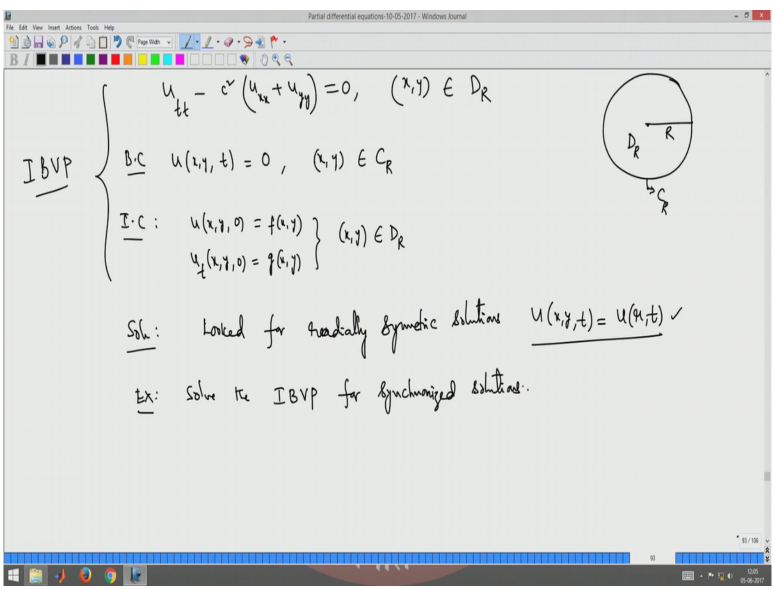
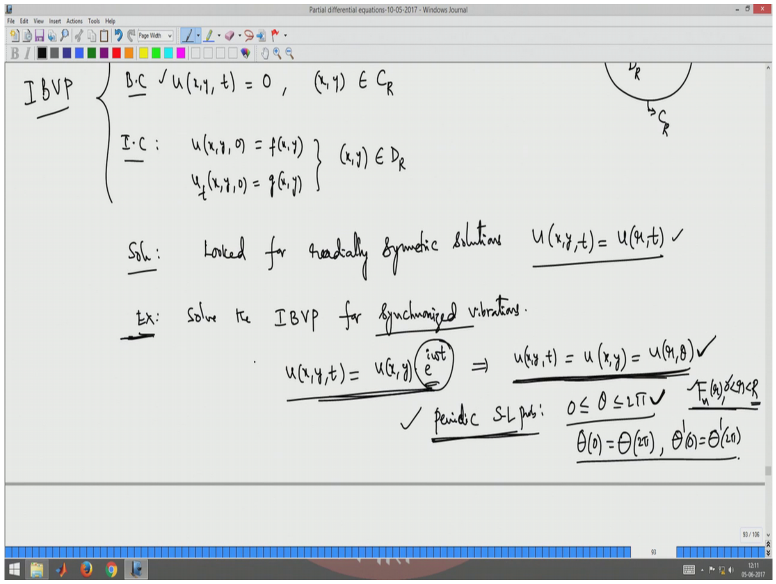
Course on Differential Equations for Engineers
By Dr. Srinivasa Rao Manam
Department of Mathematics
Indian Institute of Technology, Madras
Lecture 53
Solutions of heat equations-Properties.

So welcome back in the last few videos we have seen we have worked out wave equation 1 dimensional wave equation 2 dimensional wave equation we have worked out with initial boundary conditions. So the initial boundary value problems we have dealt for both 1 dimensional and 2 dimensional wave equation. For the 2 dimensional wave equation we have seen the drum problem vibration of a drum problem that is in a circular drum vibration of a circular drum.

Last video we have seen how to solve that initial boundary problem, so you can also workout similar problem what we do is you I will just give as an exercise before I move on to the heat equation I will just give you as an exercise the same problem vibration of a drum problem but in a different solution we can look for.

(Refer Slide Time: 1:08)





So that is you have utt minus C square uxx plus uyy equal to 0 xy belongs to that disk R, okay so that is your drum of radius R, okay. And then you have boundary condition that is u at on Cr this is Dr and this boundary is actually Cr the circle of radius so you have x, y, t equal to 0.

So on the that is what is the boundary condition so that x, y belongs to Cr, now initial conditions or u at x, y, 0 is f x, y and ut velocity of the drum x, y, 0 is g of x, y. So this is a actual problem for every x, y belongs to Dr, so at initial time this is what is happening so you know initially what is displacement of the drum the membrane and velocity of the membrane. So you can see that this is the this problem we have solved earlier by choosing by looking at the solution, solution what we have seen is the specially what is that symmetrically radially symmetric solutions.

So look for in the last video we looked for radially symmetric solutions that is u at r, theta or rather u x, y, t I look for x, y if you write it in the polar coordinates it will be r theta so and it does not depend on theta so you have you look for solution only in this form u of r, t so these are radially symmetric solutions. Now you can also look for as an exercise I can give you as an exercise the same problem solve the problem I have a problem so this is the problem boundary value problem initial boundary value problem IBVP with solution with you can also look for solutions solve this initial boundary value problem for synchronized solutions, okay.

Solutions you look for as a synchronized vibrations, okay synchronized vibrations. So what is a meaning of synchronized vibrations? So you look for your solution u x, y, t as you simply look for u x, y of into some exponential of i omega t something like this, okay. So you just bring in some (())(4:38) in the time variable. So if you look for solutions in this form then basically what you are looking for is your solutions is x, y, t t would not be there so you have x, y, t will be x, y, t so this solution is actually what you are looking for is t would not be there once you bring in once you substitute this into the equation this is your equation if you substitute this equation.

