Differential Equations for Engineers
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Lecture 32
Introduction to Sturm -Liouville theory

So in the last video we have seen how to solve Basil equation and we have seen certain properties of Basil equation based on so one of those are (so set) using those two properties we have shown we have given the orthogonal properties of Basil functions, Basil functions of first kind, ok. So those solution of Basil equation which are bounded at 0.

So today we will just have we will look into relook into these properties and then and then make use of them to show that to find all the j half j (minus) j 3 by 2 like. So we also have seen j half and j minus half as a nice explicit analytical terms of expressions in terms of sin and cosines. And once you know j half and j minus half you can get j 3 by 2 j minus 3 by 2 j minus 5 by 2 j 5 by 2 and so on, ok.

So all j half of the odd n plus 2 2 n plus 1 divide by 2 and is running from 0 1 2 3 onwards. All these things you can get just if you know j half and j minus half that we know as analytical expressions. So this we make use of the properties of Basil functions and then we can get all these j 3 by 2 j minus 3 by 2 and so on, ok we will look at that.

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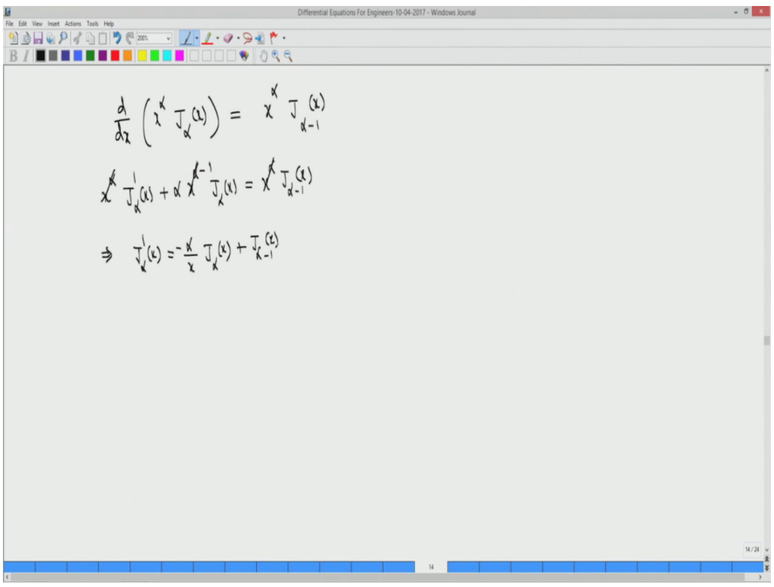

So consider the basil properties what we had, look into the basil properties so these are d dx (x 4 alpha j alpha (x)), this is given as which we have shown already so x 4 alpha comes out j alpha minus 1(x) this is 1 if you actually differentiate which you have seen x power alpha j alpha dash (x) plus alpha times x power alpha minus 1 j alpha(x).

So I just expand at the left hand side and right hand side you have j alpha minus as it is, ok. So now you cancel both sides x power alpha, so what you have is j alpha dash(x) equal to alpha by x j alpha(x) plus so minus of this, this if you bring it this side plus j alpha minus 1 (x), this is 1. So this is the property 1.

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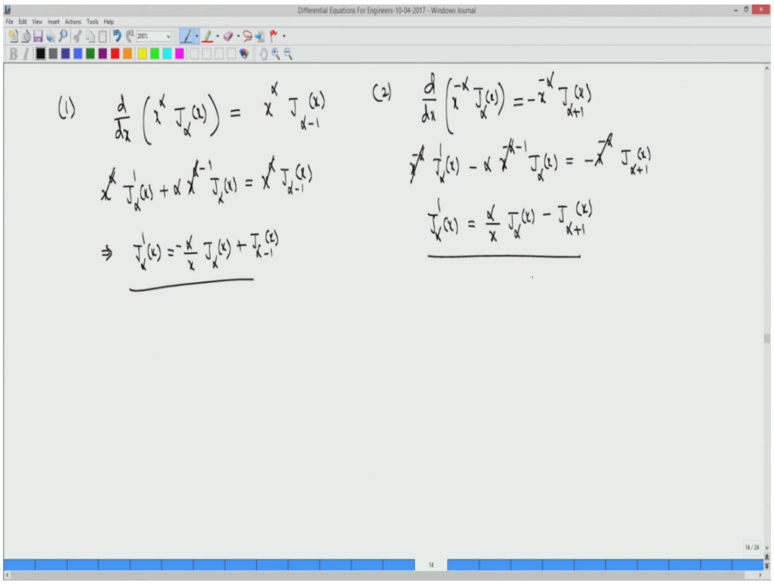

And you also have the property 2 that is derivative of x power minus alpha you multiply to the j alpha (x). And if you differentiate what we have seen is x power minus alpha comes out with a negative sign and now you have j alpha plus 1(x). Again you do the same thing expand the left hand side if you do that x 4 minus alpha j alpha(x) and this will be minus alpha x4 minus alpha minus 1, ok into j alpha (x) equal to x 4 minus j alpha plus 1(x). That is what we have, so these are 2 these are the properties these are nothing but same as the properties of these basil function which we call 1 and 2.

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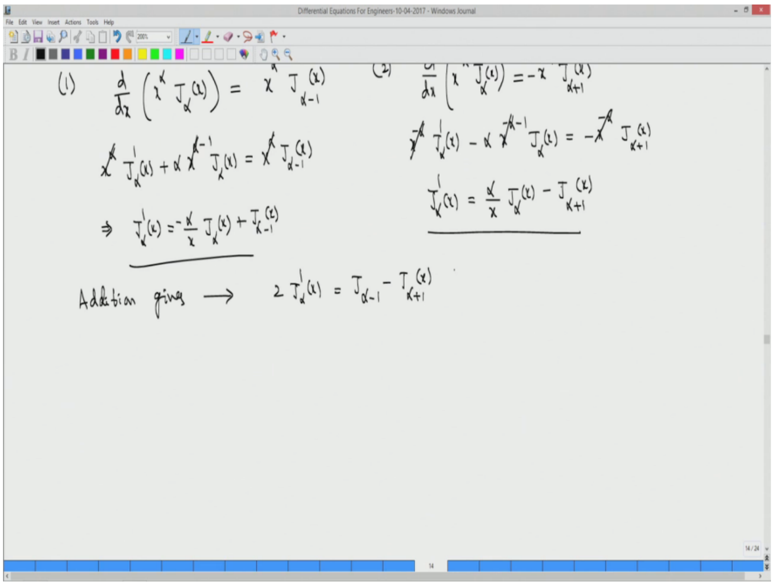

If you add and subtract them the addition will give adding, adding these two, so let us say adding these these two addition will give you what if it gives you addition so if you add it 2 j alpha dash(x) equal to, so this this goes and what you have some place alpha minus 1 minus j alpha plus 1 (x) that is 1, ok.

