Discrete Mathematics
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Lecture - 30
Modeling of Problem (Problem 2)

Welcome back. So we have been studying the various techniques for modeling of problems and we are looking at the Linear Programming approach to model a problem. So basic idea is that given problem if we can model it in another Mathematical language then it might be useful for solving the problem. There are various different Mathematical languages is that we have and usefulness of converting it or modeling it in a different Mathematical language is that we can use the technique from those Mathematical tools to solve the problem.

In this set of videos, we have been looking at how to model discrete problems using Graph theory and Linear Programming. So we are doing it by looking at a few examples.

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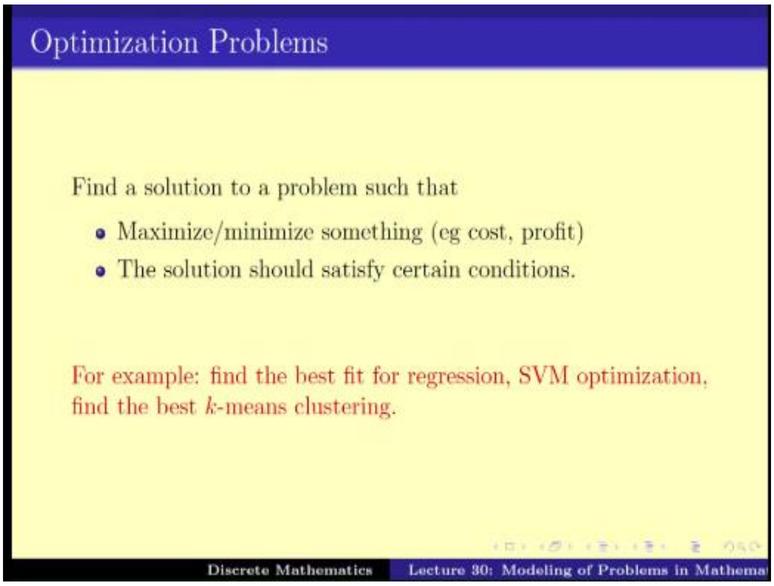

In the last video we looked at a particular problem we call that optimization problem. Now what is an optimization problem? So optimization problem is something where you minimize or maximize something like cost or a profit under certain set of conditions. Now this kind of problem arise a lot in real life in industry and so on. So in fact in the modern world in the subject of big data and so on.

We do have very obvious problems which are optimization problems. Now we have picked up 4 such optimization problems and we will see how these 4 optimization problems can be modelled in language of Linear Programming and Graph theory and that will help us to solve the problem. Now the first problem that we looked at and this is something that we studied in the last video is when we have looked at the best way of loading 3 wagons using by 4 different kinds of cargos when we have a restriction on the weights of where each wagon can carry and the space that each wagon can carry.

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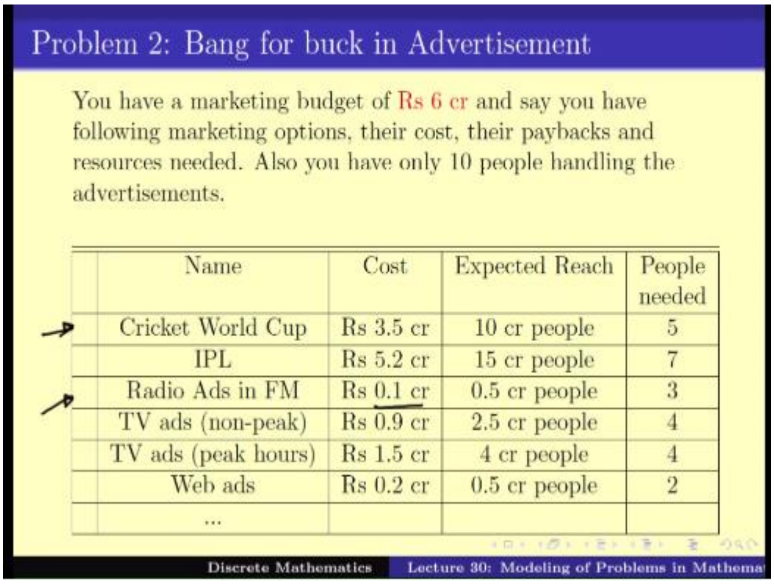

