An Invitation to Mathematics
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Unit – I
Polynomials
Lecture – 1A
Introduction

Hello and welcome to this course titled An Invitation to Mathematics. So, we will start out by talking about polynomials which are one of the very basic objects in Mathematics. So, we have polynomials, so let us start by giving some examples of polynomials.

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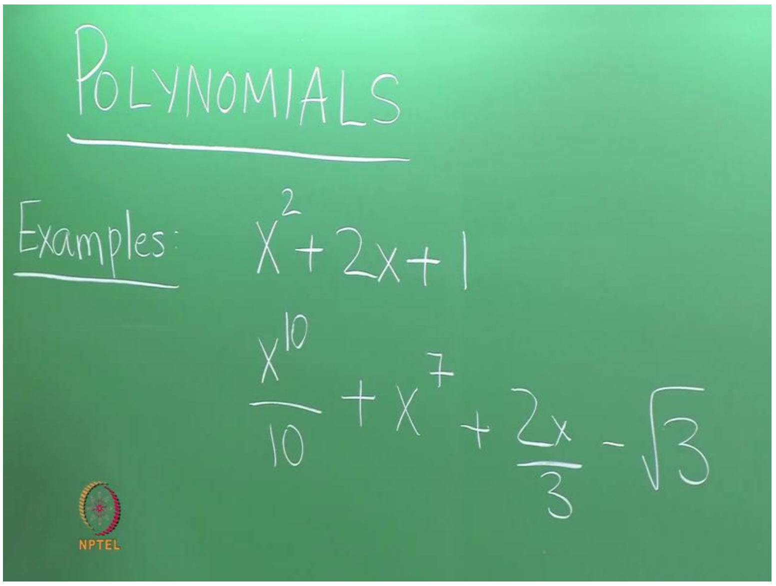

So, I wrote down two examples of polynomials. So, typically a polynomial looks like combination of various powers of a variable, this case the variable x and notice that in addition to the variable itself. So, in addition to powers of the variable, what you have are some coefficients, some numbers which multiply them. For instance, you have the number 1 by 10 multiplying x power 10, we have 2 by 3 multiplying x, you have a constant root 3 and so on.

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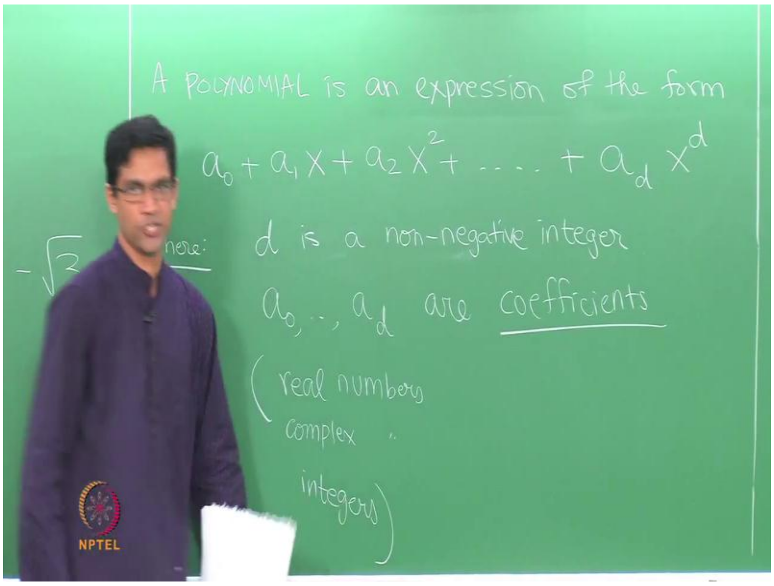

So, the general formal definition of a polynomial is well, it is an expression of the following form. So, polynomial is an expression of the form a 0 plus a 1 x plus a 2 x square and so on till a d x power d, where what are all these various things. Where, so firstly, what is d, d is greater than equal to 0 an integer. So, it is greater than equal to 0 and so may be a better ideas to say it is d is a non negative integer and what are these a i. So, they are what are called the coefficients, so and what are a i's. So, a 0, a 1, a 2 till a d are what are called the coefficients and they can be numbers of various kinds.