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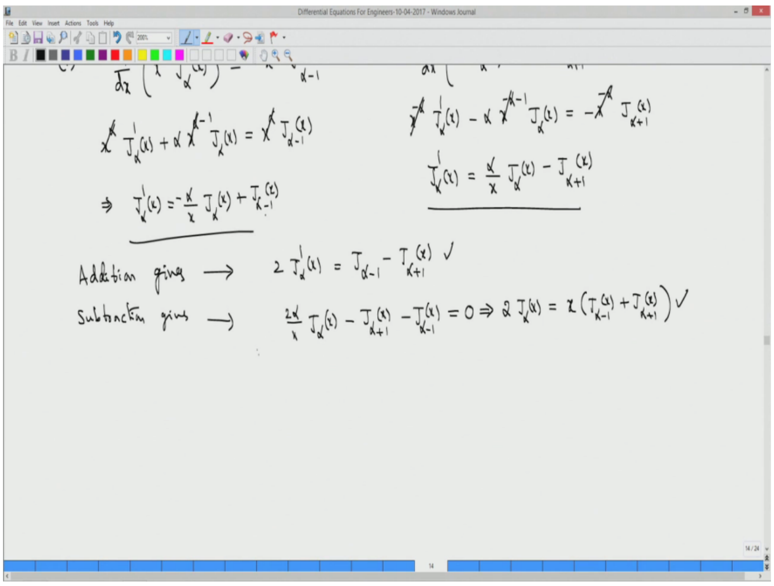

And subtraction gives you so if you subtract it so derivatives goes to your left is simply 2 alpha by x j alpha (x) minus j alpha plus 1(x) minus ok minus j alpha minus 1(x). So this minus this plus what is this minus of this and minus of this will give me this 1 equal to 0.so this is nothing but what you have is simply j alpha(x) j alpha(x) equal to 2 j alpha (x), ok subtraction will give you this. 2 alpha(x) equal to x times j alpha minus 1(x) plus j alpha plus 1(x), ok. So this is what you have, so this is the second kind. You can call them as a recurrence relations ok. For the first kind basil functions ok.

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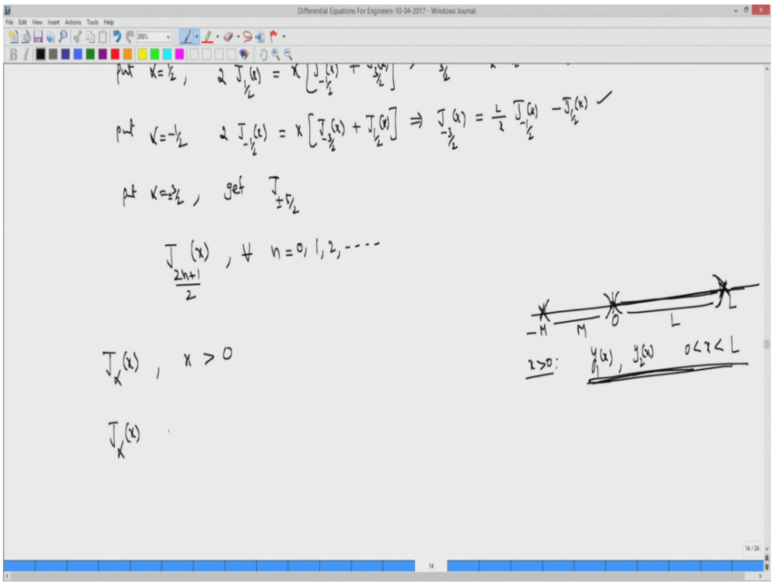

So make use of this we already know j half (x) we already know what it is. And similarly j minus half also we know the last video we have seen that this is root 2 by pi j minus half is cos x so this is sin x pi x sin x ok. And then j minus half is root 2 by pi x cos x. So once I know this nicely as analytical expressions so now I make use of this one second relation that will give me j minus so if I put here.

So if I take 2 times j alpha (x) I take this second relation x times j alpha minus 1 plus of x plus j alpha plus 1(x), ok. So here I put alpha equal to half what you get you get j half(x) equal to x times now you have alpha is half j minus half (x) and the third one will give you j 3 by 2(x). So this will give me j minus j 3 by 2. So I can get j 3 by 2 (x) in terms of j half and j minus half that is 2 by x j half (x) minus j minus half(x).

So this is my j 3 by 2. So if I put alpha equal to minus half ok then what happens I have j minus half (x) which is equal to x times. Now alpha is minus half so this is equal to j minus 3 by 2 (x) plus j half(x). This gives me j minus 3 by 2 as 2 by x j minus half minus j half (x). So like this I can go on. So having known j 3 by 2 j minus 3 by 2 now I can put j alpha as alpha as 3 by 2, ok. Plus or minus 3 by 2 let us say.

Then you can get j plus or minus 5 by 2 ok. So you can get like this and so on. So you can get 5 by 2 7 by 2 9 by 2 and so on, ok. So this is how you can get all j 2 n plus 1 by 2(x) suffix. For every n 0, 0 is known so you can get 1 if you put so 0, 0,1,2,3 and so on. So everything is known, ok. So these are Basil functions. This is how we can get all this special case of the second case 2.

That is where 2 all 5 are odd integer this is the case you could see that j alpha j minus alpha as series solutions. So you can also get them as a nice expressions an alkali expressions in terms of cosines and sin and root x, ok. So this we move on to Sturm-Liouville theory. So this is, so far what we have learned is you know how to solve second order when you have a second order homogenous equations with variable coefficients.

When you have when you look at 0 is singular point regular singular point. So you can find the two linear independent solutions ok because you see the basil equations 0 is the only singular point. There is no other singular point ok if so whatever you calculate you find this feboni solutions y 1 and y 2.

They are actually and uniformly continuous, uniformly converging as a series. This series are actually converging uniformly within the interval any interval you take all x positive because there is no other singular point. Suppose for certain equations 0is a regular singular point and there may be some l, l is a some l is a regular singular point and this side let us say some M is minus M is some regular singular point.

Then the way you calculate from this stubbiness method you get this y 1 y 2 this stubbiness series solutions converge absolutely and uniformly from this point to this point whichever is the minimum. You calculate this distance this is M you calculate this distance L, so these are the regular singular point, some singular points forget about the regular or not 0 is a regular singular point L and M are singular points.

if you know now you can calculate for x positive if you calculate for x positive you look for solutions, you look for solutions who are x positive. If you get y 1 and y 2, what you see is upto the next singular point that is L. So y 1 and y 2 are valid between x between 0 to L. So this is a theorem with a proof. So you have this y 1 and y 2 are finite whenever you have x is in within this next singularity between the 0 and next singular point L.

