Basic Algebraic Geometry

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Mod-12
Lecture 32

Fields of Rational Functions or Function Fields of Affine and Projective Varieties and their Relationships with Dimensions

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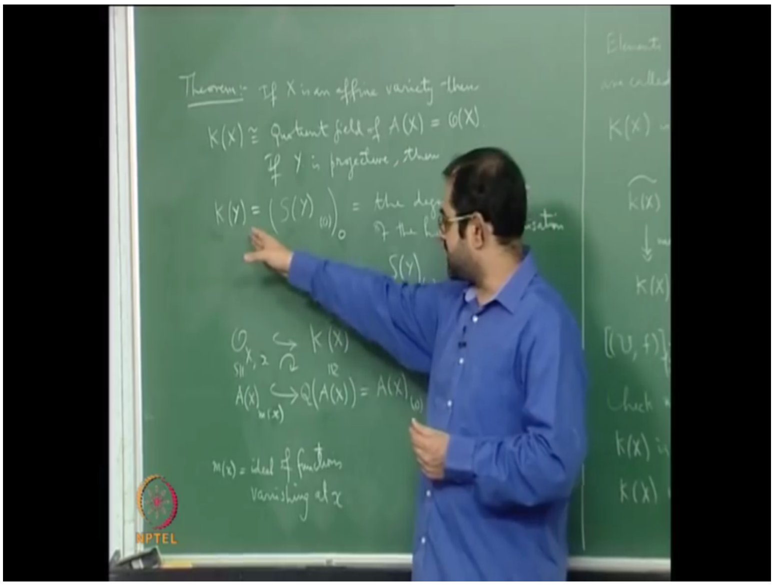

Let's continuing what we are doing in previous lecture I am trying to do this theorem that if X is a affine variety that the function field of X is just a quotient field of the a affine coordinate ring which same as a being an regular functions on X. and Y is a projective variety then the quotient the function field of Y is actually the degree zero part of the homogenous localization at zero ok and off course SY localize at zero, it self will have a being a homogenous localization it have a (())(02:05) and its degree zero part of that.

Fit it the degree zero part you get field ok right now, I will explain what about this soon, so but this is a the point, the point is that being define something in a Geometric ok and then we try to capture it using competitive Algebra I mean that's the translation from Algebraic Geometry to competitive Algebra and that and what this is, what this, the theorem tells you what this translations means what a function field ok.

If it's affine variety then the function field is quotient field of the affine coordinate ring. What is the projective variety then this given by this which also produces a fit ok. So let try to proof that and then I can use this to go and show also that there is no non-constant regular function on a projective variety ok.

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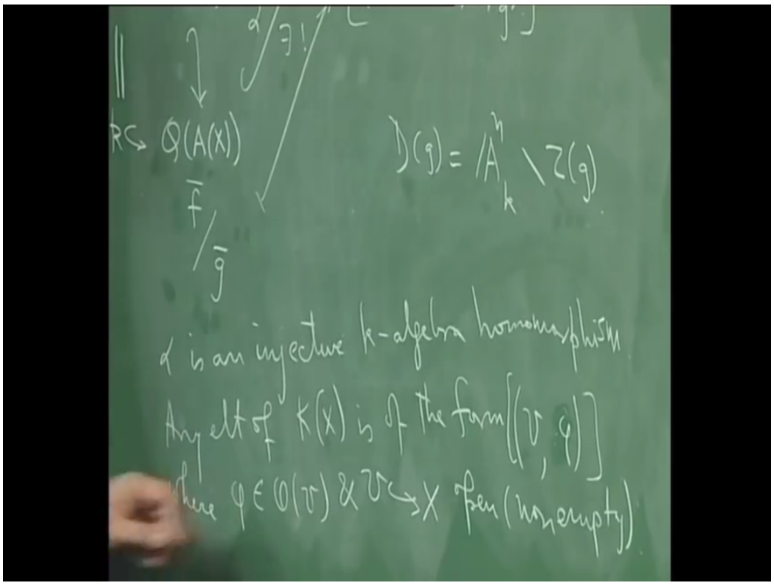

So here is aproof, so we start with X, I will fine variety, X is irreducible closet in some A of AN and well AX is, we have AX is just be definition this is affine coordinate ring of X, it's just ring of polynomials in N way which namely the polynomials function on this affine space restricted to X ok. So this is just, this is by definition A of AN divided by the ideal of X which is just and A of AN is just can be identified polynomials in invariables over K ring of polynomials invariables over K these variables excise are the coordinate function on the affine space the end coordinate functions and here is, here is your AX and by the way since X is affine there is same as OX, these two are on the same ok.

