Differential Equations for Engineers.
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Lecture-1.
Introduction to Ordinary Differential Equations (ODE).

In this introductory video we will have an overview of the contents of the course. So we will be teaching differential equations, will be learning, you will be learning differential equations basically, both the ordinary differential equations and partial differential questions. Okay.

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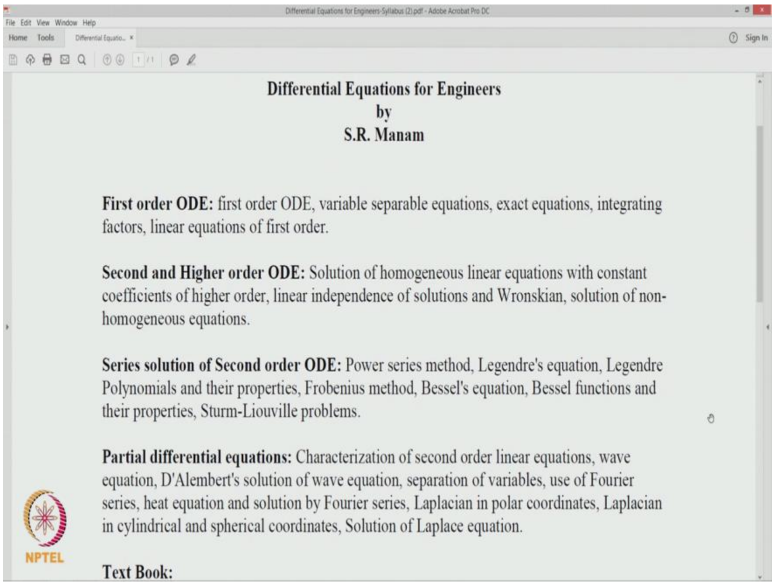

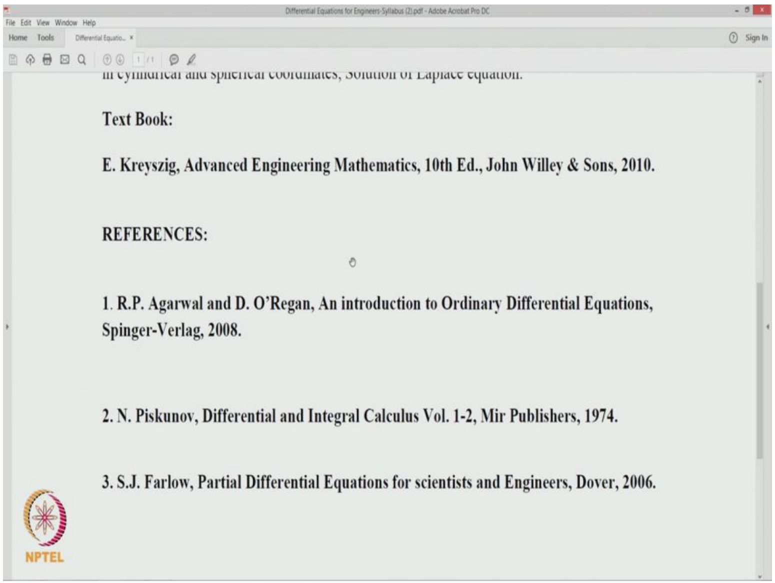

So this is the syllabus, so basically what we consider is the syllabus is divided into few sections, a few modules. So if you look at, in the 1st module you can have first-order ordinary differential equations. We will study these 1^{st} order ordinary differential equations and its methods, both linear and non-linear. Most of the nonlinear equations cannot be solved and vertically. So those are the questions for which you can solve completely, we will have those solutions or we will look at most of the equations that can be solved in a closed form will be tested this 1^{st} order ODE.

So we will see all the methods, what are the available methods that are available for the first-order ODE, we will see in the 1^{st} module. In the 2^{nd} model then we will move onto, 2^{nd} and higher-order ordinary differential equations. So in this mostly, initially we look at the some form of some nonlinear equations of second-order, those you can directly integrate. We will see only one or 2 methods is possible, later on we will move on to linear 2^{nd} order ordinary differential equations at a later date.