So between 0 to L we have our solutions are valid. Because for Basil equation there is no such L. L is infinity so for all x positive your solutions are valid. Similarly for the negative side, ok. So just for the sake of completeness you can write solutions j alpha(x) j alpha (x) and whatever j alpha(x) for the negative for this is fine for x positive. What happens if x is negative, x is negative you can write j alpha(x).

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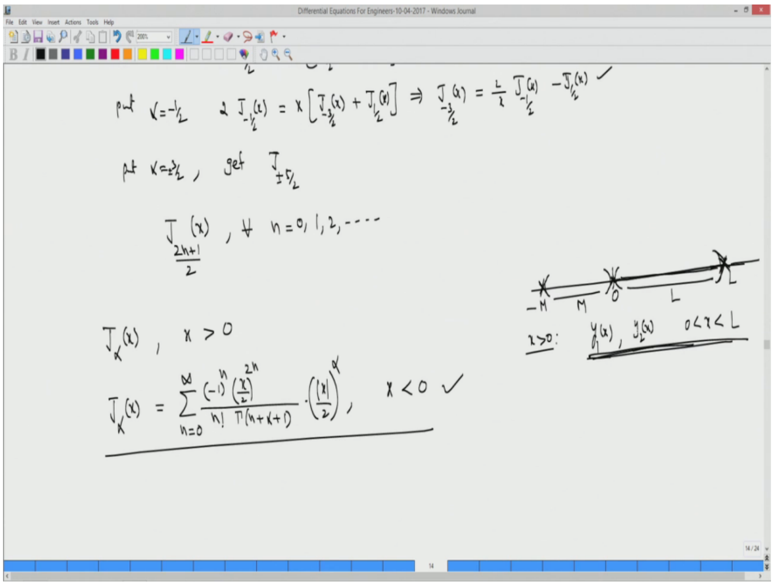

For x is negative is actually you can write this is given as summation n is from 0 to infinity x power alpha x by 2 minus 1 power n, ok. So that is what is your x j minus 1 power n x power 2 n plus new 2 n plus alpha, ok by 2 divide by n factorial and gamma of n plus alpha plus 1, this is what you have.

So here if you remove this n is 2 power n so I can make if it is negative or positive does not matter. But I have a problem if I have x power alpha. So this alone you can write into x pi mod x by 2 power alpha for x negative. So this is how you should look for your solutions. Always look for solutions in this fashion in the febinas method when the x is negative, ok.

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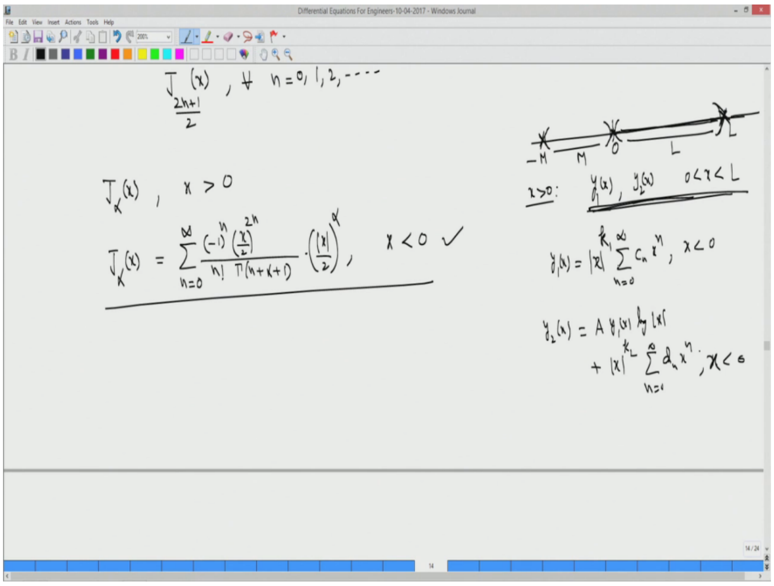

So that means you always look for your solutions in this fashion so either this way so initially you look for solution mod x power alpha power of k, ok into power series solutions, C n x power n and y 2(x) depending on the case you may have for example in the case 2 you can write some arbitrary constant and y 1(x) this is for x negative. Y 1(x) log x, log x is now is not it defined only for positive values.

So now you are defining for x negative, so you have to take mod x plus x power so you power of alpha that is k 2 right. So you have this is for k 1 which is bigger root and if you want k 2 so mod x power k 2 now you make a sum so n is from 0 to infinity some d n x power n. So this is how you look for solutions in the negative interval, ok.

So with this we move on to Sturm – Liouville theory. So this is how you can solve any second order linear you can you can you can actually solve second order linear homogeneous equation. So you can find two linearly independent solutions. Once you find two linearly independent solutions ok you know if it is if your equation is having right hand side it is a non homogeneous term you can find a particular solution by the method of variation of parameters, ok.

So with that method you can find a particular solution and then you combine with this general solution of the homogeneous equation to get the general solution of non homogeneous equation is what we have seen.

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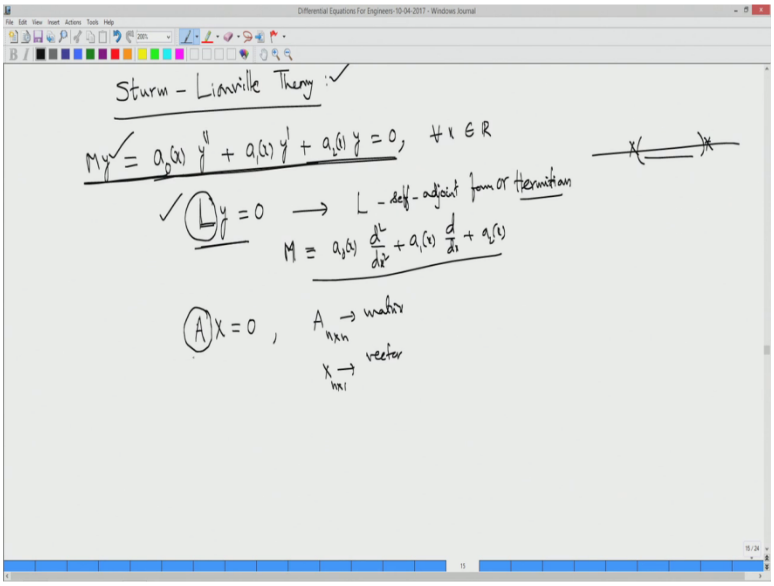

So we will start with the Sturm- Liouville theory, Sturm- Liouville so what is this theory about, so what is a Sturm- Liouville theory see so far we have a general equation, general equations are a 0 (x) y double dash second order equations a y double dash plus a 1(x) y dash plus a 2(x) y a equal to 0. So depending on whether 0 of this a 0(x) are a singular points or not.

