Basic Algebraic Geometry
Dr. Thiruvalloor Eesanaipaadi Venkata Balaji
Department of Mathematics
Indian Institute of Technology, Madras
Lecture – 11
Why Should We Study Affine Coordinate Rings of Functions on Affine Varieties?

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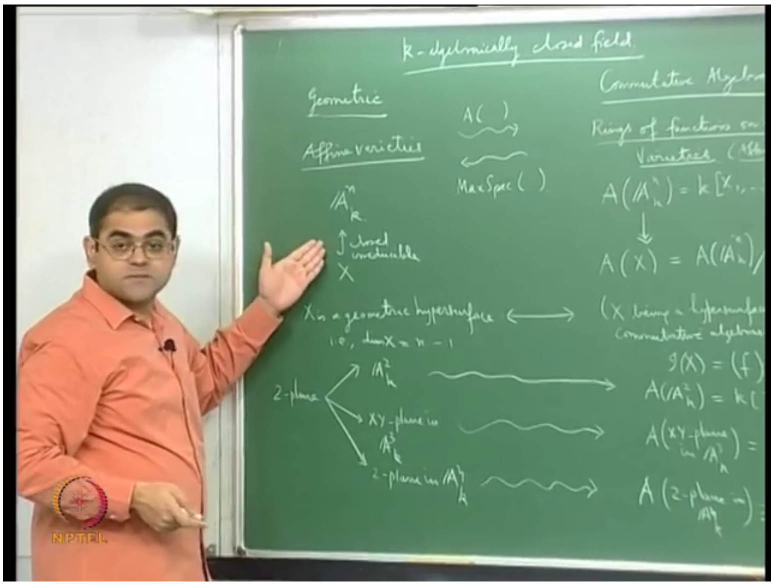

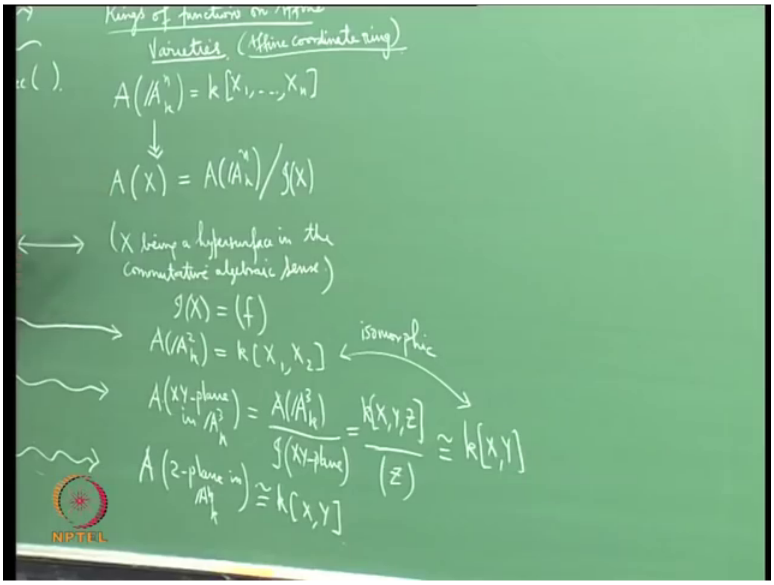

Okay, so let us continue our discussion about you know rings of functions of varieties okay, so let me recall the picture as it was explained in the previous lecture so on one side we have the, that is the geometric sign, we have if you want affine varieties and on the commutative algebraic side we have these so called rings of functions on affine varieties.

So let me recall, you see an variety is supposed to be a closed reducible subset of some affine space okay and affine space is of dimension n is just kn, k cross k cross k n times given the zariski topology right? And by the zariski topology of course if you recall if just (())(02:57) that the class close sets to be common 0 rho si of a bunch of polynomials okay in the appropriate number of variables so.

So here on this side you have close subsets irreducible close subsets of affine space so standard examples are affine space itself is irreducible of course we are always working with k, an algebraically closed field, k is an algebraically closed field okay if you give, if you take the affine n space then the ring of functions for affine space is A of An which is the polynomial ring in n variables over k okay.

So there is this association that takes to every affine variety string of functions okay, and of course the functions we are interested in are polynomial functions right, and more generally if you give me a closed subset of an affine space which is also irreducible, this is what a general affine variety is, a general affine variety is defined to be an irreducible closed subset of some affine space.

