Differential Equations for Engineers
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Lecture 34
Regular Sturm-Liouville Problem

So in the last video we have seen (how to) we have seen the properties of symmetric or skew symmetric matrices and given matrix A if it's skew symmetric matrix we have seen certain properties the same way if you are given a matrix you can check A star A transpose bar equal to A so if you check that if you verify that then it is skew symmetric so that's what you do to check the matrix is skew symmetric.

But for the differential equations if its general differential equation if it's a non linear it may not be select a general matrix so if it's a linear equation of second order that can always put in the this form that's kind of second order that can always put in the this form that's kind of a Hermitian form or self skew symmetric form ok, skew symmetric self adjointor symmetric depends on a coefficient itself ok.

So we say that it's a we just show that it is Hermitian as an operator so we need second order we are only looking at the second order linear homogeneous equation so the operator if for the matrix we replacing with second order linear differential operator so which is almost Hermitian so we need to give certain boundary conditions to make it really Hermitian that's what we have seen so we will just saw we will do that today.

So we will just show that operator is actually self adjointor Hermitian ok so we will look in to the equation again.

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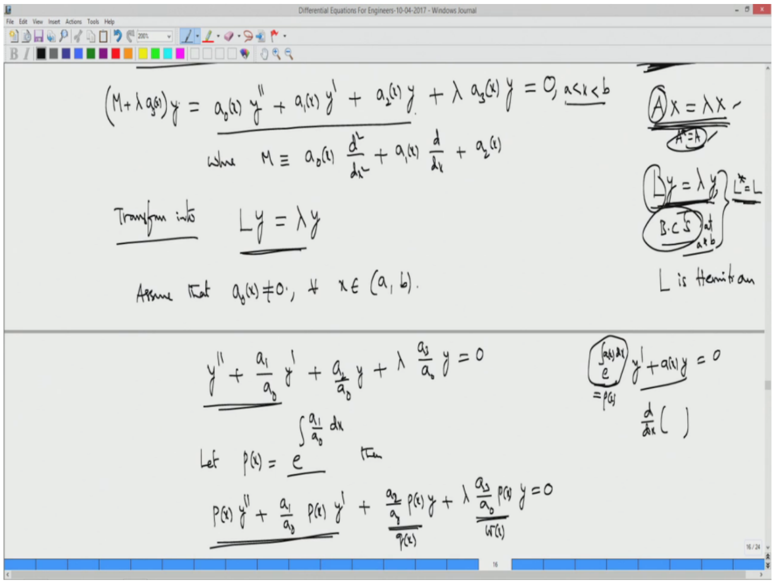

So we will just consider so this is the differential equation part so this is the differential equation this is replacing this is the operator A instead of ax you have lmy is here plus lambda sum function of xy so that I want to put it in this form ly equal to lambda y where I want to make this l is self adjointor Hermitian.

So what I did is in the last video we have seen that we calculated so we want to show if you want to show that l is Hermitian this is the definition we have seen.

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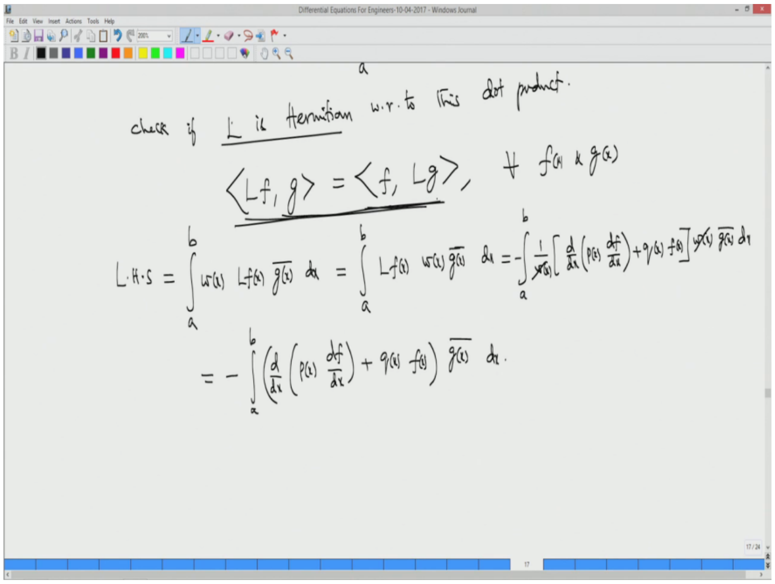

So this dot product should be same for every function f and g you take any functions f and g this is what this should be satisfied so we have taken the left hand side and we also have seen what is the dot product once you know what is the differential equation.

So once you know what is this l which is this ok so once you know what is l we defined the dot product in this fashion ok.

