Basic Algebraic Geometry
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Module 4
Lecture 10
Geometric Hypersurfaces are Precisely Algebraic Hypersurfaces

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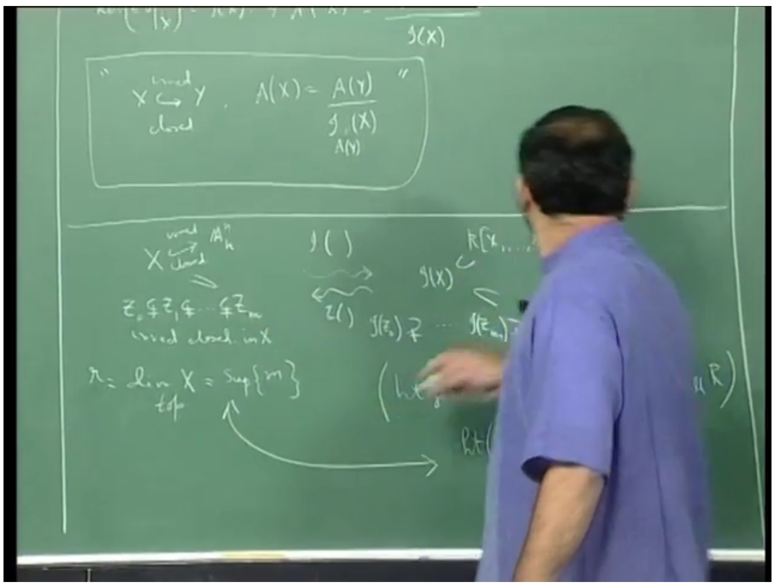

Let me continue with this discussion that I entered with last time what I want to say is when you take the dimension of X to be r, okay so you are taking X to be an irreducible closed sub-variety I mean irreducible closed subset of An an affine variety in An. So it is a common to use a word sub-variety when you look at a variety inside another variety so since X is an irreducible closed subset it is a variety and An you already know is a variety so we say that X is a sub-variety closed sub-variety of An, okay.

And the point is that if you start with if you take any chain strictly increasing or decreasing chain of irreducible closed subsets of X and if you take if you start with indexing it with 0 and go on up to m and if you take the maximum supremum of all those m's that is going to give you the dimension of X, okay and what you must understand is if that dimension of X is r and here by dimension of X I mean the topological dimension of X this is topological dimension of X, okay that is how a topological dimension is defined, if that is r then you know I will that will correspond to we have strictly increasing chain that is it will start with 0 and it will go on up to r.

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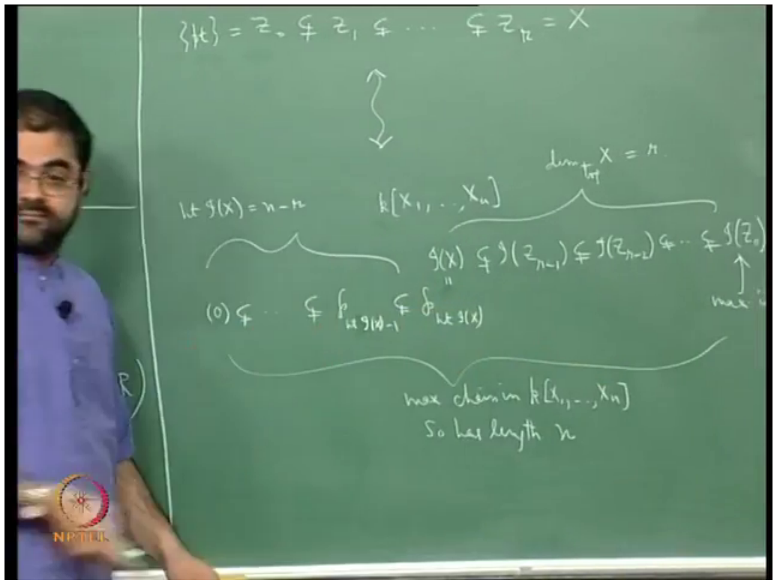

So you know if dimension of X is r we have a maximal chain Z not properly contained in Z1 and so on Zm Zr, okay and what you should understand is that Z not has to be a point, okay because if Z not is not a point and it is maximal chain I can get a contradiction by making this chain bigger by putting a singleton point in Z not, okay and that will contradict the maximality of the chain so this is the maximal chain Z not has to be point and Zr has to be X, okay Zr has to be X.