And for such a irreducible closed subset what is the ring of functions the ring of functions is defined to be the ambient ring of functions, ring of functions sorry the, that is the ring of functions of ambient space, the largest space in which x sits maybe the ring of functions on this divided by the ideal of that subset and the ideal of this subset mind you is the ideal of functions namely the ideal of polynomials which vanish on all of x, okay and you know that that is prime because x is irreducible right.

And therefore this quotient is an integral domain okay because you are going, you are taking the polynomial ring and you are going model of prime ideal okay. Basically you are going model of prime ideal so the result, the quotient is an integral domain and you see this this closed, this irreducible closed subset here shows up as a quotient here this is the quotient and this is just the, you are just going mod the ideal of x okay.

And well, see the point is that what is actually happening is whatever is happening here is on the geometric side is an exact image of mirror image of what is happening here on the commutative algebraic side and algebraic geometry derives from this translation basically okay so that problems or geometric problems can be translated into commutative algebraic problems and vice versa.

And the problem that you cannot solve here maybe you can solve it here and the other way around okay, so the first thing I wanted to tell you is of course on this side it is affine varieties, on this side it is rings of functions on affine varieties and I am saying that there is some kind of a bijection so there is also a map that is going on this side about which I mentioned the existence of but I did not tell you what it was in the last lecture I would like to tell you what that is and that is the so called maxspec.

So I will come to that and these two put together will tell you that whatever is happening here and is exactly a mirror image of what is happening on this side okay, and so let me recall see for example if you have that X is a geometric hypersurface that corresponds to on this side X being hypersurface in the commutative algebraic sense okay and in fact on this side I should not write X actually I should write something about it being a functions.

And the fact that X is hypersurface in the commutative algebraic sense is the fact is that X is hypersurface in the commutative algebraic sense is something that is defined by a single equation, so it is a locus defined by a single equation and so I of X, so the condition is I of X is generated by single polynomial f and so X is just the zero locus of single problem here and since I of X is prime okay, it will force that f has to be an irreducible polynomial.

And it has to be irreducible non constant polynomial okay so that is the condition for hypersurface in the commutative algebraic sense okay and x being a geometric hypersurface is what is the geometric meaning of being a hypersurface? The geometric meaning is via dimension, so it means that dimension of x is actually 1 less than the dimension of the ambience, the bigger space so it is n minus 1 okay where n is the dimension of the larger space and of course here by dimension I mean the topological dimension right.

And we saw last time that these two are equal and that was basically an application of both the Krull's hauptidealsatz and also the important theorem that you know and that the noetherian internal domain is UFD unique factorization domain, if and only if every ideal every prime ideal height 1 is principle okay.

So you know both these theorems we do need that the ring involved in noetherian okay. Krull's principal ideal theorem says if you take a noetherian ring and you take an element f in a noetherian ring which is not a unit and is also not a 0 devisor okay then every minimal prime ideal that contains this element has height 1 okay that is Krull's principal ideal theorem.

And so the geometric content is that you know while both these theorems tell you that these two are actually equal okay and the other beautiful thing is that you can, so you can ask the question of when the ring of functions are an affine variety is a unique factorization domain and the answer to that is that you know you take any affine variety and look at its ringer functions the condition that it is any factorization domain is equal to saying that every geometric hypersurface in that variety namely every co-dimension 1 irreducible subset is defined by a single equation okay that is a geometric translation of the fact that every prime ideal of height 1 is principal okay.

The prime ideal of height 1 condition tells you that the dimension of your locus goes down by 1 okay, that is it has co-dimension 1, goes down by from the dimension of the bigger space in which you are considering okay and so you know therefore the geometric content is that if you have unique factorization domain I mean when are rings of for which affine varieties are the ring of functions unique factorization domains.

They are precisely the affine varieties for which every co-dimension 1 irreducible subset is defined by the locus of a single equation that is what, that is the geometric content okay so this is one particular example. So you know I still I want to say a little bit more going from this side to that side.

So you see let me give you two examples one is the, let me take the two plane okay so the two plane, there are many avatars of the 2 plane so for example I can take well I can look at A2 k this is a 2 plane okay and if I look at the corresponding you know ring of functions I will get the A of A2 k which is just k if you want x1, x2 okay.

It is a polynomial ring in two variables then I can also consider two plane as if you want the XY plane in 3 space that is also after all the 2 plane only that you are considering it in a bigger space so you know for example you can take XY plane in A3 k that means you know you are looking at 3 dimension space K cross K cross K you are calling the co-ordinates as XYZ few odd and then you are looking at the XY plane.