And what I am trying to show, I am trying to show that KX is same as the quotient field of AX alright so, uhh so uhh so you know we are already have that OX sits inside KX there is something that we have already seen ok in fact any regular function is also rational function after all KX is suppose to be consist it suppose to be represented by regular function on open sets and OX is regular functions define on all of X and all of X is also open subset of X ok.

Therefore OX is a subring of KX ok there is something we have already seen we have, I have mentioned here OX sits inside any local ring and that further sits insides KX which is the function field of X. and on the other hand OX is same as AX ok and AX has its quotient field Q of AX ok any integral domain sits insides its quotient field which is a quotient field is same as field of fractions any integral domain sits insides which quotient field ok and there is property which says that you know the quotient field smallest filed in which the quotient field of integral domain is smallest filed in which the integral domain sits ok.

That's also called as universal property of the quotient field or filed of fractions and therefore since QX is a quotient field of AX which is same as OX, its smallest field which contains OX and since KX is a field which contains OX ok this has to be contains in that therefore you know get, you get a unique Ring Homomphisons from here into this ok so this Ring Homomphisons is a unique ring Homomphisons which just express the fact that an integral domain sits insides of its fields of fractions or quotient field and the quotient field is a smallest field which contains a integral domain so if some other field contains an integral domain then this quotient that isomorphism copy of this quotient field.

Inside that K ok that's what is given by this need of this map, this is an injective map in fact its pretty easy to write out what this map is, see the what is the quotient field of AX? You see AX is given by polynomials F and G, AX is just OX, OX is a regular function but here the regular function jargons restrictions of polynomials ok so you take an element of AX it's a polynomials module of this ideal ok so you can think you can write as a F bar ok so if I take a polynomial in N variables namely F then there is a quotient which is going module of ideal of X and F goes to F bar.

Which is an element here ok, and off course F bar is the co-set F plus IX ok so you have this F bar so this quotient field consist of things like this, there will be things of form F bar by G bar ok so because of the field of fractions of an integral domain consist of all fractions ok quotients of two elements of integral domain with the denominator element not being zero ok, so G bar cannot be zero.

So this how an element here will looks like ok and what is element to which you will send it after all F bar by G bar if you think of it as a, after all as a there is function there is just F by G ok after all F bar if what if F bar in AX, what it is mean? It is simply restriction of the polynomial F which is a which define function on affine space to the subset X so Geometrically F bar is actually F restricted to X ok F bar what you get Algebraically it is an it is co-set it is an, it is write off, it is reading F module O, IX.

But Geometrically what is it, it is simply restricting the polynomial F to the close subset X, close subset X, so you know so if you take F bar by G bar you are just actually restricting F bar, you are just evaluating F by G, and where this F by G make sense G does not vanish ok.

and where G does not vanish it is an open set, it is open set in affine space so if it enter sec F you will get an open subset of X, so moral of the story is at just we, we will be define this X inter section DG comma F by G so this is, I have this element which makes sense here.

So what we was understand you see DG is is the basic open set in affine space where the polynomial G does not vanish ok. DG is just the complement of Z of G which is a zero set of G ok, the zero set of G is a zero set of point where G vanishes and mind you that you know that is hyper surface ok off course G need not be reducible but even if, in fact of G into it is reducible factors.

Then the ZG will be even if of those Z of those GI's, where the GI's reducible factors of G and each ZGI will be hyper surface ok which means a co dimension ones of variety ok that is N minus one dimension close sub variety of KN ok. so the ZG will be union of hyper surface and it is compliment DG which is a basic open set mind you this is a basic open set in the affine space and this is it self isomorphism to a affine variety ok.

By Rabino wishtric this is isomorphic to a close subset of AN plus one ok, so the DG is an open subset and X inter section DG is an open subset of X and there F by G make sense right and so this is how you define this map ok and so that's how this map is define this is how you will define this map naturally you can set that this here you have define this map in a very Geometric way by thinking of ever things functions but then I told you this map also comes because of the universal properties of the field of fractions of integral domain ok.

You can check the there also the definition is same ok, so this map you get either Geometrically or via competitive Algebra it is one of the same map alright. And so all these thesis that the quotient field of AX is certainly sub field of KX ok. What I am to proof that they are equal ok, so what will have to show, we have to sow that this map is surjective alright and how I do show that this surjectivewell Tic so in valet will call this map something ok let me call this Alfa ok.

