# Dynamic Data Assimilation

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## Lecture - 23
## Relation between FSM and 4DVAR

In the previous couple of lectures we covered two distinct methods for assimilating data into dynamic deterministic models. The first one was called the 4-D Var or join first order of join method, second one is called forward sensitive method. A careful analysis of the details of these two methods would immediately reveal that the both the methods rests on considering first order variation or first variation that is why it is called the adjoint method is called first order of joint, the forward sensitive method is also called first order for forward sensitive method.

So, when you say first order there is also a corresponding second order, there is a second order adjoint method there is also a second order forward sensitivity method. The question may arise who uses first order when do you use second order. If the model and the observations are very strongly non-linear for starter methods may not be as accurate as a second order methods, but, so second order methods are generally preferred when the models are very strongly non-linear or highly non-linear, but second order methods are computationally more demanding than the first order method. So, one way to be able to use the first order method in the context of strongly non-linear system is to be able to repeatedly apply the first order in a iterative fashion effectively it is equivalent to doing a second order. So, either you do second order one shot or first order repeatedly I think the effect would in mathematically be very similar to each other.

So, having seen different versions of different types of data assimilation algorithms 4-D Var and FSM I think it is time for us to be able to find the relation, the intrinsic relation between the FSM and 4-D Var. It is we are going to now demonstrate that both the methods talk about different aspects of the same fundamental philosophy the gradient of the J function using adjoint method that we computed is called adjoint sensitivity. We are going to now relate the adjoint sensitivity to the forward sensitivity at and that is the topic examining the relation between FSM and 4-D Var.

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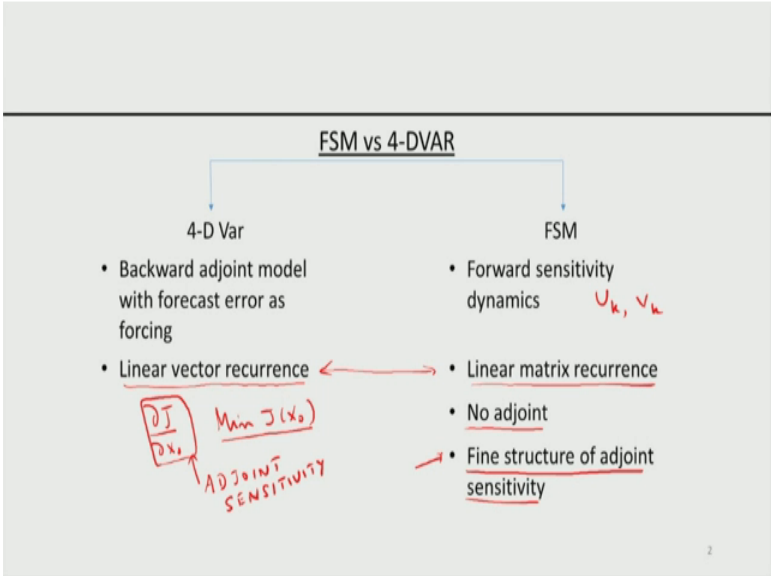


So, FSM verses 4-D Var to start with; FSM needs backward adjoint model with forecast errors is forcing you can see the backward error you remember the f k which is the normalized forecast error viewed from the model space which are the forcing for the linear backward dynamics.

The structure of that dynamics is that is the linear dynamics is the matrix it involves matrix vector multiplication, so we would like to call it linear vector recurrence. So, I have to run the model forward and run the adjoint backward this forward backward loop is repeated several times and is also combined with a minimization procedure. So, that is the overall structure of the 4-D Var.

In the case of forward sensitivity we run the model forward, we also run the forward sensitivity dynamics the U k dynamics and the V k dynamics. The difference here is that these are linear matrix recurrences as opposed to matrix vector recurrence that is the difference. Linear vector recurrence is cheaper computationally matrix recurrence is computationally a little bit more demanding it is, its much more demanding in fact, but the advantage here is that I do not need any adjoint we considered simple problems, but writing an adjoint and developing an adjoint is considered to be not an easy task in general.

So, the advantage FSM is that it doesnt require an adjoint. The disadvantage of the FSM is that it requires solution of the matrix recurrence. Another advantage of the FSM is that using forward sensitivity I can express the adjoint sensitivity I can I can decompose the adjoint sensitivity to ah an explicit as a function of forward sensitivities and for cash errors. In the case of 4-D Var our aim is simply to be able to get the adjoint sensitivity what is the adjoint sensitivity the adjoint sensitivity is essentially delta J by delta x naught that is the adjoint sensitivity because my aim is to be able to minimize J of x naught, in order to minimize J of x naught we have to compute the gradient with respect to x naught and this sensitivity is the one that we computed using 4-D Var or adjoint method. So, in this parlance this is called adjoint sensitivity, as opposed to forward sensitivities U k and V k and here.

So, what is the fine structure of the adjoint sensitivity? We should be able to express the adjoint sensitivity using U k and V k and forecast errors. So, it is this ability to express the adjoint sensitivity as a function of the product of forward sensitivities and the forecast error relates to the fine structure of the adjoint sensitivity and this has lots of advantages that we will talk about in a minute.

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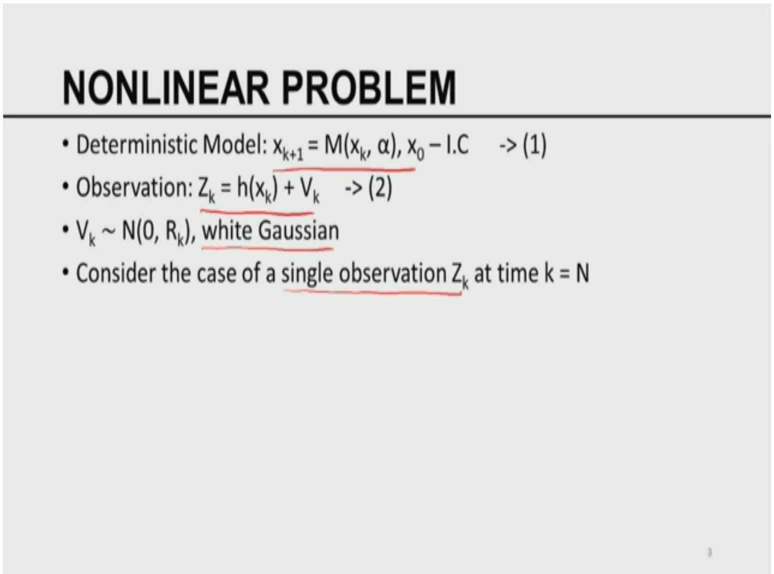


So, again consider the non-linear problem I have a deterministic model. Again x naught is the initial condition alpha is the parameter I have the observation non-linear observation I have a white Gaussian noise. In order to make things simple I am only going to consider in this lecture the case of single observation at time k is equal to N. So, if I can do the comparison for a single observation similar analysis apply for multiple observation ah. So, it is without loss of generality to get to the crux of the matter it is enough if you consider one observation.

