Differential Equations for Engineers.
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Lecture-40.
Second-order Linear PDEs.

Welcome back, we are trying to find solutions of ordinary differential equation. So now onwards we will move onto partial differential equations. So in this we only study a linear or shall differential equation of second-order. Only second-order linear partial differential equations you take and will try to give you, we classify them, so we will talk about what is the classification. We will classify these equations and put it into one of these forms, one of the 3 forms, so we will give you each of these forms, we will give you some of the well-known equation is, those equations with the initial and boundary value boundary data, we will solve them in the next few lectures. Okay.

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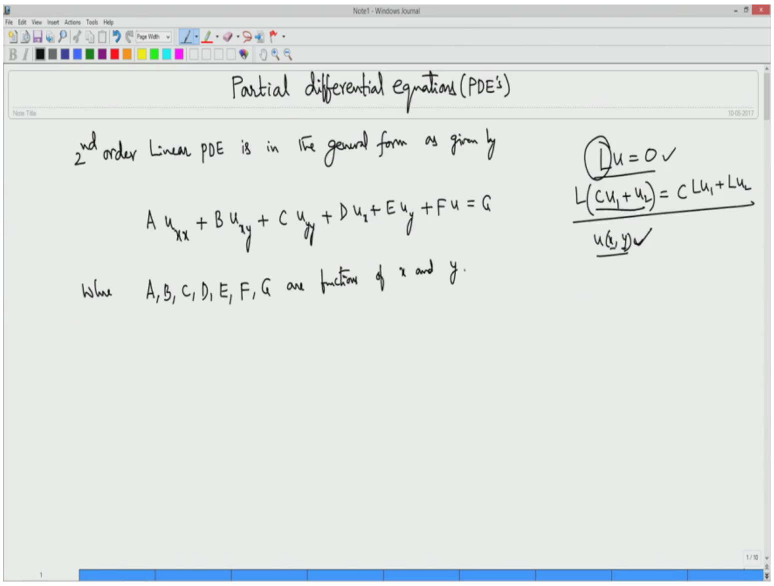

So what is the partial, so what are the second-order linear shall differential equation? So 1st of all it may read what is the linear, so linear means you know, you should not have the dependent variable u should not have, so u square, only u and only u should be there. So that means crudely what is the linear, otherwise you have Lu equal to 0. If this is your equation, L is the differential operator involving the derivatives are partial derivatives or whatever derivatives, okay. So once you involve mathematically, you take a combination C1 u1 plus u2, u1, u2 are 2 solutions of this then you apply this, then this will come out as L u1 C into L u1 plus L u2.

So if it comes like this, then you say that this L is a linear differential operator, okay. So we will see what is the second-order, second-order linear pdes. So this is actually pdes, that is general, as in the general form as given by this. You take, so this A, some portion of x, y, all these A, B, C what I write and the coefficients of this partial derivative, so it is second-order, so you have 2 x derivatives which I did not like this, as subscript. U is dependent variable, u of xy is a 2, if you have 2 variables, so 20 dependent variable is x, y and u is the dependent variable. So this is that is why partial differential equation, second-order because you have 2 derivatives, 2 partial derivatives in the coefficient is functions of xy.

So A is this plus, so I do not write now onwards A of xy, so this is simply A, B u xy plus C uyy plus Dux plus E uy plus F u, if it is a non-homogeneous equation, so you some function of x, y. So this is what it is, you have this as a linear function because you try to put, you try to replace u with this, it will always come out, okay. This, these are differential operator left-hand side, you can always write in this form. Okay. So that is why it is, this is a linear, you can see that there is no u square, only u, u terms, okay.

The derivatives, and derivatives of u and u only. Okay. And the coefficient should not have any functions of u. So all these A, B, C, F, A, B, C, where A, B, C, D, E, F, G are functions of x and y. And you say that this is a linear partial differential equation. So how do we solve this, if it is in general form? Okay, it is not easy, so we cannot solve in general, any partial differential equation is given a general form. So what we do is 1st of all to study these equations we will try to, we will try to classify them. How do we classify them? So the motivation comes from the quadratic equations.

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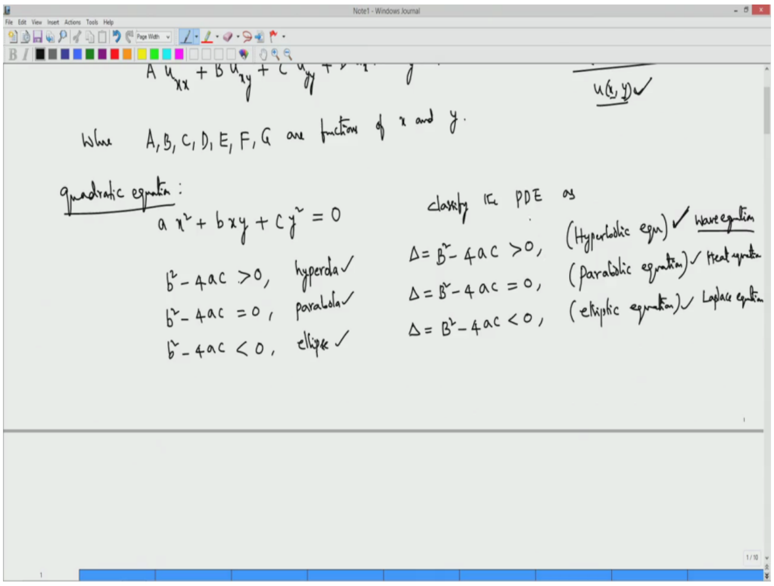

So quadratic equations, if you consider quadratic equations, okay, so what is the quadratic equation? Ax square plus B xy plus Cx square let us say equal to 0. So if you have this quadratic equation, if you have quadratic equation like this and this is y square, xi square. Quadratic equation is this, how do you classify, the Colic sections, okay. This is depending on this discriminant B square - 4 AC if it is positive, you call this hyperbolic, hyperbola, okay. If this is any quadratic equation, when you have discriminant is positive, it is actually a hyperbola.

