Discrete Mathematics
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Lecture - 49
Modelling: Graph Theory and Linear Programming

Welcome back. So we have been revising whatever we have been doing for the last few weeks in this discrete math course.

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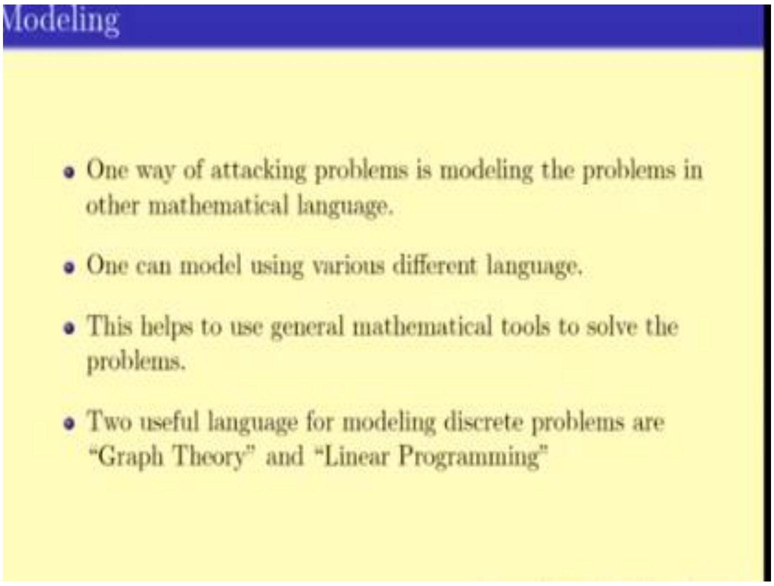

So today, we will be focusing on graph theory and the linear programming methods. The most important thing here is that, is modelling one of the ways of attacking problems is modelling the problem in a very nice usable mathematical language. Now, one can use different mathematical languages to model or one can use different models and depending on the structure of the problem one has to decide which model to you.

We, in this course, we have looked at 2 particular models, number one is Graph Theory, number 2 is Linear Programming. We have spent quite a lot of time on graph theory but graph theory as a whole deserves a full course in itself and hence just a few weeks on graph theory is not going to be enough to do justice to this subject but (()) (01:22), we did an introduction to graph theory.

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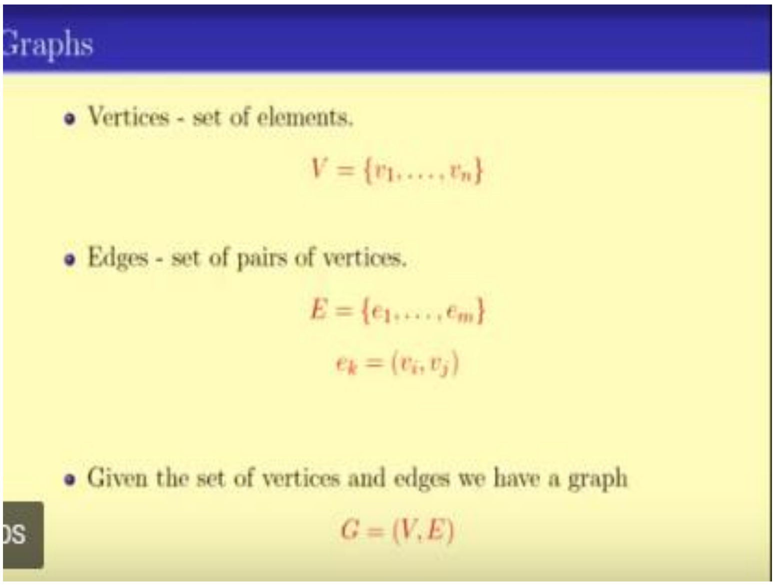

So, let us quickly recap what we have. So first of all, graphs are a set of vertices and the set of edges. The edges kind of denote the relationships, the binary relationship in vertices. The vertices are elements of some set and given the set of vertices and the set of edges, we have the graph.

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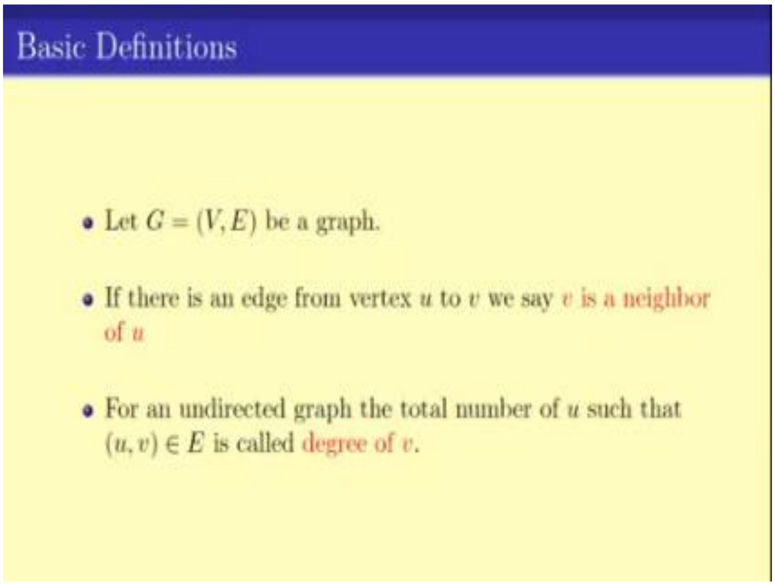

So, the graph is used to represent binary relationships if the graph is symmetric, we call it undirected as the relations symmetric. We can also have weights assigned to each of the edges and that is called a weighted graphs, we have the notion of what is the neighbour is and the degree of a vertices, vertex.