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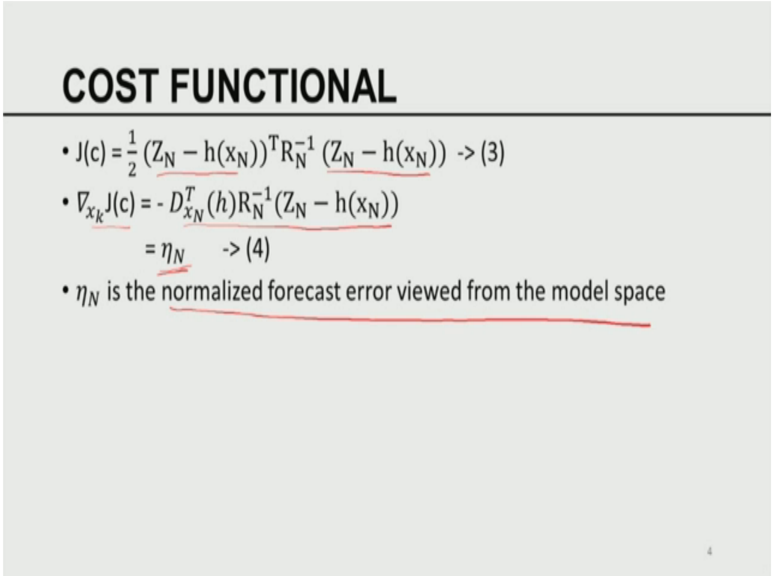


So, if I have only one observation J c has only one term. So, this is the forecast error time N, this is the forecast error time N, this is sum of squared errors weighted by R N inverse. We have already seen from 4-D Var method the derivative of J c with respect to x k is given by this and we are going to call this as eta N.

This is related to f of N I am simply using another term we could have called it f of capital N as well. So, eta N is the normalized forecast error viewed from the model space which you have already seen. I am now going to consider the first. So, I have now completed the derivative of J with respect to x k.

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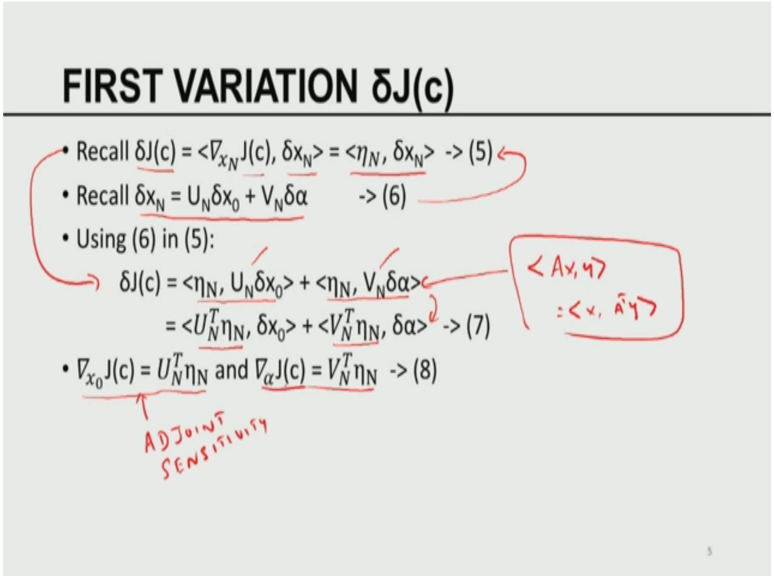


Now, I am going to consider the first variation. So, recall the first variation delta J is given by the gradient times the x of N and the gradient with respect to x of N is eta N that is what we have seen in from the slide previous slide equation 4. So, the first variation is essentially the inner product of is essentially inner product of eta N sorry is essentially inner product of eta N, the delta x N. From the forward sensitive equation we already know please recall delta x N is equal to u N delta x naught plus V N delta alpha.

Now, substitute 6 and 5 the delta x N in here that gives raise to delta c is equal to, delta of J c is equal to this term as well as some of these two terms because from 6 delta x and consists of two parts one from arising from the change in the initial condition, another of change in the parameters. So, this term accounts for the change in the initial condition, this term accounts for the change in parameter. Now I am going to use the adjoint property, please remember the adjoint property, x I am sorry (Refer Time: 10:04) a slightly different fashion there are several ways of stating this I would I would like to state it in a slightly different fashion. So, what is the adjoint property? x comma y the inner product is equal to x comma a transpose y that is the basic adjoint relation.

So, using this adjoint relation applying to 6, I get 7. So, the U N from here goes to this V N from here goes to that therefore, from basic first principles you can readily see this essentially is the gradient of J with respect x naught and this essentially is a gradient of g with respect to alpha. So, you multiply the gradient with respect to x naught, delta x naught gives you the adjoint sensitivity. So, this is the joint sensitivity we are looking for. This is the sensitivity of this with respect to the parameters. In our 4-D Var we never considered this part this is simply the sensitivity of the cost function with respect to the parameter. So, from h we now know how the gradient of the J function with respect x naught an alpha are structured.

You can readily see the sensitivity is the product of U N transpose and eta N. What is eta N? Eta N is simply the model error the normalized model error viewed from the model space. The numeral is a forecaster I am sorry not a model error I misspoke, it is the normalized forecast error viewed from the model space you are going to multiply that by the transpose of the forward sensitivity. So, you can readily see that the adjoint sensitivity is the product of transpose of the forward sensitivity times the forecast error. Likewise the adjoint sensitivity with respond to the parameter is also the product of the transpose of the forward sensitivity times the forecast error.

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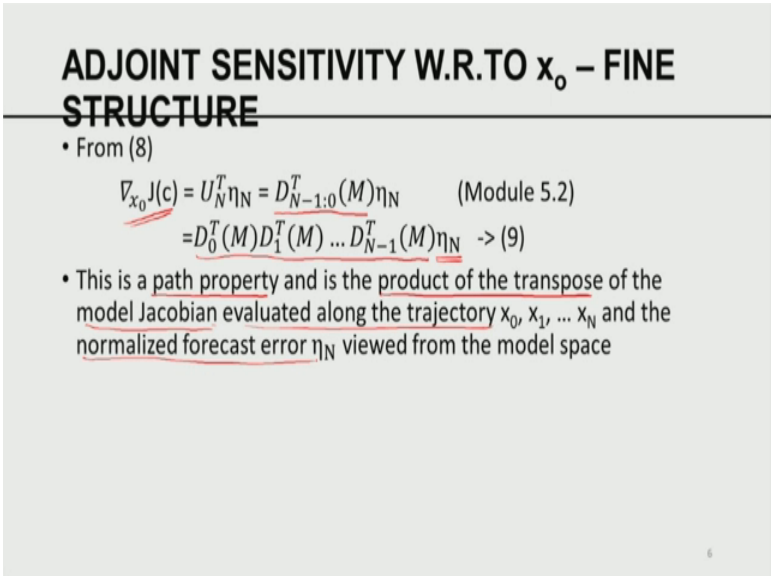


Now, please remember that from 8 we already know that U N transpose is simply product of the is simply the product of the Jacobians along the trajectory eta N is the, eta N is the forecast error at time N. We already know transpose of the product as the product of the transpose taken in the reverse order. So, this is the transpose of the products eta N is a forecast error.