If this is 0, it is called parabolic, this is parabola, okay, that is how you classify. And B square -4 AC is less than 0, this is ellipse, right. This is how you classify, this quadratic equations. In the same way we say, we can see this equation, instead of x square you have x 2x derivatives. If you have xy, you have mixed derivative with xy, 2 derivatives. And this is quadratic and I have second-order linear and I have the coefficients are functions of x, y, these are constants and these are of this form. So analogally, even for the PDE, so we will classify, so we classify the equation, okay.

Classify the PDE on the similar lines as B square -4 AC, discriminant, we call this Delta, okay, I have Delta is this, if it is positive, we call this, the equation is then hyperbolic, hyperbolic equation. If the discriminant is 0, parabolic equation and if the discriminant is negative, it is called elliptic equation. Okay, much similar to how you classify the quadratic equation. Because each equation when it is, where it is positive, its characteristic, characteristics are different, okay. So in each of these cases, each equation is having different characteristics.

So in the same way, so we also expect, can expect, so that is why we coin these names, same names hyperbolic equation of the discriminant is positive, otherwise parabolic equation elliptic equation, we classify them. You can expect each of these cases, so each different equation, so each equation here will be different, having a different characteristics, okay. So that is what we see, so how to get them, we classify such an equation, so we try to classify these equations, second-order general equations to one of these 3 types. That is putting into one of these forms is called, so if it is a hyperbolic form, if it is hyperbolic equation, that is this discriminant is 0, if it is positive, this hyperbolic equation.

So in that case if we try to look for new variables, okay, so if you are harm, you are having independent variables xy now, so you try to look for new independent variables, some xi and Eta and you work with, you do the change of variables and the equation becomes in this case is similar to a wave equation, okay, you get the wave equation type, wave equation. Second-order terms look like were equation. So that is what is hyperbolic equation, it is a typical equation that we will get here, you will get heat equation. Okay. So if discriminant is 0 and you try to look for, you try to change this equation into this form, parabolic if you work out, reduce, you look for new variables, some xi and Eta, then those new variables you get heat equation.

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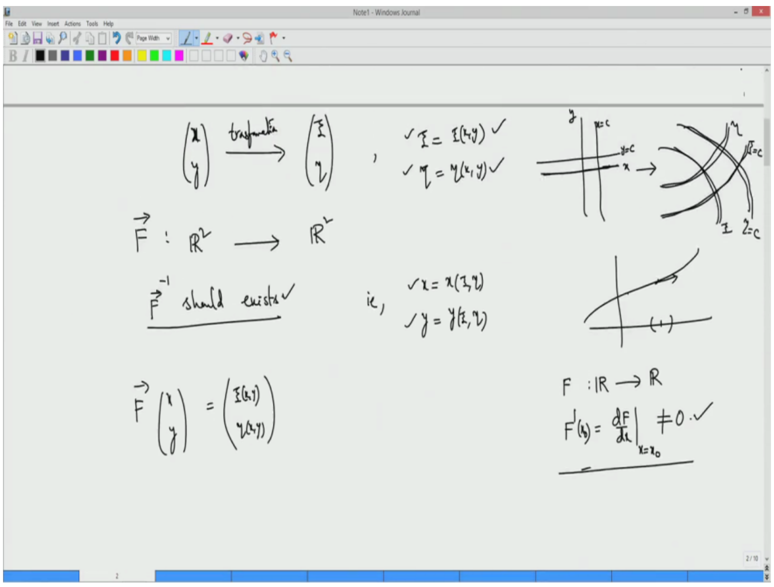

In this case, in terms of these new variables, you get Laplace equation. This is how you classify, when you classify, these are the types you take, typical equations are wave, heat and Laplace equation. So we will try to see how do we classify these equations. So let us take this general form, so what you need is, you are looking for, you are looking for basically change of variable. So x, y, to so you want to take these xy variables, so let me write x, y is a vector in this way. So x, y variables to a new variable xi and Eta. Okay. This is the transformation, change of variables transformation xi and Eta, so you go from xy to xi y, xi and x, xi and Eta functions of xy.

Eta is equal to eat of x, y, so this is of, you look for new variables. So when you go to, whenever changing these the variables x y to xi and Eta, because x and y are linearly, they are independent variables, okay they are independent variables and they are actually. So in the same way when you go to the new variables xi and it also should be independent variables, should be independent variables, okay. The transformation, so x and y are from R2 to so xi and Eta, so you have a transformation, the transformation is actually is you call vector value function, okay from R2 to R3.

So when you are in the plane xy to a new plane, so xi and Eta, something nice, you take the Cartesian coordinate, you convert this into curved linear coordinates, maybe something like this, this is your xi and Eta, something like this, okay. So that is how you try to see. What is this, this is your x, y, if x equal to 0 is y axis y equal to 0 is x-axis your lines are, if you fix y equal to constant, some constant is this line. X equal to constant is this line, okay, the same way, so xi x, xi equal to constant is this line, okay, eta is constant is this line.

