Basic Algebraic Geometry
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Mod-14 Lec 37
Geometric Meaning of Isomorphism of Local Rings – Local Rings are Almost Global.

Alright so we are discussing the importance of local rings and what we saw in the last lecture was that the you have seen two theorems to tell us the importance of local rings so the first one was that a morphisms is an isomorphism if and only if it's a homeomorphism and induces isomorphism at level of local rings so that is the characterization of isomorphism in terms of local rings ok then the other thing we saw was we saw that a rational function ok and element of the function field of a variety function which is which maybe define only on a proper open subset that can be I mean you can insure that is actually a global regular function if and only if it occurs in every local ring consider as a sub ring of the function field ok, so in this regard, what I want to tell you is that.

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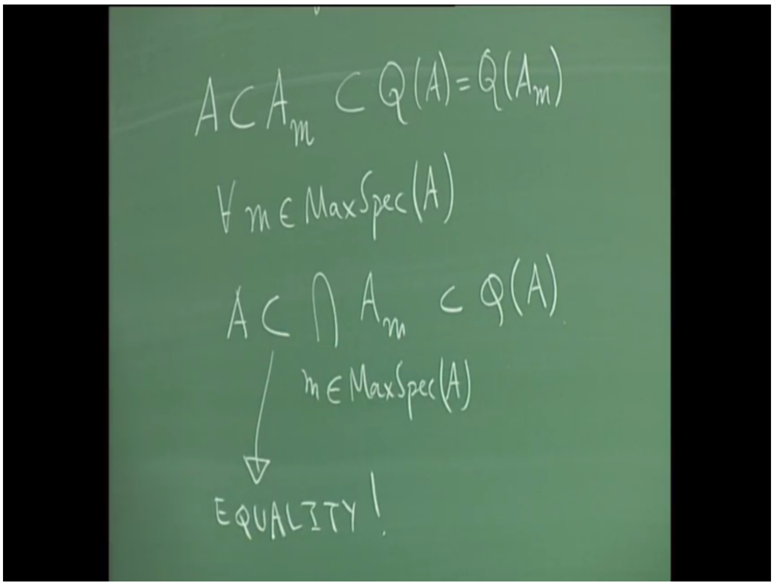

There is Algebraic fact which is simply Algebraic fact and this what we proved last time you see if a is an integral domain, a is an integral domain, and then off course a is contained in its localization of its any maximal ideal ok, so max spec a is the setup of all the maximal ideals of a ok, and off course any integral domain contained is, in its localization at a maximal ideal this map a to, it's just localization, is just a localization map ok, and this injective and that is why we consider as a sub ring of the localization of a at m, and further this is contained in the quotient field of a which is the same as the quotient field of a of the localization of a item ok.

So this is, what this step, I mean this always happen alright and what this tells you is that a contained in therefore the intersection of all the localization at various maximal ideals inside the quotient field but the fact is that, this is actually an equality if you go through the proof of the statement that you need last lecture that a rational function is a global regular function if and only if occurs in every local ring ok then we have, actually we have if you see the commutative Algebra that we did there.

The proof actually proof this that if a is an integral domain, then a is exactly the intersection of all its localization at various maximal ideals the intersection being made sense of in the quotient field of a so, so this is completely you know Algebraic, commutative Algebraic fact but it has a geometric mean, geometric mean is that if you take for a the if you take a to b, the affine coordinate ring of an affine variety ok, then this quotient field of a is nothing but the function filed of a variety and you are saying an element of the function field which is in every local ring has to be in has to be in global regular function ok.

Because the Am will then be the local rings at the points Am for example of the local ring of the variety with the affine variety with coordinate ring a, at the point corresponding to maximal ideal m, ok and that the fact that intersection of all these local rings is a, is a nice statement that when you translate it geometrically it means that a rational function which is in there every local ring is actually global regular function ok, so basically I just wanted to point out the, this an Algebraic fact but it has a geometric meaning ok.