That is also the plane, 2 plane anyway and if you look at the ring of functions on this you will get A of you know well let me write that A of XY plane in 3 space, what am I going to get, I am going to get the ring of functions on an irreducible close subset is defined to be the ring of functions of the ambient space which in this case is the affine free space so it is A of A3 okay modulo the ideal of that irreducible subset, okay that is the definition.

So it goes by model, we have to go modulo the ideal of the XY plane and if you see, you see what this will give you is you will get A of A3 is of course k XYZ because we agreed to call the co-ordinates on this 3 plane as XYZ okay and it is XY plane you are interested in and what are the functions that vanish on the XY plane it is the Z co-ordinates where Z, XY plane is simply written as Z equal to 0.

And so it is actually the ideal of XY plane, the XY plane is just the ideal by Z okay and so it is the polynomial ring k XYZ modulo Z which is actually isomorphic to k XY, okay. So you see if you watch these two are one and the same, geometrically they are just the 2 plane, here it is a 2 plane considered as the whole space and here is the two plane considered as XY plane, okay and see what is happening on this side what you get is these two are isomorphic you get that ring of functions are isomorphic.

This is also polynomial ring in two variables, that is also polynomial ring in two variables so they are isometric okay so let me give you one more example, in fact let me take more generally you can take some plane, a more general plane in n space okay. If you take 2 plane in An okay and if you associate to it the ring of functions A of the 2 plane in n space okay what will happen is you will still find that it is still isomorphic to polynomial ring in 2 variables, it will happen.

So what is that I am trying to say, I am trying to say that geometrically you are looking only at the two plane whether you are looking at the two plane completely the two plane itself or whether it is embedded as a plane in 3 space or whether it is embedded as a plane in n space, no matter how you look at it, these definition of the affine co-ordinate ring, the ring of functions on an affine variety.

This definition gives you isomorphic rings it does not give you something different and the fact that you are looking all the three cases give you upto isomorphism the same ring is an indication of the fact that you are looking at the same geometric object, you are looking at only the plane, okay.

So the moral of the story is that the ring of functions okay is really captures the object that you are looking at it is not going to change just because, it is not going to distinguish between 2 plane in 3 space or a 2 plane in a n space because the 2 plane is always a 2 plane there is nothing different geometrically about it, the only thing is the way it is embedded right.

So the moral of the story is this is this gives you one rational as to why we should study rings of functions, the ring of functions is a geometrically intrinsic object when you take an affine variety okay, intrinsic means upto isomorphism it depends only on the affine variety not on where you see it okay so that is one of the justifications of studying rings of functions on affine variety.

So I should now tell you that you know there is another common name in the literature, people also use the word affine co-ordinate ring okay, so people instead of saying rings of functions of affine varieties people just use the word affine co-ordinate ring so this is this is the word that you are often seen in the literature okay.

So the moral of the story is that, so you see, so you can expect what does this isomorphism here corresponds to here what it is telling you is you know you should expect that this should be isomorphic to this and this should be isomorphic to this and this isomorphism as varieties as affine variety is precisely what is getting translated on that side as isomorphism of their affine co-ordinate rings that is what you should expect and that is exactly what happens in fact this association is actually an equivalence okay.

I will tell you how it is an equivalence, how strong it is, on this side if you put if you take only isomorphism classes of affine varieties okay then on this side you should put isomorphism classes of such rings, affine co-ordinate rings okay then this is a bijective association because if you change the affine variety upto isomorphism then its affine co-ordinate ring will also change only after isomorphism okay, that is one point, then the other point is if anything that you that happens on this side has a meaning on that side.

So a closed sub variety corresponds to a quotient okay, suppose I do not take the set of isomorphism classes of affine varieties on this side and there also I do not take the set of isomorphism classes of affine co-ordinate rings okay then if you give me a closed sub variety like this, this closed sub variety will automatically correspond to a quotient.

So you see this closed something being a closed subset of the, irreducible closed subset of the another thing that will, this is called the closed immersed, the word used is the closed immersion some affine variety being thought of as a closed irreducible subset of some other affine variety okay this closed immersion reflects on this side as a quotient okay.

So quotients here corresponds to closed immersions okay, it does not stop there, isomorphism here corresponds to isomorphisms there that is exactly what we saw here okay, more, what do open subset corresponds to okay, the beautiful answer is the open subset here will also correspond to ring homomorphisms here in fact they will be k algebra homomorphisms and you know what they will be localizations.

So the beautiful thing is closed immersions will correspond to quotients, open immersions namely the inclusion of an open, inclusion of a variety being sitting inside another bigger variety is an open subset okay and this is something I will explain soon okay that will show of there as a localisation of rings okay.

