Discrete Mathematics
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Module - 03
Sets, Relations, Function and Logic

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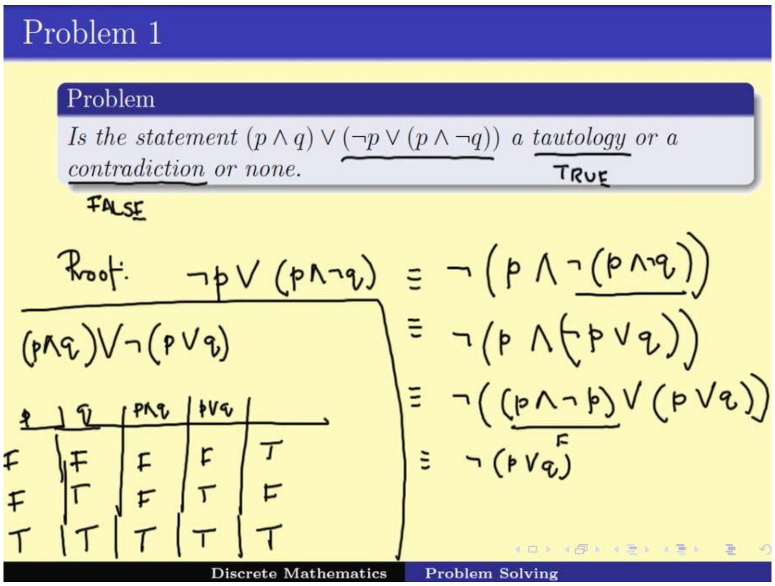

Welcome everybody. So today we will be doing some problem solving. Today we will be solving problem related to sets, relations and functions. So last week there was an assignment posted I hope you have tried them at least and this particular problem solving session will be kind of working on problems related to that. I have selected some four problems for this session. I will be solving them.

In case there is some particular that you would like me to solve you can ask me or request me in the forum. I will be happy to take it in to consideration. Now let us start with the first problem. The problem says that “Is the statement p and q or not p or p and not q a tautology or contradiction or none”. Now what is a tautology? A tautology basically is that it is always true means that does this statement for any evaluation of p and q or any whether p and q are true or false?

Always comes out to be true. A contradiction basically says that is it always false. And the third one says none basically is that it is that for some setting of p and q the statement becomes true and for some set of statement it becomes false and for some assignment of p and q to true and false. Now there are two ways of doing it of course this problem. First problem is to work out the truth table. Now the truth table here will not have too many rows because p and q are there were two variables.

Hence they have only four rows namely where p takes true and false and q takes true and false but evaluation of this particular expression might require some hard work and there is always a tendency of making mistake. So instead what we can do in such problems is that we can simplify them. So to start with let us first try to look at this expression. Can we simplify this expression? And the idea is that of course we can because given that there is a not here and not in the q.

We can of course try to pull this one out using De Morgan's formula. So this will become p and not of p and not q. now I mean this particular formula part we can again apply De Morgan's formula and we can get not of p and this will become not of q or q not of not of q is q. Now once we have this thing then we can of course apply the distributed law and put this p inside and this will become first part p and not p or p and q. Now let us see here, what is this one? P and not p.

Remember that anything and false is false right. So p and not p if one of them is true then the other one has to be false. So this how they form true and false or which is of course false and this expression p false or something. Now note that false or something is nothing but that thing itself. And hence this is same as not of p and q. Thus this whole expression turns out to be p or q and not of p or q. Now this is slightly easier to handle and though we can now write down the truth table of this one.

Say this is p this is q, now what is this p or q? You of course have p and q and the final expression right. So if both of them are false let us see what happens. Both of them are false then p and q is of course false. P or q is also false and in that case so this is not of false means true but there is an or so that will be true. Similarly, now we can keep on writing the statement as false and true where false and true and here p and q is false p or q is true so negation of true is false.

False and false is false. Now this already implies that this one is true at some place and false at some place hence it is neither at tautology nor a contradiction it is in third category. Just to complete our understanding of this particular expression let us see. Note that if I keep p as true and q as false this is same because this is an expression that is symmetric so you will still get false. Your last one is both of them are true.