Geometric meaning is that a rational function which is in every local ring is actually define everywhere it's a global regular function alright ok, so now what I stated in the end of the previous lecture was a fact about a birationality let me again recall the statement so the this another nice result that tells you about the importance of the local rings so the result is that if you have two varieties which may not be related at all, but you know suppose I know the local ring of at, of one variety at one point is isomorphic at K Algebra to the local ring of another variety at another point then the result is that these two varieties are isomorphic on open subsets containing the respective points which we called as birational ok.

So just isomorphism of local rings produces an isomorphism over a large open set because you know open sets are large in Zariski Topology so this also take to the local ring is not so local it also contains lot of global information ok, it contains information a large open set and that is in inevitable because Zariski topology all open sets are huge because they are all dense any open set is dense ok for a variety so off any non-empty open set so let me write this down so theorem.

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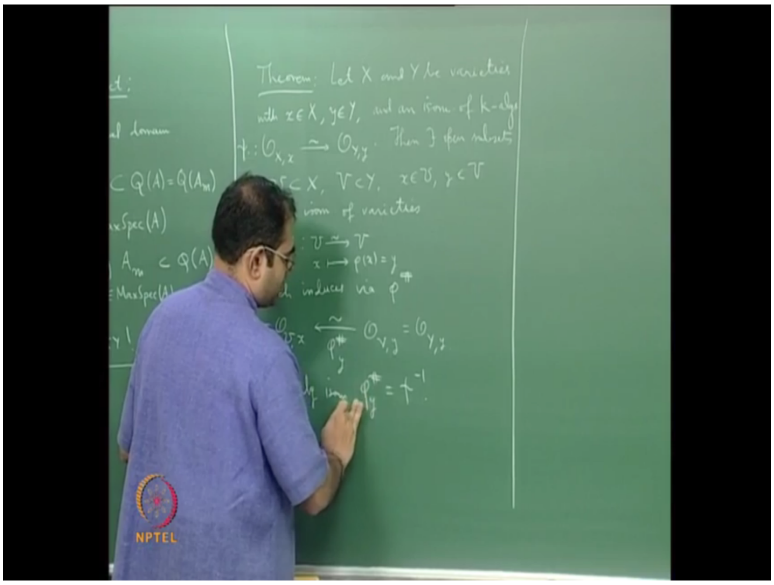

Let x and y v, varieties with x small x of point capital x, small y point of capital y, and an isomorphism of K Algebra form the local ring of x capital x at small x, to local ring capital y small y, so this is maybe you can call as I do not know what do I called it in last lecture so let me call it as Fi, I do not know what I did called it, I called Si, ok so let me called it Si, let me use the same notation so I have these two varieties now I have points where the local rings are isomorphic by k algebra isomorphism Si, ok.

Then there aces open subsets u, of x, v of y, with x and u, y in v, such that and an isomorphism varieties Fi, from e to v that takes so that x goes to Fi of x is y, k x goes to y, and which induces via FI hash so I will get a map from local ring of v at y, to the local ring of u at x, and this is this map is p hash sub y ok, and mind you the local ring of v at y, is same as local ring of y at capital y small y because the local ring does not change if go to an open set and similarly this is a local ring of capital x small x ok.

So Fi is an isomorphism of varieties therefore this also be an isomorphism the fact is that this isomorphism is none other than Si inverse ok, K Algebra isomorphism given K Algebra isomorphism Fi hash y equal to Si ok, so in other words any isomorphism so what I am saying is you see if a local ring of one variety at one point is isomorphic to a local ring of an another variety at another point then that isomorphism is actually induced from an isomorphism on an open subsets of on a open neighborhood of u of the point small x and an open neighborhood v of the point small y, they are isomorphic ok.

So the isomorphism of local rings comes from an isomorphism of huge open sets which contains those point s so in other words see isomorphism at the local ring if you think as local ring as concentrate attention on a point what you are saying as isomorphism of these local rings has can actually be spirant to an isomorphism on open sets ok, so that's the power of so you know and you know when we have two varieties which have, which are isomorphic on open subsets ok.