So and this is a very beautiful equivalence of categories so I should only tell you one thing what are the objects here so what are see if I say they are just affine co-ordinate rings of affine varieties that means I am giving the definition here which actually depends on this side but if I want to give a definition only on this side then the correct definition is these what you should put on this side is finitely generated k-algebras which is integral domains okay, so what you should put on this side okay.

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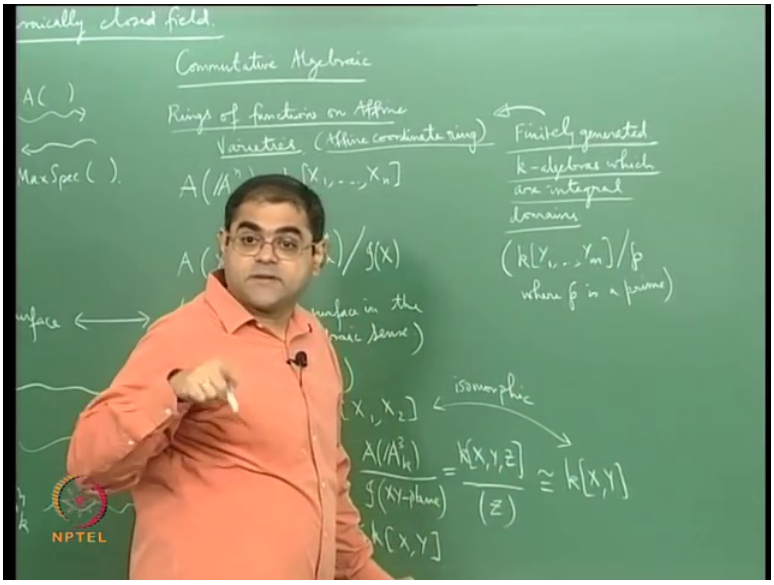

So on this side so let me write that down somewhere here, I am a little short of space but anyway let me write it, so let me write it here finitely generated k-algebras which are integral domains, so what does this mean? This means they are algebras of this form k of some polynomial ring k of Y1 etc Ym modulo a p where p is prime ideal, let me write down below also that I can write modulo p where p is a prime ideal, let me write it below so that I can, modulo p where p is a prime, okay and you know how it is going to go from here to here okay.

Do not worry about this max spec for the moment but you know what, you know the object on this side for which this is the affine co-ordinate ring you know what it is, it is actually Am okay whenever you write a quotient I told you it is the affine co-ordinate ring of the zero locus of the denominator which has to be a prime ideal considered in the ambient space for which the functions are the numerator.

So the numerator here is a polynomial ring in m variables that is the ring of functions on Am okay and p is a prime ideal there therefore zero locus will be an irreducible closed subset of Am and that therefore defines an closed sub variety of Am okay m dimensional affine space over k and (())(23:45) ring of functions will be pricesly this.

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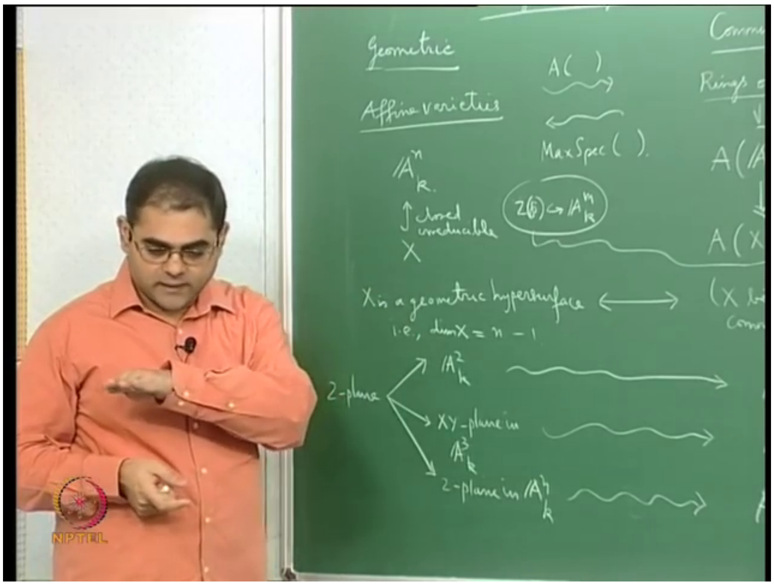

