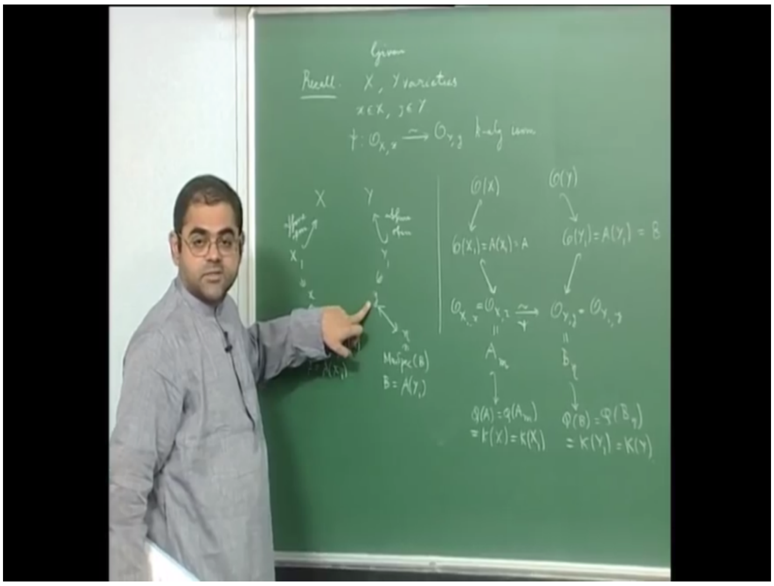
Basic Algebraic Geometry
Professor Thiruvslloor Eesanaipaadi Venkata Balaji
Department of Mathematics
Indian Institute of Technology Madras
Mod_14
Lec_38
Local Ring Isomorphism Equals Function Field Isomorphism Equal Birationality

Ok you see let me tell you let me recall the notation that I used ok.

(Refer Slide Time: 02:00)



This is just to make sure that I do not accept the notation so you know they have the situation that so here you recall what we have was, we had so we had x, x and y varieties and you had small x and capital x, small y and capital y, and you had Si from the local ring at capital x and small x to the local ring of capital y small y, you had an isomorphism K Algebra isomorphism ok, we had this and then what we did was, we wanted to show that if you have two varieties and theirs points where the local rings are isomorphic we want to show that the they are the open neighborhood surrounding those points which are, actually isomorphic is variety ok.

So in other words there is an open set, you want affine open set containing x, and another open set containing y under isomorphism between these two sets ok, and the point is that that isomorphism at local ring will induce this isomorphism ok that's what we wanted to show, so what we did was, so this is given to us ok, and what we did was so off course became any variety is covered by finitely any open sets each of which isomorphic to affine variety what we did, we took affine variety we took an open set containing small x which is an affine variety an open set containing small y, which is also affine variety and therefore you we had the diagram like this we had x here we had x one inside x which is an affine open and we had small x belonging here then we had capital y and we had this capital y one which is an affine open sub variety which contains the point small y ok and if you look at it in terms of the this is a geometric point if you look at with commutative Algebraically you got o, x here that corresponds to a regular functions on x.

We have o y here and then you have this restrictions of regular functions on x to regular functions on x one so you have o, x one but that is a same as, a x one, ok and and off course this is inclusion because that's because if a regular functions is zero on a proper open set which is non-empty then it is zero everywhere ok and another way of saying it is off course that if two regular functions co inside on a non-empty open set they constant everywhere that's reason this is a, this map is injective it is and off course it is map corresponds to the restrictions of regular functions to the open set x one and similarly you have inclusion this will prove o, y, one and o y one is same as, a y one, because and o, x one is same as a x one this is because x one and y one are affine varieties for affine varieties the ring of regular functions is same as a affine coordinate ring alright.

