Discrete Mathematics

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Lecture - 29
Modeling of Problem (Problem 1)

Welcome back. So we have been looking at how to use Graph Theory to model various problem. Now modeling problems is Mathematical languages is in general a very powerful tool. In this set of 3 videos, we will be focussing on that particular aspect of problem solving. Namely, how one can model different problems into other Mathematical language.

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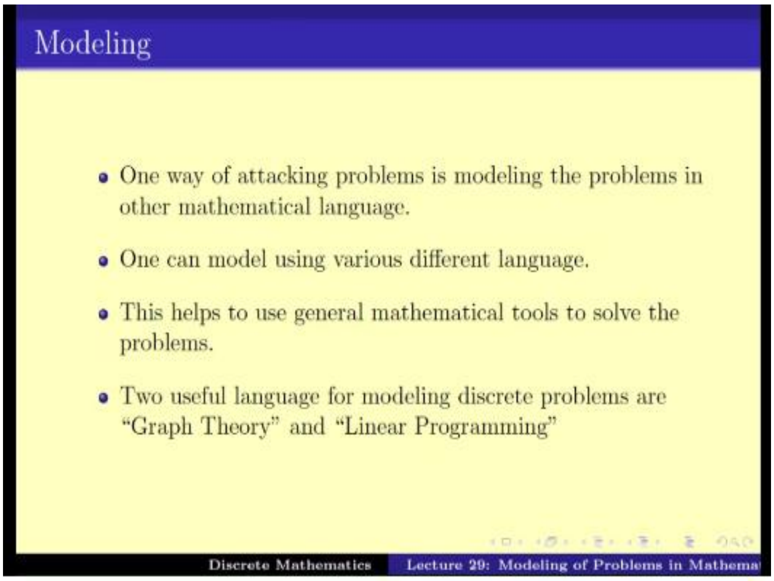

So first of all one way of attacking problems is modeling the problem in other Mathematical language.

We have done so even in our high school when we used to use polynomial or factorization of polynomials, quadric equations and so on to solve the equations or even when we use general technique of solving linear equations to solve various problems. In our high school, we saw how to use calculus for obtaining the maximum and minimum of various functions. Similarly, we saw in the last few videos how to use Graph theory to model various problems.

Now the main use of using modeling problems in different Mathematical language is that you can then use the powerful tools that are developed in various other Mathematical subjects for solving your problem. Now there are various different Mathematical languages that one can use to model one problem depending on the problem what is the right language to represent the model the problem can be asked.

Till now, we have looked at Graph theory. In this video, I will be looking at something called Linear Programming, another very powerful Mathematical model using which we can model various discrete problems. As I told you, you should not think that these are the only two ways of representing problems. There are zillions of other ways representing problems, but graph theory and Linear Programming are the most common used technique of representing problems.

So why Graph theory we saw is mostly used to represent binary relations. The Linear Programming is used to solve optimization problems.

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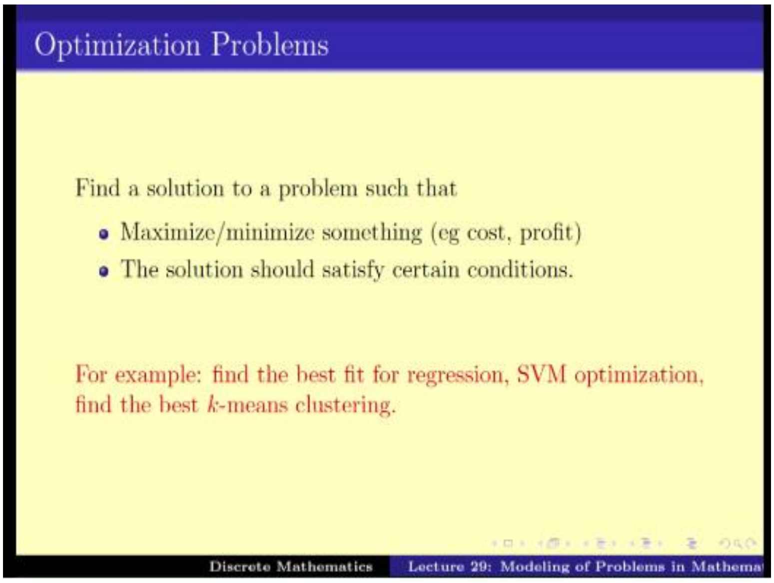