So again for the same reason if Zr is not X then I can add X here and I will get a bigger chain, okay. So corresponding to this if you look at it if you look at the corresponding diagram in the polynomial ring namely the ring of functions on affine space what is happening is a following you have in KX1 etcetera Xn you have I of X so this from X to this point of X that translates to an increasing sequence I of Zr minus 1 properly contained in I of Zr minus 2 and so on and it goes on up to I of Z not this is how it goes on and mind you since Z not is a point I of Z not is a maximal ideal because points correspond to maximal ideals, okay.

And this smaller the smaller the irreducible closed subset the larger the ideal, okay and therefore this point should correspond to a maximal ideal so this is a maximal ideal in fact this maximal ideal you know it is generated by X1 minus lambda 1 X2 minus lambda 2 and so on where lambda I are the coordinates of this point you know that already, okay. And then what is the what is the height of IX if height the height of IX is going to be you start with IX and then you get a strictly decreasing chain of prime ideals and look and the height is supposed to be the one that gives you a chain of maximal chain, okay.

And so in fact what will happen is that this will be you know p height of IX properly contained in p height of IX minus 1 properly contained in and so on and the smallest one will be 0, okay because 0 is a prime ideal so the smallest one has to be 0 and the fact is that since this is a maximal chain, okay and this is also a maximal chain you put these two together that will be the maximal chain that you can get for the polynomial ring and the whole thing will add up to the Krull dimension of the polynomial ring.

So this whole thing will be this will be a maximal chain in k X1 etcetera Xn so as length n which is the Krull dimension, the Krull dimension you know of a ring is supremum of the heights of its prime ideals, okay and of course the height of the prime ideal because the height is being measured from the 0 prime ideal the ring here is an integral so 0 is a prime ideal so you are measuring you are starting from the prime ideal and you are going down all the way to 0.

So if the ideal becomes bigger the height becomes bigger so you can imagine the height is maximum for the maximal ideals, okay. So this is the maximal possible height and that is the Krull dimension of this ring and that is equal to n, okay and therefore but you see if this is r this part is r then this has to be n minus 1 so this is what essentially tells you is that this part is corresponds to the fact that dimension topological dimension of X is r this is the part that tells you that the height of IX is n minus r and this whole thing is n and this n minus r with r adding up to n is what this formula tells where r is the ring of polynomials in n variables and what is r mod p, p is of course the ideal of X here, and what is the r mod p? r mod p is the ideal I mean is a ring of functions on X, okay.

So what you must understand is that this IX this part this is also equal to the so what I want to say so I wanted to say that this is also equal to the dimension the Krull dimension of the functions on X, okay so this is exactly what is happening in this case, okay.

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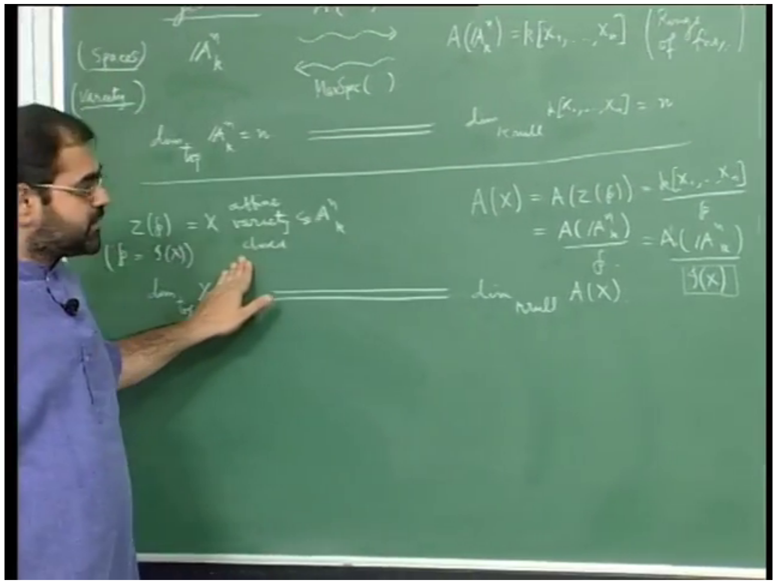

So now what I want to say is I want to also look at some special cases of sub varieties, okay and but let me make one more statement here you see you know Ax is what, this Ax is actually the polynomial ring modulo the ideal of X, okay.