Then we call those varieties as birational and what we are saying is here is that off course if two varieties are birational then the are going to be local rings here which are going to be, they are going to be isomorphic local rings ok, wherever the isomorphism is define but conversely if you want two varieties birational all you require is one isomorphism between local rings ok, so that is the power of local rings, so let's try to prove this fact, the proof also involves just commutative algebra and several facts that we have already seen.

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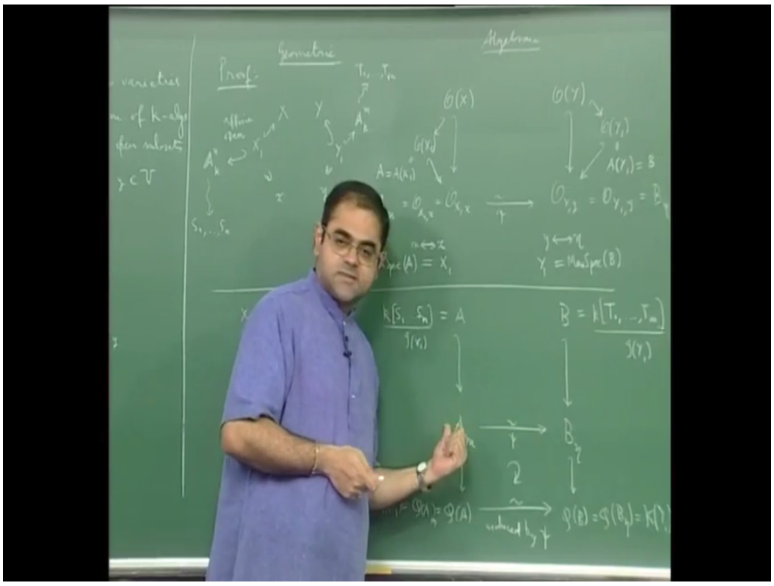

So here is proof, so the first, what is given to me is that, I am given, x, I am given y, and I am given a point in small x and capital x, and I am given point small y and capital y, ok, and what I am given is the at the level of local rings and I am given isomorphism so well if I draw the diagram corresponding to this here ended, well x corresponds to o, x, and y, corresponds to o, y, so you see here I am writing the if you want this is the geometric side and this is algebraic side ok.

So then I have corresponds to the point x, I have the local ring o, x, x, capital x small x, and here I have capital o, capital y small y, and then I have this isomorphism which is given to me, which is given to me Si, so K algebra isomorphism okthis is what I have alright, and what I am suppose to do, I am suppose to find an open set u here which contains x an open set v, here which contains y, an isomorphism between u and v which induces at the point x this isomorphism ok.

So the first fact I will use is that any variety is covered by finitely many open subsets which are isomorphic to affine varieties, so what I will do instead of x, I will look at x one inside x where x one is affine open, x one is affine open which contains the point small x, ok, and similarly I can find affine open y one, you can find affine open y one, an affine open subset y which is affine open means this is an open subset which is isomorphic to affine variety and which contains the point small y, ok.

So here I am using the fact that we have proved earlier that any variety can be covered by finitely many open sets each of which is an isomorphic to affine variety right and well what happens to what happens to this diagram on this side see I will have I will have this so what you must understand is that there is a inclusion of the regular functions are included in the local ring ok, and in fact what will happen is that if you go to, o, if you go to x one that will be that will sit inside.

That will sit in between ok, because you know for any variety restriction of an regular functions is an injective homomorphism ok, because if you have an regular function on a bigger open set I can restrict regular function to a smaller open set and this restriction is injective because if two regular functions agree on a smaller open set then they agree on the bigger open set ok. so this will and mind you this o x will be well it is going to a x one, I can write o x one equal to a x one because x one is affine because x one is affine and well I have a same kind of situation here.

I have o y one, this is sitting inside this, and this is sub ring of this and this is a same as a y one, ok, and off course and what about the local rings see the local ring of x capital x small x will be the same as the local ring of capital x one small x, because a local ring will not change if you go to enough affine open set and then in this identification with a x one this local ring is going to correspond to some a, so you know let me do something let me call this a, I must had little bit space to write things so remove part of this diagram to left side so that’s easier.