So Alfa is an injective K Algebra Homomphisons, when actually its more actually you know the quotient field of AX contains K quotient field of X is a field it contains K the scalar they are all part of constant functions mind you K is sitting inside here, K is, every element of K defines a regular functions namely constant functions so its same K here so the constant functions are there and there is a field which contains this smallest field so there is a field extension and this again another field extension ok.

KX is a bigger field extension in fact you we want to show that these two fields are the same ok. Wise Alfa surjective because you know how dose an element in KX look like an any element of KX is of the form well, is of the form of U comma F, U comma Fi where Fi is OU and U inside X is open nonempty ok.

I mean this is how we define KX it is equivalence classes of pairs consisting of an open set and the regular function on that regular set ok, and this open set has to be taken to be open set inside K alright and but then you see little bit of that part tell you that this is the image of an element from the quotient field of AX and what is that elements it's very easy to guess by just coinciding attention on a single point.

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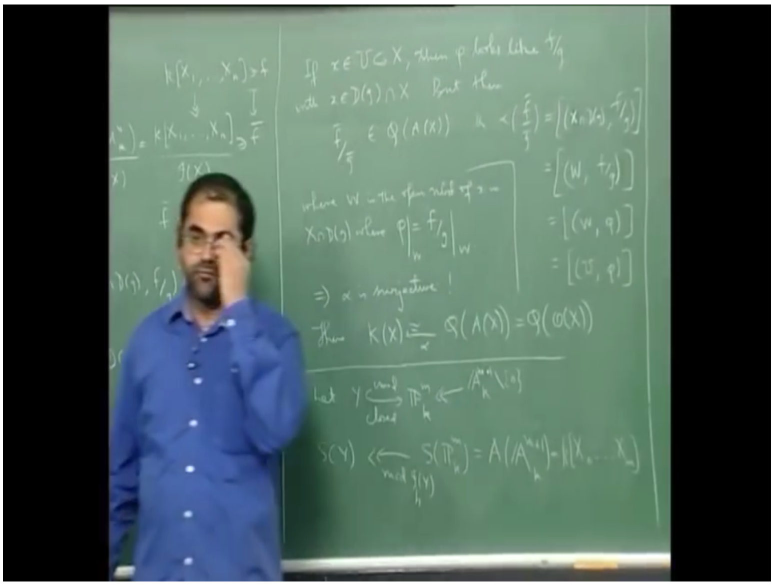

So you know if X belongs small x is a point of U inside X ok then FI looks like F by G right with X belong to DG, this how regular function is define, a regular function something that locally surrounding each point looks like a quotient field of polynomials so Fi looks like F by G means the function Fi from U to K can be identified the function F by G this is also define on a open neighborhood of point X to K and off course that open neighborhood should be a neighborhood where G dose not vanish so it has to be DG ok.

Suppose X in DG inter section X which is an open neighborhood of small x into capital X right this is just a definition of regular function ok but then, you have F bar by G bar this element if you take F bar by G bar that is certainly an element in the quotient field of AX certainly and Alfa of F bar by G bar will be you will get this as I have define this map Alfa, it is just going to be X enter the X section DG comma F by G this is what I am going to get it ok .

But then noticed that this is same as uhh and off course you know but this is same as W F by G this is two are the same where W is the open neighborhood of small x in X inter section DG where Fi is equal to F by G say after all Fi looks like F by G locally at every point so I am taking only one point you have to take this one point, take all the point as small x, so for this point small x I have a an open neighborhood W ok which is in X inter section DG ok and there on that W, Fi is same as F by G ok that means you know, so F by G make sense in W.

Fi also make sense on W, so this is same as W comma Fi and that is a same as your U comma Fi because that is evanesce, this is W comma Fi restricted to W so I should write Fi restricted to W alright restrictions are obvious let me not write this ok.

So in other words so this implies Alfa surjective thus the quotient field of X is the same as well is isomorphism this Alfa is divided, this is a quotient field the function field X, KX is same as the quotient field of AX ok which is same as quotient field of OX because OX is a same as the X ok.

So it is very simple to get the function field of a affine variety simply take it coordinate ring and take the quotient field that's it, ok, that proof the first part ok. now you are going to proof second part which about a which is a projective case right so how do you prove that, so let start with Y be a reducible closet then PN ok why is a projective variety and I am now calculating the ratio the function field of Y alright and well then you know off course the projective space is a quotient of the function affine space above and you know the homogenous coordinate ring of the whole projective space is the same as the affine coordinate ring of the affine space above and this is just polynomials in N plus one variables ok, and the homogenous coordinate ring of Y is just this module O ideal of Y, by ideal of Y is the, all those homogenous polynomials which vanish on Y so this is IH, I put a sub script H lets remind you it is s homogenous ideal, it is ideal generated by homogenous elements namely it is generated by all those homogenous polynomials in these N plus one variables which vanish throughout the locus Y ok.