In this particular video, we will be looking at the second problem namely Bang for Buck in Advertisement. Now what is this problem about? So it is about the marketing budget. So imagine that you are one of the big person in the marketing for a company. Now you have been allocated 6 crore and 10 people under you and your goal is to advertise your company in such a way that you maximize the amount of number of people that you reach.

Now you can spend the 6 crore in different ways. Now here are the set of options you can spend it on an ad during the cricket world cup. It will cost you 3.5 crore and expected reach will be 10 crore people. But for getting the advertisement working and dealing with various logistic issues, you need 5 people or you can decide maybe sponsor the IPL in which case you will have to pay 5.2 crore.

And you will expect to reach something like 15 crore people, but the number of people required to pull off this advertisement in 7. Similarly, maybe for an ad in the FM radio, you will require only 0.1 crore and you expect to reach only 0.5 crore people, but the number of people required is just 3 maybe you can think of TV ads during peak hours or non-peak hours and so on. So you have this various options.

Of course, your goal is to spend this 6 crore in such a way that this total expected reach is maximized and you don't want to neither over spent more than 6 crore nor can you invest in things for which you will require more than 10 people, for example, you cannot expect to fund both cricket world cup and IPL. First of all, because you do not have that much fund 3.5+5.2 is way more than what your budget of 6 crore is but also this 5+7 is more than the number of people that you have.

But maybe you can spend in IPL and the radio ad in FM because then the total expenditure is less than 6 crore and the total number of people required is just 10 people. So under all this circumstances what is the best thing to do? What is the best way of spending this money? So this is a typical optimization problem. One important thing here is that I cannot say that okay let me spend half I will buy half and ad in cricket world cup that cannot be done.

Either you decide to invest in cricket world cup or not. It is a 0, 1 setting. Similarly, for something like in radio ads. It cannot be that okay I only spend 0.5 crore in radio ads. No, you either spend 0.1 crore or nothing. So this is like you do not have an option of picking any one of this ad options partially. So you have either go for it or not and this is what makes the problem interesting. We will see it soon when we look at this problem in a short while.

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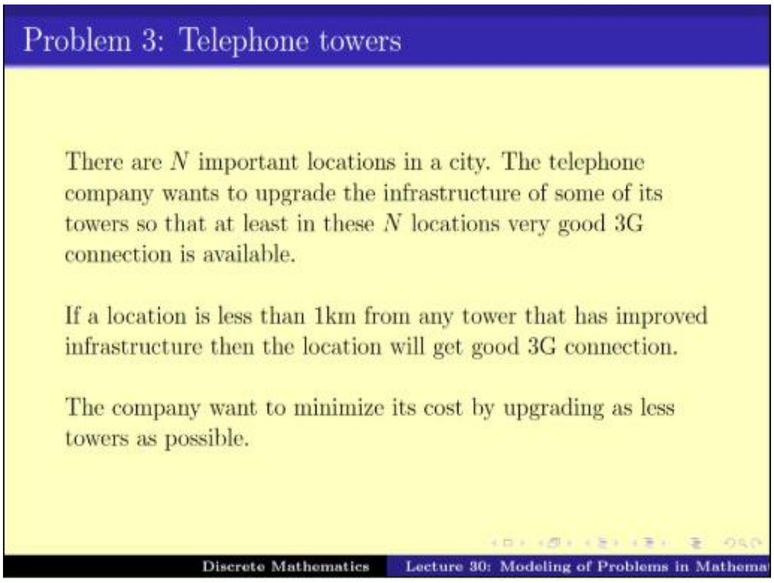

So before I go on so let me quickly talk over the other 2 problems that we will be doing in the coming videos. Namely, the problem number 3 is the telephone tower problem. You have N important location in the city and you want to ensure that every up to the N location has good 3G connections. So for getting a good 3G connection, there must be at least 1 tower within the radius of 1 kilometer that has the 3G options.