So here is y and here is y one, here is small y, ok, so this a x one let me called as a, let me a y one as v, ok and see what you must understand is that this x one is an affine variety therefore you know this x one can be embedded into some affine space into some a n, ok, off course we are working over small k which is algebraically close field always and similarly this y one sit inside some a, a n, ok.

So when I write like this what I mean is a is that x one is isomorphic to an open I mean a close reducible close subset of a n, and y one is isomorphic to an reducible close subset of a m, ok. And therefore what you must understand is that the, the points the point s of x one correspond to maximal ideals in a and the points of y one correspond to maximal ideal of v, and the point small x will correspond certain maximal ideal and the local ring it will be just a localize at that maximal ideal ok and where the maximal ideal m, correspond to the point small x ok.

In this identification of capital x capital x one with max spec a, after all max spec a set of all maximal ideals in a by the (())(19:48) with the points of x one because a is a of x one, alright and similarly the so x corresponds to the point x, small x capital x one corresponds to maximal ideal n of a and the local ring is then just the localization of a at m, ok, similarly y, y corresponds to maximal ideal mita which corresponds which comes again by this corresponds (())(20:22) and have max spec of v, the maximal ideal of v corresponds to points of y one ok.

And the point small y, y one corresponds to this maximal ideal mita and therefore the local ring here will be the same as local ring of y one at y and that can be identified with b nita ok, so we have already seen this ok the local ring of an affine variety at a point is just given by taking the localization of the affine coordinate ring may be the ring of regular functions at the maximal ideal that corresponds to that point ok.

So basically you know at the, at the end of all rings my diagram, and you know what I am going to do is, I am going to find an open subset of x one and I am going to find open subset of y one, and I am going to find isomorphism between them ok and much realize it is good enough because an open subset of x one is also open subset of x, and open subset of y one is also open subset of y, and therefore you have also found an isomorphism between an open subset of capital x and an open subset of capital y, ok.

Which contain the point x and y ok so in other words you know to say into incentive words without loss of generality I could I assume that x and y already affine ok, so any way so my so my diagram is now so I simplify my diagram so I have this a, and I have this v, and I have a sub n, I have b sub n, and I have the Si which is the isomorphism of local rings ok this is a situation and well you know and therefore looking at only the I am only looking on the affine case.

So here this so on this part of the diagram you know so let me so let me put a line here and look at this part of the diagram I have x one I have y one, x one corresponds to n, y one corresponds to b right so I have the situation now what I want you to undeserved is that the what about see if you look at quotient field of a this is a same as quotient field of a m, ok, the quotient field of a is same as quotient field of a m, and what is that it is a function field of x one ok.

So this is just k x one ok and similarly the here the and off course a sits inside its localization and the localization sits inside its quotient field ok similarly b sits insides its localization and that sits insides in the quotient field of b which is a same as quotient field of b nita which is the same as function field of y one ok, so this is an end this again this is a fact that the function field of affine variety is just a quotient field of its ring of regular functions ok.

So x one is affine variety its ring of regular functions it's a which is also the affine coordinate ring of x one and if you take its quotient field what will you get is function field of x one ok, and function field of x one is off course the equivalence classes of rational functions where rational functions are regular functions on open subsets ok, so what you must understand is that whenever you have the isomorphism between two ring to integral domains then it automatically induce an isomorphism of quotient fields.

If two so this is very natural fact which comes out ofthe, if you want the universal properties of localization or the universal properties of quotient fields, which says that given a ring given a integral domains its quotient field is smallest field which contains the given ring main so if the ring is embedded in another integral domains ok then if a ring is embedded in another field ok then its quotient field will also get embedded in that field so the quotient field is a smallest field in which an integral domains can sit ok.

So because of that property Si will induce an isomorphism here I will call this I will also call Si so maybe I will so let me write this induce by Si which is just induce by Si ok, and this diagram commutes ok so what you must remember is that it is even very easy to see what this map is ok, what is an element here an element here is fraction the numerator an element of a, and the denominator an element of a which is outside m, ok and that's what an element looks like and what you will have to do is.