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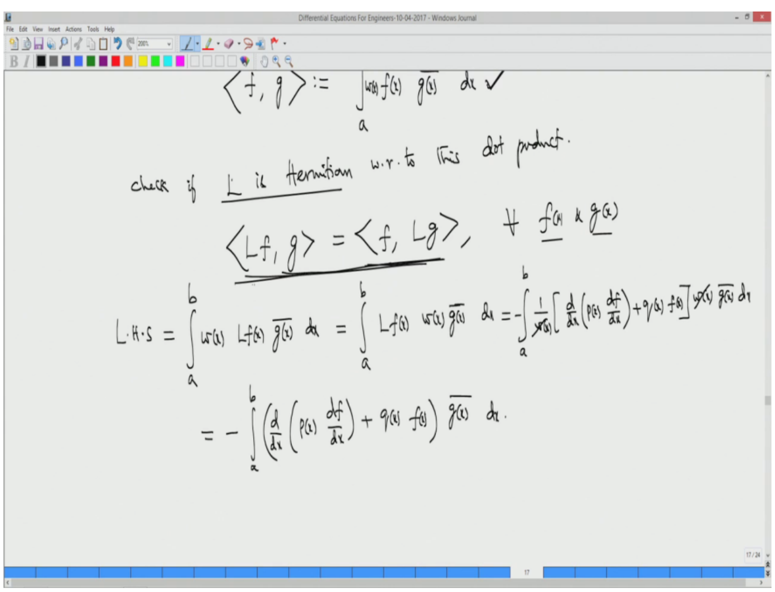

So this is the dot product we have defined with respect to this dot product we want to check whether L is Hermitian or not that means we want to verify whether this equality is true or not so we calculate the left hand side so which is like this if you so we have seen that it is actually this.

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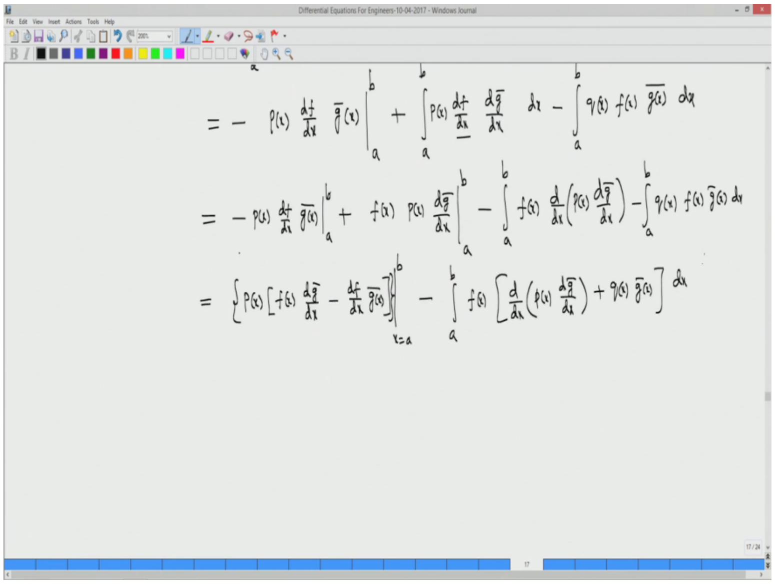

So now next step is just use integration by parts bring this l this differential operator so integration by parts basically what it does is simply take the derivatives on to the other function if you have a derivative of f into g as a integrant so by integration by parts what it does is I basically take the derivatives on to the other function so that's what we do now ok, so here so you have so you can do the integration by parts.

So just by integration by parts what we get is integral so you have a px df by dx so that's anti derivative into g bar of x so this you take limits a to b minus minus plus that's going to be a to b what you have is px only first term I am doing so px df by dx into g bar of x dx ok so this what you have after first integration and the other one you right as it is the other term that is simply integral a to b qx fx into g of x bar dx.

So now what happens is so this you cannot do unless you don't know anything about f and g so the leave it as it is this one so this is minus px df by dx into g bar of x at a to b plus one more integration by parts here so if you do that what you get is simply anti derivative of this is one function px into g bar of x is another function so you think of this px into g bar of x is another function and df by dx is one function.

So if you do the integration by parts here so you get fx and you have px px g bar of x sorry wait wait so when you do the integration by parts what you get is df by dx (and you have a) you need to differentiate other part that is actually giving d dx of px into g bar of x this is what you have the first integration by now what you have is you do this, this is one function and this is another function ok, so if you do this what you get is .

Fx and you have the derivative so d dx of px, just a minute I think I made a mistake this is what is px df by dx gf, this is the mistake so wait so actual thing is if you do the integration by parts px dfx so you have px df by dx you have to differentiate this g bar so this that will be g bar by dx ok g bar derivative so that is what is missing so this is what you have so this one now you think of this is one function, other function is px into dg bar by dx.

So if you do that integration by parts you have fx into px dg bar by dx this you take the limits from a to b and you get minus sign and now we have integration from a to b and what you have is fx and the derivative of px into dg bar by dx so you have a d dx of px dg bar by dx so this is what you have ok, so this into this minus this is as it is you have qx fx g bar of x dx so this is what you have so this is equal to.

So if you can put all these boundary terms these are the boundary terms first two so this you can px is common px times so what you have is fx dg bar by dx minus df by dx into g bar of x so this is what this for this whole thing you take a limits x is from a b so this at b this whole quantity at b minus whole quantity at a and this two terms I can put it like that and then what you have is now this you can put it together so this is from a to b fx, fx is common.