If 0 of a 0(x) if they are regular singular points, ok. If there is no zeroes of a 0(x) suppose a 0(x) is non zero everywhere. I can divide it I can actually get the power series solution. There is no other singular point so power series solutions if you look for you can get the power series solutions everywhere for every x in R, ok.I can get both y 1 and y 2 two linearly independent solutions.

Then I can solve it I can write the general solutions as linear combination of this y 1 and y 2. What if a 0(x) is 0 at some point at certain points, ok. So they are singular points we have seen already, so around that singular point if it is if it is a regular singular point the point is a singular point is a regular then you can look for from either side of the singular point you can look for solution as a febonacci series of solutions even.

So even there if these are the two singular points I can look for solution in this interval and I can get two linear independent solutions by the febonacci method in this which is valid in this interval. Again you can write the general solution of this homogeneous equation as a linear combination of those two linearly independent solutions ok. I can get the general solution. So basically I can solve this second order equation.

May be for all whole interval or whole real line or some ordered intervals where it is defined, ok. Then what is this Sturm-Liouville theory? This actually what we do is we try to look make this equation ok as some operator let us say a differential operator L y equal to 0. So imagine a matrix L is like a matrix. Matrix y equal to 0, ok.

Then this is this matrix this operator L, L I can always put it in the sulphate joined form, ok. What is a sulphate joined form? We call this sulphate joined form. We can always put in this form sulphate joint form. We will see what it is. What is the meaning of sulphate joined? This is like if you see L is I said L is operator differential operator. So you can think of L as a 0 (x) d square by d x square a 1(x) d dx plus a 2(x) it is clearly what it is, ok.

So if I put this L in this sulphate joint form this is not the sulphate joint form. So this is what we have already, let us call this M y, ok. M y is this and M is this ok. Now this M I can rewrite and put it in this form L y equal to 0 where L is the sulphate joint form. So before I say what iss the sulphate joint you imagine it is like a L is a like a matrix. So you have a matrix a x equal to 0.

Look at the matrix equations, ok. This is a differential equation this is a matrix equation where is a n by n matrix, ok. N by n matrix x is the n by 1 vector ok. When you have this you want to see what is this a if a is, so I am saying I can always make this M y equal to 0 this differential equation I can always put in this form which is sulphate joint form that are Hermitian form. Call this Hermitian, Hermitian, Hermitian if you see if you look at the you know what is the Hermitian of a matrix.

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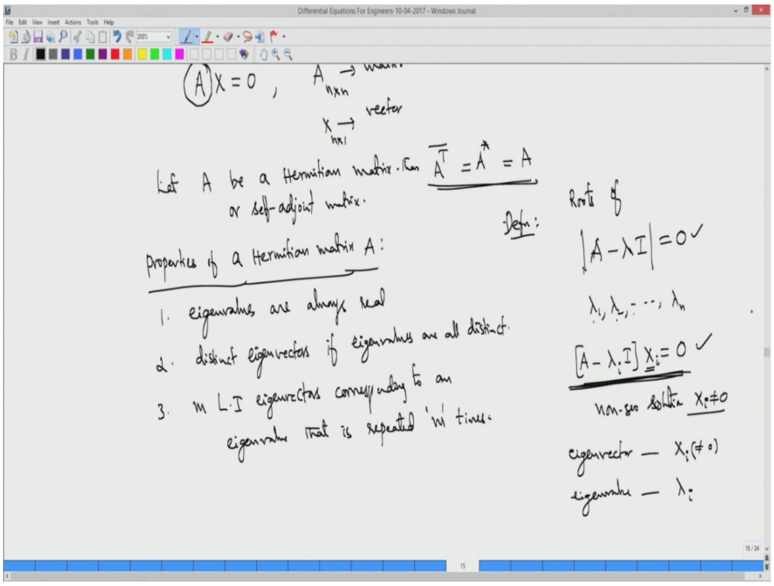

If a is the matrix let us say a is let a be Hermitian matrix. What is the meaning? So you take this transpose and then you make a bar which is equal to that is how you write a star this is actually a star. This is same as a itself. If you have a real entries it is nothing but a transpose equal to a.

If your matrix is having only real element a transpose equal to a, ok. If your matrix is having complex numbers then the bar makes sense. So a transpose bar that is a star equal to a, so if this is the meaning this is the definition of Hermitian. So if this is true you have this matrices Hermitian, ok or self adjoin ok or self adjoin matrix, self adjoin matrix, ok.

So what are the properties of this matrix if you actually see the properties, properties of this a self adjoin this Hermitian matrix a, ok of a Hermitian matrix. The thing is you can see once you have this self adjoin matrix that means a transpose equal to a or a transpose bar equal to a. If you have such a matrix if you calculate its Eigen values ok.

You know what is Eigen value, Eigen value for matrix a is a minus lambda, lambda is an Eigen value make it i this determinant equal to 0. So this all the routes of routes of this polynomial equation a minus lambda i is identi matrix n by n matrix the determinant equal to 0 will give you n routes routes of this equation that is you may have n routes. They are same or (comp) they are distinct or same can be same.

Some of them can be same because you will have this n by n matrix this equation determinant will give a n th order n th degree polynomial in lambda so that will have n routes lambda 1 lambda 2 and so on lambda n. So these are Eigen values because a minus you put take any of these lambda i any of these value of i. This as a matrix you take into x equal to 0. This will always have a non zero solution because a determinant is 0.

For this lambda the determinant is 0. That is what is the condition. So determinant is 0 so you will have a non zero solution. Solution you can call as x, non zero solution ok. So non zero solution is called Eigen vector, Eigen vector is x which is non zero and Eigen value is lambda i, ok. You can call this i if you want. Corresponding to i you can call this lambda excise.

Excise Eigen vector corresponding to a Eigen value lambda i. So this is what you see so first of all if it is a (homog), if it is a general matrix this is the definition of Eigen value and Eigen vectors, ok. If A is a general matrix, if it is a Hermitian matrix Eigen values are always real you can actually show that ok so first property is you can check it verify Eigen values are always real that is the first thing ok.

Eigen values are real and Eigen vectors you can so once they are Eigen another property of the Hermitian matrix is second important property you find all the Eigen vectors ok. If you actually calculate the Eigen vectors for each of these Eigen values you will get suppose all are distinct lambda 1, lambda 2, lambda and all are distinct then you will have n distinct Eigen vectors ok so you will have n distinct distinct Eigen vectors.

If lambda I's are distinct ok if Eigen values are distinct they are different from each other ok if suppose they are repeated suppose lambda 1 is repeated thrice you still you can find three solutions suppose lambda 1 a minus lambda 1 i into x equal to zero if you consider for lambda 1 because lambda 1 is repeated thrice so lambda 1 lambda 2 is also lambda 1 lambda 3 is also lambda 1 in that case if a is a Hermitian matrix.