So this is how you try to see after the transformation, these lines become, these parallel becomes the parallel curves, okay. So that is how you look for the transformation. So I can always find some transformation to take from this to some other figure, some other things. If you really want this, you should be able to come back, okay. When you want, when you transform, you should be able to come back whenever you want to, okay. So you want to have a transformation, it should have inverse transformation.

So to get this inverse transformation I should have F dash, this F inverse should exist, okay, at least locally, okay, should exist. Then I have issues, so I can I can always come back, that is what is the meaning of this, I have xi and Eta in terms of x, y. When F inverse exists, I should be able to x, x in terms of xi and Eta. I should be able to get x and y in terms of xi and Eta, there is a meaning, okay. That is, so I should be able to get this x in terms of xi and Eta, y also in terms of xi and Eta. So what is the best way to do this? So if you are given this one F from xi and Eta over the functions, okay.

So actually what you have is F bar of x, y equal to, is actually this vector is x, y, okay this is a vector x bar of xy is xi of xy and Eta of xy, this is how this transformation takes. Now this is a vector and this is also vector at the end. So when you apply the function of this vector is a vector, this is how we have. So I have xi and Eta in terms of xy. So when can I say that F inverse actually exist, I can go back, I can actually write in terms of xy in terms of, I can have inverse, okay.

I can write x, y, I can write F inverse exist. So this is through derivative of a function. So I degress here, so suppose you have a function F which is from real valued function, this is not a vector valued, this is only from real valued function and which one variable, okay, just take one variable. What is its derivative, suppose differentiable function, the derivative of x is actually that df by dx, right, df by dx is this at x, okay. This is equal to, this is, this value at x, at x equal to x0. You want a derivative with x0, x0 is real, this is what it is, right. What if you have, suppose this, okay, so let us, suppose the derivative is nonzero, what is this one, what is actually this F?

F is actually a curve, curve in this line, okay. So if the derivative, you always, so what is the derivative is nonzero means it is not parallel to x-axis. The tangent at any point the tangent is not parallel to x axis. If it is parallel to x-axis, then it is 0, okay. So if it is not parallel to x-axis, I can always, say look at this point, I have, you look at the neighbourhood, in this neighbourhood we find all the tangents, okay. If none of these tangents, if none of these tangents, say, let us say at this point, at this point you look at the tangents.

In the neighbourhood of this, all the points here, you look at the, for the curve look at the tangents. The tangents are not parallel to x-axis, that is the meaning of x 0. Now if F is, F is continuous, F is differentiable and it is also continuous. So the tangents are moving continuously along the curve. Suppose if you assume that is what this, if all the tangent to the curves are continuously moving, okay, that means derivative is continuous and we know that it is nonzero at one point, by the property of continuous functions, it should be in the neighbourhood of this with all the neighbourhood.

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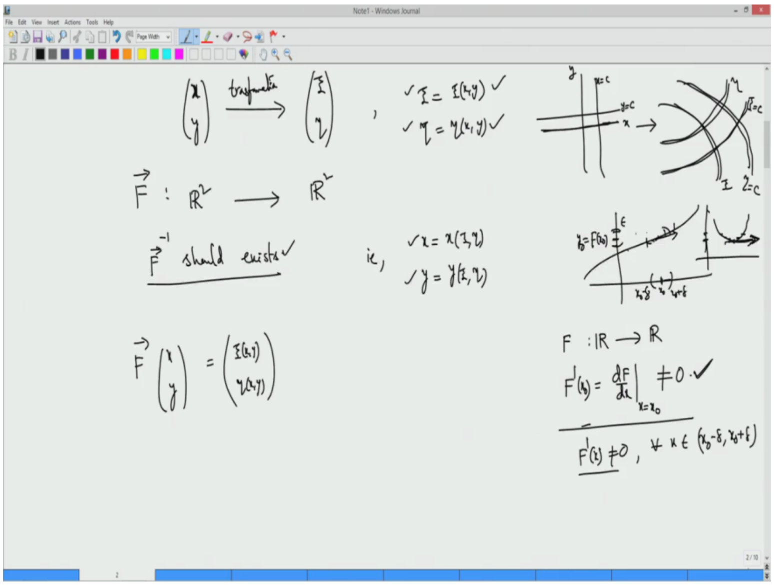

That means upto here to here, you take the condition between or for the curves they are all nonzero. Okay. So you have F – of x is actually nonzero. If it is one point it is nonzero, which is nonzero for every x in the neighbourhood, x0, okay. Say, let us say x0 minus Delta, I 0+ Delta. So some Delta neighbourhood you have this, this is your x0, x0 plus Delta, I 0 minus Delta. So within this interval, if you consider, so the derivative is also nonzero, so if you see that at one point as it is nonzero, then in the neighbourhood it is nonzero. So once this derivative is nonzero, you can see that these tangents are not parallel to x-axis.

That means you can actually see that inverse actually exist. Inverse, inverse function actually exists, okay. Only problem is that the derivative is 0, it is parallel, here at this point it is parallel to x-axis, you cannot have inverse because at this point, for the y axis, if you look at the y axis, at this point I have in the neighbourhood any point you pick up, I have 2 values, okay. And the close you have 2 values, it is not defined, okay. But if the, because derivative is 0, that means the tangent is parallel to x axis. If it is not, the derivative, the derivative is nonzero tangent is not parallel to x-axis, we can see that, you can have an inverse, okay.