And then we will study its properties and then how to find its solutions, both homogenous and non-homogeneous equations, okay. This is what we see in the 2^{nd} model. Even for the 2^{nd} order and higher-order equations, not all equations can be solved in a closed form. So what we do is, when there are linear equations with variable coefficients, if the coefficients of the linear second-order higher-order equations are functions of x, there are not constants, then you can expect power series solutions.

So we will introduce power series solutions and special equations in physics which you study as a special case of this power series solutions, okay, that will study in the 3^{rd} module. And we develop theory of Sturm Liouville based on the properties of second-order equations, 2^{nd} order linear equations and its solutions. That is basically finding Eigen values and Eigen functions corresponding to the differential operator and then its properties. Based on using these properties, we will make use of these properties and we will solve a linear partial differential equation, equations in simpler domains such as rectangular or circular or elliptical domains.

Simpler domains, you can solve linear partial differential equations by simpler techniques called separation of variables. We make use of this Sturm Liouville theory of ODEs, for ODEs and use that to solve partial differential equations, that is what you do in the last module. So this is what briefly what we are going to study or you going to learn in this course on differential equations. Okay. So Textbooks that we follow are basically Kreyszig, I may not follow exactly contents of Kreyszig but you may find most of the material in this book.

And for reference books you can look into this Aggarwal and O'Regan, introduction to ODE and Piskunov for part of the certain calculus, also you can look into any partial differential equation, for example Tang, there is a book called Tang, I did not show here, you can also look into this book, this is also okay, partial differential equations for scientists and engineers, Dover publications, it is Farlow, SJ Farlow, you can look into this book. There is a book called Tang, mathematical methods for scientists and engineers by Tang. So we have 3 volumes, I can 3rd volume you can find partial differential equations, okay, I have not shown here.

So like there are many books you can go through, so much of the contents, will be available in those books. So in this introductory lecture, we will just introduce what is the differential equation, so start with a definition and its solutions, what do you mean by solution, what is its general solution and what are the solutions other than general solution one may find for the ordinary differential equations. Okay, that we will see in this video.

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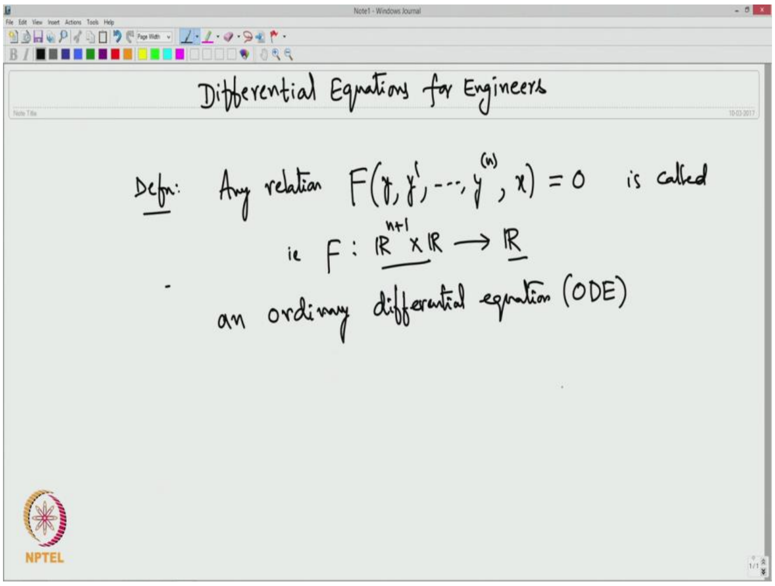

So let us start defining what is a differential equation. Differential equation is a relation between a function and its derivatives. Function is depending on the domains, so function is defined on the domain, the domain space, that the variable x, that is called independence variable, independent variables. So F is the function defined on the real line, so FX is a function, x is the independent variable, F is view it as F of x as a variable, F is dependent variable.

