Basic Algebraic Geometry
Professor Thiruvslloor Eesanaipaadi Venkata Balaji
Department of Mathematics
Indian Institute of Technology Madras
Mod-12
Lecture 31
The Field of Rational Functions or Function Field of a Variety - The Local Ring at the Generic Point

So I have, you know we are what we are looking in Algebraic Geometry is trying to look at things are which have an intrinsic meaning ok, so intrinsic means these are certain definitions and properties at you make or define which depend only on the object and not on other extraneous factors so for example you know what we are worried about is we are worried about varieties ok and then of course alright and then of course the varieties could be you know they could be you know they could beaffiner quasifiner or they could be projective or quasi projectiveand when you say, when you say these things you are trying to say that the varieties either setting as an reducible process some set of some projective space or it is setting as an open sub set of such a set ok.

So when we define variety we are already thinking of eat a setting inside somewhere either affine space or projective space so there is this and there this fact you making it set into in set some things which gives a certain ambiguity because you can use for example we take the line, the line can set inside any affine space in any way ok. So you can think of the line just a A1, can also think of it as A one of coordinate axis in a two ok or even other line in a two.

And you can also think of it as a line three space A3 ok but in any case line is a line ok. So what is incent rick about the line is a visible line all right and what is incent rick about is the way you are putting it inside, you are putting a line just as a line or putting it inside of a line the plane A 2 or you are putting a line as a three face so these are you know when we start working in algebraic geometry start building a theory of varieties.

We always start by embedding your object into some fine space or projective space, even to define variety you have to think of sitting inside of some fine space or projective space irreducible process of sub set of or as a open sub set on a such a irreducible sub set. Let it find what we want to really analyze, define and analyze studies not I am you want to define and analyze study only the properties which are intrinsic to the touch we are not interested in, they should depend only upon the variety not the way which is embedded ok.

So the fact for example you know saying that the line is one dimensional does not depend on whether the line is being part of is line A 1 or whether is being part of line A 2 which is plain or whether it is part of the line in three space A 3 whatever be the space you are embedded it's a still one dimension so we say that dimension is a very intrinsic or courteous and in fact our definition of dimension now you will see the definition of dimension in two ways minus as a topological space we have define dimension variety to be topological dimensionand we also proved that you know this dimension is same as variety is affine variety then it same as the curl dimension of the affine coordinate of the variety ok so moral of the story you know the differ mission that the topological dimension is a very intrinsic dimension it just depend on the topology of the variety does not depend on the anything else ok.

Whereas the definition that the dimension is curl dimension of the fine coordinate ring the coordinate ring of polynomials on the variety ok that ring, the curl dimension of that ring is a dimension of the variety is lies on another definition but that some more seems to depend on that ring because that ring is depend on the way is variety embedded ok if the fine variety is could be embedded it soon be affine space in so many ways.

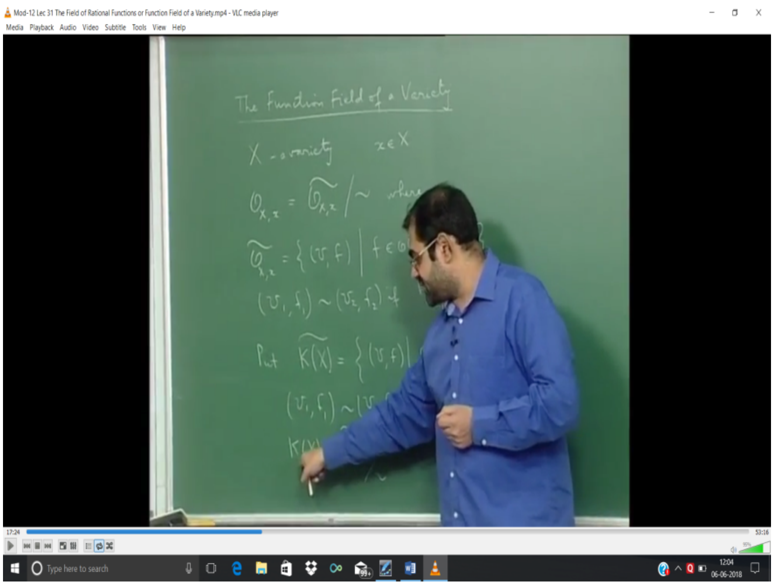
As a reducible process of sub variety and in of those cases you will get coordinate ring, ring of polynomials ok but thankfully for affine variety is we have seen that the coordinate ring itself is a invariant is a intrinsic ring namely you should take two affine varieties then the isometric deferent only if we have affine coordinate reasons for the same, so which means that no matter in which, in what way you embed the fine variety the movement you say affine variety you are considering affineisometric is a reducible process of sub set of affine space.