So the company now has to decide to minimize the number of towers if it wants to upgrade so that every location in the city get same 3G options. What is the best set of towers to upgrade. This is again another problem that one can imagine happens a lot of time in the telecom industry and similar industries and we will be using our various modeling technique to solve this problem.

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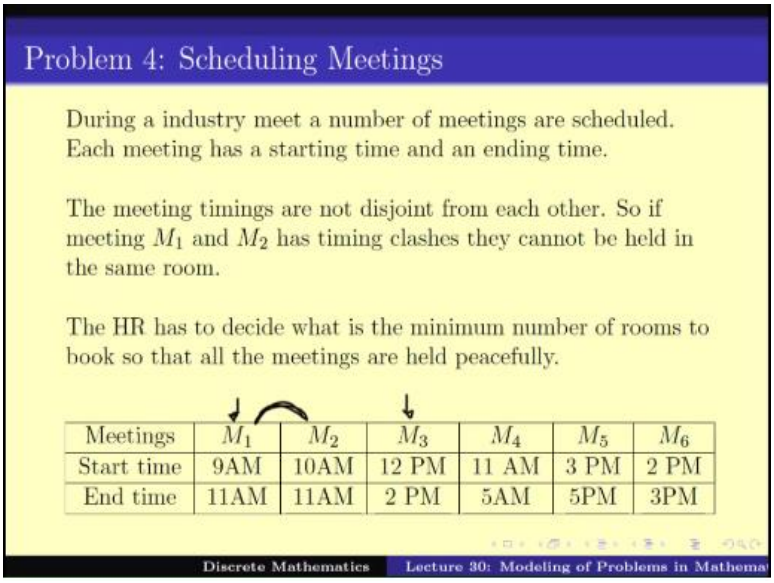

The fourth problem is the Scheduling meetings. It is like an HR is trying to organize a collection of meetings in a particular hotel and they have to book certain meeting rooms and there are certain meetings that will be there, for example, the meeting 1 will be from 9 a. m to 11 a. m. meeting 2 will be from 10 a. m. to 11 a. m. and so on. Now if 2 meetings, for example, meeting 1 and meeting 2 are in this joint times then you can have this two meetings in the same room one after the another.

But if there are 2 rooms meeting 1 and meeting 2 and if they have a conflict in time then you cannot have both the meeting in the same room in that case you have to book to different rooms. Question is that how many rooms do we need to book to ensure that there is no clash and everything happens perfectly fine. So this is again another example of minimization or maximization problems and you look at the minimization problem.

We will be seeing how to model this problem also to get a proper solution.

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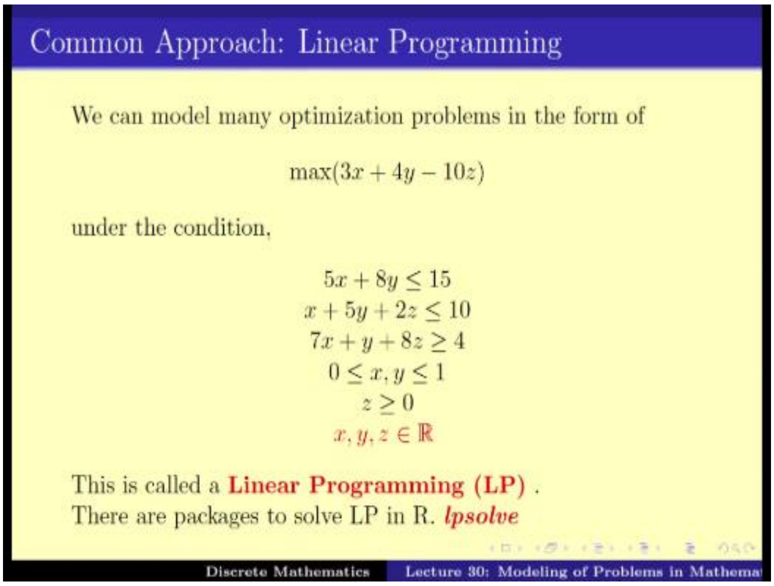