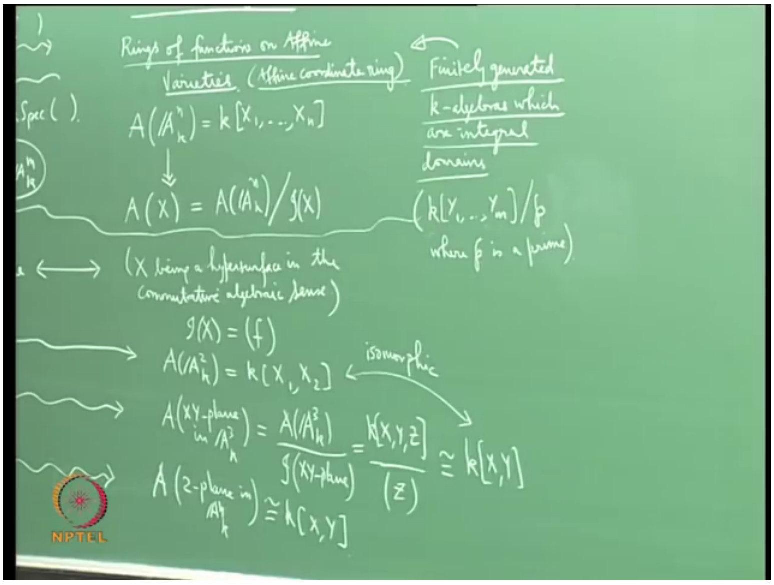

So you know this will correspond, so if I draw an arrow like this, this will correspond to, let me write it here, it will be some this will correspond to Z of p as a irreducible closed subset of An that is what it corresponds to okay. So on this side what I am putting is so called finitely generated k-algebra which are integral domains and the definition of that is actually simply a polynomial ring in finitely many variables of k modulo a prime.

I mean this prime ideal comes because I want the quotient to be an integral domain so you have to go modulo a prime and finitely generated is because it should be quotient of a polynomial ring in finitely many variables because T YI is they generate the polynomial ring in m variables therefore the images in the quotient namely the YI bars okay the co-sets YI plus P which are the YI bars in the quotient ring they will generate the quotient ring as an algebra over k okay.

So that is why this is a finitely generated algebra over k which is an integral domain and conversely every such algebra is defined to be something like this okay, so what I am trying to say is we have an equivalence of actually categories we have an equivalence of categories.

On this side we have affine varieties and morphisms would be in affine varieties, on that side you have these finitely generated k-algebras that integral domains and the morphisms are k-algebra homomorphisms and it is an equivalence of categories okay. I will explain all these things soon.

But there is one thing I should say at the outset I have been saying morphisms, I said isomorphisms of varieties, I said morphisms of varieties so that something that I will have to define I will define what is morphism between morphism from one variety to another okay so that is what is going to come very soon but I am saying that this is the general picture in which you get a complete equivalence of categories and in that sense everything that you see here is actually a mirror image of that and vice versa okay.

So all these tells you that studying the affine co-ordinate ring, ring of functions on affine variety is a very important thing because that captures everything okay whether you look at it as co-ordinate ring of affine varieties or whether you look at on this side the affine varieties it is one and the same because of this equivalence.

So this is, why I am saying all this I am saying this is why you can say that studying rings of functions on affine varieties is important and it also kind of you know it also justifies in a way Felix Klein statement that you know the geometry of a space is control by the function on the space okay.

So you know somehow this functions they control the geometry, mind you we had only affine m space which came from km and all we normally know about km is only it is an n dimensional vector space over k but we forgot the vector space structure and the geometry came in by looking at polynomial functions and using them to define the zariski topology.

So you see that even the topology on this side was gotten by functions on that side you must understand the beauty is the functions on this side completely capture the topology, okay. They in fact not only the topology the most deepest statement is that everything that functions on this side completely captures the geometry on this side so geometry at the lowest level is the topology then the next level is more deeper properties like manifold theoretic properties, smoothness, regularity and things like that okay.

So everything on this side, geometric side is captured by the competitive algebra which is the ring of functions okay so all this is to tell you why it is so important to study rings of functions okay. So I will elaborate on all this or whatever I said very soon but now what we will do is I wanted to you know, okay so let me tell you about this, this max spec thing.

So let me explain this max spec, that is the arrow that goes from this side that side alright so you know so let me again repeat the following thing, I will show later on that if you have two affine varieties on this side and you have a morphism that will give rise to morphism in the reverse direction of their co-ordinate rings and conversely.

Conversely if we have two co-ordinate rings of two affine varieties then you have a k-algebra homomorphism that will give rise to a morphism in the other direction okay that this will be a pakka equivalence and under this equivalence a closed subset will correspond to a quotient and open subset will correspond to a localize issue, okay this is what will happen.