If you go to the quotient field, the quotient field is simply quotients of elements of a to the denominator not zero ok, but you can take you can represents such a quotient, as a quotient of two quotients here ok by taking the denominator as one and then you can define what this map is simply by using this map ok so it is pretty easy to easy what this map is it is straight forward to define ok even if you will not want to worry about the universal property and things like that ok.

You can directly write out this map it is a most natural map that you can think of ok, so this isomorphism induces this isomorphism ok, and now what we are going to do is now we are going to make using the fact that that this two affine ok, so you see a which is well a is what a is the affine coordinate ring x one is ring of regular functions on x one and that is the polynomials in a n, the polynomials functions on a n restricted to x one ok.

So a is just k of x one etcetera x n, module o, the ideal of x one, ok this is the affine coordinate ring of x one alright and similarly b is b is affine coordinate ring of y one, y one is embedded in a n, so b which is the affine coordinate ring of y one is just the polynomials in a m, ok polynomials m variables restricted to y one and that's given by k of y one etcetera so there is a conflict of notation on somehow, so let me ok.

So I have conflict of notation by x one is also a variety and by x one is also a variable so it is a pretty so let me do the following thing let me change this one thing which more docents so let me use s one etcetera up to s n, where Si are there variables for coordinates and here let means t etcetera t one, divided by ideal of y one, ok, you must remember that this ideal of x one is the ideal inside this ok it is ideal in a n, and this is the ideal of y one inside this which is an a m, ok.

This a m, here is taken with coordinate s one through s n, which a m is taken coordinate a one to, p m, so this is p m, ok, so this so here the coordinates are s one, s m, and here the coordinates are t one t, ok so that's the situation and now to see now I am going to do is I am going to look at so you know this a is generated by the images of the Si's ok therefore I am going to look at what happens to the effect of Si on the Si's and I am going to also look at what happen the effect of Si inverse on the Ti's ok.

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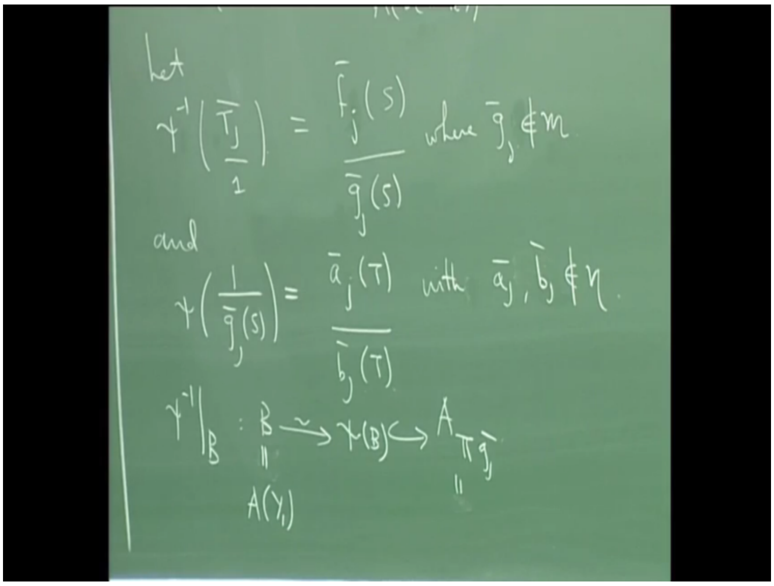

So situation is like this I calculate so let me write here, calculate Si of let's look at si of Si bar divided by one ok, so what you must understand is Si is an element in this polynomial ring ok, it's a variable in quotient you get Si bar ok, and if you go to the localization you write it as a Si bar by one so the natural map from ring to its localization takes any element of the ring to the to that element divided by one ok this is the standard map into any localization alright so any element here Alfawill go to Alfa by one so if I take the element Si bar will go to Si bar by one so I am looking at it here alright but then you know you can treat Si bar by one also Si itself, Si bar itself because if you think everything is happing in this quotient field I don't have write this divided by one but any way I will write so that you remember that I am taking this image here and applying Si to it alright.