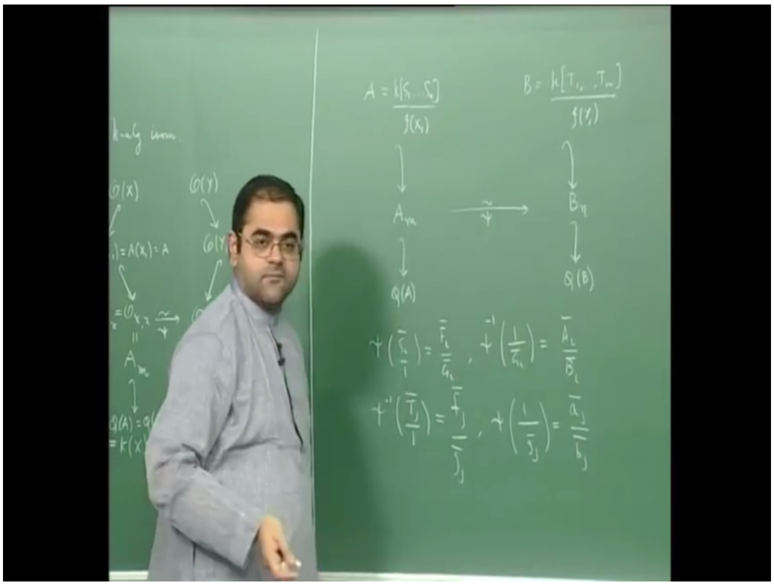
And then off course then we go all way to the point by going local ring at that point and the local ring will not change if you go to an open set, if you got to a smaller open set which contains point so this is a same as local ring of x concentration attention on a point of the open sub variety x one right and similarly we have the local ring of capital y and small y which is the local, also the local ring of small y as a point of capital y one, alright and we know that you see the we have given names for these rings we call this as a we call this ring as v ok and the point small x as a point of x one will correspond uniquely to the maximal ideal of a and we call that maximal ideal as m, and the point y, small y, will be will correspond uniquely to maximal ideal of v we call that as m, or Nita and so you know x so this is this local ring becomes it can be identified with a localized at m, ok this is the expression of the local ring at a point you take the if you have a point of a affine variety then the local ring at that point is given by simply taking the affine coordinate ring which is the same as the regular functions then localizing at the maximal ideal that corresponds to that point.

And similarly this is identified with b localized at m, ok, where m corresponds, so this x corresponds to m in the maximal spectrum of a, where a is a of x one and y corresponds to m, which is in the maximal spectrum which is set up maximal ideals of b, where b is off course the affine coordinate ring of y one right and then off course we have then all this is happening inside the respective quotient fields all these things have the same quotient field I mean this the quotient field of a, m, is same as the quotient field of this is a same quotient field of this which is a quotient field of a, and there is also functional field of x.

So what you must understand is that, this is happing inside quotient field of a, which is a same as quotient field of a, sub m, this is also equal to well function field of, this is also equal to quotient field of, , some equal to functional field of x the field of rational functions on x is also equal to rational functions of x one the field of rational, rational functions does not change if you got to an open set ok, and similarly here I am going to get this is, all this is sit inside by the way I should also tell you that the when you go from the ring to its localization the ring is an integral domain this is also injective map ok from domain to its local ring to any of its localization the natural map given by localization is an injective map because it sources is domain when you localize at domain you get the bigger sub ring of the quotient field of the domain ok, so similarly these are the domain and if localize b at the maximal ideal Nita you are going to get a bigger sub ring of the quotient field of b, which is k of b, that is a same as a quotient field of any localization in this case its question also the quotient field of field which is a field of fractions of the local ring and this is also equal to the function field of y one, the function field of y they are all the same ok.

And the point is that its at this level so I have I have an isomorphism, isomorphism here so I have Si here this SI is given to me alright, somehow I have to use this Si to produce isomorphism between an open subset of x one containing the point x and with an open subset of y one containing the point y so, so I have sum up, took up those open sets ok.

(Refer Slide Time: 10:45)



So well so what, what we have done is that a we are assume that, see you are a which is affine coordinate ring of x one if well ring if I look at notation a was k s one etcetera s n divided by the ideal of x one this corresponds to you know this corresponds to pitting because x one is an affine variety ok x one sits inside, x one sits inside some a n, ok, and if you take the affine coordinate ring of a n, as a polynomial ring in the variables s one to s n, ok then the affine coordinate ring of x one which is a x one which we call as a is going to be just this polynomial ring divided by the ideal of x one ok.

Where off course here when I say x one is being concerned in a n, here which being considered as a reducible close subset of a n, ok, which identified as affine variety right and then we also have similarly b, if also a polynomial ring in m variables divided by the ideal of y one where similarly what we are doing is that we are think since y one is affine open subset of y it means that y one is an open subset of capital y but it is also abstractly isomorphic to an affine variety therefore you can identify it you can embed it as a reducible close subset of a m, and on this a m, if you take the T's, t,one through t m to be the coordinate functions then the affine coordinate ring of that a m, which this polynomial ring in the t, one through t m, and if you go to module of the ideal of the y one, you will get the affine coordinate ring of y one alright and now in terms of these rings we have this inclusion of a into a m, and this is b into b n, ok and off course a m, sitting inside the quotient field of a and b n, is sitting inside the quotient field of b and off course the isomorphism that is given to me is between a m and b, ok.

