Discrete Mathematics
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Lecture-04
Propositional Logic and Predicate Logic (Part 2)

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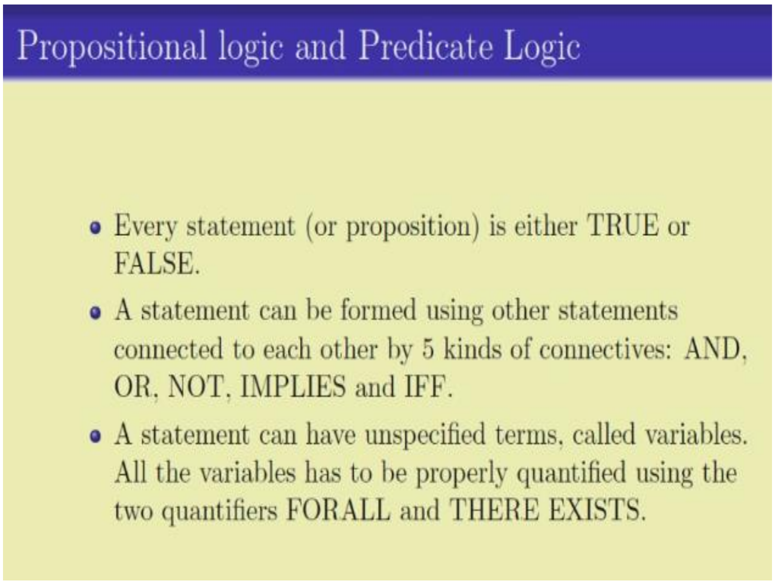

Welcome to the fourth video lecture in discrete mathematics. In this lecture, we will continue with our study of propositional logic and predicate logic. To recap, we have seen that in propositional logic and predicate logic, every statement is either true or false. A statement can be formed of other statements connected to each other using 5 different kinds of connectives, AND, OR, NOT, IMPLIES and IFF anomalies.

We also saw that the statement can have variables, unspecified terms.

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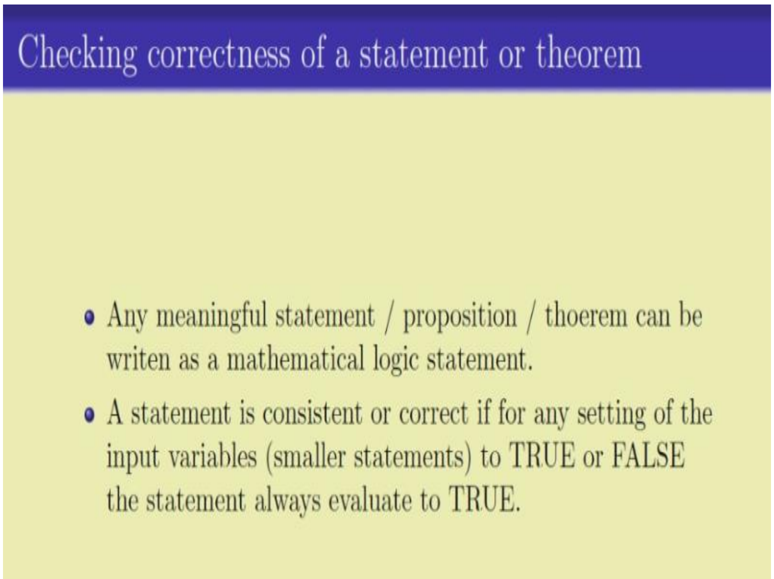

But all the variables have to be properly quantified using the two quantifiers, FOR ALL and THERE EXISTS. We also saw that any meaningful statement or proposition or theorem can be written as a mathematical logics technique. A statement is called consisted, if for any setting of its variable, which means small statement to true or false, this statement always evaluates to true, so this gives us a way of checking with the theorem or the statement is logical or not.