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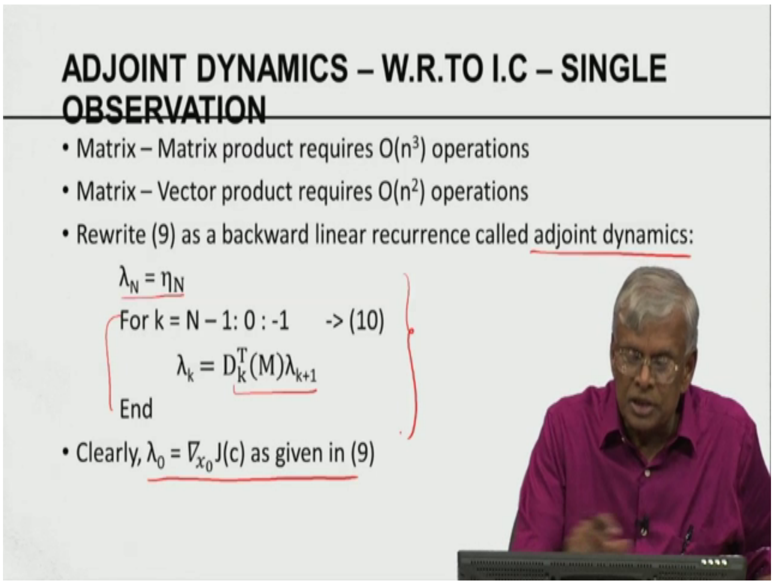


Now, you can see the adjoint sensitivity with respect to the initial condition it is a path property. Why it is a path property? It is a product of the Jacobian along the trajectory starting from x naught x 1, x 2, x N minus 1 times the forecast error at time N.

So, it is a product of the transposes of the model Jacobian transpose evaluated along the trajectory and the normalized forecast error viewed from the model space. So, you can this is what we call the fine structure, this is what is called the fine structure of the adjoint sensitivity. So, left hand side of the adjoint sensitivity, right hand side is an expression for the forward sensitivity we are able to relate the forward sensitivity and the adjoint sensitivity using this relation in 9. So, that is an important connection between 4-D Var and FSM.

Now, with respect to some computational considerations matrix matrix product requires N cube operations matrix vector product requires N square operation. So, adjoint dynamics with respect to the initial condition that we saw earlier can be written for the sake for the case of one observation, the backward dynamics is the linear recurrence which is called the adjoint dynamics. Since there is only one observation there is no other forcing for this I start with lambda N is equal to eta N I simply integrate through this de loop lambda is the vector I am interested in the matrix vector multiplication. So, this is simply a matrix vector multiplication and when you come to the end you get the forward sensitivity and which is I am sorry adjoint sensitivity and that is what we are seeking for. So, this is the adjoint dynamics that gives raise to the evaluation of adjoint sensitivity. So, this is a summary of the 4-D Var.

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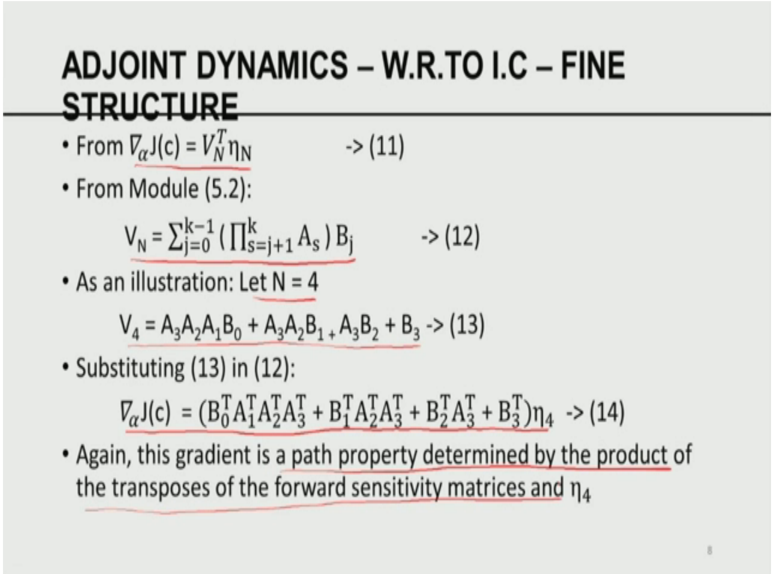


So, the fine structure is what we have talked about. So, I have already talked about the fine structure with respect to the initial condition. The fine structure with respect to the initial condition is given by 9, we also talked about how the adjoint sensitivity calculated in 4-D Var now I am going to go to exploring the adjoint sensitivity with respect to the parameter.

From the previous module we already know the solution for V N is given by this we have already solved the forward sensitivity this parameters. As an illustration let us pretend N is 4 when N is 4, V 4 is given by this expression as we saw it is a sum of the products of the model Jacobian with respect to the state and the model Jacobian with respect to the parameters a s are the model Jacobian with respect to the state bs are the model Jacobian with respect to the parameters. So, substituting this in 11 the adjoint sensitivity with respect to the parameter takes this particular form, again you can see it is a product of matrices, product of matrices, product of matrices, the whole thing is multiplied by eta 4. Again this is the gradient this gradient is a path property is determined by the product of the transposes or the forward sensitivities along the path with eta 4.

So, that is an important consideration. This reveals the importance of the Jacobian along the along the trajectory, along the forecast trajectory. Now we are going to express the computation of the forward sensitivity with respect to the parameter in the form of a pseudo code. I would like to remind you I have already in equation 10 or yeah 10 in slide 7 gives you the pseudo code for the back the pseudo code which represent the backward recurrence in the computation of adjoint sensitivity.

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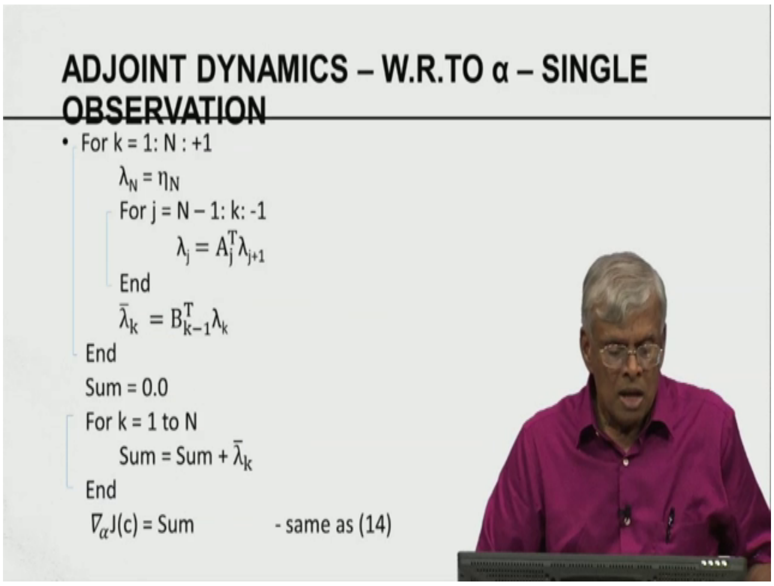


Now, this represents the backward code for the computation of adjoint sensitivity with respect to the parameter initial condition there parameter here. Because of the complexity of the expression in 14 this program is slightly a little bit more more involved. I am not going go over this line by line you can readily verify the correctness of this program and this is the backward adjoint code for adjoint sensitivity computation with this parameter. And what is the end result of this? The end result of this it computes delta alpha with respect to the parameter alpha that is the adjoint sensitivity with respect to the parameters.