Now what do we mean by optimization problem? This mean that we want to maximize or minimize something, so for example, cost of something or profit of something for under certain conditions we satisfied. So here are couple of examples from the industry world which is used in which an optimization is used a lot. In this video, we will be looking at some problems, some naturally occurring problems in industry which are optimization problems.

And we will see how to use LP Linear Programming or in general Mathematical languages to model this problems. So here is the first problem.

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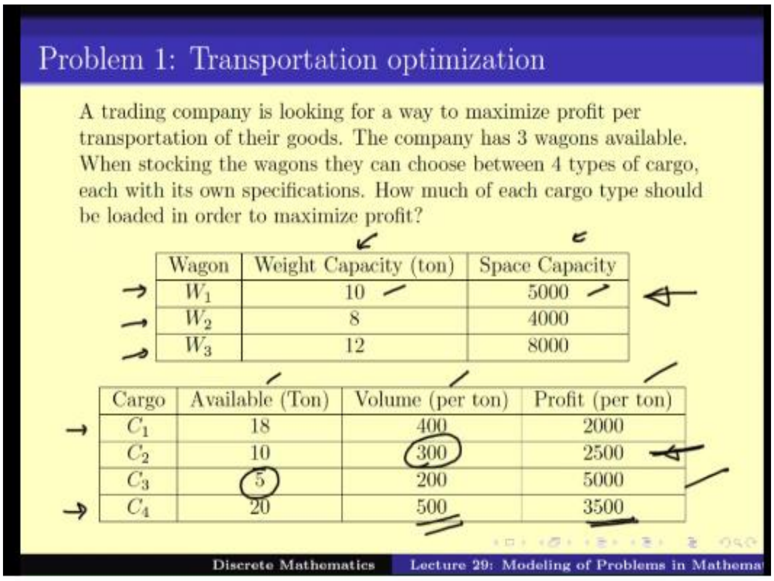

The first problem a trading company is trying to maximize profit for transportation of goods. This trading company has 3 wagons available. Now when stocking the wagon they can use 3, 4 types of good. So here there are the 4 different cargos C1 to C4 and here are their wagons. Now each wagon has a weight capacity and a space capacity. At each cargo has a volume per ton how much is there and if you manage to skip that much amount of thing what is the profit there is there.

Now this is a typical problem that a shipping industry faces all the time. So it has 3 ships, each ship has some weight capacity, some space capacity and there are various kinds of cargos C1 to C4. Each of them has a certain amount of available quantity and each of them takes certain amount of volume and each of them has certain amount of profit involved it. Now how can you pack this quantity into different wagons so that you optimize or maximize the amount of profit.

So there are various humanistic problems that one can come up with. So maybe the cargo that has the maximum profit per weight, let say that one, but maybe if we take some object which are the maximum profit per weight namely say this one C3 that I can only fill up only 5 tons and total volume available is just pretty low whereas if I take next one say it is better to take the C4 or is it better to write C3.

Although, C4 you get more profit per ton. C4 also takes up a lot more volume whereas while for C2 I have slightly less amount of profit, but I can pack more objects in it in the ship or in the wagon. So this is the typical problem that is faced all the time in the wagon industry. If you are bit more, if you want to think of it likely easily think of just 1 wagon. You have just 1 wagon with the weight capacity and the space capacity.

That means the weight capacity meaning you cannot exceed the weight. So how much amount of quantity can you store in wagon 1 if you stay in touch and how much space can you use in that wagon it is a 5000 liters. Similarly, wagon 2 you can have the most pack 8 tons and space capability is 4,000 liters and wagon 3 you have weight capacity of 12 tons and space capacity of only 8,000 tons, 8,000 liters and the cargos have different.