So if you give me for example if you take this maximal chain and you go mod if you go mod IX if you take the image of this this whole chain in the quotient, okay this is the maximal chain in the polynomial ring and this is the quotient of the polynomial ring, okay if you take the image of this chain there you will get this, this will become 0 IX will become 0 in when you quotient out by a X and this will still be your maximal ideal in the quotient because you know given a ring and its quotient the maximal ideals in the quotient correspond to the maximal ideals in the original ring which contained the kernel.

So this maximal ideal I Z not in the quotient will also correspond to a maximal ideal and you will l get a from that ideal you will go to 0, okay but what is that that is precisely the height of a maximal ideal in the quotient ring but that has to be the Krull dimension of the quotient ring and that is exactly r because this is length is r that is what is being reflected here this is equivalent that is what I want you to understand, okay. If you read this whole chain mod IX in the quotient ring this will become 0 and you will get from 0 to this maximal ideal which is gotten by dividing each of these by IX that will give you a maximal chain in the quotient ring so its length has to be the Krull dimension of the quotient ring and therefore this is also equal to r because this length is r, okay so I want you to understand this.

Okay, now so let me come back to what I was looking at there are special sub-varieties that we are interested in in some sense in algebraic geometry there are many theorems there are questions that are proved by just looking at the case when you are looking at the locus of the single equation the locus defined by a single equation namely you look at the 0's of the single equation, okay. So and this is called the hyper surface case so many theorems in algebraic geometry can be proved by first looking at what happens to hyper surfaces.

So the point is that you know from the point of view of commutative algebra why this is nice is because this corresponds to studying one equation at a time because hyper surfaces supposed to be the locus given by a single equation, right? So that is the importance of studying hyper surfaces, right? So but I will give a different definition of a hyper surface, so what I will say is hyper surface is let us give a definition which comes from dimension, okay. So I will define a hyper surface to be an X like this whole dimension is one less than the dimension of the big space, okay.

That means it has co-dimension 1 in the big space, so co-dimension of a subspace in a bigger topological space is just the difference of the dimension of the bigger topological space minus and the dimension of the smaller topological space. So hyper surfaces so X is called a hyper surface if X has co-dimension 1 so X is called a hyper surface if dimension of X is n minus 1, okay of course when I write dimension of X I mean topological dimension, okay so I am not going to keep writing the subscript top you must always remember that whenever I say dimension over a topological space it is always topological dimension that is something that you should not forget, okay.

So and why the word hyper surface is because you know well if you are in one variable then there is not much because in the one variable case you are looking at A1 and the only close subsets are finite subsets of is a finite subsets of points you know that very well. So hyper surface is just a if you want just a single point, okay that is what you will get is not really anything very interesting.

If you go to more than one variable if you go to two variables then you get a curve, okay if you go to two variables then essentially you are in a two dimensional space and you are looking at the 0's of a one equation the dimension should come you are essentially trying to look at 0's of one equation that is what you expect to happen the dimension comes down by 1 so in a two dimensional space a hyper surface is a one dimensional closed irreducible closed subset and of course a one dimensional object is always called a curve, so when n is equal to 2 you get a curve in two space, okay when n equal to 3 you actually get a surface in 3 space, okay and if n is greater than that you do not you no longer call it a surface you call it the hyper surface so the word hyper is reserved for n greater than 3, okay.

Of course if n is 3 you call it just a surface, okay if n is 2 then it is a actually a curve in two space, okay. Now you see there are so again this is another very important thing in algebraic geometry you can make a definition from the on the geometric side you can also make a definition on the commutative algebraic side and then the question is the natural question is are these two definitions equivalent. So if I want to define a hyper surface as co-dimension 1 irreducible closed subset then that is the definition on the geometric side. On the other hand if you want to intuitively use the fact that something that has one dimension less than the bigger space has to be given by a single equation then that will tell you that the you know the commutative algebraic definition will be you are looking at the 0 locus of a single polynomial, okay.

So I can give a commutative algebraic definition, okay that X is a hyper surface in the commutative algebraic sense if the ideal of X is generated by a single polynomial, okay so you see now I have two definitions, so this is definition 1 then let me write let me also put it like this X is called hyper surface in the commutative algebraic sense if ideal of X is equal is generated by single polynomial f for f in the polynomial ring, okay.

So these are two ways of defining what a hyper surface should be, this says hyper surface is given by only one equation the fact that you are using one equation is says that you are using only one polynomial so it is commutative algebra and the topological are geometric idea of a hyper surface is that you are cutting it is a dimension based definition you are cutting by 1, okay and you can ask whether these two are the same the answer is yes and that the proof of that involves significant amount of commutative algebra and I will tell you what the results are that leads to the proof of that.