So you have this is satisfied and t, t will be simply u it will become u, okay. So utt the second derivative time derivative second order time derivative will become simply u itself. So you simply look for in this form so x, y that means you already have x, y in polar coordinates for this circular drum problem so you have r, theta, so you look for solution in the variables r theta. So by writing your solution like this you can remove this t time dependence. So if you remove the time dependence so how the time so over the time it is synchronously synchronized vibrations are all actually synchronized so between 0 to 2 pi so with radius so (())(5:53) was the frequency. So with some so over a time so fixed time 0 to t with the time period so we will have some vibrations and it repeats periodically. So you have a synchronized vibrations you can look for in the drum.

So if you look for solutions again you have you get a solution so the solution depends on r and theta so once you have this r and theta so you look for solutions in this again so if you just look for solution you follow the same procedure you will see that you will get a Bessel solution Bessel equation and because theta is between 0 to 2 pi so you will have a problem for theta theta boundary value problem is actually it will give you Sturm Liouville so periodic Sturm Liouville problem you will get for theta variable for theta variable so it is a periodic because 0 to 2 pi it repeats, okay so it is same.

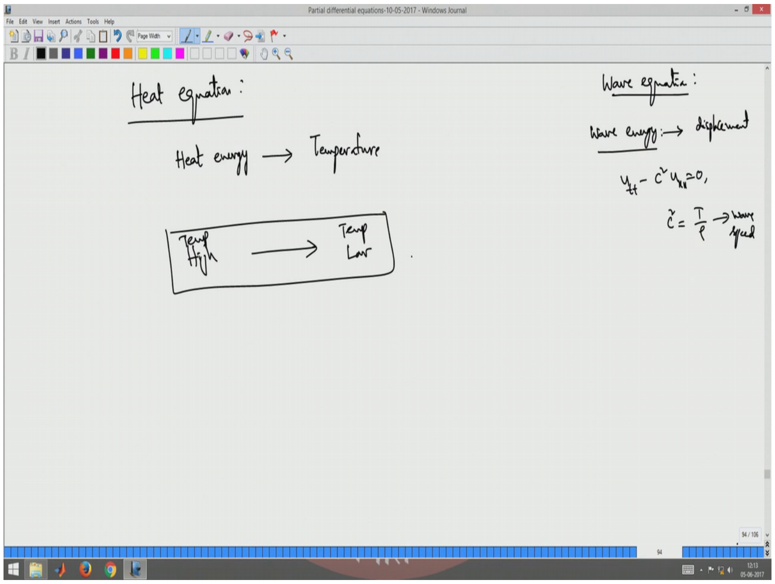
So you look for so if I say big theta of theta as your function of the separable solution in the separable solution and theta will satisfy some differential equation and you have this theta is actually theta at 0 is same as this theta at big theta at 2 pi, so and this one and you have a periodic boundary conditions that is theta dash at 0 is same as theta dash at 2 pi, so these are the Sturm Liouville periodic Sturm Liouville system you get it here for the theta variable.

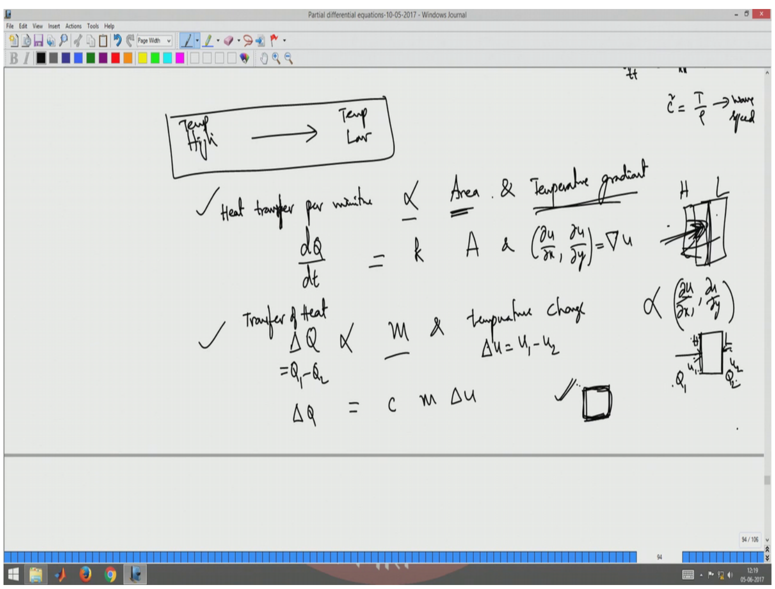
And then for so that you have eigen values and eigen function for the periodic Sturm Liouville system which we already know those are cos and theta n is n or n square so these are your eigen values cos and theta sin and theta are eigen functions they are already orthogonal form. So they are already orthogonal so you do not have to orthogonalize by (())(7:48) process so that is what we have seen when we and we worked out the periodic Sturm Liouville system.

So when you have this kind of periodic system you will get as a Sturm Liouville problem you can extract Sturm Liouville problem that is periodic type, okay. And then now for each of this n you substitute into the r variable r of r is between 0 to big R r if you call r of r is a function of or you can call this r dependence you can call F of r. So F of r will have so for each n you will have an Fn of r so because n is a eigen value so this is actually Fn satisfies a Bessel equation so you have Bessel solutions and you have a so is because is a Bessel function at 0 it is singular point so it has to be bounded boundary condition is bounded and at r it has to be you know this boundary condition will give you what is that r what is the boundary condition at r.

So that you see that Bessel function Jn of r will be your Bessel functions corresponding to for each eigen value n and then you can combine you can product take this product Fn into theta n cos and theta into An cos n theta plus Bn sin n theta. So for each n this is a general solution of this periodic Sturm Liouville system and then you can multiply with this Fn r and make a superposition of all these solutions un of theta and take a sum and then apply the initial conditions to find those two constants An and Bn so that the same way procedure is same so I leave it as an exercise and I give you in the assignment so the same problem I can give you as an exercise in assignment problems that we will see later, okay.