So I am combining this whole thing together so this will give me d dx of px dg bar by dx plus g bar of x qx g bar of x so this is what you have for this function

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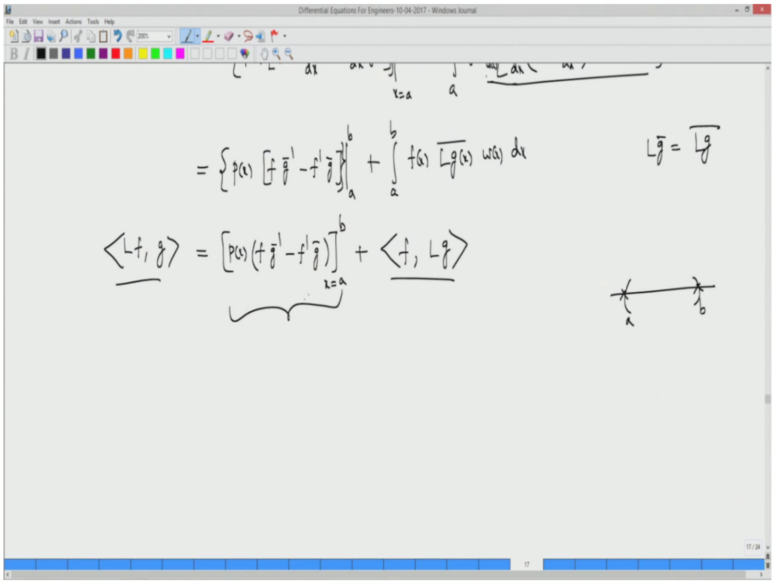

So what is this one this is exactly so what is this one this is actually equal to px I will write f into g bar dash minus f dash g bar ok, so from a to b the limits ok so this is what you have minus integral a to b fx into what is this one, this is actually plus and with the minus if you want to put take this minus inside.

For the whole thing ok minus and minus and if you look at this one this is exactly your lg ok so lg is that is actually lg bar ok x dx ok it’s not actually lg so you have to divide with one by omega x once you have one by omega x you have to multiply also ok so with one by see you have this operator one by x one by wx minus ddx of this whole thing up to here will give me this and what you have is wx that’s a weight function dx.

So this is exactly so this is nothing but px this is fine so fg bar dash minus f dash g bar this whole thing from a to b ok x is from a to b so just limits plus this is nothing but flg because this l is actually differential operator, so differential equation with the real coefficient so those are for those functions a1 a2 a3 or p p q r pq right you have pqw is all real valued functions there also bar that doesn’t make them anything so lg bar is actually lg whole bar ok.

So if you put it thing of like that so this is lg bar so that is same as lg bar so that is same as lg bar actual this is same as lg whole bar so that will give you this dot product so you see that (second ord) if you start with the second order equation that is lhs is left hand side is lfg is actually same as this one lfg so this dot product you want this to be this ok for a second order differential equation linear second order differential equation.

So for that I have this boundary terms unless I make them zero I cannot get, I cannot make this second order differential equation as skew symmetric or Hermitian ok so it will be Hermitian if I can remove this boundary terms ok, so for that what we do is we take this system this equation differential equation and I define certain boundary condition so I have a to b that's the domain so I have only finite domain it a b this is the boundary.

A and b these points are boundary points so at these points you give the boundary conditions so that I can make so if what kind of boundary conditions that will make this quantity zero.

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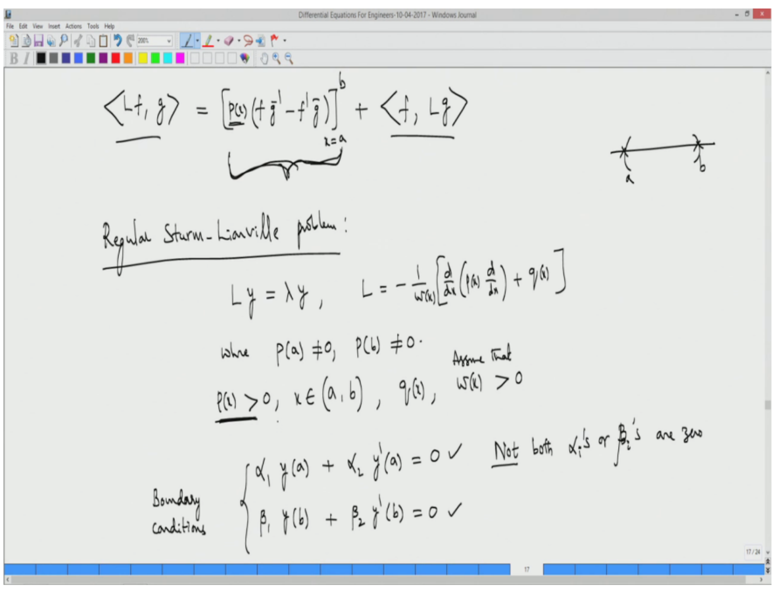

So let us start with regular Sturm Liouville system, so if I do this regular Sturm Liouville so there are different cases different boundary conditions we will give you different Sturm Liouville problems ok so let's see usual one that is our regular Sturm Liouville system. Regular Sturm Liouville problem ok?

So problem so here what you have is you have this ly equal to lambda y ok so where l is minus one by wx ddx of px ddx is operator ok minus plus qx so this is what is your l so if p is not zero p never be zero by the way bp if what is p? p is actually equal to exponential of integral a one by a so this will never be zero so you can see that this will never be zero wherever x is there x is in ab open interval ok.

So in the open interval p of x will never be zero so if p of x is non zero for every x in the open interval anywhere its non zero so include the points ab ok then its regular ok so if you can write like this where it's a regular Sturm Liouville system if where a p of a is non zero and p of b is also non zero ok so in this case we say that this is regular Sturm Liouville system q what is q? Q Is actually a2 by a not into px.

So a2 can be so it's a2 of a2 by a not that can take both positive and negative values so that means qx can take both positive and negative it can take any value but p is always positive, w you want it to be positive and for the reason you will know if it is not positive why it will has to be positive ok so you will see the reasons so you will see that px is always positive ok x belongs to ab and you don't know here p of ab ok.

