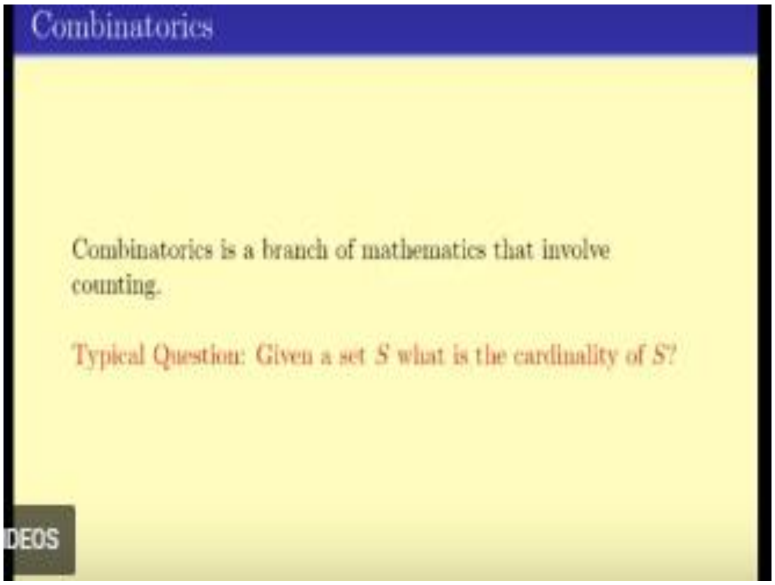
Discrete Mathematics
Prof. Sourav Chakraborty
Department of Mathematics
Indian Institute of Technology - Madras

Lecture - 50
Combinatorics

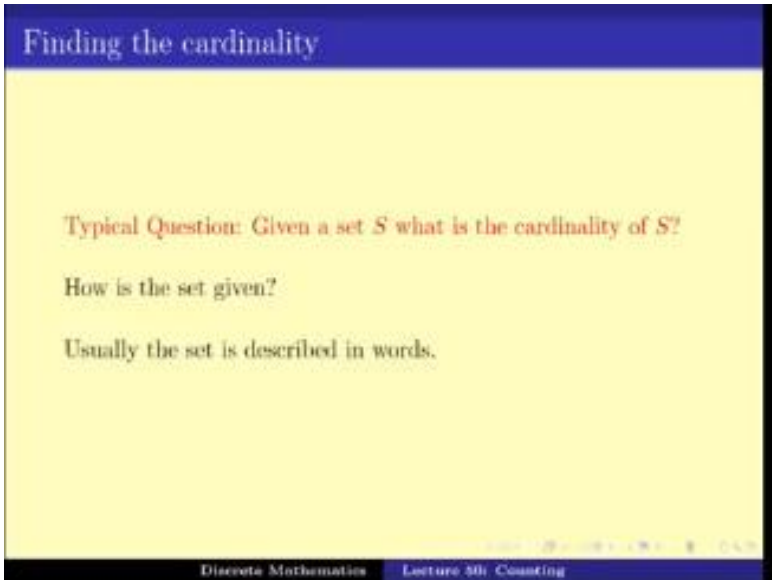
Welcome to the last video lecture in this course, so today we will be revising on the topic of counting.

(Refer Slide Time: 00:09)



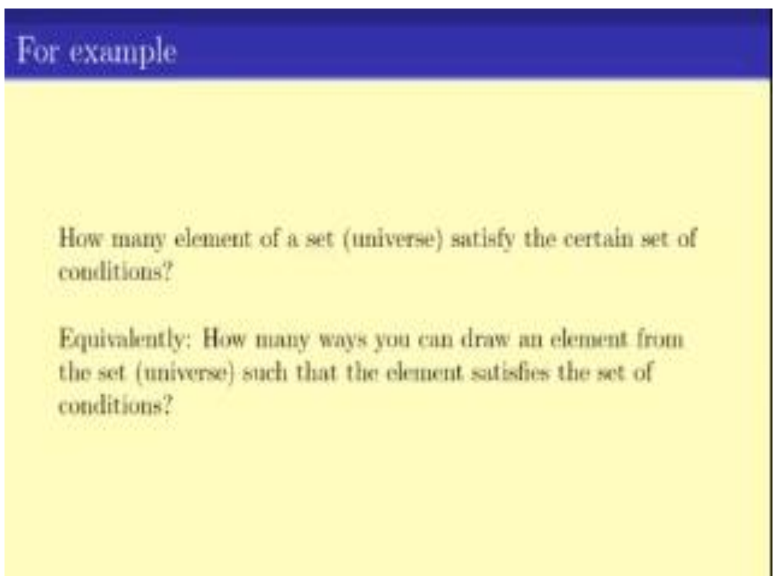
So, combinatorics is a branch of mathematic that involves counting the very big part of mathematics and we have seen that big names like Ramanujan and many other people have worked on this particular field quite a lot. Typical question is given a set S what is the cardinality of the set S?