It could be a reducible process of sub set and affinespace of any dimension ok and the dimension could be different like a same line being embedded in a plane or a being embedded in a three space so the ambient the bigger affine space in which your variety is embedded as a close sub variety that could be deferent but still the ring of the polynomials on that variety they are all the rings as a isometric the fine coordinate rings are isometric so we space as we seeing that the fine coordinate ring is anintrinsic object that is define connected with the variety.

Now you, so this what I am trying to say we are frank to it is very important that we being by using intrinsic things ok but then finally we try to find out what are the things are intrinsic ok so in that is terms affine coordinate ring of affine variety ok which is intrinsic then off course the other thing that I introduce was a local ring that a part. The local ring that a point is a something that is define that is a in a very intrinsic way ok it is just define by the using of regular functions ok and the point that is if you change the variety isomorphismthen the local ring will also change up isomorphism, ok only if you have variety and a point with isomorphism carrying to another variety and this point going to another point then local ring of the original variety at the given point is a isomorphism to the a local ring of the target variety isomorphic variety at the image point that is got by the image of this point ok.

So we say that the local ring is also something like very intrinsic unique if you change the variety up to isomorphism the local ring will also change up to isomorphism ok. So what we talk about I saw you know in that direction I want to introduce another important ring associated with the variety in fact it not just a ring it is field it is called functional field of the variety ok.So I want to introduce that and I want to tell you that a, I want introduce a very intern way then show found to compute it for affine projective varieties ok.

(Refer Slide Time: 09:20)



So let me put you guide lectures the function field of the variety and so what is we are going to do? So you know the idea is very similar to looking at the way which we got a local ring at a point ok. So you know what we did is see we start with X a variety ok and off course which we will see (())(10:04) will be affiner quasi finer, projective or quasi projective and we have a point then you know how do you define the local ring at the point of the variety, so normally what we did was if you have a point small x and capital X then what we did is we define O capital X and small x there is local ring of X at X to be O capital X small x Telda module of equivalence relation.

And how is the equivalence relation, what was this Telda O capital X small x Telda this was just you know as a functions which were open in some open, in some are risky open neighborhood to the point X. so you know this consist of the pars of the form U command F such that F is in OU and X belong to U, so you are just looking at functions which are defined in open neighborhood of the point X and which are regular alright.

And you took such pars what is the equivalence relation, the equivalence relation was at two such pars identified they define same function on the inter section ok. So you know U 1 comma F1 is equal into U2 comma F2, if well F1 restricted to U1, is equal to F2 restricted to U1 inter section U2 is equal F2 restricted U1 into section 2. Where off course you know you have to remember that X is in both in U 1 and in U 2, so axis in U1 inter section U2, so U1 inter section U2 is a non anti open X so it is in fact dense ok and you just saying that is F1 restricted to U1 inter section U2 is equal to F 2 restricted to eveninter section U2 in other words these two functions can be glued together to give a bigger regular function on U1 union it ok.

So some sense the sense of complex analysis we say that these are two functional elements which are direct analytic continuations of each other ok. So now, off course you can, we can this condition you could just say that F1 is restricted to W is equal to F2, restricted to W, but W is a open neighborhood of a point X contain in inter section U2 that is also enough because you know we have a keep always you proved this keep using all the time that two regular functions if they are equal into non open sub set then they are have to equal everywhere ok this is true always right.

So, now you know this we got this local ring like this alright and off course we have seen that this how to compute this local ring ok and what is affine case ok and what is the projective case we have seen ok alright but now what I am trying to say if you want to function field it is very simple what you just do is, you just remove these restriction of coinciding attention at a point ok just do not worry about the point ok.