(Refer Slide Time: 9:38)





So we will start now we can move on to heat equation so what we have seen so far is a wave equation that is that models string infinite string is actual finite or infinite string and if you consider infinite membrane if you consider two dimensional membrane or two dimensional membrane that is actually that so that is the and you get a two dimensional wave equation that models this membrane vibration of a membrane vibration of a string is one dimensional wave equations.

So you now heat equation so you should know what is heat, heat is basically in the wave equation we have the wave equation we have some energy, so energy is a wave energy energy you cannot quantify so it is just wave energy so how we quantify is through displacement, okay so through this displacement of a string displacement. So same here heat equation you have a energy that is called heat energy, so wave energy becomes heat energy here, heat energy so we measure this heat energy as at every point as a temperature, okay.

How we derived this heat equation? So I will just outline this is not our intention to so in this course we do not intend to derive the model equation heat equation, so instead I just give you briefly what are the constants involved here in the heat equation in the wave equation we have a speed so you get utt minus C square uxx equal to 0 where C is actually T by rho so T by rho so C square is T by rho that represents wave speed, okay so wave speed so this is a wave speed.

So here what we see is we have some fundamental principles so heat actually so if you consider any rod heating, okay you consider any rod suppose it is a different apertures, okay heat actually flows the energy actually flows from higher temperature so at one point suppose you consider this rod suppose you have a higher temperature here its temperature is high here, okay and temperature is low here. So heat actually transfers from here to here so that is the, okay from that is by experiments you can easily see that this is what is true, okay.

And also the rate at which this heat flows, okay so rate of rate of heat flow that is heat transfer per minute actually proportional to the area and area and temperature gradient, what is this? Suppose you have the area so if you see that this is the some kind of wall you so the heat transfer is more heat transfer is transfer of heat energy through this body. So if you increase this area the transfer rate of transfer per minute is more, okay. As you increase it may be more depending on, okay.

So that rate of transfer heat transfer is actually proportional to this area and also the temperature gradient, so the temperature gradient is because see it may be so heat transfer may be decreasing or increasing so that depends on the temperature gradient so if your temperature is higher here and the temperature is low here so heat is actually transferring from this side if it is high here if it is high and if it is low here then the heat transfer is from this side. So is actually that gradient so that spatial derivative gradient of this is the spatial derivative this is dough u by if it is a two dimensional dough x u and dough u by dough y this is the gradient.

So this is actually proportional this is the area and this gradient that is dough u by dough x and dough u by dough y this is actually gradient of u, u is a function that is what is like displacement so you are measuring that energy so that is u of x, y if it is a two dimensional if it is u of x if simply take one dimensional rod is only u of x so that gradient so this heat transfer rate, okay that is so derivate dv t of q is actually proportional to this.

So once you remove this proportional constant that is called thermal conductivity of the body so that thermal conductivity of the body this body thermal conductivity that depends on the area and the temperature gradient, so that is called k. Also another principle from the experiment is quantity of heat gain by this body or quantity of heat loss by the body depending on, okay depending on how which direction the heat is flowing.

Temperature change, okay and the temperature is changing what is the quantity of heat gained and the temperature is different, so suppose high is here low here what is the temperature that is gaining by this body, okay is actually proportional to so this Q is actually proportional to mass of the body so this is the mass m is the mass of the body and like here you have one more principle here so you have one more principle it is the temperature proportional to this is the heat transfer or transfer of heat is proportional to the mass of the body and to the temperature change, okay.

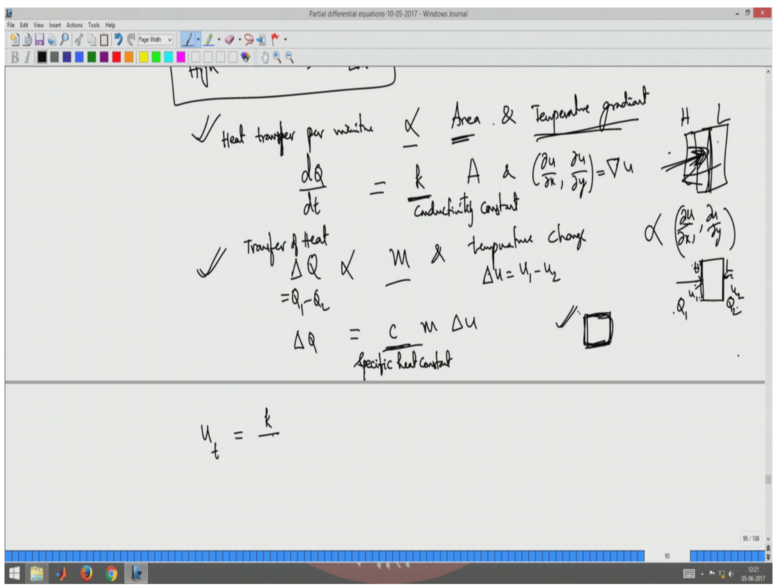
That means simply temperature it depends on the temperature how that is not simply gradient is simply how much so transfer of heat is proportional to the temperature of this change, okay this means simply the difference between 1 and 2 the other end of A a plate like this suppose you have a body three dimensional body so this to this, okay if you consider this as one thing so here so high temperature and low temperature here, okay.

So the transfer is the temperature here the temperature difference is this one u1 here u2 so that difference and this is a Q1 so that rate flow so this is heat from here the difference between heat here this is heat is Q1 u1 is the temperature Q2 is here is that heat and u2 is the temperature at this point. So this difference Q1 minus Q2 is actually proportional that is what I am writing as delta Q, so the change in the heat is actually proportional to the change in the temperature that is I am calling delta u, okay.