So only in the open interval you know that it is a non zero ok now what happens pq q can be anything q can be anything so I am not imposing anything you see qp is always positive q is this wx is also you need to be assume that is positive ok so need to be positive this is you can assume that, we assume that w is always positive reason is if it is negative what happens to this dot product if you look at the dot product which you define if w is negative.

When you take the same function g equal to f what you have f square into wx the integral constant so dot product you want it always to be positive quantity ok so but that is going to be negative if w is negative so for that reason we can always assume that w is zero so you for your problem so all the differential equations with you know when you take this right hand side this third function with the parameter so you always assume that a3 is always positive ok.

So it takes the positive values a3 by a not will always be positive function ok that is your w, so with this then it say that is the Sturm Liouville regular Sturm Liouville problem if mainly px is positive so in this case the boundary conditions you make this regular Sturm Liouville problem the boundary conditions are like this you consider some alpha 1 y is the unknown function so y at a plus alpha2 y dash at a equal to zero so this is the one boundary condition ok.

So these are the boundary conditions, what are the boundary conditions we have this is one at a so other one is you call this some beta one y at b so at b I am giving beta 1 beta 2 values and y at b equal to zero if both alpha 1 and alpha 2 are zero so there is no boundary conditions so at least one of them not both of them are zero ok so not both alpha I's or Beta I's are zero ok so you can say that this is not both the alpha I's both the beta I's are zero.

Otherwise there is no boundary condition there so these are the boundary conditions if it prescribe what happens to this term so now we always look at this term I want to make this (I defy) I give the boundary conditions regular case these are the boundary conditions because px is non zero at ab so I have to make this quantity ok so this quantity I want to make it zero so if f and g are two solutions of this system ok.

You take f and g are two solutions ok, so if you have f and g are two solutions corresponding to some lambda ok.

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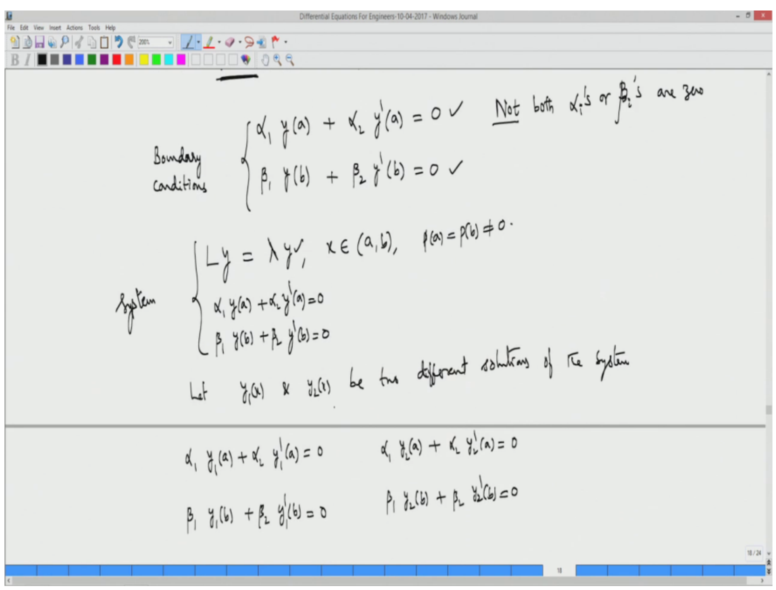

What happens to that so we will see with these boundary conditions the moment you give this boundary conditions now this equation ly with this boundary conditions let's write this regular Sturm Liouville again ly is lambda y and these are your alpha 1 y at a plus alpha 2 y dash at a equal to zero and beta 1 y at b plus beta 2 y dash at b equal to zero.

So this is a regular Sturm Liouville system x in ab and you have a p or at a and p at b are non zero so this is the condition so these are this is the regular Sturm Liouville system so once you have this so I want to see any solutions of this system this problem any solutions if you put it here so that is what right you take any solutions f and g and you take this operator lfg has to be flg any functions that are the solutions part of the solutions of this equation ok.

So let us consider two solutions let y1 of x and y2 of x be two different solutions of the system, system means differential equation at the boundary condition this is your system ok, regular Sturm Liouville system so if you take that so what you have is y1 satisfying this equation and these boundary conditions if you apply that boundary conditions alpha1 y1 at a plus alpha 2 y1 dash at a equal to zero.

And similarly beta1 y1 at b plus beta 2 (y2 y1 at) y1 dash at b equal to zero, and similarly you have y2 is also satisfying other y2 is also satisfying the equation ly2 equal to lambda y2 ok for some lambda ok so I am not saying that y1 and y2 are two solutions for the same lambda so for some lambda these are the solutions ok, for some parameter values of lambda y1 is one solution but satisfying the same boundary conditions y2 is also satisfying for some lambda.

Sum of the equation but the same boundary conditions so if y2 is having satisfying the same boundary conditions you have y1 alpha1 y2 at a plus alpha2 y2 dash at a is zero and similarly beta1 y2 at b plus beta 2 y2 dash at b equal to zero ok.