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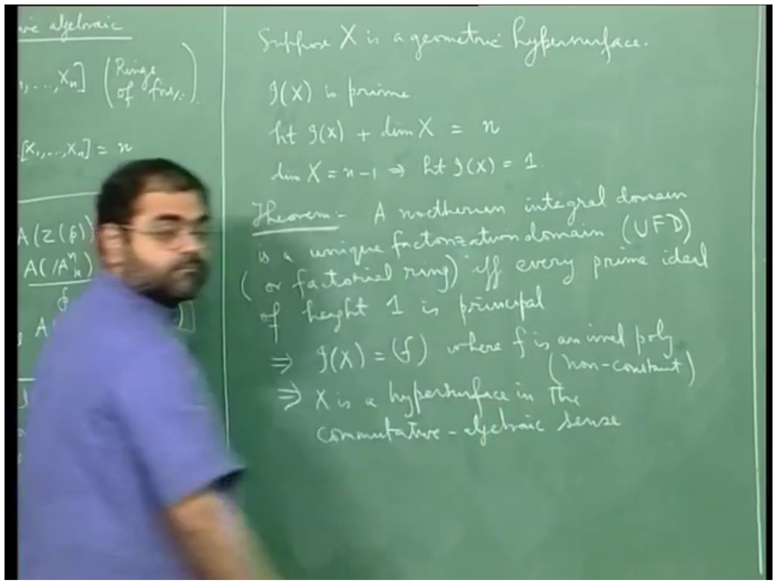

So you see suppose X is a geometric hyper surface suppose X is geometric hyper surface so mind it geometric hyper surface means is a hyper surface in a geometric sense according to this definition, okay. So again let me reiterate this is what I mean by geometric hyper surface something that has dimension one less than the dimension of the ambient space, right? I would like to show that X is commutative algebraically also a hyper surface namely that X is defined only by one equation, why is that true because of the following thing, see I of X is prime of course because X is irreducible so I of X has to be prime we are only worried about varieties, okay.

Then you also know that height we have just now seen the height of I of X plus the dimension of X is n, right? The height of I of X plus the dimension of X is n which is the dimension of the Krull dimension of the whole polynomial ring which is same as dimension of the affine space, okay. So let us write that height IX plus dimension of X is equal to n we know this, alright? Of course we know it means because of this theorem here this formula I have written down is a theorem if r is a finitely generated k algebra namely a quotient of the polynomial ring in n variables over a field and suppose its quotient by an ideal is a prime ideal so that the quotient is actually an integral domain then the Krull dimension of the quotient plus the height of the prime ideal by which you gone modulo to get the quotient should add up to the Krull dimension of the ring, okay that is our theorem, okay that is what is being used here.

Now what is given is since is geometric hyper surface dimension of X is n minus 1 so dimension of X is topological dimension of X is n minus 1 we will tell you that height of IX is 1, okay. Now this is a theorem in commutative algebra, okay so theorem a Noetherian integral domain is a unique factorization domain it is written UFD for short or some time it is also written as factorial ring in some books, okay if and only if every prime ideal of height 1 is principal, okay this is the theorem from commutative algebra, okay.

So what it says is you start with a ring commutative ring with 1 which is Noetherian, okay which means every ideal is finitely generated or the ideals satisfy ACC ascending chain condition and assume that it is also an integral domain that means it has no 0 devices it is same as saying the 0 ideal is prime then to conclude that it is a UFD unique factorization domain, okay namely that the there is a notion of irreducible elements prime elements and any element can be uniquely factored into a finite product of irreducible elements to certain finite powers and this factorization is unique up to permutation of the factor and up to units, okay.

So an example of unique factorization domain is of course the polynomial ring because you know the polynomial can always be factored, okay. This so the condition that a Noetherian integral domain is a UFD is equivalent to every prime ideal of height 1 being principle that means you take a prime ideal if it has height 1 then it has to be generated only by a single element, okay this is the theorem now if you use this theorem use the fact that this IX is prime and it is a prime ideal in this polynomial ring the polynomial ring is a UFD and there is a prime ideal it has height so 1 its principle. That means the ideal is generated by a single element, okay and that element has to be irreducible, mind you because if that element breaks up as f1, f2 then IX the X will breakup as 0 of f1 union 0 of f2 it will so it will not be irreducible.