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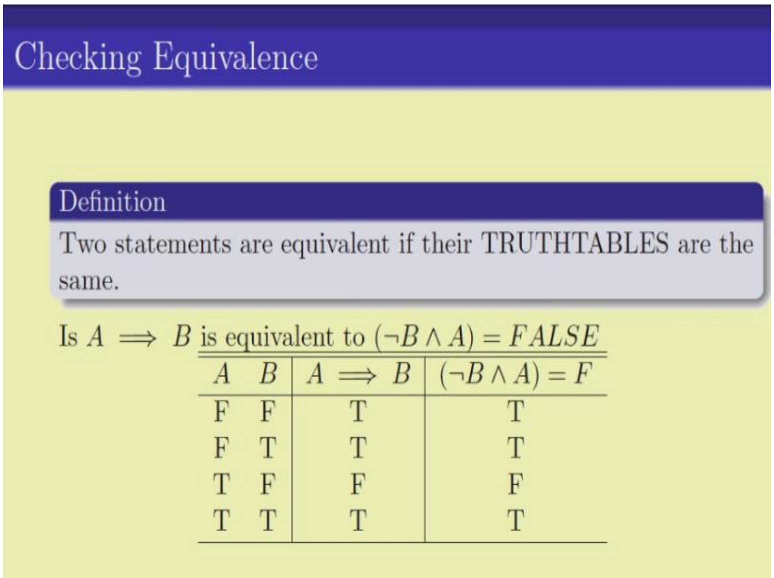

Now, there can me multiple different ways in writing a particular statement or in other words, there can be multiple different mathematical logics statement, which basically mean the same thing. So this statement we call as equivalent statements. So we say that two statements are equivalent if the truth tables are same. For example, consider this statement A implies B, I claimed that this statement A implies B is same as a non B and A is false.

So how we checked that? So of course the way of checking that is via the truth table. So for example here, I have written truth table, so for all the possible inputs of A and B, that means A can be either false or true, similarly B can be either false or true, we would like to write the truth table or what is the evaluation of A implies B and NOT B and A=x, so let us do it one by one. If you remember, the rules of implies, a false statement always implies, a false statement implies or true statement, both of them are true.

So if A is false and B is false, false implies false, yes it is true; false implies true, yes it is true. But true statement whereas implies only a true statement. The true statement cannot imply false statement. So true implies false is false and true implies true is true. So, now let us look at the truth table of NOT B and A. Now let us see, let us try to do it quickly. If B is false, NOT B is true; so in that case, if A is false; true and false is false.

Yes, if we remember that for the case of AND, only time it is true, is only when both of them are true. So, this itself false, when A is false and B is false; so it is true. Similarly, when A is false and B is true, again false and anything is false, so it is false to the statement is true. But A is true and B is false, then what happens? Then NOT B is true; true and true is true, true cannot equals to false. This sentence is false, to this is false.

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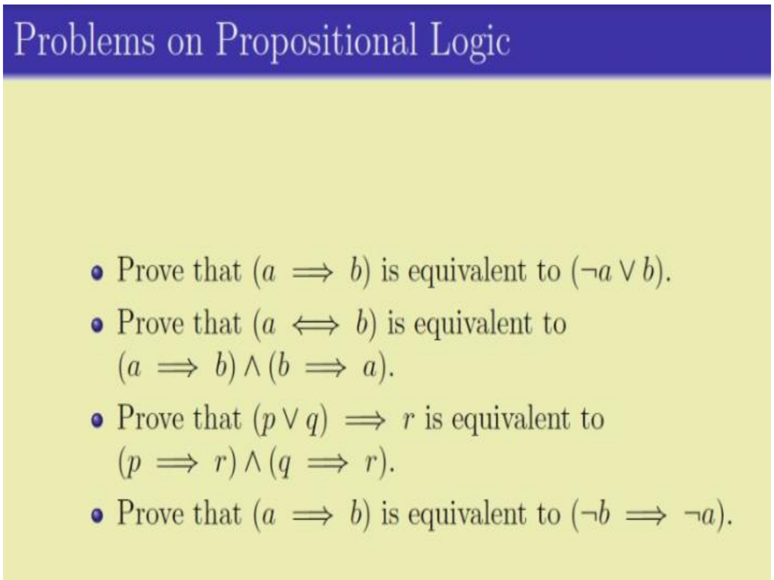

