Basic Algebraic Geometry
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Module 10
Lecture 25
Expanding the Category of Varieties to Include Projective Varieties and Quasi-Projective Varieties

Ok so we were discussing projective spaces ok.

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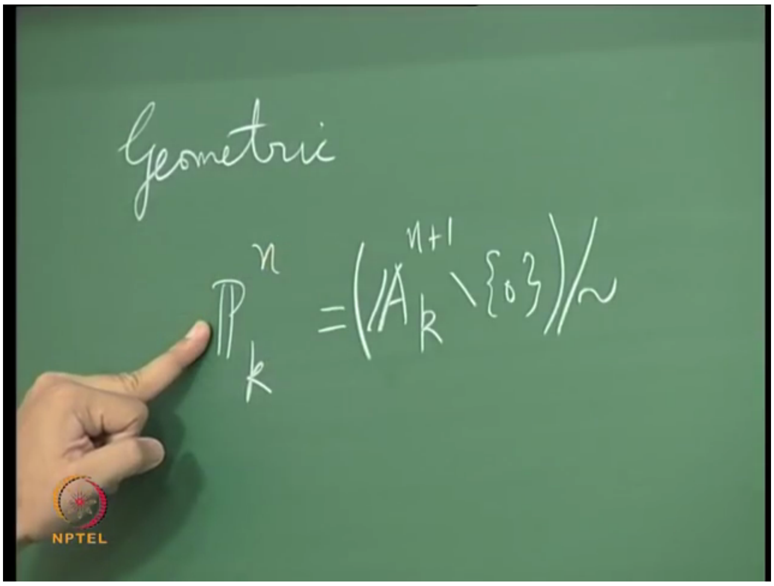

So if you recall we have here projective space, projective N over K which is where K is offcourse an algebraically closed field and this is offcourse this thought of a space of lines in N plus 1 dimensional affine space alright and we have given the Zariski topology on this ok and infact if you remember the, so this so there was a very nice picture, one giving the geometric side and the other the algebraic side. So for the algebraic side, so this is the geometric side and this is algebraic side. So this is the picture very similar to the case to the affine case.

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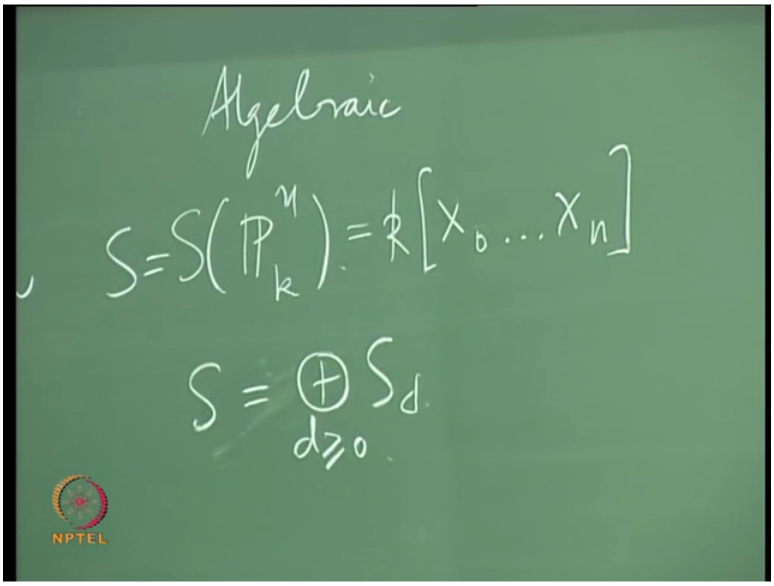

So on the algebraic side you take the so called homogeneous coordinate ring of this projective N space which is defined to be the ring of polynomials in N plus variables and it is customary to start the indexing of the variables from zero ok and offcourse you have N plus variables and this is the affine coordinate ring of the projective space above of the affine space above ok.

So this projective space is afterall affine (space) the N plus 1 dimensional affine space punctured at the origin a modulo the equivalence which identifies all the points passing through on a line passing through the origin ok, all the points on a line passing through the origin in N plus 1 dimensional affine space or identified as a singly equivalence class ok and therefore in other words an equivalence class is just a line passing through the origin and therefore this is the space of lines in affine N plus 1 space ok and this is the homogeneous coordinate ring which is the (aff) which is also the affine coordinate ring of the affine space above alright.

And what you do is, that offcourse the important structure here is that we are interested in the so called graded structure of this ring. (the) this ring so let me write it as S, this S is the graded ring since that S is the direct sum of its degree D piece is for D greater than or equal to zero ok, where S d corresponds to homogeneous polynomials of degree D, S0 is offcourse is going to be just K, K the constants which are homogeneous polynomials of degree zero and it is this so called graded structure which is very-very important and infact what we do is that Zariski topology is defined like this on the projective space.

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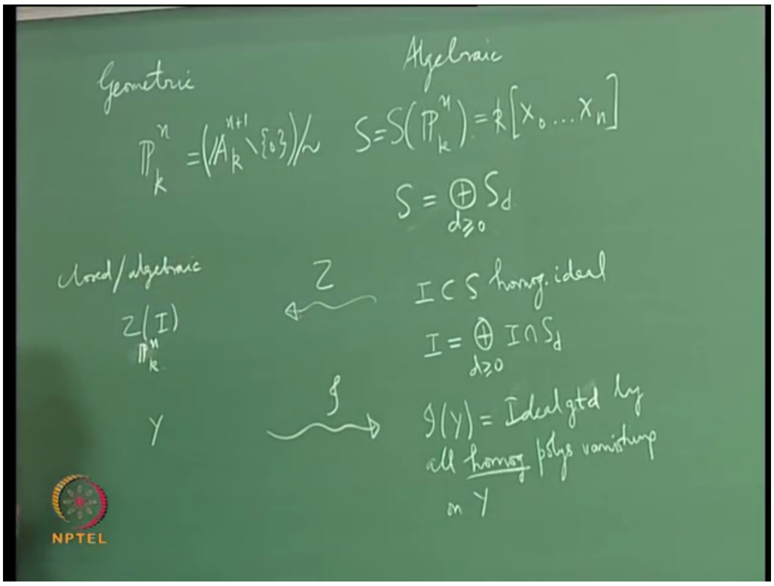