So, for instance, we can have coefficients which are real numbers, they could be complex numbers; they could be integers and so on. So, what are these coefficients? They are could be real numbers, complex numbers, integers. So, it could have various restrictions on what kinds of coefficients are allowed. Now, this number d here, so observe that, when you have a combination of powers of x, you can always assume that d is the largest power of x which appears.

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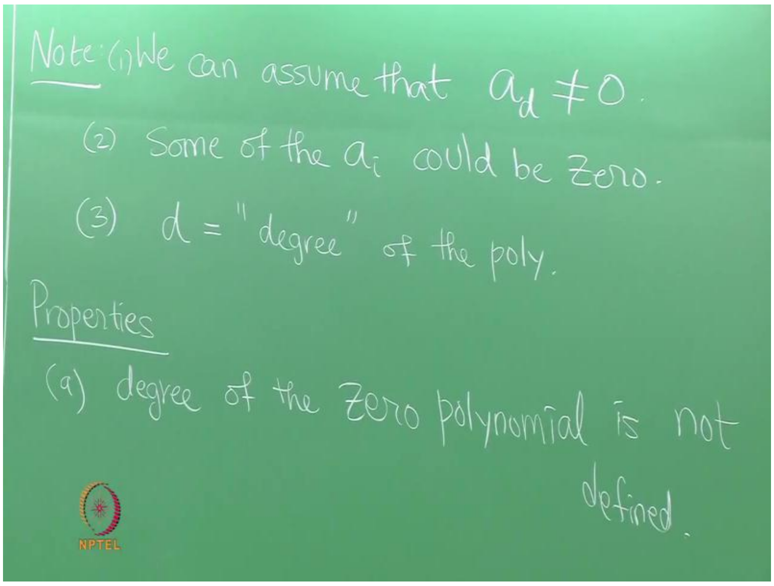

So, here are various small observations note that, we can assume that a d is not 0, so that is first observation. We can always assume that d is the largest coefficient which appears. Point number 2, some of the other a i's could very well be 0, so we are not forbidding that. For instance ((Refer Time: 04:24)) in the second example here, the degree d, so this number d is called the degree, d is 10, because it is the largest power of x which appears, but you do not really have all powers of x appearing. So, you do not have x power 9, you do not have x power 8 and so on.

So, you could have other powers. So, some of the other a i's could be 0, some of the a i's could be 0, there is no problem with that and I just want to say, the terminology d is what is called the degree of this polynomial. So, largest power of x which appears and so, let see what else is there about the degree itself. So, here are some simple properties of the degree that will often be useful. So firstly, the degree of the 0 polynomial is not defined. So, what do we mean by the 0 polynomial?

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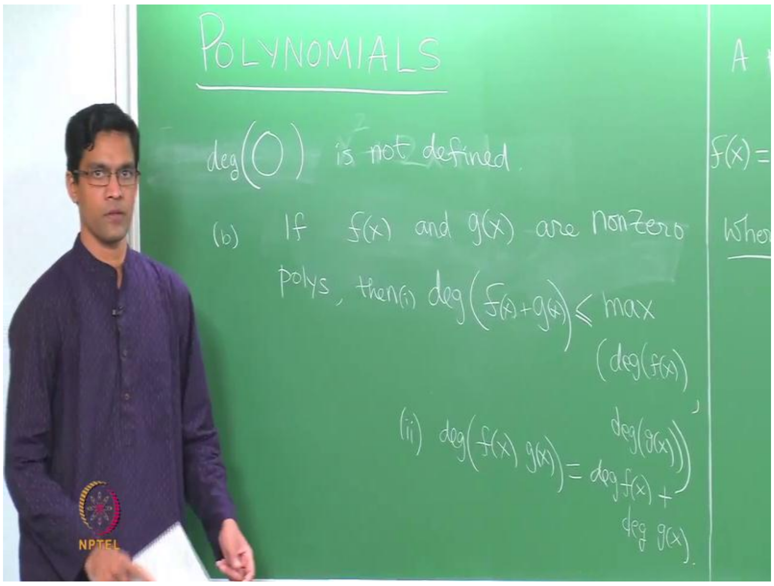

