Differential Equations for Engineers
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Lecture 26
Frobenius Method of Solutions (Continued)

We were looking at case of solving a differential equation of second order with variable coefficients by Frobenius method. So we have already seen we were doing one example where the indicial equation has two roots, first root will give you one solution. Second root when you look at the difference between these two roots k 1 and k 2 and their difference will be integer which is non zero (dif) integer.

So we have given first solution, in this video we will give you a second solution. How to get the second solution? There we have already seen what kind of form it takes for the second solution. So we will proceed how to get that second solution. So this is your second solution and the solution is of this form y 2 is A or some arbitrary constant times y 1 log x and x power this k 2, so k 2 is this. So finally you get this form.

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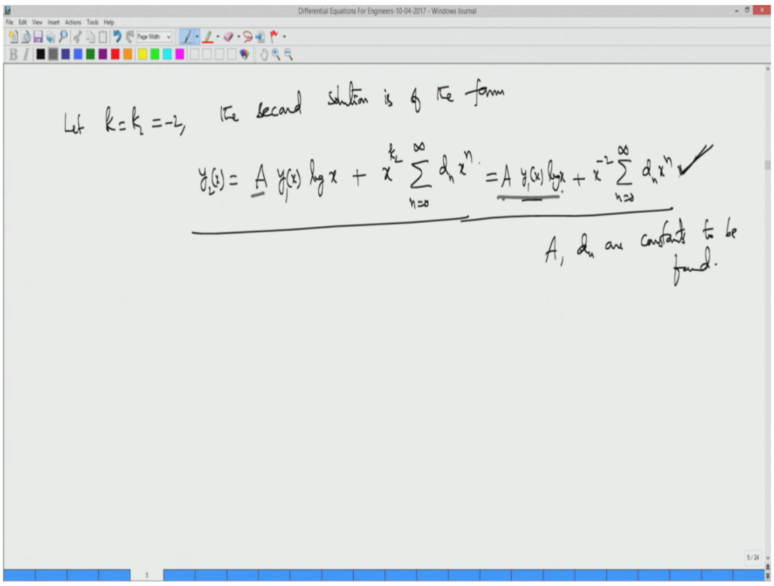

So y 2 is A times some arbitrary constant y 1 which. You already wrote it for k equal to k 1 which is 1 log x plus x power, k 2 is minus 2 and we have a power series representation d n x power n and d ns are arbitrary constants along with A. So these are constants we have to find out by simply substituting into the equation. So the equation is given here so that is x square y double dash. So this is your equation. So we will just substitute into this equation for y 2.

So for y you replace this y 2 so calculate y 2 of x. So I will use black. So y 2 is A times y 2 dash, y 2 dash you calculate so you get y 1 x by x. So that I am differentiated log x, okay, and then plus A log x y 1 dash of x plus here. So you simply differentiate once that is n is from 0 to infinity d n x power n minus 2, okay. So if you differentiate this is going to be n minus 2 times x power n minus 1. This is what you will get because k 2 is integer.

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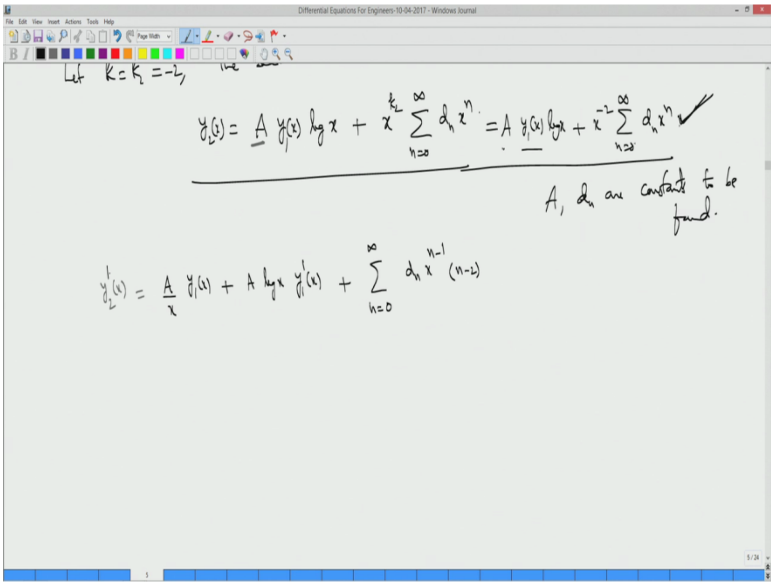

So once you differentiate you will see that n equal to 0. So I think you cannot do this. So let us differentiate. So if you look at this series that is d 0 x power minus 2 plus d 1 x power minus 1 plus d 2 x power 0 plus d 3 x and so on. So d 4 x square and so on, okay. So or you can differentiate as a two functions, okay.

As if this is one function and the series is another function so differentiate this once so you have x power minus 2, this will give me n is from 1 to infinity, n into n d n x power n minus 1, okay. Then plus keep the series as it is then differentiate the other part that is x power minus 2. What is the derivative of 1 by x square? 1 by x square derivative is minus 2 by x cube, okay, minus 2 by x cube.

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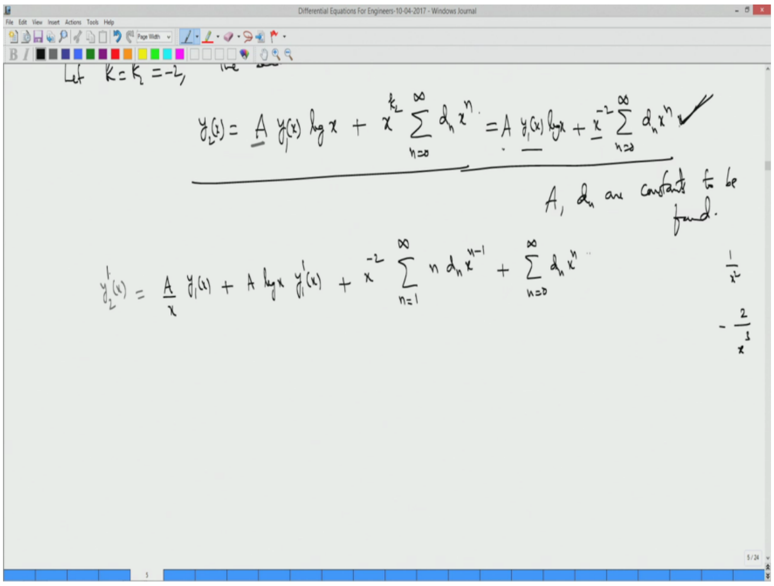

So you have minus 2 by x cube. So this is what is your y 2 dash. And you need y 2 double dash. So second derivative will be A by x square y 1 x plus A by x y 1 dash of x, so I differentiated this term one more time and it will be the second term you have A by x y 1 dash of x plus A log x y 1 double dash of x plus again this will give you so one more derivative here.

So if you do that x minus 2 and this is from n is from 2 to infinity, n into n minus 1 d n x power n minus 2. So I differentiated the series first. I had to differentiate x power minus of that will give me minus 2 by x cube where this n is from 1 to infinity, n d n x power n minus 1, okay.

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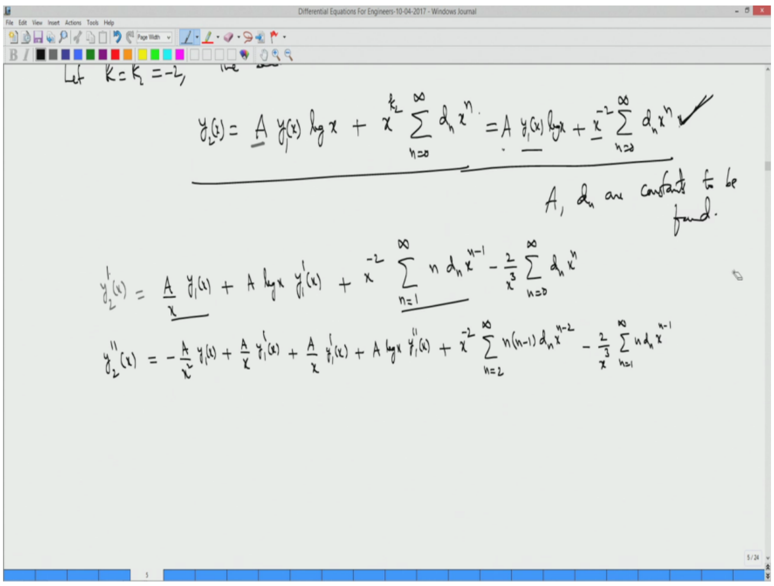

And then now you differentiate the last term so that will give you minus 2 by x cube. If you differentiate this series n is from 1 to infinity, n d n x power n minus 1. And then you differentiate 2 by x cube that will give me, so derivative of 1 by x cube, 1 by x cube derivative is actually x power 5, right? So you have 3 x power 4 I guess. You get x power 4, right? So you have this is x power 6, x is minus so we have 3 x square.