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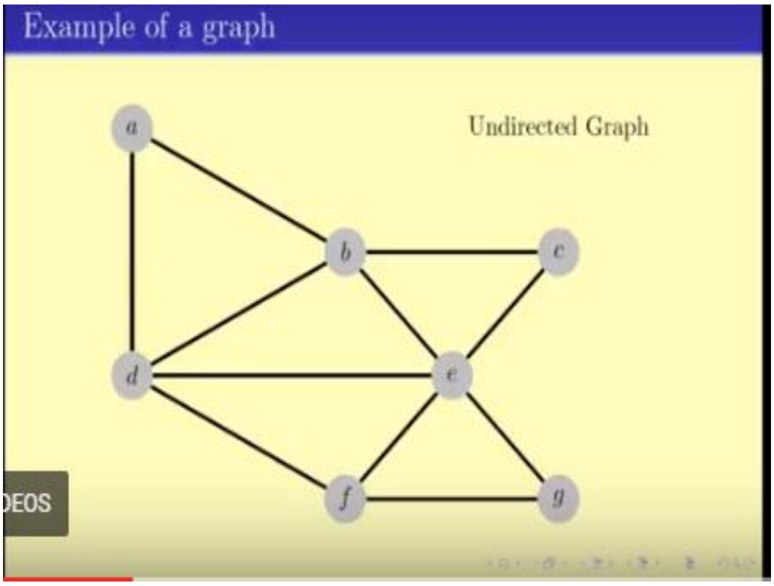

So, in fact, we represent these graphs using by drawing on the plane where these blocks represent the vertices, the edges between they are drawn by lines that join the 2 vertices which they are supposed to represent.

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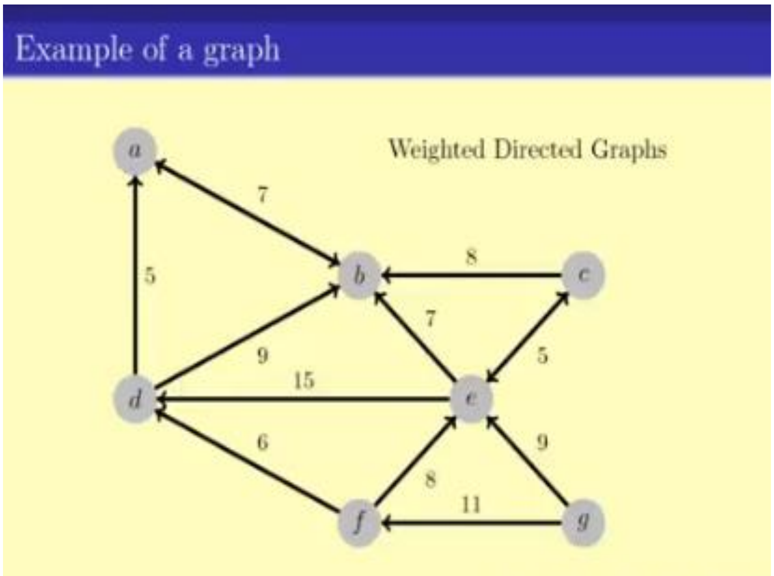

We can have weights from the edges; we can also have direction on the edges if the original binary inflation is not symmetric.

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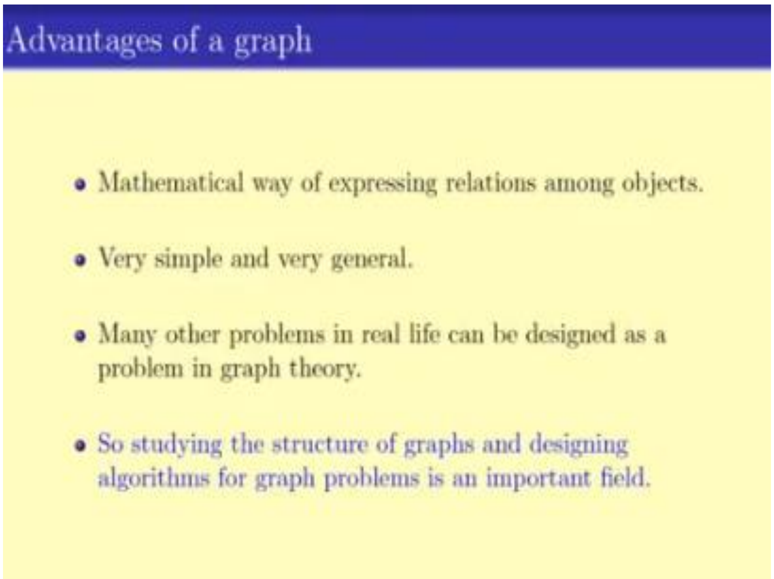

The advantage of the graphs is that it is the nice mathematical way of expressing relationships between objects. They are very simple and very gentle. We have seen this example of many problems in real life, can be designed as a problem in graph theory and hence studying the structure of graphs and designing algorithms of graph is an important field in the modern world of algorithms and Complexity.

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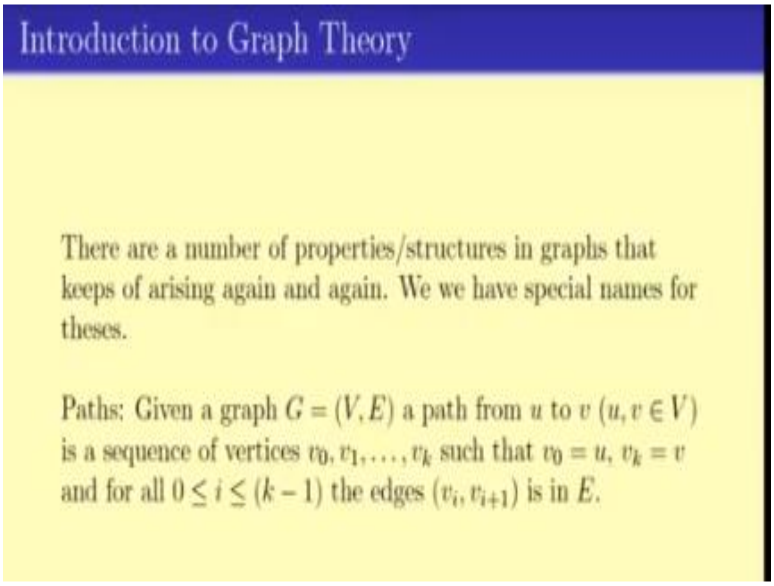

Now, there are quite a number of properties that keeps on arising again and again and we have special names for this.

