Basic Algebraic Geometry
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Module 3
Lecture 6
Understanding the Zariski Topology on the Affine Line

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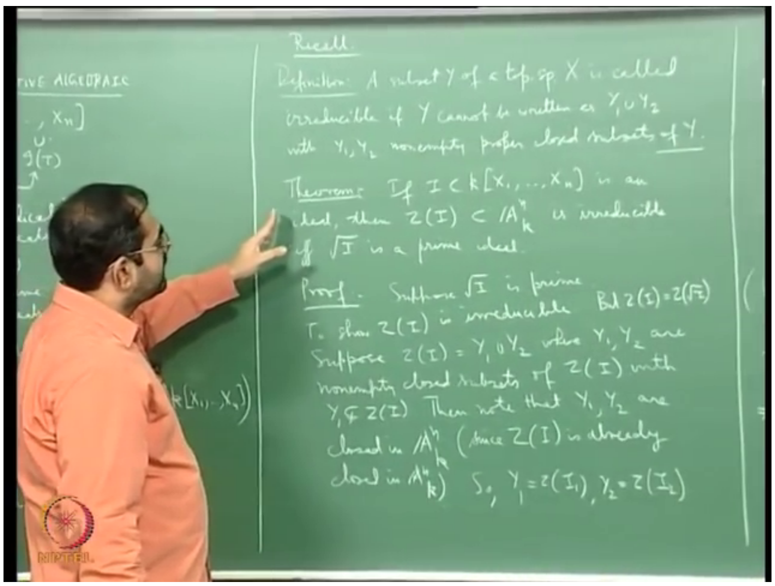

So let us continue with our study of irreducible subsets of affine space, so we have just seen in the last lecture that the condition that the 0 locus of an ideal is irreducible is that the radical of the ideal is a prime ideal, okay and so let us look at some examples in fact some examples of Zariski topology how it looks like, so in fact Zariski topology is very special when you take A1 namely the affine line which is k itself and (())(1:57) with Zariski topology, okay.

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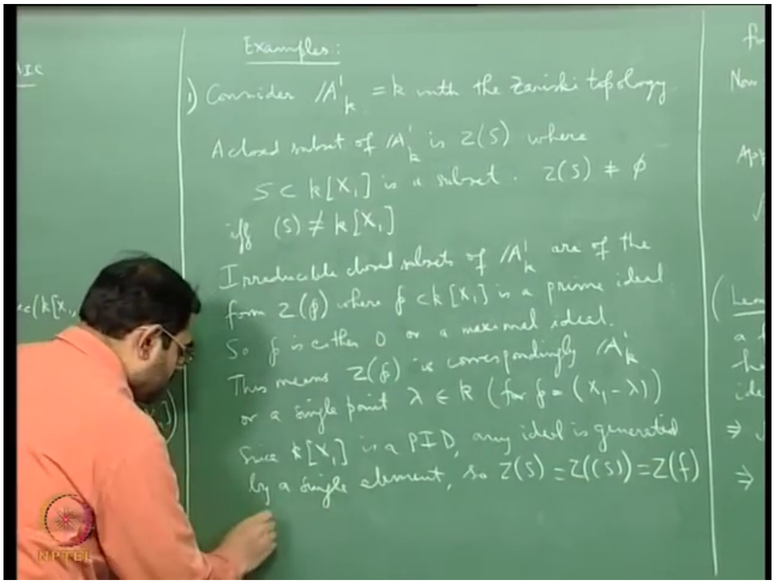

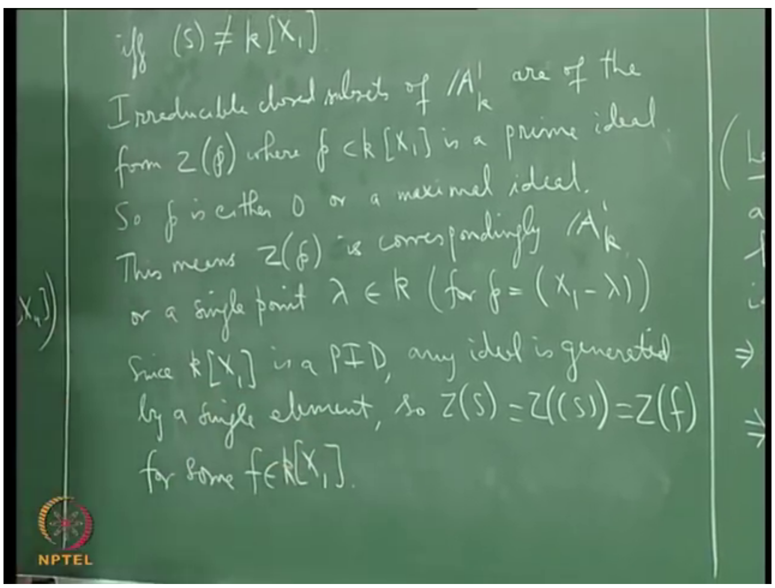

So let me make a few statements so consider so let us look at some examples these are general examples about the Zariski topology and also about irreducibility. So you first consider A1 this is just k with Zariski topology, okay now so what is the 0 set of A1 what is the close subset of A1 close subset of A1 is Z of S where S inside k of X1 is a subset, okay this is the definition of close subset, okay so S is a bunch of polynomials in one variable, okay and of course you know if you want this close subset to be non-empty you should assume that the ideal generated by S is not the whole ring Z of S is non-empty if and only if the ideal generated by S is not the whole polynomial ring, okay this is essentially one of the version of the Nullstellensatz, okay rather weak form of Nullstellensatz which says that you know the 0 set of an ideal is non-empty if the ideal is a proper ideal, okay.