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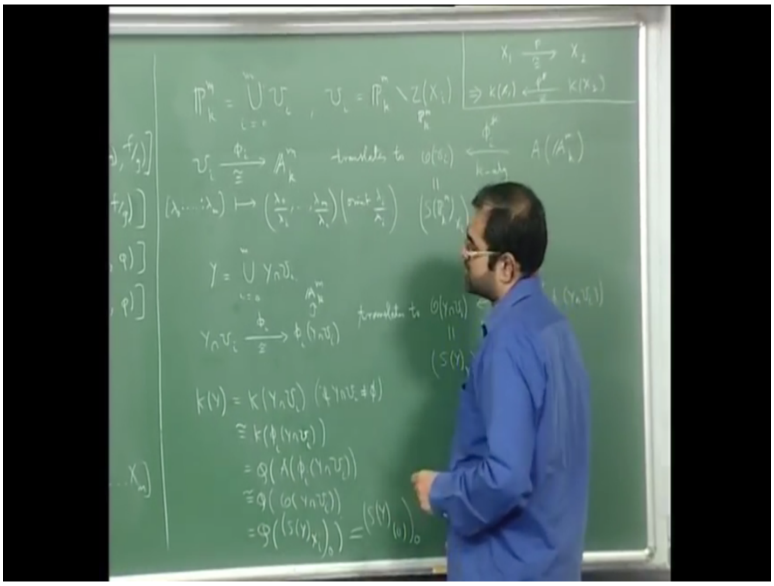

Now we have seen in earlier lectures that you know that the projective space has a affine cover consisting of M plus one open subsets which each isomorphic to affine M space ok, so you know we have PMK is union of UI, I equal to zero to M, UI is just the complement of the zero set XI ok this is a zero set in this is projective zero set this is zero set in PM.

Each excise homogenous polynomials of degree one it is zeros set will define a close subset of projective space it is called a hyper plain projective hyper it is called a projective hyper plain define by Xi, vanishing of the XI, and it is compliment is UI ok and this UI are open sets we have already seen that there are isomorphism Fi from UI to AM get the isomorphism like this and you know how the isomorphism are written UI corresponds to all those points projective, homogenous coordinates with IF coordinate mod zero.

So what you do is you take a coordinate like this this is written as lambda not through lambda M with colons to indicate that this is a common that it can be skilled by a common ratio it is homogenous coordinates you simply send it to lambda not by lambda I, lambda M by lambda I where omit lambda I by lambda this is the map, this is the map that define (())(23:33) and we proved that this map is isomorphism of varieties and under the isomorphism of varieties this translates to this is a Geometric map and it translates to competitive Algebra to an isomorphism of the affine coordinate rings namely AM of AMK so this sis by pull back this pull back of regular functions and give me a regular functions here you compose with this you will get a regular function here that is a map like this.

The opposite direction and this is K Algebra isomorphism, this is off course isomorphism variety ok, and here I will get O of UI and we have proved that what this OI's is actually it is the coordinate ring of PN, PM localize at XI and then you take the degree zero part ok so we proof this, so we proof this in an earlier lecture right, and now this is the picture with the whole projective space now you rewrite the picture with corresponds to this close subset Y ok.

So what you will get, if you intersect with Y you get Y also a union of I equal to zero KM, Y inter section UI ok so Y is covered by this Y inter section UI and each Y inter section UI is an irreducible close subset of UI and under isomorphism Fi it will give you an irreducible close subset of affine space, so it will corresponded affine variety so this just express the projective variety as union of at most M plus one affine varieties.

So you know, we use to show that any projective variety is a union of affine varieties, and we later on proved that any quasi affine varieties also union of affine varieties so therefore we proved that any variety is affinitive in affine varieties so the affine varieties building box of all varieties any way so, you have this and what happens is that, what happens this isomorphism if this Y inter section UI you will have an isomorphism Fi I, and the Fi I will give you isomorphism with Fi of Y inter section UI which is a variety there and what will happen is that the and if you this should be translate if you go to competitive Algebra this will translate to AF the fine coordinate ring of Fi of Y inter section UI which is affine sub variety Fi, F, Y, inter section UI is a irreducible close sub variety of AN ok.