You start with I in S homogeneous ideal, a homogeneous ideal namely I the condition for homogeneity is that if you take the ideal I should be the same as it should be the same as direct sum of all its pieces intersected with S d. So you take the ideal I and intersect with S d what you will get is all those elements in the ideal which are homogeneous of degree D and offcourse we take a direct if you take the sum it will offcourse be a direct sum because this is already a direct sum ok and this will certainly be a subset of this always for any ideal.

But then the condition that the ideal should be homogeneous is that this is exactly equal to this ok, which is the same as saying that given any polynomial, in given any element here each of its homogeneous components is also back in this ideal ok, that is what it means and you know we saw this as a geometry condition for a polynomial to vanish on a line it is necessary a line passing through the origin it is necessary that the polynomial has no degrees has no constant term that it is constant term is zero and every degree D piece, every homogeneous piece of the polynomial should also vanish on that line ok and that is exactly the this homogeneity condition ok.

So once you have homogeneous ideal then you can define the zero set of this ideal in the projective space, which is the set of all points here which are common zeros of all the polynomials here ok and offcourse it is the homogeneity of the polynomial which allows you to decide for sure that the polynomial vanishes at a point on the projective space, it is, because it is the homogeneity of a polynomial, it tell you that if it vanishes on a line passing through the origin then it will vanish at every point on that line ok (and).

So we get this and these are the so called closed algebraic subsets the in projective space and this gives the projective space Zariski topology, a topology which is called the Zariski topology. The proof that this is a topology is very similar to the affine case ok you can check it and offcourse you have to remember that the property of an ideal, the property of homogeneity of an ideal behaves well under some product intersection and taking radicals ok.

Namely sum of homogeneous ideals is homogeneous, a product of homogeneous ideals is homogeneous and intersection of homogeneous ideals is also homogeneous, a radical of a homogeneous ideal is also homogeneous ok. This are simple facts that you can check and moreover you can also check that, to check that homogeneous ideal is prime you can check the prime must condition only for homogeneous products ok.

So to in general if you want check-up ideal is prime you take a product in the ideal and show that one of the factors of a product is also in the ideal but then you can restrict this checking to homogeneous elements. If you wanted to check a homogeneous (ele) ideal is prime ok. So the fact is that as it the affine case you get a very nice picture, you get you have an arrow going in this direction and well there is also an arrow that is going in this direction.

So this is the Z , this is the I and this is give me any subset Y of projective space then I have I of Y this is the set of all so you take the set all those homogeneous polynomials which vanish on Y and then you take the ideal generated by that. So this is the ideal so this is well, so this is the ideal generated by all homogeneous polynomials vanishing on Y ok. So and I have also told you if you recall in the last lecture that yet another way of saying that an ideal is homogeneous is by saying that it is generated by homogeneous elements ok.

So since this ideal is generated by homogeneous polynomials which are offcourse homogeneous elements it is obvious that this ideal is a homogeneous ideal. So you get a kind of you know mappings back and forth in this on this side you can have you know closed subsets or algebraic subsets of projective space and on this side you can have homogeneous ideals and you have mapping going in this direction and the reverse direction but then if you want to make this into bijective correspondence you will have to restrict offcourse uhh where infact on this side I can take all subsets ok but and here I can take all subsets and here I can take all ideals but offcourse the point is I can't take all ideals here I have to take only homogeneous ideals ok.

But then this map always gives me something closed here ok because that is how Zariski topology is define whereas if you give me any subset this always gives me homogeneous ideal ok. So if you want a bijective correspondence, what you will have to do is that, just like the affine case you will have to restrict here to you know, you have to restrict here to radical ideals ok and on this side you will have to restrict to closed subsets and then you have bijective correspondence ok and uhh as before this is as in the affine case this is an inclusion reversing bijective correspondence between radical homogeneous ideals on this side and closed subsets here.

The only thing you have to remember is that you should take homogeneous radical ideals that is the first point. The second point is you will have to leave a one particular ideal and that is the so called the irrelevant maximal ideal. That is the actually the maximal ideal that corresponds to the zero in the affine space above of which has been thrown out when we considered the projective space ok. So this is the fact that I told you last time and offcourse we have nice things like, we have the statements like.

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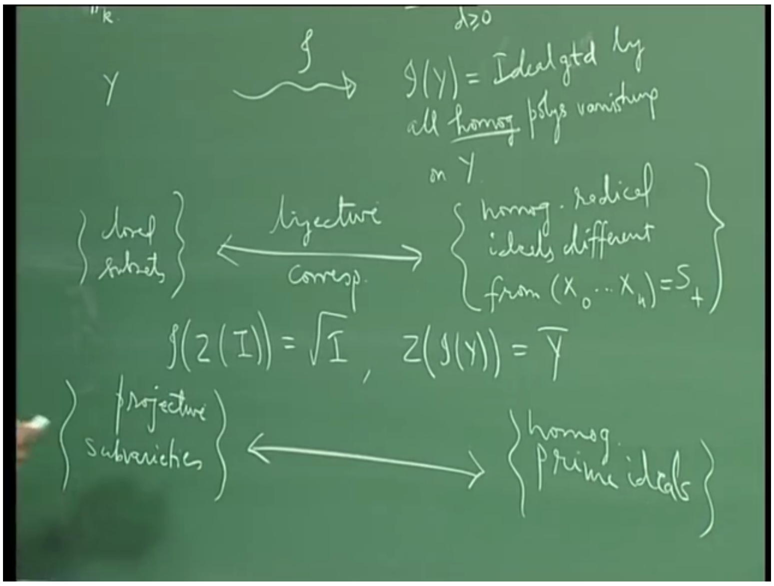

So let me say that just repeat it here you have closed subsets on this side and you have a bijective correspondence.