This is a K Algebra isomorphism that's given to me off course all the ring homomorphism we are considering al K Algebra isomorphism's ok, and essentially they all as for as the as for as the same varieties concern the all corresponds to going to I mean going from here to here corresponds to taking germs of regular functions and then going here all way to the quotient field is trying to look at an equivalence class trying to look at regular function as rational functions ok and off course from one side to the other we always think of ring homomorphism is coming because of morphisms via pull back of regular functions but the point here we have to concept the morphisms we have a concept of isomorphism between an open subset of x one and an open subset of y one ok.

So what we did is if you remember so we have put all these assumptions we and put Si of Si bar by one is equal to Fi bar by Gi bar ok the Si's are in, are polynomials here and the Si bars live in this quotient and you are considering the because this is sub of this ok this is sub ring of this so you considering the mass elements here ok Si bar one is an element here and then I apply Si I get the quotient I get an element here a quotient Gi bar is not in the, where Gi bar is certainly not in the maximal ideal m, ok, and then this has to be proved a Si inverse one by Gi bar loose here this has to be Si inverse ok and similarly I do for Si inverse.I take the image of coordinate the generating coordinate variables, variable generating functions the Ti Tj's and so this is what I have.

(Refer Slide Time: 15:52)

Now what I want to, want to you to check is that see you, if you take the set, if you take this localization a localized at product of Gj bar, product of Ai bar Bi bar product of Si inverse of the Bj bar ok and you say you take this you take this localization ok and then you take this localization on as far as so this is you know he is going sit inside this localization and this going to sit inside this localization and in this localization you take product of, product of capital Gi bar to product of Aj bar Bj bar and I think you will also have to take Si of Bi bar product of Si of capital Bi bar ok.

So now the now the fact is that now the fact is that if you off course this localization will sit inside a m, and here inside the n, ok, these localization will sit here and what you can check is that the way we have define Si and Si inverse you will get an isomorphism like this ok you will get an isomorphism like this and that just because of the universal property of localization will get a you will get a map like this and you will get a map in reverse direction ok, you will get an isomorphism between these two alright and such that this so you will get this map here which I would like to call by some name may be you called as I do not know what I have done so it was probably Fi.

So you know you have, you have this map so let me put this arrow in this direction call this is as Fi inverse you will get a map like this where K Algebra isomorphism's Fi inverse, Fi inverse will just come because universal property of localization and the way because of the way in which this is defined and what will happen is that, this Fi inverse you know if you take this Fi inverse then I also get Fi inverse what this Fi inverse will do is that it will also induce an isomorphism here ok and this isomorphism will be the same as the isomorphism Si is started with so you will get something like this will get this ok.

You see everything, everything which is constructed in Si alright therefore the point of going inverting all these elements is to go to a proper, I mean it is to go to a proper open subset to go to an open subset of the affine variety with coordinate ring a, ok, and the same way inverting all this things here is the point is to go to an affine open subset of the variety y one which coordinate ring is b ok so this, this is a affine coordinate ring of an affine open subset of y one and this is an affine coordinate ring of an affine open subset of x one ok, and this isomorphism of affine coordinate rings gives an isomorphism between open sub sets.

So you know you, therefore, so hence you will get you see you have x and inside you have x one and inside x one you have this u ok and what is this u, this u has affine coordinate ring given by this whole expression so this is a u ok and similarly so this is affine open and this aslo again affine ok and similarly at for y you have y one which is affine open and here you, we called as v that also be an another affine open and what you will get is that, you will get, you will get a morphisms so I think so notation y, let me make, let me do something that I will say that I will get an isomorphism Fi ok such that if you take the isomorphism that is induced by pull, that is induced by pull back of regular functions.

Because isomorphism between affine varieties corresponds to an isomorphism between their affine coordinate rings which is an K Algebra isomorphism's so this will induce of Fi hash so let me change this to Fi hash ok and then the direction is correct so I will called this is as Fi let me call this into this as Fi hash and then this isomorphism will take the point x to the small x to the point small y so it will induce an isomorphism of the local ring of y to the local ring of x and that's exactly this so that is just Fi upper hash at y and mind this is Si infrared o, whole v y, ok.