So observe, when we say the 0 polynomial, we just mean think of in all the a i's as just being 0. There is no power of x which appears, it is only a constant term, the constant term is 0. So, in this case, the degree is not really defined, because you know remember the degree was suppose to be the largest power of x which appears with the non-zero coefficient.

Now, since no power of x really appears in the 0 polynomial, you do not really define the degrees for the 0 polynomial. So, this guy does not have a degree. So, here is always saying this, we just shorten degree to deg, deg of 0 is not defined, only defined degree for non-zero polynomials. Now, if f and g, so often we will use some symbol to denote a polynomial. So, for instance, we could call ((Refer Time: 06:53)) this polynomial as f of x or you know we will use other symbols as well g of x and so on.

So, if f and g, if f of x and g of x are non-zero polynomials, then degree of their sum. Let us call it f of x plus g of x is less than or equal to the maximum of the degrees and so this is property 1. Property 2 says, the degree of the product f of x times g of x is exactly the sum of their degrees, degree of f of x plus degree of g of x. So, what we have done so far, we just defined what a polynomial is, it is an expression of that form. We have said, what the coefficients mean, the coefficients are just these numbers in front.

And we have said, what the degree means, is the highest power of x which appears and what we have done is written down some simple properties of the degree. So, observe that, while we did this I have sort of silently split in two things, two operations here. I said something about a sum of two polynomials and the product of two polynomials. So, these are just the natural operations on polynomials.

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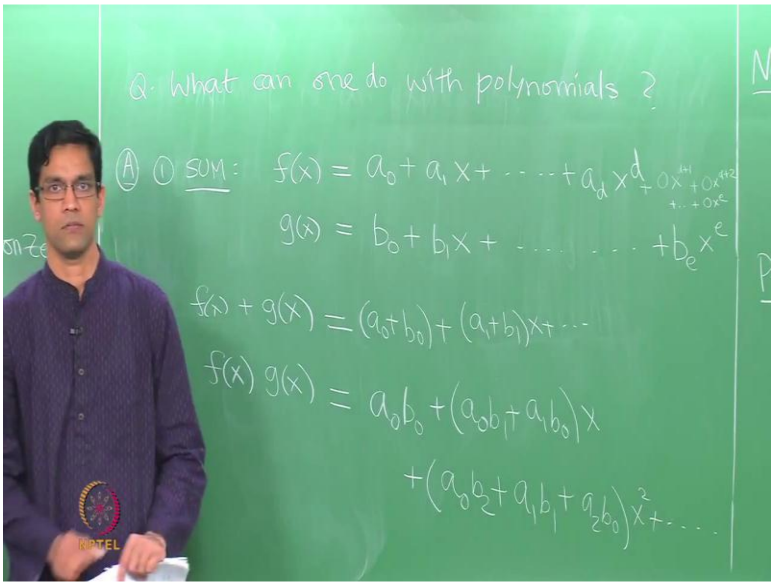

So, here is a basically the following question. What can you do with polynomials? What can one do with polynomials? What sorts of operations can you subject them too? So, there are several different operations. So, one is the sum, the other is the product. So, let us write each of them, one sum. So, what is the sum of two polynomials mean, suppose I have two polynomials f of x and g of x, so with the assumption that a d is not 0.

And I have another polynomial, let say b 0 plus and it would go to a, you know I could have different degree. So, let us call that degree something else. So, I will call it, let us say b e x power e. So, d is the degree of the top polynomial, e is the degree of the second polynomial. Then, the sum is just defined sort of term by term. So, how do you define the sum of the two polynomials in the obvious way? So, I assume this must be familiar.