On this side you take homogeneous radical ideals different from the irrelevant maximal ideal, which is usually written as S plus, it is written as S plus because it is the sum of all the it is the direct sum of all the S d's for D positive ok, if you take S 1 plus, S1 directs sum S2 direct sum and so on, what you will get is exactly the ideal generated by all the variables, so it is written as S plus ok and this called the irrelevant maximal ideal ok. So you have this bijective correspondence and then offcourse as you, we have these facts like I of instead of I is rad I and you have E z of I F Y is Y bar ok.

This are all facts that we have in the affine case. We have the and offcourse in one direction this is trivial, the other direction is the so called projective Nullstellensatz ok and so you have projective version of the Nullstellensatz and you have these two facts and you also have as in the projective as in the affine case you also have this fact that if you take prime ideals, homogeneous prime ideals that is a subset of this because a prime ideal is always radical. So if you take the subset of homogeneous prime ideals that will, that under this correspondence will go to what are called as projective varieties.

This will be irreducible algebraic sets in projective space. So on this side I get projective sub-variety here and my projective sub-varieties I mean closed or algebraic subsets of projective space which are irreducible. So in other words what I am saying is just in affine case a subset here is closed subset here is irreducible if and only if its ideal is prime ok and so these are all things that we have just as affine case.

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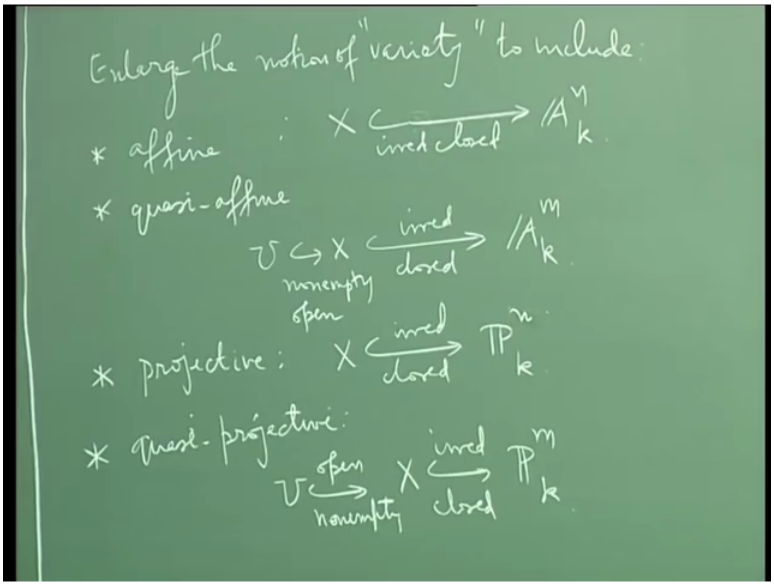

Now what we do is that so we define, so we uhh so what we do is that we enlarge the notion of variety to include so far our varieties were either affine or quasi affine so affine meant that you are looking at an irreducible closed subset of some affine space and quasi affine means you are looking at an open subset non-empty subset of an irreducible closed subset of affine space.

So this is X irreducible closed in some affine space and quasi-affine meant something that is an open subset of such an X that is an open subset of affine. So you can think of it as U sitting inside X this is non-empty open subset of X which is an irreducible closed subset of some affine space ok. So this is what is meant by a quasi-affine variety. So we have already dealt with these two. Now you extend the definition to include projective varieties and quasi projective varieties.

So what a, so projective varieties are similarly irreducible closed subset in projective space. So it is some X which is irreducible close, in such projective space and offcourse you can again define quasi projective varieties and quasi projective varieties are open subset of projective varieties. So quasi projective these are open subsets of projective varieties so they will look like an open subset open non-empty inside X which is irreducible closed some projective space.

So now variety it means any one of the following four possibilities. So it is either an irreducible closed subset or an open subset of that in affine space or in projective space ok. Now so you, we enlarge the notion what a varieties and then you have to note that talking about irreducibility outing out that projective space is also noetherian. So I just wanted to remind you that projective space is just the quotient of the punctured affine space and offcourse you know if you take and the punctured affine space is noetherian.

So the projective space is noetherian. For example how do you verify that a space is noetherian you show that it satisfies the D C C descending chain condition for closed sets. So if you give me a descending chain of closed subsets in projective space, you simply pull it up by the projection map to the affine, the punctured affine space above and then you add the origin ok, so that you will get a descending close sequence of subsets in affine space.

But then you know that the affine space is noetherian, therefore that's sequence stabilizes and therefore its image below will also stabilize ok. Offcourse you will have to remove when you take the image below you will have to remove the origin and then take the image under projection from the punctured affine space to the projective space. So it is obvious that projective space is going to be noetherian ok and then you know the moment you have a noetherian topological space then every closed subset has a noetherian decomposition namely decomposition into unique decomposition into irreducible finitely many irreducible closed subsets.

The decomposition being unique except our for permutation of the elements appearing occurring in the decomposition except provided you assume that there is no redundancy in your decomposition namely no irreducible closed subsets in the decomposition is a subset of some other irreducible closed set in that decomposition and therefore and the such sets irreducible closed subsets though finitely many irreducible closed subsets the union of which in a unique sense is the given closed subset of projective space. They are called the irreducible components ok.