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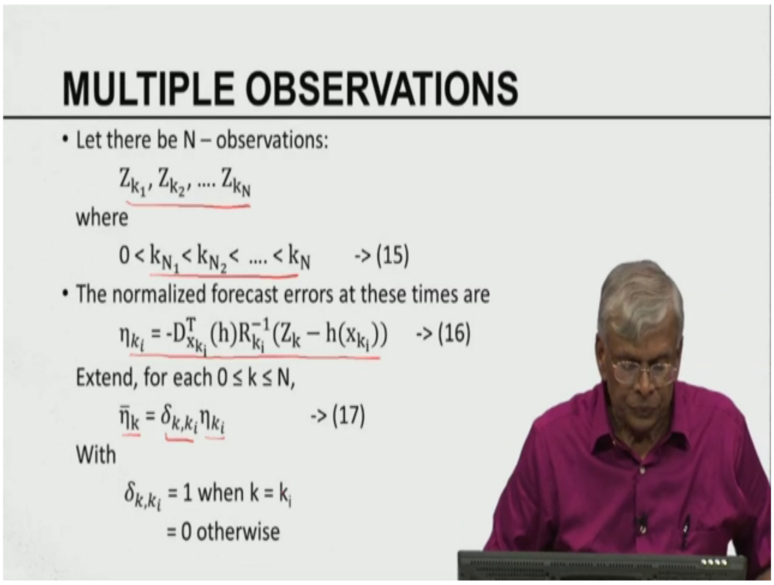


When I have multiple observations it is simply an extension of what we have shown again I am going to have N different times I would like to show this for the sake of completeness. There are N different times, N different observations each of the eta k i which are the forecast errors normalized forecast errors viewed from the model space each one of the time instances since the forecast errors are not at every time there are gaps, I am now going to define eta bar k which is related to eta k eta, eta k is defined only at discrete instance in time eta bar k is defined at every instance in time it is multiplied by a delta function k k i. What does it mean? When k is equal to k i it will be 1 otherwise it will be 0. So, it is a kind of a selector function that maps the observations at certain intervals of time to continuous observation. So, delta k is the standard delta function.

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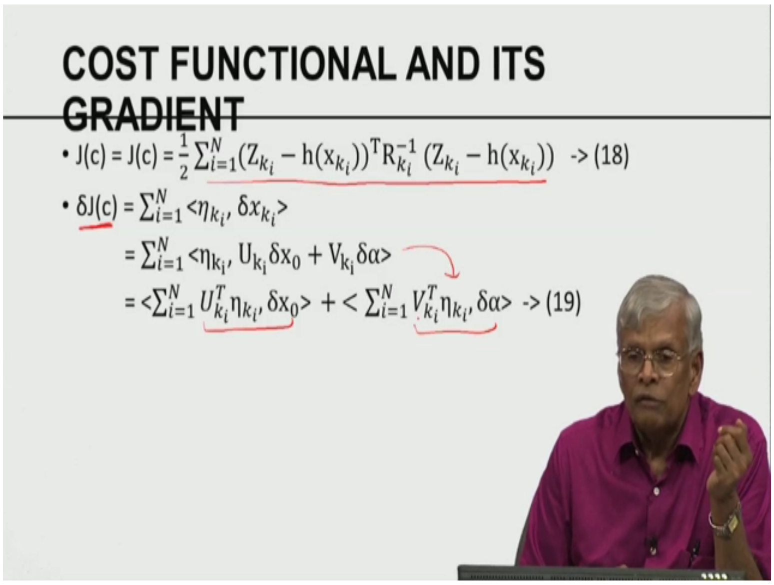


With this standard delta function we can now express the cost function again delta c this is the sum of all the weighted squared errors over all the observations. We have already seen delta c is given by this the forward sensitivity part with respect to the state this is the forward sensitivity part of the parameters. I know I am skipping some of the details, but the details are already explicitly given. We have already seen that the move from here to here uses the adjoint property the U k tank becomes U k bar and so on. I think a bracket is missing in here again a bracket is missing in here.

So, this 19 is simply an extension of what we did for one observation now in this case because there are multiple observation there are summations from I is equal to oh I am sorry the I think I have, I made an error, I do not think this is needed, I do not think that is needed sorry, I do not think that is needed that is correct the summation is already there I am sorry I missed it. So, we have utilized the linearity property as well the adjoint property to get this 19. So, 19 essentially tells you how your first variation of delta J is related to the forward sensitivities with respect to state and initial conditions.

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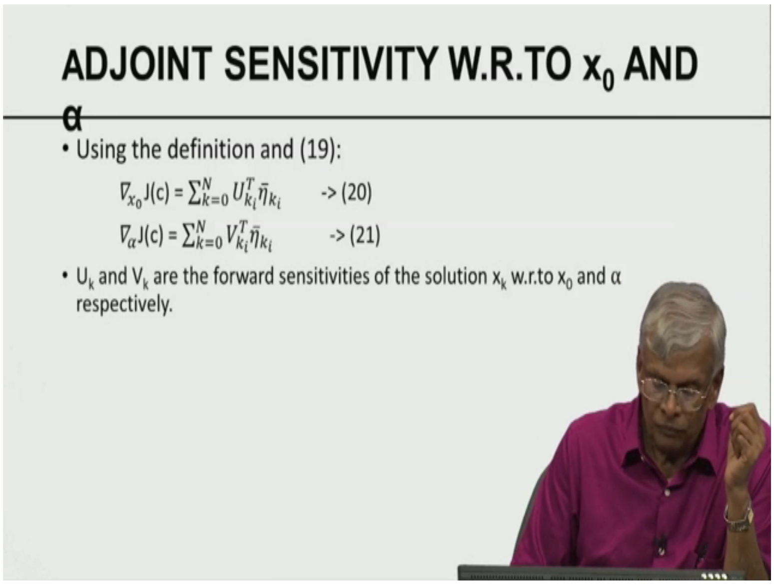


Therefore from using the fundamental definition the expression for the forward sensitivity with respect to the expressions for the forward sensitivity transpose times the observation summation over, gives you the adjoint sensitivity. The expression some of the transpose of the forward sensitivity with respect to the forecast errors gives raise to the adjoint sensitivity these two are simple extensions of the one observation case U k, V k are the forward sensitivities of the solution x k with respect to x naught and alpha.