You can define the neighbourhood of this point, this is x0, this is my fx0, Fx 0, that is your y0. So there is some Epsilon neighbourhood, so some neighbourhood I can always find an inverse. Okay. Within this interval I can define, I can see that I can define this function, okay, which means, I can have an inverse here, you can see this, okay. So, so what is that we are trying to say is, if the derivative nonzero, you have inverse exists, okay, for a continuously differential function.

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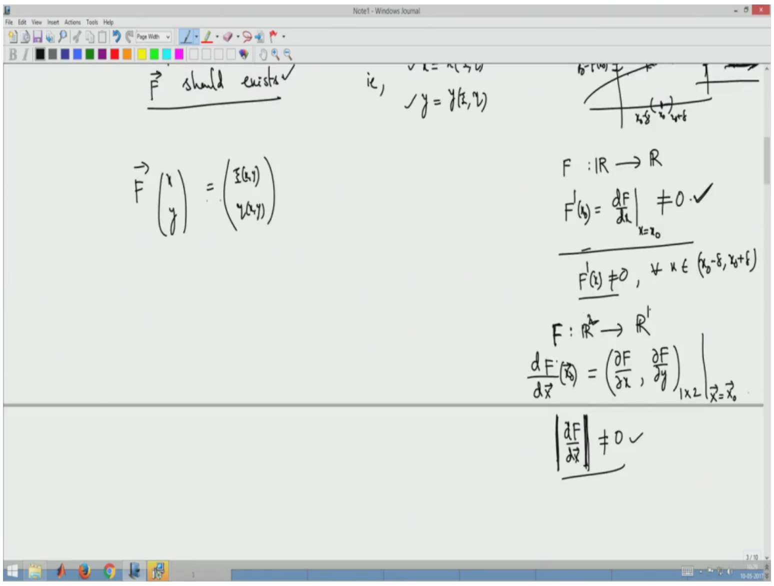

Same thing now you consider from F is from R2 to R, R2 to R multivariable function. So that means x is the function of x, y. Now what is its derivative? Df, so if you call this vector, you call this D F by Dx bar, x bar is a vector in R2, plain vector, okay. If this is at x0, x0 is also vector, it should be equal to, this is dow, what is, what is this one , this is actually , what is the derivative here. So you have, you have 2 variables which is, from calculus you can see that doe F by Doe x and doe F by Doe y, dis what is your derivative. Okay, 2 partial derivative.

So this is as you can see this has a matrix, this is 1 and 2. So if your function is from 1, R2 to R1 and you see this range, range is, range is Euclidean space with 1, 1, this 2 here. So 1 by 2 matrix, so you can see that the derivative is actually 1 by 2 at x equal to 0. So you consider x bar equal to x0 vector for suffer you see the derivative is actually, it is an operator, linear operator. You do not know what is the linear operator but you can think of it is a matrix. It is a matrix, 1 by 2 matrix, range, whatever R Power, that is 1 by2, that is what is, that is how you represent this derivative.

So what is the meaning of this derivative is nonzero, that means this, what do you mean by this is nonzero? This means you have to consider df by Dx bar, modulus, okay, this quantity you have to take nonzero, okay. Then you may have some inverse, inversion here is not possible because this is from R2 to R1, so conversion of this you cannot have, R2 to R1 real, maybe it is not, maybe it will be cumbersome to talk about the inverse. So in this way if at all you consider the conversion, the derivative is nonzero, this is what you have to take. This is simply I want to tell you that now how you say is the distance, is a vector, so the distance, so the modulus value of the vector should be nonzero.

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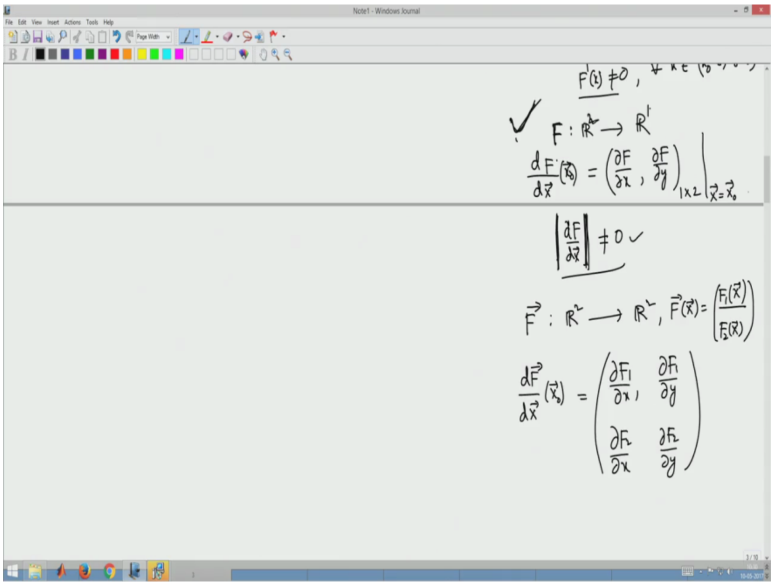

That is the meaning of nonzero derivative. Now what we have is a function from R2 to R2, a function from, it is a vector valued function now, so that file output a bar, R2 to R2, okay. So what is its derivative? Df bar by dx bar at x0. This is, this is actually equal to, now what is my F bar, F bar is actually, F bar of x bar equal to, let us say F1 of x bar, this is like earlier function and similarly F2 of x bar. So you are 2 components, each of this is like earlier type. Now what is this one, so this is a vector, so you should write, this is a vector, so you should write like F1, F 1 of x bar, F2 of x bar, okay, so it is a vector, so this is how it is.