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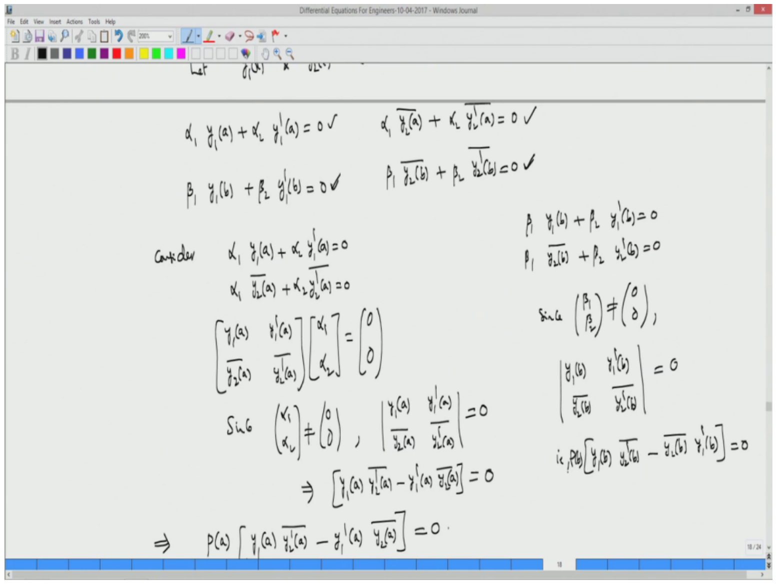

Now you consider these two equations together, these two equations if you take together so you write it together so consider these two or these two ok so either this these two or these first two solutions so if you consider.

Consider alpha1 y1 at a plus alpha2 y1 dash at a equal to zero similarly alpha1 y2 at a plus alpha2 y2 dash at a equal to zero if you consider these two equations you get this one so you know that we have seen we have already assumed in the boundary conditions that not both alpha I's are zero so that means for this is like a system y1a y1dash a ok and y2a y2dash a this is the matrix and this is your unknowns alpha1 alpha2 equal to zero zero.

So this is like you have system ax equal to zero and I know that alpha1 alpha2 is non zero ok alpha1 alpha2 as a vector ok is non zero so this is you we know this is the assumption we made not both alpha1 alpha2 are zero since this is non zero then the determent has to be zero because this is the homogenous ax equal to zero when you have a non zero solution that means when you have non zero solution determent of a has to be zero.

So that is that means the determent since this is this you have y1 at a y1 dash at a and y2 at a y2 dash at a so this determent has to be zero we have this so what is this determent this is nothing but y1 at a y2 dash at a minus y1 dash at a y2 at a ok so this is equal to zero so this is exactly what you have here so p at a say the lower limit p at a that is anyway not zero and what you have is f and g are two solutions of this system ok.

If you take f is y1 g as y2 so what you have is so this derivative so you can see that this derivative this is nothing but y2 is if y2 is a solution y2 bar is also a solution ok because the homogeneous equation so lambda, lambda is always real ok so you can take think of this once you make this problem alpha1 alpha b2 are real ok alpha I's and beta I's are real so if you take y1 as one solution y2 is also another solution.

y2 bar is also a solution satisfying that corresponding to some lambda bar ok if y2 is a solution of this system what you have is y2 bar is a solution of ly2 bar equal to lambda bar some lambda ok if y2 is a solution for some lambda y2 bar is a solution for some less for the same lambda bar ok for the same and so conjugate of the same lambda so with that so it's equation is generally satisfied and the boundary conditions we always assume that they are real.

So if because they are real it satisfies same boundary conditions by y2 bar also so you can think of y2 as y2 bar as well so you can think of y1 and y2 be two solutions ok y1 and y2 bar be two solutions you can think then you have this one you can make bars here ok because alpha1 alpha are real so you don't have beta1 beta2 all real so this you can put it as bars so if you do that so what you have is these are bars so these are bars.

So that exactly what you have in a form so this is y2 y2 bar so this is exactly the lower limit of this special term where p at a when you put x equal to a f and g are two solutions f and g bar two solutions here I have taken as y1 and y2 and the lower limit will be exactly will become zero because this is zero this implies p at a into y1 at a into y2 dash at a bar minus y1 dash at a y2 at a bar this is zero ok so exactly so the same thing with these two other equations.

Do the same thing with the other equations you consider them put it as a system since beta I's are different beta I's both are non zero ok that means instead of alpha1 alpha2 you have beta1 beta2 so you do the same thing what you get is you have beta1 y1 at b plus beta2 y1dash at b is zero this is one equation other one equation is (y1b) y2 b bar plus beta2 y2 dash b equal to zero so if you consider this and you can see that since beta1 and beta2 is not actually zero zero.

So what you have is a determent of this matrix y1b y1dash b y2 b bar and y2 dash b bar this determent has to be zero so what is this determent? This is that is simply y1 at b into y2 dash at b bar minus y2 at b bar y1 dash at b so this has to be zero you can multiply with pb that is also zero.