I can always find three linearly independent solutions for this system so i will have three linearly independent solutions (for the vector) for the matrix a ok for this system you will always have three linearly independent solutions so that means you have three linearly independent Eigen vectors corresponding to the Eigen value that is repeated that many numbers of times ok.

So third point is you will have m linearly independent Eigen vectors corresponding to an Eigen value that is repeated m times ok so if you have an Eigen value that is repeated m times you can get m linearly independent Eigen vectors and again once you have so the important property here is one more is observation (once you) suppose (there Eigen value) all the Eigen values are distinct.

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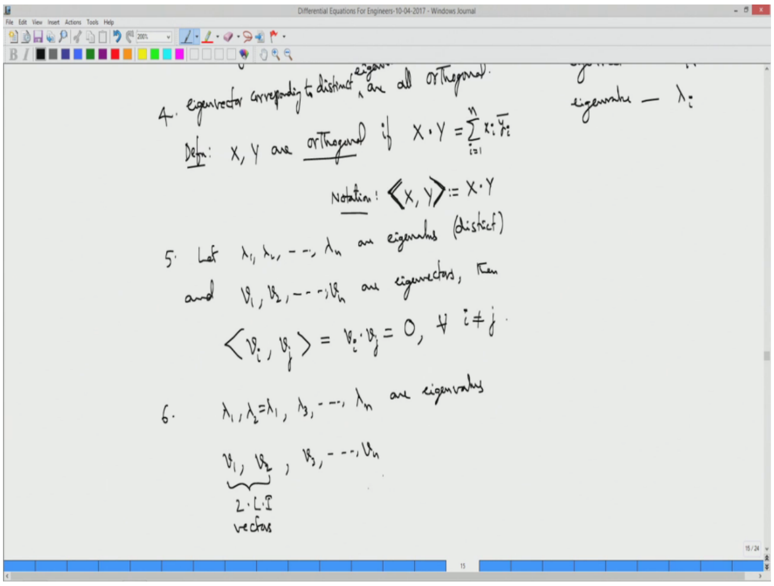

So you will get n distinct Eigen vectors not only they are linearly independent they actually they are orthogonal to each other ok. So distinct Eigen vectors are all orthogonal, what you mean by orthogonal? Orthogonal with respect to so what is the meaning of orthogonal so suppose you have a two vectors ok, so let's write two vectors x and y ok xy are orthogonal if it's the definition ok.

If x dot product with this vector dot product ok, so what is the vector dot product if your vectors having only real entries you can simply have xi yi, if you have a complex entries you can just make a bar ok, that is i is from 1 to n so this is what you have. So this is the dot product you can call that you can with the notation you can write x,y in this bracket if you put it which is actual by definition this is a dot product of y ok.

Notation, this is just a notation. So if you have a Hermitian matrix all the Eigen vectors which are distinct, all the Eigen vectors corresponding to distinct Eigen values ok Eigen vectors corresponding to distinct Eigen values you should say Eigen vectors corresponding to distinct Eigen values.

Are all orthogonal so that means if i have all are distinct you would take any on other so lambda 1 lambda 2 these are all distinct lambda n are Eigen values ok.

Corresponding Eigen vectors let's call this (let's call me) let's call this as v1 ok v1, v2 when i write v small v that is actually represents a vector ok v1 v2 vn is all Eigen vectors distinct here these are all distinct none of them are each other same ok. So there are nothing is repeated here they are all each is if take any pair they are all different from each other so you have these Eigen vectors they are all orthogonal.

(So you) that means you take any vi dot product that is dot product now we have a notation that is vj this dot product which is vi dot vj which is zero for every i is not equal to j that is what is the meaning ok, so if this is what is a property that's you can write it as property file if let Eigen values that are distinct and these are Eigen vectors then this is true then you see that all Eigen vectors are orthogonal that is what is the meaning of orthogonal.

So orthogonal means this is true this inner product sorry this dot product which I represent with this ok so this is what is the case so what if they are repeated so the last point is what if they are repeated, if they are repeated suppose say lambda 1 and another one lambda 2 equal to lambda 1 ok and then lambda 3 and so on lambda 4, lambda n. So only lambda 1 is repeated twice ok are Eigen values.

And then corresponding Eigen vectors I know that v1 is Eigen vector v2 is Eigen vector two linearly independent these are two linearly independent vectors ok. Vectors corresponding to lambda 1 and i will have v3 v4 up to vn these are all Eigen vectors corresponding to distinct Eigen values, so what happens (these are all) if you consider any pair here they are all they both are orthogonal ok. Any pair is orthogonal to each other but these are not ok.

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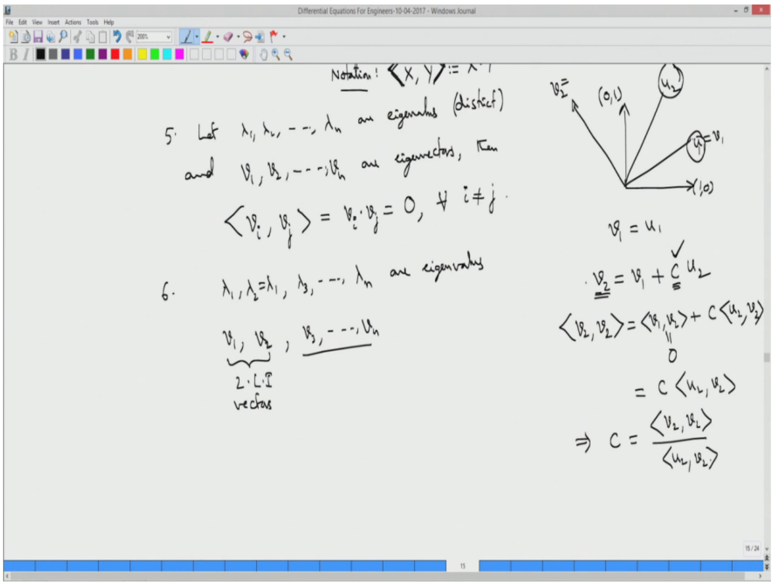

These two but you can make them if they are suppose you consider two vectors here if they are like ij x axis and y axis they are already orthogonal their dot product is zero, one zero and zero one ok. And if they are like this (if they are) if your vectors are this one v1 and v2 so you can always make them orthogonal this way you consider so by just notation you call them so let us call this u1 is 1 vector, u2 is another vector (which is) they are not orthogonal.

Clearly you can see that u1 and u2 are not orthogonal, so you want to make them (orthogona) orthogonal you define it like this u1 ok i want to define now you take v1 as u1 ok this itself is v1 and what is your v2 i want v2 in such a way that i want to take a linear combination of these two your u1 and u2 so that, that is orthogonal that is my v2, v2 i want a linear combination of this and this, how do i do we will see this.