So if you actually see, now I know that what is the derivative of this now, this you know from here, this is the derivative, so you write it, so doe F 1 by Doe x and doe F1 by Doe y, okay, that is 1 And similarly for F2 you write Doe F2 by doe x, Doe F2 by Doe y. So what you have, this derivative is actually, earlier is 1 by 2 matrix, now we have to by 2 matrix, 2 by 2 matrix. So the derivative is actually 2 by 2 matrix. What do you mean by the derivative, the derivative is nonzero which means this distance, the modulus, what is the modulus here?

You can take a determinant, okay. This is the determinant of this matrix, okay, this is nonzero, this is nonzero means, means determinant, determinant of df bar by dx bar which is F, doe F1 by doe x, doe F1 by Doe y, Doe F2 by doe x, Doe F2 by Doe y, this has to be nonzero, that you can calculate, right. So if the determinant, if it is from R2 to R2, the function, the derivative is actually, is a matrix, 2 by 2 matrix and it is nonzero means the determinant has to be nonzero. That means then you as like analogous to this curve, when you have the derivative is nonzero, the tangent is not parallel to x-axis, you can have, you can see that there is an inverse in this case also, in these 3 cases also.

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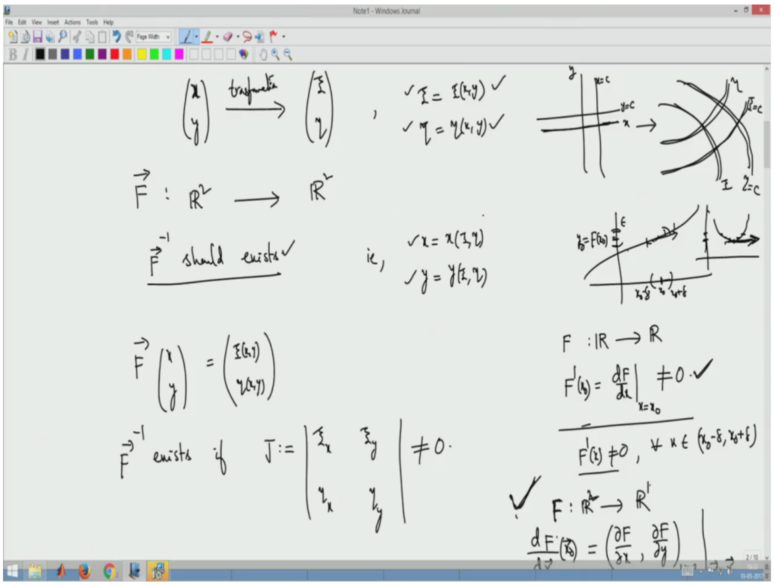

In this case from F is from R2 to R2 and the derivative which is the determinant, this is called Jacobian, Jacobian of the matrix, okay. Jacobian is nonzero, we write J, this Jacobian is nonzero, that means you can have inverse, F inverse, F inverse, okay. So this exists, so I can write what is my F1 is my xi and F2 is my Eta. So what should I say, if I have this exists, okay, F inverse exists, F bar inverse exists, if Jacobian which is, the definition is a determinant of xi xi x, xi y, Eta x, Eta y, which is nonzero. If this Jacobian is nonzero, the determinant of this matrix, then you have in this inverse, inverse exists.

So you should always look for your xi and Eta functions in such a way that this Jacobian is nonzero, I can anytime if I want I can go back to the old variables x and y, okay. You do not lose anything. So one-to-one correspondence between xy variables and xi and Eta variables, this is what you should know before you transform this equation into a new variable xi and Eta. Now having known, having observed these things, what we do is, we are looking for a new variable xi and Eta, so we are trying to change transformation xi with this xy to xi xy and Eta xy.

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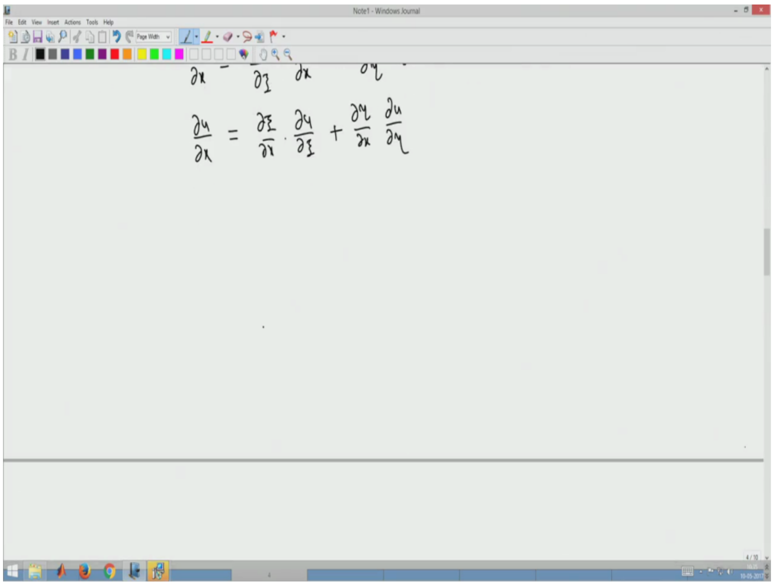