Again if A is true and B is true, then NOT B is false, NOT B and true is false, so this is again true. Thus we see that the truth table of both the statements are same, which means that they are equivalent, and here is the truth table of that. Now I have listed a number of problems which are very important in coming up lectures, so proves that A implies B is same as NOT A or B, proves that A IFF only IFF B implies A implies B and B implies A, proves that P or Q implies R is equivalent to P implies R and Q implies R.

Similarly proves that A implies B is equivalent to NOT B implies NOT A. So leave you guys with these problems, I use this problems, the statement of this problems in the coming video, very crucial. Now let us go back to the definition of equivalence, as I told you checking equivalence is same as checking truth tables are same. As you can imagine, when these mathematical logics statements are the - or in other words, when there are quite a number of variables.

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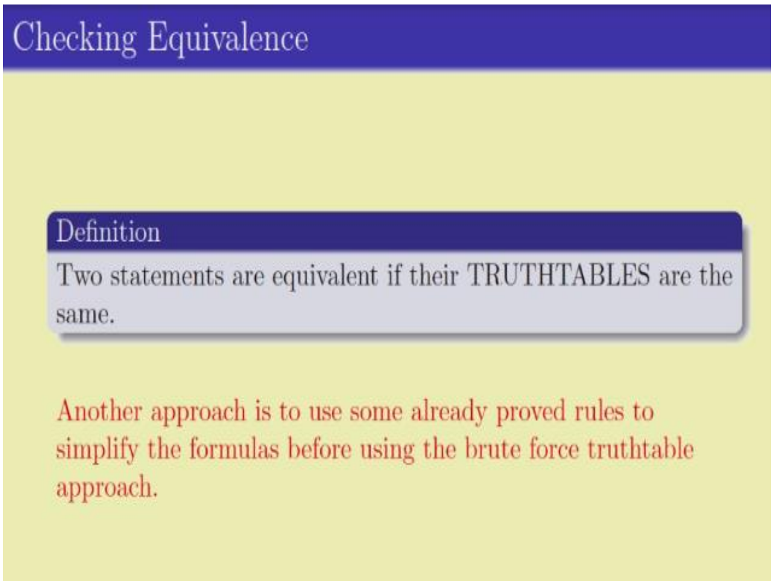

In that case, doing the truth table can be quiet a tedious process. There is one more way of checking equivalent namely, instead of using the brute force truth table approach, we can simplify the formulas using some rules. So here are some useful rules, any of you check for yourself that all these rules are correct, that means this rules can be proved from the truth tables.

To remember these rules, let me give you a small tip, this rules are identically same as the rules for set operations.

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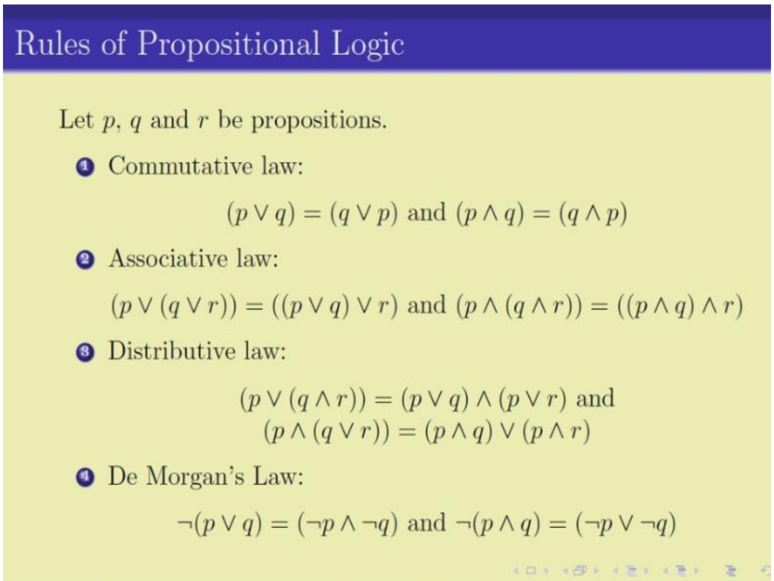