So the fact that X is irreducible which is equivalent to the fact that IX is prime tells you that this f which generates IX has to be an irreducible element, okay. So put all this together so this will tell you that I of X is equal to f where f is an irreducible polynomial and of course non-constant it has to be non-constant because you know if it is constant you know if it is a non-zero constant then it is a unit the ideal generated by that will be the whole ring, okay it cannot and the whole ring is not a prime ideal and if it is the 0 constant polynomial then if you take the 0 prime ideal its height is 0 because there is nothing smaller than that.

So f has to be a necessarily an irreducible polynomial it has to be non-constant polynomial. So what this tells you, this tells you that X is commutative algebraically a hyper surface, okay. So what this tells you is that if you require a close variety to be a hyper surface geometrically namely it has to have one dimensionless than the ambient space the ambient affine space then it is then it means that it has to be also commutative algebraically a hyper surface namely it has to be defined only by one equation and that one equation has to be an irreducible non-constant polynomial, okay.

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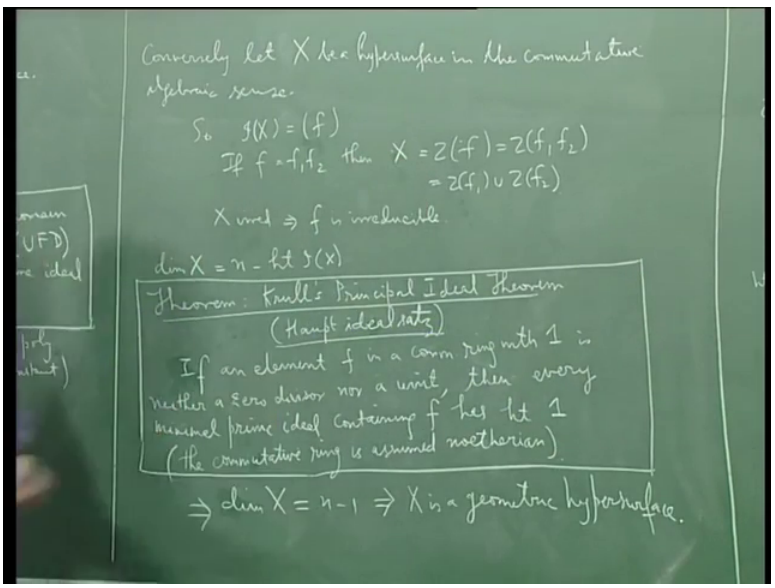

Now we can go the other way also, the other way is probably a little easier conversely let X be a hyper surface in the commutative algebraic sense start with this, so ideal of X is f it is defined by hyper surface in a commutative algebraic sense means it is defined by a single equation. So ideal of X is f I will again reiterate that f has to be an irreducible polynomial if f is equal to f1, f2 then X will become X which is 0 of f will become 0 of f1, f2 which is actually going to be 0 of f1 union 0 of f2 and X irreducible will tell you that this cannot happen one of them has to be the whole space, that mean it has to be X itself.

So X irreducible will imply that f is irreducible X irreducible as a topological subset you will imply f is irreducible as a polynomial, okay. Now what I want to prove? I want to prove that X is geometrically hyper surface that means I have to show that it has height I mean it has dimension n minus 1 what is dimension of X again we use this formula dimension of X is equal to n minus height of IX, okay.

So now we need another deep theorem another deep in the sense rather fundamental theorem it is called the Krull's Hauptidealsatz, okay it is called the Krull's principle ideal theorem, okay so let me state that theorem Krull's principle ideal theorem it is also written as Hauptidealsatz, and what does it say? It says let if an element f in a commutative ring with 1 is neither a 0 device nor a unit, then every minimal prime ideal containing f has height 1 I just say that the commutative ring is assume Noetherian is assumed let me put that for safety sake, okay.

So this is another important thing algebraic geometry the fact is that the most sophisticated part of algebraic geometry is supposed to work over anyway not even over Noetherian rings but at least when you are doing decent amount geometry you really want to work only with Noetherian rings because you get Noetherian decomposition for example, okay. So for example you know the Noetherian decomposition in affine space which told you that any algebraic set can be decomposed uniquely into a finite union of affine closed sub-varieties which are unique if the decomposition is not redundant that is no such closed sub-varieties contained in any other in the decomposition, this comes out of the Noetherian property of the topological space, okay.