So, we just take a 0 plus b 0 as the constant term and then you collect coefficients of like powers of x. So, for x, the coefficient is now a 1 plus b 1 and so on and observe that, while doing this, you might at some point run out of powers of x. Because, for instance if the second polynomial has higher degree than the first, adding them, for instance, until degree d, you could of course; you will be able to collect coefficients.

Beyond that, it will happen that g has more terms than f. In which case, you just treat the coefficients of those things as 0 here. So, for example, if b is bigger than d, I would short of pretend that the other powers of x, do appear with coefficient 0 for instance. So, you think of it a 0 x to the d plus 1, 0 x to the d plus 2 and so on till 0 x power e. So, that is a way you do the addition, you just keep going all the way to you reach the highest power of x.

So, I am not being to formal about this and similarly, the product again is the familiar operation f of x times g of x. So, you write these two things out and then, you just use the distributive law and just expand the whole thing out. So, when you do this for example, I multiplying these two things out I will get, so a 0, b 0 will be the constant term. To get a coefficient of x, I need to multiply a 0 with b 1 x and a 1 with b 0 x and b 0 with a 1 x. So, this is plus a 0, b 1 plus a 1, b 0 times x and then to get a question of x square, similarly notice will have to have a 0, b 2, b 2 x square.

So, let us write the down a 0, b 2 x square a 1, b 1 will be an x square and a 2 b 0 also be the x square and you keep going. You just multiply the whole thing out completely and whatever you get is call the product of the polynomials, so these are both just the obvious operations on polynomials.

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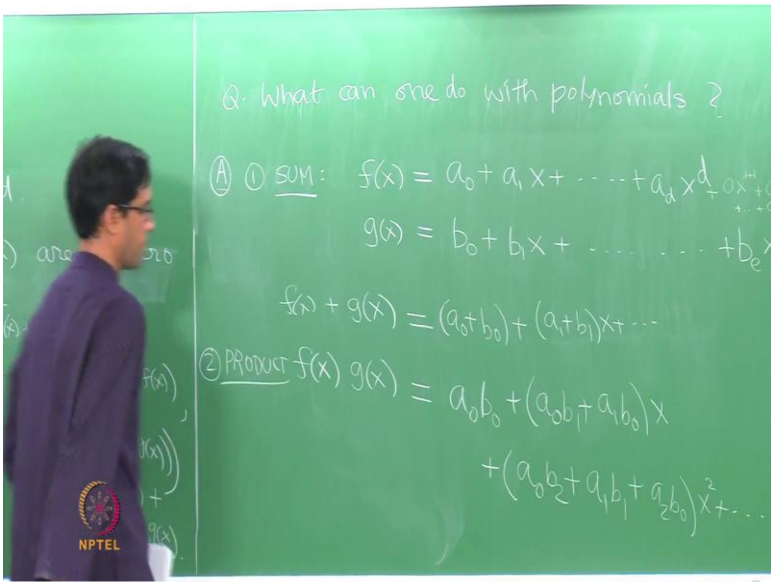

So, one other operation is you can substitute, so I sort of also root product here.

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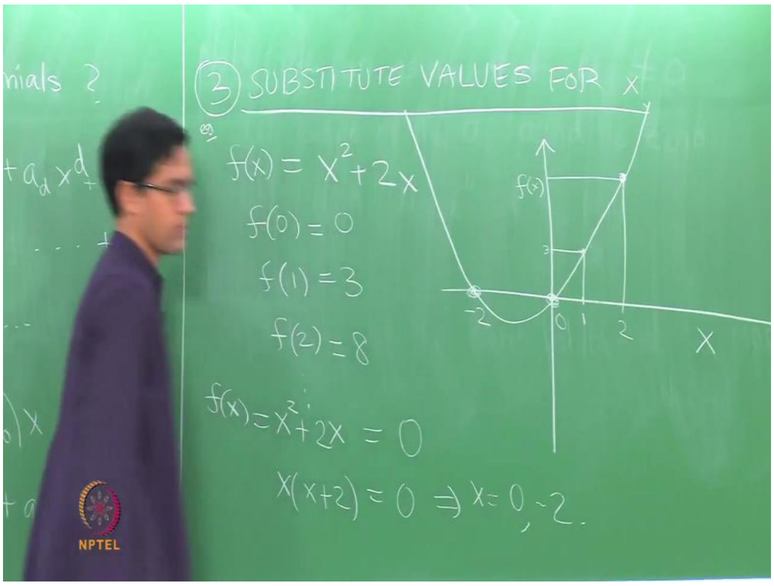