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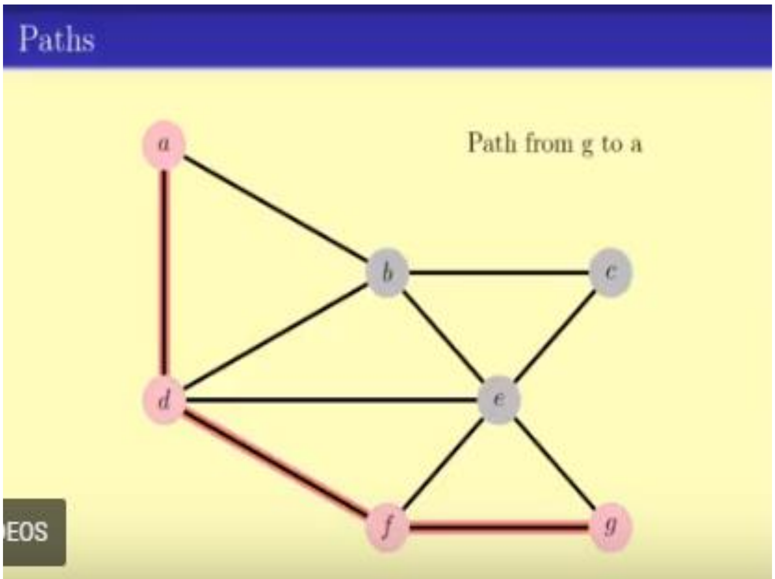

So, we have (()) (03:46) of paths which basically - if you have to look at a path from g to a, it is a set of edges is a path from g to a. They can be various a path from g to a. Paths can be directed or undirected.

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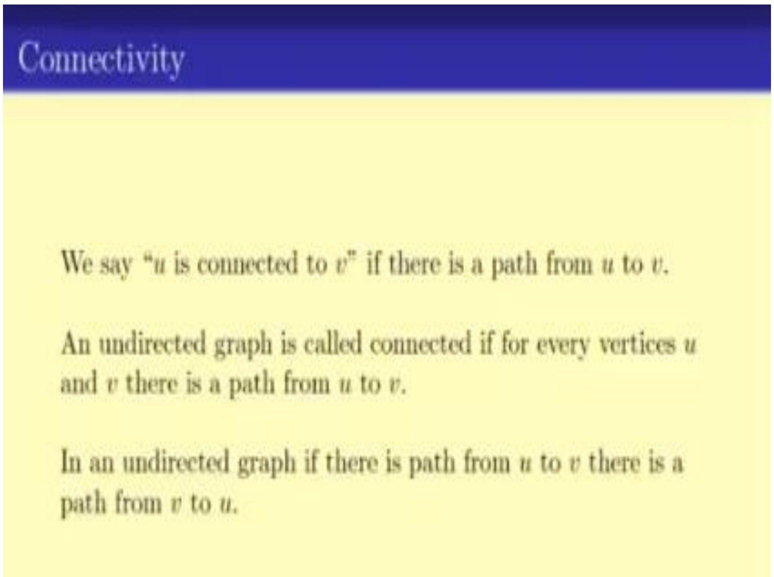

And we have a notion of connectivity that we looked at.

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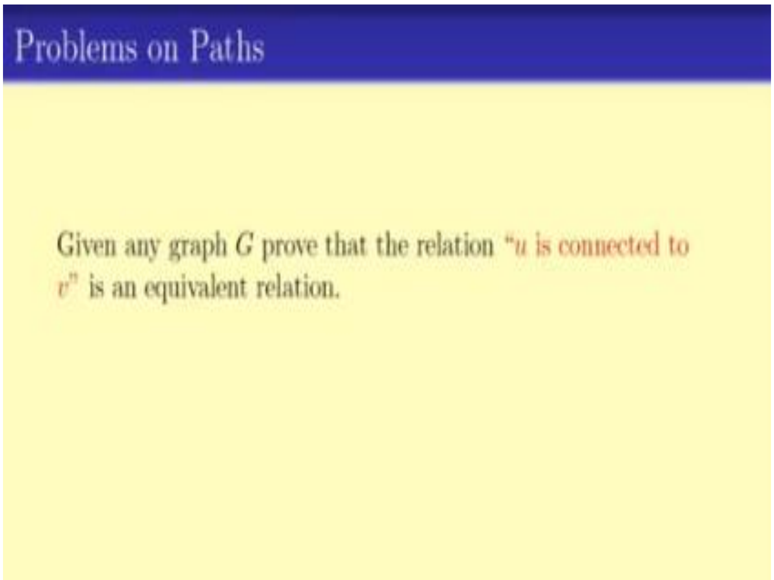

And the notion of whether u is connected to v that if there is a path from u to v is an equivalent relation.

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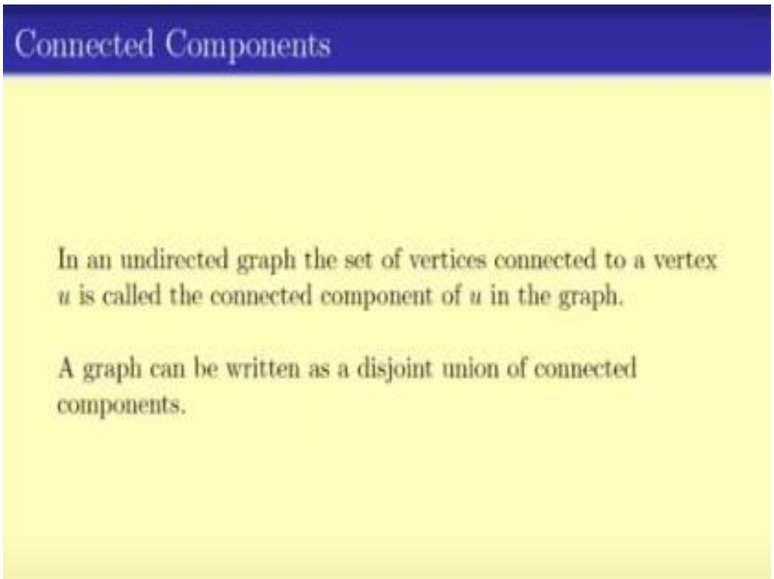

And hence can be written as a - hence the graph can be written as a disjoint union of connected components.