So, of course, the goal will be to maximize profit such that none of the wagons are overloaded either in terms of weight or in terms of space. Now question is that how much do you fill in each of the wagons and what cargo would you pick? So we will be looking at this problem and there are 3 more problems. Let me first describe those 3 problems and I will go back to this problem after that.

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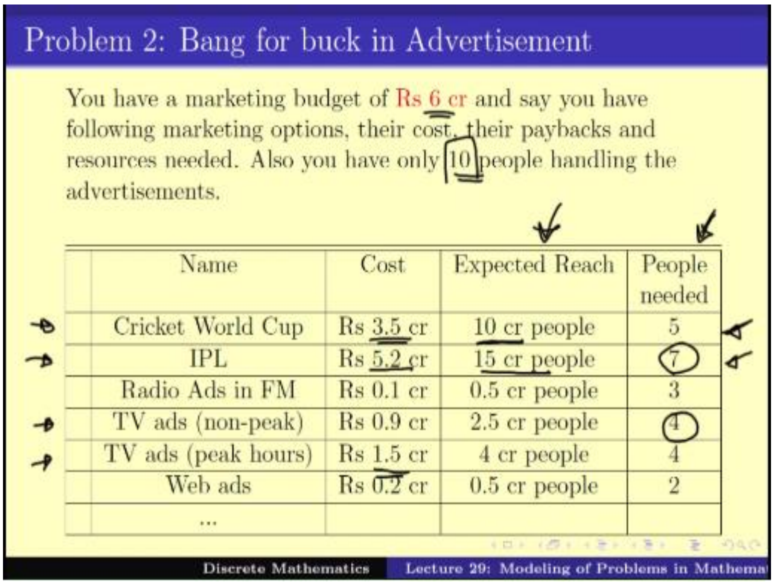

The next problem is another problem from the industry world namely Bang for Buck in Advertisement. So, for example, say you have marketing budget of 6 crores and even spend this money in various fields. Now you have only 10 people handling the whole advertisement for the whole company. So one thing that you can do is that you can maybe invest 3.5 crores to rate an advertisement for cricket worldwide.

In that case you will be expecting to reach something like 10 crore people, but you will need 5 people to make sure that the advertisement works. Well you can have the IPL maybe you will be reaching 15 crore people the cost will be more and also the number of people that you will use it or needing is 7 people. Similarly, for Radio the cost is very low 0.1 crore, but you expect to reach half a crore people and you also need only 3 people and similarly like this.

Question is that how much or which of this thing should we investing. If you decide to invest in cricket world cup you have to spend 3.5 crore in which case, you have to allocate 5 people for it. So if you, for example, I have totally 10 people what it means is that I cannot invest in both cricket world cup and IPL or I cannot invest in both IPL and TV ads or non –peak times because IPL will require 7 people and TV ads will be for 4 people so all total I need 11 people, but I only have 10 people.

At the end of the day I also have only 6 crore so I cannot invest, for example, in IPL and TV non-peak ads because IPL will require 5.2 crores and this will take 1.5 crore, so all total I require more than 6 crore. So the goal at the end is to maximize the expected number of people you are reaching under the condition that you spend less than 6 crore and you all total number of people require is less than what you have 10.

So again this is a typical problem in optimization problem, but the optimize time to optimize the profit or here in this case expected reach under the condition of certain conditions which is here upper bound of department and number of people being used. Then look at the third problem.