So this is just, so you have noetherianess of projective space you have irreducible (decom) you have the noetherian decomposition for any closed subset ok that is because noetherian property and then you will also have this fact that topologically you know that any noetherian space is quasi-compact therefore you will get that projective space any a projective space is offcourse quasi-compact and infact direct demonstration of that is that we have seen that the projective space is actually a union of N plus 1 affine spaces ok.

So there is already a finite cover by affine spaces. So P n has a union, P n is the union of finitely many A n's, N plus 1 A n's ok. So but infact any closed (sub) any subset of P n being a sub set noetherian topological space will be noetherian and you know and since the noetherian topological space is always quasi-compact any subset will be quasi-compact ok. So well now so these are all nice things that are going on here. So in particular you must remember that if you take an open subset, if you take a quasi-projective variety then that is both irreducible and dense in its closure which will be a projective variety.

Just like if you take a quasi-affine variety it will be both irreducible and dense in its closure which will be a affine variety ok. So this is the situation. Now what I you wanted to do is, I have enlarged the objection the category of varieties like this ok. I also want to enlarge so I am thinking of the category of varieties which means I am thinking of both, I have to think of both objects and morphisms ok. Objects offcourse I have enlarged because I have added projective and quasi-projective varieties. But then I have to enlarge the definition of morphism and you know definition of morphism the affine a quasi-affine case is that it is a continuous map that pulls back regular functions to regular functions.

Therefore if I want to enlarge if I want to define morphisms which involve even projective or quasi-projective varieties I have to tell you what are meant by regular functions for projective or quasi-projective varieties ok and the answer is very-very simple. Just like the affine case where the regular functions is just a quotient of polynomials locally. In the projective case you only require that it is a quotient of homogeneous polynomials of the same degree and you put the same homogeneous degree so that you get a valid function ok.

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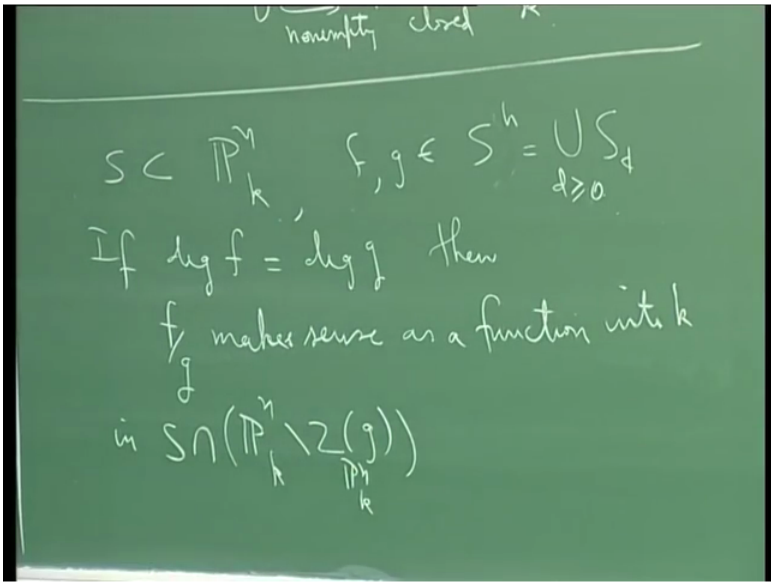

So let me say that so you know so suppose you are having a subset S in projective space ok and you take F and G in the S to upper H this is the union of all the S d's D greater than equal to zero ok. Let me alright, so let me take D greater than equal to 1 ok, let me not take non-zero constants ok. See you take two so I am taking two homogeneous polynomials. Offcourse you know I cannot evaluate a polynomial on even if it homogeneous I cannot evaluate a polynomial at a point of projective space.

The only the homogeneity of the polynomial will only tell me that it is, what I can uniquely always say is whether the polynomial vanish at that point of projective space or not but I can't give but if it doesn't vanish it can't give you a particular value ok. That is because if I plug in at point from projective space here then you know there is a common multiple which is floating around because the points in projective space are common ratios they that is why they are called homogeneous coordinates and that whatever constant multiple can always be pulled out of the valuation and it will come out to the power Z equal to the degree of the homogeneity of the polynomial.

So but the point is that if degree of the homogeneous degree of F is equal to the homogeneous degree of G then you know ok so maybe there is no harm in including zero also because anyway constant functions will make sense. So if both of them have the same degree then you know F by G makes sense as a function into K in S intersection the compliment of E Z of

G ok. So the point is that if I plug in problem is that if I plug in a point of projective space into a homogeneous polynomial then if I change representation of the point then a scalar will come out and it will come out with a power which is equal to the degree of polynomial.

But if I take such quotients ten these powers will cancel ok therefore you get a well-defined function. So all this is to tell you is that you know homogeneous polynomials are not enough to define functions but quotients of homogeneous polynomials with the same degree certainly define functions on appropriates subsets of projective space, offcourse appropriate (subject) subsets I should by that I mean the denominator polynomial should not vanish ok. For me to be able to evaluate F by G at a point ok. So now the, now this is the prototype of what a regular function if for a subset of projective space.