You can see the morphisms are going in the, this is an arrow reversing the equivalence because you see from X to An if you have a closed embedding okay then you have from the functions on An to functions on X this is just restriction of functions which is seen as the quotient okay.

So this is what will happen and when if you have U an affine variety U sitting as an open subset of An what will happen is from A of An to A of U, you will get a homomorphism which will be actually localization of this ring at a suitable multiplicative subset in the sense of commutative algebra okay so everything on this side will translate to something on this side and conversely okay.

But all that needs to be proved we are going to prove it, okay. But I am just telling this to you in at the outside to tell you that why it is so important to study rings of functions on affine varieties okay which are of this form finitely generated k-algebras which are integral domains.

Now I will go one step further, what I am trying to do is go one step further and say that suppose I forget the picture on this side suppose I completely forget affine space, I forget the points on affine space I completely forget everything suppose I have only the function suppose I had then from the functions I can get back my space okay that is what this arrow from this direction to that direction is all about that is just from the function I can spook up the space.

That means the whole space is already here in the function okay and that is a much stronger proof of the veracity of the statement of Felix Klein that the geometry of the space is not only, so you know what he said is geometry of space is controlled by the function on a space but more seriously if you give me the functions then I can give you the space, I can cook up the space from the spacer function from the if I know the functions on a space then I know the space I can construct the space from its functions okay it is a very very deep thing.

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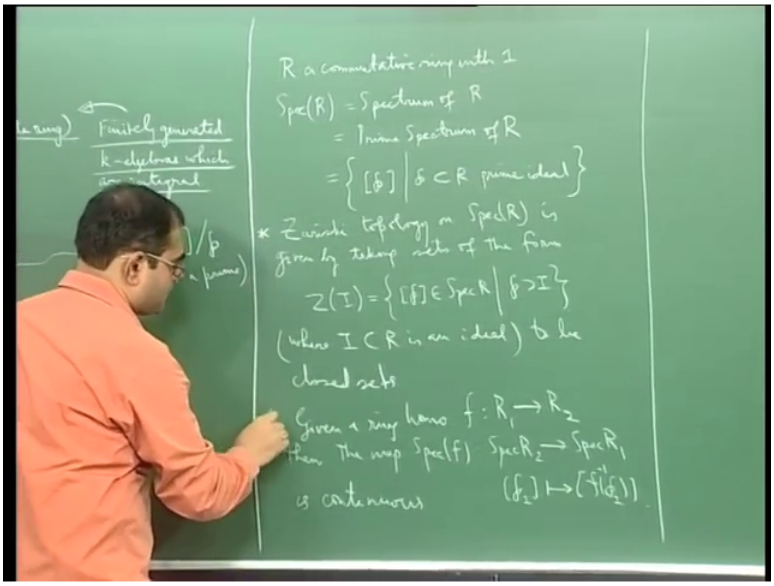

So that is what this max spec is and that is what I want to explain so what I will do is I will start with something that you must have seen in the course in commutative algebra but nevertheless it is not very difficult to recall this things so you know you start with R a commutative ring with 1.

So what you do is you look at spec R, this is called the spectrum of R, in fact it is called sometimes it is also called the prime more explicitly as a prime spectrum of R and this supposed to be the set of all prime ideals in R okay. So what you do is you look at the set of prime ideals of R okay and regard each prime ideal not as a prime ideal regard it as a point so this square bracket script P is supposed to tell you that you think of, here you are thinking of the prime ideal P as a point in spec R.

When you think of it as a point in spec R you are not thinking of it as a prime ideal inside R, you are thinking of it as a point on a space okay so this spectrum is just the points which corresponds to prime ideals okay and then so you should not confuse this with equivalence class because normally you have you put a square bracket around in element to say that it is equivalence cause of that element so you should not confuse it with that there is no equivalence class.

It is just to distinguish between the prime ideal as a subset of R and the prime ideal as a point of spec R they are different okay so my space is spec R now the spec R will become a topological space, okay how? So there is a Zariski Toplogy on this just like you have Zariski Topology on affine space you have Zariski Topology on spec R and in fact you can say both ways this is inspired by that and that is also inspired by this okay it depends on various thought okay.

They are equally one and the same so let me, so let me define this, Zariski Topology on spec R is given by taking sets of the form Z of I okay so defined to be equal to set of all p in spec R such that p contains I where I and R is an ideal to be closed sets okay, so you see the zariski topology is defined like this, the zariski topology is defined as the to be it is defined by specifying collection of closed sets, what are the closed sets?