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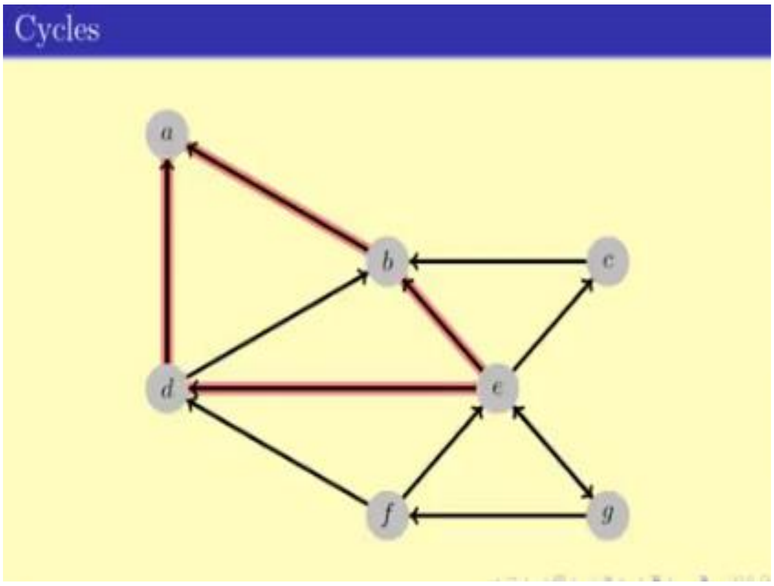

We have also seen cycles like this.

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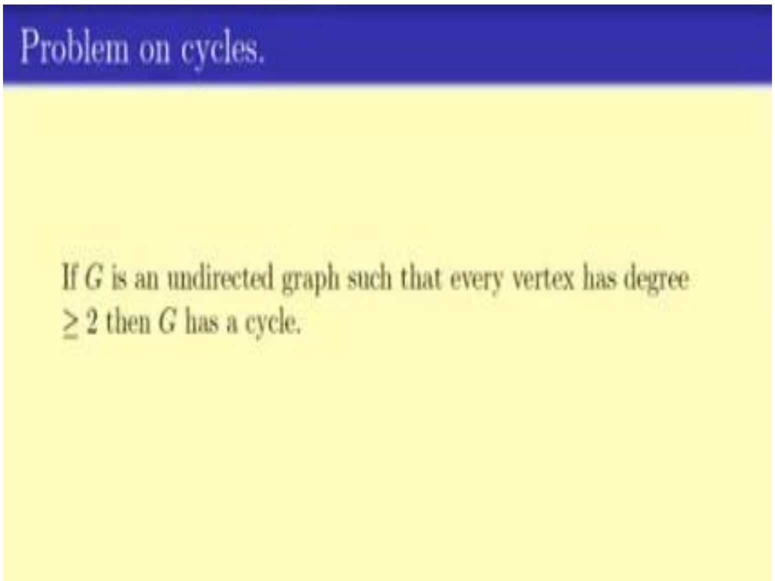

And we also have looked at some cases when cycle exist.

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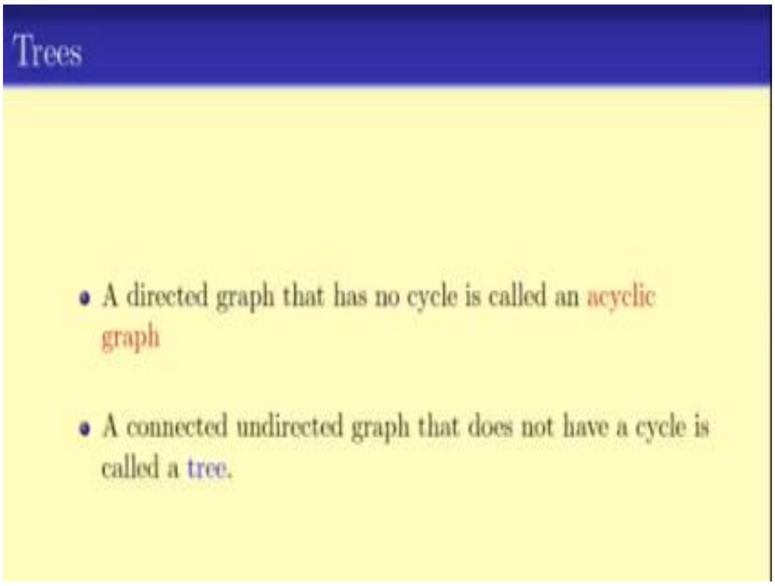

And if graph is not cyclic or does not have a cycle, it is called a tree and a connected graph without a cycle gives the tree.

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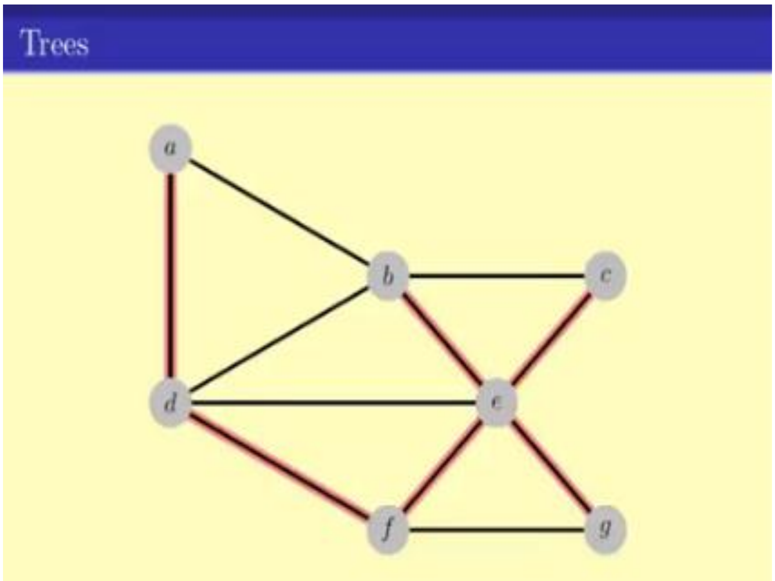

The tree is a very useful notion again. So, these are small of small notions of graph that we have seen for various properties of graphs that we have seen in this course. We have gone through various of these examples, right.