V2 equal to so this is called Graham Schmit orthonomal process ok, so how do i do this you simple thing this is v1 ok which is u u1 minus some constant so let us say some constant times u2 simply take the linear combination with some arbitrary constant c, so i want to find this i want to choose this constant c in such a way that v2 is orthogonal to v1 ok that means v2 inner product you take with v2 itself ok.

If you take this becomes v1 v2 and i am choosing my c in such a way that this is zero so this is zero ok so this is zero and then c equal to, so let us say v2 is some vector which is already orthogonal to v1 so that is what you my v2 ok. So (if i have) if i want such a v2 what is my c that is what is the point so v2 you choose in this fashion v1 plus c times u2 ok then you have c times inner product with sorry dot product with v2.

So (i why) i have chosen my v2 in such a way that this is zero ok, so this is equal to simply c times u2v2. So this means c equal to simply i will have v2v2 divided by u2v2.

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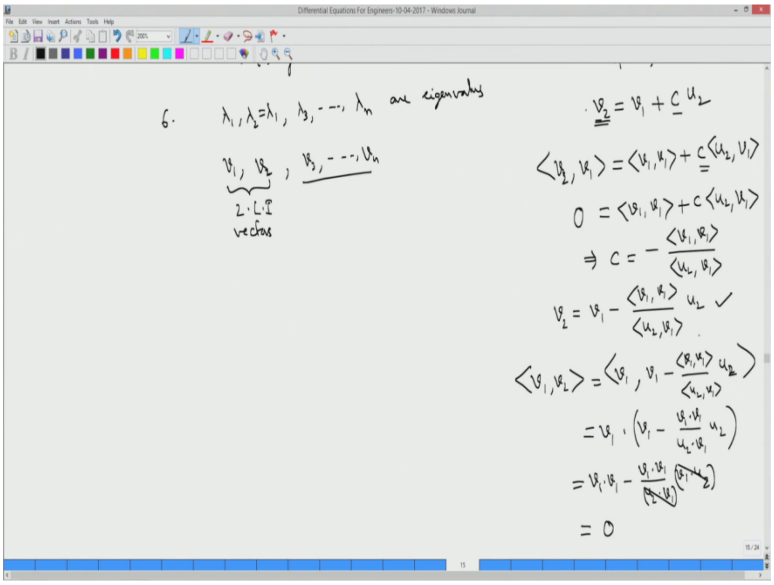

So with this c if i choose my v v2 is orthogonal to v1 that is the meaning ok so that implies v2 equal to v1 plus v2v2 divided by u2v2 into u2 so you see that this is and v1 equal to u1, so i already u1 so v1 wherever v1 is the that is actually u1 so i can write in terms of u1u1 .

Not like this so let's redo it, so you choose c in such a way that v2 is orthogonal to v1, so how do i choose this (you take the inner product with) you take the dot product with v1 then i will have v1v1 plus c times this is simply making the dot product with each of this left and right side ok then c times u2 dot product with v1, v1 is actually u1 so and i want my v2 to be zero for some c value ok.

So if i want the left hand side zero (36:38 that canted to my c) so this is equal to v1v1 plus c times u2v1 this implies c equal to minus v1v1by u2v1 so u2 i know u1u2 are known so this is what is my c so with this choice of c my v2 is v1 minus v1v1 by u2v1 into u2 you see that this and this are orthogonal to each other that is if you actually choose v1v2 if you do that actually so you will see that v1 v1.

And you take the (inner product with this) dot product so basically dot product ok. So dot product i replace v2 with this v1 minus v1v1 by u2v1 into u2 ok, so this is the dot product so this is nothing but v1 dot v1 minus v1v1 ok so i just repeating divided by u2 so if you want you can write dot product as this v1 dot v1 u2 dot v1 into u2 ok. So this is nothing but v1 dot v1 minus v1 dot v1 by u2 dot v1 into v1 dot u2.

So think of just real values you can see that this this cancels what your left with this zero ok, so (this is how you) what if these are if you uu and v, u1 and v1 are having complex entries what happens so you see that v2v1 v1v1 u2v1 so this is what you have is this fine ok.

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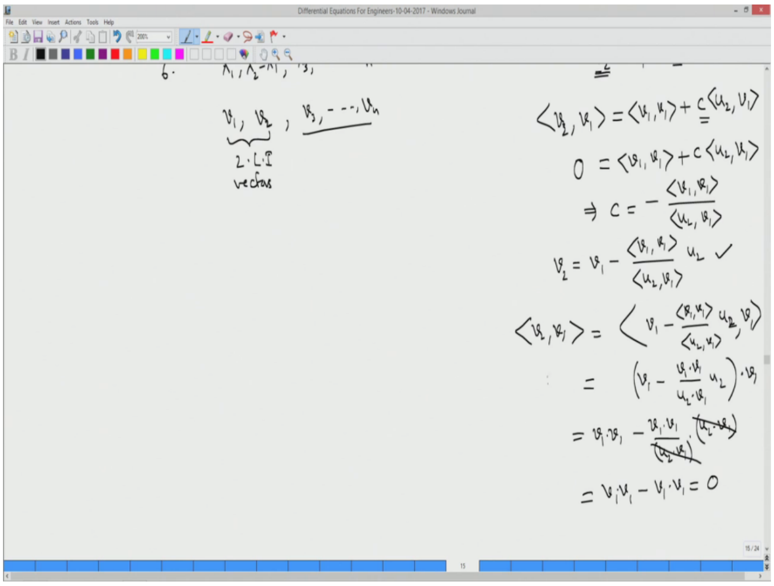

So if you have v2 you can rewrite if you want v2v1 ok, if you do that this (you are making inner product with) you are making dot product with v1.

So this is also same so this is same as v1v1 dot product with v1 such easy to see if you do like this, so this is v1 dot v1 minus v1 dot v1 by u2 dot v1 into u2 dot v1 ok. So this this cancels what you are left with is v1v1 minus v1 dot v1 which is zero.

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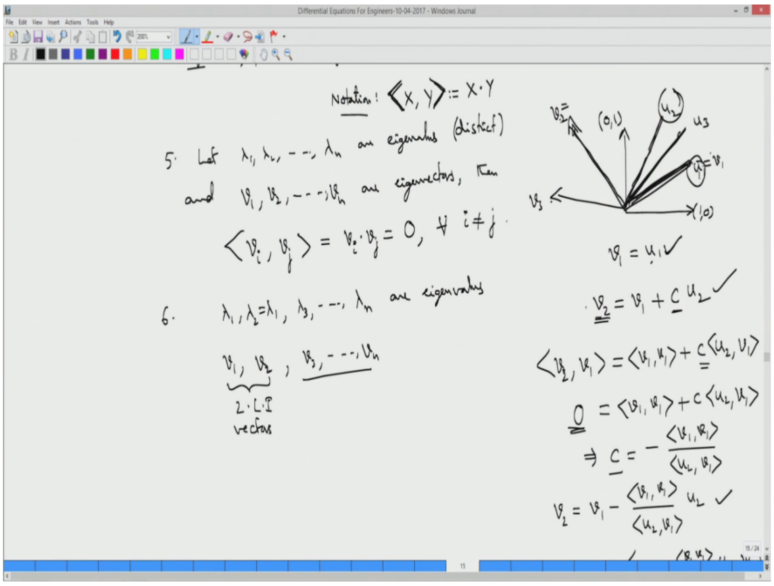

So you see that v2 is perpendicular to orthogonal means perpendicular to v1 so that is what is meaning. So if you are given two linearly independent vectors even in the plane you can actually by this process.