And therefore in it is usual that you always work with at least in the first course in algebraic geometry you always work only with Noetherian rings so it is harmless to assume things are Noetherian, of course it is a matter of technical expertise to see which of the theorems will still go through if you remove the Noetherian hypothesis, okay but the point I want is to say is that look at what this says, see the beauty with commutative algebra is that if you translate it to algebraic geometry it actually has a meaning, okay and it is as follows see at least in this case see you see if so what it says you take a commutative ring with 1 assume it is Noetherian if you want take an element f, okay assume the element is not a 0 device, okay and assume it is not a unit, right?

Then you look at the ideal generated by that single element f, okay and you can talk about the minimal you can talk about prime ideals which contain that element f there are such in fact any non-trivial ideal in a commutative ring non-zero commutative ring is always contained in a maximal ideal any proper ideal is always contained in a maximal ideal this is if you want consequence of zones lemma, okay but therefore you know if you take the ideal generated by f it is a proper ideal it is a proper ideal because f is not a unit, alright? And the ideal generated by f contains a it certainly it is contained in a maximal ideal more generally you can look at prime ideals also which contain the ideal f ideal generated by f and what the theorem says is that if you look at any minimal prime smallest amongst this set of primes which contain f the height of such a prime is 1, okay.

So it is what it says is that you take a minimal prime containing f then between then from that prime that prime has height 1, okay between that on the smallest possible prime, okay f is caught there in between these two, okay of course the smallest possible prime will be the 0 ideal if the ring is in integral domain, okay in which case you are saying that the ideal is generated by f is caught between the 0 ideal and the smallest prime which contains f each of the smallest primes which contain f.

So there are two facts that I want to tell you, let us first apply to our situation in our situation our commutative ring is of course the polynomial ring in n variables, okay and here is and the element f is the single non-constant irreducible polynomial, okay. If you take the ideal generated by f that is already a prime ideal mind you, the ideal generated by f single element is always a prime ideal in fact the truth is that if you take any unique factorization domain and you take a irreducible element there that is an element which cannot be factored into smaller elements, okay into non-trivial factors which we tend to call as smaller elements, okay smaller factors such an irreducible element if you take the ideal generated by that that will be prime, okay.

So since the polynomial ring is unique factorization domain and since you have started with an irreducible polynomial which is an irreducible element in a unique factorization domain the ideal generated by that element will be a prime ideal, so this is certainly a prime ideal. So if you look at this any minimal prime ideal which contains this it has to be this itself if when you look at the minimal prime ideal generated by an element f it will be different from f it will be different from the ideal generated by f only if the ideal generated by f is not a prime ideal, if a ideal generated by f is a prime ideal then the minimal prime ideal which contains the ideal generated by f is at the ideal generated by f itself.

So if you apply Krull's principle ideal theorem what it will tell you is that the height of f is 1 the height of the ideal generated by f is 1 but the height of the ideal generated by f is the same as height of IX, so it will tell you so this will tell you that height that dimension of X is actually n minus 1 which will tell you that X is geometrically a hyper surface it will tell you X is a geometric hyper surface, okay. So what you have got is that if you start with a hyper surface in a commutative algebraic sense you get a hyper surface in geometric sense and we have already seen the other way we start with the geometric hyper surface then it is a hyper surface in a commutative algebraic sense so both these definitions are the same this definition is completely geometric it is gotten by saying that it is one dimensionless its co-dimension one this definition is commutative algebraic you are saying you are looking at only 0's of one equation and they are one and the same, okay this is again I mean this is what you should always appreciate there is something going on here which has complete translation in this side, okay.

And of course these two theorems here which come into the picture they are very very important and at some point if you have not already seen them in a course in commutative algebra you can you should make it a point to set aside some time for extra reading and if you can when you can do that try to look at the sketch of a proof of theorems like this but what I want to tell you is that there is geometric significance, so for example suppose f is not so let me explain more generally what this statement is saying suppose f is not an irreducible polynomial, okay what does this statement say it has a geometric meaning what is it it is the following let me write that.