So minus 3 by x power 4, okay. That is how minus 3 into 2, so we have 6 by x power 4. So we have minus minus plus, so it is going to be plus and we have this n is from 0 to infinity, d n x power n. So this is what is your y double dash.

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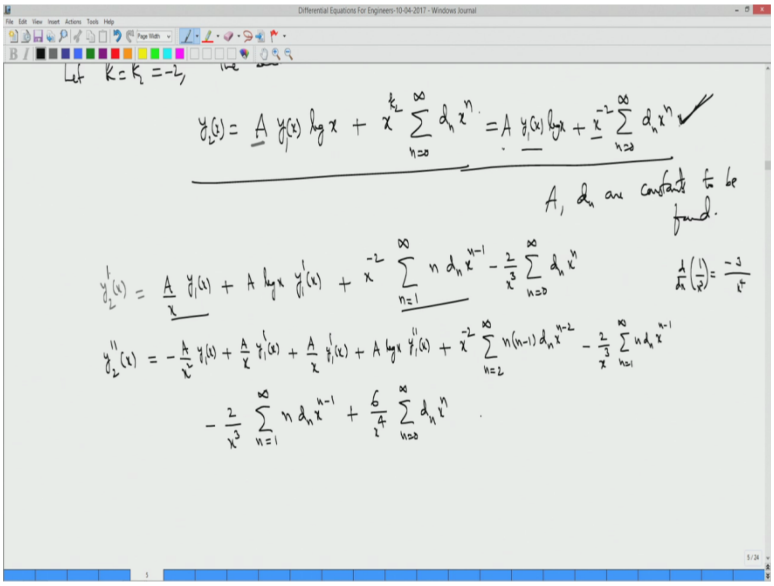

Now substitute this y 2 here, y 2 dash and y 2 double dash substitute into the equation. Given equation is if you substitute so you have x square into y double dash first of all. First term is x square into y double dash . So if you multiply you need this x square. So I multiply here itself x square y double dash will be you remove x square here and you have x here, x goes and you have x here, A by x, so x into A and you have x square and this goes and here you have simply x, okay. And then here again x and here x square.

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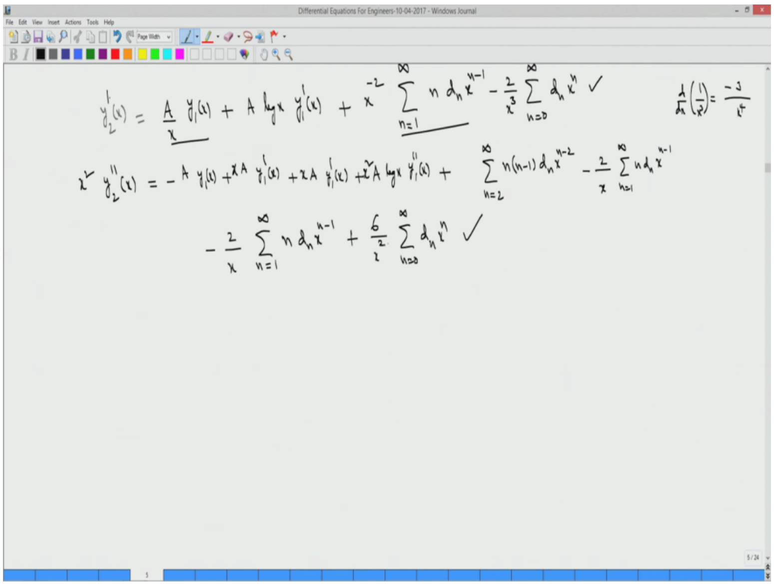

This is how you will get x square y double dash. So second term is 2 x plus x square y dash. So that is my second term of the given differential equation. So this if you do so for this you need x y dash, x y 2 dash and x square y 2 dash. So so let me rewrite again.

So you have x y 2 dash of x is, from y 2 dash you can get it as A y 1 A x log x y 1 dash of x plus x power minus 1, sigma n is from 1 to infinity, n d n x power n minus 1, minus 2 by x square, sigma n is from 0 to infinity, d n x power n. And this is by x y 2 y dash. So you need 2 x y dash, so multiply 2 so it will be 4. So this is what you have. So what you need is x square y 2 dash.

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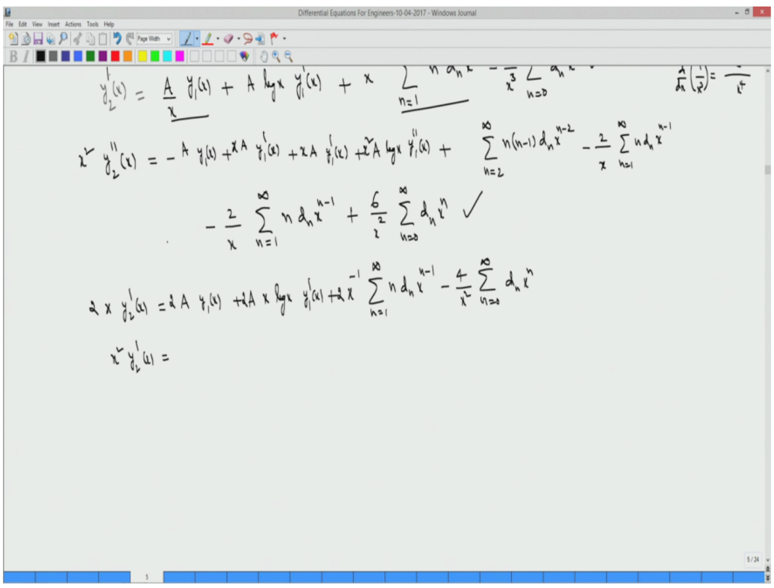

Now you multiply x square to this y 2 dash. So that will give you A x y 1 x plus A x square log x y 1 dash x plus you get simply the series n d n x power n minus 1 minus 2 by x, sigma n is from 0 to infinity, d n x power n. So these are the three things you have to substitute. So these three and then for y into the equation. So given equation is x square y double dash plus x.

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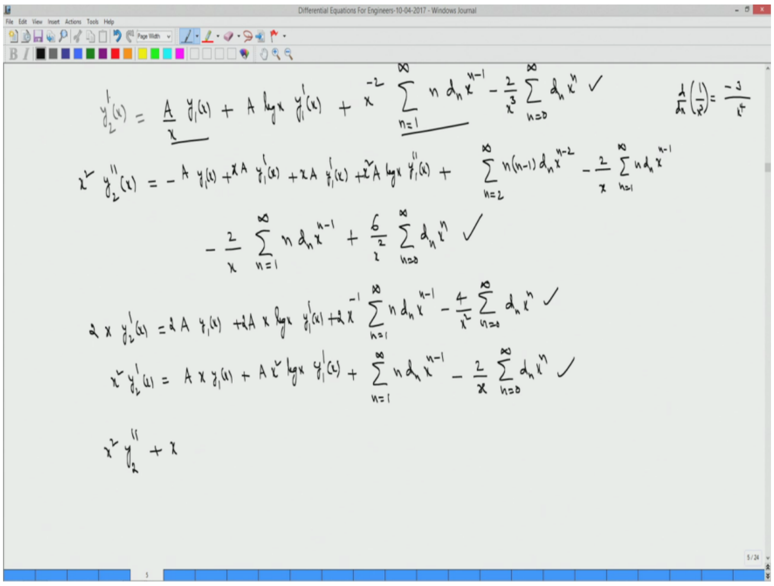

So if you want solution y 2 so x 2 y double dash, x into 2 plus x square is 2 x plus x square into y 2 dash minus 2 times y 2 has to be 0. So this is the given equation, okay. The given equation is substitute y 2 dash and y 2 double dash and y 2, okay, from all this. So x square y double dash if you see you can take this one and bring it here. So you substitute here the whole thing if you replace so what you get? So you get minus A (())(10:29) so y 1 x plus x here y 1 dash.

So you have 2 x A y 1 dash of x. So second and third term together is that plus A x square log x, okay. So A x square log x y 1 double dash of x plus this is as it is, you have n is from 0 to infinity, n into n minus 1 d n x power n minus 2 minus 2 by x, n is from 1 to infinity, n d n x power n minus 1. So this is my x square y 2 double dash.