So actually so here if you remove this proportional constant that is called specific heat of the specific heat, okay is called the specific heat so C times m into delta u so that is delta Q. So based on these two principles heat is transferring from high temperature to low temperature you can actually derive and if you consider a typical element of a rod, okay heated rod so you can just find the heat so heat finding the temperatures and heat using that temperature and heat based on this experimental facts you can actually derive the heat equation like this.

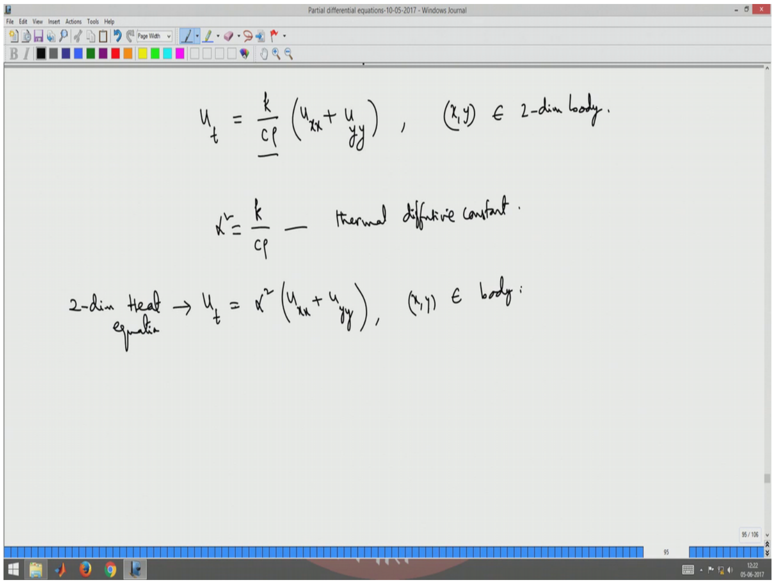
So if you have a two dimensional heat equation so you will see that if you combine this two it will (())(18:05) C m and k, okay. So m is the mass that means it involves the row of this density of the body, okay. So finally you see that our intention is not to derive this heat equation just give you the basic underlying principles based on which this is derived so you have a ut.

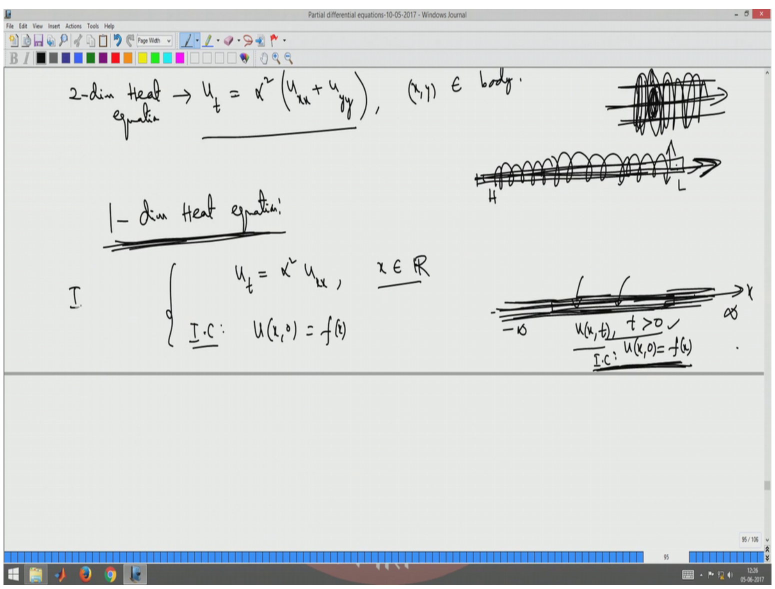
(Refer Slide Time: 18:33)

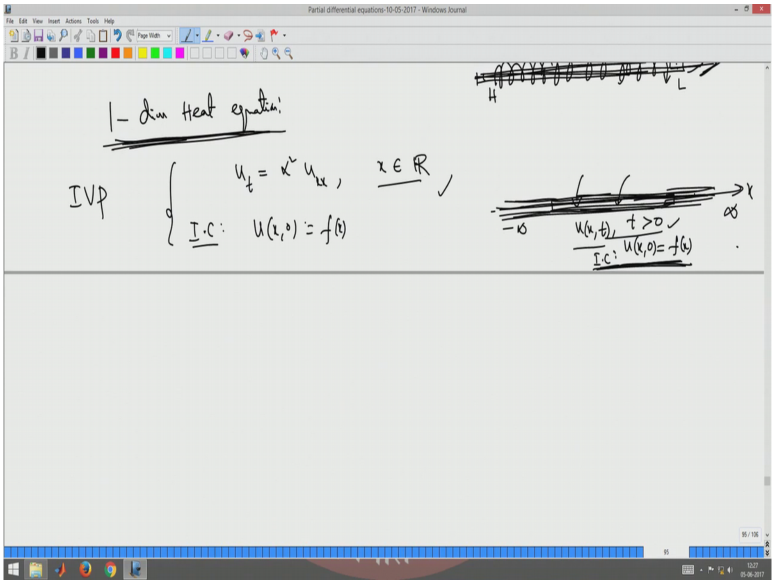


So only one derivative equal to so this will be that constant k that is thermal (())(18:35) thermal conductivity, okay so this is a thermal conductivity you can call thermal conductivity or you simply say conductivity constant this is specific heat constant, okay C is specific heat constant k is thermal conductivity heat conductivity constant. So k divided by C times C is the specific heat constant and you have that mass is depending on mass area so this is a row.