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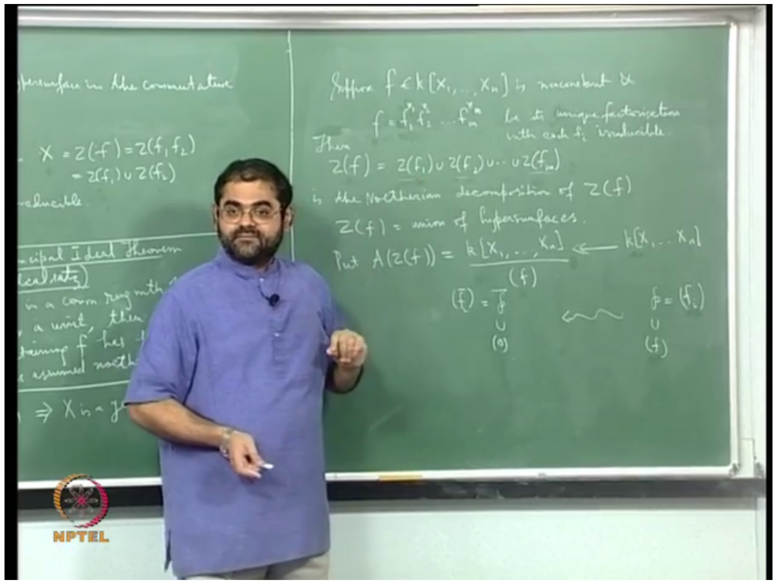

Let me try to explain that you see suppose f in the polynomial ring is non-constant and f is equal to let us write f1, f2 etcetera fm be its factorization unique factorization, okay with each fi irreducible.

So this is again the fact that any polynomial can be if it is not irreducible you can break it down into a product of factors each one of which is irreducible and of course I am writing it like this but there could be some factors could repeat, okay. So you know let me write it so you know maybe I should put powers to be very accurate so you know if I will have to put something like n so n1 f1 power n1 f2 power n2 fm power m sub m, so if I write it like this then you know I mean that no fi is the same as any other fj, okay and these you know these powers are all uniquely determine this is just like the fundamental theorem of arithmetic where you say any integer can be uniquely factored into product of prime powers the primes occurring are unique and the powers of each prime that occur are unique and it is the same thing that is happening in the polynomial ring (())(40:17), okay and that is way it is unique factorization domain.

Now of course here I can always push in or pull out here constant so this factorization is unique up to a unit which is a non-zero element of k, okay. Now if you look at the 0 set of f, okay then this you know it is just going to be 0 set of f1 union 0 set of f2 union 0 set of fm this is what is going to be this is what you are going to get, right? Because 0 set of f will be 0 set of f1 power n1 union 0 set of f2 power n2 and you must understand the 0 set of a power of f is a same as 0 set of f itself because the set of point where a power of a polynomial vanishes is same as set points where the polynomial vanishes.

So when I go to the 0 set all these powers are gone I do not care about the powers, now if you watch look at each Z fi each Z fi is irreducible as a subset it is an irreducible closed subset, why? Because each fi is already an irreducible polynomial and you know since each fi is an irreducible polynomial and it is an irreducible element in the polynomial ring which is a UFD the ideal generated by an irreducible element is a prime ideal so the ideal generated by each fi is a prime ideal and the 0 set of that prime ideal is therefore an irreducible subset, okay.

Therefore each Z fi is irreducible and what and of course no Z fi is contained in some other Z fj that is because no fi divides any other fj they are all distinct irreducible polynomials, okay that is what unique factorization means when you write in product of factors the factors are not repeating suddenly, okay it is only to take care of reputations that you put the powers, right? Now watch these are our irreducible closed subsets, okay if you look at this what is this? This is actually the Noetherian decomposition of f this the way I have written it this is the Noetherian decomposition of f mind you Z of f is an algebraic set Z of f is not an irreducible algebraic set because f is not the ideal generated by f is not a prime ideal that is because f is not irreducible, alright? And by and you know the Noetherian decomposition is unique the Noetherian decomposition says that if you have a Noetherian topological space you have a close subset then the close subset can be written as a finite union of irreducible closed subsets and this decomposition is unique if you assume that none of the subsets is contained in any of the others, okay.

So this is the Noetherian decomposition of course up to a permutation of the Z fi's, okay. If you watch if you take the if you take the if you go to this case go to what we have proved so far that geometric hyper surface is same as a hyper surface in the commutative algebraic sense what will tell you is that each of these has dimension n minus 1 each of this is a hyper surface. So what it will tell you is Z of f is a union of hyper surfaces, okay it is a union of hyper surface and if you look at let me look at the following let me do let me give a tentative definition how did I define the coordinate ring I mean the ring of functions on a close subset I simply defined it as the ring of functions on affine space modulo the ideal of that set, okay.

Now what I will do is I will just put A of Z of f, okay so as polynomial ring modulo the ideal generated by f make this definition, okay make this definition this is not a very good definition for the reason that since the ideal generated by f is not prime you are going the ideal modulo which you are going is not a prime therefore this crazy thing is not an integral domain this quotient ring is not an integral domain.