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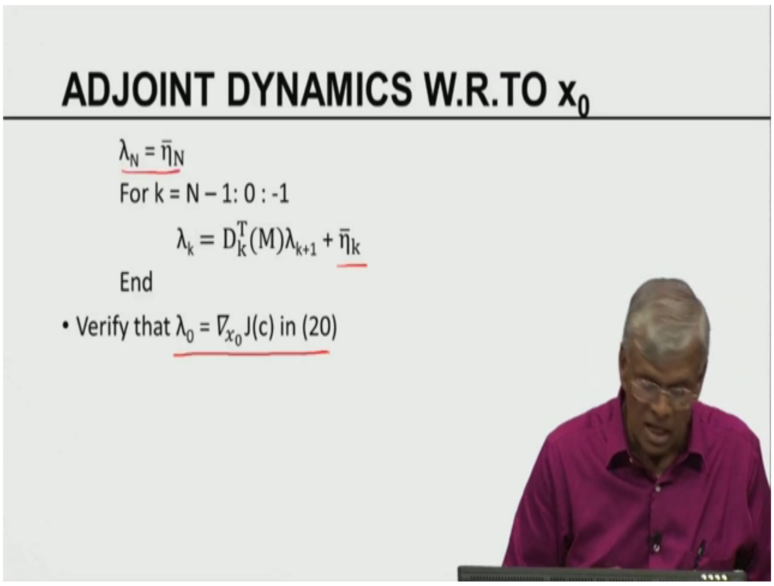

So, adjoint dynamics with respect to x naught is given by this in the case of multiple observation. So, please remember in the case of single observation we simply have the final condition there is no forcing when there are multiple observation I have a final condition and I also have a forcing.

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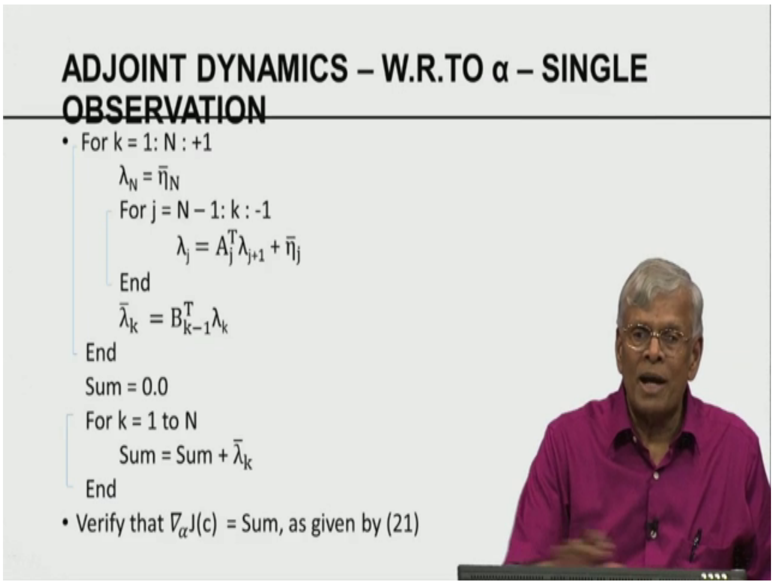


So, in the end when we calculate this I calculated adjoint sensitivity. So, this is the backward dynamics likewise the backward dynamics for the multiple observation is given by this structurally they are not too different from each other. So, we have given the pseudo code for computing the adjoint sensitivity in the case of multiple observation both with respect to x naught and alpha. Slide 13 gives the adjoint calculation for multiple observation with respect to x naught. Slide 14 adjoint sensitivity with respect to the parameter.

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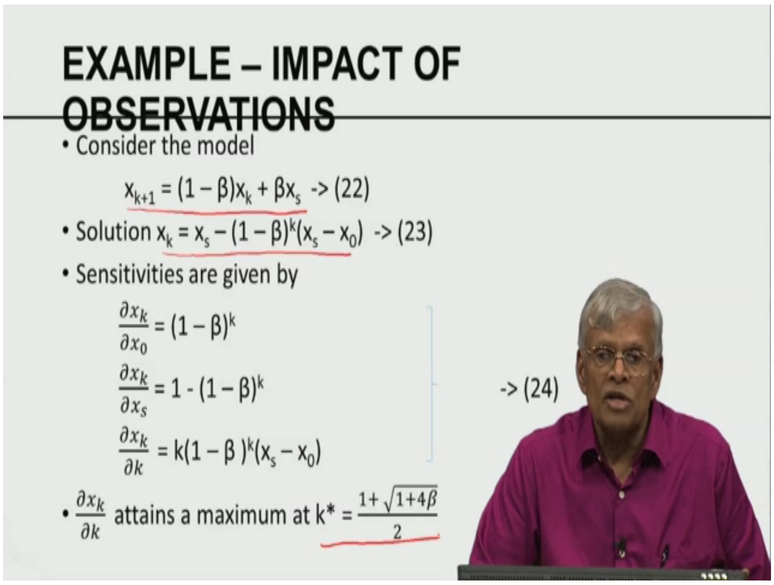


Again an example now I am going to talk about their example one of the standard questions in data simulation is how do I distribute my observation temporarily how do I distribute my observation stations spatially. So, the fundamental question relates to how do you distribute the observation in a spatial temporal domain where from I can get the maximum amount of information that can be derived from the observation for being assimilated into the model. That is one of the fundamental question that is continues to be of great interest in data assimilation because there are physical processes going on in nature development of models and model analysis continues separately.

In order to be able to fit the model to the data I need observations data as our observations. So, and then when the data are the model available we are going to do the data assimilation part, but the data collection is an expensive business what to measure, how to measure, where to measure these are all fundamental question that have to be satisfied that has to be um argued and the decision has to be made. I am going to make observation that this time because, I am going to make this observation at these spatial location because, the reason is I would like to get I would like to be able to maximize the transfer of information from observation to the model through the process of data assimilation.

Now, we are going to talk about that particular aspect of how the distribution of observation impacts the computation of the adjoint sensitivity because that is that is one of the fundamental question one has to be concerned with. In order to answer this question I am going to be concerned with the same model that we talked about in the previous lecture this is the discrete version of the cold air moving over the hot sea surface and there is temperature transfer because of turbulent mixing and this is the solution, the solution can be rewritten 22 is the model 23 is the solution the discrete time sensitivities of the solution with respect to the initial condition, sensitive respect to the boundary condition and sensitive respect to the parameters are all given.

From the parts of the solution we you may remember that the sensitivity of the solution of this parameter exhibited a maximum the maximum occurs when k is equal to k star k star is this value. So, delta x k divided by delta the partial of x k with respect to k times the maximum, when k is equal to k star which is given by that. And you may remember beta is related to k beta is equal to delta t times k delta t is the time discretization k is the original parameter in the continuous time version.