Namely, if you can replace and with intersection or with union and not with complement, you get the same set of rules as you solved in the case of set operations. So remembering this rules is not a hard job. Other than the set of rules there are couple of rules that are for the quantifiers, particularly negation of quantifiers, namely if the proposition is of the form, FORALL x P(x), where P(x) is the proposition using the variable x and it is quantified in FORALL x.

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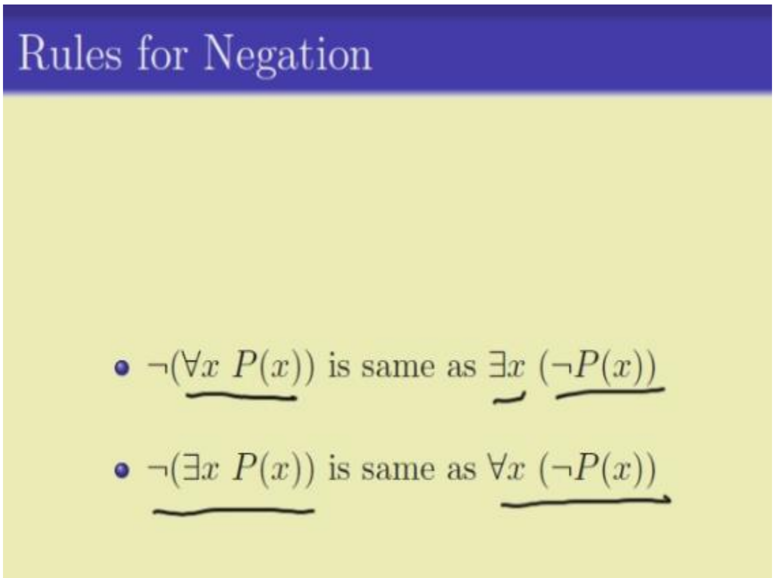

Then negation of this is or the opposite for the statement FORALL x P(x) is THERE EXISTS and x such that P(x) does not hold or in other words, THERE EXIST and x, such that, negation of P(x) is true. Similarly, the negation of THERE EXISTS x P(x), THERE EXISTS x is that P(x) hold or P(x) is true. The negation of this is FORALL x; P(x) does not hold. This set of rule is very useful for various purposes; we will see some of them right now.

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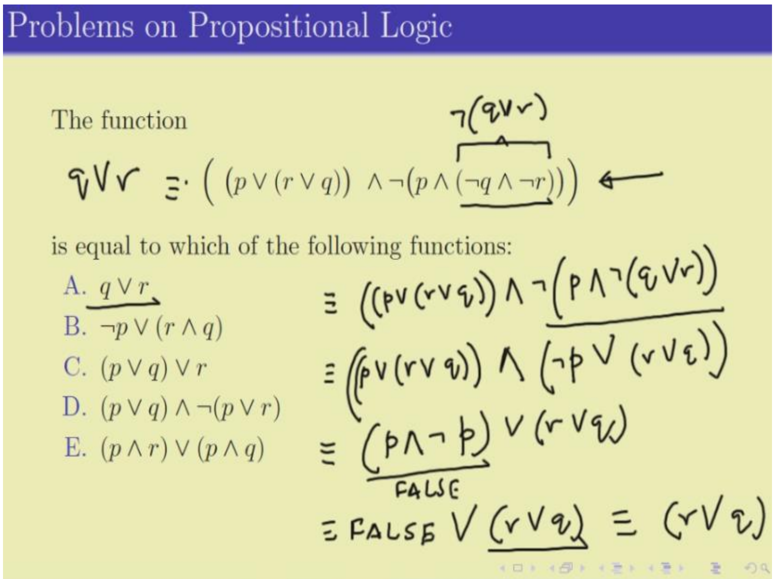