If this would be true and then true or not a true means false, but true or false is true. Thus you can see that for two of these cases it becomes false and two of the cases it becomes true right. Now for the exercises as well as the assignments that will be posted you will be asked to solve multiple choice questions. There are of course easier to handle but if you can understand how to prove this statements or this type of statement it will be helpful for you to attend the problems.

And that could be right way of doing it. Because of some biggest constants that we have for holding this course online we cannot ask you to solve problem which are objective type but we encourage you to solve them at home.

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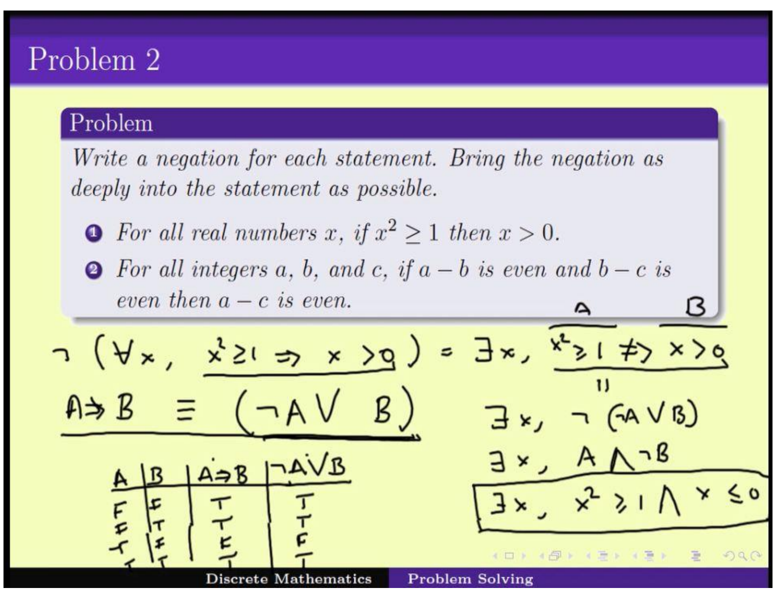

Now moving on, okay now moving on let us go with the second problem. The second one says that write a negation for this following statements okay. So let me do the first one. The first one says that for all the real number x, if x square is greater than or equal to one then x is not equal to zero. Now there are multiple ways of doing it. Firstly, let us look here, so this statement is on the form therefore all x, x square greater than or equal to one implies x is strictly less than zero.

If I want to take negation of this now the negation of this which will become of course there it is x such that x square greater than or equal to one does not implies x is greater than zero. So its negation for all x changes to there is and this expression is the predicate and this one negation of this. So a implies b opposite of that is a does not implies b okay. But now how can you write this statement a does not implies b now there are various ways of writing a does not equal to b.

Since one of the ways of seeing is that how do you write A implies B? We have seen this one in our in you know our lecture notes on proof by contradiction namely A implies B is same as A and B no this is not true it is same as not A or not B. Now why this is true? The basic idea is that A implies B in other words if A has happened then B must happen so either A does not happened which is not A or B does not happen sorry or B happens.

So either A does not happened or B happen. So A implies B is same as not A or B. Now see as you can see even I have bit confused over what is the right one? So the best we are checking it actually to write down the truth table. Let us quickly write down the truth table and we convince that this is the correct statement. So if it is A implies B and this other one not A or B. So if A is false both of them are false.

If A implies B is true because A false (()) (14:25) and this one is of course not A and this is true. If it is false and this is true then again A implies B is true, the false (()) (14:37) and again same logic this is true because not A is always true. They see if this is true and b is false then A implies B is false but now here not A is false then B is also false so this is false and false is false. And the other last one is true and true. If both of them are true, true implies true is true.

And what is A? When not A is false but B is true so true and false is true. As you can see both these columns are same and hence we can say that A implies B is also same as not A or B. Now here you see useful ways of this propositional logic. Sometimes we ourselves get confused you do not have to memorize it. But propositional logic the understanding of truth tables helps us to quickly check with that the thing that we remember is correct or wrong.