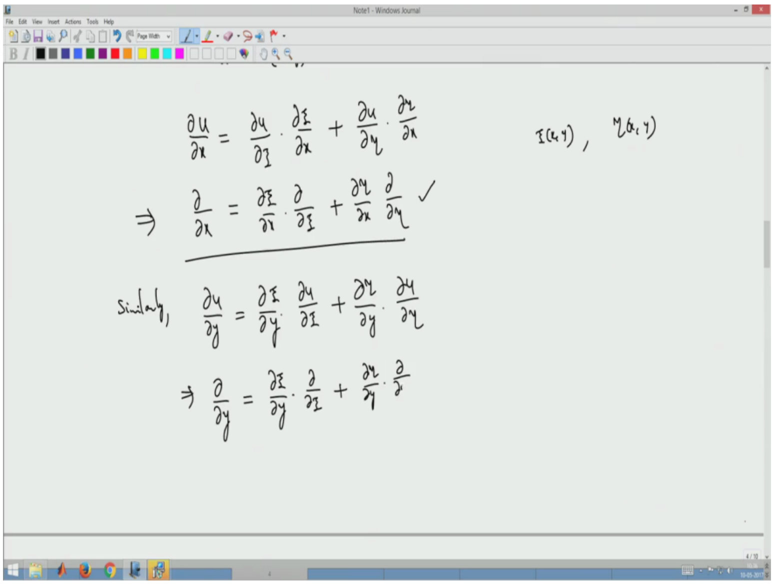

So this is a change of variable, so we want to replace all the derivatives of x with the derivatives of xi and Eta. So to do this, so we have to replace the derivative ux, what is ux, I want, u is basically the function of x, y, 1st of all. U is the dependent variable which is earlier function of x, y. Now with the Jacobian is nonzero, okay, it is xi of x, y, Eta is eta of xy, okay and the Jacobian is nonzero, that implies I can write x in terms of x of xi Eta, y in terms of y of xi Eta. So x I can replace, can replace in terms of xi Eta, y in terms of xi Eta.

So finally it becomes u is a function of xi Eta, xi Eta but xi is a function of x, y, it of x, y. Okay, you can simply say that this is xi and Eta. So what you have is ux and doe u by doe x is actually equal to doe u by this variable doe xi and doe xi by doe x plus doe u by Doe Eta into doe Eta by doe x, that is what is my doe u by doe x. You try to see this one, this u can replace with doe xi by doe x and this we write, okay, this you put it in front and this you write next. Doe u by doe xi plus doe Eta by doe x, this you write 1st and this is the next, doe u by doe Eta. So if you write this, this is what is your doe u by doe x.

And you remove this u, this is just like operator, if you remove this, what you get is from this you have doe doe x, doe doe x is in this form, I know what is my xi, xi is a function of x, y, I can calculate xi x, I can calculate Eta x, okay, Eta is a function of x, y, I can calculate my Eta x, that will be my doe doe x. In the same way you can similarly, I can get doe u by Doe y and doe u by or doe xi by doe x into doe u by doe xi, doe xi by Doe y plus doe Eta by doe y into doe u by doe Eta.

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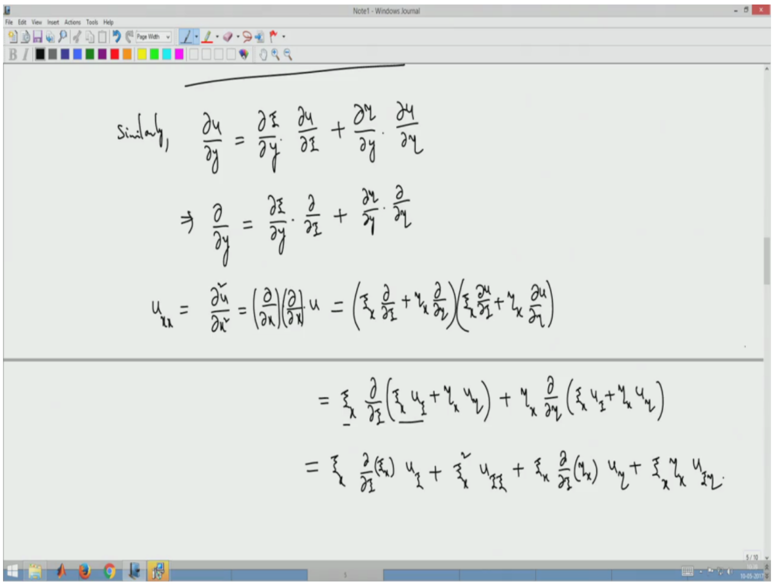

So that implies doe doe u, doe doe y is actually equal to doe xi by Doe y into doe doe xi plus doe doe Eta by Doe y into doe doe Eta is an operator, okay. Now you take your equation. So what you have is uxx, what is this one, doe square u by doe x square, this is doe doe x of on the doe u by doe x. Okay, this itself is acting on u, so this is something like this. Doe u, doe u, you apply to u, so doe doe x, doe doe x you apply to u. Now we have replaced doe doe x with what I have. Okay. What I have is, so I can have xi x, doe doe xi plus Eta x doe doe Eta, that is for doe doe x.

You repeat one more, xi x doe doe xi plus Eta x doe doe Eta acting on u. So if it is acting on u, first one will give you doe u, doe u, simple, okay. And now you expand it nicely, what you see is, if expand it, xi x doe doe xi of this one. So that will give you, what you get, so xi x, doe xi by xi x, okay. I will write directly, so you see that u x x will be, okay you can write it, so doe doe xi of acting on xi x u xi plus Eta x u Eta. Okay, this I am writing like this plus Eta x doe doe Eta of xi x Eta xi plus Eta xu Eta. When you look at this one, this will be xi x and what you have, this you are different setting this first term, so that will be doe doe xi of xi x into u xi.