Now the common approach to all of them is the Linear Programming approach. The idea is that you maximize or minimize a Linear equation something like 3x+4y-10z. Under the condition that some set of Linear equations are satisfied, for example, 5x+8y<15 or 7x+y+8z>4 or something. So this is a typical instance of a Linear Programming. You maximize or minimize a linear optimization function under some we call them as linear constraints.

Now crucial part is that what are this x, y, z? So these are variables, but are they taking Real numbers, can they fractional value, can they take only integer value and so on. Now the most simple one is when they take in value in the Real numbers and in that case we call that one a Linear Programming. Now usually we can use Linear Programming to model acquired number of problems.

We say it in the last video how to use Linear Programming to solve the problem 1. The good advantage of using Linear Programming is that there are various nice packages in various software to solve Linear Programming. It is a very well studied subject of course it is beyond the scope of this course to tell me how to solve a Linear Programming in its full glory, but there are packages or software or whatever that can use to solve Linear program.

So from the point of view of solving a problem it is pretty much good enough to reduce a given problem to a Linear Programming or use Linear Programming to model a problem. So in this case language art here is a way in which you can fit this Linear Program into language and ask for a solution.

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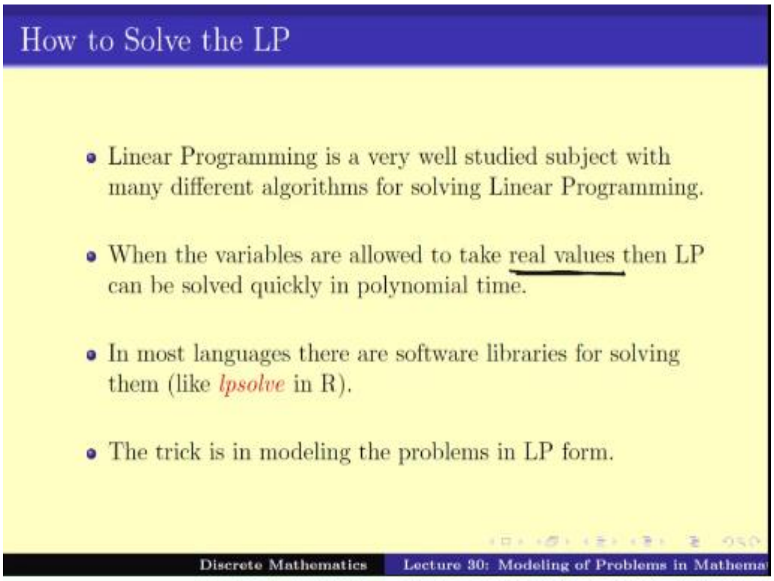

And the best part of it is that Linear Program particularly when the variables are taking real values they can be solved in polynomial fact. There are nice software libraries that can use to solve it. The biggest trick is how to model this problem in the LP form.

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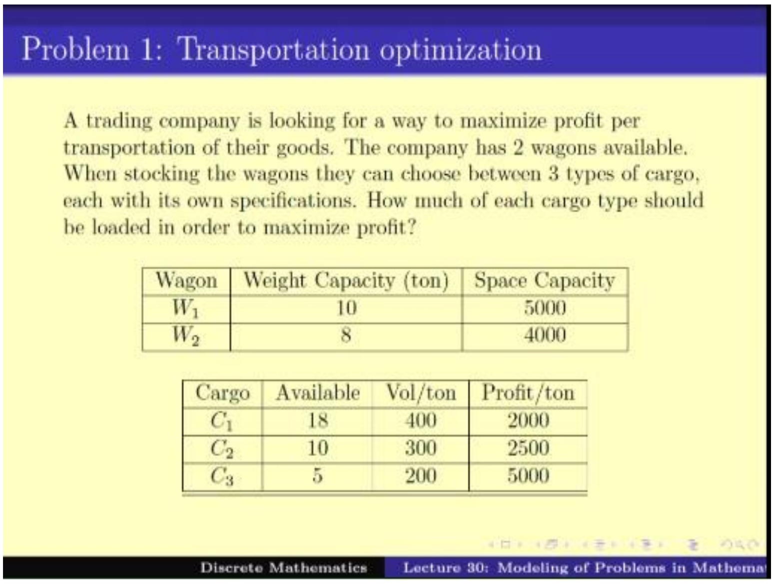