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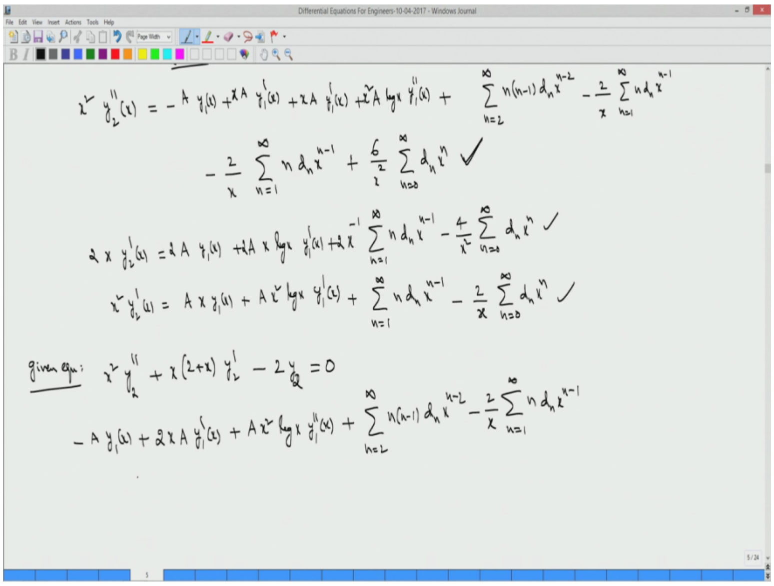

Now you add these two. So this 2 x y 2 dash that also if you add so basically I am adding these two. So 2 x y 2 dash plus x square if you add it so you can see that 2 A x log x if there are some common terms you can add it, okay. So if it is not, maybe you have to rewrite everything as it is. So let me write. So you have 2 A y 1 x plus 2 A x log x y 1 dash x, y 1 is the first series solution which we already obtained, okay, 2 times x inverse, n is 1 to infinity, n d n x power n minus 1 minus 4 by x square, n is from 0 to infinity, d n x power n.

That is my 2 A x y 2 dash plus x square y dash that also you add. A x y 1 x, A x square log x y 1 dash x plus sigma, n is from 1 to infinity, n d n x power minus 1 minus 2 by x, n is from 0 to infinity, d n x power n. And then minus 2 into y 2. So what is your y 2? Y 2 is the form which you chosen, A y 1 log x, okay. So 2 into A y 1 x log x minus 2 x power minus 2 sigma, n is from 0 to infinity, d n x power n. That is what is the form, okay, this one. So n is from 0 to infinity. So d n this is equal to minus 2 times of that is 0.

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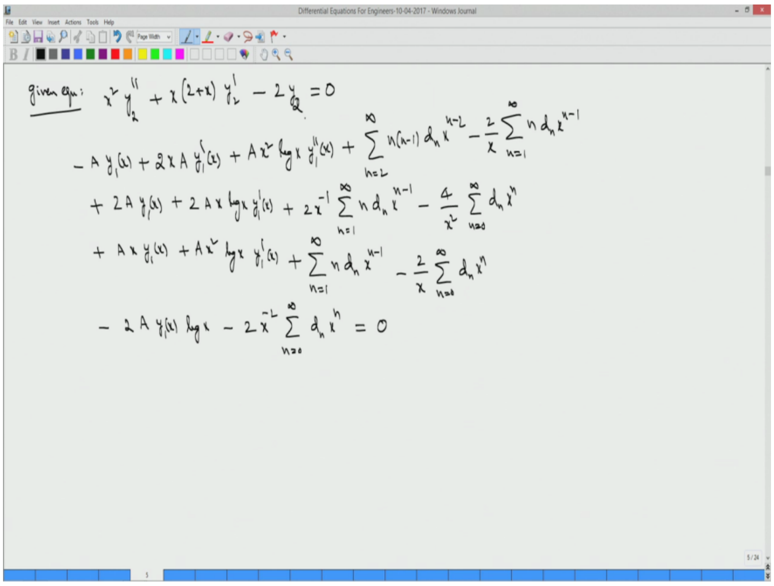

So this is what is the reaction. Once you substitute y 2, y 2 dash, y 2 double dash into this into this given equation this is what you got. You can observe the coefficient of log x. What is the coefficient of log x? If you see the coefficient of log x, A log x for example. A log x is the coefficient, you take A log x out, you see the coefficient what you have. So x square y 1 double dash of x, okay. So this term take n. Where else you have? So you have here.

So and this will give me 2 x y 1 dash of x and then here this will give me plus x square y 1 dash of x. (())(14:11) So this is going to be minus 2 y 1 of x. But now that this is 0 so this is actually the exact equation. This together if you see this together, this is the actual equation. So original equation is this is actually given equation satisfied by y 1. So we know that this is actually equal to 0, okay. So because of that these three terms will be 0, these four terms, okay. That is what we have done now.

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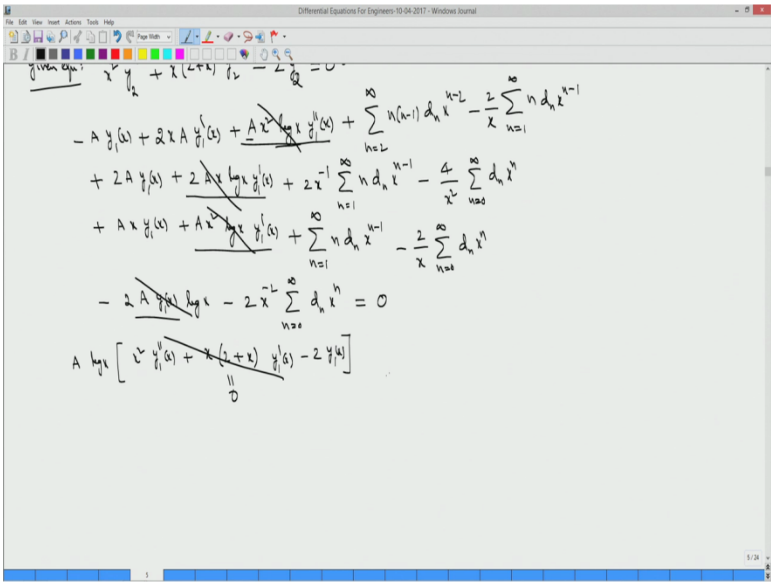

Now what else is left? what is left here is minus A y 1, okay, minus A y 1 so you can write minus A y 1 of x, then 2 x A y 1 dash. Is there any other thing? There is nothing so you have to write it. So 2 x A y 1 dash of x and then 2 A y 1, 2 A y 1 and minus A y 1, so that is going to be plus, okay. So this is together will give me only A y 1. So that is why I have written here and you have written this part here.

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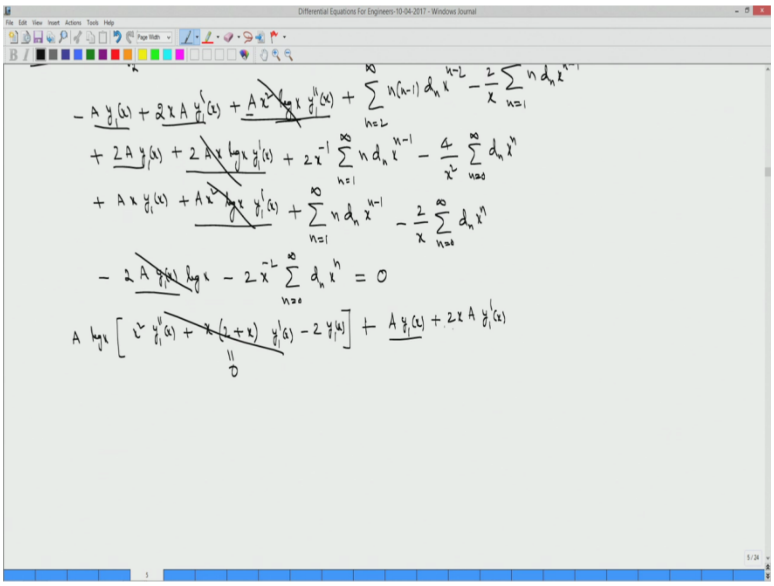

Now what is left is A x y 1 so that this together A x y 1 so you have A x y 1 of x, okay. In this one now you write all the remaining series, okay. So what is the remaining series? So take all this 6 terms, okay, 7 terms. So if you write this n is from 2 to infinity, n into n minus 1 d n x power n minus 2 minus 2 by x, n is from 1 to infinity, n d n x power n minus 1 plus 2 by x, n is from 1 to infinity, n d n x power n minus 1. So this is actually this is also cancelled, okay. So 2 by x n d n x power n minus 1, n is from 1 to infinity so this and this will go.

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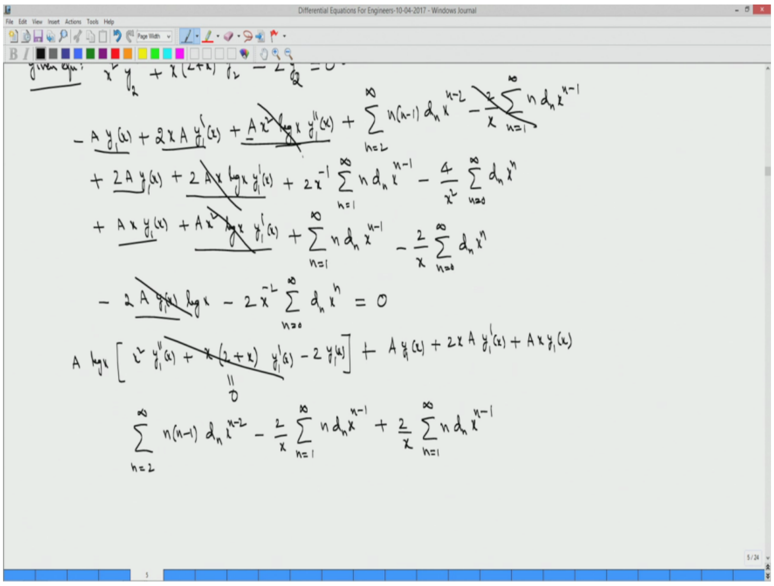

So I do not have to write, okay. So this I have written these two gone now these two. So if you look at this you have minus 4 by x square sigma, n is from 0 to infinity, d n x power n plus sigma, n is from 1 to infinity, n d n x power n minus 1 minus 2 by x, n is from 0 to infinity, n d n x power n. And then here 2 by x square d n x power n, so minus 2 by x square minus that is going to be 6. So these two together. So this one and this one you put it together that is going to be minus 6 I have written here and these two as it is here. So that is equal to 0.