So we call y as dependent variable, x as independent variables. If you have a relation between y and its derivatives, say N derivatives, Nth order, then any relation between y, y dash up to yn derivatives and x relation between all these variables is actually called a differential equation. So formally we write the definition. Any relation capital F of this variable, dependent variable y, y dash up to N derivatives. My notation is like this y, in the superscript, if I put a bracket, that means a derivative. Dash, this is dash, so this.

Now we have independent variable x, so this is equal to 0. Any relation, okay, any relation F like this, if you have a relation like this what exactly this means that is F is from, you have 1, 2, 3, to N, so R power N Plus1. I have how many variables, how many dependent variables that can take real values, are y, y dash and y N derivatives, so R N Plus1, N cross, x, x takes also real values. So you have R to R. So any relation F, that is from this domain to this domain satisfying F of this relation, F equal to 0 is called, is called differential equation, is called an ordinary differential equation or ODE, now onwards we call ODE.

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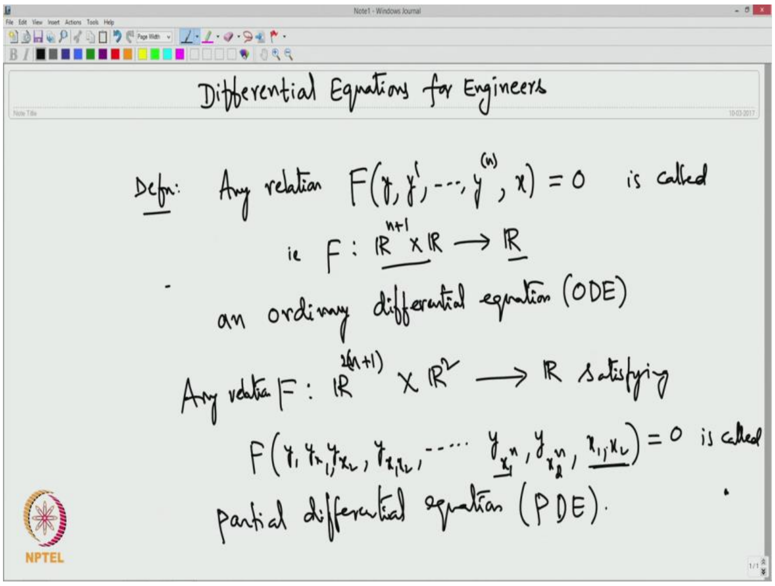

Why it is ODE, because you have one linearly independent, sorry you have one independent variable X. If you have more than one independent variables, let us say if I have a relation like this, if I have a relation, any relation, F from R, RN, I do not know exactly how many have. So if I take 2 independent variables, so I have 2, so 2 independent variables, instead of x, I have x1 and x2, that takes 2 values here, to finally R. And y, y is the dependent variable, dependent variable is always 1, okay, it is scalar differential equation.

So you have y and its N partial derivative with respect to x, with respect to, with respect to x1 and with respect to x2. So you can have for each x1 I have N Plus1, so you have 2 times. So any relation like this satisfying F of y, yx, y x1, y x2 and y x1 x2, so like this and so on, all combinations. Similarly y, so you have only 2nd derivatives, right, so you have, so like that you go up to N derivatives, N derivatives of a depending on the order.

So if you have N derivatives of y means y, let us write y x1 power N. y x1, x2 power N. x1, that means y x1, x1, x1 up to N times, like that, all the N derivatives. Dow N square, Dow NY divided by Dow x1, Dow x1, Dow x1 N times, that is what is this one. So like this if you have, and x1, x2 are 2 linear independent, are able to independent variables, this is, this relation is called partial differential equation or PDE.

So what we do is, we do only 2 derivatives, variables can be 2, okay, so it is going to be variables dependent, independent variables are going to be 2 and if the order, I can go to only 2 derivatives, there is second-order partial differential equation in 2 variables. Because if you have more than one variables if you have, then it is more than 2 independent variables if we have, more than one, so you have 2 here, so it is the partial differential equation.