Which is off course you know the same as o y one and this also this also the same as o y y, right and this is identified with o u x, ok and y is off course y is off course Fi x ok, so this Fi hash it y this is a isomorphism the level of local rings this is the original Fi that you started with ok, so moral of the story is, if you start with an isomorphism of K Algebra isomorphism between the local rings at points of two varieties then two varieties are birational that's means the open subsets on each which are isomorphic to one another ok and told you that the property of varieties being isomorphic on open subsets it's called birationality ok and so all you are saying is that if the local rings at two points of two varieties are isomorphic then the two varieties are birational.

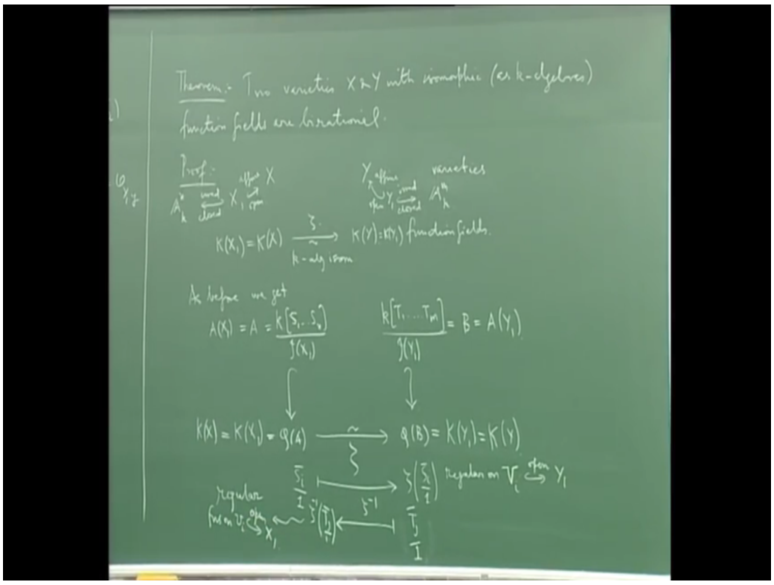
So this is just tell you that the local ring carries a lot of information ok it carries lot of global information also so global information in sense does not carry information about the whole variety but it certainly carry information about an open set containing the given point where you considering the local ring but that open set is off course dense open set so it's a non-empty open set which is reducible and dense because Zariski Topology open sets are huge ok therefore the local ring also contains information over a huge open set ok.

So that's a, you that asa matter of fact you know you must also notice that the dimension of the local ring is same as the dimension of the variety so the movement the local rings at, to given points of two varieties are isomorphic it means first of all it means the varieties are having the same dimension ok so this one happen if the varieties do not have the same dimension ok if the varieties are different dimension then you cannot find an isomorphism even I means you cannot find isomorphism between local rings of, from a point one variety to the point of the another variety ok.

Therefore the first thing you should notice is that once you have the isomorphism of local rings it already tell you that the two varieties having the same dimension but the story is more the fact is that the isomorphism of local rings actually makes the, makes them birational ok, so that's one, that is to say about the power of local rings but then I also wanted to tell you about, I also wanted to tell you about in connection with this about the power of functional fields ok.

So the next result that I am going to talk about is, about the fact that you know if you have two varieties such that their functional fields are isomorphic as K Algebras ok then also they are birational ok, so isomorphism of function fields of two varieties is also you know it also has a same effect as having isomorphism of local rings ok, so let give this note book back to you.

(Refer Slide Time: 26:36)



So let me make, so here is a theorem so this is kind of because it is connected with this, I am continuing with this if two varieties so this tells you about the importance of the function fields ok if two varieties are isomorphic function fields they are birational and conversely ok. Two varieties x and y with isomorphic as K Algebras function fields or birational ok, so this is you know now the, now I am for a movement I am trying to tell you about the importance of the function fields alright.

So you know the function fields of a variety is an invariant in the sense that it will only change up to isomorphism if you change the variety up to isomorphism ok, so off course if two varieties are birational ok then there is an isomorphism on open sets ok and this isomorphism on an open sets will need to isomorphism of the function fields ok and you know the function field will not change if you go to an open set ok, so if two varieties are birational then certainly their function fields are isomorphic that's obvious ok but what is not obvious is the other way round.