Now, here is the problem; this problem is saying that let us say p OR, r OR q and NOT of p and NOT of q and NOT of r. This particular expression is equivalent to of quick of the following. Let us try to solve this problem. We will use the rules of propositional logic that have been listed out here. To start with, let us use the De Morgan's Law we says that NOT of p or q is same as NOT of p and NOT of q, so let us how can we use that particular law?

Let us apply the De Morgan's law in this particular term, that means NOT q and NOT r will become NOT of q OR r. So this term, the whole term congruent to p OR r OR q, AND, NOT of p, AND, NOT of q OR r. Now I can apply the amount of law again in this term and I will take the negation inside and I will get the first term saying p OR r OR q and I will get NOT of P, OR, NOT of NOT of q, OR, r. So NOT of NOT is negation of negativity is same, saying this is nothing but q OR r.

Now I can use the commutative law to replace this q OR r to r OR the q, so let us do that. Let me just remove this and this can be written as r OR q. Now I can apply the distributive law, which means then I can take away r OR q away, then I get, p and NOT p OR r OR q. Now what is p AND NOT? Note that when the P is true, NOT p is false and in that case, I get true and false is false. Similarly, when p is false, NOT p is true, so then it again becomes false and true is false, so this term is always false.

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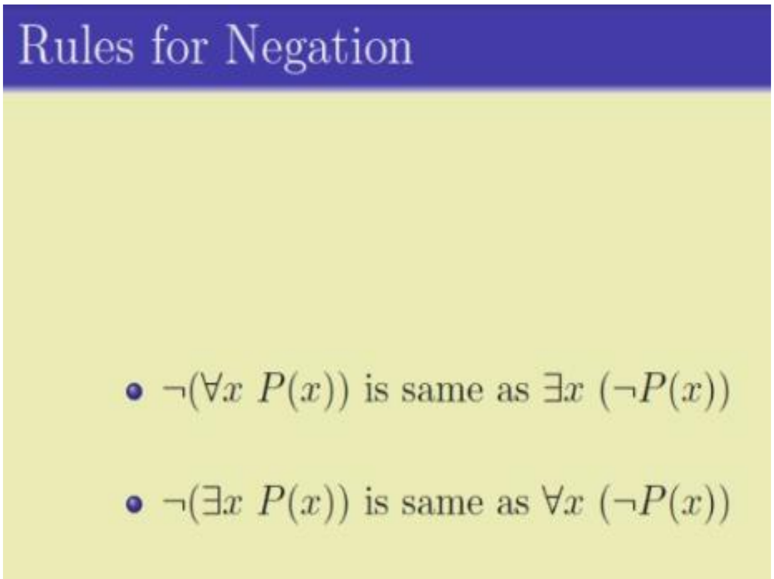

So I have false OR r OR r q, now false or anything is same as false or true is true, false of false is false. So this is nothing but r OR q, okay and r OR q is nothing but q OR r by the commutative law. So that means this one is congruent to q OR r. I applied the distributive law, the associative law and the De Morgan's law, multiple times to check that given equation is equivalent to q OR r okay. Now moving on, let us try to see, if we know how to apply the rules of negation.