So in other words you can say here that A implies B is same as not A or B. So now this statement is we can write it as of course there exist x and negation of not A or B which is this one is A of course and this is B so which means that this is not of A and now we can apply De Morgan's Law not on not of A and not of B or in other words there exist A such that x square is greater than equal to one and x is not greater than zero or less than or equal to zero.

Right so this is the negation of the statement. While the implication does not imply it is also that is thing but writing it in this form of not A or B helps us to solve it. So the negation of this following statement is this.

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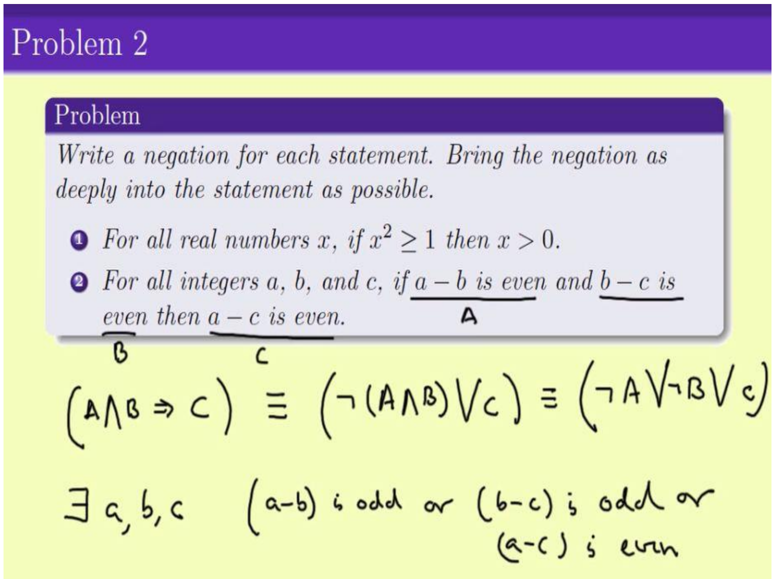

Now let us go to the second one. The second one says for all integer a, b and c, a-b is even and b-c is even then a-c is even. Let us apply the same rule this time here of course this one we have two parts right A and B and C. So we have A and B implies C. By this same logic as we just now see this should be congruent to not of A and B or C. Now by De Morgan's law this is same as not of A or not of B or C.

So when we take negation of all becomes there exists, so it will be there exists a, b, and c such that not of A means a-b is odd or b-c is odd or a-c is even right. We have known that there can be multiple ways of writing the negation of this following statement. And I encourage you to discuss on the forum what are the various different expressions for these statements that can come.

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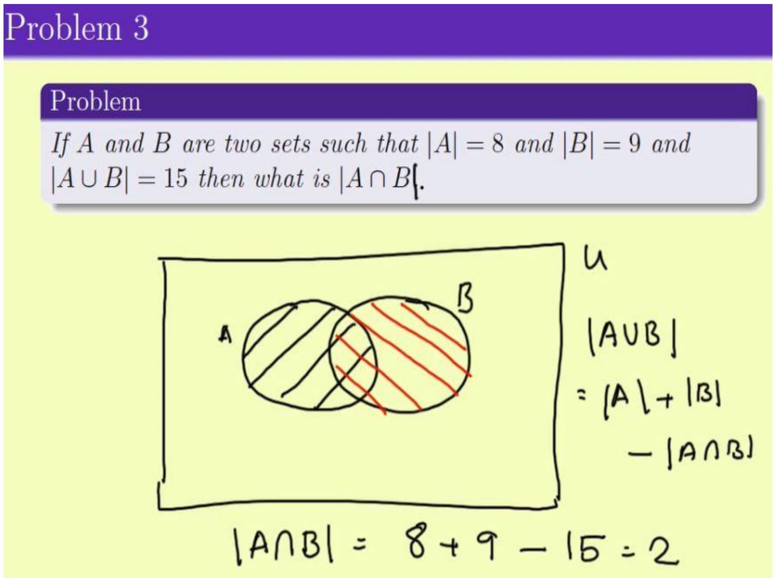

Now going to the third problem so this problem states that A and B are two sets such that size of A is 8 and size of B is 9 and A union B is 15 and then what is size A intersection B? Of course I have missed a bar here this should be A intersection B. Now to get to solve such kind of problem the best way is probably the Venn diagram. We have discussed this in the class. We call this one the universe U. Here is the representation of the set A and here is the representation of the set B.