And so in other words the elements of S do not generate a unit, okay and in fact you see what are the irreducible close subsets of A1? Irreducible close subsets of A1 are going to correspond to prime ideals in k X1, okay and so irreducible close subsets of A1 correspond to are of the form Z of p where p inside k X1 is a prime ideal, okay but then you know if k is a field then the polynomial ring in one variable over the field is a PID it is a principle ideal domain that is every ideal is generated by a single element and you also know that in a PID every non-zero prime ideal is also maximal, okay.

So the only chances for p are either that it is the 0 prime ideal, okay and the other chance is that it is a maximal ideal, okay. So p is either 0 or a maximal ideal, okay this means that the 0 set of p is correspondingly A1 because if you take 0 set of 0, okay instead of all points which at which the function 0 vanishes then you will get all the points so in 0 you get A1 or a single point lambda belonging to k, okay because the maximal ideal will look like X1 minus lambda, okay.

So if p is 0 then the 0 of p will be A1 it include all the points that in particular tells you that A1 is irreducible, okay if you use the fact that the 0 set of a prime ideal is irreducible and if p is a prime ideal which is not 0 then it has to be a maximal ideal and a maximal ideal is of the form X1 minus lambda for some lambda in k I told you this is also another version of the Nullstellensatz but actually in this case because it is just in one variable you do not need the Nullstellensatz, okay you just have to use the fact that k is algebraically closed. So you know the fact is that if you take any ideal, okay which is a proper ideal then you see because it is principle ideal domain this ideal will be generated by a single element and there this 0 set of this ideal will be just the 0 set of a single polynomial, okay.

So and then the 0 set of the single polynomial will be just the union of the 0 sets of the linear factors of the single polynomial any single polynomial is can be broken down into a product of linear factors because k is algebraically closed and each linear factor will correspond to point, so what you will get is that the only close sets are just finite subsets of points, okay. So that means the open sets are just compliments of finitely many points and from this you can see that you take any two open sets they will certainly intersect which is what you should aspect because any two non of course I mean any two non-empty open sets because you already know that A1 is irreducible so any two non-empty sets have to intersect but the fact is any two non-empty open sets are just compliments of finitely many points and but there are infinitely many points the reason that there are infinitely many points is because an algebraically closed field is always infinite a finite field cannot be algebraically closed.

So let me write all that down since k of X1 so let me write here it is for p is equal to ideal generated by X1 minus lambda, okay since k of X1 is a PID principle ideal domain any ideal is generated by a single element any ideal is generated by a single element. So Z of S is just Z of ideal generated by S is just Z of f for some element f in the polynomial ring, okay but since k is algebraically closed f can be written as a product of linear factors the number of factors will be equal to the degree of f of course I am assuming that f is not the constant polynomial is not a constant polynomial because if f is of course f is constant polynomial 0 then you will get the whole space if f is a non-zero constant then the ideal generated by f will be the unit ideal will be the whole ring and the 0 set will be empty, okay and the empty set is always a close set by definition, okay.

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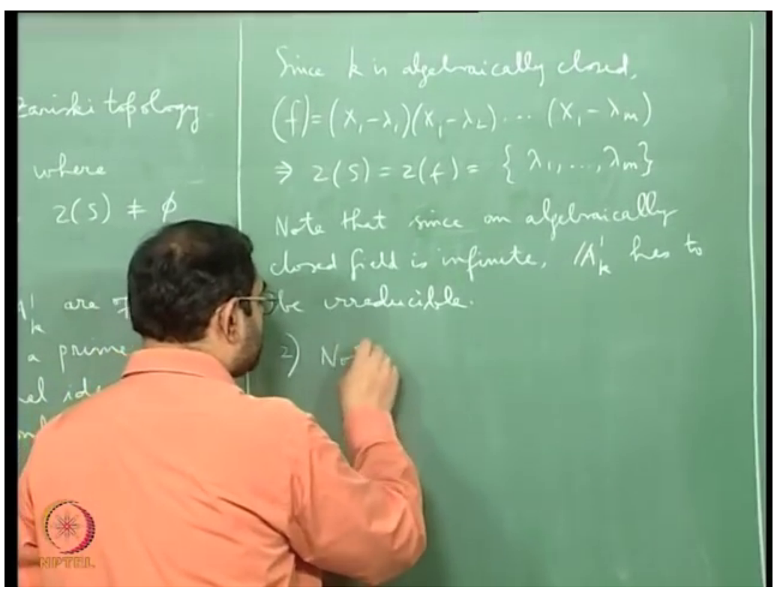

So since k is algebraically closed the f is some ideal generated by f is just ideal generated by X1 minus lambda 1 times X2 minus X1 minus lambda 2 times X1 minus lambda m, okay and which implies that the 0 set of S the 0 set of f is just the union of all these points lambda 1 etcetera up to lambda m of course I am putting this (())(11:56) notation some of the lambda could be repeated there would be multiple 0 zeros of a polynomial but the fact is therefore that any closed set is just a finite subset, okay and you know what this will tell you is that see this will tell you that you do not at least as far as A1 is concerned you really do not need to use the Nullstellensatz to get all these results, okay you do not need to use the Nullstellensatz.