And beautiful thing is it you, if you do not wait coinciding attention at the point you go global ok, you get something global ok. So what you do is pull K X to be KX Tilde to be a set of pars U comma F so just F belongs to U and now there is no restrictions on you, U inside X as well open sub set to open monist ok. Off course here also whenever I say F belongs to O U, I am yes it is implicit understood gives open sub set of X. ok. U use as SK open neighbor ok fine, so now you see, so what is the deference between this and this, the deference is here you are only taking open neighborhood but here you are simply taking any non into open sub set and you are taking a regular function on that ok you just drop attention to a point and you will be hold what you get is you prepared same procedure what you get is not a local ring you need a field ok.

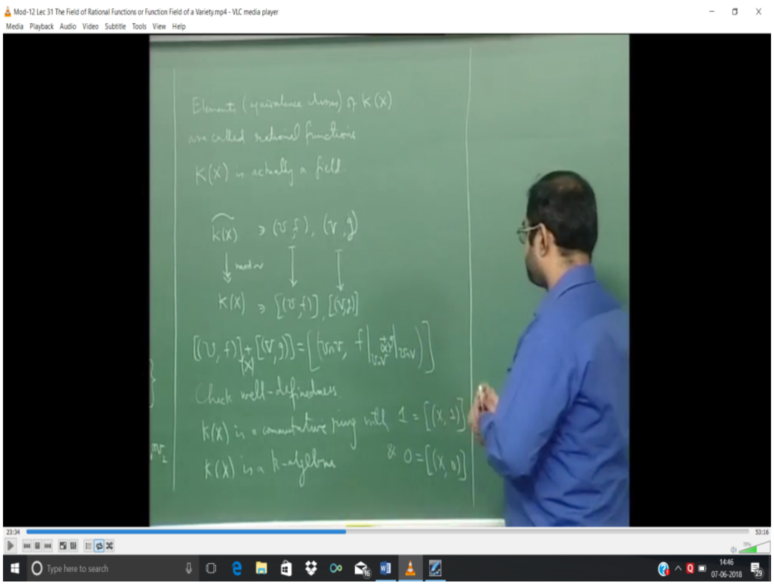
Now what you do is again you define the same thing you define same equivalence relation U1 comma F1 is equivalent to U2 comma F2 if and only if F1 is restricted to U1 inter section U2 is equal to F2 restricted to U1 inter section U2. Off course for this, I mean you are not required it on all of inter section U2 is enough to unit require to open on into open sub set of U1 inter section and you must always remember that the very important fact that variety by definition or variety is all reducible and therefore you take any non into open sub set of a variety it will continue to be reducible it will be dense and any two non into sub sets of variety was always inter set ok so this is the problem that the (())(15:50) problem is that the open set is huge ok the dense the closure is a whole variety, I mean off course non into open sets they huge and need to dabble intercept alright.

And well so it just to enough to require that F1 F2 co inside in a open sub set of U1 inter section U2 ok not into open sub set of U1 inter section U2. So these definition these equivalence same as equivalence here only T is here not, focus and attention at a point alright and then you put KX to be KX Telda mode T equivalence ok so off course when I write mode equivalence I means equivalence crosses ok so this is a set of equivalence classes this is also set of equivalence classes.

Now the beautiful thing that is just like in this case set of equivalence classes, local ring ok you got a competitive ring which is a local ring which had any maximal ideal which represented by functions which or regular in a neighborhood of X which is 9X at X ok and in fact the representatives the local ring namely the equivalence calls as Germs a function ok so this is the set up Germs at the regular point X and every Germs of a regular function represented like this it is represented by regular functions on a open neighborhood of the point ok.

Now here we call the elements here they are kind of they are rational functions ok, and basically they are to be thought of us functions are regular or in a open set ok that why we should think of rational functions. Rational function is nothing but the regular function or an open set. It is not a regular function of whole variety but it is a regular function it may be regular function only on a open sub set which may be a proper open sub set ok so that, which means there could be a point it is not regular ok where you cannot think of where might explain the function but may not be regular alright.