The closed sets correspond, they come out of ideals, ideals of R you take an ideal I of R immediately you can write down the closed set Z of I okay which you should think of as a 0 set of I okay if you compare with the zariski topology on the affine space and the 0 set of I is all those prime ideals, all those points which correspond to the for which the corresponding prime ideals contain I okay, this is the way it is defined and you can check that this satisfies the conditions for a topology, namely sets of this form satisfy the (())(35:03) for close sets in a topology and you get thus spec R becomes a topological space okay.

And then you see you can also check that you know if you have, if you give me given a ring homomorphism f from R1 to R2 okay so R1 and R2 are commutative rings with 1 and f is a ring homomorphism from R1 and R2 then the map spec f going from spec R2 to spec R1 which is given by give me a prime ideal of R2 and simply send it to its inverse image which is a prime ideal of R1 okay.

If you give me a ring homomorphism from R1 to R2 then it induces a map in the reverse direction on the spectra, on the prime spectra and then it simply given by pulling back prime ideals because you know the inverse image of prime ideal and ring homomorphism is again a prime ideal it is not true for maximum ideals but it is true for prime ideals inverse image of maximum ideals need not be maximum but inverse image of prime ideal is always a prime ideal okay.

So what will happen is well the fact is you get this map that is not the point, the point is this is now a map of topological spaces, see it is a map between two topological spaces, most obvious thing we can expect is that it should be continuous and it is okay. You saw this map is continuous, so this is another fact that you can, that you should have come across in a course in commutative algebra if you have not you can it is very easy to check okay.

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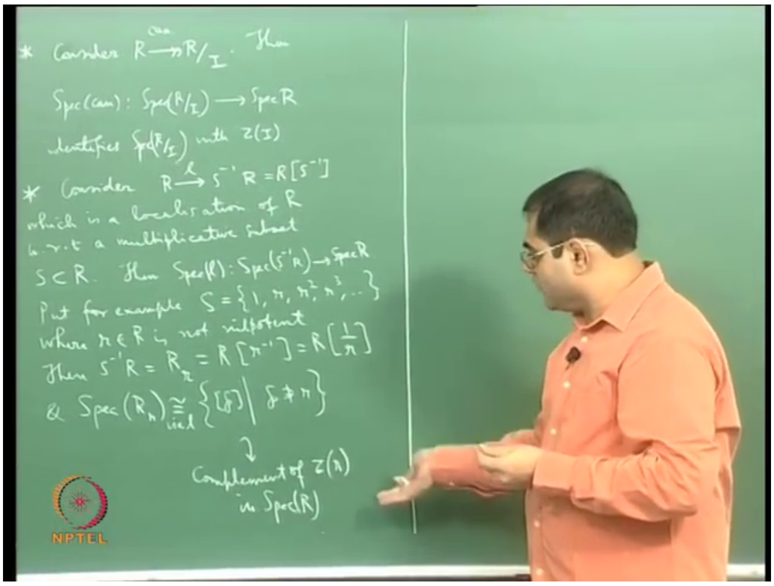

And in fact what happens is if you look at, so let me maybe I will put a star here, I will put on the star here, then here, so it is like a bulleted list, so but instead of bullets we are using stars. So here third thing I want to say is that what about a closed subset like this, so what I want to say is that Z of I so if you take, consider R to R mod I the canonical so I am writing CAN for the canonical quotient map from R to R mod I okay.

Then if you look at spec of that which goes from spec R mod I to spec R okay, because you know I have already told you if you have a ring homomorphism from one ring to the other, you are going to get a map on the other spectra in the reverse direction. In particular you take the ring homomorphism to be their canonical ring homomorphism that you get from ring to its push then the corresponding map on spectra what will be, this will actually identify spec R mod I with the close subset Z of I spec R.

So what will happen, so identifies spec R mod I with Z of I, so mind you Z of I is a subset of spec R by definition it is a close subset of spec R because of the zariski topology we have defined on spec R and spec R mod I the image of spec R mod I under this is identified with that okay, so in other words Z of I is also spectrum of ring.

The moral of the story is that the closed subset Z of I also is a spectrum, so every, we define a close subset inside a spectrum and it turns out that every close subset itself is a spectrum can be identified with the spectrum, what is a spectrum? It is spectrum of the quotient by that ideal okay and in some sense you know you must think of it like this you must think of it, think like this if R is thought as a ring of functions on spec R okay abstractly then R mod I is a ring of functions on Z of I which is identified with spec R module okay.

So you see R is a ring of functions on spec R alright, the functions in what sense is something that has to be made formal, more precise but do not worry about it, R is a ring of function in spec R then what is the ring of functions on ZI? It is R mod I okay because spec of R mod A can be identified with Z of I okay, and there is one more thing you see what is the situation with respect to localization so two particular kinds of ring homomorphisms are quotients and localization okay.