So because in Nullstellensatz is kind of is redundant it is super flows in the case when n is equal to 1, okay when n equal to 1 Nullstellensatz is just the definition of what an algebraically closed field is, okay so you really do not need the Nullstellensatz so all this can be directly seen the moment you define Zariski topology even without using Nullstellensatz, okay. So in particular for example how do you see that A1 is irreducible one way is of course from the general theory you can say A1 is irreducible because the 0 set because ideal of A1 is 0, okay and 0's are prime ideal therefore the 0 set of 0 ideal is irreducible namely A1 is irreducible that is one hi fi way of seeing it.

But more easier way of seeing it is that you see if A1 where reducible then we should be able to write it as a union of two closed sets two proper closed sets but you see any close set is finite so if you are able to write it as union of two proper close sets that means you are saying A1 is finite but A1 as a set is just the field, so saying that A1 is reducible is same as saying that the field has finitely many elements but that is not possible for an algebraically closed field an algebraically closed field is infinite, therefore the fact that an algebraically closed is infinite we will automatically tell you in this case that A1 is irreducible, okay.

So note that since an algebraically closed field is infinite A1 has to be irreducible, okay. So in this case everything is very easy to see just because of the fact that every ideal is just generated by a single element because it is a principle ideal, okay. And you know I want you to really think you see let us put for k complex numbers, okay put for k complex numbers and take complex numbers usual topology, okay complex numbers is just the or to the X, Y plain if you want, okay you can also think of it as an (())(15:15) plain, okay.

The point is if you take if you think (())(15:21) at the usual topology see the so many close sets, okay there are so many close sets, for example you take disk and then you take its closure that is a closed set you take a rectangle and take the closure that is a close set you get all kinds of closed sets but you take see Zariski topology then the closed sets are only finitely many points you see there is so the open sets are compliments of just finitely many points it is just the open sets in the Zariski topology for complex numbers is just the plain punctured at finitely many points the huge open sets, okay and the close sets are just finitely many points.

So you see it is a you see there is such huge difference between the ordinary topology and the Zariski topology and you can from this also see the Zariski topology is highly non (())(16:25), okay because (())(16:28) is a condition that given any two points I can find open sets which contain these two points and which are disjoint from each other, okay I can do that in the usual topology you give me two distinct points on the complex plain then you know I can find two small enough open disks surrounding those two points which will separate these two points the two open disk will not intersect but if you try to do that in the Zariski topology you will not be successful, okay that is because any two non-empty open sets will intersect so you cannot find two disjoint non-empty open sets containing two given distinct given points.

So you see Zariski topology is highly non (())(17:09) and you know this leads to thinking in a philosophical kind of way you see the topological space which are not (())(17:18) are not well behaved, okay because the (())(17:23) is if you look at it from the topological point of view it is a fact that you know if it is the uniqueness of limits, okay see in topology the existence of limits and the uniqueness of limits put together what is called as completeness, okay the completeness is the property that limit exists, okay for example when you take subspaces of the Euclidian space you know completeness is just the fact that you take any Cauchy sequence in that sub space it limit should also belong to the sub space so and of course the limits limit of a sequence is unique so you get the existence not only of the limit but also that they limit is unique, okay.

But the truth is that the uniqueness of the limit are low and if you take it out as a property and forget the existence of the limit it is the uniqueness of the limit that is promised by (())(18:19) space, okay. Now so the fact that complex the complex plain with Zariski topology being (()) (18:31) you see the feeling that it is somehow there is no uniqueness of limit if they exist. Now the answer to that is that you should not think that way in algebraic geometry the notion of uniqueness of limit is not (())(18:52) is another property and that property which is the analog of (())(18:57) is called separatedness, okay.

And so the nice thing is though the complex plain is not (())(19:06) it is what is called separated, okay and separatedness is the right analog of (())(19:12) in the you know in the Zariski topology. So what you should understand is that though at the outsight it looks non (())(19:22) but for all practical purposes there is the analogous property of separatedness which I will explain later, okay. Yeah, so fine so the next thing that I should look at is, yeah in fact let me also remind you of fact from you know field theory if you have a finite field, okay.

Then the algebraic closure of the finite field is countably infinite, okay this is one fact so it is infinite and if you in general if you have any field then the algebraic closure if the algebraic closure will have the same cardinality as that of the original field if it is infinite, okay then the algebraic closure of the field will have the same cardinality as the field itself, okay this is a fact it is just a counting argument because you get the algebraic closure by putting in 0's of all polynomials in the field after all algebraic you go to the algebraic closure of a field because you want to solve all equations you want to get 0's of all polynomials with coefficients in that field and but then the polynomials when you look at 0's of polynomials, polynomials can be counted by counted degree, okay.