(Refer Slide Time: 18:15)



So let me write that down elements equivalence classes of KX are called rational functions, they called rational functions they represented by regular functions or an open sub set alright and further you see this KX this is actually is a field, KX is a field and why is that so because you see, you take two elements in KX so you know I have this KX Telda to KX portion map it is surjection that's why I am putting double arrow head and this set of equivalence classes this is mode equivalence ok and what you are doing is taking if you give me rational function U1 comma F1 you send it to its equivalence class which (()(19:37) putting a square bracket ok and will you give another rational function U2 comma F2 I will send it to rather let me use U comma F and V comma G so this will go to V comma G round bracket for square bracket, the square bracket indicating the equivalence class here and then how do you add these two guies its very simple you add them like this,

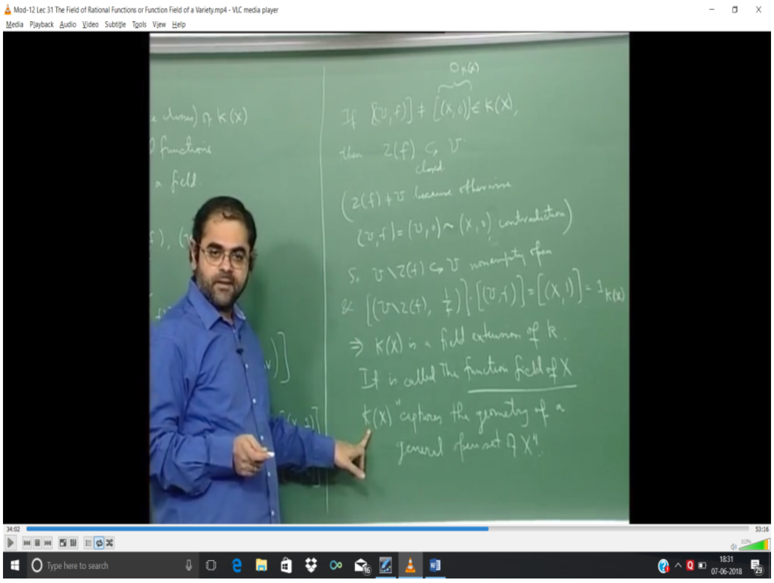
You give me two rational functions you just add them on the derivative section because you know as I told you (())(20:23) fact any two into non into sub set of will intercept on the inter section both functions make sense so I could I them so I can do this I can take this equivalence class and define to be the sum alright and then the same way I can define product, I can put product here and I can put product here ok so depending up on what you are either some more product can be define like this and now I want you to check, it is easy to check as you must have done in the case that this definition of addition and multiplication easily depending on choice of representatives because I have define the sum.

When I have define the sum of two rational functions I am actually taking some of the representatives and I am taking the some of the representatives on the inter section, on the inter section of the dominance by the representatives functions define ok so and similarly when I do, for a product I also use representatives but they, whenever you know you use representatives you have to make sure that your definition is correct its well define so you have to check ok so check that, check well define as that’s simple exercise that same kind of exercise you have under gown for this case over, you for local ring.

So this will make, KX into quantitative ring in fact you will have the KX K is a competitive ring off course it is competitive ring because addition of functions multiplication of functions is point wise and that is quantitative because if they are, the functions are taking values base field K which is competitive alright. So it is a competitive ring with one to be, one with be given by the pairs X comma constant function one and zero element given by pairs X comma zero ok so it will be competitive ring with unity and with this is zero element and in fact it K algebra because you know regular function multiplied by regular function multiplied by KLR it is also regular function because KLR off course regular function we have taught of strong regular function.

So KX is K algebra off course you know in our discussion we always fix small k to be an algebraically close field where we working that’s why we do study of variety so this KX is KL require ok. All function take value of small k, and well know the point is fine the point is a well in this case after you took if you took open neighborhood of a point and then you define these equivalence and when mod equivalence equal is max you got a local ring. You got a competitive ring which also small k algebra but use maximal it had only one maximal ideal and therefore you get the local ring, the point is in this case you do not get maximal ideal zero and so will get a field so why is a field to show that the field I will have to take every on zero element here is invertible ok.

(Refer Slide Time: 24:20)



So if you take U comma F not equal to zero element, zero element is X comma zero in KX ok so then you know what I want to understand is that you see, you take zero take Z of F ok. See Z of F inside X will be a close sub set ok so what I want to understand that whenever you take a variety and you take a regular function on it ok and you take the set of, the local vary vanishes ok that always a close sub set ok. So in fact you know I should say it is a close sub set of U in fact because F is define only on U which not right X here be careful alright so F, F is regular function on U and mind you U is a variety.