So now what is this now the image of this is going to be something here alright so but something here is a co, it is a fraction it is of numerator and denominator, the numerator is an element of b, the denominator is an element of b, which is not in Nita maximal ideal of Nita so what I am going to get is, I am going to get Fi bar by Gi bar ok where off course Fi bar and Gi bar are just polynomials T, Tj's, ok and the Gi bar is not in the maximal ideal Nita that correspond, that maximal ideal Nita of the that correspond to the point small y, ok.

So let me wrote GI bar does not belong to Nita so what you must understand is if GI bar is I mean Gi bar is not in Nita therefore so once one by Gi bar make sense here one by Gi bar make sense here because it is after all an element which is inverted what is b sub Nita, b sub Nita take all the elements of b and then you divide all of fractions where you invert things which are outside the maximal ideal Nita and GI bar is not in Nita so one by Gi bar n=make sense in b Nita right.

But then one by Gi bar is here ok, so it has to come from something there because after all Si is an isomorphism ok, so I can look at those element here so let me write Fi bat of Tj by so I will just put Fi bar of t just to remind you that the Fi's and Gi's are you know polynomials in that P's, ok and the S and off course something here will be a polynomial in the SS ok, so you let this also let Si, Si inverse of one by Gi bar of T, to be something here and that something there is going to be an element in the in here and if I write this as well let me write this is as Ai bar Ai, Ai of S bar by Bi of S bar, when I write S I mean the triple of variables to Sn and when I write T, I mean to prove m to variables T one, to T m,ok.

The things on this side are all the polynomials in the SS and the things on this side are all polynomials in the T's and if you go to another quotient field here, the things are in this side or essentially taught of quotients of polynomials in the SS and here the elements of the quotients of the polynomials in the T's, ok. So now what I want to understand is that see whenever you have an isomorphism of local rings then you know local ring has only one maximal ideal ok.

So whenever you have isomorphism from one local ring to another local ring then under a isomorphism image of maximal ideal is again a maximal ideal therefore what it means is that the maximal ideal of this local ring the unique maximal ideal of this local ring has to be caring by this isomorphism on to unique maximal ideal of that local ring ok, and that means that the an isomorphism of local rings will map given isomorphism of the corresponding maximal ideals and that fact that one by Gi bar is, that fact that you know so I want to say that this when I write this expression ok I want to say that this fellow here is invertible ok.

Because you see Gi, if you want one by Gi bar of T, is a unit here one by Gi bar of T is a unit here, because Gi bar of, GI bar is the Gi bar are outside the maximal ideal therefore it already inverted here so it's a unit and an isomorphism will carry unit to unit therefore it means that Ai bar by Bi, this Ai bar by Bi bar is unit ok and since Ai bar by Bi bar is unit it mean both Ai bar andBi bar are units ok so the up short of this is both Ai bar and Bi bar do not belong to the maximal ideal m here ok.

Where Ai bar Bi bar do not belong to the maximal ideal m here so this is an observation this an observation right this is an observation so now if, let interpretate this lets interpretate this little carefully now you see, somehow you know we have to go, you know you want, you have at level of small x, we have at small x small y at the local ring level you have isomorphism, you have to somehow lifting to its open sets to I have to go closer to x one and y one so which means that I will have to come go away from the local ring I have to come closer to this ring ok.

So you know if you look at what happen to image of Si so the first thing is that you know if I take you see the Si take each Si bar into an expression like this alright therefore if you take any but any element of A is a polynomial in the Si bar's ok, because A is just polynomials to SI bar module o which I will get so any element of A can be taught of as a polynomial in the Si bar's ok other words we say that Ai is generated by the Si bar's ok, therefore any element of A is a polynomial in Si bar's and if you take it image under Si ok then what I am going to get is a polynomial in these things with K quotients ok, and you see the denominator is given by the Gi's ok, therefore you can see it very clearly that the image of A and the under the image of sub ring A under Si lands inside this localization namely b localized at the product of all the Ji bars ok.