So in the last video we saw how one can use the LP to formulate or model the problem 1.

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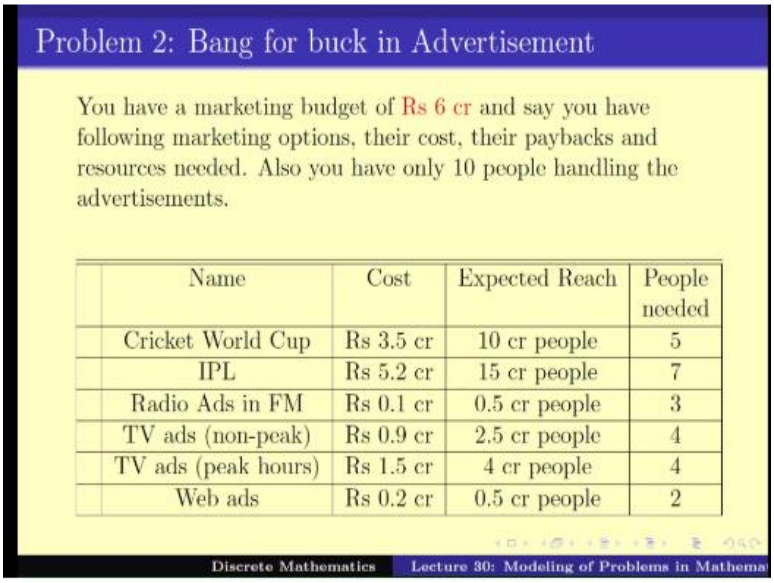

In this video let us look at the problem 2. Now just quickly recap what the problem 2 is. You have 6 crore in hand and 10 people and you have to invest in various advertisement options. Each advertisement options has a cost associated with it and the number of people required and each of them has an expected number of people you can reach. Your goal is to reach as much people as you can.

Under the condition that you do not spend more than 6 crore and you do not need more than 10 people. So what is a typical way in which we can start modeling?

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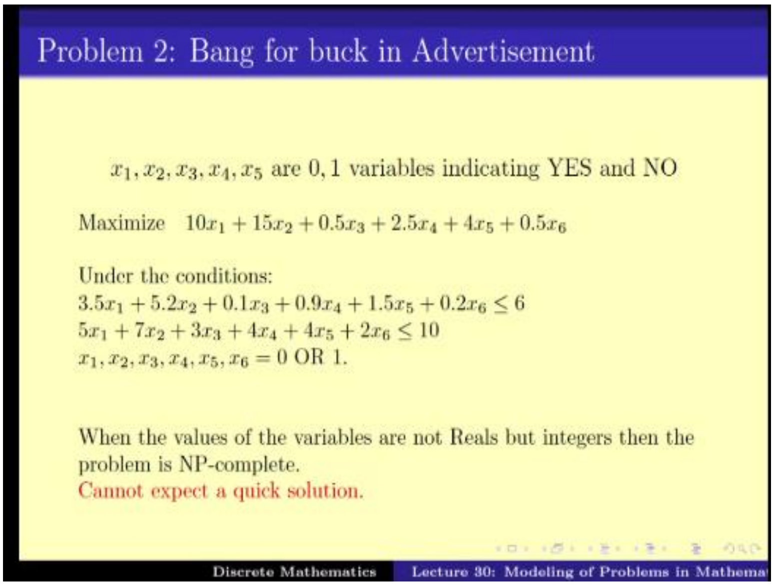

Now one point is that here we can decide let X1, X3, X3, X4, X5 these options basically are saying that what is the proverb, I mean when (()) (14:10) basically saying whether I am going to invest in this particular ad or not. So if X1 is 0, that mean, I am going to investing cricket world cup, if X2 is - I do not invest in cricket cup, if X2 is 1 it means I am investing in the IPL and so on. So if this is the case, now what is the expected reach of people? Now if X1 is 0 then number of people I get, reach is 10 crore times X right times X1.