(Refer Slide Time: 00:35)



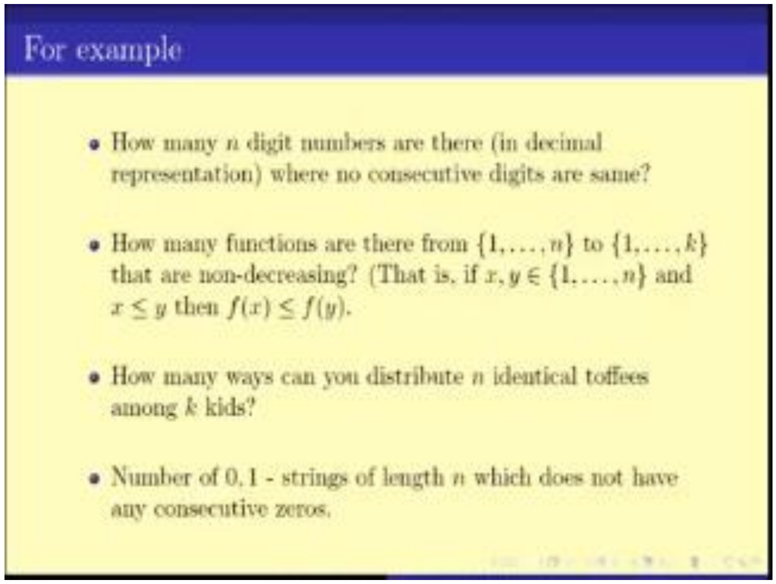
Now, the main question is how is this set given? Usually the set is given in terms of words and you have to count it.

(Refer Slide Time: 00:42)



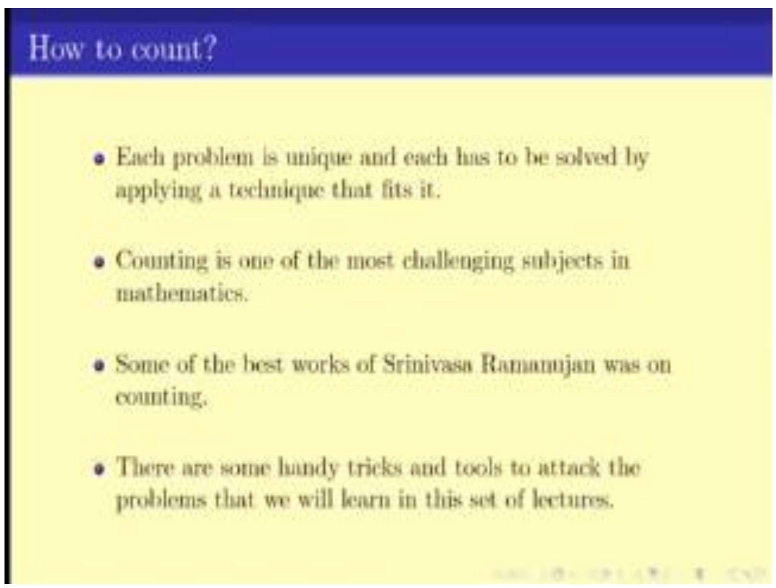
For example, one can ask how many elements of a set satisfy the certain set of conditions or if you draw, how many ways can draw an element from the set such that the elements satisfied set of conditions.

(Refer Slide Time: 01.00)



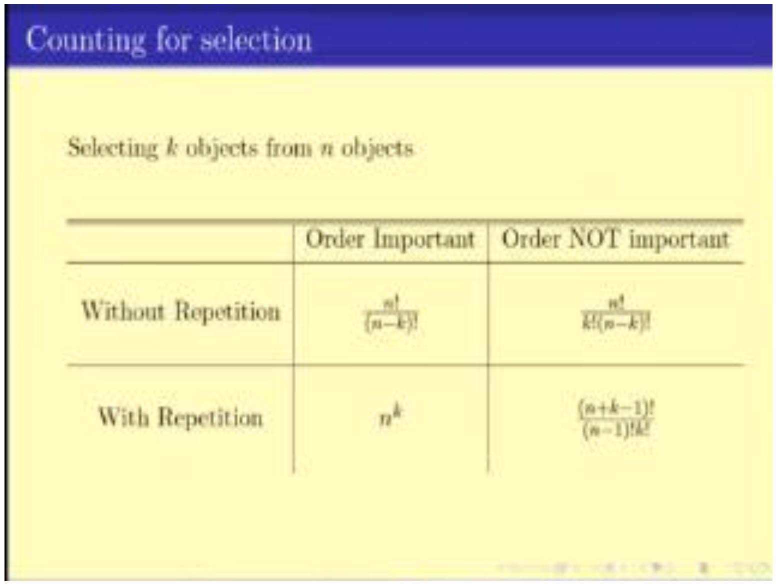
There are quite a number of examples that we have already seen in this course.