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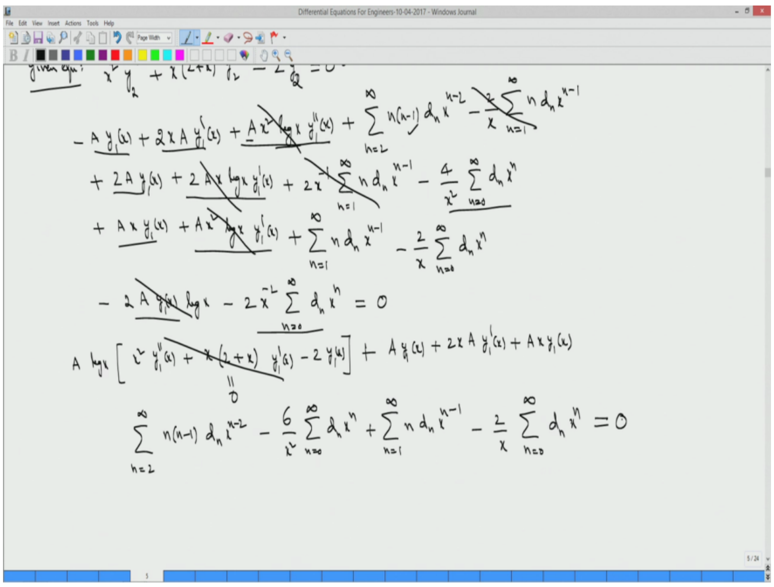

So this is what we have. So anyways this is gone so what is left is only this part. So this is the equation. Now we know what is y 1. So y 1 is a series we know, y 1 dash . So you can rewrite and substitute your y 1 here as a series so remaining parts are all series. You substitute here and you will be able to find, once you get a series you replace y 1 as a series. You will see the once y 1 you replace with the series which you know as a first solution and you will get all the terms one, two, three, four, five, six, seven terms.

Among the seven terms in the equations you have last four terms or series. First three terms if you replace y 1 as series, y 1 is a series solutions. So you calculate y 1 dash, substitute in the first three terms you will see that seven series together equal to 0. So now that you expand and make the coefficients of each of this coefficients of x power n 0 that will determine all the unknown coefficients so which are A and d ns, okay.

So we can see that. So what is your y 1? For that you need y 1. So what is the y 1 you got? so this is what is your y 1. So y 1 is x minus x power x square by 4. So y 1 is x minus x square by 4, x cube y 4 5, like that, okay. So we will see exactly. So we will calculate that y 1. So what is your y 1? Y 1 is y 1 x since y 1 x is given as x minus x square by 4 plus x cube by 4 5 minus x power 4 by 4 5 6 and then plus x power 5 by 4 5 6 7 and so on, it will go on like this. This is your series solution which you got from the first solution, okay.

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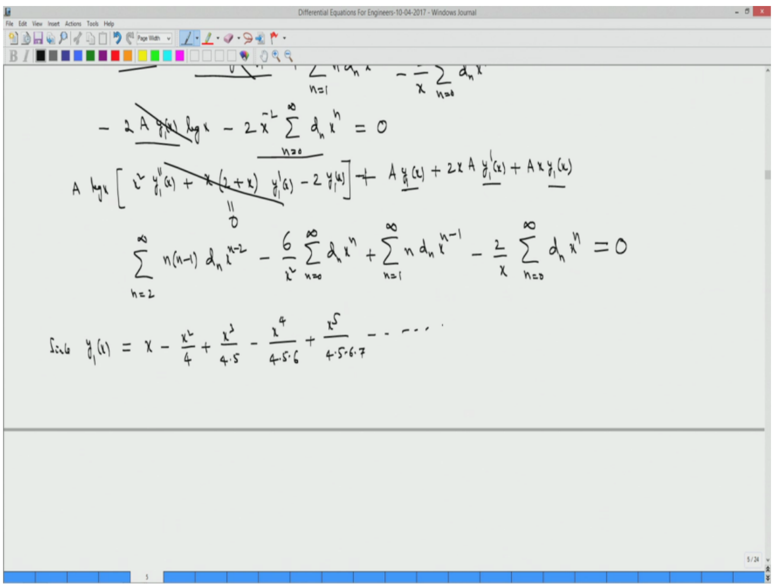

So this is this one. So once you get this y 1 so you need to substitute here. So for that you need y 1 dash so you calculate y 1 dash of x. So that will give me 1 minus x by 2 minus 3 x square by 4 5 minus 4 x cube by 5 6, 4 goes and then plus 5 x power 4, 4 5 6 7 and so on, okay. these two if you try to substitute that into equation. So if you substitute A times A x minus A x square by 4, A x cube by 4 5 minus A x power by 4 5 6 plus A x power 5 by 4 5 6 7 and so on, okay. So this is my first term A into y 1 x.

This is your A into y 1 x that I substituted now. Now plus 2 A x into y dash. So 2 A x into y 1 dash is 2 A x. So just multiplying 2 A x to your y 1 dash. 2 A x minus 2 A x square by 2 minus 6 A x cube by 4 5, okay. 2 A x is going to be x cube so minus so this is plus, right, this is plus. It should differentiate so one plus minus plus minus.

So you have this as plus and now minus 2 A x power 4 by 5 6 plus 2 is 10, so 10 A x power 5 2 A x divided by 4 5 6 7 and so on. So that is your second term. Third term is A x y 1 dash x. So you multiplying A x 2 y 1. So that will give me A x square minus A x cube by 4 plus A x power 4 by 4 5, A x power 5 by 4 5 6 plus A x power 6 by 4 5 6 7 and so on, okay. So these are the three terms. Now I wrote it as series.

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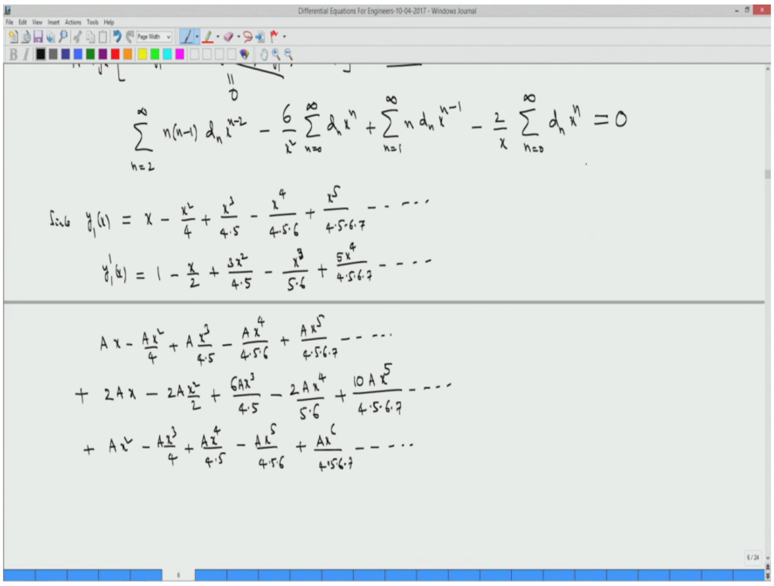

This also I can expand. So even this series terms also you can now expand and write it as a full series. So if you write that so you will get this series if you write n equal to 2 so you will get 2 times, n equal to 2 if you put 2 d 2 that is 0 plus n equal to 3, 6 d 3 x plus n equal to 4, 12 d 4 x square. So I will write four terms, okay. 5 n equal to 5 that will give me 20 d 5 x cube and so on.

If required you can add it later here, okay. So that is my this series. So next one is minus 6 by x square, n equal so d is d 0, n equal to 0, okay, and then minus 6 by x square, n equal to 1 will give me d 1 that is going to be x. So that x this x goes so 6 by x. Again minus 6 by x square. So we have d 2. This x square and d 2 x square will go so will give me minus 6 d 2. And then minus 6 x d 3 minus 6 x square d 4 and so on like that you get the second one.