If X1 is zero, I get zero, If X1 is 1 then I get 10 crore people. So we want to maximize the expected reach which is of course this number, but we have some conditions. What are the conditions? Now if I decide to spend in X1 if X1=1 I have to spend 3.5 crore. If X2 = 1 I have to spend 5.2 crore. So that means we have to ensure that total cost is met, total amount of money that I have to pay is less than 6 crore and which can be modeled using 3.5 times X1+5.2 and X2 and so on is less than 6.

And similarly number of people used can be modelled like this. Now this is a very much of empty problems, we have to maximize some condition, maximize this under some set of linear constants. So we have to maximize some constants under some set of Linear constants, but the problem is that this X1 to X6 are not Real numbers they are 0 or 1. So this is what makes it big problematic.

When the numbers are 0 and 1, we do not know how to solve the problem easily and in the language of algorithm it is told that it is an NP complete problem meaning we do not expect to get a very quick solution to this. So although we have managed to convert the (()) (16:50) problem into an LP formulation, but since the XY cannot takes fractional values hence I do not know how to solve this problem quickly.

So you cannot expect to have a quick solution to this problem. So can we do something else.

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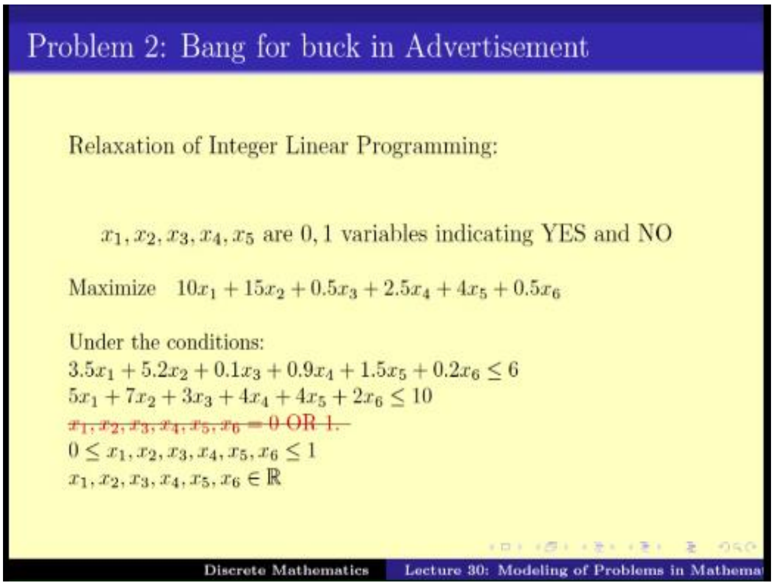

So the idea is that we can relax this integer Linear program by saying that instead of having this condition of 0 and 1 that the variables are either 0 or 1. I can say that let this variable be between 0 and 1 and now I will solve this problem. The problem is that it might give me an answer something like X1 = Half which mean that okay X1 = Half meaning in this case by half the advertisement in cricket world cup which might not be an option which is not an option for me.

So it is a not a useful solution for my case. If I get a fractional solution to this LP.