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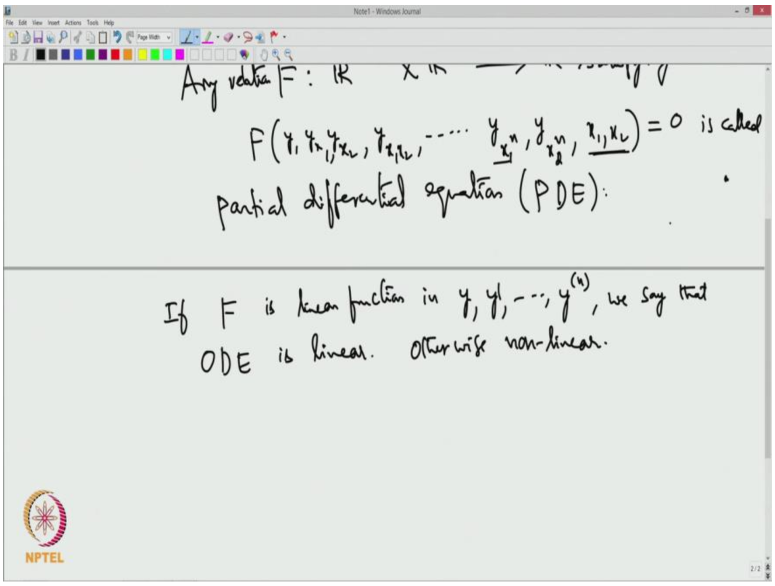

So when do you say a partial differential equation is linear? A differential equation is linear if, just giving… so when you say that F is linear in y, y dash are all N derivatives, these variables, if F is linear, something is linear, if it is, linear in y means you will see only y variables just as y with some coefficient with a function of x, then it is linear. If it is y square, root of y, y Power anything, it is all non-linear. Okay, once you see that, immediately you can say that it is non-linear.

So we can define what is linear and non-linear. If the function F is linear, linear function in the variables, in y, y dash, YN, for ODE, for ODE we can say this. If this is a linear function in these variables, we say, we say that the ODE is linear, otherwise, otherwise non-linear. Same thing with the PDE, okay. So it is all, it is all dependent variables. y, yx1, that means Dow y by Dow x1, Dow y by Dow x2, Dow square y by Dow x1 x2, all these N derivatives, these should be, these should stay in the equation as as it is, not in the squares, not in power, then it is linear PDE.

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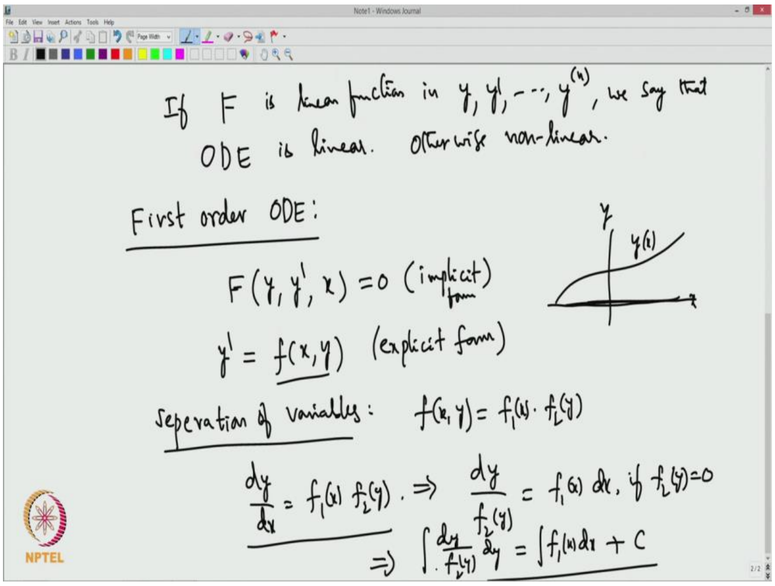

So this course we start with the first-order, first-order ODEs, first-order ODEs will try to give the solutions methods, how do we solve it? If you have F y, y dash, so I have a first-order. y the dependent variable and x is the independent variable which is equal to 0. So this is a general first-order ODE, when it is giving, nothing is given means what is the domain, x belongs to full R, y Y dash belongs to full R. So x and y in the plane, x takes the full values, y takes the full values.