U is any open sub set of variety is again a variety ok. So U it self a variety F is regular function on U take the zero set of F in the set up points of where is F vanishes that's gone be a close sub set of U ok because actually you know roughly the idea is that the close sub set that define by vanishing of polynomials alright in fine case it is, in the fine case there common zeros of bunch of vanishing, bunch of polynomials and if you are the projective case then it is common zeros of bunch of homogenies polynomials ok.

In any case it just vanishing polynomials that gives a zero the close sub set the for Zariskito policy ok. Therefore if you take a regular function on a variety U is also variety and F is a regular function on that locally F is a point of polynomials therefore looking at zeros of is regular function like looking at locally at zeros of the numerator polynomial because locally F is looks like a numerator polynomial divided by denominator polynomials and zeros of F will be just zeros of numerator polynomial ok therefore essentially you are just looking at zeros of polynomials and therefore again you are going to get a close sub set ok so this is a close sub set and mind you Z of F you know cannot be, U ok Z of F is not equal to U because if Z of F is U then that's means identically zero on U but then if F is identically zero on U then F is equal to zero function which identically zero on X, and then contradict the fact this, these two are different elements of the functions field ok.

Z is not equal to U because otherwise you will see that U comma F is this same as U comma zero this is equivalent to X comma zero it is a contradiction ok. So I should say contradiction, it is a contradiction to the fact that the, these two are different alright. So that means zero, Z of F is topper close sub set of U so it's complimenting U it is again sub set of U ok, so you know minus Z of F inside U non empty open, and off course you know where regular function dose not vanish, its reciprocal also function ok.

After regular function locally of the form portioned of polynomials where it will not vanish whether numerator polynomial dose not vanish if the numerator polynomial dose not vanish ok already writing the portion denominator polynomials cannot vanish, if the numerator dose not vanish then its reciprocal, the reciprocal of these two numerator and denominator will give you also a rational again give you a regular function at that point locally ok therefore wherever a regular function dose not vanish its reciprocal also regular function so what happens is that you will get this pairs U minus ZF which is an open sub set of U proper it's on T open sub set of mind you this, this is actually mind you it is dense in X axis ok it is non open sub set of U and U is non empty open sub set of X, so this is non empty open sub set of X, so it is reducible it is dense ok.

And on this I have the function one by F, one by F is certainly regular function on that, ok. So this element if you multiply with the original element U comma F what you get just X comma Y you will get this ok by the definition of the multiplication alright after all one by F and F are going to multiplied one wherever dose not vanish ok, and wherever F dose not vanish it is certainly it is a huge open sub set alright. Therefore, so what does tell you is that whenever U comma F is not zero element mind you this is zero element, this is a zero element in KX.

So whenever something is not zero it has a reciprocal there is something by which you can multiply it to get one, this is one, this is the zero in KX and this is the ine in KX ok. So here also when I write one equal to X comma one I mean this is one in KX and this is zero in KX ok. So what is true every non zero element is? I have complete you ring in which every non zero element is invertible so its field ok.

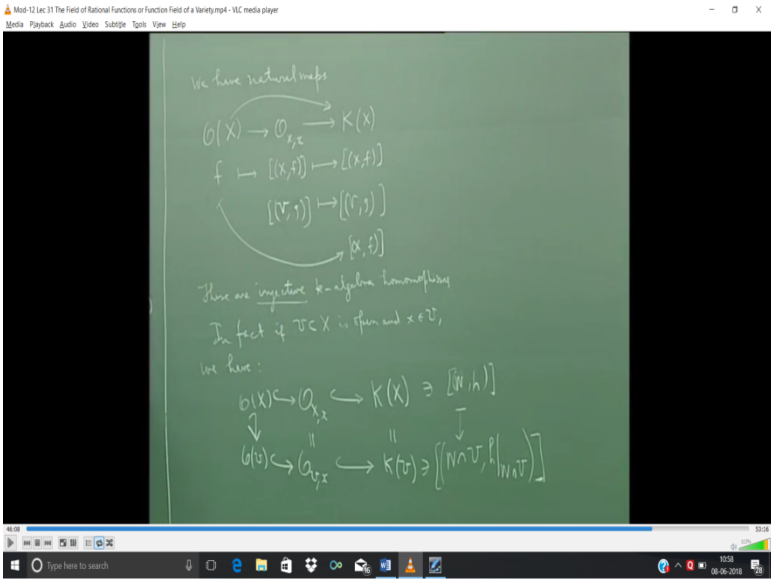
So moral is that the KX is a field and is a field extension, extension of K ok. Because it is K algebra so you know it is a open ring of K it contain small K as a sub ring and when you were have larger ring which is whenever you have a, when the smaller, when you have ring extension both of field you it is actually field extension ok. Smaller ring it is K the larger one that you have defined it constructed is, the also field ok this is a field extension ok.