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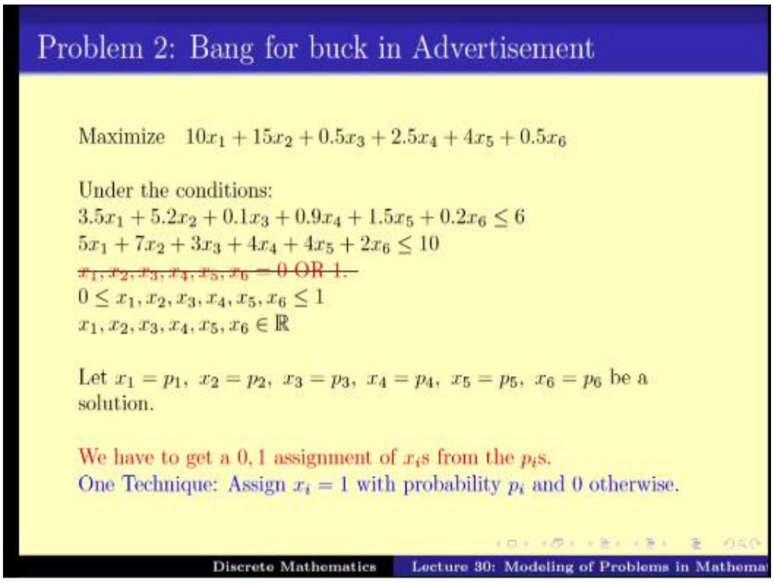

So if I have a fractional solution to the LP I might get a solution like X1=P1, X2=P2 and so on till X6. X6=P6 but P1 to P6 are not necessarily 0 or 1 but some fractions. Now what do we do with this P. For our purpose we have to get a 0, 1 assignment to the exercise. Now here there are various techniques to solve it and I am just giving you one of them. This is the most case the only option, but here it is the assign X1=1 with probability PI and 0 otherwise.

Now this basically means that I first solve the relaxed LP get the solution and depending on the fractional values of the variable that I get I get a 0, 1 assignment. And one thing to see is that the expected return now for people who are familiar with probability I am talking about this expected return if you are not familiar with probability do not worry this is not something to be too much cares about.

It basically says that this still gives us a reasonably good solution. It basically says that on expectation this is a pretty good solution and we get a good outlook. Now the main reason why I started this problem is that to pickup the example is that again this is an example where I took the problem, you converted to an LP you could not solve the LP, but still there was the whole literature using which you can still get a good enough answer to the LP using various ethics.

So in general, whenever you have to solve a problem the good technique to is that how can I use Mathematical languages to solve these problems or reduce this problem or model these problems. And LP is one of those nice tools to have.

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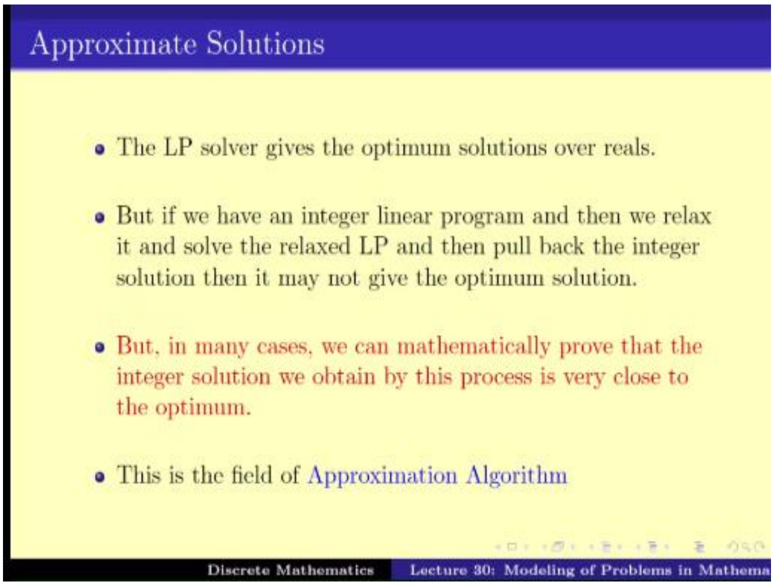

The good thing of this thing is that the LP solver gives us optimum solutions and kind of the LP solver gives him optimum solutions over Reals, but if we have an integer Linear program then we can somehow get a solution that is not too bad from the optimum solution and this basically talks about a new kind of algorithm called approximation algorithm. I do not want to go into it. This is not in the scope of this particular subject.