So you can see that somehow you will get a countable union of subsets which are just as a same cardinality with a field and then you will see that you will again get the union will again be the same cardinality as that of the field, okay. So this is a fact that you can look up from any you know standard algebra book, okay so let me repeat, the field is finite then its algebraic closure will be infinite and countable be the field is infinite then algebraic closure will be infinite and have the same cardinality is that of the original field.

And why I am saying this is also since I brought in idea of algebraic closure I should tell you why this is important, this is important because you see we somehow in this course we are only worried about algebraically closed fields but you know you can write equations over fields which are not algebraically closed, okay for example real numbers, then what is a theory? The theory is that first of all whatever equations you write you can always go the algebraic closure of that field and work there and for this you need the existence of an algebraic closure and this is the theorem in algebra from field theory which says that gives any field you can find an extension field a larger field which is algebraically closed and which is algebraic over the given field namely that it is gotten from the given field by just adjoining roots of polynomials with coefficients in the given field, okay.

So this existence of algebraic closure tells you that no matter what bunch of equations you are trying to look at 0's of over a field that will always make sense over the algebraic closure of that field and when you come to the algebraic closure of the field you are in this situation and you are doing variety field, okay. So this gives you an idea how to proceed if you are having equations over a field which is not algebraically closed, okay that is one thing and the other thing is that many of this results that I have written down are not true if you remove the algebraic closure property, okay that should be obvious to you because you know this algebraic closure property that gave rise to that is needed for Nullstellensatz, so and you know the Nullstellensatz is somehow you know in this principle in all the main statements that we are making.

So if you remove algebraic closeness that is if you take a field which is not algebraically closed one could be in lot of trouble. So the simple example is you know if you for example you know if you take an equation like X1 square plus 1 equal to 0 which makes sense over the real field you know that first of all the 0 set is empty if you consider the affine space over the real field because there is no 0, the 0's are they are complex roots. So the 0 set of a polynomial might become empty, okay might turn out to be empty and that demonstrate the field of the Nullstellensatz if you work with the field which is not algebraically closed.

The other thing is this fact that if you state with the prime ideal then the 0 set of the ideal is irreducible that also kind of goes wrong, for example if you take the if you take the polynomial ring in two variables over r the reals the field of real numbers let us call the variables as x and y, okay then if you look at the polynomial xy minus 1 whose 0's are the that is a locus of the rectangular hyperbola, then you can see that of course it in two pieces, okay it is certainly not connected, right? But the ideal generated by xy minus 1 will certainly be a prime ideal because xy minus 1 is an irreducible polynomial, okay.

And you know in a unique factorization domain an irreducible polynomial an irreducible element will always generate a prime ideal, therefore the irreducible polynomial xy minus 1 will generate a prime ideal in r of xy but the 0 set in A2 of r which is r cross r will be the rectangular hyperbola and that is not connected, okay. So here is so the fact that the 0 set of a prime ideal is irreducible is contradicted is just because you are working over real numbers which is not algebraically closed.

So you should always remember that when you are all these results strongly depend on the fact that the field is algebraically closed, okay.

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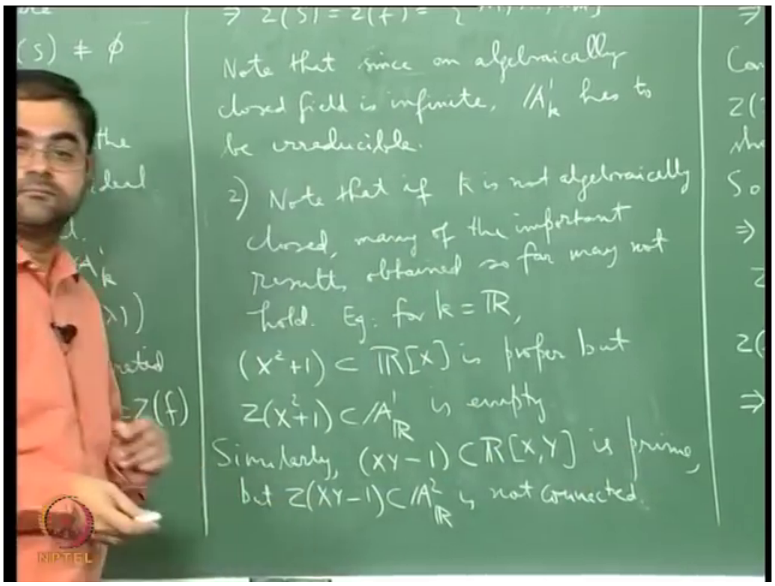

So let me write that down note that if k is not algebraically closed many of the important results are obtained so far may not hold, examples is for k equal to r real numbers, okay X square plus 1 ideal generated by X square plus 1 so let me write it as in Rx is proper but Z of this in A1 R is empty, okay. Similarly XY, ideal generated by XY minus 1 in R x, y is prime but Z of x, y minus 1 in A2 R this is not connected, okay.