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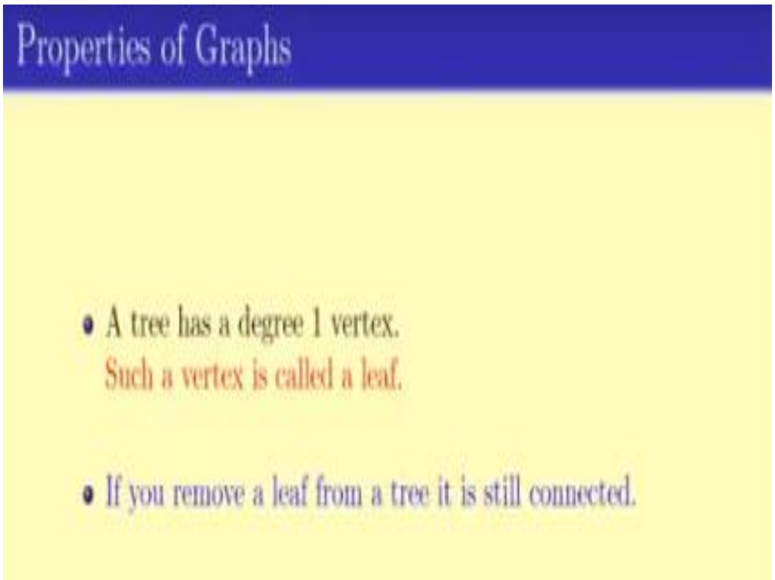

We have seen nice properties of trees, for example, a tree has some - has a degree one vertex, if you remember leaf from a trees in still connected.

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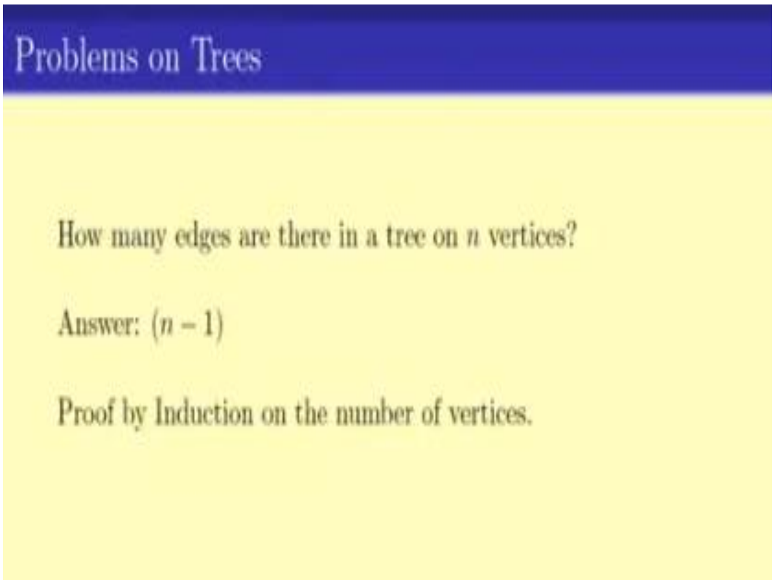

And we have also prove that every graph has a spanning tree that the tree that touches all the vertices.

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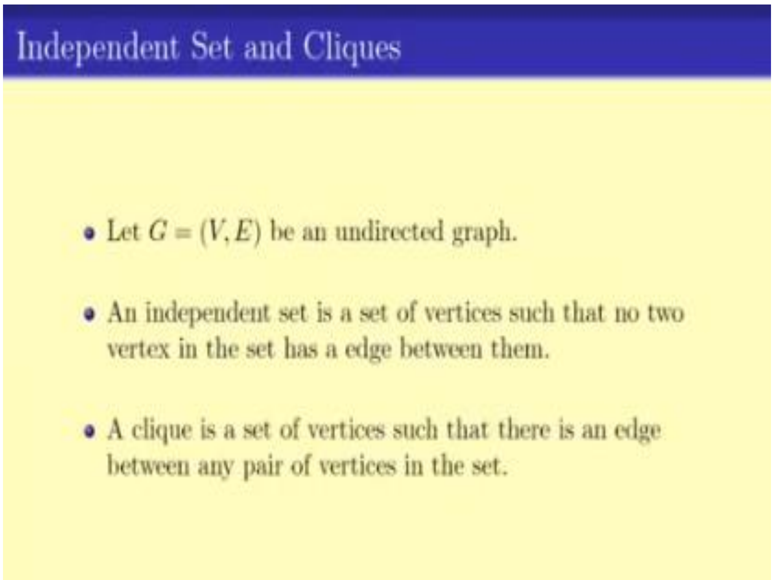

So, we can also define other structures in graphs namely set and cliques that we have defined. So independent set is a set of vertices, such that there is no edge between any pair of them and a clique is the opposite of that.