And again this is KI start which is full back regular function of Fi I ok this is also K Algebra isomorphism, and what you are going to get here is the end of regular functions on Y, Y inter section UI and you know we had proof that this is, this can in fact by identified with S of Y you localize at XI and then take the degree zero part ok.

So it is, this thing when you intersect with Y you get this alright and now, here is a point, the point is well you know KY is same as K of Y inter section UI if Y inter section UI is not zero, this is nonempty ok because I told you that the function field is not going to change if you go to nonempty open subset Y inter section UI is nonempty open subset of Y, and off course I am choosing an I such that Y inter section UI is nonempty open subset of Y, if some Y inter section UJ, I do not choose that J ok.

There is certainly at least one I such that Y intersects UI ok and you take the Y inter section UI that is an open subset of Y, nonempty open subset of Y it has a same function field but then K of Y inter section UI is same as K of, KI of Y inter section UI this is equal to this because Y inter section UI and Fi I inter section UI are isomorphic off course Fi I isomorphism but then, so this is a function field of Fi of Y inter section UI, Fi inter section UI, is if Fi inter section UI, is a sub variety of AN ok and so it is affine affine variety.

And we have just now proof, we just proof here in the first part that is you have affine variety then the rush, then the function field is given by taking the quotient field of the field of, quotient field of ring of the regular functions namely the affine coordinate ring, so this si a same as quotient field of the affine coordinate ring of Fi of Y inter section UI, ok and off course here you know when I put a called T maybe I should put here isomorphism because I missing that the fact if two varieties isomorphic then there function fields are isomorphic.

So I have these isomorphism between,Y inter section UI, and Fi of Y inter section UI, these are so these two are isomorphic varieties. Isomorphic varieties have isomorphic function fields ok, so that is the fact that is pretty easy to understand I did state it but let me write that down suppose you have X so let me put you some other rotation uhh if X one is a variety and X two is variety and Fi is a isomorphism of varieties then I have Fi start the pull backup functions will induce an isomorphism of KX two with KX one ok so if two varieties are isomorphic then there function are also isomorphic.

The idea is very simple because you see if you give me an element of K X two it is going to be a rational, it is rational function so it is a regular function on open subsets of X two and if you pull back you will get a regular function on the inverse image of the at open subset in X one and that is going be define in KX one and you can check that there is isomorphism it is trivial check that if two varieties are isomorphic then there function fields are isomorphic ok.

So an isomorphism of varieties induces the isomorphism function fields in the opposite direction always they isomorphism at the level of regular functions or at the level of local rings or at the level of function fields ok all isomorphism that involve functions either locally or globally they are all induces by pull back that’s something you should not forget ok.

So isomorphic varieties have isomorphic function fields and since Y inter section UI, is isomorphic to Fi I of Y inter section UI, they have isomorphic function fields but the quotient field of Fi I of Y inter section UI, is a quotient field of affine coordinate ring ok that what you have seen in the first part and then but this is the same as well, K, so this is a same as quotient field of this isomorphic quotient field of O of Y inter section UI, because again if two rings are isomorphic, if two integral domain are isomorphic, then there quotient fields isomorphic.

So the quotient field of this is isomorphic to the quotient field of this because these two integral domains are isomorphic, but I have already identified O of Y inter section UI, with SY localize at. So this is the quotient field of SY localize at XI take the degree zero part and now I want you to check it is very simple check competitive Algebra that this is a same as simply SY localize at zero and take the degree zero part ok.

So it is last equality that has to check and it is kind of it is very very easy to check ok because you know there is only thing that needs to be checked, there is only thing that needs to be checked, so you must understand what I am doing here, here I am using the fact that if two varieties are isomorphic we have, function fields isomorphic, and here I am using the fact, if two rings are isomorphic, two integral domains are isomorphic, then there quotient fields are isomorphic and then I come to this ok, and uhh so how does we have proof last equality that's also little bit of competitive Algebra so let me rough this part so I continue.