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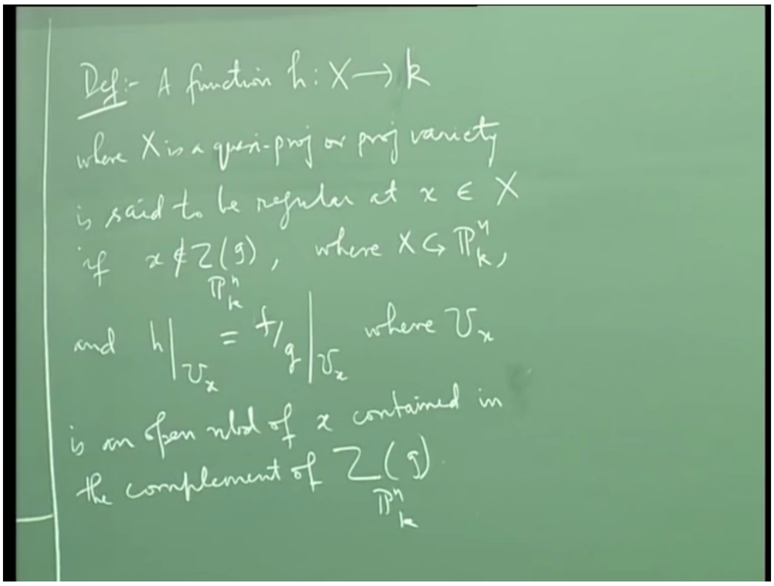

So we make this definition that so here is the definition, a function H from X to K where X is a (variety) quasi-projective or projective variety is defined is said to be regular at X (()) (27:31) if it locally looks like a quotient of two homogeneous polynomials and the number of variables is equal to one more than the dimension of projective space in which exists ok.

So if X belong to Z of G where X is subset of A n and H restricted to U x is equal to F by G restricted to U x where U x is an open neighbourhood of X contained in said oops X should not be in (())(28:44) I don't want I want to divide by G and I want evaluate it at X. So G should not vanish at X so this has to be corrected X should not be in the zero set of G and this neighbourhood should be contain in the compliment of this, zero set of G, the compliment of. So this is the definition of what a uhh function regular function at a point means.

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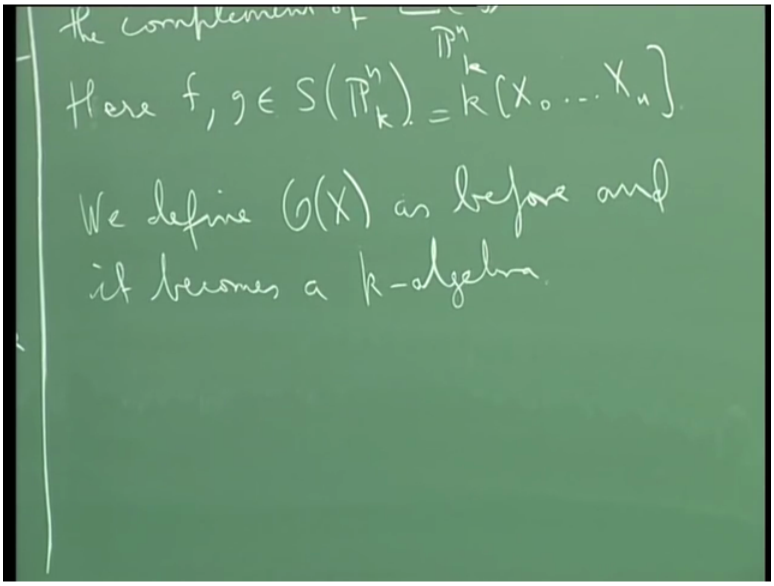

Offcourse here F and G are in the homogeneous coordinate ring of the projective space ok, which is a well polynomials in the right number of variables. So this is going to be K of X knot ok. So the idea is very simple and offcourse you know its regular a point automatically means regular and neighbourhood of a point ok. So because you are requiring this not only at that point you are requiring it in the neighbourhood of the point so the definition of regular function as in affine case already says that regular at appoint if and only if its already regular in a neighbourhood of a of that point ok.

So now what we do is again define the ring of regular functions on the projective variety or quasi-projective variety. We define O X as before and it becomes K algebra ok. So regular functions O X is offcourse set of all global regular functions namely function which are regular on the whole of X which are that means functions are regular which are regular at every point ok and if you take the set of all such functions that is a K algebra because sum of regular functions is regular, product of regular functions is regular and that (multi) when you multiply a constant function constant with regular function that is again regular, because the constant it also set of as a constant regular function ok.

So well so we have this ring of regular functions on the on your quasi-projective or projective variety and the point I want to make is that as before every a regular function is always continuous ok.

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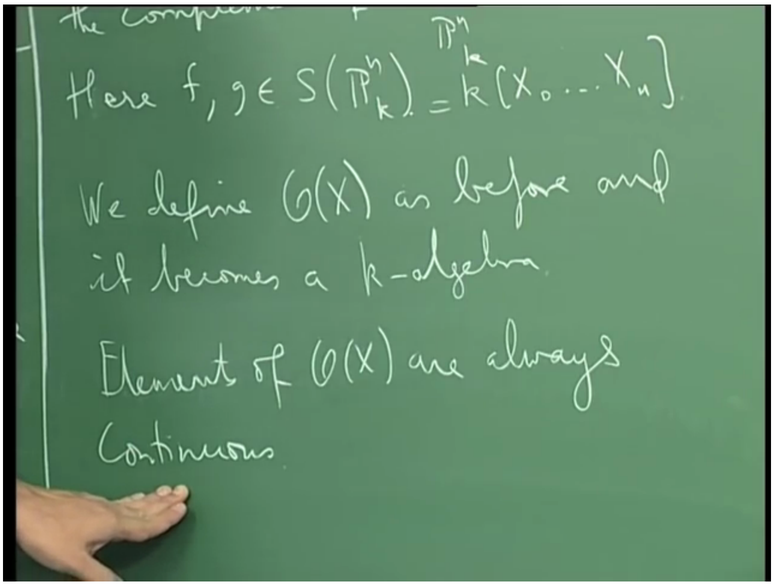

Regular functions elements of O X are always continuous and that is again something is continuous offcourse for Zariski topology. So elements of O X are going to give (morhp) I have still not defined morphisms, so let me come to that later.