So there is a localization b invert all the Gi bar's and the product of all the Ji bar's, you have a further localization this will be a intermediate because see all the Gi bars are not in Nita ok, all the Gi bar's not in Nita and that means that the product of all the Gi bars not be in Nita ok, because Nita is a maximal ideal so its prime therefore B localized at all the products of Gi bars which is which by the way is a same as inverting each of the Gi bars ok.

So this is b localize the product of Gi bars and that is a that means sit inside a subring between b and b Nita ok and the fact is that the if you follow a by this it actually land inside this Si restricted to a ok, Si restricted to a will be will land inside this because after all what is an element a, it will be a polynomial in the Si bars if you apply Si to each Si bar I am going to get Fi bar by Gi bar so if I have polynomial expression the Si bars I am going to get a polynomial expression in the Fi bars by Gi bars and that some polynomials divided by some products of Gi's ok.

Some products of powers in Gi's ok and therefore it will like here alright so the moral of the story is that Si restricted to A, will give you injective K algebra homeomorphism from this this right and, and you know the point I want to make is that what I have we done, what have we done geometrically, see geometrically what you have done is you see you have gone to each Gi bar is a polynomial is a regular function each Gi bar is an element here but what are the elements here, the elements are here just regular functions on y one ok and what you mean by inverting Gi bar, inverting Gi bar I wants to going to the open set the basic open set given by the non-vanishing Gi bars ok.

When you invert a regular function ok you go to when you invert a regular function the ring that you get is the affine coordinate ring of the basic open set define by that regular function namely the locus where that regular function does not vanish so what you are getting here is the basic open set divide, given by the product of the Gi bar that's, this is the locus on y one where none of the Gi bar vanish ok and that's locus contains the point small y ok, that's because none of the Gi bar vanish at y and that's just restatement of the fact that none of the Gi bar is the maximal ideal Nita which is, Nita is a maximal ideal corresponding to the point small y ok.

Therefore what is happen is that you know you got an open set here alright we have in fact morphisms of varieties from this open set to x one ok, we have morphisms like this and that is because you see Si restricted a, Si restricted to a is just from a to Si of a, SI is a isomorphism after all Si is an isomorphism so the restriction is an a isomorphism and you see and Si of a is contained in b sub taught of Gi bar ok and mind you this is a this is just a of x one ok and this is the thing here is a of d the basic open set define the product of Gi bar ok and the movement here K algebra homeomorphism from affine coordinate ring one affine variety to the affine coordinate ring of another affine variety it corresponds to a morphisms in the reverse direction because we have seen this there is a bijective corresponds between the morphisms of affine varieties and the K algebra homeomorphism between the affine coordinate rings.

Therefore the Si restricted to a you know that is a K algebra homeomorphism from a to Si a, and therefore it is a K algebra homeomorphism from a to b localize at product of Gi bar therefore it induces map like this which is a morphisms varieties and that is this map that is this map and so so this is well I do not know where I should give given M to this, so let me write this just write this induce by Si restricted to a ok, that is a map like this and now whatever I did with after all a and b Si is an isomorphism what are I did with a now I conclude with b ok.

So what I do, how do I do the same thing? Wit b again what you do is that you know you look now instead of looking Si look at Si inverse ok so again calculate lets write Si inverse on generates of b what are the generates of b? the Ti bar, the Tj bar generate b ok so calculate the image of Tj bar under Si inverse ok so Tj bar is the image in this quotient ring ok and it start up as the element Tj bar by one here and you up sing inverse and you are going to get something here alright and that is going to be simply a quotient of polynomials in SS with the denominator outside the maximal ideal ok.

So you are going to get this equal to Fi of Fj of S, Ui bar by Gj of S, where Gj bar is not in the maximal ideal M, you are going to get this ok where I take, where I take the image of Tj bar here right it is going to be element here alright an element here is fraction, it is a quotient of two elements the numerator any element of a and the denominator, an element of a which is outside M, and off course elements of A are just polynomials in S ok write module O the ideal of x one ok.