So solution means any relation for y on x, that is any function in a curve YX that satisfies this relation is called a solution of the differential equation, ordinary or partial, whatever. If we take the partial differential equation, you simply look for y acting on x1 and x2, okay. In the domain where x1 and x2 are the variables, y of x1 and x2 is the solution of the partial differential equation, if it satisfies y and its all derivatives, when you substitute into the, into the relation, it has to satisfy. Okay.

So here, it is a ODE, so domain a single variable, so it is a curve. If it is PDE, if it is a PDE, you will have 2 variables, so it is solution is defined over a plane, so that means you can, it is like a surface. A surface, surface that satisfies equation, surface, right, that is equal to F of x1 and x2, so function of x1 and x2, now what is that, x1 and x2 is the plane, 3rd I mention is Z. So Z equal to F of x1 and x2 or y of x1 and x2, if that satisfies this relation, that means you have a surface that satisfies partial differential equation, this is called solution. Okay.

So we start with this first-order ODE, this is what it is. So how do you solve, we do not know the methods, if it is in implicit form. If the equation is in this implicit form, it may be difficult to solve this equation. So if we can solve for y dash in an, in an explicit form, so still you, so if you have an explicit form like this were still I replace this F, F is still not known, x I will write independent, wherever I am writing 1st, it is the independent variable. So if you have in this form, okay, this is in implicit form, implicit form you may not know in solutions, not always okay. Very few methods, very very few are available to solve the implicit equations.

Okay, only standard equations you can solve, just one or 2, Clairot's equation, Lagrange's type equations. You can look into the syllabus, you can look into the textbook, you can look into the reference books, you can see implicit equations, certain implicit equations, how they can be solved, okay, if you are interested. Otherwise if you want, if you want solution methods for many equations, you want them in an explicit form like this.

Explicit form for y dash, y dash equal to F of xy, where x is arbitrary function. So now you start with the method, separation of variables method, 1st method separable, separable method. Separation of variables. What does it mean? So if I have F of xy, if I have something like F1 x into F2 y, so these variables x and y are separate it. So I have separate nice forms. If you have in these nice forms, if you have, general F, if it is in this nice form, I can easily solve.

So your equation is DY by DX equal to F1 x into F2 y. So whenever you see in a equation, you have to see domain, so this implies, domain is everywhere it is defined. So whenever, I do not write the domain, whenever you see the equation, it is defined on a certain domain. So domain means values x takes wherever it is defined, okay. x takes all the values, if nothing is given, x takes all the values and the dependent variable can be, it can be there, it can be any value in a in a real line.

So mostly independent variables, what the values it takes is the domain. Okay. So nothing is given means x belongs to full R. So if I divide this DY by F2 y is equal to F1 x DX. You can see that this immediately when you write like this F2 of y should not be 0. So you, you are in some domain in the xy plane where F2 y, certain values of x square F2 y is 0 you should avoid, that is not in your domain, that is the meaning, when you divide it. Okay.

So formally we do like this, so once you do this, you can integrate this equation, so we have DY, so variables are separated, these are functions of y, these are functions of, right-sided is function of X. So you can divide it F2 y dy and integrate. So integration will give you F1 x the x, once you do the integration, so you will have integration constant C. So this is actually or general solution, this is the general solution of equation.