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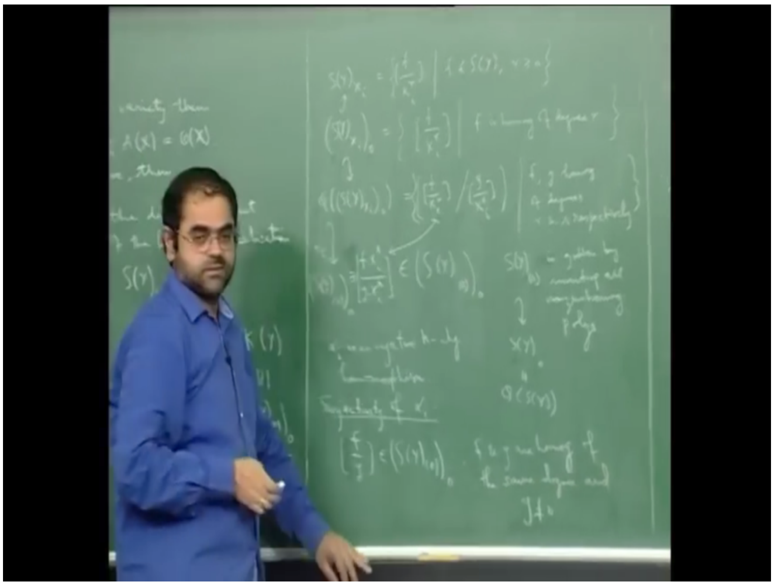

So well, so what is, what is SY localize at XI is just elements of the form F by XI to the power of R, equivalence classes like this where F is, F is polynomial in this N plus variables and R is greater than or equal to zero I mean this is what it is, and the square bracket is equivalence classes ok this is for the localization, is SY is localize with XY is just invert XI ok, invert XI means you are allowing denominators with positive integral power of XI right and what it is SX, SY, XY, XI localize at zero this, well it is such things which with homogenous degree zero which means that you know the numerator show also be homogenous denominator show also be homogenous and they should be homogenous of the same degree so that the when you take the difference of degrees it zero ok.

So this is this will consist of things of the form well sums of final sums finite sums off course also finite products all at mater well I could simply write it in some saying all that I can simply write it so, as F by XI to the power of R such that F is homogenous of degree R there is a degree zero part right and off course degree zero part which insides this ok and what about this that is and you know if I take the quotient fields of this ok thisis an integral domain alright and it take it quotient field so if I take quotient field of S, Y, XI at zero this is what this will sit inside and this will consist of quotient of things like.

So it will be something like F by XI power R divided by some G by XI power S, it would be quotient like this where F G homogenous of degree R and S respectively this is what it will be alright if I take the quotient field of that right, and well, and what you can understand that you know I can identify element like this with the element F times XI power X by G times Xi power R ok this you just divide these two you know very namely this what you will get ok.

The numerator will have degree r plus S denominators will degree R plus S, so this is a quotient of homogenous polynomials of the same degree ok so what you are going to get is, you are going to get an element here ok, so this belongs SY localize at zero degree zero part I will tell you what this means see SY is localize at zero is equal to is gotten by inverting all non-zero homogenous polynomials ok this is the notation is ring localize at prime ideal ok.

So the point what I want to notice is that this is homogenous localization so normally you localize a ring at a prime ideal you are suppose inverting everything put side the prime ideal ok but here it is homogenous localization what you are doing is you are not inverting every non-zero polynomial but you are inverting only non-zero homogenous polynomials the reason why you are inverting only homogenous polynomials is because only then this localization will get gradation ok.

If I mind you this sits insides SY localize at zero so you know this when I write like this, this is the usual localization zero is a prime ideal and relocalize it the prime ideal is invert everything outside the prime ideal so this is actually the quotient field of Sy this is quotient field of SY this is very big this is very big because this consist of quotient where in the denominators you put any polynomial which is not zero in fact you are inverting non-zero homogenous polynomials even I mean sorry in fact you are inverting even non homogenous polynomials alright.

But this is smaller you are inverting only homogenous polynomials only those homogenous polynomials which are non-zero you are inverting them, so this gives you here sub ring of this is a quotient field ok, but this is graded because the gradation can be define as degree of numerator minus degree of denominators because denominators consist of only a homogenous polynomials which is non-zero because here you are inverting only there non-zero homogenous polynomials and in these if you take the degree zero part that is what this is, so this consist of exact elements like this ok.

This consist of exact elements which are given by numerator homogenous polynomials denominators is also homogenous polynomials they are both of the same degree ok. and therefore you can see that this is certainly setting inside this homogenous localization degree zero part to which this belongs and that is the map you send this guy to this guy alright and you can check that this map is injective alright.

What we are trying to say is it also surjective that what we want prove you want to show that this, there is an isomorphism between this and this I written equality but in most cases the equality is just isomorphism now with proper identification an isomorphism treated as equality I mean if you think of this function as the same as this ok then you get equality but you prefer like this with all these weird brackets here and another wired bracket if you want ok then you can think of a formal isomorphism ok but if you just think of them as functions they just equality right.