(Refer Slide Time: 19:13)







So this into uxx if it is a two dimensional body you will two elements to spatial variables x, y, okay. So x, y belongs to that body, okay two dimensional body this is how you derive, okay. So we can see that this k by C omega is called k by C omega is actually called thermal diffusive constant. So I write directly know ut equal to know I write this as k kappa, okay so I simply represent as alpha that is write this as alpha square, okay so let me write this as alpha square uxx plus sorry this is yy, okay uyy so this is a two dimensional heat equation so two dimensional heat equation is this so where x, y belongs to the body so what we do is we will as usual we try to solve one dimensional heat equation.

So this is heat equation two dimensional heat equation so we consider first one dimensional heat equation, one dimensional heat equation we will try to solve first so that is ut equal to alpha square uxx alpha square is positive so these are constants which are positive so is always positive, okay uxx x belongs to so initially full infinite rod so before I say this one dimensional heat equation this models one dimensional it is a rod you consider a rod so rod what you do is laterally you insulate it.

So if you laterally you insulate it insulate this rod laterally so that heat is not flowing in this lateral directions, okay not in this one so if you consider this rod so heat is not propagating in this kind of lateral directions, okay only direction only heat is actually flowing so that if you insulate like that heat is only flowing from only one direction that is this one. So you will have if it is a temperature is high here temperature is low the heat is flowing from high tem to low temperature that is this direction.

So heat is flowing in this direction only one direction you insulate the lateral part so outside of the rod outside the rod you insulate it so that heat is not actually you can ignore the whatever even though it is small heat may be transferring from here to here so that if you assuming that the rod is thin you can ignore those transfers lateral transfers, okay and here it is insulated both sides of the rod outer part of the rod so that the only flow is through inside that is from one direction.

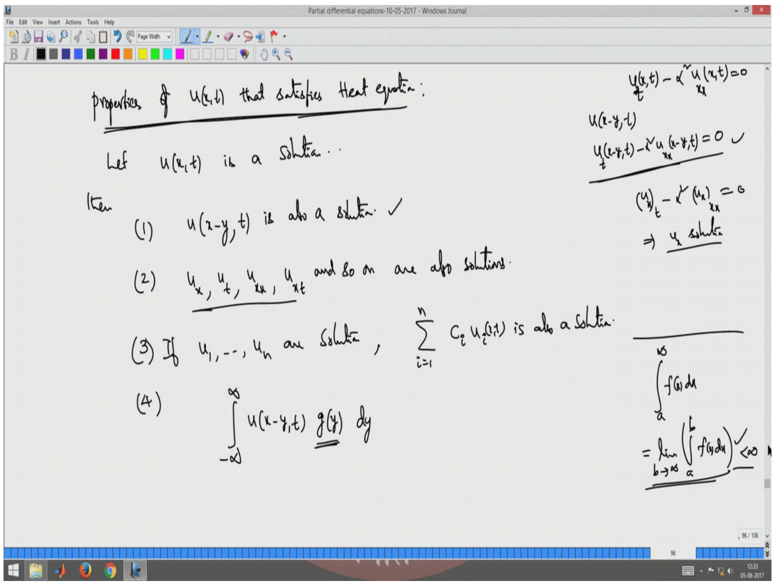
So that is why it is one dimensional heat equation you consider and you have a rod rod of small thickness so if you consider this this is a rod you can consider now infinite rod that is why x belongs to r so this is your x direction so x is one dimensional rod so that is minus infinity infinity that is the rod taking the position, okay and you want to see what is that temperature u at x, t so x is spatial variable and t is the time variable so you want to know what is the temperature across this rod.

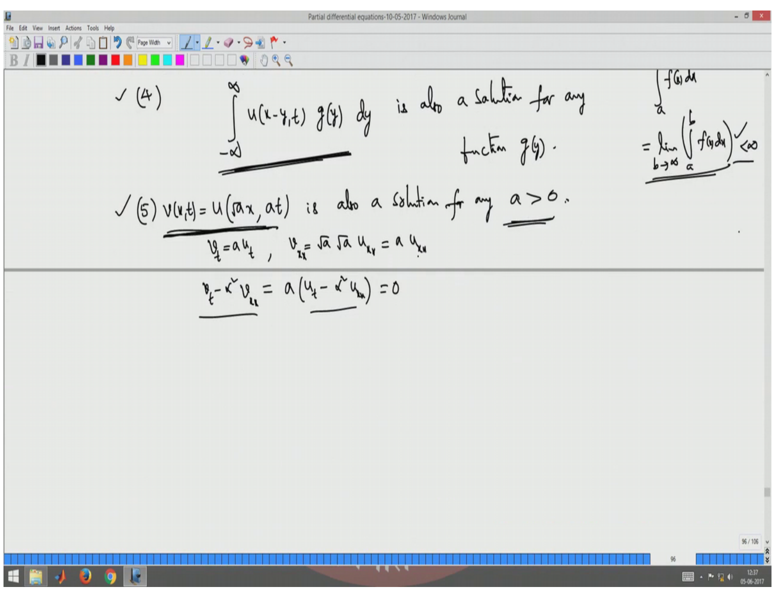
Initially if I know something, okay initially at t equal to 0 if you know the temperature so let us say fx this is the initial condition just like a wave you have the initial conditions like for a given displacement and the velocity of the string if you give you can find the solution for all times so here also we give the initial condition there is no boundary here if it is infinite rod there is no boundary you can find solution because you know that it is a rod heating rod heated rod, right? So you can actually find how after all of after some time after all times initially if you know the temperature of this rod has fx at all positions what is its temperature at all times that is you can find u of xt, okay u of xt for all times t positive so you can find that is your solution of this heat equation.