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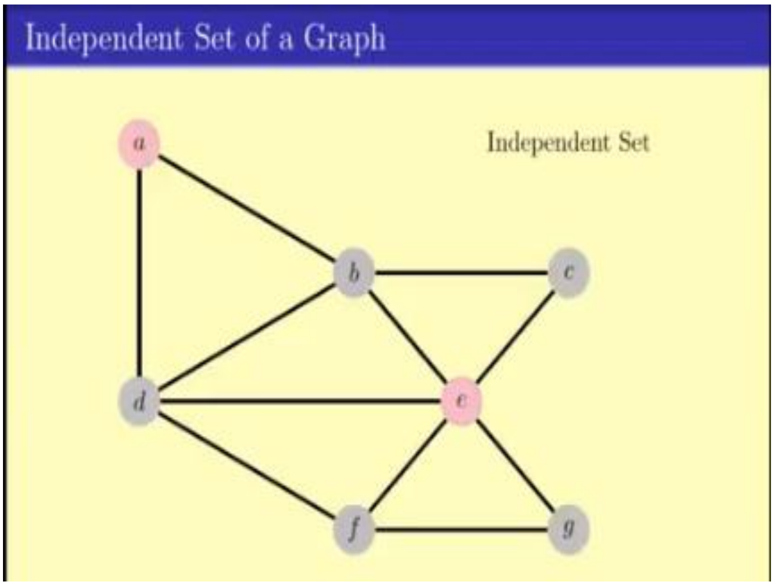

So, in other words, here is a set of delayed vertices or sets of independent set and we can have clique and now there are a lot applications, that one can apply or one can model using graph theory and I am not going to go through this application one by one.

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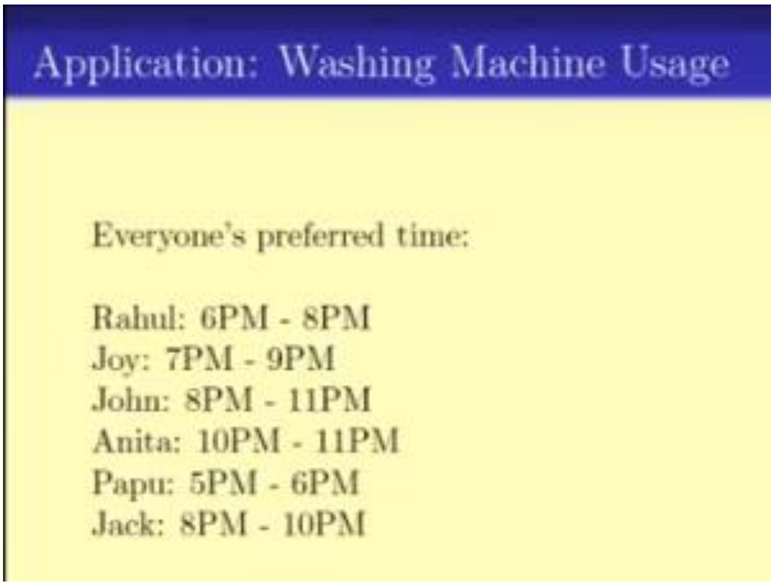

But please go back and refer to the original video in the original places.

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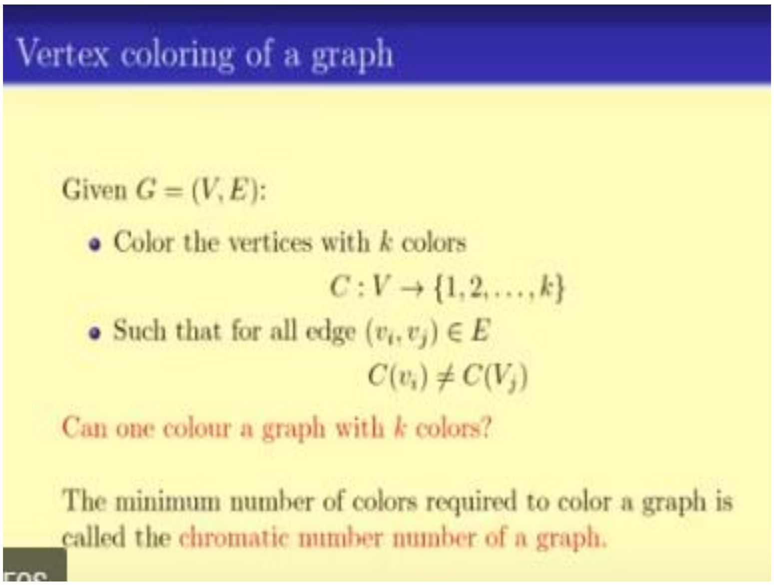

So, we can use the concept of colouring whether when we are allowed to colour a vertex with some K colours such that move to neighbours at the same colour. This is one more concept in colouring that is very useful and very handy and it used to and you study a lot. We called the chromatic number of graphs.

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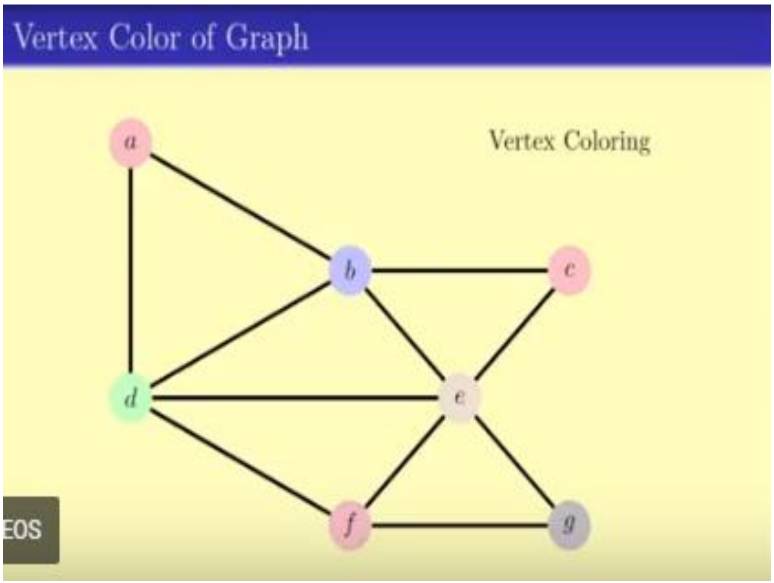

So here for example, one can colour this one with 4 colours.

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Colouring is having applications in drawing on colouring of maps and we also looked at some problems and properties of colouring.