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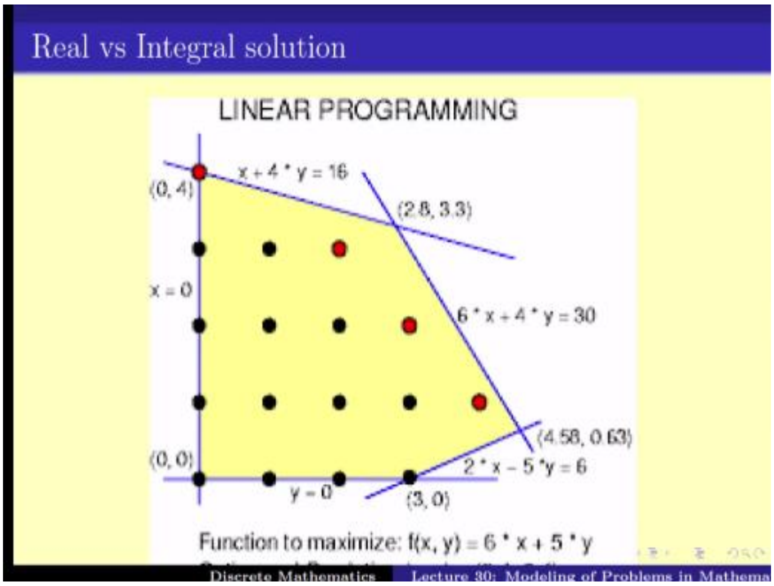

Now why is an integral solution harder to get in a real solution let me not go into too much, but this is one of the picture that kind of explains why finding an integral solution can be a harder job than finding a real solution.

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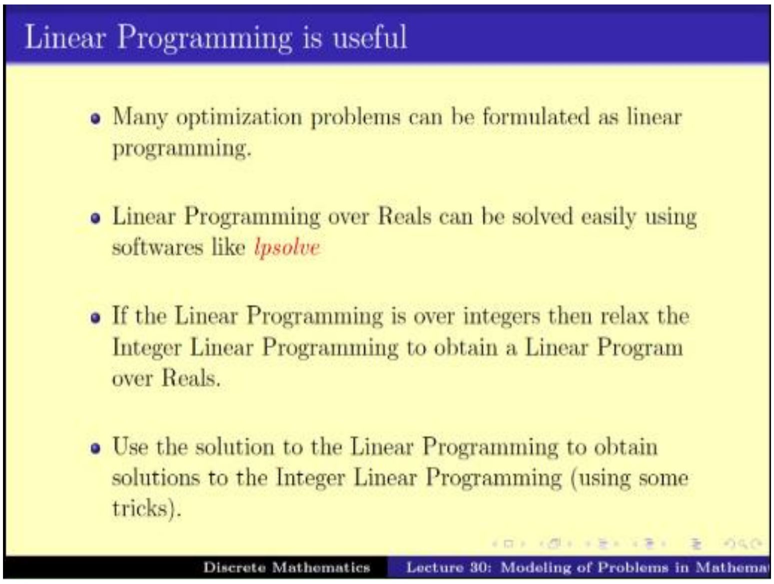

So anyways wrapping up many optimization problems can be formulated as Linear program. Linear program over Reals can be solved quickly. If the linear program is over integers then one can relax it to get a linear program over Reals and then make some use of that linear the solution for the linear program over Reals to understand something about the actual problem. So this is a quick introduction for you guys to how to model using Linear Programming.

Let me also say that this is a very rich area officer and a completely different course is possibly required for this thing. There are certain other various some conditions that can be used to handle this optimization problem and I am not going to talk too much about it. Now we are still left with these 2 problems the telephone tower problem and the meeting scheduling problem. We will be discussing this one these two top problems in the next video. Thank you.