Regular functions of O X are always continuous ok and the continuity is obvious because of , it is obvious if you look at the if you remember the fact that the Zariski topology on the projective space is a quotient topology of the topology above ok so if you give me a regular function on a subset here ok then if you compose it with the projection ok you will get a regular function on the affine space ok above on a subset of the suitable subset of the affine space above and that is continuous and that will tell you that the inverse image of closed sets are closed because of the definition of the quotient topology and therefore what will happen is that regular functions are it is very trivial to see regular function are continuous ok.

Now that we have defined this what we can do next is now with this paves the way to be able to define morphisms, so now how do we define morphisms between two varieties is just this definition the same as before it is a morphisms between two varieties is just a continuous map that pulls back regular functions regular functions ok, so definition remains the same. Only thing is now you have your objects are more you are not only considering affine or quasi-affine varieties you are also considering projection or quasi-projective varieties.

So you can think of morphisms from an affine or quasi-affine or a projective or a quasi-projective variety into another variety which is again one of the one of these for types ok.

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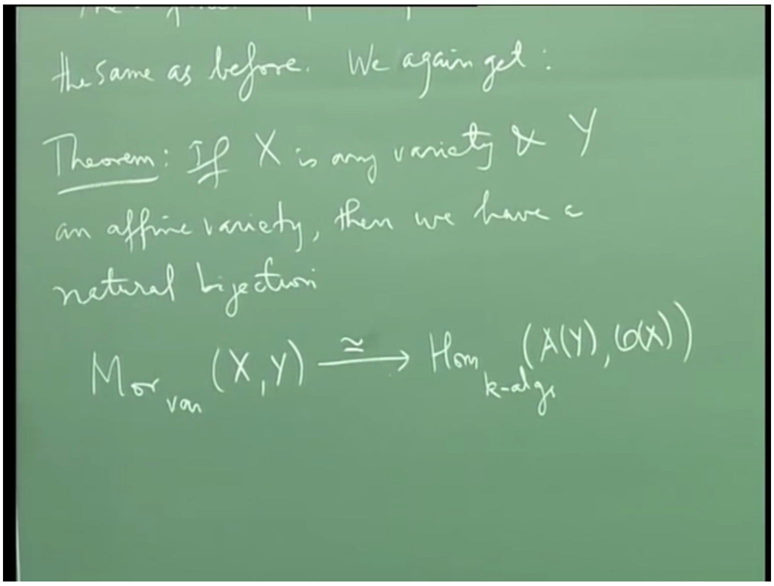

So the definition of a morphisms keep the same as before , the definition of a morphisms is the same as before and again what will happen is that we again get the following important theorem. If X is any variety and Y and affine variety then we have a natural bijection from the set of all morphisms of varieties from X to Y to the set of all homo-morphisms of K algebras from A Y to O X.

We saw this theorem where we thought where we were thinking of X only as a final quasi-affine variety but then the same theorem will the proof will go through now if you back and look at this proof you will see that the same proof will work even if X is a projective or quasi-projective variety ok.

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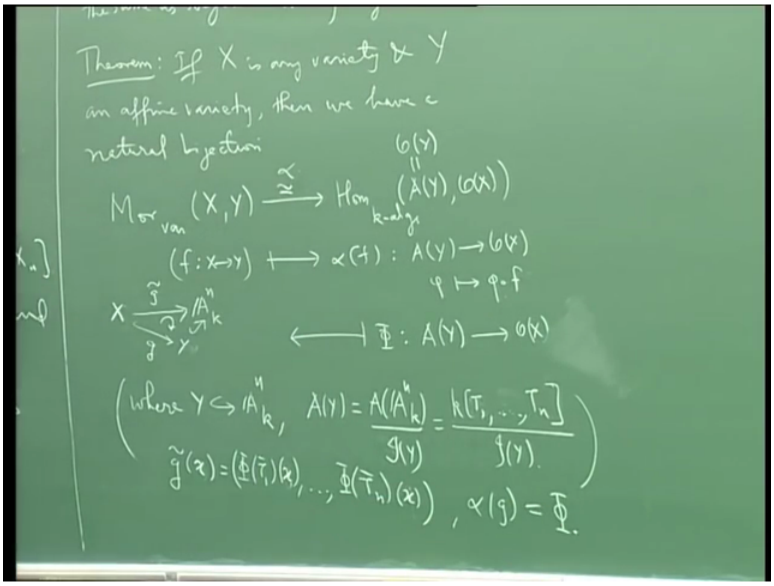

So this theorem still holds and if you remember I think I call this map as a alpha and what was this map well if you give me morphisms from X to Y then it goes to alpha F which is just a pullback of regular function it is a map from A O Y to O X which will pullback regular function Phi to you give me a regular function on, you will give me a regular function Phi on Y the if you compose it with F you will get regular function on F on X.

So first apply F then apply Phi ok, this is just the pullback of regular functions and you must remember that A Y is a same as O Y, A Y and the O Y are the same because Y is an affine variety ok. So the affine coordinate ring is the same as the global regular functions ok and so this is the map we defined and then you also have the inverse map which goes in this direction and what is the inverse map if you start with Phi here K algebra homo-morphisms from A Y to O X then what you do is that you recall that Y is an affine variety so Y sits inside some A in, so A which it so it means the affine coordinate ring of Y is just the affine coordinate ring of A n modulo the ideal of Y, this is how we define the affine coordinate ring of a fine variety.

And then and this is well this is going to be identified with K X1 etc upto X or let me put K Y1 or maybe T1 etc upto T n, T n modulo I Y and so you are, you have the T I bars here which are regular functions they are just the (globe) they are just the coordinate functions on the affine space in which Y sits ok and you are just a T I bar means just it can also be thought of just as T I restricted to I ok, because afterall taking this quotient amounts to restrict in polynomial functions the closed subset Y ok.