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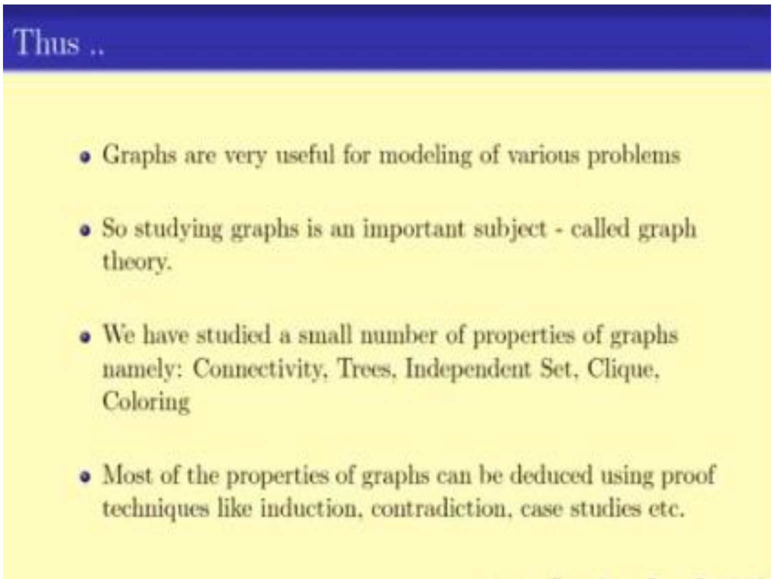

Thus, graphs are very useful for modelling various problems, studying graph is an important subject called graph theory. People who have (()) (07:25) attended this course, I strongly recommend you to go and learn a little bit more on graph theory, possibly attend one more course on graph theory. It is a subject that deserves a whole course and it is a beautiful subject that is very useful for solving their problems, mathematical problems as well as real life problems.

We have studied a small number of properties of graphs and we have of course, use the proof techniques of induction, contradiction, case study, etc. to solve (()) (08:07).

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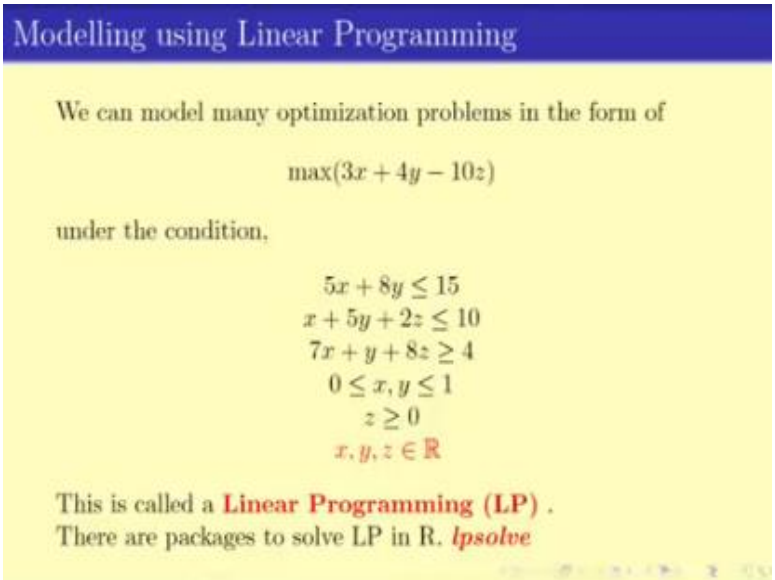

Now one more model that is used extensively is linear programming model. The idea is that you can maximize or minimize a linear equation under a set of conditions and this is called the linear programming model.

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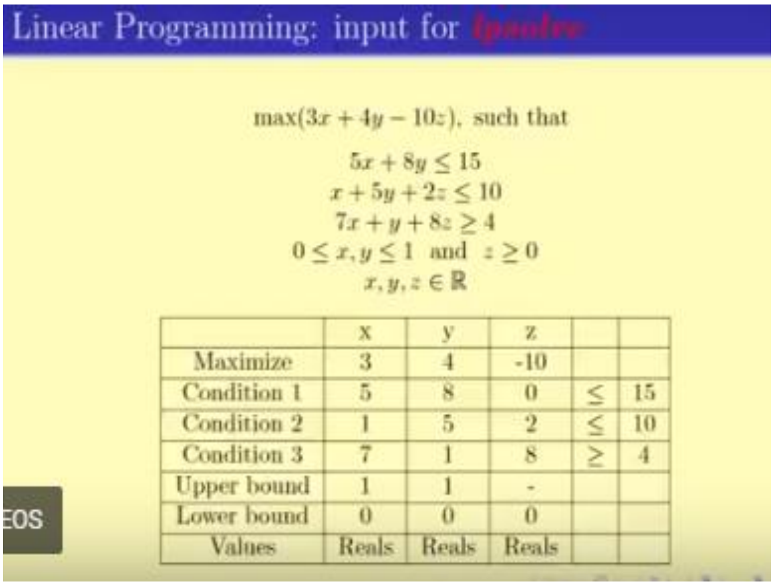

Now, it is linear programming model can be, linear program can be solved if the variables can take real numbers. This is called linear programming and it can be solved very quickly using various softwares possibly in (()) (08:49), there is a very simple way of solving it.

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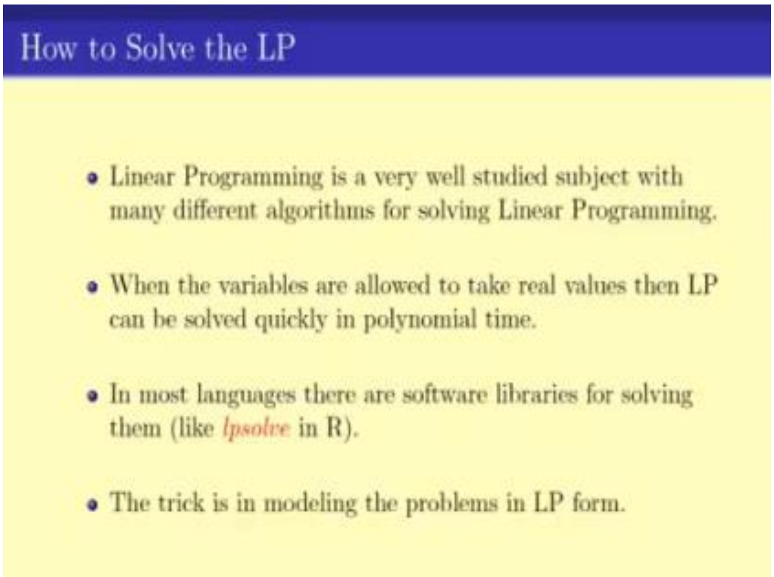

Now linear programming is again another subject that is very well study and many different algorithms are solved using linear programming. If the variables are reals can take the real value, then it can be solved in polynomial time but the trick is of course, how do you model it in the LP form.