You can actually make them or that can always look for v2 as a linear combination of this u1 and u2, u1 which is as v1 linear combination you choose for some constant, that constant you can find out in such a way that v2 is orthogonal to v1 so that is what i use to if i make it zero i can find this my c, with this choice of c if you put it here you can verify that they are orthogonal. Now i may have (three) i can make them orthogonal.

Suppose you have three vectors let us say you are in the space u1 u2 u3, u1 u2 i make it v1 u1 is v1 u2 is v2 now u3 is also you can make so i can now take u3 i want v3 as a linear combination, now you have v1 and v2. Now the third vector is v3 this i want another v3 so i look for v3 which is orthogonal to v2 and v1.

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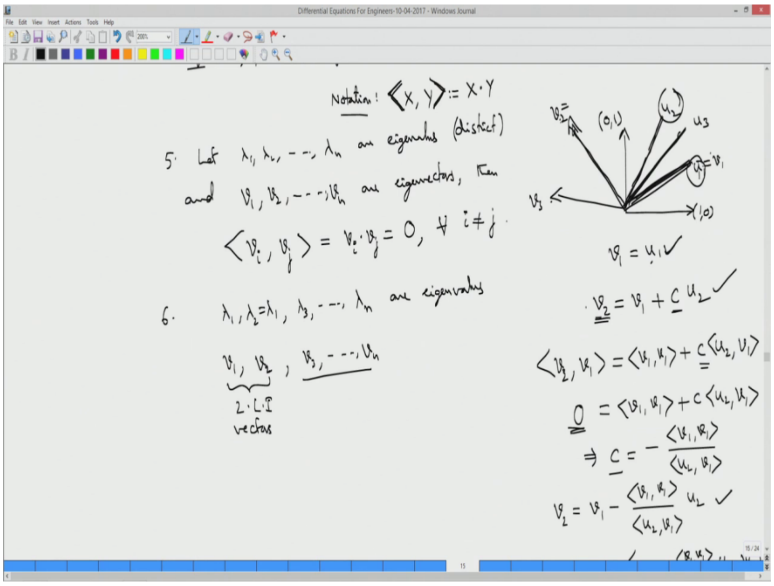

But i write it in terms of v3 i write in terms of v1 v2 and u3, so once i have v2 i can remove this u2 gone ok.

So i have v3 i want another perpendicular vector, so if i look for (the) such a thing i can always look i can rewrite as a linear combination of v1 v2 and u3 v1 and v, so how do i do this so v1 plus c1 v2 plus c2 v3 but there is no v3 so you have u3 ok, so again you want what is this how do i find these constants c1 and c2. See i have two constants i have to find right, if i look for v3 i choose v3 in such a way that which is perpendicular to v1.

And v3 is also perpendicular to v2 so i have two conditions here so these are the conditions with these two conditions i can get two constants ok. So with these constants if you do that you can find these two constants (that) so the whatever you come back and substitute here i can get v3 then v3 is the way you get that v3 that v3 is actually perpendicular to v1 and v2, so like this you can go on get any number if you have n linearly independent vectors.

You can do this process and create you can make them orthogonal that means they are perpendicular to each other ok. So such a thing you are doing so you have this so what you have is so you can make them you make them by that process by the (explain) above explained process, this is called Graham Schmit process ok make them orthogonal by the Graham Schmit orthogonal process.

You can make them so by this process this is (genas) i just explained the space or the plane ok, so this you can do in any finite dimensional space,

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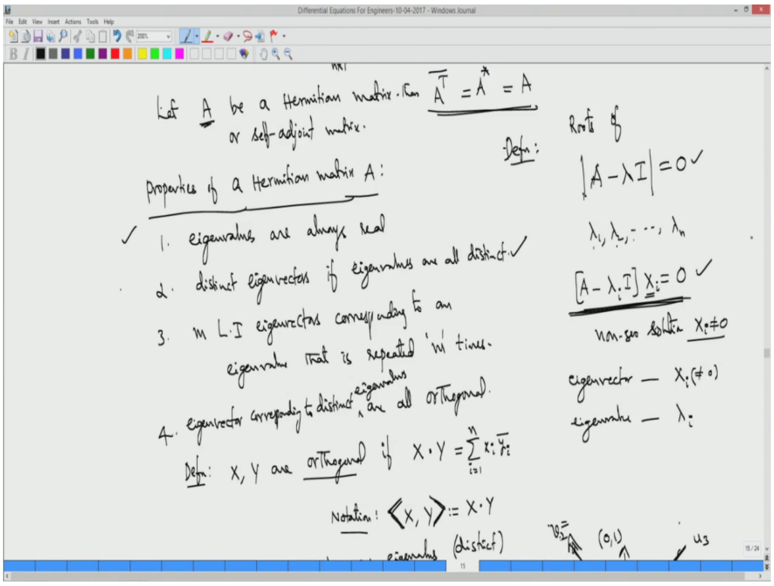

So you can do this you can go on now ok, any given linearly independent vectors you can actually make them orthogonal ok by this Graham Schmit process ok. Only thing is you have to find every time once you get now you have v1 v2 v3 are orthogonal.

Now suppose you have one more vector you make them one more time v4 is v1 v2 v3 are already orthogonal now so now you make a linear combination v1 plus c1 v2 plus c2 v3 now you have one more vector that is not orthogonal that is u4 you write c3 (u3) u4 you can get this c1 c2 c3 because you want v4 which is our condition is v4 has to be linearly independent with v1 v4 has to be orthogonal with v1 v2 v3.

So those are the three conditions now i have three unknowns three conditions, now i have three unknowns three conditions that v4 v1 is zero v4 v2 is zero v4 v3 is zero so with these three i can find my c1 c2 c3 what you get is v4 that is orthogonal to v1 v2 v3. Now i make u1 u2 u3 u4 as orthogonal vectors v1 v2 v3 v4 in that v1 is nothing but u1 itself ok, so that's how we draw this, this is a Graham Schmit (orthonomal process) orthogonal process ok.