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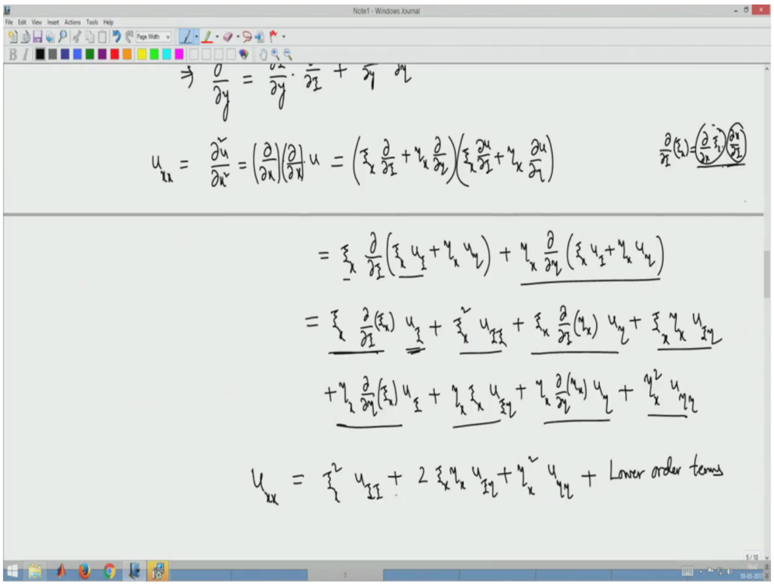

Plus now, you differentiate this one, so you have xi x here, xi x here, xi x square u xi xi, okay. Similarly you get xi x doe doe xi of Eta x, okay, into u Eta plus xi x Eta x into doe xi Eta. This is what you get when expand this one, similarly when you do this side, you will get, now this term if you do it, 4 terms you will get, okay. So you simply write xi you replace with Eta. So Eta x, now look at this one doe doe xi, doe doe Eta of xi x u xi plus Eta x xi x u xi Eta now plus this one. So Eta x Eta x, Eta x doe doe Eta of Eta x u Eta plus Eta x square doe u, so doe doe Eta of u Eta, so that is u Eta Eta.

So this whole thing you can see this one, so you look at the second-order terms, you write separately, okay. So if you write this, what you get is u x square u xi xi plus this one together, so these are, these derivatives, you can use that they are continuous functions u 2 minutes derivatives so you have 2 xi x Eta x u xi Eta, okay. U xi Eta and then plus Eta x square u Eta Eta. Okay, this is what you have plus some coefficient, what is actually this xi x, xi x into, so doe doe xi of, doe doe xi of xi x, this you can say xi is a function of x, y, so you can rewrite this, so doe doe x of xi x into doe x by doe xi, that is what this is.

Doe x by doe xi is possible because x I can write in terms of xi y because of inverse, okay, because Jacobian is nonzero. So that we you can actually have this, you can further simplify the things but you do not need to worry. So you have u xi, the coefficient of u xi is this one, that will be some function of x, y, okay. So you have u xi and you have u Eta terms and okay, u xi and u Eta terms, those are simply lower order terms, lower order terms. You can say they are not really important, okay. So uxx, finally what you get is this one, so you can write in terms of xi xi, xi Eta and Eta Eta plus lower order terms in terms of u xi, u Eta, okay.

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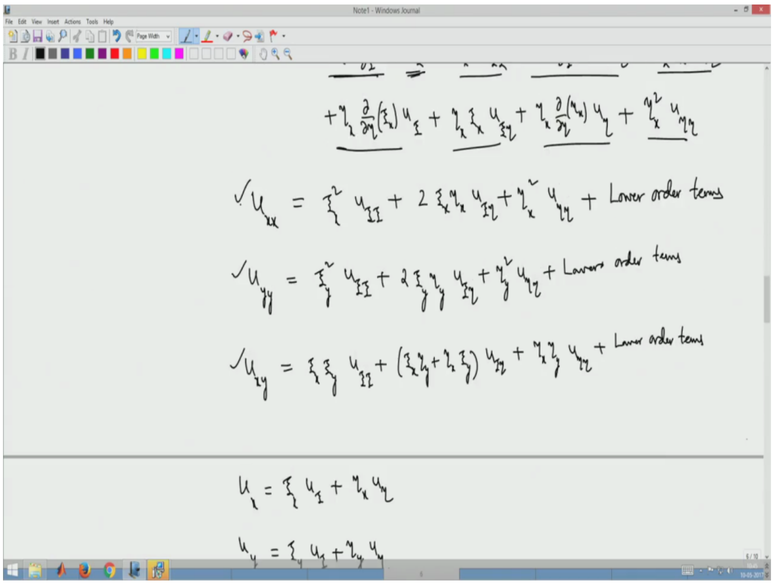

So if you do similarly, you can get uyy is also, you can instead of x you replace with y, so you have xi y square u xi xi 2 xi y Eta y u xi Eta plus Eta y square u Eta Eta plus lower order terms. Okay. Now what happens, we will also have in the general equation uxx, uyy and u xy. So this also you have to write. U xy in the same way, what you do is, you just have to doe doe x operating on Dodo y operating on u finally, okay. Something like that if you do, again you will see that by the same calculations, if you collect these higher-order terms, second-order terms, what you get is u xi xi, so you get xi x xi x, xi y u xi xi, here you get xi x Eta y plus Eta x xi y of u xi Eta plus Eta x Eta y u Eta Eta plus lower order terms. Okay.