So give the initial condition as u x, 0 equal to fx initial condition, so this is your initial value problem for a heat equation, okay. So how do we solve this so earlier we solved this wave equation for infinite string just by reducing into using canonical form we reduced it into (()) (24:46) you brought a new variable xi and eta and you simply integrate it out, okay so that you have a solution that is D'alembert's solution nicely you derived it. So but for the heat equation you cannot do this is already it is already in that canonical form heat equation, right? So we do not have another type of canonical form even if you have another type, okay.

So you cannot integrate so if it is like this it involves a first order term ut and uxx. So we cannot integrate this is a typically partial differential equation so you do not know you do not have any method to integrate directly so what we do is this heat equation with this initial data we can nicely get the solution by simply by looking at the property of this properties of this heat equation, so we look at the properties of this equation, okay.

(Refer Slide Time: 25:38)





Properties of solutions u of x, t so let us say u of x, t is a solution of the heat equation solution ut properties of u of x, t that satisfies heat equation only equation heat equation, okay only heat equation you consider only this one, okay. If u satisfies this there are certain properties which you have for the solutions so we will see how we do that, okay so we will just write these properties and then first after that we will just show that the solution is unique unique solution is required.

So before I show that uniqueness of this problem, okay let me give you the properties how we find the solution is just by based on the properties of the solutions of the heat equation so first thing is you suppose ux is a solution let u of x, t is a solution then property 1 is you take a spatial translations that is spatial variable is x x minus some constant y let us call this y x minus y t is also a solution of heat equation you can easily verify so if you say this u x minus y y is constant t, what is ut? Same, ut of x minus yt, okay plus uxx that is also same uxx of x minus y t is also 0 you can easily verify this, okay.

Suppose you have ut u is x, y x, t plus minus minus alpha square so this is minus alpha square, okay minus alpha square uxx of x, t equal to 0 suppose this is (())(28:02). So you want to find dough u by dough x at x minus y if it is a constant it is also 0, okay. So this is 1 maybe we will prove this we can actually show we can prove it, okay one can easily show or we can actually prove. So first let me write the properties all these things so 2 is second one is if u is the solution u of x, t I can simply take the derivatives that are the functions of two variables u of x, t I can ux I can differentiate with respect to x, I can differentiate with respect to t, I can differentiate two times with respect to x, I can differentiate mixed derivative and so on, okay and so on or also solutions.

If u is the solution its derivatives are also solutions that also we can see, okay you consider ux ux so if you have this one this is satisfied you simply differentiate with respect to x x, x so this is a solution of so ux is satisfying now ux of t u t, x is same as u x, t so u x, t minus alpha square ux of x, x equal to 0 that implies ux is a satisfying ux is a solution so once u is a solution you put it in the equation ut what you get is ut minus alpha square uxx equal to 0 now you differentiate with respect to x and you can show that uxx is also solution like that because it is a homogeneous equation you can differentiate any number of times that shows that all these are also solutions that is a second property.

Third property is you take a linear combinations suppose u1 is a solution u1, u2 un are solutions if these are solutions some of this linear combination of all this so Ci ui, okay u1 of xt u2 of xt and e1 of x, t so these are all combination uy of x, t with constant i is from 1 to n is also a solution solution of the heat equation so this is the third property. We can write the fourth one now you simply if u is the solution I can simply write I can take some integration I write u of x minus y, t that is the solution we have seen from property 1 this I multiply with some g of y dy and integrate from minus infinity assume that such a function g for which this makes sense, okay this improper integral.

So if you have if you do not know what is improper integral so anything you know you learnt in calculus a to b, a, b are finite that is a proper integral if it is a to infinity of fx dx this you have to view as limit of a to b fx dx as b tends to infinity. So if this limit exists and these are proper integrals, okay this is defined this you know what it means and you take this limit if this is finite then you say that this is improper integral is also exist that is finite, okay.

So such a thing you have both sides limits so you can consider here to infinity and minus infinity to a, okay. So that way you split it and you see that both of them exists as a sum, so it is not true for all gy, okay so this is also a solution, okay is also a solution for where g is for any function g you can write, okay for any function g of y only thing is this should make sense this is a finite integral.

And the last one so based on this simple property is you can actually find the initial solution of the initial value problem. So fifth one is suppose u is a u of x is a solution you simply take a scaling of this variables, okay. So I can simply write u of some constant that is say let me call this ax square root of a into x and a into t if I simply multiply some root a with x and a with t for the function of u of xt, okay.

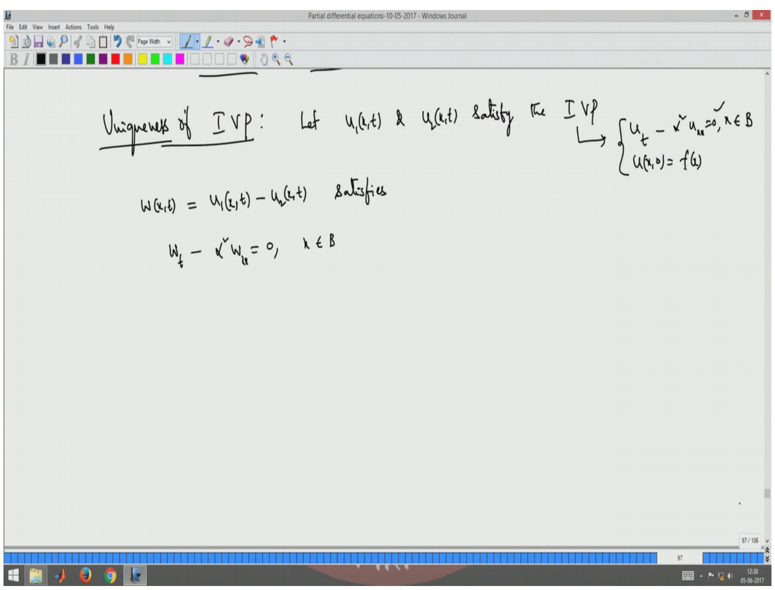
So if I call this as v of xt is also a solution so far I am not writing as this solution never, right? So you have infinite there are many solutions, okay. If u is solution is translations is also a solution translation of the spatial variable is derivatives are also solutions, okay and you can take the linear combination is also a solution and finally you take this this is called what you call this this kind of integral, okay. So you take this convolution, convolution integral with g of x minus this translation in the spatial variable and then you multiply with this constant this function g of y and integration this integration is a function of xt is also a solution.