If you just say that field of rational functions on one variety is isomorphic as a K Algebra ok to the field of rational functions on another variety that's good enough to say, to saying that as off course saying that the varieties are them self-isomorphic on open subsets namely that they are birational ok, so the proof for this also once in a more or less in the same I mean the ideas are nearly the same as in this proof ok.

So what you do is see you have seen you have so I have I have x I have yvarieties alright and I have k x and I have k y, they are the function fields and I given and I am given isomorphism let me call this as cita ok this is a K Algebra isomorphism so I have K Algebra isomorphism between the function fields of two varieties ok and what I want to say is that just as in this case the when you have an isomorphism of local rings as you able to took up two open set's where they isomorphic you can also took up open sets here and her where the two varieties are isomorphic ok just an isomorphism at the function field level is enough ok and how do you do that again what we do is.

Will go to so just asa we did there we went to affine open sets ok which is helps us to translate everything into commutative Algebra ok so we can do the same thing here we can assume x one is an affine open in x and you know y one is an affine open in y, and well if you go to affine open off course whenever I say open set is an non-empty open set not worried about the empty open set ok, so if you go to affine open set then well what happen is the function field does not change ok.

The function field does not change so this is same as k of x one this is same as k of y one ok so it is enough to show that the x one and y one are birational ok because that will mean that x and y are also birational just because x one is just an open subset of x and y one is an open subset of y, so you know again I follow the same I follow with the same notations so what I do is I think of x one as sitting inside a n, ok, and I think of y one sitting inside a m, alright and I take on a n, I take the coordinate to be s one through s n on a m I take the coordinates to be t one through t m, ok and I write a to be the affine coordinate ring of x one b to tb the affine coordinate ring of y one I use the same notations.

But the only think that is missing is I do not have these two points and the local rings and the isomorphism between local rings I do not have that but I have the, I have the isomorphism here I have the isomorphism at the quotient field because after all the quotient field of the affine coordinate ring of variety is same as the function field of the variety so what I have now is as before as before what you, we get, we get the following diagram, you get diagram similar to this only you do not get this ok.

This is not given to you there is no point here going to point there such that the local rings are isomorphic there is only an isomorphism between the quotient field of a and the quotient field of b, so what happens we get, we get this diagram so let me write it down here, so a is the affine coordinate ring of x one and you know what I have done is as in that case I have embedded x one as an reducible close subset of a n, so that is affine coordinate ring becomes the affine coordinate ring of a n, which is the polynomial ring in the, in the coordinates s one through s n, module the ideal of x one in there affine space ok, and I have on this side I have y one also embedded as an reducible close subset of a m, and here again I will get if I take the coordinates on a m, to be t one through t m.

Then if I take k of t one through t m, this is going to be the, this is coordinates ring of the affine space a m, and then I go module o, the ideal of y one I end up with b, which is the affine coordinate ring of y one ok.

And then what I now have is, well this is sitting inside the quotient field of a and but the quotient field of a is, is just a function field of x one and that is a same as function field of x and similarly this is sitting inside the quotient field of b which is the quotient the function field of y one which is also same as function field of y and now I have this isomorphism zeta which is an isomorphism between these two function fields ok now you know now I have to play nearly the same game as I did here and well the in fact the truth is that you know somehow in way if I had proved this statement first ok then that statement would have been easier to prove because you know if I had given you this statement first then if you go back to this situation if isomorphism between local rings will induce an isomorphism between the quotient field and once I have isomorphism between the quotient fields.

If I grand that I have already proved that is birational I am already done so in a way this could have been reduce from here but then this no harm in doing like this because you learn some techniques ok so basically what we do in this situation is the following so you know you take you look at all these Si's which are the polynomial ring their images in a r the Si bars ok they are all elements of this quotient fields and you apply zeta to them ok so what will happen is that you get the all the Si bar each of Si bars going to I mean you are thinking of Si bar as Si bar by one, that is the way you think of elements of the domain as elements of the quotient fields ok and you apply zeta to this.