So this is what you see if your matrix is Hermitian matrix this is important ok, so not only this any vector so this is a is n by n matrix so (any once) you have a Hermitian matrix with these n orthogonal vectors ok, so you get (n linearly) n Eigen values either repeated or not repeated distinct, either distinct or they may be repeating in any case you have n linearly independent Eigen vectors.

If they are distinct they are actually they are nothing but they are directly orthogonal if they are not orthogonal you make them orthogonal ok. So they are always have if you have n Eigen values for any matrix if it is a Hermitian you will have n linearly independent Eigen vectors you make them orthogonal so in any case implies you have (n linearly independent) n orthogonal, orthogonal means obviously they are independent.

So n orthogonal Eigen vectors ok so once you have orthogonal vectors any vector you take any x in rn i can write in terms of all these Eigen vectors all these orthogonal vectors i can make them some linear combination of these vi's i is from one to n so this is true for any x i can write we can write we can have this representation ok. Because any vector i can write in terms of ijk in r3.

In the plane nay vector i can write in terms of ij, so same way any vector here in rn i can write in terms of these vi's which are orthogonal vi's are like ijk like that in r3 ok, so this is what is (analogal) analogous to this matrix what we do is we convert this equation my equal to zero into a form that is called orthogonal so there is a Hermitian form we have to make ly equal to zero ok.

(I) we can always do this because i can convert any second order equation into this form so that differential equation is always i can make them Hermitian we will see that ok. So i can always make them Hermitian now you because analogous to this matrix which is Hermitian you have all these properties the same thing you can do here also ok, so here (is a) because it's a matrix the solutions are only finite.

(The space) the solutions are all vectors, vectors are in the finite dimensional space that is rn either space or plane ok these are all two dimensional is plane, three dimensional it is space ok, so if n equal to 3 you are in the space n equal to 2 you are in the plane, otherwise it's a finite dimensional means n is finite, so once you have this you have (n linearly independent) n orthogonal vectors you have you can always get but if it is a differential equation.

(The solutions are all) solutions can be the functions their functions cannot be in a finite dimensional vector space ok. They are not finite dimensional vectors each vector let us say 1 x, x square x cube and so on we have infinitely many linearly independent they are all linearly independent if they are solutions they are also so ok, So because they are all infinitely linearly independent functions.

So the space you can call this just like imagine your rn it is like r infinity, you have infinite dimensional space those functions belongs to this polynomial themselves belongs to infinite dimensional space so in general if you talk about general functions exponential all these, they are actually form infinite dimensional space ok. So the solutions are also belongs to infinite dimensional space, so if you have a Hermitian form of that differential operator.

Differential Equation, now what you do is you analogous to these properties of a matrix you find the Eigen values Eigen vectors and make them orthogonal.

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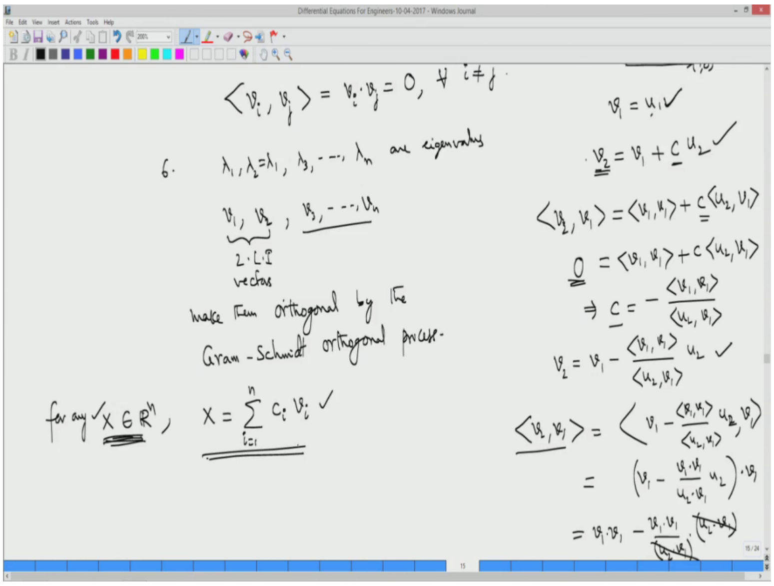

And then so that you can write at the end like this any function in the infinite dimensional space i can write in terms of this Eigen vectors ok that is actually your fourier series, this is what is a Sturm Liouville theory so that is what we will see ok.

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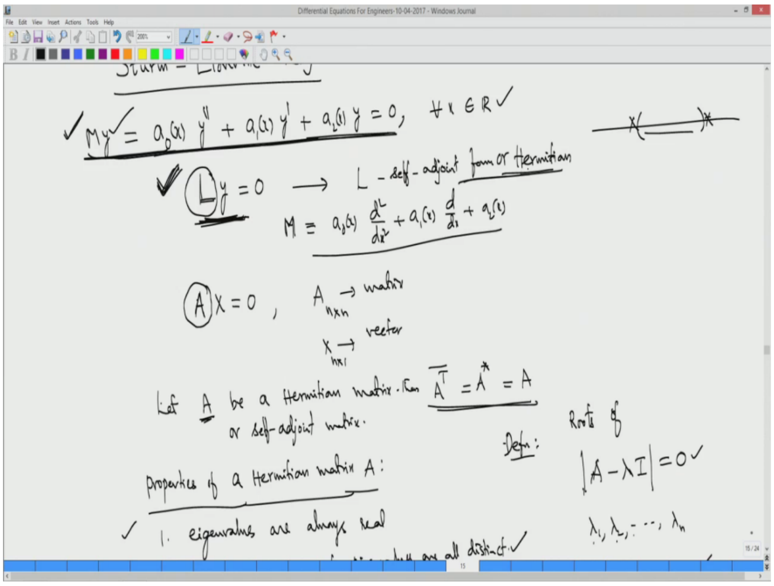

So how first of all i have to make (what is) how to make a general second order differential equation into in this form ok, so main idea is to make this equation into self adjoint form or Hermitian form so that we will see in the next video ok, so we will see that how to make the second general second order equation homogeneous equation into self adjoint form or Hermitian form so in order to do this.

If it's a matrix you can easily see it's just a entries a transposing equal to a or a transpose bar equal to a, that is what you have to see but (for a matrix) for a differential equation if you want this to be in the Hermitian form you need certain boundary conditions so for if you want a boundary, what is the domain? Domain is full r so you want if you want a boundary, boundary means (for a) the domain is full real line.

What is a boundary there is no boundary minus infinity, infinity so you need a finite domain so that you can make a boundary, boundary terms are ab ok, if you have a interval if your differential equation depend only on the interval finite interval your boundary is a and b that is your boundary, so you need certain boundary conditions at a and b so that l will be Hermitian so that is what we will see in the next video.