And finally you once you have ux is solution just simply take this x variable you multiply root a now this is for any a positive because root a is defined only for positive number. So for any a positive this is what is also solution. So this also you can easily verify so vt you want to find out that is simply ut and a comes out, okay and then you need vxx vx one x will give you root a into ux other x one other x if you do you have one more root a into uxx.

So that is actually nothing but a uxx, so you see that vt minus alpha square vt minus alpha square vxx is actually equal to that a comes out which is ut minus alpha square uxx this we already know because u is the solution which is 0. So that implies v is also this scaling the scaling with variables this is called dilation also, okay. So this is the dilation function or scaling scaling these variables with the constant a is also a solution.

So based on these five properties we actually can get the solution of this initial value problem, okay is what we see before I find this solution we will just show the uniqueness of the initial value problem.

(Refer Slide Time: 35:32)



Partial differential equations-10-05-2017 - Windows Journal

$$w(x,t) = u_1(x,t) - u_2(x,t) \quad \text{satisfies}$$

$$\text{IVP} \begin{cases} w_t - \kappa^2 w_{xx} = 0, & x \in B = (-\infty,\infty) \text{ or } (0,L) \text{ or } (0,\infty) \\ w(x,0) = 0. \end{cases}$$

"Energy" $E(w)(t) := \frac{1}{2}\int_B w^2(x,t)\,dx$

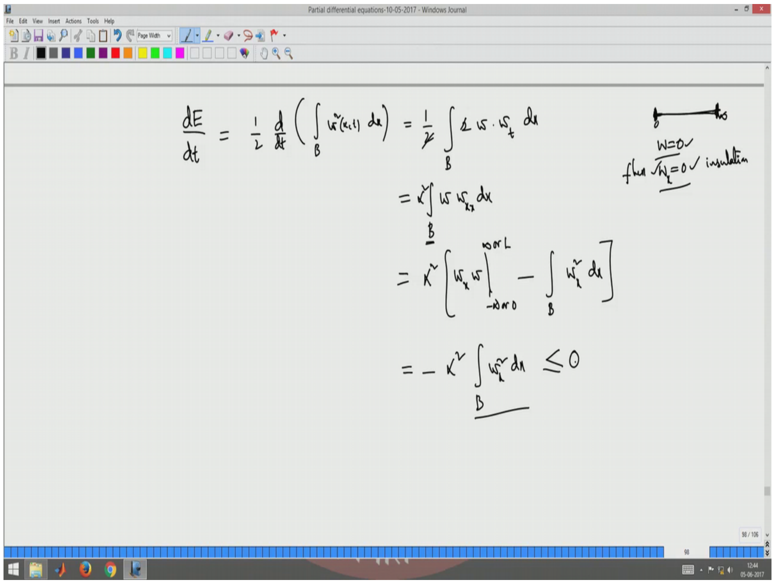
Partial differential equations-10-05-2017 - Windows Journal

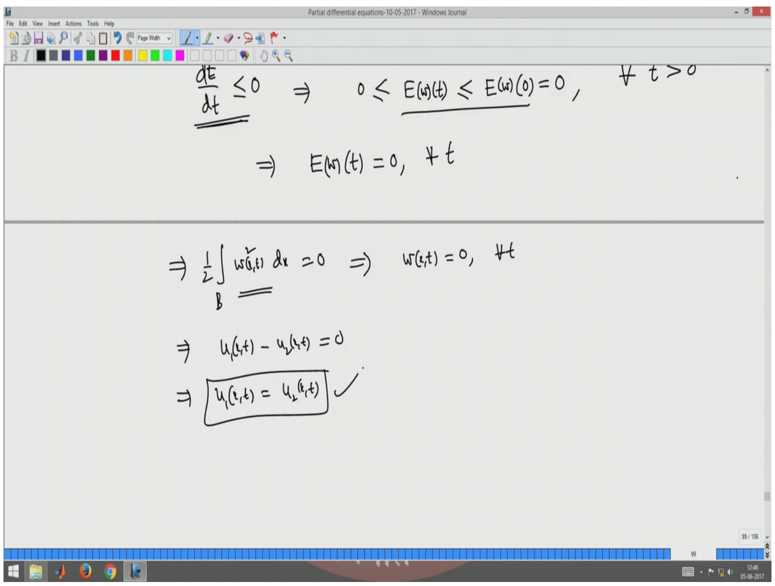
$$E(w)(0) = 0$$

$$\frac{dE}{dt} = \frac{1}{2}\frac{d}{dt}\left(\int_B w^2(x,t)\,dx\right) = \frac{1}{\cancel{2}}\int_B \cancel{2}\,w \cdot w_t\,dx$$

$$= \kappa^2\int_B w\,w_{xx}\,dx$$

$$= \kappa^2$$





So we will first show this uniqueness uniqueness by the energy argument uniqueness of heat equation of the initial value problem. So consider let u1 of x, t and u2 of x, t satisfies and satisfy the initial value problem, what is that initial value problem? That is this is you ut minus alpha square uxx equal to 0, okay so and then x belongs to some body 1 dimensional is a 1 dimensional equation so some 1 dimensional body B I am calling this say rod, okay and you have initial data is x0 equal to fx, okay.