Now this says that this particular area is A, so this is A and this is B. So if I draw this B with these red lines. Now what is A? Size of A is 8 right. So this black area is 8. Size of B is nine so that area is nine. And the union is 15 so the area of the whole shaded area is 15. Even when looking at this one you can see that what is A union B. Now a union B is of course if you take the size of A plus the size of B.

You count the black area once, the red shaded area once and by doing so you count the area that is shaded both with red and black twice. So if I subtract that area with the area of A intersection

B I should be getting exactly the area of A union B and that's what all is equal. Just by looking at the Venn diagram you can decide how to draw this one. Now this one is 15 and this is 8 and this is 17 so what is A intersection B?

Now A intersection B is nothing but A plus B which is 8 plus 9 minus A union B which is 15 which is nothing but 2. The Venn diagram is the very useful thing to do. By doing the Venn diagram you will be able to understand how the steps are distributed very evenly.

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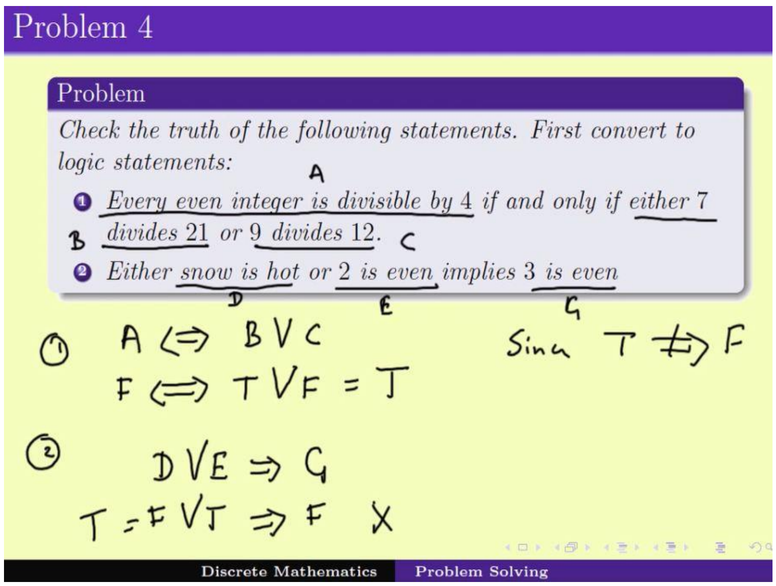

Now let us go to the fourth problem here. It says that to check the truth of the following statements. First one, if every even integer is divisible by 4 if and only if either 7 divides 21 or 9 divides 12. Now to understand this statement you have to understand what are the sub topogenous? This one every even integer divisible by 4 is say A. If and only if either 7 divides 21, this is B and 9 divides 12 is C. So this statement is on the form A if and only if B or C.

Now here is every even integer divisible by 4 of course not. For example, 6 is an even integer and does not divisible by 4 so this is false. So for all the if and only if now what is B? 7 divides 21 and this of course 7 does divides 21 which is true and this one is 9 divides 12 so this of course false 9 does not divides 12 false. So false if and only if true or false now what is true or false? True or false is nothing but true so false if and only if true or false or of course implies true but true does not implies false.

And since true does not imply false this statement is false the whole statement right. So just converting and understanding which of this statement or the subset means writing the preposition and understanding whether that is true or false gives us the right answer. Now let us do the second part so this was the first part. Now the second part, it says either snow is hot or 2 is even implies 3 is even.

Again let us look at the sub part they call this one say D, 2 is even is E and 3 is even F. Now F is not a good idea may be G so what this says is that D or E implies G. Now let us see D is snow is hot no it is not. Snow is seriously not hot so these are false or what is E? 2 is even. 2 is even is of course true and this implies 3 is even which is false. Again true or false is true. True does not implies false hence this is also a false statement right.

So these are the four problems that we did in this particular problem solving session. If you have any more questions that you would like to discuss prepositional logics, sets relations so on. Just feel free to request in the forum and I will try to act more repeatedly next problem solving. Thank you.