(Refer Slide Time: 01.07)



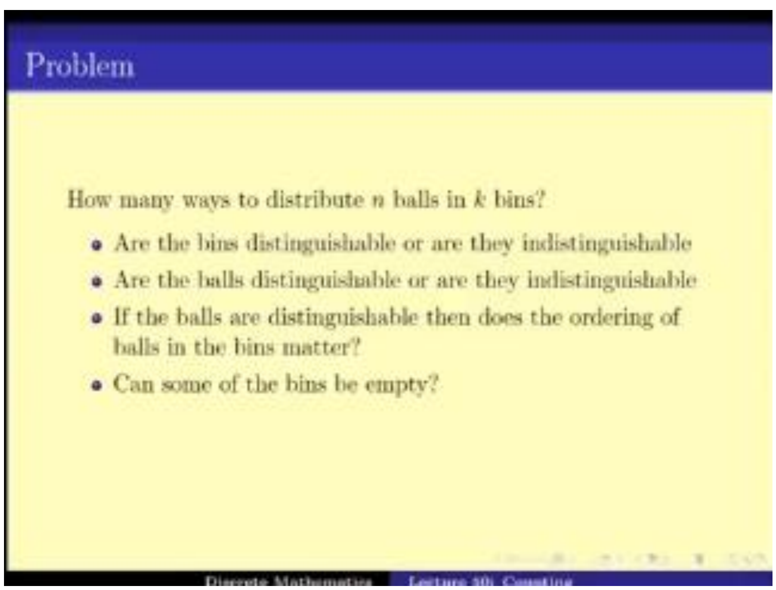
Now, each problem is unique and each has to be solved using applying a technique that fits it. In fact, combinatorics is possibly one of the most challenging subject in the mathematics and hence some of the most creative ideas come out in the field of combinatorics. Some of the best work of Srinivasa Ramanujan was on count. There are quite a number of handy tricks, and tools that one can be used to attack these problems and that what we have learnt in this picture.

(Refer Slide Time: 01:57)



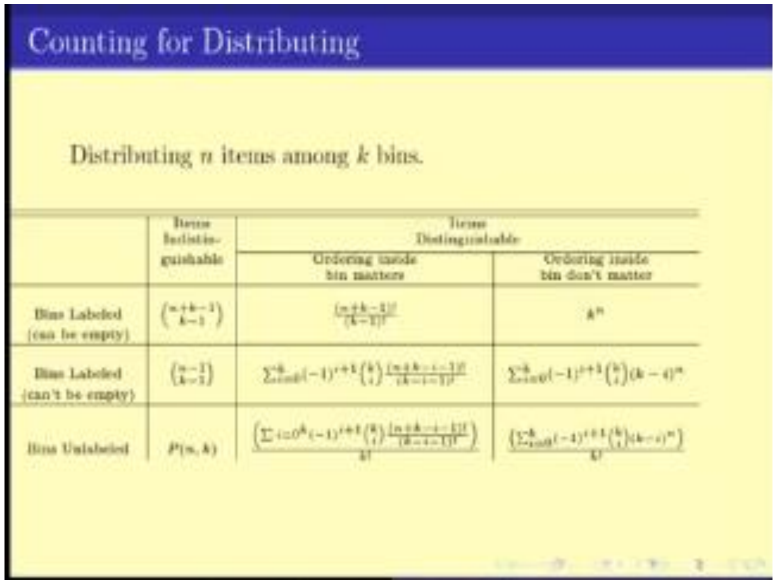
In particular, we looked at 2 special cases, case 1 was how many ways can you select k objects from n objects and the idea was that is all depends upon whether the k objects are identical, n objects are identical or not? Whether repetitions are allowed or not? Whether the ordering once we chose it matters or not? and so on.

(Refer Slide Time: 02.31)



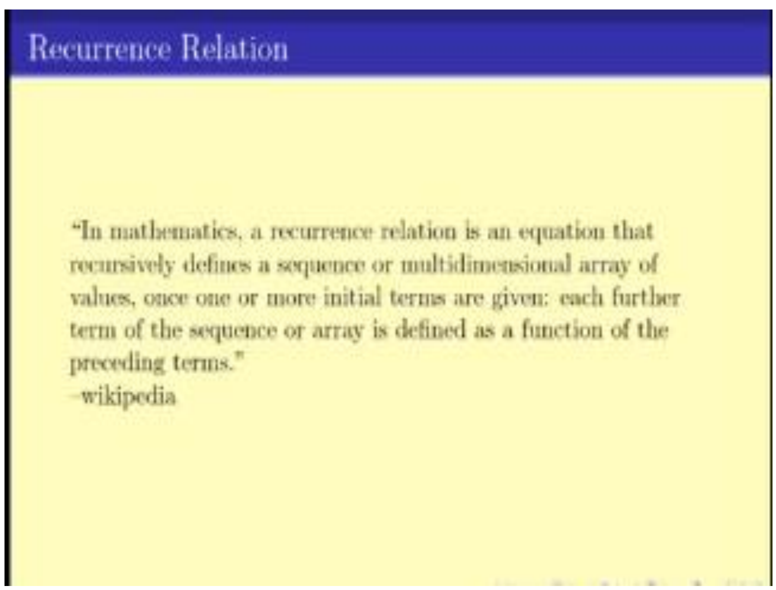
And the other thing is that how many ways to distribute n balls into k bins and whether the bins are distinguishable or indistinguishable relative, balls are indistinguishable or distinguishable whether the matter ordered inside the matters can be bins be empty.