So now each T I bar will go to a certain regular function in X and use this bunch of N regular function in X to defy a morphisms from X to affine space and show that morphisms actually that map is actually a morphisms which factors through Y and for which the alpha is Phi. So you know so the diagram is that from Y what you do is you get a map into A n and this is given by so here is G and G is G of Y is just Phi of T1 bar of Y dot-dot-dot Phi of T n bar of Y it is a (())(39:12) so and the fact is that this factors through so I rather call this map as G this as G tilda if you want I need factors through X oops my this should have been X.

So this should have been X, so this all should have been small x's so this map in from X to A n and it factors through Y and through a morphisms like this and Phi is actually Phi, so let me write that below alpha the alpha of this G is actually Phi ok. So this is the inverse map, this is the alpha inverse. This is how we got this bijective correspondence. You can check that the whole proof goes through if you allow X also to be a quasi-projective or a projective variety there is no difference ok. The proof doesn't I mean really the proof really didn't depend on the fact that X was affine quasi-affine ok.

So you can check this theorem so in particular you know if I take Y equal to A1 it will tell you that the morphisms from X to A1 are the same as the regular functions on X ok. So just as in the affine case regular functions are the same as morphisms into A1 there is no difference ok, there is really no difference.

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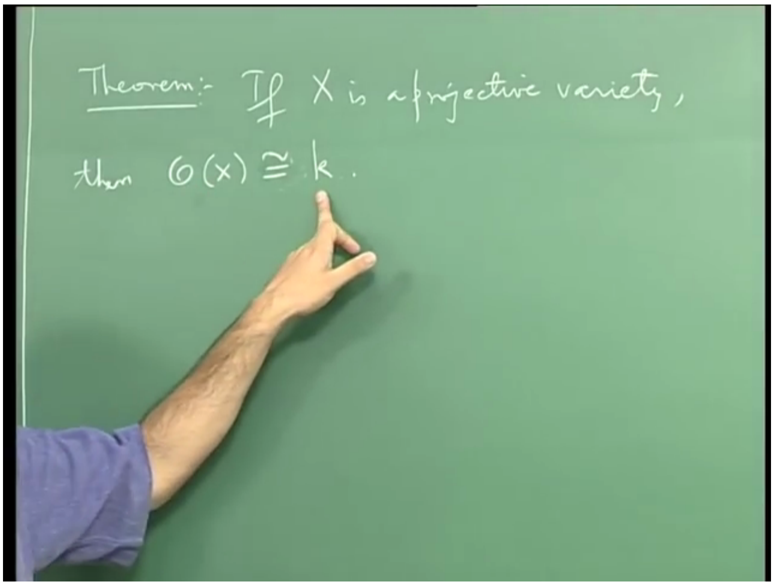

So the same proof works and the point I want to make is here is very important theorem which is I would like to say in this connection, see, we saw that if you put Y to be A1 you will get regular functions ok.

But more importantly we saw that you know the if you take any affine variety which is different from a point ok, the fact that is different from a point means that its ideal is different from a maximal ideal and therefore then you take its affine coordinate ring it will have lots of polynomial functions ok. So it is going to be polynomial ring modulo some ideal which is a prime ideal but it is not a maximal ideal ok. This is finitely generated K algebra which is integral to main this has lots of polynomial functions.

So if you give me a affine variety which is different from a point they are lot of global regular functions which are given by lot of polynomials ok. Whereas this is not the case for a projective variety ok. So the theorem is that if X is a projective variety then O of X is isomorphic to K, O of X is just K ok. So maybe I will let me put isomorphic ok, where by isomorphism I mean so what I mean by this is that, every global regular function is the function that corresponds to a constant, it is a constant function.

The only global regular functions are constant. So you must think of this as an analogue of the fact that you know if you have a compact complex manifold then the only global holomorphic functions on that will be constant and that is just because of (())(43:22) theorem ok, that a bounded entire function is a constant. So it somehow you must think of X as being compact and therefore it doesn't admit any global functions which are not constant ok.

But the proof of this will require some more definitions so I will differ that ok. But what you must understand is that, if your varieties are projective variety then it has no global regular functions which are no non-constant global regular functions. Offcourse constant functions are always there, but if you want non-constant regular functions there are none ok. These makes life a little bad in the following sense because you know you can be what we have seen that if you have two affine varieties then they are isomorphic if and only if their fine coordinate rings are isomorphic.

And you know for an affine variety the affine coordinate ring is the same as the ring of regular functions ok. So an affine variety can be kept track of by looking at its affine coordinate ring and the affine coordinate ring doesn't change no matter in which projective space you are embedding the fine variety as a closed subset of ok. But this is not going to (happen) uhh so let me repeat that, if you take an affine variety if you takes its affine coordinate ring that is the same as its ring of regular functions that ring is independent of the embedding of this affine variety as a closed irreducible closed subset of some affine space.

If it change the affine space and you embed the same affine variety into some other affine space as an irreducible closed subset then if you compute the affine coordinate ring there you will still get an isomorphic ring ok. So you can keep track of an affine variety by looking at its ring of functions that is what is says, the ring of functions completely controls and keeps track of their affine variety. But this is not true for projective variety because for a projective variety you take two different projective varieties they unfortunately the ring of regular functions is just K it is just the constants.

So there is no way to it becomes hard for you to distinguish between two projective varieties ok. Then offcourse so this leads to other problems and infcat this is what leads you to study, so infact this should tell you, you should expect that if you take a projective variety and try to define the coordinate ring of a projective variety which is you know analogous to what you would do for affine variety namely take the homogeneous coordinate ring of the ambient projective space if you have projective variety embedded in an ambient projective space.

What you do is you take the homogeneous coordinate ring of the ambient projective space and go modulo the ideal of this projective variety and the result id again a graded ring because you are taking a graded ring ok and you are going modulo an ideal which is a homogeneous ideal its homogeneous prime ideal.