Now this is called the function field, it is called function field of X ok, and what so special function field of X so the answer that geometrically following that the function field of X will give you all the information that is true on a large open sub set of X ok you, so, this is the philosophical important, this is KX will contain all the information about X which you can findon a large open sub set off course any non-open sub set is large ok, so you take any general open set next you take properties that are valid on general open sub set of X then all those properties they will be captured by KX ok so KX geometrically captured what is happing on large open sub set of X ok.

So KX captures the geometry on a general, geometry of ok, so the point is you see what I want to understand is both in the local ring case and the function field case you are just looking at pairs of functions ok I mean you are just looking at pairs which consist of regular functions defined on an open set ok. here you are only concentrating on open set of which is contain of given point and you get the local ring, and the local ring contains all the information in a neighborhood of the point ok all information in a neighborhood of the point how the variety behaves in a neighborhood of that point all that information control by this and captured by this ok.

Whereas if you want to know what is happing on a general open set of X that information is captured by KX ok so this is you know if you want a study on general open sub set of X if you have geometric question about general open sub set of X ok then what you have to studies, this. If your question about what happing at a point of X then the objectives of the studies is this, the local ring ok. These are two extremes ok although how they are connected, we are going to see how they are connected? Ok. Off course all the y are connected to each other essentially have everything is done by algebra ok so, so now come to following question.

(Refer Slide Time: 34:30)



So you can ask, how these things connected so the first thing that I want to tell you that you know we have maps so before that yeah we have maps, we have match hole maps, O, OX OXX, KX ok, there is maps like this, so what is maps? Let F BF regular function on X, you just send it to see F is a regular function on FX then I have pair U X coma F and its equivalence class will give me a point it give me element of the local ring at this point X because the element of the local ring at a point are generally Germs on a regular functions defined on neighborhood of the point and since F is global regular function, it is also regular function at that point so I then take it Germs so this is a Germs of F at the point and well you what I can do is on the other hand you know this is this map alright mind you that I can also send a map from here to here namely if you take U comma G let me write, let me write V comma G.

What is V comma G, G is a regular function on the open set V contains a point X ok this is a Germs of V, this Germs of G at the point small x ok and off course G this open set V may be a proper open set G may not extend to a regular function on the whole of X it may be restricted it might just be define on this open set ok. Now but anything here I can further sending to the something here mind you and you seem the same notification but the point is this equivalence is equivalence in local ring, this equivalence in the function field ok but I using the same square bracket ok now the point is that off course you know there is also map which goes like this correctly and that is simply all the way going to X comma F, which is the compassion of these two maps.

This will also go the X comma F where again this square bracket is the equivalence class here, and this square bracket is equivalence class is here ok alright, so I have maps like these, now the fact is the these maps are well the factis the these maps are injective K Algebra Homomphisons so these are injective K Algebra Homomphisons this injective K Algebra ok.

So off course they are K Algebra Homomphisons namely they are ring Homomphisons which are K Lenia ok that's no problem it is easy to see. The point is, the point is injective, so the injective comes like this you see that's was something uses again this fact you know if two regular function are equal and open set then they are, equal everywhere ok.

Off course the non that open set has to be non-empty open set off course you need a point to test you need points to test whether functions are equal ok so well you see why this is injective it is very clear because you know if X comma F is zero ok then it means that F vanishes a neighborhood of small x ok but that means F is equal to the zero function in a open set containing small x okbut then if you use the fact that two regular functions if they are equal or into non anti open set there equal throughout so if F is, small f is vanishes in a neighborhood of small x then it vanishes everywhere that means F it self zero ok, so I will tell you why this is zero.