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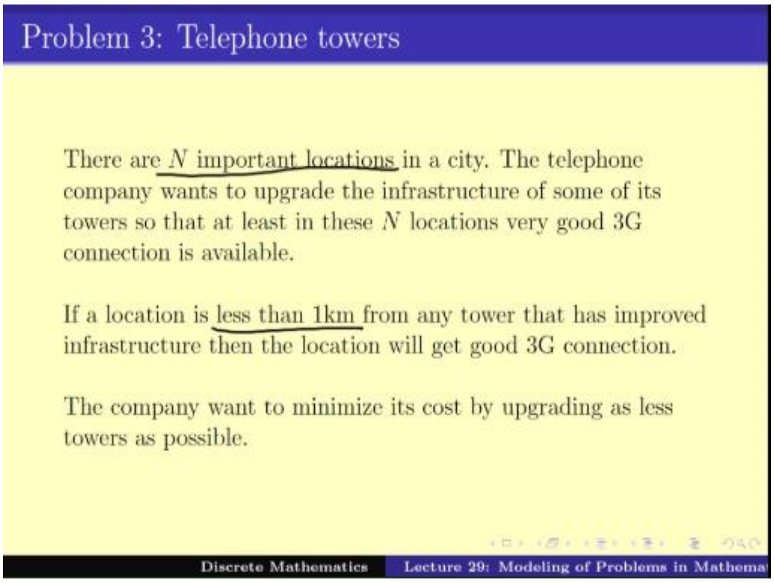

The third problem is about telephone towers. You have N important location in the city. Now you want to upgrade some of the infrastructure of some of the towers to some 4G connections. Now if a location is less than 1 kilometer from any tower that has improved location then it will have good 3G connections. The company of course wants to minimize the number of towers it has to upgrade.

So that at the end it also want to ensure that everybody in the city or everybody in all this N important locations in the city have good telephone connection or with good 3G connection. Again this is optimization problem, it is a minimization problem why do you want to minimize the number of telephone towers to be upgraded under the condition that every person or all the N important locations is less than 1 kilometer from any tower that is upgraded, some towers that is upgraded.

So again this is an optimization problem, a problem that occurs in their time in industry world quite often.

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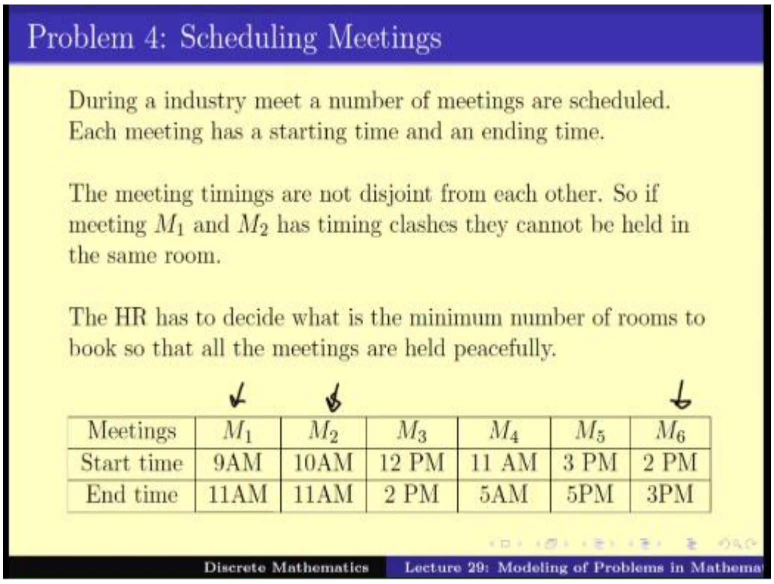

The fourth problem is a problem that industry HR has to decide so when there is a meeting that has to be decided and they have to book hotel rooms for various meeting. So here is a list of meeting that will happen. There is meeting 1 that will start at 9 a. m and end at 11 a. m, meeting 2 that starts at 10 a .m and meeting end at 11 a. m and so on. Now of course that clash in the same time for meeting 1 and meeting 2 cannot be held in the same room.

So the HR has to decide what is the minimum number of room to book so that all the meeting are held peacefully. So we will need to solve this problem also so here again it is a optimization problem. Its minimized the number of rooms require for ensuring that all the meeting happen without any clash. So here are 4 problems that I have picked up and we will see how all these problems can be written in the language of Linear Programming and Graph theory and that is how one would like to model this areas real life problems.