So this is setting in side this and conversely you know you can check that if you give me any element here it come from an element here ok, so only fact that need s to prove, be prove as a surjective alright and how does one show that is very very simple conversely, inverse if you want let me call this map as Alfa sabic so, I just want to say that Alfa sabic is, well and injective K Algebra Homomphisons.

Off course you know see the fact is that this is non-zero Homomphisons from a field and you know non-zero Homomphisons from a field is always injective so you do not have to worry about verifying injectivity because the source is a field alright being the quotient field of this integral domain alright I have to only very about the surjectivity so how do you get surjectivity, surjectivity is also pretty easy well take an element here how dose an element look like, it will look like a quotient of a two homogenous polynomials of the same degree alright.

So take F by G take a quotient in this which means that F and G are homogenous of the same degree and off course G is not zero because whatever you out in denominators whatever invert is not zero so any element here looks like this and I want to tell you that this element actually come from an element here ok and that’s perhaps pretty easy you can easily see I just have to divide by the same power of XI as the degree of F which is the same degree of G so it is obvious you can see that so it is surjective obvious.

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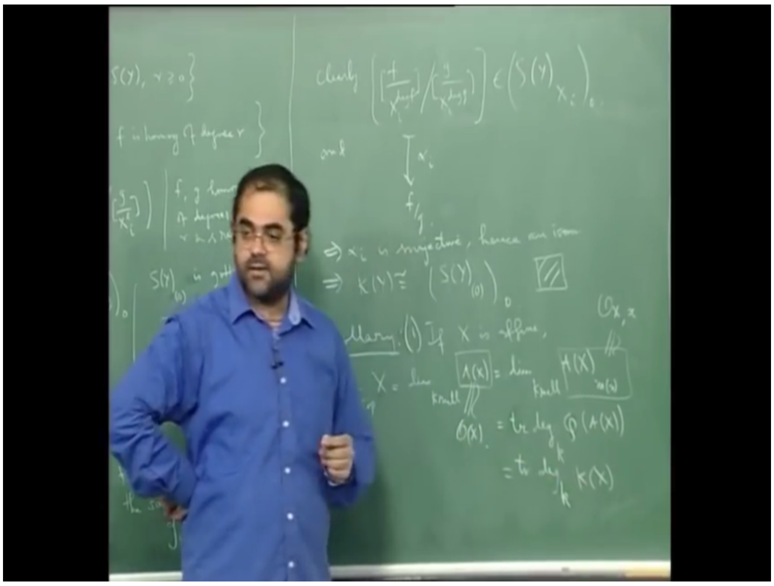

So clearly this element F by Xi degree F degree of F divided by G by XI degree G this element this fellow is in SY localize at XI took you zero and it is image under Alfa I is F by G ok so that implies that Alfa I is an isomorphism it is surjective hence an isomorphism ok. as functions if you consider everything as functions then these two are not just isomorphic they even can treat them equal ok.

So this proves the other fact the KY is nothing but take the homogenous coordinate ring of Y you localize at you take homogenous localization at zero and then take the degree zero part this is an isomorphism ok so that proves the theory ok, so it is very clear that you can you know how to write the competitive Algebraically what the function field is for affine variety or for projective variety it helps you with calculations ok, and there is an another fact that comes with in connection with these two diagrams the fact is following the fact is off course the function field is an extension of the base field small k ok and the fact is that transcendence degree of function field O of variety over small k will give you the dimensions of the variety that is also true.

So you know so here is a that is very very well, corollary is, if X is affine dimensions of X this topological dimensions of X is a dimensions krulls of AX this is also the dimensions krulls of KX at MX where X is the local ring this also equal to transcendence degree over small k of the quotient field of AX which is the same as transcendence degree over small k of the function field of X and mind you, that this guy as the same as this is isomorphic to OX small x and this guy is isomorphic to OX.

So this is one corollary, so you know you have so many formulas for the dimensions of affine variety, it is either the topological dimensions of X or it is a krull of AX is the ring of polynomials is restricted to X ok or it also the krull dimensions of the ring OX this is a K Algebra ok, it is also the krull dimensions of AX localize at MX because you know AX localize at MX is the local ring of the capital X small x ok and this is the same as the transcendence degree of quotient field of this and that is same as transcend degree over K of over small k of capital KX.