So and if a subset is not connected it cannot be irreducible because irreducible is very very strong. So if the field is not algebraically closed you will not get the many of the important results here, okay.

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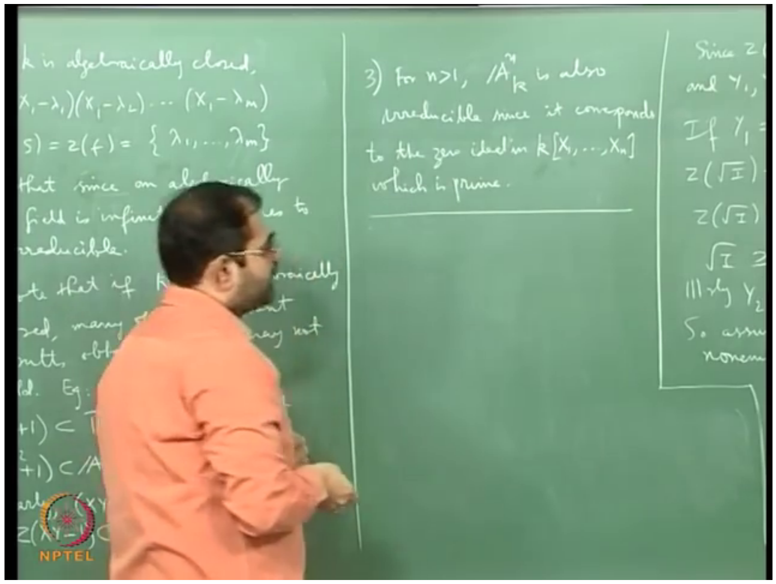

Then the third thing is to look at I mean just to look at the whole affine space if you look at the whole affine space then we will of course get that the whole affine space is reducible for n greater than 1 Ank is also irreducible since it corresponds to the 0 ideal in polynomial ring which is prime, okay and you know of course the ideal in the ring is commutative ring with 1 is prime if and only if the ring is in integral domain and you know that the ring of polynomials in n variables over a field is an integral domain that is a product of two polynomials cannot be 0 without either of the polynomials being 0 you cannot have two non-constant polynomials whose product is 0 that can never happen, okay.

And more generally the fact from commutative algebra is that you know if you have a commutative ring and you look at the polynomial ring over that commutative ring then the if the original ring is an integral domain then the polynomial ring in n variables over that integral domain is again an integral domain this is a fact that you can check. So more generally instead of k if I put an any integral domain then the polynomial ring over that will continue to be an integral domain, okay.

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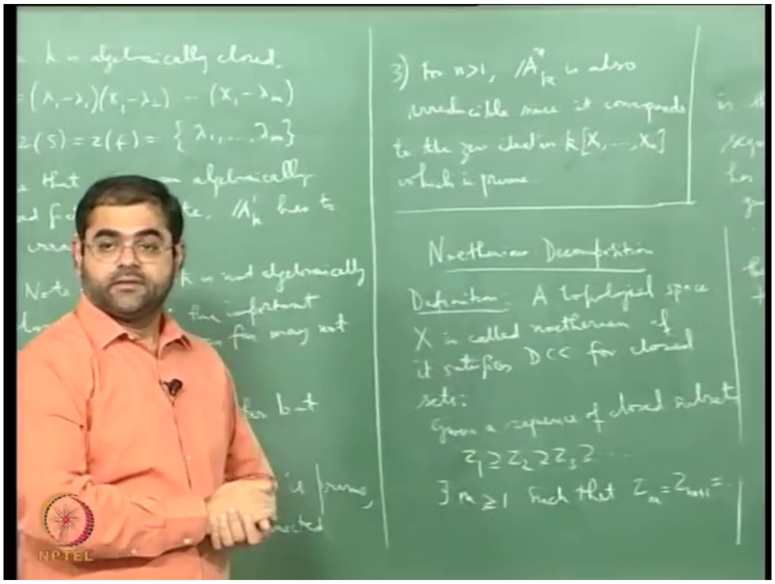