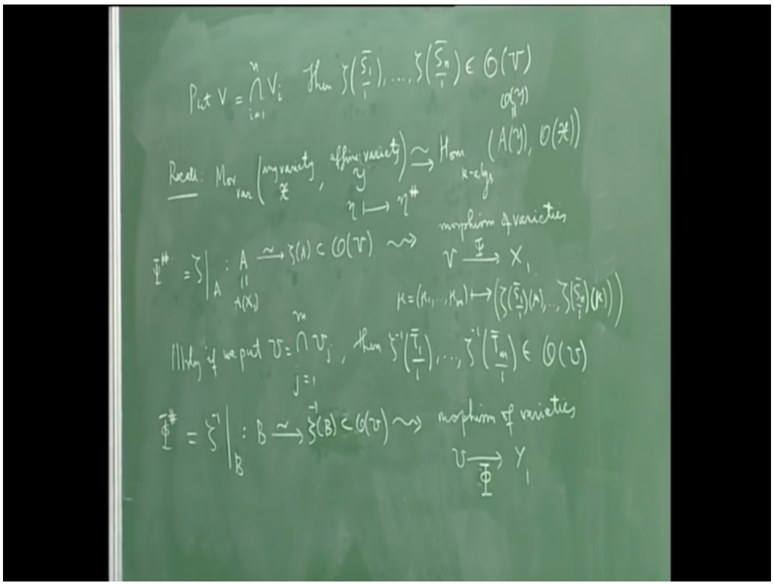
So I will get zeta Si bar by one alright and the point is that all these, what are the zeta Si bars by one, well the rational functions on, they are rational functions on y one, they are rational functions on y one, and you know what is, so you know you should remember that k y one is rational functions y one up to an equivalence ok so rational function is given by a regular function on an open set and two such pairs consisting of a regular function and an open set which its define are set to equivalent if the intersection those two open sets the corresponding regular function co inside ok and mind you any two non-empty open sets will always intersects because irreducibility ok.

So you know any rational function here is an regular function on an open set so you know each Si zeta I bar by one is regular on some open set it is an regular on some open set and there only n of them ok so if you take the intersection all of those open sets then these fellows become regular functions on that common open set ok so you know so let me write that this is regular on a u, let me call this as these open sets v I open inside y one ok these are all regular there and similarly I can do it the other way round.

So what I can do is you know I can also look at the T's ok, the Tj's are all coordinate functions here and their image is in b will be the Tj bars and then you take the their images is here the quotient fields then follow by zeta inverse ok and I am going to get an rational function on this side ok so the same argument is apples so if I take the Tj bar divided by one and then you know if I apply if I apply zeta inverse mind you then I will end up with zeta invers of Tj bar by one and these fellows are going to be regular functions on Ui's where Ui's are open subsets of x one.So these are going to be regular functions on Ui which are open, open in x one ok so I am going to end up like this.

Now you see if you look at that carefully so let me rub this so you know what you do, you do the following thing so this zeta of Si bar by one is a regular function on Vi foe each I, I equal to one to, n, ok now you take the intersections of all those Vi's ok.

(Refer Slide Time: 39:49)



Put v, put v equal to intersection or v is equal to intersection I equal to one to, n, Vi ok put this then you see then the zeta of s one bar by one and so on zeta of s n bar by one they are all in o, v, ok they are all regular function on v alright because each one is regular on Vi and all of them out to gather are going to be regular on v alright so you know the point is that you see we use this fact here so if you recall if you recall the there is a canonical bijection between the morphisms of varieties from any variety if I call it as so I am fraud of notations so let me use some script notation I am short of symbols so let me use some script notation you take any variety x and you take an affine variety y ok, and you look at the set of morphisms of varieties from script x through y alright script x to script y then this is in an natural bijection with the set of all homomorphism's of K Algebra’s from a y to, o x and mind you a y is same as o y because y is affine ok.

See we already seen this we have seen this bijection set, set of all morphisms from a variety into to affine variety is in bijective corresponds with the set of all K Algebra Homomorphism's from the affine coordinate ring of the target affine variety to the regular functions on the source variety ok now you know you see now you see what you do with these guise ok what you do with these guise is that you know I can use this to define I can use this, these regular functions n regular functions to define a morphisms from v into x one.

So what you do is you see, you see what you do? Is the following I mean it is exactly if you go back and look at it that’s how we establish this bijection ok so what you do is you see you have see zeta restricted to zeta restricted to a, will actually go from a into zeta of a which is sub of o v, ok you see zeta if you see the zeta is from quotient field of a to the quotient field of b, alright and what is any element of a? Any element of a, is a polynomial in the Si bars ok any.