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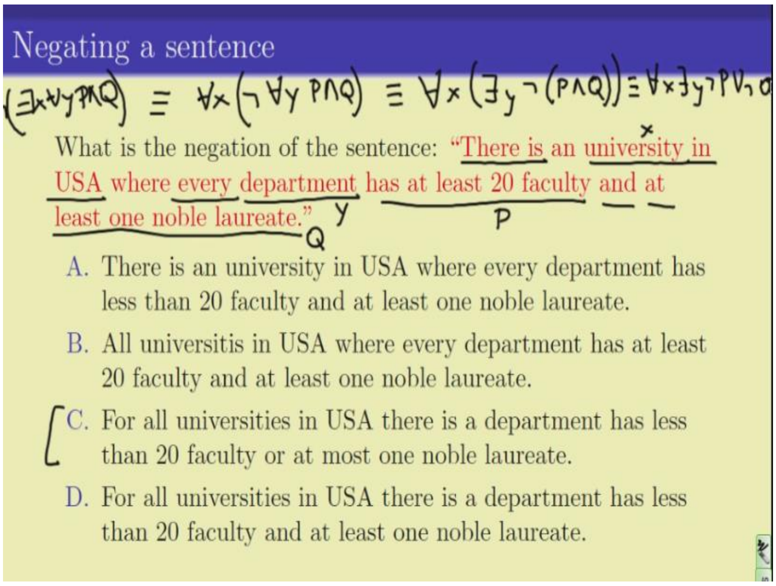

So here is the sentence, there is an university in USA, where every department has at least 20 faculty and at least one noble laureate. So what would be the negation of this? So out of this 4 choices, which one is the correct negation? Now if one negates these sentence, first thing to do is, identify the quantifier and the variable. So in these case there are 2 quantifiers and 2 variables. There is at least, with the quantifier, they are exist with the variable university, is called university at x.

Then there is another quantifier namely, every, every FOR ALL and the variable here is the department and this is called as y so the equation is of the form THERE EXIST x, FOR ALL y, now the 2 propositions. What are the 2 propositions? has at least 20 faculty, so let me called it for as p and at least one noble laureate, which is, that we called as at least one noble laureate, has become q.

So this is the equation that we have, THERE EXISTS x, FOR ALL y, p and q. Now if I want to negate these term for (()) (17:14) apply the rule of negation then this one becomes, therefore the first one, THERE EXIST x, note that NOT of THERE EXIST x means FOR ALL x, negation of (()) (17:35) thin size FOR ALL y P AND Q. Once again, let us apply the rule of negation, (()) (17:49) NOT of FOR ALL y becomes THERE EXIST y, NOT of P AND Q.

Once you have NOT of P and Q, we can apply De Morgan's law and we see this has become FOR ALL A and THERE EXIST y, NOT of P OR NOT of Q. So if I have to negate, it should P, for all University in USA, THERE EXIST a department such that opposite of, either it has less than 20 faculty or does not have a noble laureate. So the decide answer is C, that means for all university, you can say there is a department that has less than 20 faculty or at most one noble laureate.

Now using this same trick or same method, any sentence can be negated, first convert into a mathematical logic statements and negate it, without on making a mistake is minimal if you follow this particular procedure. If you have any doubts, how to negate a sentence; the English sentence or maths sentence, find out the quantifiers, find out the variables, find out the propositions and negate it using the set of rules.