So this is an one important corollary and if you have and you know the point is that if it is projective all the statement are correct accept that you do not bringing the global regular functions because as I told you we are going to prove that projective variety all the global regular functions, are all the only the global regular functions are constants ok, so if Y is projective then you know I will get I will have to forget this ok there is only think that I have to throughout but everything else can be written so let me write that down.

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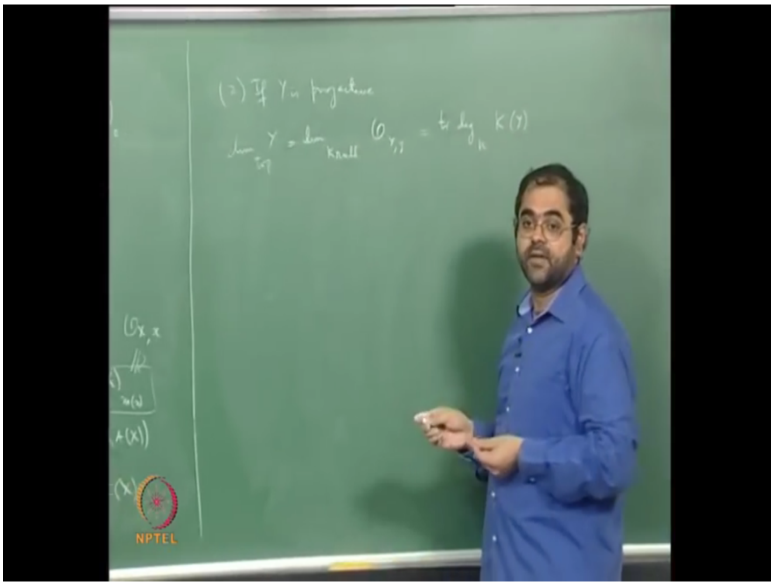

If Y is projective then also you will see that dimensions Y topological dimensions Y is the same as T krull dimensions of the local ring of Y point small y at these also transcendence degree over small k of the function field of Y this what will happen ok, and so Y is the corollary all that we have seen I have to explain that and the see we have already proved the topological dimensions is same as krull dimensions of the affine coordinate ring for a affine variety, and but you know but then you know if you take the krull dimensions of the local ring ok then you see the krull dimensions of the local ring is just the krull dimensions of this ok.

But the krull dimensions of this is actually same as the height of this maximal ideal ok, see because when you localize at a prime ideal in the local ring you are only prime ideal that survive are the prime ideal which are contain in the localiest prime ideal in the prime ideal you are localizing ok, so what is one thing in competitive Algebra that you should remember about localization if you take ring if you take a competitive ring A and if P is a prime ideal, if you go to AP what are the prime ideals in AP the prime ideals in AP are preciously the prime ideal seen which contained in P by going to AP you are thronging out you are removing all the prime ideals are outside P which have elements outside P.

So by going to AP you are actually only focusing attention on prime ideal which contains P, so you know the krull dimensions of this thing is a local ring so the krull dimensions will be it has only one maximal ideal and you know krull dimension of ring is just supreme of the heights of all its prime ideals so it will be height it will be here the biggest ideal is the maximal ideal, so the krull dimension of the ring is same as the krull dimension of the height of this maximal ideal but then this local ring module of this maximal ideal is just the base fields K ok this AX is localize at MX module O, MX is just K ok and dimension formula will tell you the dimensions of K is zero therefore the dimensions of AX mod therefore the dimensions of AX is localize at MX, MX is same as the height of MX ok and but then that height will be equal to the krull dimensions of AX ok.

And that is the reason why you get the equality ok and these equality comes because any integral domain and it's that if you have the, so these equality comes because of the fact that you know, so this also comes from a former I have sited earlier if you have finitely generate K Algebra which is an integral domain then the transcendence degree of the quotient field of the integral domain is same as the krull dimensions of that Algebra ok.

So, if you applied that to AX you will get the equal to this ok because AX is a finitely generated K Algebra which is integral domain ok and its so transcendence degree over K has to be equal to the krull dimension of X ok. So that give this equality alright so you do not get this is equal to this rather you get this is equal to this and therefore this is equal to this because these two are equal ok and off course we have proved AX is same as KX so this also equal to transcendence degree over K of KX ok and same kind of argument will give you will these equalities in the affine case ok and you have to only remember that when you if you want to do this for Y its enough to do it for YI where YI is affine piece of Y which gotten by intersecting with UIok and if you do it, to do this competition its enough to do this competition with Y replaced by Y inter section UI which YI if you want ok and then that case is already covered here therefore this follows ok therefore that's the proof of this statement ok, so I will stop with that.