Now most of this problem or at least few of them have some common properties in them.

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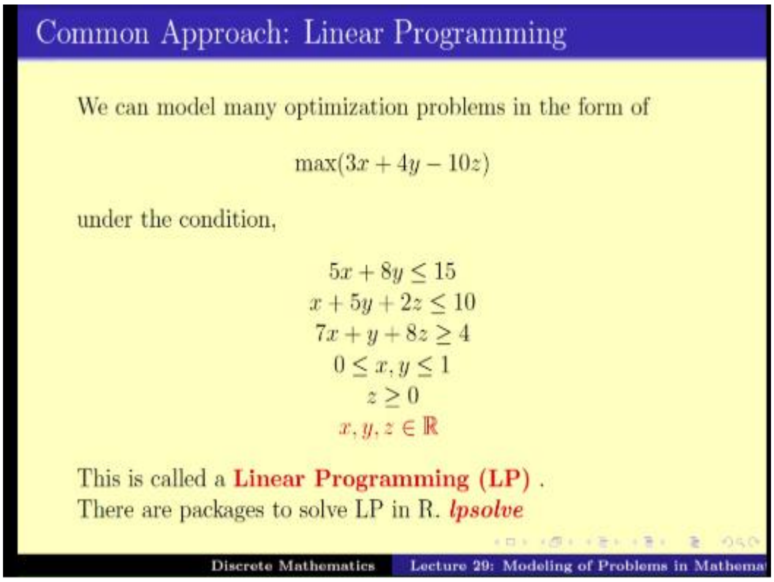

Common approach is what is called a Linear Programing. Now what is Linear Programming? So Linear Programming is the simplest form of optimization problem that you can think of where we want to optimize in this example to maximize some equation, for example, here 3x+4y-10z where some equations are satisfied for example under this condition. So I have been given this set of equation 5x+8y≤ 15, x+5y + 2z ≤ 10 and so on and so forth.

And of today we have told that x and y are number between 0 and 1. Z is a positive number and under such condition what is the maximum that we can have. Now this is a typical form of Linear Programming. Linear because both the constraints as well as the optimizing function are linear when you get degree 1 polynomial. So then we do not have something x square XY be coefficient. So I have 3x+4 y – 10 z. So it is a linear term.

Now again one thing to notice that when we add this condition that xyz are Real numbers and not just integers that are forming from the Real number then this is called the Linear Programming. We will be talking about the case when they are not Real numbers possibly next video, but in this video let us assume that x, y, z are Real numbers and in that case this structure or this whole thing called a Linear Programming.

Now useful as a Linear Programming is that Linear Programming can be solved pretty quickly and they are well known structure on that. In fact, in our one of the very useful machine languages tools that is there, there is a package that can solve it. So if you have such a thing you can feed the program in a particular way and in which case you get the Linear Program solution.

So if I can convert those problems into an instance of linear program then I can use the concept of Linear Programming to solve the problem.

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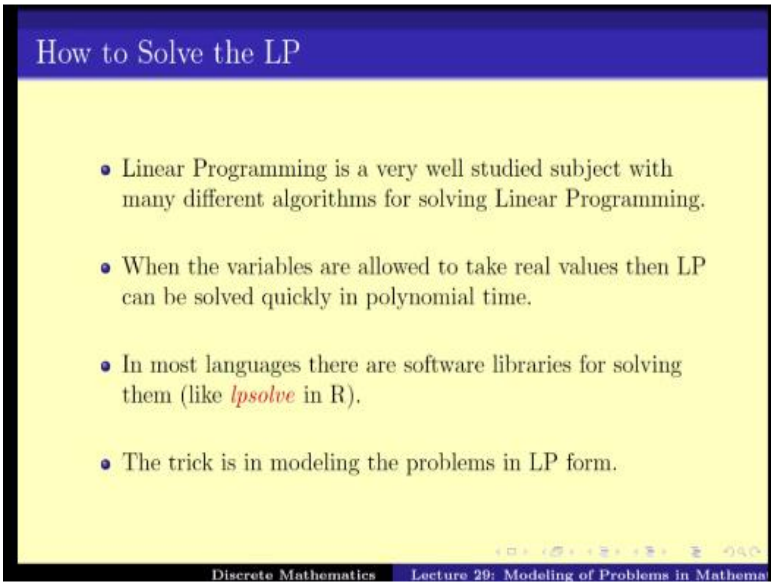