So this is what u1 satisfying this problem and u2 satisfying this one, so let us consider w which is difference between this u1 of w of x, t as u1 of x, t minus u2 of x, t satisfies what happens if satisfies the same equation this is the homogeneous equation so this is a linear equation so with the difference also satisfies so this implies wt minus alpha square wxx is 0 and x belongs to body 1 dimensional body, and then what happens to the initial condition? u1 at x0 is fx, u2 at x, 0 equal to fx so the difference is 0 so the w at x, 0 is actually 0 so this is what is the initial value problem for the difference w.

Now I define a so called energy this is not actually from physics, okay for the wave equation we started with kinetic energy at half mb square, okay that is how we started here I defined it in terms of this solution w so energy if I write it as E that is equal to half integral over a body, okay this body B, B means this can be anything B can be either infinite rod that is minus infinity infinity or it is a finite rod let us say 0 to L or anything, okay semi-infinite so 0 to infinite anything you can take as a B.

So on this B if you simply write half w square, so w so E is a function of t for all times if you want to define as a function of t w x, t square I square this w as dx B is infinite rod minus infinity infinity so you are integrating integrating all the temperatures a blue square you have taken so Et is like that, okay. So E is depending on w so I write like this E w of t so this is what I define as energy. So clearly this is a square of w so implies the temperature is always positive this integral is always positive.

Now what happens at E of w this function at time 0 is because this is satisfying 0 this integral is 0, okay. So at initially this energy is 0 so what is that derivative the derivative of this energy you calculate a time derivative of this energy dE by dt this is equal to half d dt of this integral over B w square of x, t dx, so you can take this derivative inside because this integral is constant minus infinity infinity or anything 0 to infinity or 0 to L. So half times this will be simply integral over B a 2 omega at t, okay 2 omega so what is the derivative of w so 2 w w and again you differentiate with respect to x w, t so that is w, t dx.

And we have 2, 2 goes integral B w into alpha square w, t I can write it as because w, t satisfy the equation alpha square wxx, so we have wxx dx. Now you do the integration by parts here so you have alpha square, okay and you get wx w over the boundary so this is so if I call this boundary is this is the body body boundary on the body terms, okay. So say if it is infinite rod you simply put minus infinity to infinity or 0 to L, okay or 0 to infinity anything, okay combinations you can take depending on what is your body body is infinite semi-infinite if it is 0 to infinity you say 0 to infinity, okay.

This minus integral over B wx square dx, okay. So now if it is infinite at infinity you can assume that a temperature is 0, okay. If it is a rod so semi-infinite rod at x equal to 0 either temperature is 0 or the you insulate that body you insulate this rod at this x. That means at temperature is 0 means w is 0 or so it is insulated at this end that means flux is 0, so no heat is going out that means the temperature difference is gradient is 0 that is proportional to the heat exchange.

So that means wx is 0 this is called the flux, insulation means that is that is for the insulator insulation part at the end if you insulate do not allow it the heat to pass go out. That means this flux is 0 that is the temperature gradient has to be 0 that is wx. So in any case that is infinite, semi-infinite or finite if it is a finite the only physical things you have to see is (())(41:37) you keep the temperature fixed, okay so w or you keep the temperature 0 at both ends and it will be 0 or you allow it to be flux is 0, okay you make the flux is 0 so you see that both cases this quantity this boundary terms will be 0, okay.

So this will become minus alpha square integral B wx square dx and clearly alpha square is positive this integral is also positive and I have a minus sign so that this is actually less than or equal to 0 that means what you found out is d by dt is less than or equal to 0. So the derivative is less than or equal to 0 implies Et ew of t is actually always less than or equal to E w of at 0 at all times t is always less than or equal to time at 0 which we know that it is 0 and for all times this we know that it is positive or greater than equal to 0, okay as you see this temperature if this is 0 so w x, t is the temperature square that is always positive if the temperature is 0 is can be even 0 so this is a term energy is either completely 0 less than equal to greater than or equal to 0. And now from this derivative is less than 0 implies it is always decreasing so this is what is true for every t positive, okay.

So this implies E w of t because you see is bound with 0 so that means it has to be 0 for every t, okay. So what is that this means this has to be this cannot positive that means energy has to be 0 that means w has to be 0 because implies over B half that is the energy that is w of x, t square dx equal to 0 because this is integrant is positive w has to be this is 0 means it has to be wx has to be 0 for every t, okay.

So this means what? So this means w is nothing but u1 of x, is minus u2 of x, t equal to 0 that is the meaning of w, so that is nothing but u1 of x, t equal to u2 of x, t so that shows that if you have two solutions of the initial value problem u1 and u2 they are actually same, okay you take the difference use this energy argument and show that the uniqueness of the initial value problem that we have considered, okay.

So the solution you have (())(44:24) solutions satisfies five properties 1, 2, 3. 4, 5 so the simple properties, okay and you have the uniqueness of the initial value problem so you have certain properties of the solutions and this uniqueness of the solution for the initial value problem. So you have the properties of the solution of the heat equation five properties and the initial value problem with the initial condition you have only uniqueness is guaranteed by this energy argument.

So with using this we try to get the solution in the next video we try to get you the solution based on this properties of the solutions of the heat equation and the uniqueness of solution of the initial value problem for the heat equation. So we will just try to derive we will just try to construct a solution based on this properties and this uniqueness of the solution of the initial value problem. So this is what we will see in the next video, okay thank you very much.