Third term you can see that n equal to 1 so d 1 plus n equal to 2, 2 d 2 x plus 3 d 3 x square plus 4 d 4 x cube and so on. Finally the last term minus 2 by x d 0 minus 2 by x d 1 x. So that x x goes, okay, so you have minus 2 d 1. So minus 2 by x d 2 x square.

So that is going to be x minus 2 x square so minus 2 d 3 x square minus 2 d 4 x cube and so on, okay. All will be minus. And n equal to 4 you have minus 2 by x d 4 x 4, so x x goes x cube and so on. So this is what I replaced all the series I expanded them. This is actually equal to 0.

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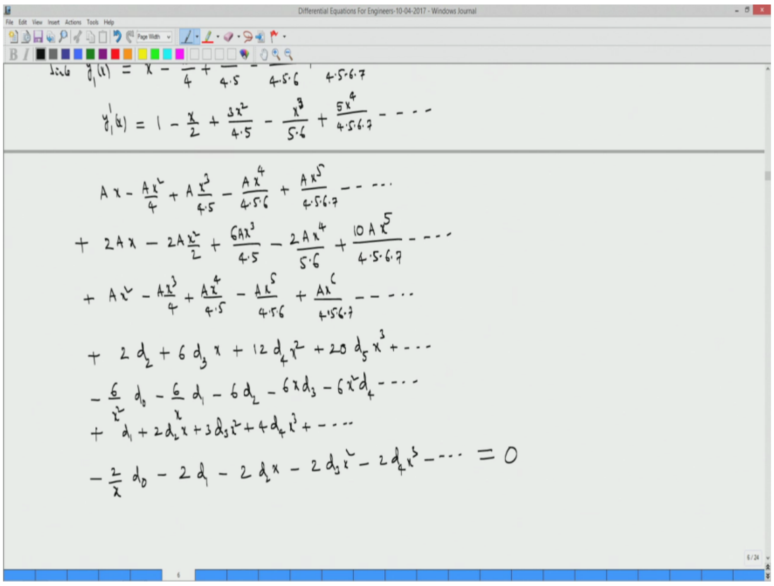

So since y 1 and y 2 are solutions, y 1 is a solution and you already know that y 2 is also going to be solution. These are absolutely and uniformly continuous in the domain where it is defined, okay. So this I have given without proof is our uniformly convergent. So that you can add term by term and so you can put it together, okay. So that is why I am able to add all these series together. Now simply get the coefficient of lowest terms, okay.

So what is the lowest coefficient here? So first series I think only starts from x, second series also starts from x, third is also like that. The fourth one is also like that. So fifth one is minus 6 by x square. So you have x power minus 2 coefficient, pickup the coefficient, okay. So coefficient of x make it 0 . What is the reason? So I have coefficient of x power so C minus 2 x minus 2, C minus 2 or rather C minus 2 I put it here, okay, x minus 1 and so on.

So C 0 x plus C 1 x plus C 2 x square, like that I am going to put equal to 0. So these are C 1, C minus 1, C 0, C 1, C 2, these are all the coefficients of x power minus 2 x power minus 1 so on.

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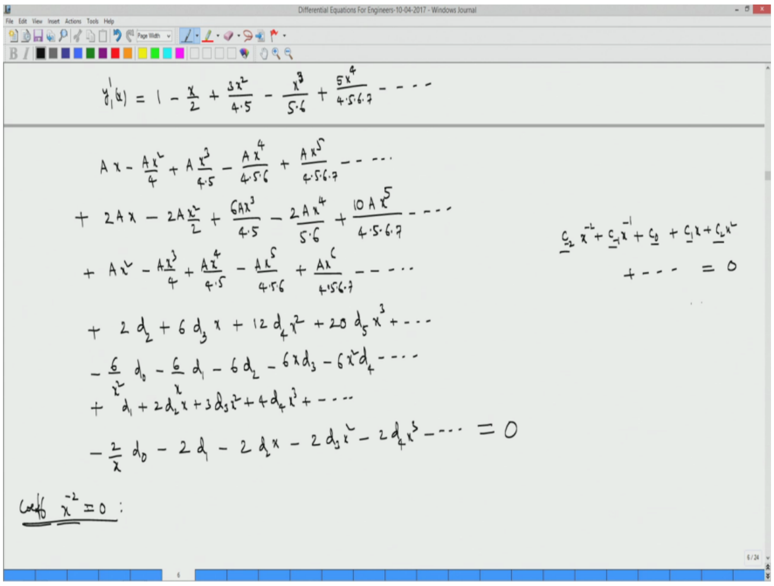

So like this if you have a series equal to 0, each of these coefficients have to be 0. That is why I am making it equal to 0. Coefficient of x 2 that is here C minus 2 which is 0 which I am calling simply whatever, okay. I am saying what this C minus 2 is. That I have only minus 6 d 0, okay, is equal to 0. So there is no such thing that is the coefficient of x power minus 2 that has to be 0. So it immediately gives me d 0 is 0, okay.

And similarly coefficient of x power minus 1 make it 0. So this will give me what is that you have so 1 by x minus 6 d 1. What is other one? So minus 2 d 0 equal to 0. This will give me I already know that d 0 is 0 that will give me d 1 is also 0, okay. And then make the coefficient of 0 is 0. Coefficient of x power 0 is 0. That will what will give? So what you get constant term.

There is the constant term in each of the series. In all these seven series terms. So you have 2 d 2 minus 6 d 2, okay. So I have here 6 d 2, 2 d 2, 6 d 2, okay. So I may not write anything here so plus d 1. So d 1 which is already 0 equal to 0. So d 1 is already 0 so d 2 is again minus 4 d 2 is 0. That will also going to make it d 2 0.

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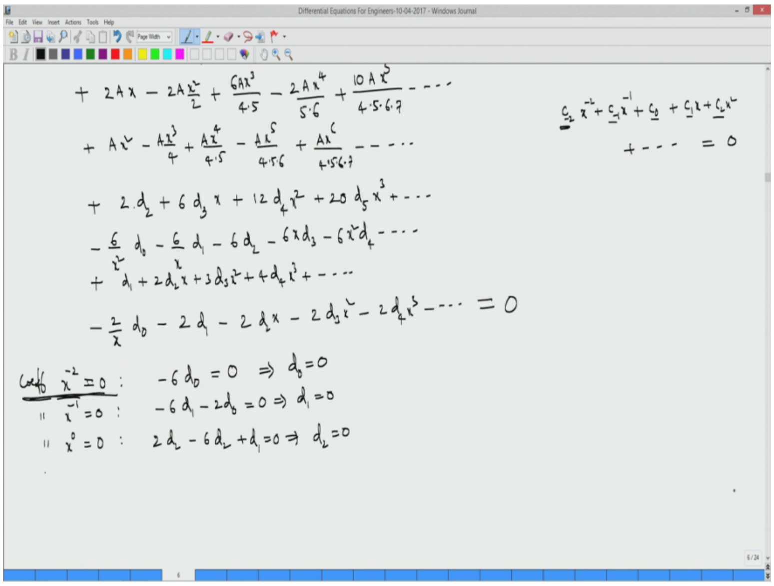

So now look at the coefficient of x 1 equal to 0. This will give me, what is the coefficient of 1 that is A, A plus 2 A so that is going to be 3 A. 3 A and then you have 6 d 3 minus 6 d 3. So it is going to be 0. So that is why I do not have anything and here plus 2 d 2 that is also 0. So that will give me A equal to 0. So we are seeing all these 0 solution something is wrong. So y 2 is satisfying this so when we substitute this y 2 double dash and y 2 dash and y 2 into this equation what we had missed is these two terms.

So when you have this x square y 2 double dash I added only this part so this remaining you can add here now. So you have minus 2 by x, n is from 1 to infinity, n d n x power n minus 1 plus 6 by x square, n is from 0 to infinity, d n x power n. So this is my x square y 2 double dash and then 2 x y 2 dash is this four terms and then x square y 2 dash is this four terms. And then minus 2 y is these two terms is 0.

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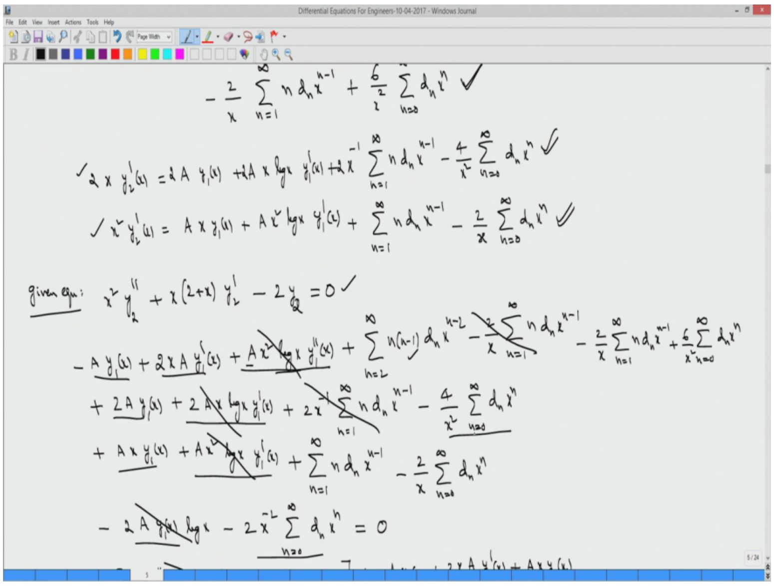

So as usual so you can remove this since y 1 is satisfying these four terms whatever I cancelled is actually giving me because satisfying the this is the say first solution satisfies the given equation. So that will make it 0. So what you have is this and I have two more. So these two terms are required. So (ex) these are extra. So it is not just this. So I have to add minus 2 by x, n is from 1 to infinity, n d n x power n minus 1 plus 6 by x square, n is from 0 to infinity, d n x power n which is equal to 0. So now this is what is equal to 0.