So Linear Programming is a well studied subjects and there are lots of different algorithms and norm. When the variables are allowed to be Real numbers then we can solve this very quickly. There are some nice packages in various languages that help it to solve which is modeling the problem in the Linear Programing form. So let us start with the first problem. You remember so just saw the sake of simplicity.

I have reduced down the number of wagons to 2 and the number of cargos to 3. Now how to solve it? Say I start with saying that we have these 3 cargos and 2 wagons

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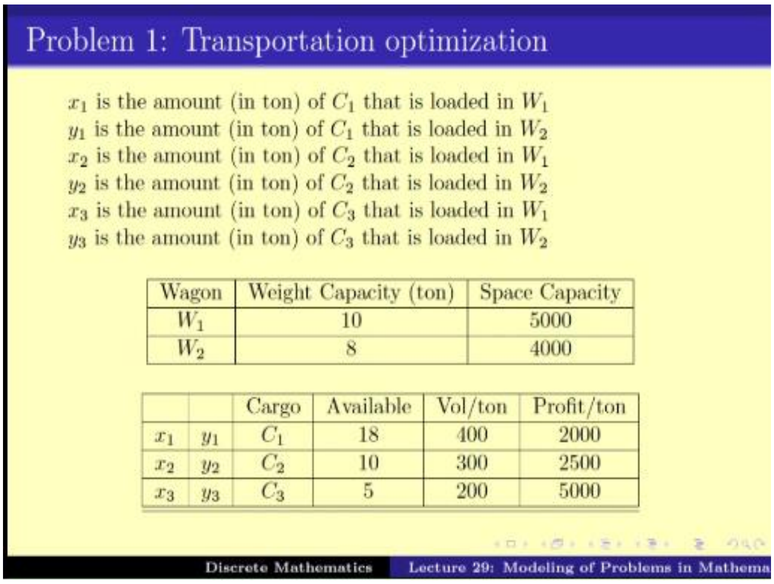

Let say that X1 is the amount of cargo 1 in wagon 1. Y1 is the amount of cargo 1 in wagon 2. Similarly, X2 is the amount of cargo 2 in wagon 1 and Y2 is the amount of cargo 2 in wagon 2 and similarly for X3and Y3. So now that this has done, we can now quickly calculate first of all how much profit we have and also what are the future. So what will be thing we maximize? We maximize the profit.

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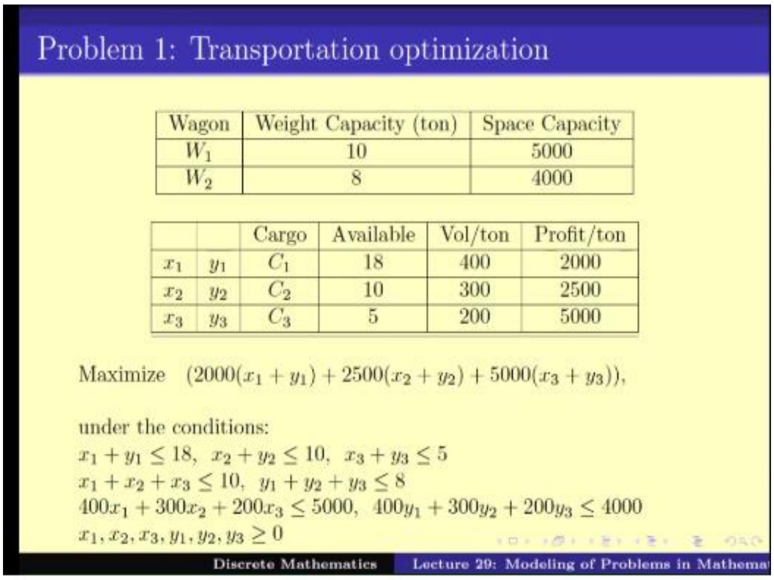