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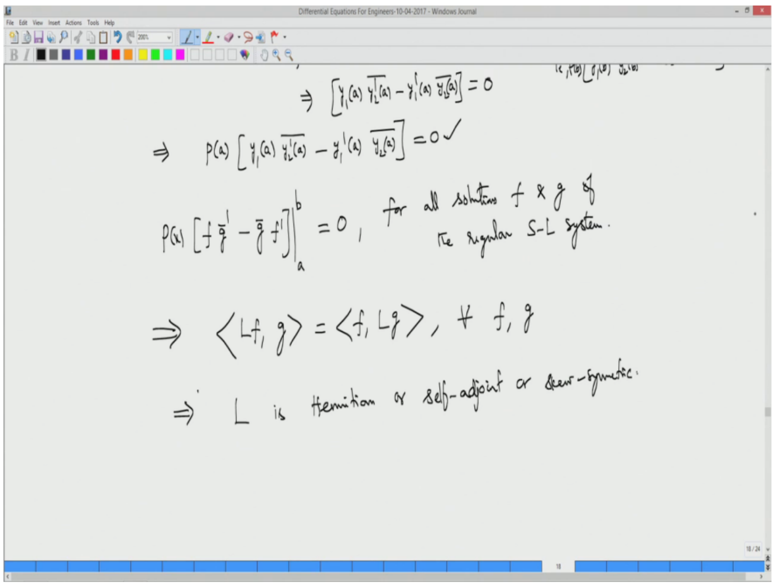

So with this these are simply lower and upper limits of this special term which you have so that makes it zero.

So if you consider this special regular Sturm Liouville system with these boundary conditions what you found is that special term px and f and g are two solutions any solutions of this system fg bar fg dash bar I will write fg dash bar minus g f dash bar right that is zero f dash g bar fg bar dash ok. So this is actually should be g bar dash and g, g bar f dash so that is g bar f dash this term and you multiply and you put this limits it is zero ok.

For every f for every solutions of or rather for all solutions for all solutions f and g of the regular Sturm Liouville system regular sl system this is wide I right now onwards ok sl system so if you have a two solutions sl system for some lambda, lambda is a parameter for some lambda if you take what you get is that special term will become zero so this will simply gives you lfg is nothing but flg that makes it zero ok.

So that makes it this is to for every f and g solutions of this system so that makes it l is Hermitian so this implies l is Hermitian or self adjoint or skew symmetric. And you are not in the finite dimensional you are in the infinite dimensional space ok you call it that space as rn there you have vectors are only, you have only n linearly independent vectors here you can have infinite many linearly independent functions in the as a functions.

If you there at the function space linearly independent vectors, vectors are functions those vectors are those functions are infinitely many ok linearly independent functions are many because 1x x square you consider these are the solutions of you can just think ok for some operator for some lambda these are the solutions where these will give these kind of boundary conditions they are all solutions.

That will make it they form infinite dimensional space ok so in this case so you have call this operator is self adjoint then we don't prove this properties in the infinite dimensional we just limit it so we just analog ally we take these properties for granted so what we have shown that the properties for the for the matrices when l is a matrix a ok.

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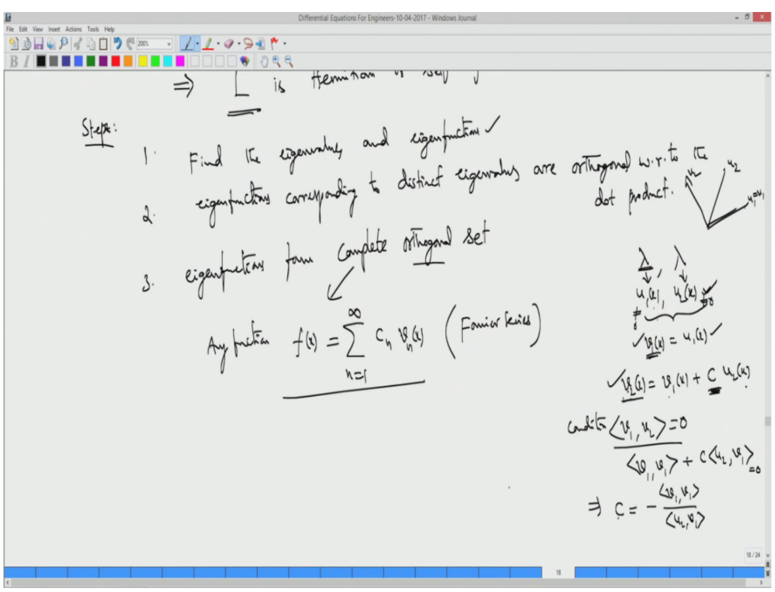

So what we have is once you have this you find what is the next step is find all the eigen values. Find the eigen values and eigen functions vector is actually here function so I can call this eigen functions that is what is the step ok so the next steps are like this so step first step is this find the eigen values and eigen functions second one is and then you can easily see that the property so different different eigen values distinct eigen values gives eigen functions this is the property which we assume ok.

Eigen functions corresponding to distinct eigen values are orthogonal ok with respect to the dot product that we define for the operator l what we have define this is exactly what we define so if you have a differential equation with this operator ok so if you have this operator so you have this inner product they have with this dot product ok so with respect to this dot product these functions are orthogonal so that's what you have.

So once you have so your job is to find is eigen values and eigen functions then you can say that these are all different eigen values, different eigen values if you take corresponding eigen functions they are actually orthogonal if they are not suppose some eigen values are repeated or for some eigen values repeated you may have two functions ok two eigen functions you may have because the second order maximum you will have two linearly independent solutions.

We know any second order equation you can have maximum two linearly independent solutions so give fix your lambda you may have two linearly independent solutions for that system ok with the boundary conditions so in such a case you may have maximum two linearly independent solutions for the corresponding eigen value so that means you have two eigen functions maximum because it's the second order for an eigen value.