(Refer Slide Time: 02.59)



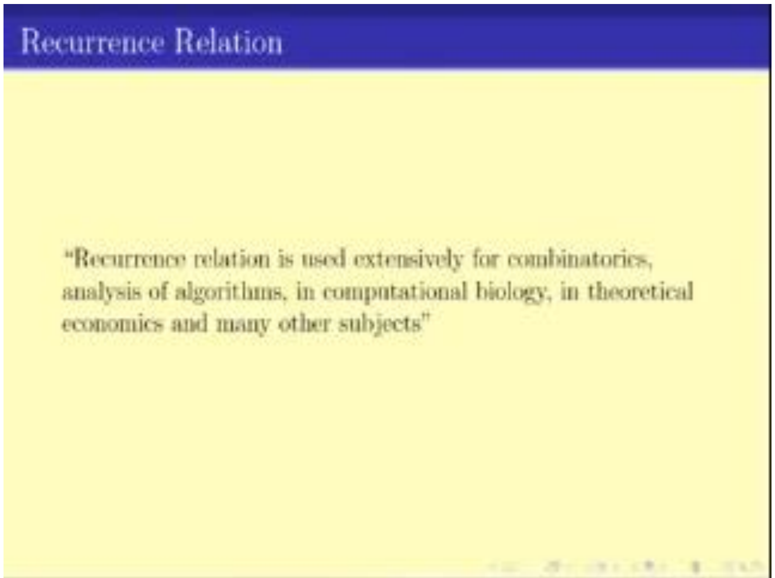
These are various cases that we handled and by doing so, we have come up with the - we kind of went to the whole of it. So we have looked the various cases and we saw how to compute for each of these for it, each of the various cases. So these are the some small tricks that we used to solve these problems.

(Refer Slide Time: 03.18)



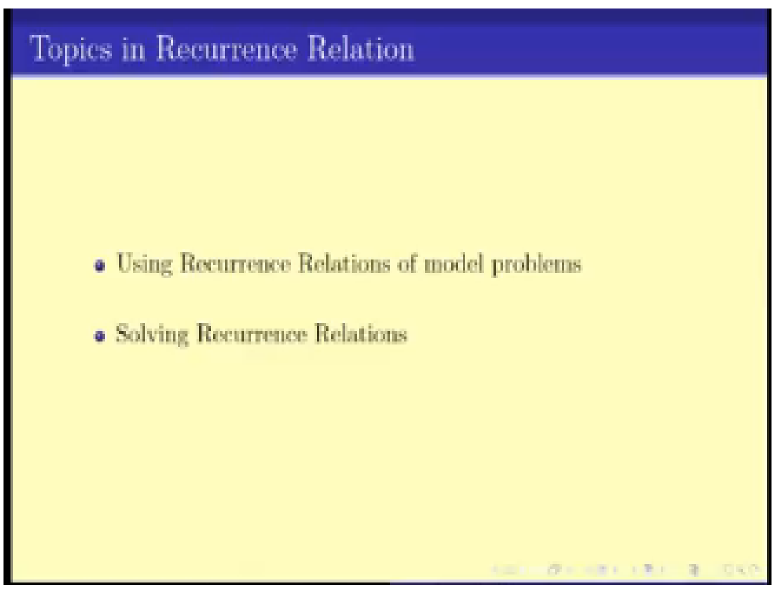
But one of the more general technique to solving the counting problem is using recurrence relations or in other words, recurrences is a very useful, is a very useful way in which, the inner term is written as a function of the previous terms and that extensively used for various other ways.

(Refer Slide Time: 03.43)



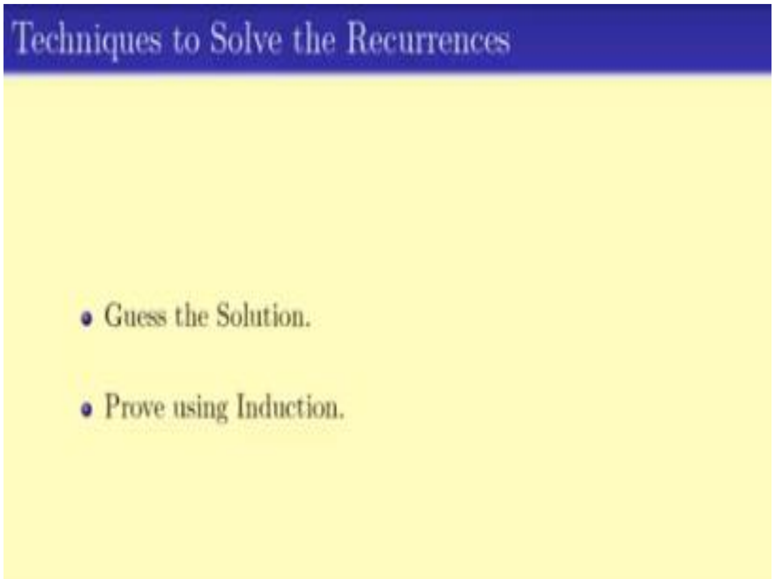
So, the main thing to study is that how to use recurrences model problems and how to solve the recurrence relations and we have in fact, seen quite a few examples of how to model problems using recurrence relations.