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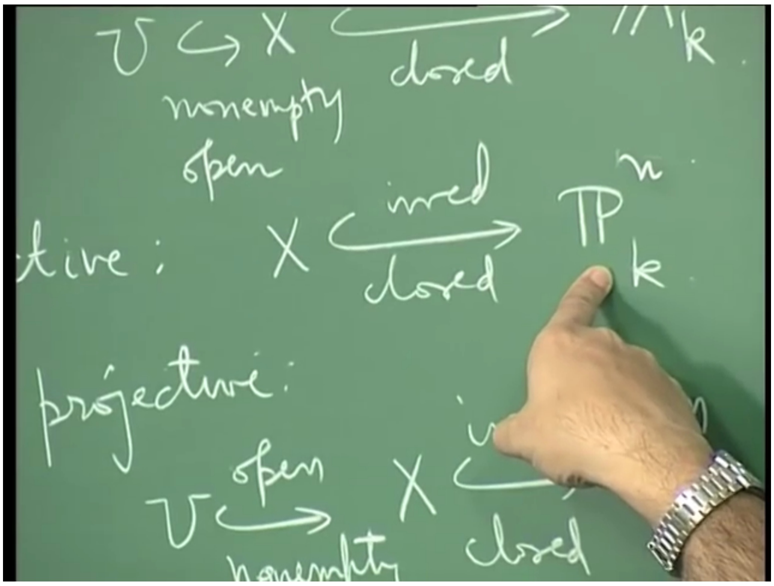

So you again get a graded ring which is an integral domain which is a finitely generated K algebra but the problem is that if you change this embedding you take the same projective variety and put it into some other projective space and calculate again look at the coordinate ring homogeneous coordinate ring it will change, it could change and it will.

So it is very so the way in which a projective variety is embedded in projective space is has a is doesn't have a uniformity about it and this tells you that you know it gives you the following fact I mean this is the following philosophy which is the basis of all higher study about projective space it is the fact that if you want to study all the functions if you want to study the geometry of projective space you want to study geometry of projective variety you must look at its embedding in various projective spaces. That should reveal its geometry ok.

The way its, the way its homogeneous coordinate ring changes as you embedded it in various projective spaces ok, that should give you a some grasp about the geometry of the projective variety. So it is but nevertheless this doesn't mean that there are not that you don't have properties of projective variety which are intrinsic to it as a variety ok. So what it tells you is that you can no longer work with global regular functions on it. Because there aren't any non-constant global relay functions ok. So I will come to the proof of this later but then I want to tell you only one thing.

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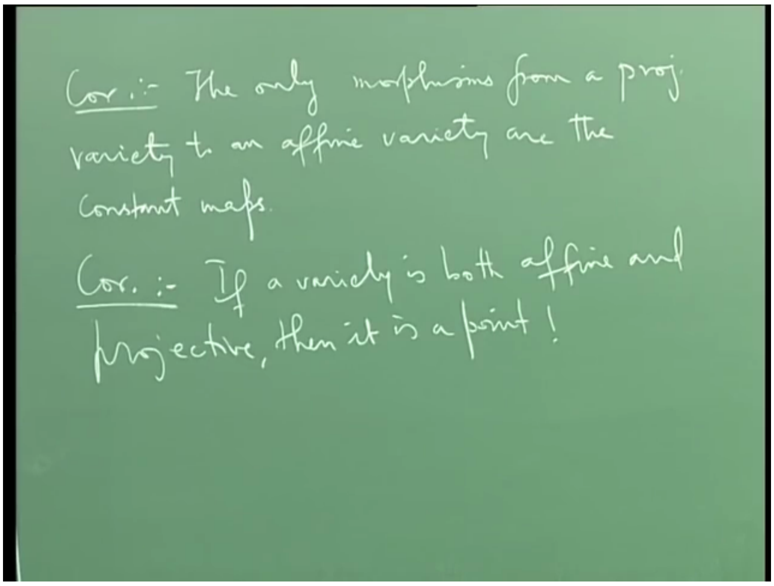

If you look up put these two together as a corollary you will get the only morphisms from the projective variety to an affine variety or the constants or the constant maps. So this is something that you can see immediately because you know in this bijection suppose X is a projective variety if X is a projective variety then O X will become K ok and therefore I will get morphisms from X to Y in bijection with homo-morphisms K algebras from A y to K ok.

But every K algebra homo-morphisms from A y to K is surjective because it is a K algebra homo-morphisms the image has to contain K. so every K algebra homo-morphisms from A y to K will be surjective which means its kernel will be a maximal ideal and therefore the set of morphisms from X to Y ok will be the same, will be in one to one correspondence with the maximal ideals of A Y but the maximal ideals of A y correspond to points of Y and therefore what will happen is that what this will translate to if you look at it, it will be that the only morphisms from a projective variety to affine variety will be the constant map that ends whose image is single point.

And how many points, how many such morphisms will you have? As many morphisms as there are point in the target variety ok and each point in the target variety which is an affine variety corresponds to a maximal ideal of A y mod which you get a homo-morphisms from A y to K that is what these bijections is. So as a corollary what you get is that the only morphisms from a projective variety to affine variety or the constant maps ok. There are no non-constant morphisms, there are no morphisms except constant maps ok.

And offcourse this also should tell you another corollary that you can get is that if a variety is both affine and projective then it is a point ok. This is also something that you can easily realise because you know if the variety is projective then its global relay functions are just constants and if it is an affine variety then you know the ring of regular functions is will now be (cons) just the constants and for what affine varieties will the ring of regular functions be constants only single turns which consists of points.

So if you put the condition affine and projective on a variety then you are reducing it to a point ok. So these are two easy corollaries of this theorem and this, these two theorems ok. So I will stop here and continue in the next lecture.