And why is this injective literary the same, same argument if you take V comma G, Germs of a function G which is regular in this open neighborhood V about the point small x if it goes to zero here it means that it is equal to zero or some non anti open sub set of V ok but in any case that will also force it to be zero everywhere in fact this goes to zero here it will tell G is the zero function on V and the zero function on V extension zero function on x ok and therefore this will it self-zero alright.

So the moral of the story is both these ring Homomphisons have zero carnal so there injective Homomphisons and what is the moral of the story is, moral of the story is beautiful thing that regular functions sit inside the local rings the local ring sit inside the function field this is how the, you know so the idea is that somehow when are looking at topographically whole you have the whole variety X you concentrate attention to a point you get a larger ring ok if you have the whole variety X you have OX is a regular functions on X ok.

Now yo concentrate attention to a point you get a larger ring that is a local ring at that point ok, which is larger than the ring of regular function and then on the other hand if you concentrate attention to a point, if you concentrate on a open set you will get a much larger ring which is in fact a field and that's a field of rational functions now what I want, you to understand is that you know in all this things nothing will change if I replace Xby open sub set U, ok so in fact what will happen is that in fact if U is an open sub set of X is open and small x is U then you have he have a competitive diagram we have equalities in fact OX to O XX I mean at the level of local rings at the level of function fields you are not going to have any difference.

So I will put a how carrot seeing that you are considering as sub rings what will happens this will be the same as O UX is OU and this will be the same as KU and this will set inside ok so if you the function field depends only on open sub set of X so the functional filed of the variety will not change if you replace X by an open sub set so KX the function field of X if you replace X by U non empty open set then KX will be same as KU by going to non-empty open set you are not changing the function filed and that's just reflection of the fact the function field captures information on the general open set so if you go to general open set the function filed does not change ok and the local ring at a point also depends only on a neighborhood of the point it does not depend on the ambient variety.

So whether you are considering X is a point of open set U of capital X and calculating the local ring of small x of the variety U or whether you are calculating this local rings of small x with respect to the bigger variety ambient variety capital X you will get the same results ok. so local ring also depends only on the neighborhood of the point ok and off course the fact, what is happenings here is at you also get an inclusion also like this, because every regular function can be restricted to get a give you regular function on U and the restriction maps, restrictions of functions they are all injective because the same old reason that if a regular function.

if two regular functions co-incident of a open sub set on into open sub set then they have to consider everywhere or another way of saying it is a regular function vanish in a open sub set then it is identical zero it will vanish on the whole variety. So this is what happens if you go to a open sub set and why these two are equal and something that you can easily, you can get these equalities in a very very easy way namely what you do is you know you and any element of KX is form U comma F where it is regular function on U and you take the equivalence and what you have to do is very very simple I should not use the same let me use V comma G or even better let me use W comma H.

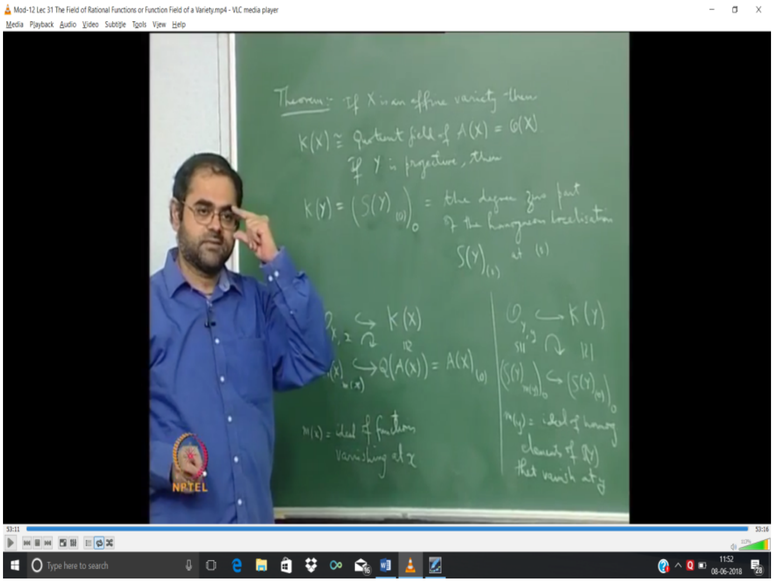
W is an open sub set of X, H is a regular function on W then I have this pair W comma H and its equivalence class is the rational functions her ok and what will I send its very simple you see W is non empty open sub set of X, U is non empty sub set of X, so the intercepts so I can make sense of H also inter section that what and that's the element I am going to send it to you, so I am simply going to send it to you W inter section U comma H restricted to W inter section U.