(Refer Slide Time: 03.48)



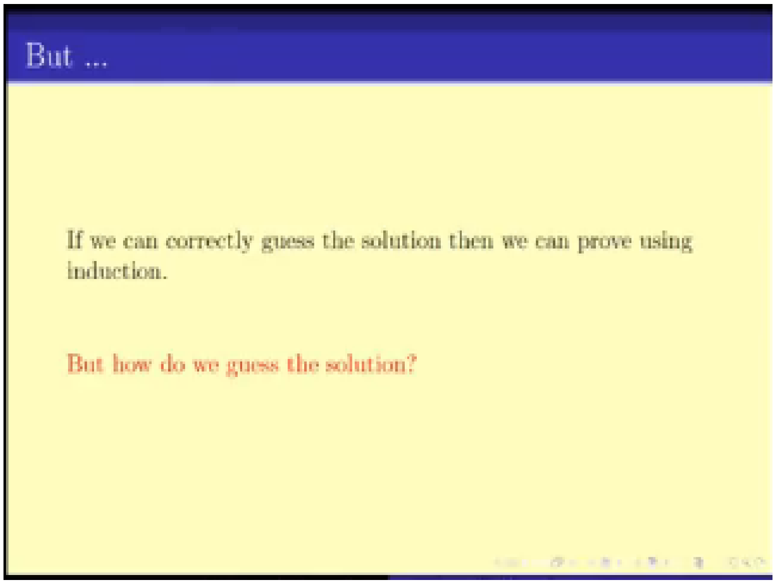
And we also saw various techniques for solving recurrence relations.

(Refer Slide Time: 04:10)



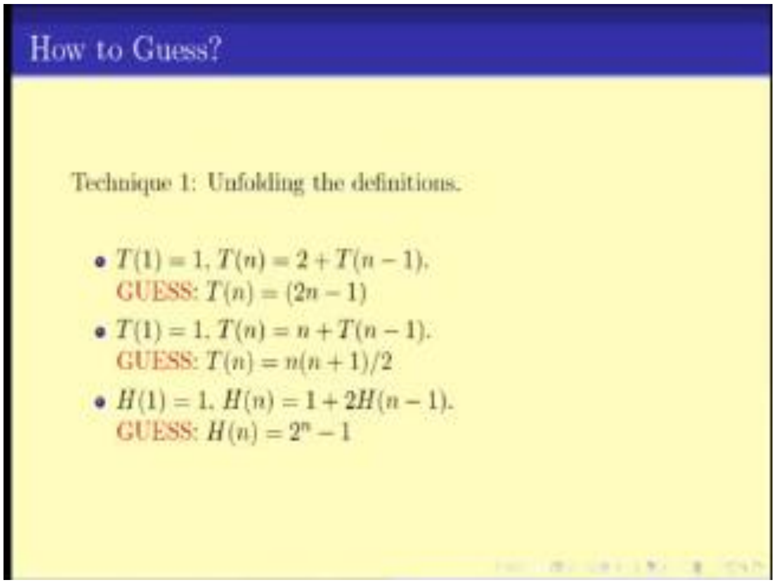
The technique that we are seen, the most obvious one was first guess the solution and then proof by induction.

(Refer Slide Time: 04.18)



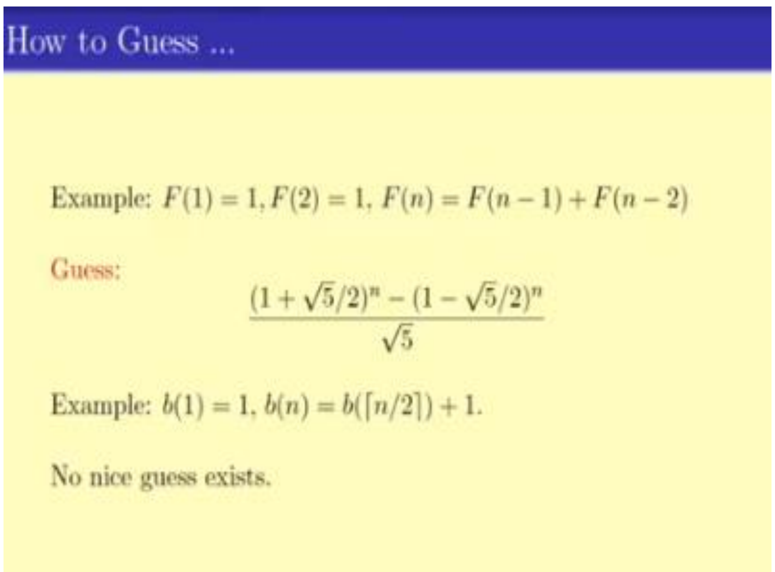
If you want - if one can guess it correctly then, that is it. But guessing the solution can be a tricky problem.