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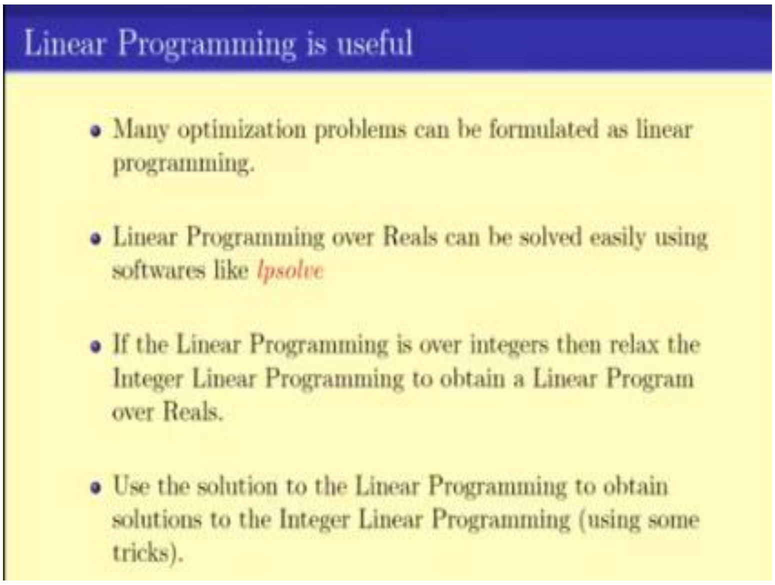

So many optimization problems can be formulated in this linear programming format. If the linear programming, these variables are over integers and unfortunately it is not necessary we can solve it using polynomial time. In that case, there are various tricks to solve them.

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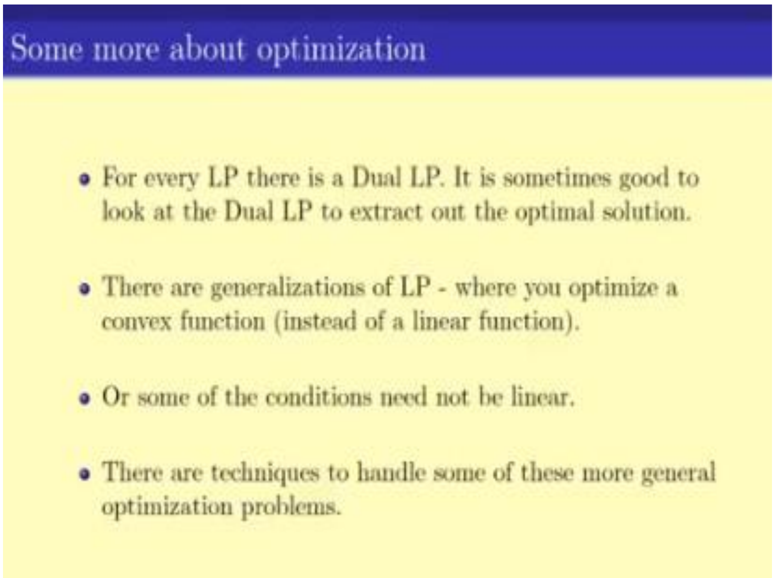

The subject of linear programming is a very vast subject and I am not going to - we did not do too much work in this, course on this. The main idea was that can we or how can we modulate using linear program. So in this course, we spent a quite a bit of time on how to model a problem using either graph theory or linear program. Now how to solve such problems is something that I have not done exactly in the class and unfortunately it will require independent set of courses on linear programming and/or graph theory to solve them.

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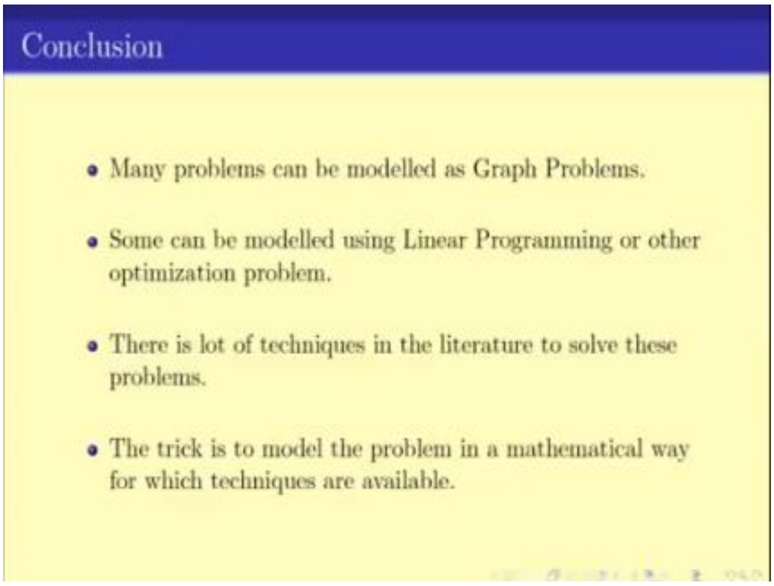

So I again, encourage you guys to attend these courses. So to conclude this particular revision, many problems can be modelled as graph problems or linear programming or optimization problems. There is lot of things known in the literature. The trick is to model the problem in our mathematical way in for which techniques are available. Thank you.