I take this class then I take it equivalence class and that is a element I am going to send it. ImpencibleI should write this as a map and to you can see that the map will K Algebra Homomphisons but then I put equality because actually as for as te functions are concern I am simply restricting the same functions so that's why I put equality here ok.

You think of this functions then you can put equality here ok so if you think of them relies functions ok off course you drum that here you should think of representatives as functions of non-empty open set ok and here you must think of representatives functions on a regular functions on a non-empty which contains point small x that's only difference ok so you have this, nice diagram alright and now what I have to do is tell you bought this, KX will access of affine variety,what is KX will be access of projective variety we need to understand what that is ok.

You already understood what local rings are when X is fine variety or projective variety ok, they are given by suit by localization ok, we have to repeat this to extend this kind of argument to the function fields ok so let me just state the results that will probably do it in a next lecture,

(Refer Slide Time: 46:36)



Here is theory X is affine variety then KX is isomorphism to quotient field, field of AX is equal to OX ok so if X is an fine variety then you know we have define AX affine coordinating of X and you have proved that the fine coordinating is equal to the ring of regular functions ok and this equality being part of the functions ok. we put these qualities because we think of a function here also as define there and conversely ok and what F is projective, then KX is let me put Y write down confuse if Y is projective then KY is just you take the homogenous coordinate ring of Y ok the homogenous coordinate ring is define the same for projective variety is define the same way as affine coordinate ring is define namely you take the rings of polynomials of the ambient space that you are considering and then go model O the ideal of functions vanishes on the variety in this case off course you will have to take.

you have to take polynomial ring in, of the fine space over the projective space in which the variety was embedded and go model of Y it will be homogenous ideal and then you take this then what you do is, you take the zero ideal ok that’s prime ideal and this is localizing at zero and then I put on the zero here saying that the you are taking degree zero part of the homogenous localization of SY ok.

So this is degree zero part of the homogenous localization SY at zero ok. so here is the formula for the function field of an fine variety and then this formula for the functional field of the projective variety ok and you know if you try to write out if try to write this kind of diagram including the you know the local rings and quotient fields then what will happen is that you know if you take O small x that is going to sit inside KX and you know this KX is just AX quotient filed of AX ok.

Q represents the quotient filed of AX that is KX that's what this first part of the theorem this is isomorphism and we have already seen that this is nothing but AX localize at MX, where MX is the maximal ideal in AX corresponding to functions that vanish at small x so I should put small x here, so MX is equal to ideal of functions vanishing at X ok.

You already prove this and we have this inclusion ok and you must recall a fact in from competitive algebra it is so easy, if you have (())(50:55) ok then it is quotient field will be the same as the quotient field of any of its localizations. The quotient field of a or filed of fractions of integral to main is the same as the quotient fields or field of fractions of any of its localizations.

In fact the quotient largest localization ok it is a localization everything accept the only element that you cannot convert to zero and zero is a prime ideal ok and so you are localizing at the prime ideal ok so this QAX is just AX local at the prime ideal so ok, this is what it is. You are inverting, everything outside zero right because localization on a prime ideal means invert everything outside the prime ideal in this case invert that is not zero that will give the quotient field.

So this is the picture you get the fine case ok and you will get the similar picture for the projective pace if you take OYY small y point of capital Y where capital Y is projective variety this is going to sit in inside KY and what's going to happen that the function filed of KY will be just SY localized at point zero degree zero part and this will contain SY localization at MY and degree zero part where here MY so this is off course this diagram is complete, this diagram this isomorphism of the local rings has already being includes ok and this isomorphism of the function fields that I have to prove and here again MY is equal to all those ideal of homogenous elements of SY that vanish at small o ok.

So this is how you get the local ring the quotient, the fine case and this is a local ring and the quotient field in the projective case ok so will prove this in next talk.