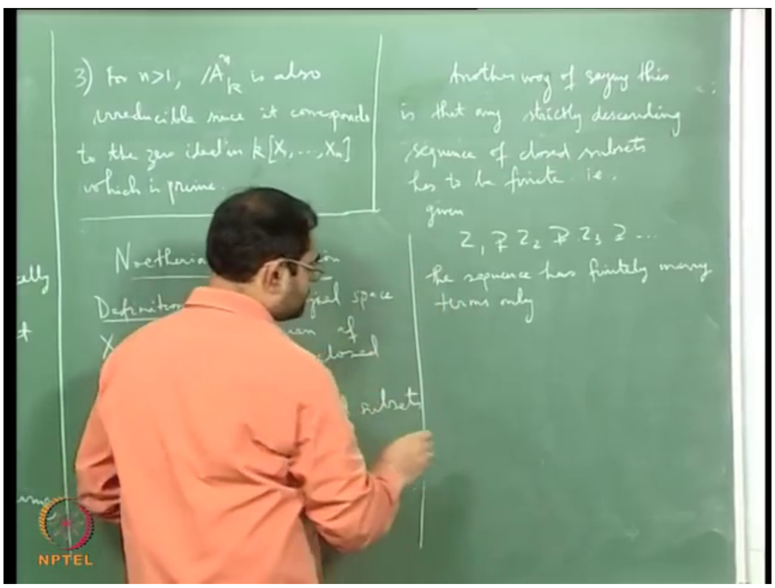

So this is one fact and okay so maybe I will continue with my discussion and for that so I will go what is called as Noetherian decomposition, so I will go into what is called as Noetherian decomposition so this is another important feature of irreducible close sets, okay it is the in some sense it is the analog of the Noetherianness of a commutative ring, okay. So you know so let me make a definition a topological space X is called Noetherian if it satisfies DCC for closed sets, okay.

So DCC is abbreviation for descending chain condition, okay and what does that mean? It means the following thing if you have a sequence of closed subsets which is becoming smaller and smaller, okay a descending sequence of closed sets then that sequence has to stabilize beyond a certain stage, okay it has so what it means is that if you have such a sequence then beyond a certain stage all the sets occurring in that sequence have to be the same, another way of saying it is that if you are going to look at only strict sequences namely a sequence in which every next set is a strict strictly smaller subset of the previous one then it has to be only finite length you cannot have an infinite length sequence like that.

And the actually I should tell you at the outsight what is importance of this topologically the importance of this topologically is the this gives you a handle on being able to define the dimension of a space, okay you can define the whole notion of dimension of the space can be defined if you have this notion, okay. So you know it is if you think of linear algebra, okay then suppose you have an n dimension vector space, okay then if you take a sequence of subspaces then you see that the sequence of subspaces has to stabilize in fact if you take a sequence of strict subspaces then you know it has to stop because as you take a stricter subspace the dimension falls and the dimension cannot go below 0 and the 0 dimension space will be just the 0 vector single point, okay and it cannot beyond that.

So you if you have a vector space of dimension n and you take a sequence of strictly decreasing subspaces then you know you can get only something of length n plus 1 starting with the whole space which is n dimensional then a hyper plain which is n minus 1 dimensional then a hyper plain in that which is n minus 2 dimensional and you can go on up to 0 so that will give you n plus 1 terms from 0 to n, okay and it cannot be any longer than that.

So in the same way what you are doing for a general topological space is that you are of course in the case of vector space you have subspaces, okay but in a general topological space what you do is you do not use subspaces but you use close subsets, okay and that is the difference but the fact is that this DCC allows you to define dimension, okay. So let me write that down given sequence of closed subsets f1 or let me use z1 containing z2 containing z3, okay there exist R or there exist m greater than i equal to 1 such that z m equal to z m plus 1 and so on.

So the sequence of a descending sequence of close subsets at some point has to become constant, okay of course the other way is that if you have a strictly descending sequence of closed subsets that has to be of only finite only it cannot be infinite, okay that is another way of seeing it so let write that also, yes I am continuing here another way of saying this is that any strictly descending sequence of closed subsets has to be finite that is given is Z1 properly containing Z2 properly containing Z3 and so on the sequence has finite length finitely many terms, okay so this is the another way of saying it, okay a strictly decreasing sequence of closed subsets has to terminate has to stop, okay.

And so you of course the main purpose of this is I told you is that it allows you to define dimension, okay and but there is something more it gives rise to what is called Noetherian decomposition, okay.

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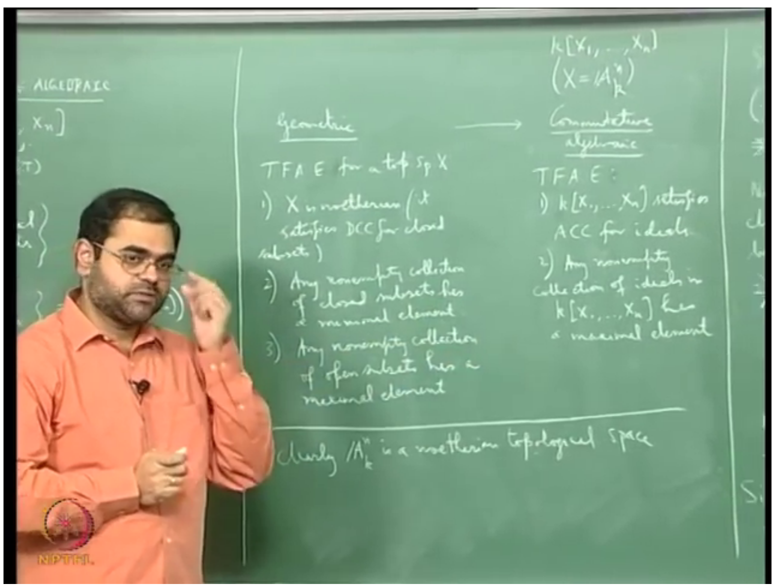

So let me write that probably I will erase this part of the board but probably before that let me do the following thing let me convince you that the affine space is Noetherian, okay so the affine space An with Zariski topology is Noetherian and the reason is because this Noetherian condition on the affine space actually translates if you apply the I function to the ascending chain condition on ideals of the polynomial ring which holds because the polynomial ring is Noetherian, okay.