And so what happens with localization consider R to S inverse R l which is a localization of R with respect to a multiplicative subset S inside R so if you remember in commutative algebra given a ring R we define what is meant by a multiplicative subset. A multiplicative subset is a subset which does not contain 0 and for which, which is closed under multiplication so if there are two elements in that set then their product is also in that set okay and we can in competitive algebra from the localisation of R with respect to the multiplicative subsets which is just inverting the elements of the multiplicative subset.

So S inverse R which is the localization of multiplicative subset sometimes it is also written as R bracket S inverse okay so it can be thought of as a polynomial ring in R in as many variables given by reciprocals of elements of S modulo the natural relations that come because of the elements of s being inside R okay and that is the reason why one writes S inverse R as R square bracket S inverse.

But essentially it means that you are just inverting S so you know simplest case is if you take R to be for example if you take R to be integers and you take S to be the compliment of 0 okay then S inverse R will just be rational numbers, more generally if you take an l far as an integral domain and S is the compliment of 0 which is a prime ideal okay the compliment of a prime ideal is always a multiplicative subset and localization by that is always going to give you a localization.

And in this case compliment of 0 if you localize the integral domain at 0 at the complement of 0 then you are going to invert that means you are inverting everything which is different from 0 and that means you are forming a field of fracture so forming the field of fraction is very special case of localization. Localization is the generalization of that.

We would have seen this in commutative algebra but the fact is that if you take this localization map okay then what does it correspond to in the spectrum at the level of the spectrum, then spec l will go from spec S inverse R to spec R okay, you will get this map. Let me take a particular case put for example S to be just the multiplicative subset given by a single element r, r square where r is in R is not nilpotent I want R to be the element r in R small r in capital R to be nilpotent because I do not want some power of it to be 0, if some power of it is 0 then the multiplicative set contain 0 and I do not want 0 in the multiplicative set there are some books which say that 0 is allowed in the multiplicative set but then the convention is there that the localization becomes 0 ring and nobody wants to study the 0 ring as far as geometry is concerned.

So we always never want 0 to come inside a multiplicative subset that is the reason why I do not want small r to be nilpotent okay, and then in that case we normally write S inverse R as R localized at small r okay we also write it as R bracket small r inverse some people write it as R of 1 by r okay these are various notations okay.

And the fact is spec and spec of, if you calculate spec of R r what you will get is, you will simply get all those prime ideals the points corresponding to the prime ideals such that the prime ideal does not contain small r, okay. And this so you will get this identification okay and this is just complement of Z of R in spec R, so you see spec R has a zariski topology as I have explained and then there is the ideal generated by small r okay it is ideal generated by single element and you take the 0 set of that ideal what is the 0 set of the ideal?

By definition it is all those prime ideals which contain the ideal R but a prime ideal contains the ideal generated by R if and only if it contains R itself so this is Z of R precisely has for its point those prime ideals which contains small r and what is the compliment it is all those prime ideals which do not contain points corresponding to prime ideas is don't contain small r and that is precisely spec R r, capital R sub small r and how do you get this identification?

This identification is provided by this map, the image of this map, the image of spec l in this case identifies spec of S inverse R with this subset which is an open subset of spec R okay because it is the compliment of a closed set so the moral of the story is you know the ring of functions on the compliment of the locus defined by a single element okay namely the name of ring of functions on the locus where a single element does not vanish is precisely localization by that element okay.

Which is if you think of small r you know if you think of small r as an element of capital R and if you think of r as the ring of functions on spec R then small r is a particular function and what is Z of r you should think of Z of r as the locus where r, small r vanishes okay and what is its compliment? Its compliment is should be the locus where small r does not vanish but where small r does not vanish if small r is a function that does not vanish one by small r should also be a valid function.

So it in other words it tells you that in this locus where small r does not vanish which is an open set, the functions are simply given by R of 1 by small r namely the functions are of the form, some function of some element of R divided by some power of small r which is intuitively correct and in fact you get it here okay so this identification is via L in this case.

So the moral of the story is the ring of functions corresponding to a closed subset is given by, corresponding to a closed subset given by an ideal is given by the quotient ring and the ring of functions corresponding to an open subset so called basic open subset where function does not vanish is simple given by inverting that function that is what it says okay.

So this is a background from commutative algebra okay and we can use this to go from this side of the diagram to the other side of the diagram which I will, with which I continue with this in the next lecture okay.