See the Si's generate the polynomial ring of a affine space and their image is may generate Si bar will generate a in any element of capital A is actually can we taught of as a polynomial in the Si bars ok and therefore you know but all the Si bars their image is under zeta are all line they are all landing in K Algebra o v, they are all landing in K Algebra o v, and therefore you see zeta will land inside o v, the image of zeta will land in o v, alright and as a result you see you get a K Algebra Homomorphism's from a which is a of x one to, o v, if you apply this bijection this will correspond to the morphisms of the varieties from v to v to x one ok alright.

So you know in this you apply with script x equal to v and script y equal to x one ok, and you will get a morphisms from v to x one so so by this result this gives rights to a morphisms of varieties from v to x one ok you get a morphisms like this and in fact you know it is even easy to write what that morphisms, it is even easy to write what that morphisms is. The morphisms is see after all this v is see this v, this v is an open subset of y one and y one, v is an open subset of y one, y one is sitting in affine space.

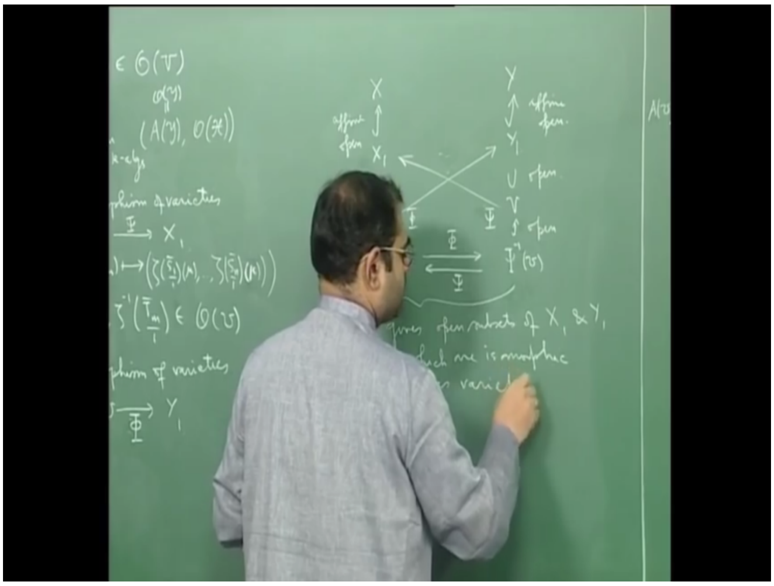
So you know any point is given by m coordinates ok and you know you just take this point of v with the m coordinates meu one etcetera meu m, and you know what you will have to send it to, you will have just send it to you know these are all regular function on v ok, these are regular functions of v, alright and therefore they are given by quotients od polynomial in m variables on v, and you substitute for those m variables these these, these values ok, so that will give you an n to pull that n to pull will preciously be an n to pull of a n it will land inside x one ok.

So if you call this as u ok then this just, this is just the map Si, s one bar by one of u and so on sorry zeta s one bar by one acting on I mean evaluated meu and so on up to zeta s n bar by one evaluated at meu so you will get this, you will get this n to pull and this n to pull will be a point of n, in fact it will point of x one that is how this bijection was established if you go back through that lecture and try to recall so you get this morphisms like this and you know if you call this so you know if you call this morphisms as again let me use some notation.

So now let me use, so let me use Si here or let me use some other notation so I will use capital Si ok I will use some block SI like that so I have morphisms like this right and what is meaning that this comes from this it means that this is actually Si hash ok the map from here to here is just Nita going to Nita hash because you know given an morphisms of varieties you have the morphisms at the regular functions level given by pull back regular functions ok and what is it mean to say that I started zeta restricted to a here and I ended up with this Si there.

It means that this Si is zeta sorry I started with zeta restricted to a here and I got a Si here it means that Si hash is that zeta ok so zeta restricted to a Si hash alright and moreover and this Si you know what you must remember is that this Si (())(49:37) Si hash is zeta and mind you this zeta is part of an isomorphism at the quotient field level this zeta from a to, o u, o v, if I go to a quotient field ok if I go to the quotient field level it is an isomorphism zeta is an isomorphism the quotient field level that something you should not forget alright.

(Refer Slide Time: 50:06)



But any way you know you can so what we have done is you know if you look at the, so let me rub this diagram lets it cost any confusion so you know situation is now what we have done is we have we have this x and we have this x one which affine opening side of x and well off course we embedded x y inside affine space where that's not important now, then there is y one this is also affine open in y and you see what I have done is I got this v which is an open set in y one and I got an morphisms like this I have got a morphisms like this alright and in the same and for that I use the fact that all zeta Si bar they are all regular functions on this open set.