So that is a giffy saying that the affine space is Noetherian, okay so let me write it like this geometric so there is again a geometric and there is a commutative algebraic and the geometric thing is so I will use the following are equivalent, okay for a topological space X, number 1 X is Noetherian, okay which is DCC for closed sets, okay it satisfies DCC for closed sets, number 2 the you know but before I come to number 2 let me take the case when X is An, okay if you take so I go from this picture to this picture on this side and writing it only for any general topological space but I am going from here to here only when I am looking at An, okay.

And what happens is that you see you will see that again the following are equivalent for number 1, k X1 etcetera Xn satisfies ACC for ideals, okay so this is I mean this is you know this is one of the definitions for a ring to be Noetherian for a commutative ring to be Noetherian ideals should satisfy the ascending chain condition, okay. And this polynomial ring is certainly Noetherian because that is because of the Hilbert Basis theorem or Emmy Noether's theorem that if a given ring is Noetherian then the polynomial ring infinitely many variables over that ring is also Noetherian, okay. So you see this corresponds to this but then if you look at the definition of there are several other equivalent formations of the definition of a Noetherian ring of course there is one other definition which says that if you give me any non-empty collection of ideals it has a maximal element, so this is another condition.

So any non-empty collection of ideals in k X1 etcetera Xn has a maximal element, okay and you know usually (())(42:24) lemma schemes of choice kind of argument that tells you that you know these two are equal, right? And you see the point is that there is something corresponding here the corresponding thing here is you see ideals if you take An ideals correspond to closed subsets so you know the translation of that will be any non-empty collection of closed subsets has a minimal element because you see the when you take An into the picture on the commutative algebraic side you will take you are actually looking at k of X1 etcetera Xn which is the ring of functions on An.

So ideals correspond to closed subsets and you know maximal means with respect to inclusion they will correspond to minimal subsets because this correspondence is inclusion reversing. So in fact it is true that for a general topological space that is correct that any this condition that X satisfies DCC for closed sets is equivalent to any non-empty collection of closed sets of closed subsets so of course here I mean closed subsets has a minimal element, okay any non-empty collection of closed subsets has a minimal element and you know but you can write something more here and what you can write more is just because in this case you have a topology the complements of closed subsets are open subsets you can also say that any non-empty collection of open subsets has a maximal element, okay because that is the complement of this open sets are just complements of closed sets.

So in fact I can also write any non-empty collection of open sets open subsets has a maximal element, okay. So you get that also in this side, okay so it is clear that it is immediately clear from this comparison that An is a Noetherian topological space with Zariski topology. So I will just draw line here and say clearly An is a Noetherian topological space and you know let me tell you again but I will come back to this in a later lecture and that is you see the whole point about this Noetherianness is to study finite dimensional Noetherian topological spaces and you know by any amount of intuition even by common sense you should expect that the dimension of An must me n, okay you should expect that.

And that is what we have to prove, okay but for that you know there are two things first of all you should know how to define dimension, okay and then you have to prove that the dimension is n, okay and it is very easy to define what dimension is what you do is that you emitted what you did for what you see happening for a vector space of dimension n if you take a vector space of dimension n if you take the longest possible strictly descending sequence it will have n plus 1 terms starting with the full space of n dimension and ending with the 0 subspace which is 0 dimensional there will be n plus 1 terms.

So what you do is you define the dimension of the topological space to be you take sequences like this strictly descending sequences and take there length and take there supremum we have to use the word supremum because there could be I mean you could have larger you could have lengths of any I mean you could have a sequences strictly descending sequences of any length, okay. See the definition of Noetherian only tells you that if you have a strictly descending sequence it has to be a finite length but it does not tell you that the lengths cannot exceed a certain finite quantity.

So you could have sequences different sequences each of different finite lengths with the lengths increasing going being unbounded. So you take all possible sequences like this strictly decreasing sequences and take there length, okay and in fact you should take length minus 1, okay because you know if you go by the vector space analogy then the length of strictly decreasing sequence with larger possible strictly decreasing sequence is n plus 1 terms and you have to take way 1 from that to get n which is dimension of the vector space, okay.

So that is how you define dimension and then after you define dimension you will have to show that dimension of An is n this is what you want, okay. So what you should try to understand is that on the geometry side you see we have Zariski topology and now we are talking about dimension which is a topological property, okay. So you know this is the this is how you start studying the topology of varieties, okay and as I told you this is the first step in geometry you look at some topology you look at the topological properties try to talk about dimension connectedness, irreducibility and things like that try to understand what are the closed sets what are the open sets and so on and so forth.

And then after you done enough topology then you start worrying about more complicated things like things that are you know connected with analysis like you know tangents, tangent spaces and smoothness and singularities and things like that, okay which also we will do in this case, okay. So what I am trying to say is that at this level we are still looking at the topology, okay and so I need to tell you also one more thing so you might wonder how this notion of dimension is going to translate into on this direction and the topological dimension on that side of An being n is going to translate into what is called as a Krull dimension, okay named after the commutative algebraist Krull of this ring, okay and the Krull dimension of this ring is n and that will correspond to the topological dimension of the affine space, okay.