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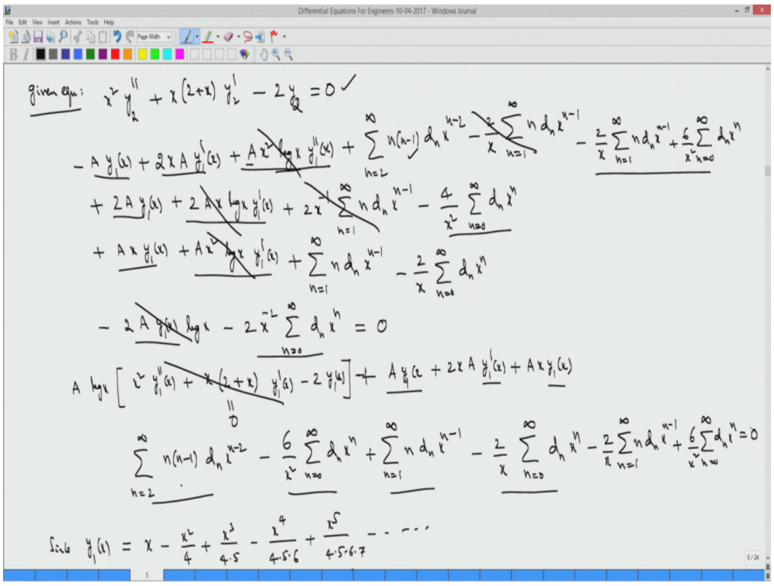

So when you are adding this equal to 0 so let me add these two terms which I missed earlier. So this is one so the first term so what you have is so the last time which we missed this if you write it also you have minus 2 by x. So let me see if they are common to anywhere. So we have they are not common. So we have 6 d n square d n. So this will go, these two terms will go.

So I do not have to write the second term, okay. So you have one, two, three, four, fifth, fifth series you do not have to write. So one, two, three, four, five so this part is actually not required because they get cancelled with the last one, okay.

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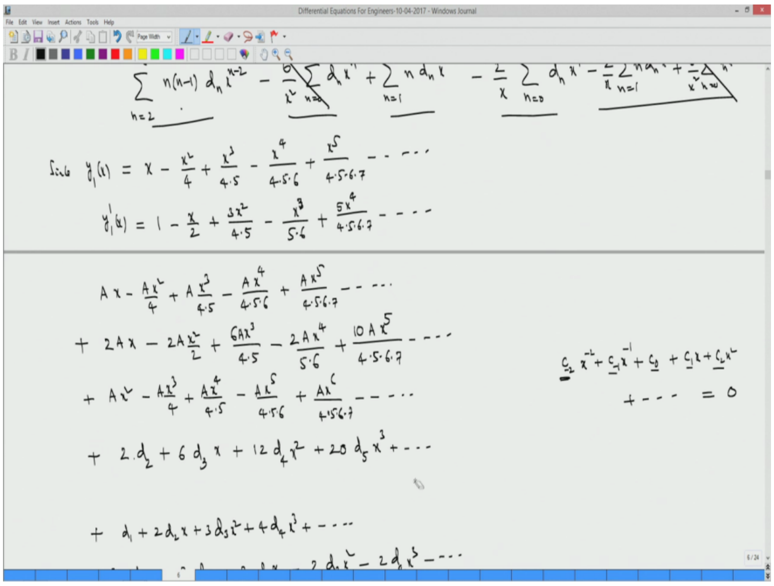

So this series is getting cancelled with the last one so only you have to add this term so which I missed, minus 2 by x n d n. So minus 2 by x, okay, so you have minus 2 by x, n equal to 1 that will give me d 1. So 2 by x d 1 minus 2 by x and then you have n equal to 2, you get 2 d 2 x minus 2 by x 3 d 3 x square minus 2 by x 4 d 4 x cube and so on. So that is what is equal to 0. So I have a now one, two, three, four, five, six, seven, seven series which are 0. So you have three terms here four, five, six, seven, okay. These two series gets cancelled.

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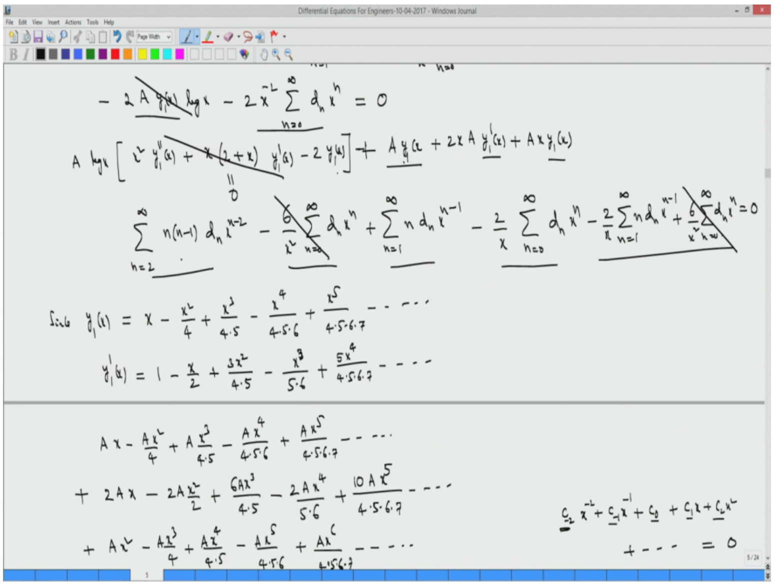

So now make the coefficients of x power minus 2 0 so what you will get coefficient of x power minus 2 is 0 will give me d 0 is 0, okay. Let me see what you have x power plus what you will get, this gets cancelled so you have x power A y 1. So y 1 2 x, x power this so you have there is no x power minus 2 term, right? So everything is powers of maximum x power minus 1. So this here x x goes, okay. Minus 2 by x so this is going to be 4 d 2.

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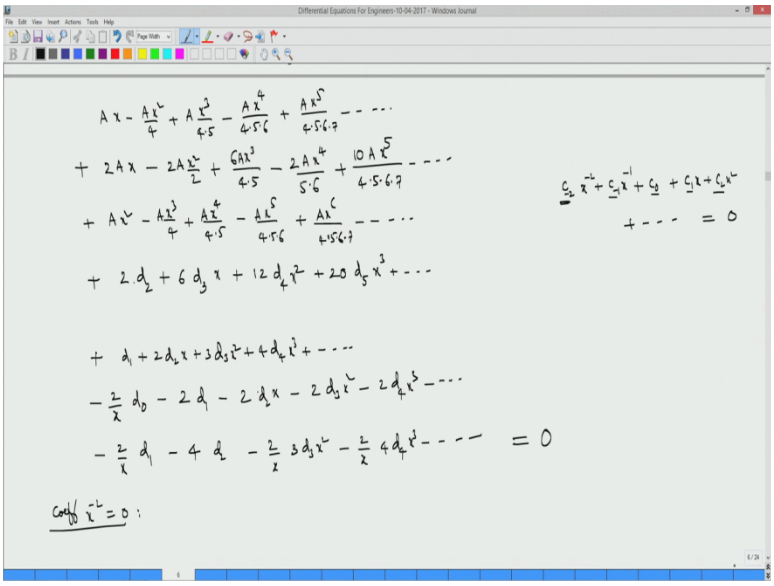

Similarly here if you remove this x this is going to be x. If you remove this, this is going to be x square and so on. So you have 1 y x terms. So you have maximum lowest term is x power C minus 1. So this term is not there. So this is how the series is going to be. So all these seven series if you add up it is going to be starting from x power minus 1 that is the first time and then so on x power 0, next term and so on. So you have x power minus 1 coefficient 0 then what you get is, so what is the coefficient of this?

You get minus d 0 minus d 1 which has to be 0. So that will give me d 0 equal to minus d 1, okay. So d 0 equal to minus d 1 and then coefficient of x is 0. So this will give me 2 d 2. So constant that is 2 d 2 plus d 1 and then minus 2 d 1 minus 4 d 2 which is equal to 0. So this will give me if you add it, it is going to be minus 2 d 2 minus d 1 which is equal to 0 so you can write d 2 as minus 1 by 2 d 1, okay.