It is not difficult to see, okay. So you could see that after transformation, in terms of new variables xi and Eta, u xx, uyy and u xi, u xy, mix derivative. So this is your, I have this one, this one and this one, okay. Now a is a function of xy, x I can write in terms of xi and Eta, so all these functions A, B, C, D, E, F, G are now functions of xi and Eta. Okay. So this way, now if you substitute, go on substitute into your equation, all these, you can also calculate, you know now all these things ux, uy and u, okay. Now u is a function of xi and Eta and now you can also have ux, that is simply, what you have, so ux is, what is ux, directly you have already, so ux is xi x xi x u xi plus Eta x u Eta. This is what is your ux.

Similarly you can get uy, that is xi y u y plus Eta y u Eta. So you have everything, so u is a function of now xi and Eta, everything I have, you go and substitute into the general form of the second-order linear partial differential equation. If you just substitute there, what you end up is, as you collect the terms of u xi xi. So when I substitute u xi xi, I have A xi x square and I have here, okay, this is C xi y square and I have here, this is B xi x xi y, okay. If you collect that, what you end up is equation, the PDE becomes, now you have the coefficient u xi xi, the coefficient is whatever you get.

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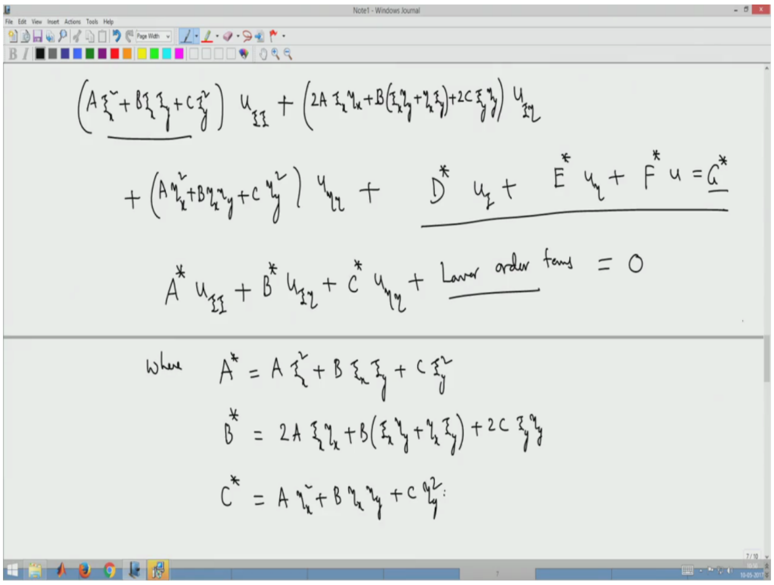

So that I call it some coefficient, what you have A xi x square plus B xi x xi y plus C xi y square, okay. What you have is simply like the terms A into this, C into this, B into this and then you add them up, okay, D, E, F, like that you add. So you collect the terms of higher-order that is u xi xi, now this this plus I will write now what I have for the coefficient of u xi Eta. That is going to be, you see that it is going to be 2 A xi x Eta x plus B times, okay, 2 A xi a, that is what we have seen, right so you have 2 xi x into x and here, this term with B and this term with C. Okay.

B times xi x Eta y plus Eta x B xi y plus 2C xi y Eta y, that is what you have, okay. Plus what you have is something similar to u Eta Eta terms, u Eta Eta terms or something like this, again u xi y, sorry, u, a, what is the coefficient of u Eta Eta, u Eta Eta is again Eta x square, again here. So u Eta Eta is this one and this one terms. If you do that, you see that it is going to be A Eta x, everything in terms of Eta, so okay, instead of xi, you replace with Eta, Eta x square plus B, Eta x Eta y plus C into y square plus some coefficient of u xi, some coefficient of u Eta, some coefficient of u equal to some, so what you have, what is the A, B, C, D, E, F, G, G you have written, right.

So when you finish, what is your general equation G, so that may become, you may have some terms that will have G star, okay. Whatever is the new term is G star. So this I call is F star, okay, this I call as A, B, C, D, E, so E star, okay, this is my whatever coefficient, I call it some D star, these are not, lower order terms, they are not important. What is required if you have to A Star u xi xi, so A Star is this and B star u xi Eta plus C Star u Eta Eta plus lower order terms equal to 0, lower order terms means including the constant, the right-hand side, non-homogeneous term is zero. This row, this is what you get, okay.

So where A Star is this, where A star is simply A, this is A, B, C, now A, B, C are known, now A Star is in terms of A, B, C. A xi x square plus B xi x xi y plus C xi y square, B star is this 2 A xi x Eta x plus B times xi x Eta y plus Eta x xi y plus 2C xi y Eta y and C star is A Eta x square plus B Eta x Eta y plus C Eta y square. So we have reduced the given partial differential equation of second-order, linear partial differential equation, you reduce to, you you using the changing the change of variables, new variables xi and Eta, so with these new variables you can go and come back. So it is actually invertible new variables, okay, invertible new variables, you write your partial, you converted your partial differential equations in terms of these new variables. Okay.

So instead of x, y independent variables, now you have new independent variables xi and Eta, so this is what is the. So the given second-order linear partial essential equation becomes this type, this is the now, this has become, this has reduced to in terms of new variables xi and Eta, okay. So we will see how to classify this in the next video, so we will look at the discriminant A square, B square -4 AC depending on the sign of this B square minus 4AC, the discriminant and we will decide how to convert, how this equation reduces, okay, how it will become, what kind of form it takes, so that your hyperbolic equation means it is in this form, you can reduce, it typically equation will be wave equation, similarly heat and Laplace equation you can get based on the type of equation, okay, you are given partial differential equations, that we will see in the next video.