So we have to translate all this two things here and to prove that the Krull dimension of this is n you need some field theory and some commutative algebra which I will recall, okay. So that is to indicate to you how this is going to go about, alright? But apart from that what is this thing I started with? I started with this Noetherian decomposition, so if a topological space is Noetherian the nice thing is that every non-empty closed subset has a Noetherian decomposition, okay and that is kind of the importance of having a Noetherian topological space, okay this break you can breakup any close subset into a finite union of irreducible closed subsets such that no which can be made unique up to permutation of the factors in such a way that it can be made if you assume that none of no one of these is contained in some other, okay.

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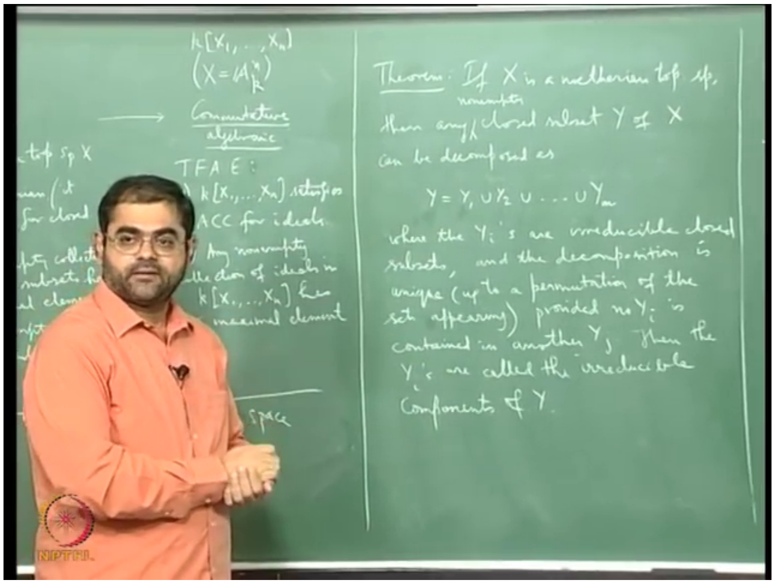

So it is a Noetherian decomposition that is very important so let me write that down that is a theorem and we will probably look at a proof of that in the next lecture. So theorem if X is a Noetherian topological space then any closed subset any non-empty closed subset y of X can be written can be decomposed as Y is equal to Y1 union Y2 union and so on Ym and where the Yi are irreducible closed subsets and this decomposition is unique and the decomposition is unique up to a permutation of the sets appearing provided no Yi is contained in some in another Yj, okay this is the theorem on Noetherian decomposition if you have an Noetherian topological space you take any non-empty close subset you can break it down into irreducible closed subsets and finitely many of them.

And you can make the decomposition unique if you assume that there are no redundancies that is no Yi is containing some other Yj, okay. And if you believe this theorem what this will tell you immediately is that since An the affine space which is Zariski topology is Noetherian it will tell you that any close subset of An namely the 0 set of an ideal can be broken down into finitely union of affine varieties any irreducible closed subset of any closed subset of An which may not be irreducible it can be broken into a finite union of irreducible closed subsets and you know the irreducible closed subsets are called the affine varieties and therefore you get a decomposition of any closed subset into affine finitely many affine varieties and those finitely many affine varieties are called the irreducible components it is just like in topology when you have a space which is not connected you know you can break the space down into union of its connected components, okay.

In the same way what you can do here is that an any subset of a Noetherian topological space any closed subset of a Noetherian topological space can be broken down into irreducible components and these irreducible components are closed so these Yi's are called the irreducible components of Y, okay then the Yi's are called the irreducible components of Y, okay. So I will end with this and I will just make a final remark to tell you that you see the notion of Noetherianness is essentially needed on one hand to make sense of dimension, okay and on the other hand it is very helpful because it allows you to break down any closed set into irreducible closed sets in a nice way, okay.

And that for as important and all this for as is important is because you can talk about dimension of affine varieties, okay and you can also break down any close subset of affine space into its irreducible components which will be affine varieties and which will be essentially unique, okay so that is the importance of the Noetherianness and so this is the geometric part of it, but if you come to the commutative algebraic part of it what it is? It is the just the usual Noetherianness of the polynomial ring, so you see this is the beauty of algebraic geometric that some nice property on one side gives rise to properties on the other side which have nice consequences.

So after all the fact that the polynomial ring is Noetherian, okay is a completely commutative algebraic property but when you translate it to geometry you see the advantage is that it allows you to define dimension, okay and we are eventually going to prove that the dimension of An is n as you would expect, okay. And it also allows you to decompose any closed subset into affine varieties, okay. So you see this is how you see how a nice commutative algebraic property translates into such nice things in geometry, okay.

See this is essentially the attraction of algebraic geometric to go to take something on one side and try to go and investigate what means on the other side you get interesting things on the other side and here we are going from this from the commutative algebra side to the algebraic geometric side, okay. So I will stop here.