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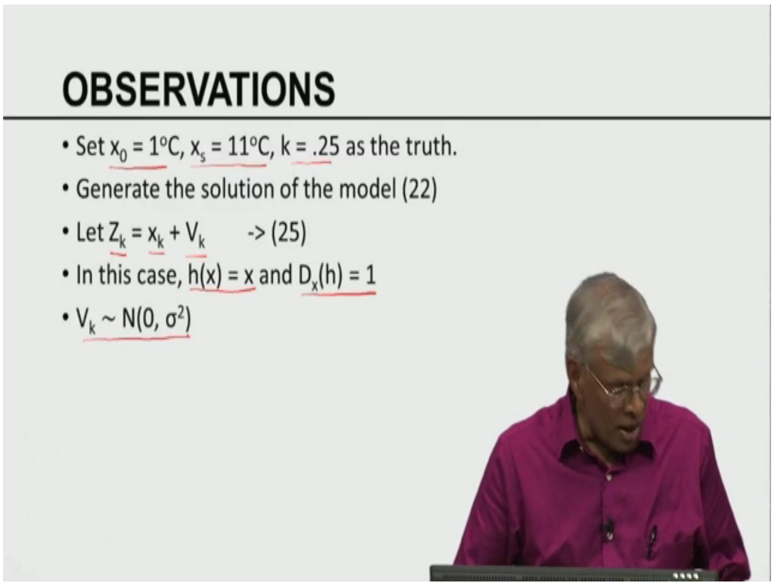


So, what is that we are going to do? We are going to do a twin experiment I am going to start with x naught is one sea surface temperature is 11. So, you can easily see the air when it comes in contact to the water for the first time has a temperature 1 degree centigrade. At that time the water is 11 degree centigrade so that is going to be a transfer of heat from the water surface to the air. The heat transfer coefficient is assumed to be

0.25 and we assume x naught is 1, x s is 11 and k is equal to 0.25 that corresponds to the truth what does it mean that is how nature has arranged matters.

So, you generate the solution by running the model forward in time. I am going to observe the air temperature I am trying to make observations the air temperature. So, the observations V k is equal to the air temperature plus noise. So, in this case which affects is x itself; that means, is the linear h of x has an identity function the Jacobian of h is a unity is a very simple problem we simplified it because we want to be able to bring out the beauty of the underlying argument, the argument being how the distribution of the observation affect the quality of the computation the quality and the value of the adjoint gradient, adjoint sensitivity I should say adjoint sensitivity.

So, V k is given by this standard Gaussian noise. So, that is what mother nature does.

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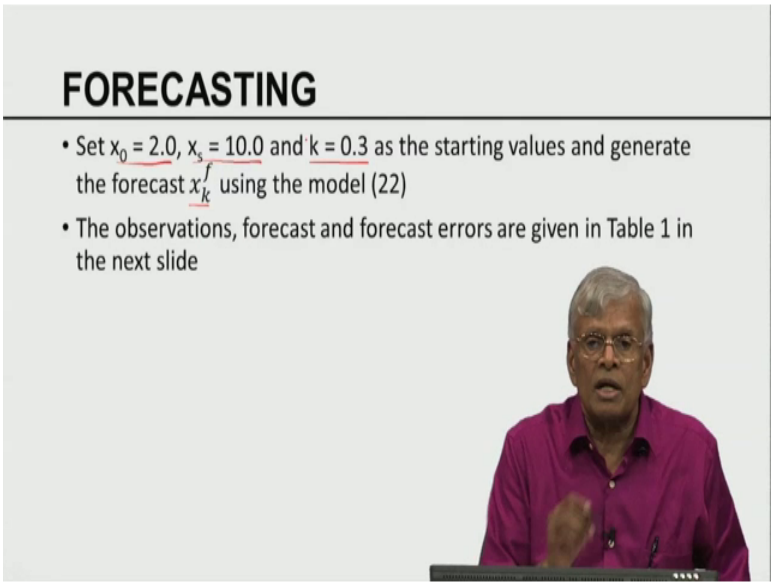


Now, not knowing what the mother nature has planned I am going to assume my initial condition is 2, the surface temperature is 10, k is 0.3. Look at this now I have errors in the initial condition, I have errors in the boundary condition, I have errors in the parameter.

So, if I am going to make a forecast by running the model forward the forecast is going to have errors. The forecast error results from the confounding of the errors in initial condition parameters and boundary condition is one of the hard cases because we are assuming everything is wrong. We summarize the observation so computed, we summarize the forecast calculated from these control value, we summarize the forecast errors all in the following table. So, you can readily see the forecast error E k the observations z k and x k f is the forecast.

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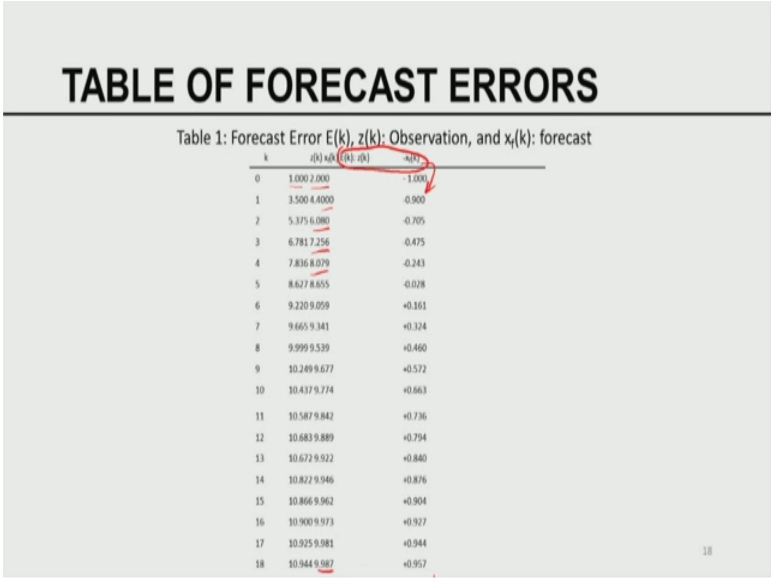


| k | z(k) | $x_f(k)$ | E(k): z(k) -$x_f(k)$ |
|---|---|---|---|
| 0 | 1.000 | 2.000 | - 1.000 |
| 1 | 3.500 | 4.4000 | -0.900 |
| 2 | 5.375 | 6.080 | -0.705 |
| 3 | 6.781 | 7.256 | -0.475 |
| 4 | 7.836 | 8.079 | -0.243 |
| 5 | 8.627 | 8.655 | -0.028 |
| 6 | 9.220 | 9.059 | +0.161 |
| 7 | 9.665 | 9.341 | +0.324 |
| 8 | 9.999 | 9.539 | +0.460 |
| 9 | 10.249 | 9.677 | +0.572 |
| 10 | 10.437 | 9.774 | +0.663 |
| 11 | 10.587 | 9.842 | +0.736 |
| 12 | 10.683 | 9.889 | +0.794 |
| 13 | 10.672 | 9.922 | +0.840 |
| 14 | 10.822 | 9.946 | +0.876 |
| 15 | 10.866 | 9.962 | +0.904 |
| 16 | 10.900 | 9.973 | +0.927 |
| 17 | 10.925 | 9.981 | +0.944 |
| 18 | 10.944 | 9.987 | +0.957 |

So, look each of these now. So, the z k, E k I am sorry z k, x k and E k. So, this must be the first column is z k, the second column is the forecast from the erroneous state, the last column is the is the error. So, please remember this must be together this, that is the last column, that is the last column. So, the first column is z k, second column is x f the lab column is z k minus z k minus x f that is there that is the condition there is a little bit of a space here I hope you understand now what these numbers are.