Now I play the same game with zeta inverse of Tj bars ok so so similarly if we out u equal to intersection I equal to one to, j equal to one to m of Uj ok, then then zeta inverse of t one bar by one etcetera up to zeta inverse of t m bar by one they are all going to be in o u, ok, where u is an open subset, u is an open subset of x one ok and therefore if you take zeta inverse and restricted to b, it is going to give you map from b to, o u ok and I should say b to zeta b which is sitting inside o u, mind you this is an isomorphism, this is also isomorphism because there image is under an injective ring home K Algebra Homomorphism's ok.

So zeta b is, so zeta inverse b off course into o u alright and so you know again by this yoga what will happen is that you are going to get a morphisms of varieties from u to y one given by some let me call as Fi capital Fi like this ok such that well when you take this morphisms and you take the pull by regular functions it gives you, you get Fi hash and this Fi hash actually equal to zeta inverse alright so this means that you know we have got one like this so we have we have u open inside x one and we have got a, we have got a so this is, this is Si then this is Fi ok.

So u is here I have use sub of this so I have Si inverse if u, this is an open subset of this ok and then here I take Fi inverse of Si inverse of u that is that here is an open subset ok and that fact is that FI will carry this into this and Si will carry this into this the only thing that you will have to worry about is that the image of Si, the image of Si has to intersect u, ok that’s an important thing because if the image of Si does not intersect u then Si inverse u, become empty alright and similarly the image of Fi should intersects Si inverse u ok.

And so this non-emptiness statements comes because the both Si and Fi, at the pullback regular functions level they induce in fact injective maps ok and it’s this injectivity that insures that first that if you take u will intersect the image of Si and that similarly Si inverse u will intersect the image of Fi ok that’s because of the injectivity of that follows from the way we have constructed them right and therefore you get this you get this map and they will be inverses to each other.

So the fact that you are using is that you know if you have two open set’s and you have if you have two open set’s I mean if you have two varieties and you have two morphisms going in opposite direction such that you know the composite morphisms so Fi followed by Si here and then Si followed by Fi there, if they induce identity at the quotient field level ok they induce identity at the quotient field, they induce identity map at the quotient field level, identity of the quotient field then they have to be inverse isomorphism’s ok.

So you know if you go to the quotient field level Si followed by Fi is just going to be applying you know this Si is going to induce you see the Si induces zeta at the quotient field level Fi induces zeta inverse at the quotient field level so you know Si composition Fi and Fi composition Si they will induce identity maps at the quotient field level if you have two morphisms alright which induce up on composition identity at the quotient field level then those morphisms, then those morphisms have to be inverses of each other ok.

So that will tell you that you have so this gives open subsets of x one and y one which are isomorphic as varieties what I want to remark is that you know this when we the way we got the isomorphism the birational in case of isomorphism of local rings see the same thing will also work in this case ok so what you can do is that you can do something like this ok which was in the local ring case so in fact you can right you can get these things more expressly by doing like this ok.

(Refer Slide Time: 58:14)

So you know if I rub this (())(58:18) try to write that out expressly so you just have to take see if you take zeta of Si bar by one this is an element here the quotient field of v so it is a fraction so it is, it is Fi bar by Gi bar and then so you know and then if you take the if you apply zeta inverse to one by Gi bar right I get some Ai, I will get I will get an Ai bar by Bi bar ok so one by so if I take zeta of Si bar by one I will get a Fi bar by Gi bar, where Fi bar and Gi bar are you know polynomial in the T bars ok.

They are elements of b and because this is after all field of fractions of b, every element here looks like a quotient of elements of b ok, and I will and if I take the one by Gi bar ok which also make sense in the quotient field of b, ok and I apply zeta inverse I will again get Ai bar by Bi bar alright and zeta inverse of if I apply zeta inverse to Tj bar by one I will end up with some capital Ai bar by Bi bar, I mean Aj bar y Bj bar where Aj, capital AJ and capital BJ are polynomial here ok in the SS right and if I take Fi plus zeta to one by Tj bar ok then that's going to go to some small fj bar by small gj bar ok.

And now what one has to do is that so they, whenever I applied zeta I am on the you know I am on the side of the b I am working with the T bars and whenever I applied zeta inverse I am working with the S bars ok so these two are connected with the T bars and these two are connected with S bars.