So if they are not orthogonal obviously they are, for the same eigen value you have eigen functions if I call so let's call this some v1x ok v2x are two eigen functions, if these are two eigen functions you cannot expect because they are from the same eigen value these are eigen functions corresponding to the same eigen value, they are linearly independent that much issue because that's how find these two ok you are simply go to solve the equation.

And with boundary conditions with by fixing this lambda so you looking for non zero solutions these are the non zero solutions just like what you do for the matrix, for the matrix how do you find the eigen vectors you fix that lambda you calculate all the solutions non zero solutions of that a minus lambda ix equal to zero so by doing that if you look for non zero solutions they are eigen vectors so you are doing the same thing here.

You fix this lambda in ly equal to lambda y and the boundary conditions you calculate the solutions non zero solutions if you can get two solutions ok they are they are certainly linearly independent solutions ok two non zero linearly independent solutions and you cannot say they are orthogonal ok what we did if it is a repeated route but that means I have two repeated x lambda I have two eigen functions corresponding to this this.

So in such a case you make them orthogonal by the Graham Smith process by redefining y to x as you can make them ok v to x into the process so this v1 of x plus so you call this something like v1 I want to if you make orthogonal I call this let's call this u u1 and u2 and you how do you make this Graham Smith orthogonal process you I want to make them orthogonal like this v1 is u1 of x similarly v2 of x you take u1 that is actually v1.

I define v1 plus some constant times u2 these are known u1 u2 are known so I choose my c in such a way that v1 v2 are orthogonal this is the condition v1 v2 with respect to this that inner product is that dot product is zero ok so with this condition you put this condition that makes it v1 v1 this dot product plus c times u2 into v1 this has to be zero, so that makes it that will give you minus v1 v1 by u2 v1 so u1 v1 is known right u2 v1.

What is the right hand side v1 u2 v1 so only u2 and v1, v1 is nothing but u1, u1 u2 are known so you can make put this c here and that is your v2 so now v1 v2 are orthogonal and v2 is nothing but linear combination of these functions u1 and u2 so they are also linearly independent so you have two linearly independent solutions but now they are orthogonal so something like if you have two vectors in the plane you are given like this you can make them orthogonal ok.

So by this is like u1 u2 you make them actually you are taking u1 as v1 so this itself is v1 now you make orthogonal to this, this is your v2 that's what you did ok so I explained earlier so you have you can make them orthogonal if they are not if you have two linearly independent eigen functions ok so what is the next step what is the other properties which you have seen for the matrices which are skew symmetric Hermitian matrices.

Eigen values are eigen vectors and any vector in that space or functions actually I can write in terms of these eigen vectors ok actually all these eigen vectors for the matrix you have if it is n by n matrix you have only you will get n linearly independent eigen vectors that is not two for the general matrix ok if it is Hermitian you can always get n linearly independent eigen vectors ok some of them may not be orthogonal so you can make them orthogonal.

By this Graham Smith process so finally you get n linearly independent orthogonal eigen vectors so here also you get the same so all these functions you get once you get all these eigen functions they and if they are not orthogonal once you fix lambda if you get two linearly independent solutions make them orthogonal by the Graham Smith process finally ultimately you get for different lambda values you collect all these are actually orthogonal functions.

So you have n linearly independent for the space rn in the finite dimension here you have infinite that is the meaning ok so once you have n linearly independent eigen vectors in rn any vector I can write it in terms of them so here also (any function now any function) any reasonable function ok I can write in terms of these eigen functions that is called complete so these eigen functions form a complete set.

Complete means any function I can write it in terms of them that is exactly over Fourier series ok so I will write just write like that so eigen functions form complete orthogonal set so orthogonal we have already seen how to get them, this complete means this means any function let’s say f, f I can write in terms of cn n is now from one to infinity because you have infinitely mean eigen functions cn un so let’s call the, once I make them vn I call them vn of x ok.

So this is what you get this is the actually fourier series what you see fourier series you get a different equation you will get a different fourier series.

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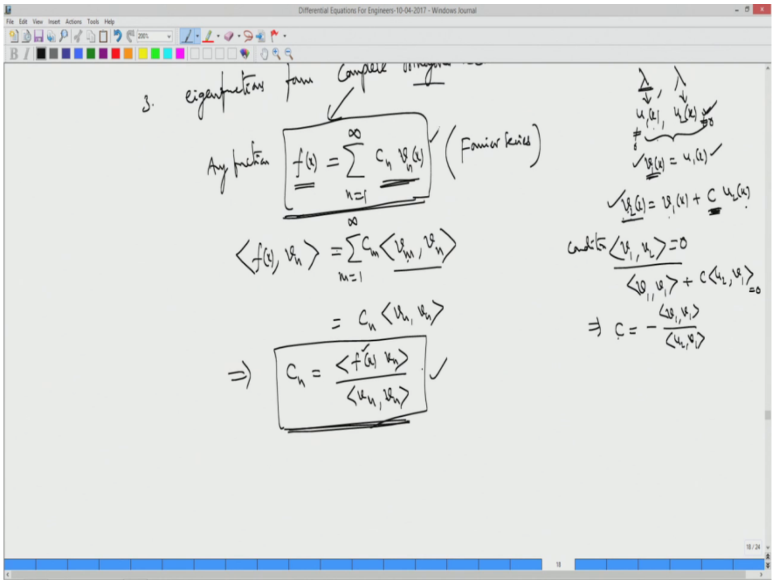

So how do I find my cn, cns I can easily find you know the inner product which are orthogonal so you multiply vn both sides with respect to this inner product you make that integral as in the (inner product) as in the dot product ok the dot product you defined as an integral.