(Refer Slide Time: 04:29)



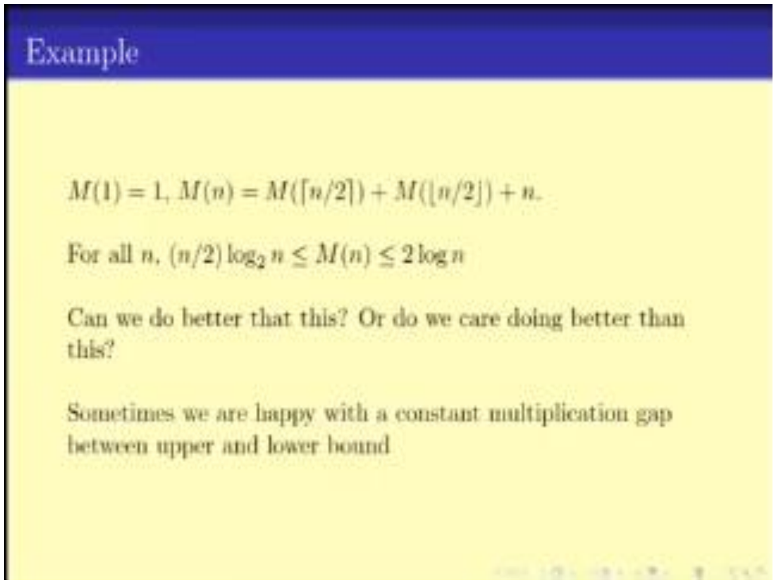
So we have looked at a few techniques, the technique one was the unfolding the definitions and we saw how this can be used to solve this or guess recurrence solutions.

(Refer Slice Time: 04:42)



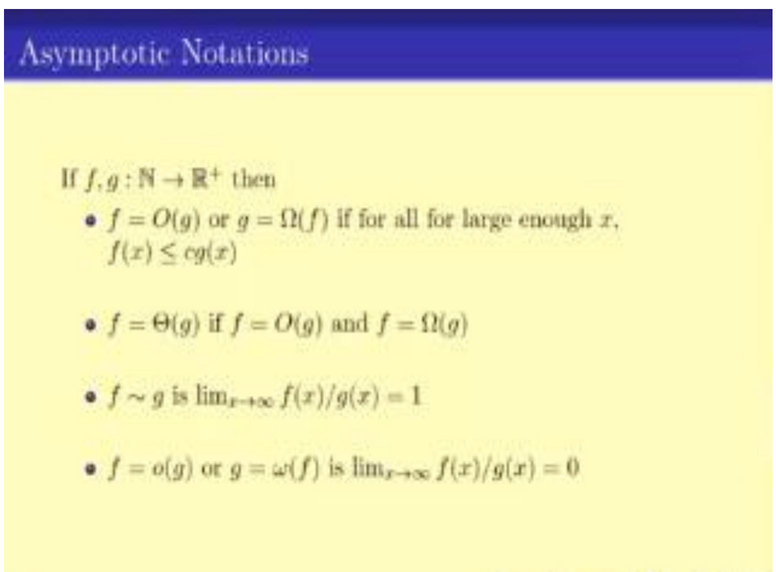
But then there are techniques where unfolding technique does not work. Problem always unfolding techniques does not work. Particularly when this functions that we guessed is very complicated or where the recurrence relation is not that, not that nice.

(Refer Slide Time: 05:06)



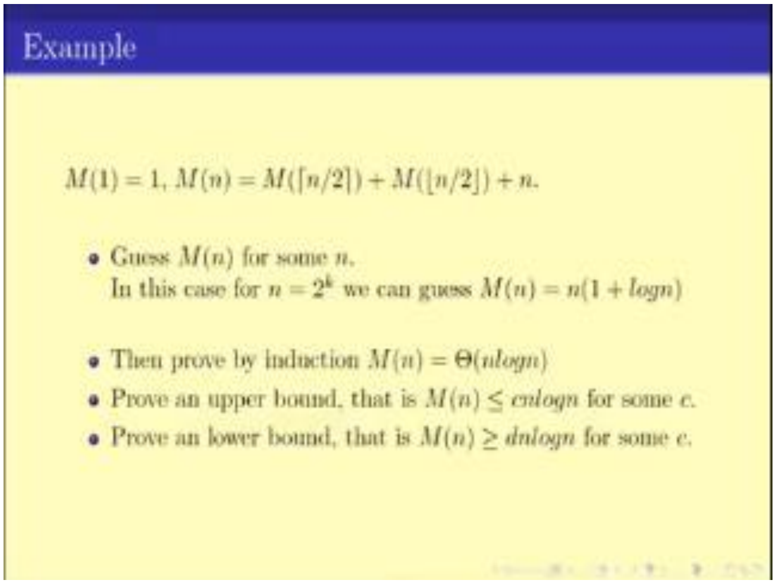
So to handle the second case, when the recurrence relations are not that nice, we what we did was that, told was that, we can come up with upper and lower bounds.