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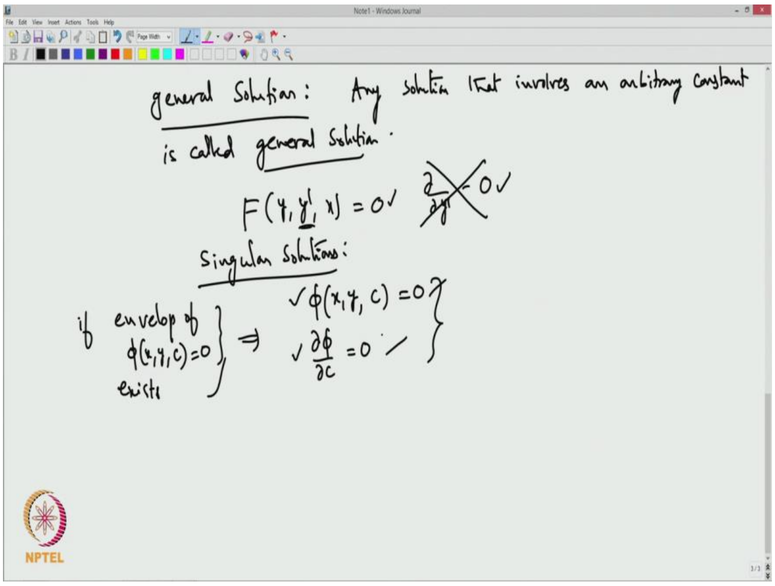

So the general solution, so general solution means, so you have an arbitrary constant, any solution that involves an arbitrary constant is called, is called general solution, okay. So when you integrate you have integrating constant, that is arbitrary constant, so this is general solution. So if it is a non-linear equation like this in implicit form, FY, y dash, x equal to 0, if it is in this form, you may have, you may not have, you may have, you may get the general solution as a certain form that involves an arbitrary constant.

It does not mean that these are the all solutions, you may have some other solutions. Therefore singular solutions, singular solutions, so what are the singular solutions, I will just explain. So what you have is, if you, if you have a general solution that involves an arbitrary constant, what are these for first-order ODE, these are solution curves with the parameter C, the arbitrary constant. So you have a parameter, family of solution curves.

If they, the envelope of these curves if you can find, if it exists, it is a solution of the differential equation that cannot be gotten by this general solution, okay. So such a solution is called singular solutions, we do not deal in this course, we only want, what we mean by solution is a general solution, finding the general solution. When I say solve the differential equation, I only want you to find general solution of the differential equation, okay, not the singular solutions.

But you, you I think you should know how to find the singular solutions, okay. So I will give you examples how do you solve this one. So whenever you have these solutions, there are many ways to, there are ways to get these singular solutions. The idea is you got rid of this y dash by just simply calculating dow F dow y dash equal to 0. So this is, this one equation, given equation and this is another equation, these 2 equations if you, from this if you remove your y dash, what you get is a solution in terms of xy.

These are curves, that curve is called singular solution. Okay. That is one way but if the envelope exists, that is the solution. Then this, envelope exists then if you, if you solve these 2 equations, then you can get, you can expect to get your envelope, that is solutions, singular solutions. But the best way is you solve the 1st one and you have a general solution, let us say phi of x, y, some constant C equal to 0. Suppose this is the general solution of the given equation, so you do not to let say, so we do not do this, so instead you have a general solution and then you try to, you differentiate this, dow phi by dow C equal to 0.

So look at these 2 equations, eliminate C will give you the singular solutions, okay. So, so if the envelope exists, okay, envelope, envelope of Phi of x, y, C equal to 0, C is arbitrary, okay. If envelope exists, is this envelop exists, then you solve this to get the singular solution, okay, then this is satisfied, okay. So this will give you singular, this together if you eliminate C, that will give singular solution. So we have just defined what is the differential equation and what is the meaning of a general solution.

So we have seen that general solution is, if you, if it is a first-order equation, if you have a solution that contains an arbitrary constant, that is called a general solution, okay. And these are not the only solutions, if it is a non-linear equation, it can have some other solutions, that is called singular solutions, that is also what we defined. So we will demonstrate with an example in the next video what you mean by singular solutions, okay. So once you have a family of solutions, general solutions, that may generate an envelope.

If you view geometrically, a general solution is a family of curves that may generate a new envelope so that can be a solution of the equation. So that is what we will see, when the envelope exists, that can be a solution, that is called singular solution, we will demonstrate with an example in the next video. Thank you.