Now if I manage to shift X1+Y1 amount X1 in wagon1 and Y1 in wagon 2. So if profit that we have is 2000 times X1+Y1. Similarly, for cargo 2 it is 2500 times (X2+Y2) and 5000 times X3+Y3 for cargo 3. And what are the conditions let us look at it. There is weight conditions. So weight conditions say that wagon 1 cannot take more than 10 kg okay sorry not this one.

Weight conditions are that there are only cargo 1 only 18 tons are available. So X1+Y1 <18. Similarly, only 10 tons are available for cargo 2. So X2+Y2<10 and similarly X3+Y3<5. So this is one of the condition some more condition. The other condition is the capacity of the wagon. The amount of weight that is going in wagon 1 must be less than 10. Namely there is X1+X2+X3<10 and similarly for the wagon 2 Y1+Y2+Y3≤8.

And third one is the space capacity. Now what is this space capacity? How much space do we end up spending? So 400 is the amount of volume per ton for the cargo 1 is 400, the amount of volume require for shipping X1 amount in wagon1 is 400 X1. Similarly, amount of volume required for shipping amount is cargo 2 is 300 X2 and so on. So we get 400 X1+300X2+200X3 this should be 5000 in the upper more on the volume that can be spend.

So this is ≤5. And similarly this one for the case on wagon 2. So here is that question we have for the set of inequality which are the conditions and what we have to maximize and we know that all of them must be > 0. X1+X2, X3 and Y1, Y2, Y3>0. Now this is typically in the form of LP and we can solve this and once we solve it using whatever program LP solver that we have we get an optimum solution for this problem.

So here is an LP you solve it and you solve the optimum solution.

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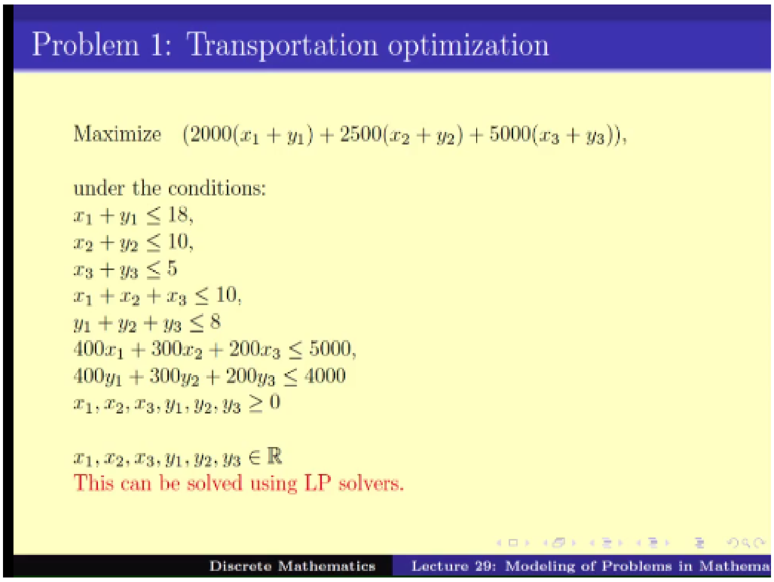

Here of course we can assume that X1, X2, X3 and Y1, Y2, Y3 are from the Real numbers. So this can be solved with LP solvers and hence we have seen how we can reduce that problem or model this problem using LP and then use this standard tools that we have for LP solver to solve this problem. Okay meaning video we will be looking at the second problem and how one can use Linear Programming again to solve this problem. Thank you.