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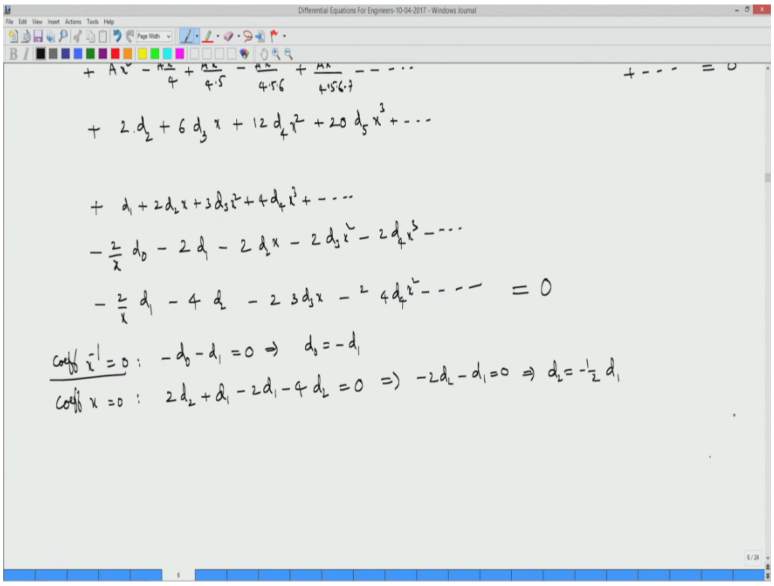

And now make the coefficient of x power 0 here so coefficient of x power 1 that is coefficient of x is 0. That is going to be, what is the coefficient of x? A plus 2 A, so first two will give me A plus 2 is 3 A, okay. So these are the two terms and then here you get 6 d 3, 2 d 2 and then minus 2 d 2 and then here again minus 6 d 3 equal to 0. So that will give me 6 d 3 6 d 3 goes, 2 d 2 2 d 2 goes, 3 A equal to 0 that will give me A equal to 0.

So immediately once A is 0 the coefficient of log x for your second solution A is 0. So you will not have log term, okay. So only x power minus 2, d 0 onwards it will be there, okay. So d 0 is arbitrary, okay, so d 0 is 0. Not able to get it, right? So d 0 is given in terms of d 2 so that you can put it like d 1 equal to so this is actually like d 1 is minus d 0.

So that will give me what is d 1? D 1 is minus d 0 means this is you can write in terms of, so it is going to be half d 2, okay. So d 2 is minus half d 1, d 1 is minus d 0. So I will make it this one.

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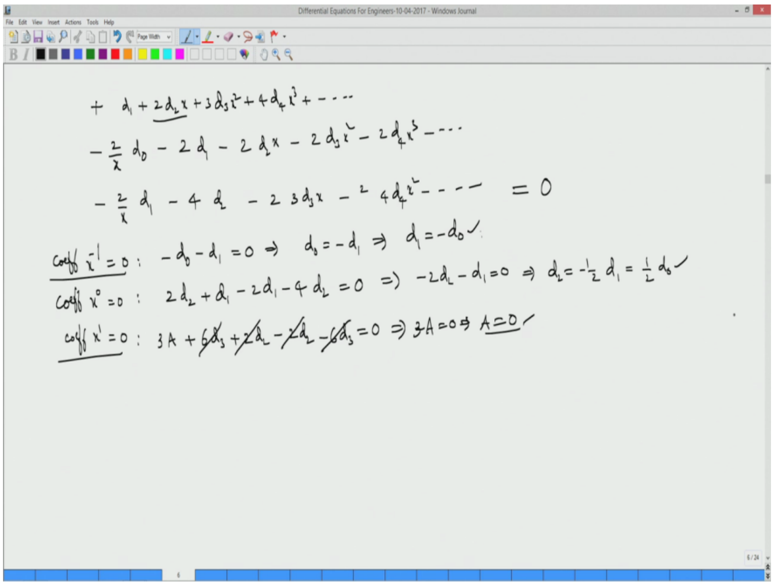

So you could get d 1 d 2 in terms of d 0, d 0 is arbitrary. So like that you can go on. I can calculate one more and make it equal to 0. So coefficient of x square if you make it equal to 0 if you do it meticulously (don) it should not go wrong. You will get everything properly so you get minus A by 4. A is anyway found coefficient of, so I do not have to whenever A is there now I found A is 0 I can simply remove this and I need not construct because A is 0, okay. So A is 0 I need not worry now anymore here, okay.

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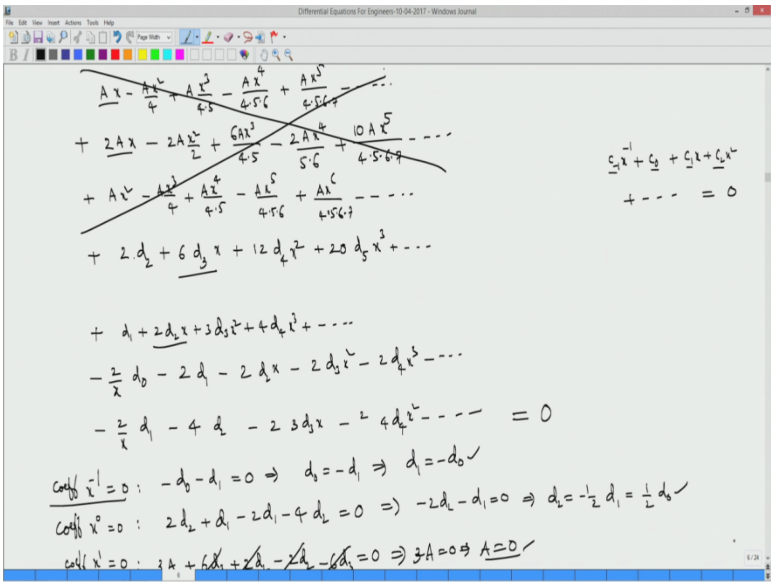

So I have to take coefficient of x square. Coefficient of x square is 12 d 4 and then plus 3 d 3 minus 2 d 3 from here on here minus 8 d 4 equal to 0. So that will give me 12 minus 8, 4 d 4 and plus d 3 equal to 0. So that will give me d 4 equal to minus d 3 by 4, okay. So what is d

3? D 3 is arbitrary now, okay, d 3 and d 0 both are arbitrary. That is what you are seeing, okay. I could not get d 3. So let me calculate one more term so coefficient of x cube equal to 0 if you make it what happens?

So sometimes you will get all the coefficients except one or if you look at this form of the second solution, okay, where is your second solution? So this is in this form. Either I may get all these constants A and d ns so that you know exactly what is your y 2 or rather A and d ns, you will be able to find all these constants in terms of one constant, okay. So that means you calculate all of them except one.

But you write everything else in terms of one constant that means so that the constant comes out and some power series, okay. That way you will get it so once you take whatever the arbitrary constant you take it as one. So once you take that as arbitrary constant one that will be a solution. That way you can get or if you sometimes you may end up getting some solutions in terms of one arbitrary constant, some solution in terms of some other arbitrary constant.

For example in this case when A is 0 that is what I am getting, okay. So if you see this A got 0 and d 1 d 2 are given in terms of d 0, d 4 in terms of d 3 because d 3 I could not find that means d 3 is arbitrary. Now if you make it coefficient of x cube is 0 from these three. So if you look at this 20 d 5, okay, so then what you have?

So it is going to be x cube is 4 d 4 and then 4 is minus 2 d 4 and what I get here so this is going to be coefficient of, so what you get is one more term is actually here minus 2 by x, 5 d 5 x cube, okay, 5 d 5 x power 4. So that is going to be x cube with removing this.

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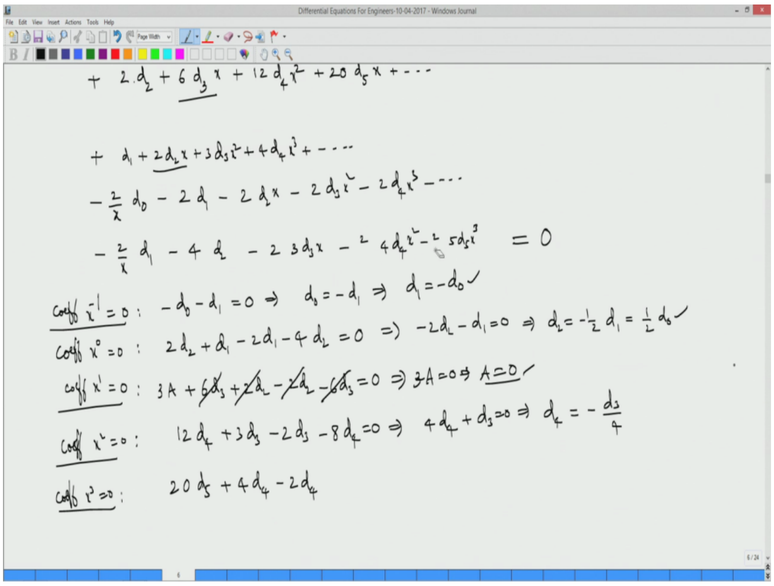

So you have 2 5 and so on. So you have minus 10 d 5, okay, that is what you get, equal to 0. So this will give me 10 d 5 plus 2 d 4 equal to 0. So this will give me d 5 as 1 by 5 minus 1 by 5 d 4. And d 4 is given in terms of minus d 3. So it is going to be plus d 3. So you have d 3 by 4 5, okay, and so on you will get it. So like this you are getting. So what you are getting finally is y 2 of x which is given as A into log x. A y 1 into log x, A is 0, A I found to be 0.