(Refer Slide Time: 05:25)



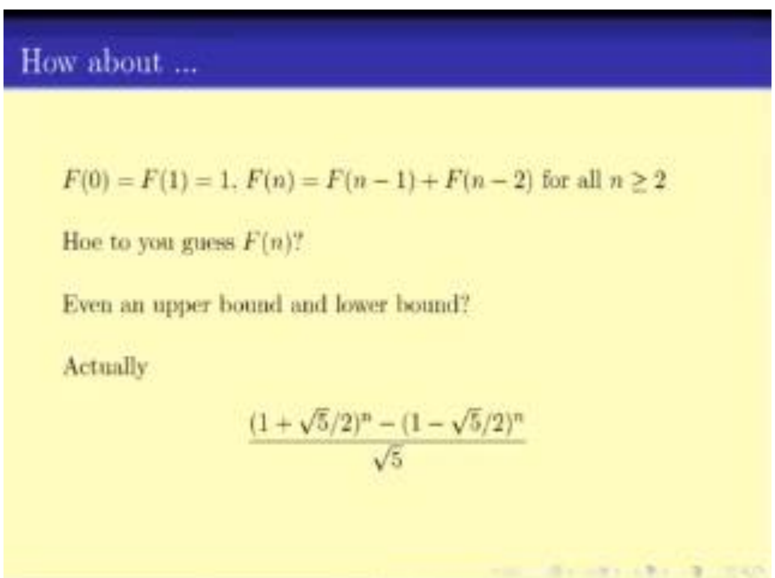
And maybe that will be good enough for us and that introduced us to the field of asymptotic notations which are extremely important mathematical language or stating relationship between functions. It has been used quite extensively in algorithms and in various other fields also. We saw that how one can use the asymptotic notations to solve the recurrence relations, not exactly but asymptotically.

(Refer Slide Time: 05:56)



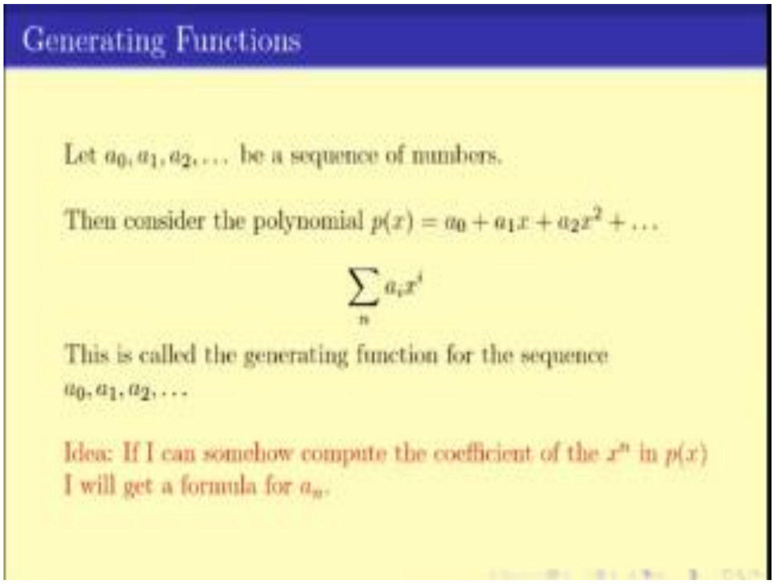
For example, problem like this, could be solved using asymptotical notations and proves that it might be as theta of nlogn and again the way to prove it is by induction. So these all the things are basically somehow guess the solution and then prove by induction, either the exact value or the asymptotic value.

(Refer Slide Time: 06:28)



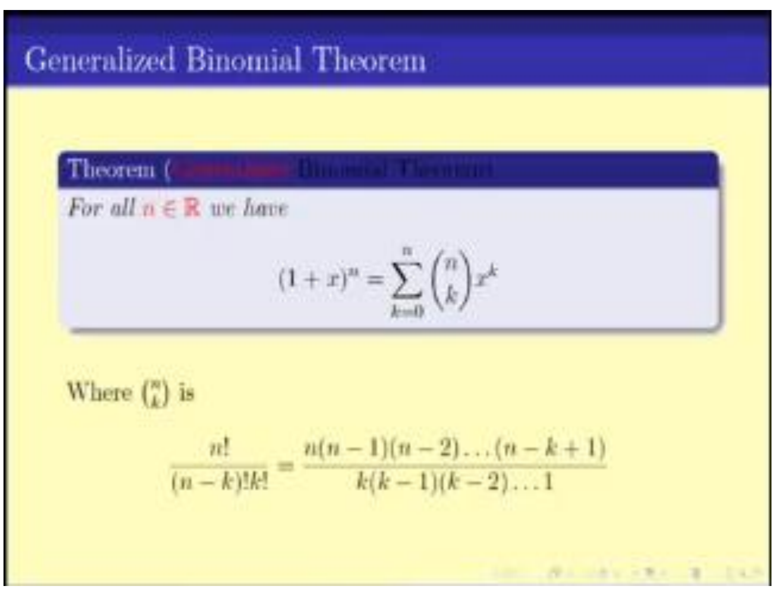
But then, there are other recurrence relations, for which situations are much more complicated and we getting even an upper bound or lower bound is not good enough.