So, we are running the model to time 18, you can see from the forecast the temperature of the I am assuming initial condition is two the temperature is rising; that means, the water is getting, the air is getting hotter the water is transferring heat to the atmosphere. The actual observation generated from the true initial conditions are given by this therefore, you can readily see this is the observation, this is the model predicted. So, the difference between two is minus 1 and that is how you are going to get the forecast errors.

So, sometimes the forecast errors are negative, sometimes the forecast errors are positive they widely vary from minus 1 to close to plus 1 as time evolves.

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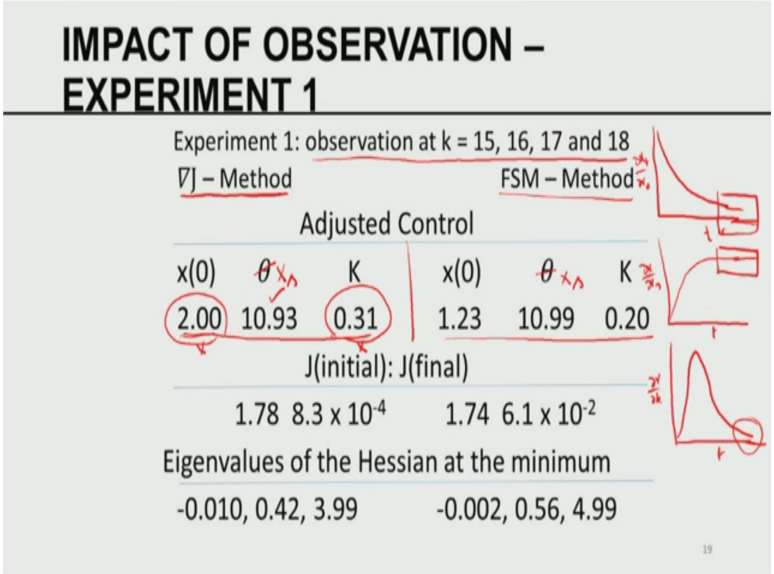


Now, we conduct experiments I am going to tell you the summary the experiments experiment 1, in experiment 1, I take observations at time 15, 16, 17, 18, it means I am taking observations at very late in the game. I am computing the delta J and FSM method I am using 4-D Var and the forward sensitive method. Initial condition the theta must be x actually initial condition x naught is 2, so when I used the forward sensitivity method these are all the results for the forward sensitivity methods this is x. So, this is actually x s this is actually x s, their adjusted value of the control is given by initial condition the sea surface is temperature and the control value.

Look at this now this is the record value using these observations. So, I use the observations 4 observations at time 15 16 17 18 if I applied the adjoint method the recovered value are these using the same observation if I use the FSM method I got the recovered value to be this. Now, you can see the true value of the initial condition is 1, that true value of the parameters was 0.2, FSM has recovered the true initial condition and the true parameter and the boundary condition I think was 11 and is pretty close.

In the case of 4-D Var the initial condition did not recover correctly there is error in the parameter recovery as well the actual parameter is 0.2, but sea surface temperature is recovered reasonably closely. So, this is good, but these two are not good. So, this gives you a (Refer Time: 33:46) comparison of the performance of the two methods. You may ask why this happens, if you look at the sensitivity functions as we plotted in the previous one the sensitivity with respect to the initial condition comes down this is t this is delta x t divided by delta x naught, the sensitivity with respect to the parameters x s goes like this the sensitivity with respect to this is again t this is sensitivity with respect to the parameter k it went up and came down. So, 16, 17, 18 the times at which I 15, 16, 17, 18 the times at which I compute the, I performed the, I collect the observations 15 is here and so on the forward sensitive solutions has already becomes very low close to 0.

But the sensitivity with respect to the boundary condition is quite large is equal to unity. They bound the sensitivity that the parameters has become very close to 0, therefore, the adjoint method is not able to recover the initial condition largely because the sensitivity with respect to the initial condition has died down to 0, the sensitivity with respect to the parameters also has died down to 0. If you record the structure of the adjoint sensitivity it is the sum of the transpose of the forward sensitivity times the forecast error the product of the two quantities if one is close to 0 that the product is 0. So, the adjoint sensitivity in the case of 4-D Var does not have enough information from the initial condition sensitivity and the parameter sensitivity that is why the adjoint sensitivity is not able to recover the true value there is larger error in here, large error in here; however, if you change the observation timing 1, 2, 17, 18.

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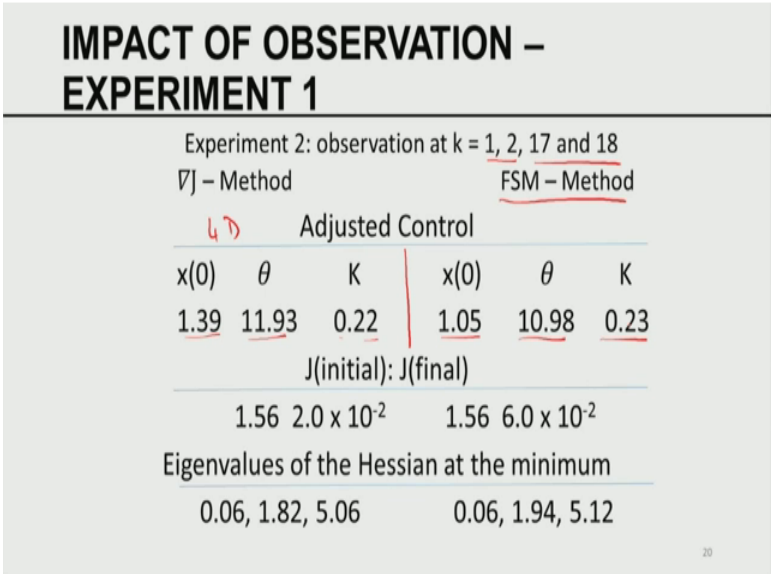


So, two observations in the earlier time all, the two observations the final time. So, what is the difference between these two experiments in the first experiment we put 4 observations all in the end here we are putting two observation the beginning and the two observations in the end.

Why this is important? If you look at the sensitive curve in slide 19 the initial condition sensitivity is large initially, the initial condition sensitivity is also large the sensitivity of the solution of this parameter is also large initially. So, by sampling at 1 and 2 I get lot more information about x naught, I get a lot more information about k, by sampling at 17, 18 I get a lot more information about x the boundary condition x (Refer Time: 37:19). Therefore, by distributing the observation some in the initial, some in the final, we are able to maximize the amount of information that can be transferred from observation to the control.