So that you make use of that dot product so if you do simply if you have f, f is any function you multiply vn here and you multiply that weight function you integrate both sides from a to b if you do that what you get is this dot product which is and you see that because vn vn let's so you check the index this is only from i is from ru from right it from m is from one to infinity cm vm this is going to be vm and you are making a dot product with vn.

And these are all orthogonal if n is m is not equal to n it will be zero what is contribute here only cn vn vn so this will determine cn, cn is fxvn by vn vn so these are your fourier coefficients ok so these are your fourier coefficients that's the fourier transform if you define if you have a signal time signal you first define fourier transform as a different frequencies, this will give you those fourier coefficients ok so frequencies are 1, 2, 3 and so on.

So these are ok so you make a frequencies this is actually fourier transform ok different frequencies these are fourier coefficients and you correspondingly you have this cn vn is a fourier so any signal fx I can write in terms of frequencies different frequencies you break it the signal you break it into different frequencies so these are your frequencies you can define given its time signal you can get this fourier transform and this is a kind of inverse transform.

If you know if you have given signal here fx you get a fourier transform that is of fourier signal fourier transform and using them you can get back so if you are given like this all these frequencies you can get back your signal from this inverse transform.

That's fourier series ok, so this is what you see from the Sturm Liouville any Sturm Liouville problem and so there you have a equation so you have a differential equation.

So given a differential equation with l second order equation for different equation you have a different fourier series so you have a kind of infinitely many fourier series ok for a infinite equations you can create any equation you can get the such fourier series ok as what you will learn ok

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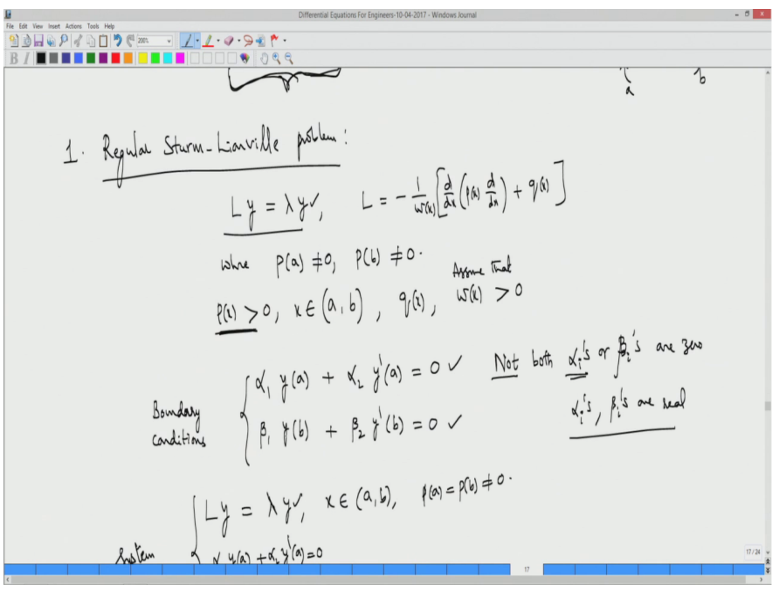

So we will see this is what I have say so if you have a second order equation with p of a and p of a p of b is non zero and these are the boundary condition match.

That special term is zero so that (operator) differential operator is Hermitian so that you can conclude the properties using the properties of this Hermitian operator ok analogous to the Hermitian matrices you can conclude that you can actually get any function you can write in terms of those eigen functions ok so these things are useful when you are solving a certain pde's partial differential equations in certain simple domains ok.

By separation of variables method when you do when you are trying to solve the partial differential equations in some certain domains what we do is we extract we try to make the partial differential equation into an ordinary differential equation problem so actually is strictly speaking is you make it a Sturm Liouville problem you extract a Sturm Liouville problem and the you make use of this and finally solve the partial differential equation.

That is where these Sturm Liouville problems are useful ok you can also see that different fourier series you can get out of this Sturm Liouville theory ok so this is only case one which you have seen for the regular Sturm Liouville system so this is type one like this you can get different things so what else, how else what is the next thing is next case is how do I see how do I give a different boundary conditions when p of a is zero or p of b is zero or both are zero ok.

Either p of a is zero or p of b is zero or (both of) both p of a and p of b are zero in that case what happens ok if p of a p of b is zero so that is one system that is or both are zero that is called singular Sturm Liouville system another case is p of a equal to p of b ok in that case what you see is you can look for some periodic Sturm Liouville system so we will see these two cases how to make second order homogeneous equations.

So that operator l as a Hermitian with the boundary condition so we will try to provide the boundary conditions in such cases ok depending on what is p of a p of b how the p of a and p of b ok if they are non zero it's a regular if they are same p of a equal to p of b then you have periodic system if p of a equal to zero or p of b equal to zero or both of them are zero that is one case that is singular Sturm Liouville system so in that case how to make this special term zero.

So that the operator corresponding the actual differential operator l is Hermitian so that properties of these operator can be used to get the fourier series ok that's what we will see so we will see the next two cases and the next video and then we will give different examples in each case if it's a regular so if you have a example for the regular system so get the eigen values and eigen functions so whatever we planned here.

So all these three steps we will see we take it and just conclude ok so that's what we will do so we will see this these next two cases how to make the second order differential operator self adjoint or Hermitian ok so thank you.