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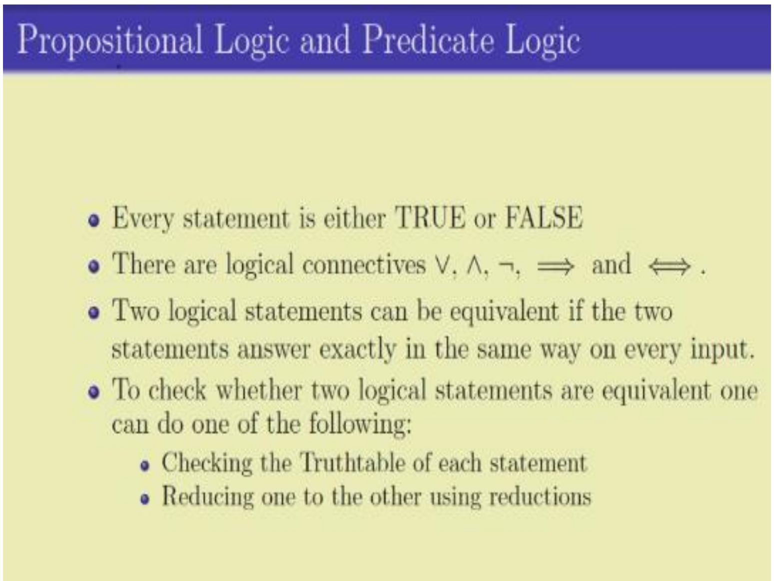

So this brings us to the end of propositional logic and predicate logic, at least to the basic rules of that. To recollect again, every statement is either True or False. There are logical connectives; AND, OR, NOT, IMPLIES and IFF. We say that 2 logical statements or equivalents or statements are identical for all input. To check equivalents of 2 statements, we would either check by writing the truth table of both of them or by reducing one to the other using a set of rules.

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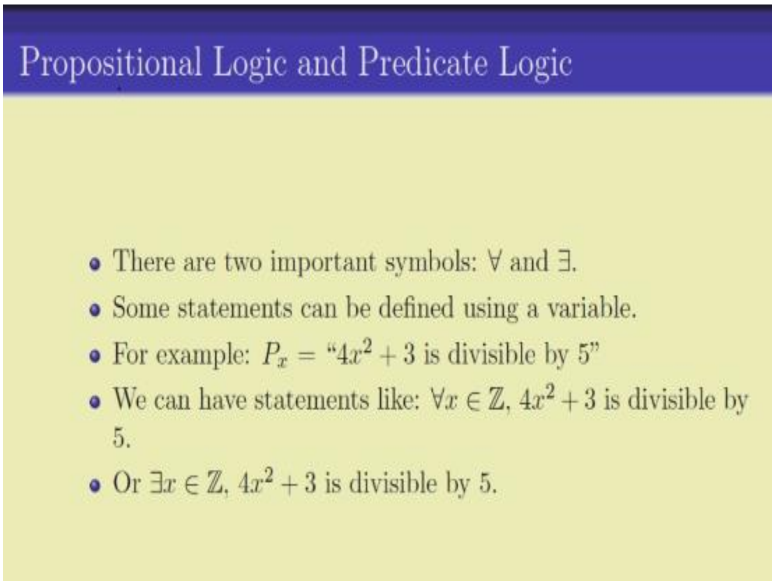

The rules, that we have discussed. There are 2 important symbols, which would be quantifiers; FOR ALL and THERE EXIST. Some statements can be defined using a variable. For example, we can say something like FOR ALL x inverse, 4 x square + 3 is divisible by 5 or THERE EXIST x Z, 3 x square - 4 x square + 3 is divisible by 5. These 2 are examples of propositions, which has variables namely x in them and they are quantify properly; the first one is FORALL x and the other one as THERE EXIST x.