(Refer Slide Time: 06:46)



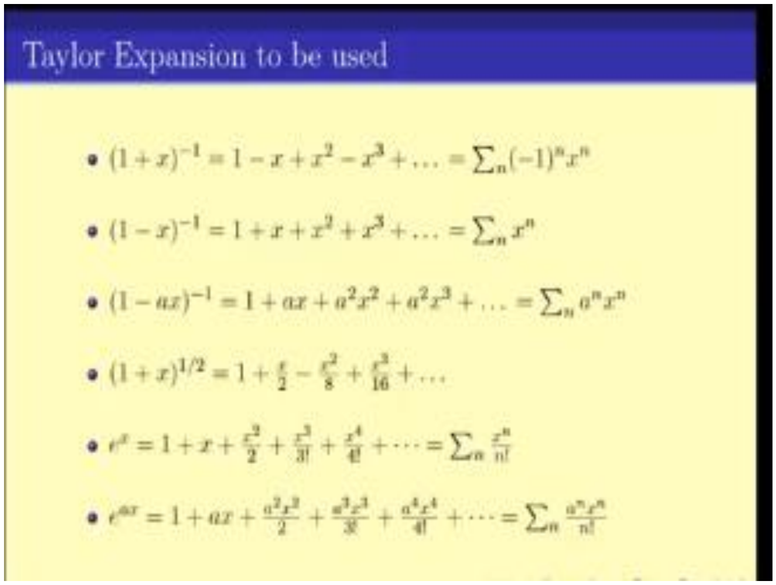
And for those, we came up with a completely different technique, it gave an extremely powerful technique and possibly the hardest thing we are talking about course till now namely which is called generating functions. So the idea is that if you have a sequence of numbers a0, a1 to a infinity, we considered this polynomial p of x as a0 + a1x + a2x square and this is called the generating functions of the equation and somehow by using the (()) (07:20) generate the recurrence relation, we can write with P of x as a function of x irrespective of the all the ai's.

(Refer Slide Time: 07.39)



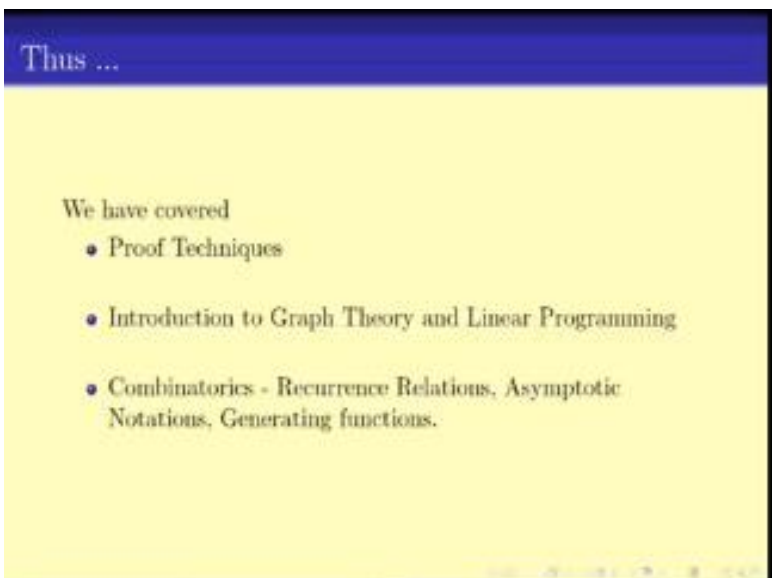
We saw a various example of this one and we use the concept of generalized binomial coefficient and the Taylor series that we have seen to solve them.

(Refer Slide Time: 07:42)



So these were the techniques that we have seen for counting problems of combinatorics problems.

(Refer Slide Time: 07:59)



To wrap up the whole course, we have seen we have covered in this course proof techniques, introduction to graph theory, linear programming, combinatorics, recurrence relations, asymptotic notations and generating functions. Now, let me be honest with you that whatever we did in this course is just a small part of what the actual course of discrete math is. The discrete math is the way more, the vast subject actually.

I have only been able to introduce you to some of the subjects and show you the nice exciting parts of them. There are lots and lots and lots of things to do outside whatever we have done. I pointed out to things like graph theory or generating functions and so on. And the world does not end there. Discrete math is the basic foundation for mathematics, it has been mathematical foundation for whole of computer science, whole of various kind of, part of mathematics and so on.

So if you are interested in discrete math, please continue with your work, finding out other subjects that will be that fall in this category as keep on reading then. Thank you.