So for example in this quotient ring f1 you see f1 bar, f2 bar, fm bar which are the images of f's the fi's in this quotient you see there if you raise them to the powers these powers and multiply them you will get 0 but individually they are not 0 they are so each f1 if the image of each f1 here is a 0 device mind you and but the point is in this ring see in the if you look at so let me write that this is a quotient of k X1 through Xn, okay now take a prime ideal p which contains f, okay a prime ideal p contains f if you go down here, okay it will give rise to a prime ideal p bar which will contain a 0, okay it will be a prime ideal p bar which will contain 0.

What you should understand is that for each of this prime ideals I can take the ideal generated by fi's, okay you take the since fi divides f the ideal generated by fi will be multiples of fi and f is also multiplied by fi, so ideal generated by fi will contain the ideal generated by f, okay anything which is a multiple of f is also multiple of any fi. So I can take a prime ideal which contains fi, okay that will correspond to the prime ideal generated by fi bar in the quotient ring, okay and the fact is that these fi's they will be the smallest prime ideals which contain f that is because of this unique factorization you will have to do a you have to convince yourself that this smallest prime ideals which contain f in this ring are precisely the fi's, okay.

And what does Krull's principle ideal theorem says? It says that in the polynomial ring itself the smallest prime ideals which contains this f have height 1. In other words what it says is if you commutative algebraically look at only 0 of a single equation but do not insist that the 0 set is irreducible you will not get a geometric hyper surface but you will get a union of geometric hyper surfaces that is what it says that is the full content of this theorem, okay see you can ask this question, right? A commutative algebraic surface if you want to just define it as a surface which is given by single equation since I already want something that is irreducible that single equation has to be irreducible but if I relaxed the condition that the locus is not irreducible then it is just an algebraic set. So you are looking at single 0 you are looking at the 0 locus of the single polynomial the polynomial is necessarily irreducible.

Then what this says is if it is irreducible then it is a hyper surface, if it is not irreducible it is union of hyper surfaces that is what it says that is why that should tell you why this statement of this theorem involves the minimal prime ideals which contain f you see they become relevant in this non-irreducible case the minimal prime ideals that contain f they correspond to the ideals that correspond to the irreducible components of the 0 set of f that is the connection geometric connection to this statement, okay.

And the importance with this theorem is that you know you can ask more generally this question, so this is also part of algebraic geometry you have something nice happening, okay for us we have always started with the polynomial ring in n variables you can work with more general rings if you work with more general rings you can ask the question when will this be true you start with a geometric hyper surface is a same as a hyper surface in the commutative algebraic sense for what kind of spaces will it be true based on the ring of functions on those spaces and that is the answer given by that is the geometric content of this theorem what it says is if you space has a ring of functions which is a unique factorization domain, okay if your space is such that it is ring of functions the unique factorization domain there is no difference between a commutative algebraic hyper surface and a geometric hyper surface that what it says that is the geometric content of this theorem, okay.

So what you should understand is this is the point about algebraic geometry you have some statements which are completely statements in commutative algebra but if you translate them they translate into something very geometric. So you know how to define what a unique factorization domain is in a commutative algebraic sense it is a you know it is an integral domain we have unique factorization every element can be written as a product of powers of irreducible elements in a unique way, okay that is what a unique factorization domain is this is a commutative algebraic definition, but what is it geometrically mean? So geometrically you can say if you geometrically you also think of rings as rings of functions on some space.

So algebraically geometrically how to define a unique factorization domain one way is you say I mean look at all you can ring of functions if you want to think of ring of functions is a UFD then the space must have the property that the geometric hyper surfaces should be the same as the hyper surfaces in the commutative algebraic sense, it is for those spaces that the rings of functions can be called unique factorization domains, okay.

So what you must understand is that this unique factorization which is a very you know completely algebraic statement it is a purely commutative algebraic kind of statement that has the geometric significance that we find loci defined by single equations are the same as loci which have co-dimension 1 you see that is the that is how you geometrically interpret the completely algebraic definition of what unique factorization domain is, okay see the whole beauty of algebraic geometry lies in this you take something completely commutative algebraic completely see what it means geometrically and you do the other way also so this is an example as to how you can make this translation, okay. So in my next lecture what I will do is I still have to explain how the inverse of this A function is the max spec function so I will have to do that so I will do that in the next lecture.