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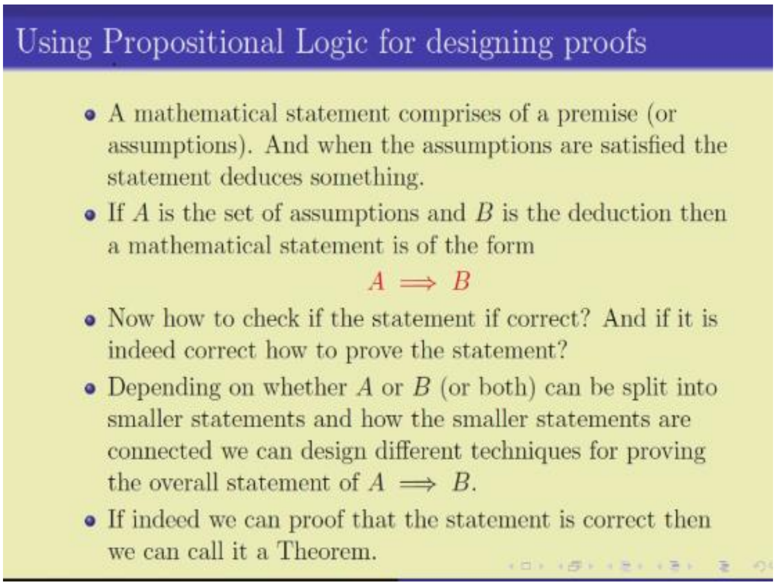

Now that we have as set of prepositional logic and predicate logic and how to check equivalence. We can use this frame work to design mathematical correct tools. So how will you design that? Again the mathematical statement comprises of a premise and a deduction, like all other mathematical sentence or theorem. So mathematical statement can be write like, if A is the set of assumptions and B is the deduction, which is of the form, A implies B.

Now A implies B, this sentence you have already seen in the exercise that I given to you in the beginning of this particular video, that A implies B is equivalent to various other forms or various other way of writing term. So depending upon, whether A can be written as union, I mean AND of 2 more statements or, OR of 2 more statements or B can be written as AND or all of statements.

We can come up with different proof techniques. Now to check whether this statement is correct or not, we need to give a formal proof, may be if A implies B will be true, then we have to prove that and as I told you depending, whether A or B or both can be split into smaller segments, we can come up with different proof techniques for solving A implies B. If indeed we can somehow prove A implies B, then we call it as a theorem.

Next we will go into different proof techniques, I request you guys to go and do the exercises that I told you in the beginning of this video, where I have asked you to prove various equivalence between the statements A implies B and other form. It helps us to set up the different proof techniques. Depending on the problem, sometimes some proof technique will be more useful than the other.

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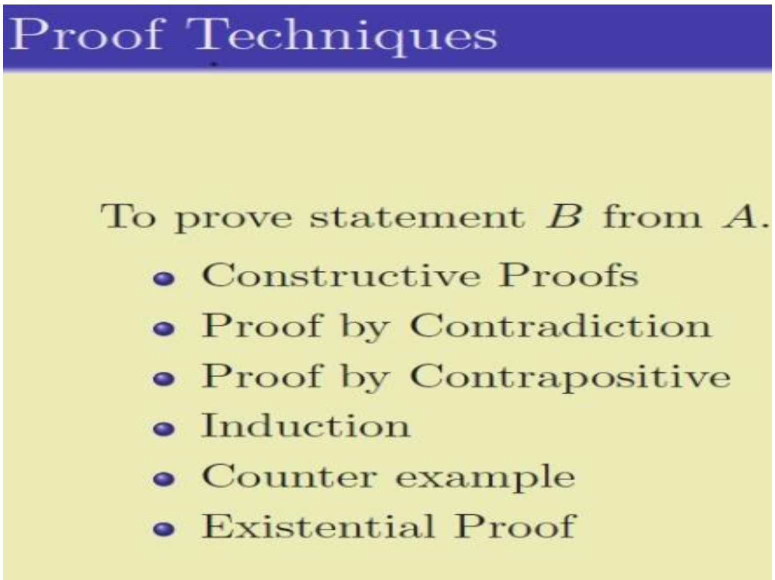

We will be going through the different proof techniques one by one, just to give you highlight or what is going to come. There are number of proof techniques, namely there are constructive proofs, proof by contradiction, proof by contrapositive, induction, counter example and existential proof. We will try to spend lot of time doing various proofs or solving various problems using different proof - of all the proof techniques.

We will describe the proof techniques and see under what circumstances which proof technique wanted. This will be basically all planned for the next two or three weeks. Thank you.