So, in the next slide you can readily see if you use the 4-D Var method I am sorry if you use the 4-D Var method, this is the 4-D Var method, this is the FSM, likewise this is the 4-D Var I am sorry this is the 4-D Var, this is I would like to say that this is 4-D Var, this is FSM. So, if I come back here you can readily see the 4-D Var method does better with respect to the distribution of observation at 1, 2 and 17, 18 this is better than the previous recovery this is better than the previous recovery, but this is even better because FSM is able to take advantage of the distribution of observation with respect to sensitivity.

So, what is the moral of the story. So, far indeed assimilation we have assumed we are given a bunch of data we never worried about the impact of data and the quality of data simulation that is what motivated us to be able to think about a method by which we can characterize the impact of distribution of observation on the adjoint sensitivity computation. Why adjoint sensitivity computation? Because adjoint sensitivity gives the gradient of the cost function with respect to the control, once I compute the gradient of the cost function with respect the control I can use it in the minimization algorithm. So, for the whole framework of data assimilation within the dynamical context to work I should be able to compute this gradient this adjoint sensitivity reasonably accurately. So, all the information in the observation and the forecast errors have to be transferred to this quantity called the adjoint sensitivity.

It is this transfer of information is very much critically dependent on the location of observation the spatial temporal location of observation. So, this experiment 1 and 2 for the simple problem illustrates the fact that if you put all the observations and one end at another end we may not be able to maximize the information because at one end the forward sensitive may become close to 0 or the forward sensitivity may become close to 0 in the beginning that for to maximize the impact of observation what is the lesson. You need to run the model, you need to run the forward sensitivity you need to ascertain the spatiotemporal regions where the forward sensitivities are not close to 0. If you put observations in those locations where the forward sensitivities are bounded away from 0 those places will have will contribute to maximum amount of information that one can utilize from the observations back to the computation of adjoint grady, adjoint sensitivity that is the moral of the story.

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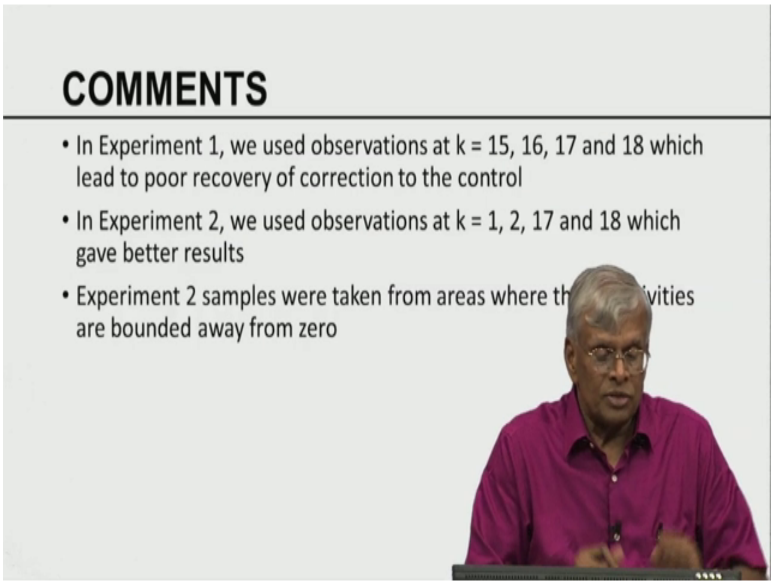


So, comments in experiment one we use the observations at 15, 16, 17, 18 which led to the poor recovery of the correction to the control this is largely because of the fact at a later time, the only sensitivity that is bounded away from 0 was the boundary condition sensitivity initial condition sensitivity and the parameter sensitivity have already died down to 0. So, we will have greater difficulty in recovering the initial conditions and the parameters if you put all the observations where these two sensitivities are close to 0 that is the moral of the story, from experiment 1.

The model of the story in experiment 2 is that I had two observations in the beginning and two observations in the end. In the end the boundary condition sensitivity is large in the beginning the initial condition sensitivity and the parameter sensitivities are quite good. So, the initial observation help you to derive information on the initial conditions in the parameters, the later observations help you to derive information about the boundary condition. So, by having a combined distribution initially and finally, we are able to maximize the transfer of information from observation to the control that is why we saw in experiment 2 the recovery was much better than in experiment 1.

So, this essentially tells you forward sensitive method using forward sensitive method not only one can do the data simulation which in principle is equivalent to the 4-D Var, it also helps you to ascertain regions in the spatial temporal domain where if I put the observations I will get the maximum benefit. It is this duel advantage of the forward sensitive method we believe is one of the strengths of the forward sensitive method, as it occurs with everything in life if there is an advantage there has to be a disadvantage.

What is the disadvantage? In the case of forward of 4-D Var the backward recurrence relation involves only matrix vector multiplication, but in the case of FSM the forward recurrence relation in needs matrix matrix multiplication. So, for large scale problem the use of FSM would require excessive computational demands. So, for large scale problem 4-D Var is still preferable, but if you want to be able to do some diagnostics as to the distribution of the observation within the framework FSM by running the model forward in time by parting the variation of sensitivity in the spatio-temporal domain one can ascertain a priori regions where the sensitivity is bounded away from 0 and hence if you put the loc if you put observation those locations you will get the maximum benefit to be able to do the data assimilation.

So, both the methods have advantages a disadvantage while they are also equivalent in some sense that is the moral of the story.

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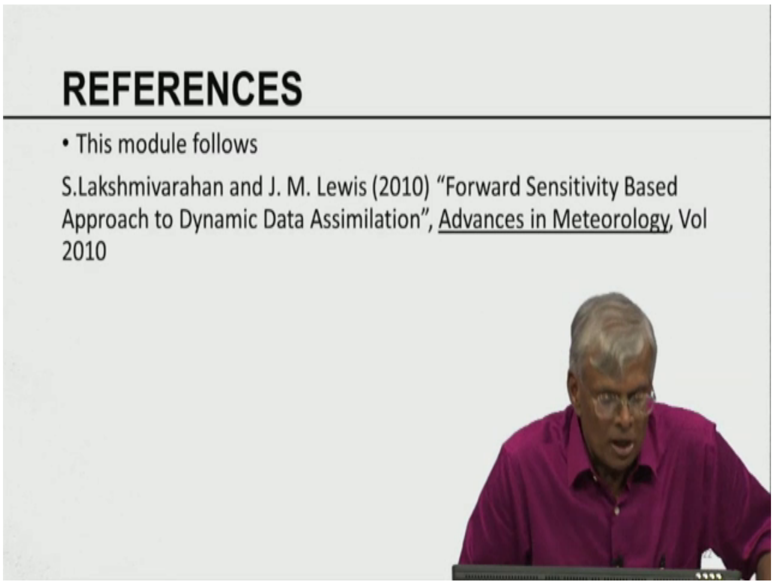


Again these modules follows from our paper Lakshmivarahan and Lewis, Advances in Meteorology, the title the paper is Forward Sensitivity Based Approach Dynamic Data Assimilation.

Thank you very much.