The third thing, you can do with the polynomial is to substitute values for x. So, you treat it like the function. So, if I have a polynomial f of x, so again the obvious thing. So, I give you x square plus 2 x, then I plug in various values for x. So, imagine I am plugging in various real numbers and I could sort of see, what happens. So, for instance, I plug in x is equal to 0, this gives me 0. So, plug in x equals 1, it gives me a 3, x equals to 2, gives me 8 and so on.

So, I could think of various values for x and of course, the usual graphical pictorial depiction of this is by the graph of the function. So, you plot the x and y axis, along the x axis you plug in various values for the variable x is case, it is call it x and along the y axis, you plot the values of the polynomial f of x. So, for instance at 0 the values is 0, at 1, we said the values 3. So, x equals 1, y equals 3, this point and at 2, the values 8, plot this point and so on.

And of course, you join all of them up by occur and it sort of get the familiar parabola in this case. So, that just the pictorial depiction of this polynomial itself, it is a graph of this polynomial. So, again something that should be very familiar and this brings us to the notion of a root of a polynomial.

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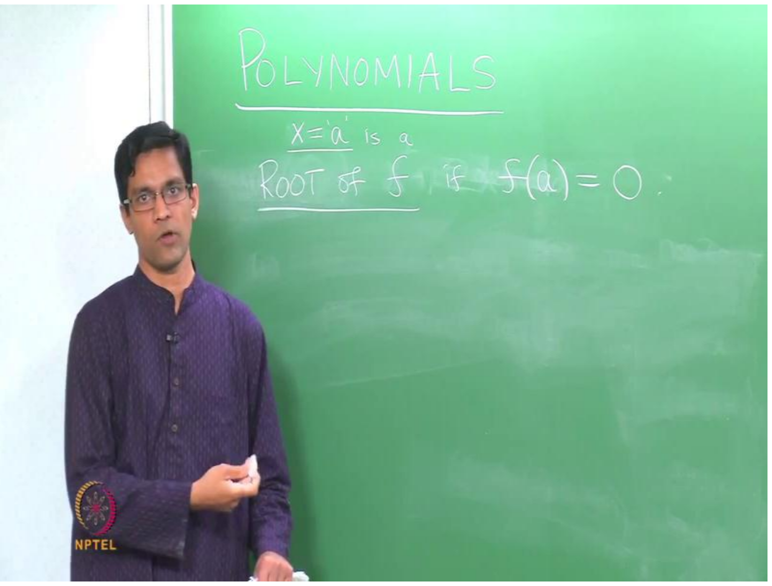

So, what is the root of the polynomial f, it is just the value at which the polynomial become 0. So, you say that x is a root of f x, x equals a, let say these root x, it is equals a is a root of f, if when you plug in the value a, the polynomial become 0. So, in this example there, there are two roots, there are two values at which the polynomial takes the value 0 ((Refer Time: 15:32)). So, one of them is of course 0 and the other is minus 2.

So, to find the roots again this is the standard procedure of solving coordinating equation, I have x square plus 2 x and I want to know, what are the values of x, which make the 0. So, we have x times x plus 2 is 0 and we solve this to find the 2 roots. So, I am just short of reviewing the various familiar facts about polynomials.