So that is going to be that term will not be there and you have x power minus 2. So d 0 is I did not find. So d 0 I did not find. So d 0 plus d 1, d 1 is minus d 0 x, okay. What is d 2? Plus half d 0 that is my d 2 x square, okay. These are all given in terms of d 0, plus remaining are d 3, I could not find d 3, d 3 x cube and d 4 is minus d 3 by 4 x power 4. What is d 5? D 5 is d 3 by 4 5 x power 5 and so on. So you can expect next one will be minus d 3 by 4 5 6 x power 6, okay, like that you can expect.

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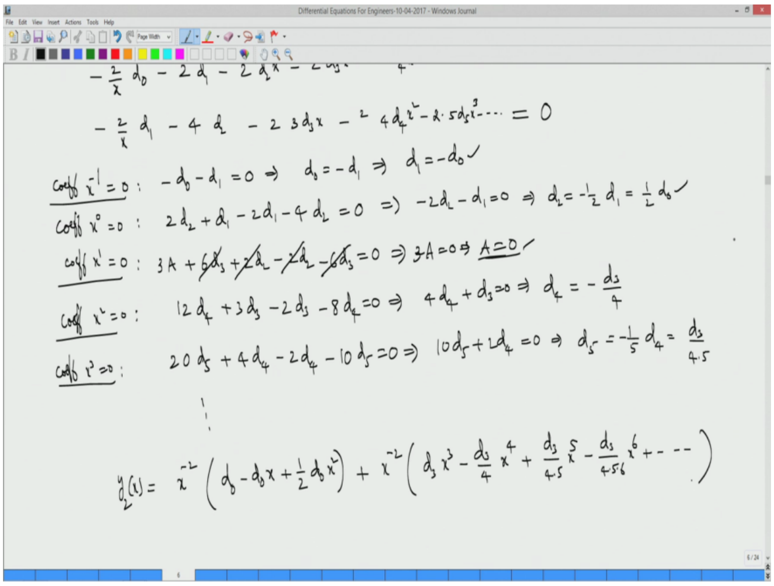

So what you got finally this is equal to d 0 times, so 1 by x square, take this x power minus 2 inside you get minus 1 by x plus 1 by 2. So this is your solution. So this is your d 0 into this plus now if you do this, this is going to be d 3 comes out as constant. This is going to be x minus x square. If you take this x square by 4 and then you have plus x cube by 4 5, okay, and then minus x power 4 by 4 5 6 and so on. So what is this? This you already know this is exactly your y 1 of x, okay.

So sometimes you may get your second solution in terms of something as your y 2 plus y 1, okay, like here. So I do not need this y 1 because I already got my y 1, okay. So this itself is second linearly independent solutions first of all. If you do not want this form and you already have y 1 you take whatever, if y 1 is part of solution into your second solution y 2 you can remove it, you can split it and remove it. So this is my y 1.

I do not want that so you take d 3 as 0 and d 0 is arbitrary that you take it as 1 so that you get your second solution y 2 as 1 by x square minus 1 by x plus half. So this is your second linearly independent solution. This is also linearly independent solutions like y 1 x, so I got some y 2 of x, some function here so this is your say your y 2 x plus some arbitrary constant d 3 times y 1 of x. This is also solution. These are two linearly independent solutions because y 1 and y 2 are two linearly independent solutions, okay.

You see that this part and this part are two linearly independent solutions. This already know so you can remove it. So this is your secondly linearly independent solution or here itself you can say this whole thing together you can consider d 0 equal to 1 and d 3 as also 1. You can also say that this is a series solution which is also linearly independent solution, okay.

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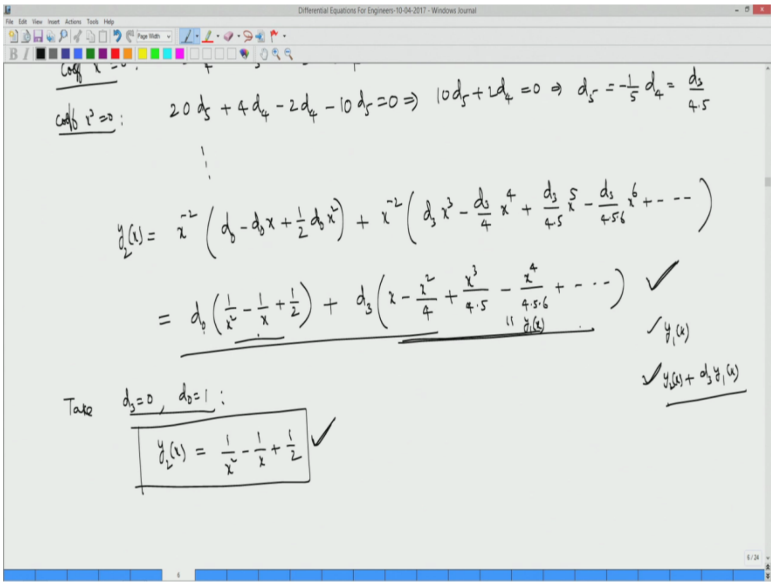

So if you take d 3 as 1 this is going to be like this. It is a series solution I have some only three terms as a solution plus already known solution together. This is your y 1 and y 1 plus y 2 as two linearly independent solutions. So you can also say these are y 1 x and y 2 x. These are two linearly independent solutions. Here I removed this y 1, okay, that is why it is only y 1 and y 2. If you do not remove your y 1 so it is going to be both are series solutions. That is how you get, okay.

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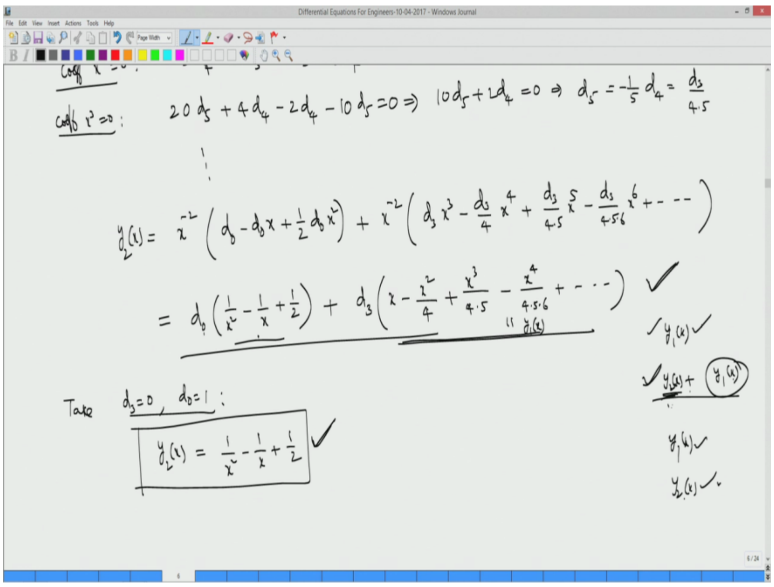

So this is how you can solve in second order differential equation and the indicial equation has two roots and the difference is integer, okay, non zero integer. So here in this particular case you have seen first bigger root is 1. That will give me the series solution. Second solution we have chosen in a certain form and the difference is a non zero non integer, okay, so difference between the roots k 1 minus k 2, k 1 is a bigger root, okay.

So when k 1 minus k 2 is integer, sorry, it is a non zero integer, okay. This is the case for non zero integer so the bigger root will give me one series solution. Second solution in certain form when the difference between these two roots k 1 minus k 2 is non zero integer which is k 1 is 1 and k 2 is minus 2 so the difference is 3 which is integer which is non zero. This case we have chosen special form for y 2 and substitute into the equation and get the unknowns.

So what we find is the second solution is actually simpler form 1 by x square minus 1 by x plus half, okay. So this is how you can get any second order equation when the indicial equation has roots and root difference is non zero integer, okay. So we can just meticulously follow this same method. You can find any (solu) equation in this particular case. Now we look at the (seve) another case. Now we have only one more case is left, okay.

That we will see in the next video that we consider an equation whose when you find the indicial roots, roots are same. So that means the difference between the roots is 0 but integer, okay. So that is away from the case 1 and case 2. So this is the case 3. You have actually equal roots, one will always give me first bigger root. Let us say bigger roots both are same.

So bigger root will give me one series solution and you have to find second linearly independent solution as special form which I explained earlier as a case 3. So now will demonstrate with an example in the next video.