So I am going to get this and again if you calculate SI inverse of, so if you calculate Si of one by Gj bar, this is going to give me one by Gj bar is mind you a unit here ok because Gj bar polynomials not to the maximal ideal M, ok, elements which are not in the maximal ideal become units in the localization in that maximal ideal and when you apply SI to that I am suppose to get a unit here ok and I will write it as a bar Aj bar of T by Vj bar of T with both Aj bar and Vj bar with both Aj bar and Vj bar not in the not in the maximal ideal Nita ok.

Because that's how (())(46:53) will look like right, so you are going to get something like this and if you repeat the same thing that you did for Si for Si inverse what you are going to get is following, so what this tells you that Si, the image of sub ring b under Si inverse will go to the localization of A, at the product of the Gj's ok so you know I am going to have something like this.

So I have Si inverse this direction right and b is a sub ring here when I take the image of b I am going to land in the localization of a at the product of all the Gj's so let me write that I have a product of Gj bar this is localization of a the products of the Gj bar's and this sits inside this and this off course sits inside am because all the Gj bar's outside, all the Gj bars are units in the, in this local rings ok.

Because they are outside the maximal ideal m, all the Gj bars are outside m, ok therefore this localization sits inside this localization and what's going happen is that if I apply SI inverse to b its going to land inside this so I am going to get something like this. I am going to get an arrow like this which just Si inverse restricted to b ok, because when I apply Si inverse to something here, something here is just a polynomial in the T bar, in the Tj bar's and polynomial in the Tj bars and if I apply SI inverse I will get a polynomial in the Fj bars by Gj bars ok.

So it is going to be some polynomial in S divided by some product of power of Gj's so it is going to land inside this ok, and again so again you know just as the earlier case you can write Si inverse restricted to b is going to give you Mal from b it is an isomorphism of b with Si b and SI b is contained in a localize at the product of Gj bar ok and but b is just affine coordinate ring of y one and this is a because is the point of Gj bar is affine coordinate ring of the basic open set where the Gj bar do not vanish so this is just a f, d f, product of Gj bar ok.

So the moral of the story is that just as in this case you get the morphisms from d product Gi bar to x one, you here so you will get a product another morphisms from the basic open set define by product small Gj bar and a identical y one ok so moral of the story is, I will, if I translate this Si inverse restricted to b, being from b to this I will get a, I will get a map like this, I will get, I have b product of Gj bar which is an open subset of x one which contains the point x, mind you the point x belongs to this open set that's because none of the Gj bars belongs to the maximal ideal corresponding to x ok so x in the locus of non-vanishing of the Gj bars and the non-vanishing Gj bar at x is geometric expression of the fact that the Gj bar are not in the maximal ideal corresponding to x so as a result I will get a morphisms like this.

I will get a morphisms like this and this morphisms is this morphisms is induces by Si inverse restricted to b, ok, so you know I am coming close to trying to find, so basically I want an open subset of x one and an open subset of y one and I want to isomorphism between them so what you have done is we have gotten open sets here and there, and we have got morphisms into the ambient space we have to make sure, you have to make this open sets smaller to check that you get morphisms between them ok.

And that's where the big deal comes so you know now what you do is you I have still things t, b inverted on the, on the side of T's I mean on the side of SS namely the Ai's and Bi's and I still have things to be inverted on the side of them of the T's namely the small Aj's and the small Bj's ok so what you do is you take you go to further localization ok you here, you go to the further localization of a at product of Gi Gj bar and you also take the product of product over all capital Ai bar capital Bi bar this is the further localization ok which sits inside this ring alright.

Similarly you go here for further localization namely you get b this already localize at product of Gi bar but you still have these fellows small Aj bar small Bj bar which are outside Nita so I can product, take the product of small Aj bar small Bj bar ok and that's the further localization that also sits inside this ok and what does it what does it, among to what is amongs to is here you are doing to smaller open set namely you are going to open set given by product of Gj bar into product of capital Ai bar Bi bar which is open inside this and x is actually here ok, and here you are going to smaller open set namely D locus where the product of capital Gi bar and product of small Aj bar Vj bar and y is here.